An Exploration of Competitive Signalling Equilibria with "Third Party" Information Production: The Case of Debt Insurance

Anjan V. Thakor

The Journal of Finance, Vol. 37, No. 3 (Jun., 1982), 717-739.

Stable URL:
http://links.jstor.org/sici?sici=0022-1082%28198206%2937%3A3%3C717%3AAEOCSE%3E2.0.CO%3B2-A

THE JOURNAL OF FINANCE • VOL. XXXVII, NO. 3 • JUNE 1982

An Exploration of Competitive Signalling Equilibria with "Third Party" Information Production: The Case of Debt Insurance

ANJAN V. THAKOR*

ABSTRACT

In markets in which sellers know more about product quality than buyers, but cannot convey their superior information either by directly issuing costly signals of the Spence type or by successfully funding the production of information, I suggest another way in which the informational asymmetry problem can be resolved; a third party can produce the necessary information at a cost and use it to price a service consumed by the sellers. Buyers can then observe a seller's choice of service consumption level and be well informed in equilibrium. In this framework I construct a model in which a borrower's choice of insurance coverage signals its default probability to lenders, and explore the properties of the resulting signalling equilibrium in a variety of cases.

SUBSEQUENT TO AKERLOF'S PROVOCATIVE paper [1], the analysis of equilibria in markets with asymmetrically informed agents has been the subject of considerable enquiry. Akerlof's novel insight was the observation that such markets can fail in the sense that the demand, at any price, is zero. This market failure can be prevented, however, if a priori imperfectly informed buyers of a given product can somehow revise their initial conditional estimates of product quality so that prices are consistent with quality in an equilibrium. Bhattacharya [2, 3], Ross [9], and Spence [12, 13, 14] have suggested that one way of achieving this is to allow sellers of the product or service to issue costly signals of the quality offered for sale: in a rational equilibrium, prospective buyers could use these signals to discriminate accurately between products of differing quality. Campbell and Kracaw [4] have recently suggested another possibility—that sellers can make side-payments to information producers to acquire the necessary information at a cost and convey it to the market. This paper explores yet another distinct way of resolving the information asymmetry problem. The market structure considered here involves three parties: a group of sellers, each aware of the quality of its own product; a set of buyers who perhaps satisfy the rational-expectations assumption that they are aware of the average quality of the products in the market, but are unable to distinguish one seller from the other on the basis of product quality; and "third-party" information producers who expend resources to produce information about the quality of each product offered for sale. These information producers recover the cost of information acquisition by using their knowledge to structure and price some fixed-quality service that they provide to

* Assistant Professor of Finance, Graduate School of Business, Indiana University. I wish to thank S. Bhattacharya and Tim S. Campbell for their very helpful comments. I alone am responsible for any remaining errors.

the sellers of the variable-quality product. Given a price for this service, each product-seller chooses a service level it wishes to consume, and this choice then acts as a signal to the market of product quality. The object of the paper is to analyze the properties of the equilibria which obtain in such a setting, under the assumption that both the product and the service markets are perfectly competitive.

As the preceding discussion suggests, the analytical approach developed in this paper is applicable to a fairly general class of information-asymmetry problems. However, in order to generate specific results it is necessary to focus on a special type of market. I explore the information signalling aspects of insurance in an asymmetrically informed market for debt. The setting used for this exploration assumes that each borrowing unit possesses perfect information about its own default probability, while the insurer can acquire the same information about each borrower by investing in information production. Investors are initially unaware of default probabilities. The borrower has a choice as to how much insurance coverage it obtains, and the insurance premium is a function of the amount of insurance coverage purchased as well as the probability of default. It is then shown that the coverage on any debt issue can act as a signal of its probability of default. A properly designed premium function will induce a borrower to obtain a coverage that signals exactly the true probability of default so that a priori imperfectly informed investors will be well informed in equilibrium—that is, mispricing of debt issues is precluded even though the yield on every bond is based *entirely* on the associated observable equilibrium insurance coverage.

In its signalling cost structure (the risk-neutral part of) the model developed here is most closely related to Bhattacharya's [3] nondissipative dividend signalling model, although the analogy is not exact. But, in spite of some similarities between the approach in this paper and that in most of the signalling literature, there is an important difference that sets them apart. In the typical signalling model there are two principal actors in the drama. One is an economic entity which emits the signal, bears the costs of signalling, and reaps the benefits which accrue from its true quality being identified. The other is (possibly) a collection of agents (or a market) to whom this true quality is revealed. However, no party directly incurs a cost of producing information. In this paper, the stage is expanded by introducing a third party *information producer* in the form of an insurance industry. The insurance industry can invest in information which identifies the true characteristics of borrowers in the market. It then faces an appropriability problem. One way to approach this is to devise a way for the third party to *successfully* sell the information to the market. This is the line of reasoning pursued in the recent Campbell and Kracaw paper [4]. I suggest another alternative: the information producer sells some other service to borrowers (firms) in the market and uses its information in pricing that service. Note that the information producer could just as well be a bank, an investment banker, or the supplier of any product or service used by the firm where the long-run profitability of the firm is important to the relationship. The question I examine is the manner in which insurance will be priced and how the markets for debt and insurance will function if insurers can acquire information about the characteristics of the insured at a cost.

The paper is organized in five sections. The first section presents the basic three party costly information production-signalling model in which insurance coverages function as signals of default probabilities. To provide a proper perspective, the actual development of the model is preceded by a brief discussion and comparison of the model with the essay by Rothschild and Stiglitz [10] on the nature of competitive equilibria in insurance markets with asymmetric information. Throughout, it is assumed that insurers are risk neutral and necessary conditions for the existence of equilibrium are derived for two distinct cases: (1) risk neutral borrowers and lenders, and (2) risk averse borrowers and lenders. In the second section, a variety of equilibria are illustrated for the risk neutrality case. Similar illustrations for the risk aversion case are provided in the third section. In both cases, the welfare implications of the different equilibria are examined. These illustrations suggest that the task of designing a separating equilibrium is made delicate by the assumption that borrowers and lenders are risk averse—in fact, for the risk aversion case, it is usually impossible to obtain an equilibrium without imposing rather severe restrictions on the information production cost function and some other parameters. The penultimate section contains a brief discussion of municipal bond insurance and some comments on the possible relationship between the demand for such insurance and information asymmetries in these markets. The fifth section concludes.

I. The Basic Model

The notion that the act of insurance purchase can, in itself, reveal something about the characteristics of the insured, has been previously exploited by Rothschild and Stiglitz [10] in their provocative analysis of the problems of nonexistence of equilibria in competitive insurance markets characterized by asymmetric information. Given the structural similarity between their model and the one presented here, it will be useful to begin this section by comparing the two approaches. Rothschild and Stiglitz construct a model which consists of a risk neutral insurer and a risk averse purchaser of insurance. They define equilibrium as a situation in which the insured has maximized expected utility subject to the constraint that the insurer breaks even, and they then proceed to explore the nature of competitive equilibria. They show that when perfect information about the insured's accident probability is available symmetrically to the insured as well as the insurer, a competitive equilibrium will exist and each customer will buy complete insurance at actuarial odds. Rothschild and Stiglitz justify their initial perfect information assumption by considering a market in which every customer has the same known accident probability. But, if the market consists of heterogeneous customers who differ in their disaster probabilities and if insurers cannot distinguish ex-ante one risk class from the other, it is natural to ask whether these different risk classes can be distinguished by the degree of coverage which they purchase. Using a two class model in this framework, Rothschild and Stiglitz argued convincingly against the existence of a stable Nash equilibrium under a wide variety of plausible assumptions. They also conjectured that for a continuum of risk classes, a stable equilibrium was highly unlikely, though in a different context, this conjecture was confirmed by Riley [8].

The similarity between the Rothschild and Stiglitz model and the model in this paper is that both are concerned with the properties of equilibria in asymmetrically informed insurance markets. However, there are significant differences. First, the assumption of risk aversion on the part of the insured is crucial to the Rothschild and Stiglitz analysis, since risk neutral customers have no reason to participate in the insurance market. In contrast, the model here can accommodate risk neutrality just as easily as risk aversion. Secondly, this model resembles the Rothschild and Stiglitz perfect information model, in that both the insurer and the insured are assumed to know the probability of disaster, with three differences: (1) instead of just one class of customers, there is a plurality of customers who purchase insurance, each characterized by a possibly different disaster probability; (2) the insurer can obtain perfect information about all disaster probabilities by investing in information production; and (3) an additional ingredient, represented by bond market investors, is introduced. This added complication, in conjunction with the special signalling role assigned to insurance, gives rise to some new results not encountered in the Rothschild and Stiglitz analysis. For example, it is no longer true that each customer will always buy complete insurance at actuarial odds. Finally, the equilibrium concept employed in this paper differs from the one used by Rothschild and Stiglitz. This distinction will become apparent when equilibrium is formally defined.

Consider a bond market in which a borrower has the option of choosing how much coverage it wants on its debt issue, in exchange for a premium that depends on the amount of coverage purchased as well as the probability of default of the debt issue. Bond market investors are unable to observe the default probability of any borrower and thus the interest yield on a debt issue is determined simply on the basis of the associated observable insurance coverage. Borrowers purchase insurance because it offers two benefits. The first is a direct benefit—even in the absence of an information-related dependence of the interest yield on the observed coverage, a rise in the insurance coverage increases the expected value of the payoff to investors at maturity for a *fixed* yield, and therefore in a competitive market any upward movement in the coverage purchased will be accompanied by a compensating downward movement in the yield. In this case, if markets are complete in the Arrow-Debreu sense (or if agents are risk neutral) and signalling considerations can be ignored (because investors know the true default probabilities), a competitive bond insurance industry will price its insurance coverages such that in equilibrium borrowers are indifferent between obtaining insurance and not obtaining it.[1] However, when the coverage functions as a signal of the default probability, there is also an indirect signalling benefit which derives from the fact that the resolution of the information asymmetry problem facilitates a more efficient allocation of resources. Thus, in this case insurance has some value

[1] This is a well known result. In a complete market, any firm or individual can obtain perfect insurance against states of nature by holding a state contingent claim for every state of nature and thus, outside insurance is worthless. Insurance is valuable in an incomplete market (with risk averse agents) mainly because it expands the space of attainable income vectors.

even in a risk neutral world.[2] My analysis of this market hinges on six general assumptions which are stated below.

Assumption 1: Each borrower knows its own probability of default, but the insurer and investors are initially unaware of this probability.
Assumption 2: Perfect information about any default probability can be generated by investing in information production. The cost of producing information about a borrower may depend on the default probability of its debt issue.
Assumption 3: Borrowers themselves have no opportunities for *directly* conveying their superior information to the market through signals of the type discussed by Bhattacharya [2, 3], Spence [13, 14], or Ross [9].
Assumption 4: The existence of information-transfer devices like credit ratings is precluded.
Assumption 5: Borrowers cannot avail of *direct* side payments to any economic agents in the market to produce information about default probabilities.
Assumption 6: Bond prices adjust in a *tâtonnement* process for the capital market to clear where prices bid by some are instantaneously known to all others.

The first two assumptions are merely a description of the market setting, while the last four assumptions ensure that the information asymmetry problem is not resolved through mechanisms other than the one I wish to focus on. In particular, the last two assumptions provide some justification for information producers to sell insurance contracts rather than selling their information directly to the market or organizing themselves as intermediaries to identify and invest in undervalued bonds. The implications of these assumptions will be discussed in some depth in the last section.

Without loss of generality, let the size of each debt issue be unity and let n represent the default probability. Each debt issue has a single period maturity. Cross-sectionally, n varies over the open interval $(0, 1)$, but a priori the value of n associated with any *specific* debt issue is unknown to the market. However, an investment of $K(n)$ in information production can reveal the true n. Information producers make these necessary investments to procure this information and then sell insurance contracts to borrowers. Each borrower has a choice with respect to how much coverage it wishes to purchase and $y(n)$ represents the "fractional insurance coverage" purchased by a borrower with default probability n. A priori imperfectly informed investors observe $y(n)$ (but not n) and use it to deduce something about n. The interest yield on a debt issue, $W(y)$, is then a function of the *observable* fractional coverage. This means that the borrower's obligation at maturity is $(1 + W(y))$ and the insurer is liable for an amount $y(n)$ $(1 + W(y))$ in case the borrower defaults.[3] The insurance premium the borrower

[2] Thus, the signal constructed here is productive in all the three ways identified by Spence [13]: (1) it is privately productive to the borrower because it distinguishes it from others with higher default probabilities; (2) it is directly productive because of its insurance function of increasing the expected value of the investors' payoff at maturity; and (3) it is socially productive because it permits ex-ante discrimination between borrowers by lenders and thereby contributes to a more efficient allocation of resources.

[3] It is assumed that the default probability n is unaffected by the interest cost $W(y)$. Illustrations in later sections will shed further light on the implications of this assumption.

has to pay at the time the debt is issued is a function, $\phi(n, y)$, of the default probability (known to the borrower and the insurer) and the fractional coverage purchased. Equilibrium is reached in this market when certain conditions, defined below, are satisfied.

Definition of equilibrium

In equilibrium, (1) every borrower has chosen an insurance coverage that maximizes its expected utility (risk aversion) or minimizes its total expected cost (risk neutrality); (2) given the optimally chosen coverage and the corresponding interest yield, the insurance premium is such that the insurer is guaranteed a payoff that *exactly* compensates it for the risk borne as well as the cost of information production for every insurance policy offered (perfectly competitive insurance industry); and (3) the interest yield on every debt issue, contingent only on the associated observable optimal insurance coverage, is correct in the sense that every debt issue is priced *as if* investors were aware of default probabilities (this is an ex-ante rational expectations constraint which is the traditional rationality requirement of signalling models).

With this general framework, I shall first develop the model for risk neutrality (Model 1) and then for risk aversion (Model 2). In both models, starred variables indicate equilibrium values, subscripts denote partial derivatives, and all relevant functions are assumed to be twice continuously differentiable in their arguments. *Model 1:* If borrowers and lenders are risk neutral, each borrower chooses a coverage that satisfies

$$y^*(n) \in \operatorname{argmin}_{y(n)\in[0,1]} (1-n)[1+W(y)](1+r)^{-1} + \phi(n, y) \tag{1}$$

subject to

$$\phi(n, y^*(n)) = K(n) + ny^*(n)[1 + W(y^*(n))](1+r)^{-1} \tag{2}$$

and

$$(1+r) = (1-n)[1+W(y^*(n))] + ny^*(n)[1+W(y^*(n))] \tag{3}$$

where r is the single period riskless rate of interest.

Let Z_0 and Z_1 be the borrower's wealth endowments at the beginning and the end of the period respectively. Z_0 is assumed to be nonrandom and identical for every borrower (otherwise it may be possible to distinguish costlessly between different borrowers on the basis of their observable initial wealth endowments). At the end of the period, the borrower's wealth is Z_1 with probability $(1-n)$ and zero with probability n. Thus, bondholders are paid $1 + W(y)$ by the borrower with probability $(1-n)$ and $y[1+W(y)]$ with probability n by the insurer. Z_1 is assumed identical for every borrower, but n varies cross-sectionally. To ensure that every borrower is capable of paying the initial insurance premium, it is required that $Z_0 \geq \sup_n \phi(n, y^*(n))$. Further, since $(1-n)$ is the no-default probability, $Z_1 \geq \sup_n\{1 + W(y^*(n))\}$ is also a necessary condition.[4]

[4] Although the variables Z_0 and Z_1 do not appear directly in the equilibrium conditions for *this* model, it is useful to bear in mind that they represent implicit constraints. For instance, if $\sup_{n\in(0,1)} \phi(n, y^*(n)) = \sup_{n\in(0,1)}\{1 + W(y^*(n))\} = \infty$, the implicit constraints will obviously be vio-

Note that the information producers incur the cost of information production and receive the insurance premia at the beginning of the period; all other payoffs occur at the end of the period.[5] The first-order condition for (1) is

$$(1 + r)^{-1}(1 - n)W_y(y^*(n)) + \phi_y(n, y^*(n)) = 0 \tag{4}$$

Taking a total derivative in (2) and rearranging gives

$$\{\phi_y(n, y^*(n))(1 + r) - n[1 + W(y^*(n))] - ny^*(n)W_y(y^*(n))\}\, dy^*/dn + \{[\phi_n(n, y^*(n)) - K_n(n)](1 + r) - y^*(n)[1 + W(y^*(n))]\} = 0 \tag{5}$$

Similarly, the total derivative condition for (3) is

$$\{(1 - n)W_y(y^*(n)) + n[1 + W(y^*(n))] + ny^*(n)W_y(y^*(n))\}\, dy^*/dn - [1 - y^*(n)][1 + W(y^*(n))] = 0 \tag{6}$$

Substituting (4) into (6) yields

$$\{-\phi_y(n, y^*(n))[1 + r] + n\{1 + W(y^*(n))\} + ny^*(n)W_y(y^*(n))\}\, dy^*/dn - [1 - y^*(n)][1 + W(y^*(n))] = 0 \tag{7}$$

Finally, adding (5) and (7) results in the equilibrium condition

$$[\phi_n(n, y^*(n)) - K_n(n)](1 + r) - [1 + W(y^*(n))] = 0 \tag{8}$$

The above condition must hold, along $y^*(n)$, for every n.

At this stage, a few comments on the nondissipative signalling cost structure employed here are in order. There are two crucial requirements for a given policy variable (y in this model) to be an informative signal in the market. First, the policy variable should be costly to the issuer (the borrower in this model), and second, the costs should be systematically related to the quality (n in this model) being signalled. In (1), $\phi(n, y)$, the insurance premium, represents the signalling cost to the borrower. To systematically relate signalling costs to quality, $\phi(n, y)$ must be designed so that 'lower quality' borrowers find it more expensive to signal. Therefore, the premium must be inversely related to the borrower's quality or equivalently, positively related to n at the margin. The equality in (8) verifies this—since $1 + W(y^*(n)) > 0$, it must be true that $\phi_n(n, y^*(n)) - K_n(n) > 0$, which means that the signalling cost, minus the cost of information production, is an increasing function of the default probability.[6]

Although (8) is a necessary condition for all equilibria, it is also sufficient (as the illustrations in the next section will highlight) only for *nonseparating equilibria* in which different quality borrowers are indistinguishable on the basis of y.

lated and it will be necessary to find a subset $\Omega(\subseteq(0, 1))$ such that $Z_0 \geq \sup_{n\epsilon\Omega} \phi(n, y^*(n))$ and $Z_1 \geq \sup_{n\epsilon\Omega}\{1 + W(y^*(n))\}$ are satisfied—in such cases it will not be possible to allow the n's to vary cross-sectionally over the entire interval (0, 1).

[5] To obviate the need to explicitly consider any problems associated with the possible inability of the insurer to pay off investors in case of default, it is assumed that the observable initial wealth endowment of the insurer, when added to the premium inflow (less the total cost of information production), is large enough to cover even the largest possible cumulative loss.

[6] Actually, it is plausible to assume that it is more expensive to generate information about riskier borrowers, which means $K(n)$ is usually nondecreasing everywhere with respect to $n(K_n(n) \geq 0)$. In that case, (8) implies $\phi_n(n, y^*(n)) > 0$ and thus the total signalling cost is positively related to n.

For *separating equilibria*, some additional conditions are needed to guarantee sufficiency. These are derived below.

The second order condition (sufficient) for the existence of a *unique* minimum is obtained by partially differentiating (4) with respect to y:

$$(1 + r)^{-1}(1 - n) W_{yy}(y^*(n)) + \phi_{yy}(n, y^*(n)) > 0 \tag{9}$$

Next, totally differentiating (8) and rearranging gives

$$dy^*/dn = [1 + r][\phi_{nn}(n, y^*(n)) - K_{nn}(n)][W_y(y^*(n)) - \phi_{ny}(n, y^*(n))(1 + r)]^{-1} \tag{10}$$

Totally differentiating (4) yields

$$\{(1 + r)^{-1}(1 - n) W_{yy}(y^*(n)) + \phi_{yy}(n, y^*(n))\}\, dy^*/dn = [1 + r]^{-1}[W_y(y^*(n)) - \phi_{ny}(n, y^*(n))(1 + r)]$$

Using (10) to substitute for dy^*/dn in the above expression gives

$$(1 + r)^{-1}(1 - n) W_{yy}(y^*(n)) + \phi_{yy}(n, y^*(n)) = (1 + r)^{-2}[\phi_{nn}(n, y^*(n)) - K_{nn}(n)]^{-1}[W_y(y^*(n)) - \phi_{ny}(n, y^*(n))(1 + r)]^2 \tag{11}$$

By (9) the left-hand side in (11) is strictly positive. Thus, the necessary and sufficient conditions for a separating equilibrium are that, along $y^*(n)$, in addition to (8),

$$W_y(\cdot) \neq \phi_{ny}(\cdot,\cdot)(1 + r) \tag{12}$$

and

$$\phi_{nn}(\cdot,\cdot) - K_{nn}(\cdot) > 0 \tag{13}$$

Therefore, the insurance premium less the information cost (which in a competitive insurance market is the expected cost of insurance to the insurer) should, cross-sectionally, attain a unique interior minimum. Those familiar with Bhattacharya [3] will note the similarity between this and the second order sufficiency condition there.

Model 2: Let borrowers and lenders be risk averse. For simplicity, assume that all borrowers are identical and all lenders are identical, and all utility functions are additively time-separable. In equilibrium, every insurance coverage satisfies

$$y^*(n) \in \operatorname{argmax}_{y(n) \in [0,1]} (1 - n) U(Z_1 - [1 + W(y)]) + V(Z_0 - \phi(n, y)) \tag{1'}$$

subject to

$$\phi(n, y^*(n)) = K(n) + ny^*(n)[1 + W(y^*(n))](1 + r)^{-1} \tag{2'}$$

and

$$Q(1 + r) = (1 - n) Q(1 + W(y^*(n))) + nQ(y^*(n)\{1 + W(y^*(n))\}) \tag{3'}$$

where $Q(\cdot)$ is the typical lender's utility function, $U(\cdot)$ is the borrower's utility for wealth at the end of the period, $V(\cdot)$ is its utility for wealth at the beginning of the period, and Z_0 and Z_1 are the borrower's wealth endowments at the beginning and the end of the period respectively.[7] For reasons cited in Footnote

[7] It is assumed that $Q'(\cdot) > 0$, $Q''(\cdot) < 0$, $U'(\cdot) > 0$, $U''(\cdot) < 0$, $V'(\cdot) > 0$, and $V''(\cdot) < 0$.

4, it is necessary to assume that cross-sectionally $n \epsilon \Omega \subseteq (0, 1)$ such that $Z_0 \geq \sup_{n \epsilon \Omega} \phi(n, y^*(n))$ and $Z_1 \geq \sup_{n \epsilon \Omega} \{1 + W(y^*(n))\}$.
Conditions analogous to those in the previous model are derived in the Appendix.

The illustrations in the subsequent two sections yield some useful insights into the nature of equilibria which satisfy the conditions derived for the two models. In particular, they highlight certain important differences between separating and nonseparating equilibria and more importantly, provide an indication of the increased complexity created by the transition from risk neutral to risk averse borrowers and lenders.

II. Illustrations of Equilibria: Model 1

In this section, borrowers and lenders are assumed to be risk neutral and three different types of equilibrium are illustrated: (1) a unique nonseparating equilibrium in which each borrower buys complete insurance; (2) a separating equilibrium which satisfies all the conditions derived for such an equilibrium in the previous section; and (3) a separating equilibrium which does not satisfy the definition of equilibrium stated in Section I but is acceptable under an alternative equilibrium concept.

A. *A Non-Separating Equilibrium*

A unique nonseparating equilibrium which satisfies (2), (3), and (8) is given by[8]

$$y^*(n) = 1 \forall\, n,$$

$$W(y^*(n)) = r,$$

and

$$\phi(n, y) = K(n) + ny$$

which implies

$$\phi(n, y^*(n)) = K(n) + n \quad \text{and} \quad \Omega = \{n : Z_0 \geq K(n) + n, Z_1 \geq 1 + r\}$$

In this equilibrium, every borrower buys complete insurance in a competitive market and there is no net default. Actually, this situation is equivalent to the information producers purchasing all the debt issues in the market and then reissuing (riskfree) debt on their *own* account. Such a solution is feasible if firms with informational investments can sell insurance without the market being

[8] Note that the function $W(y^*(n))$ represents the *equilibrium* interest rate. In solving for the optimal coverage, one should be careful *not* to insert this equilibrium value in the borrower's objective function given by Equation (1). The nonseparating equilibrium described in this section can be generated by a function of the form

$$W(y) = ry + M(1 - y), \quad \text{where} \quad M \epsilon (\text{Max}_{n \epsilon \Omega}(r + n)(1 - n)^{-1}, \infty)$$

That is, such a function will give $y^*(n) = 1 \forall\, n \epsilon \Omega$ and consequently, $W(y^*(n)) = r$. It is easy to check that this equilibrium satisfies (8) as well as the constraints represented by (2) and (3). Interestingly, although the nonseparating equilibrium given here is *unique*, the function $W(y)$ compatible with such an equilibrium is not unique—it is easy to construct a variety of other similar functions that will lead to the same equilibrium.

aware of the premium terms, so that no insurer is prevented from recovering its information cost $K(n)$. In the next subsection, I assume a different pair of functional forms for $W(y)$ and $\phi(n, y)$ and demonstrate the existence (and explore the properties) of the resulting separating equilibrium.

B. A Separating Equilibrium

Suppose in a given market setting, cross-sectionally default probabilities lie in the interval $(0, \gamma]$, with $\gamma < 1$, i.e. $\Omega = (0, \gamma]$.

If $1 + W(y) = (1 + r)(2y - y^2)^{-1}$ and $\phi(n, y) = 2ny(1 + n)^{-2}(1 - n)^{-1} + K(n) - n(1 - n)(1 + n)^{-2}$ then the following separating equilibrium satisfies (2), (3), (8), (12), and (13):

$$y^*(n) = 1 - n \ \forall\, n \in (0, \gamma] \subset (0, 1), \qquad 1 + w(y^*(n)) = (1 + r)(1 - n^2)^{-1}, \qquad \phi(n, y^*(n)) = K(n) + n(1 + n)^{-1}$$

and

$$\gamma = \mathrm{Min}\{\gamma_0, \gamma_1\}, \quad \text{where} \quad Z_0 \geq K(n) + n(1 + n)^{-1} \ \forall\, n \leq \gamma_0 \quad \text{and} \quad Z_1 \geq (1 + r)(1 - n^2)^{-1} \ \forall\, n \leq \gamma_1$$

Verifying that (2), (3), and (8) are satisfied is trivial. To check whether (12) and (13) also hold, note that

$$\phi_n(n, y) - K_n(n) = 2y(1 + n)^{-3}(1 - n)^{-2}[1 + 2n^2 - n] + [3n - 1](1 + n)^{-3}$$

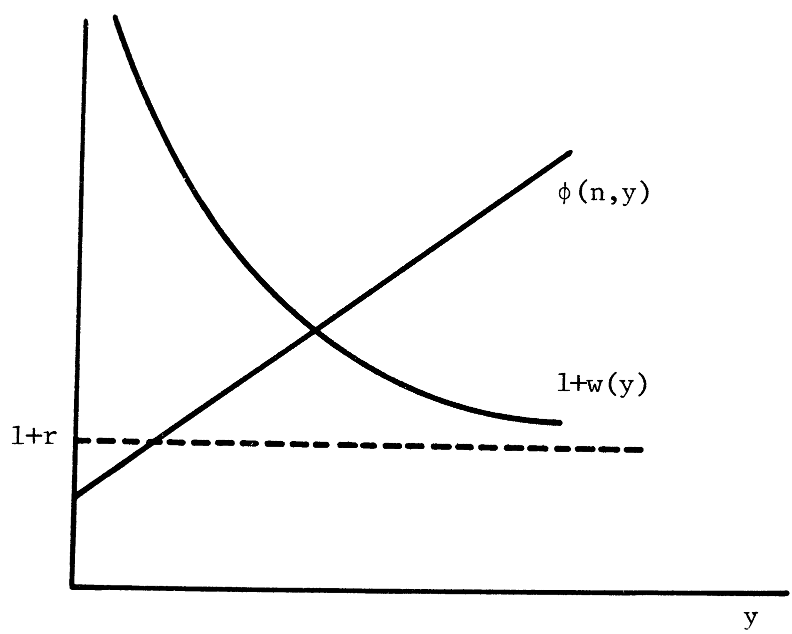
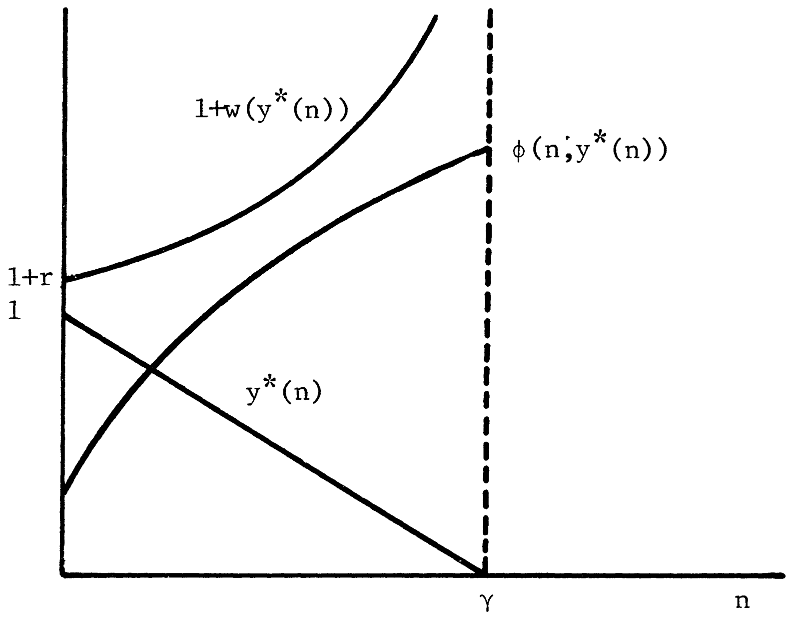


Figure 1. Insurance Premium and Interest Yield as Functions of Insurance Coverage for a Fixed n (Risk Neutrality).



18

Figure 2. Interest Yield, Insurance Premium and Insurance Coverage as Functions of n in Equilibrium (Risk Neutrality).

and

$$\phi_{nn}(n, y^*(n)) - K_{nn}(n) = (6n^2 + 2)(1 - n)^{-2}(1 + n)^{-3} > 0$$

Further, $W_y(y^*(n)) = -2(1 + r)n(1 - n^2)^{-2}$
and

$$\phi_{ny}(n, y^*(n))(1 + r) = 2(1 + r)(1 - n + 2n^2)(1 + n)^{-3}(1 - n)^{-2}$$

which means $W_y(y^*(n)) \neq \phi_{ny}(n, y^*(n))(1 + r)\ \forall\ n \in (0, \gamma]$.

For a fixed n, $\phi(n, y)$ is an increasing linear function of y with a positive intercept, and $1 + W(y)$ is a decreasing convex function with

$$\sup_{y \in (0,1)}[1 + W(y)] = \infty$$

and

$$\inf_{y \in (0,1)}[1 + W(y)] = 1 + r$$

These relationships are graphed in Figure 1. Figure 2 is a graph of the relevant *equilibrium* relationships—$y^*(n)$, $1 + W(y^*(n))$, and $\phi(n, y^*(n))$ are all drawn as functions of n. $y^*(n)$ declines linearly in n while $1 + W(y^*(n))$ is an *increasing* convex function of n with the same infimum value as $1 + W(y)$. For illustrative purposes it is assumed that $K(n) = \sqrt{n} + 0.1$. With this specification, $\phi(n, y^*(n))$ is an increasing concave function of n, bounded below by 0.1 and above by $0.1 + \sqrt{\gamma} + (1 + \gamma)^{-1}$.

This solution has a nice intuitive appeal: in equilibrium, borrowers with higher intrinsic default risks choose successively lower levels of insurance coverage[9] and face both higher insurance premia and market interest rates. In the limit, as $\gamma \to 1$, the borrowers who are certain to default buy no insurance and face an infinite market interest yield. In other words, the lowest quality member does not signal, a result encountered in numerous signalling papers.

C. *Optimality of Equilibria*

Given that feasible cost structures exist for both separating and nonseparating equilibria, I now examine whether one type of equilibrium Pareto dominates the other.[10] Define

$$R(n, y, W(y)) \equiv (1 - n)(1 + W(y))(1 + r)^{-1} + \phi(n, y)$$

Combining (2) and (3), it is easy to see that for *both* the separating and the nonseparating equilibria illustrated in the previous subsections,

$$R(n, y^*(n), W(y^*(n))) = K(n) + 1 \equiv S(n)$$

where $S(n)$ is defined to ease comparison with Bhattacharya [3]. This means that every borrower, insurer, and investor is equally well off in either equilibrium and thus, the two equilibria are Pareto equivalent.

The fact that a nondissipative signalling model has *both* a separating and a nonseparating equilibrium, with the *same welfare* for *each n* in both, is novel and surprising. It is a result not encountered in other nondissipative signalling models, namely those of Bhattacharya [3] and Ross [9]. In the application of his analysis to the labor market turnover model of Salop and Salop [11], Bhattacharya [3] finds that with the wages of all workers contractually vested, there is *no* separating equilibrium with a continuum of quit-probabilities, but a unique nonseparating equilibrium exists. This contrast should be viewed in the following perspective. In Bhattacharya's model, the specification of the $R(\cdot, \cdot, \cdot)$ function is *exogenous*, for instance, in the aforementioned application to Salop and Salop [11]. In this paper, *both* $W(y)$ and $\phi(n, y)$ are *endogenously* determined functions, and hence the function $R(\cdot, \cdot, \cdot)$ is *not exogenously* specified. This creates a fundamental difference in the set of equilibria.

D. *An Alternative Equilibrium Concept*

There is a small problem with the above analysis. Since the insurer incurs a positive cost of information production for *every* potential borrower in the market,[11] unless *all* borrowers participate in the suggested scheme it will not be

[9] This is slightly counterintuitive if signalling considerations are ignored.

[10] Rothschild and Stiglitz [10], among others, refer to nonseparating equilibria, of the type explored here, as "pooling" equilibria.

[11] If $K(n) = 0$ for any n, it simply means that information about that borrower is costlessly available and all investors would acquire that information. Consequently, there would not be any information asymmetry problems to be resolved with respect to *that* borrower.

possible to satisfy (2) over all contracts. This means that even a borrower purchasing no insurance should pay the insurer a positive amount. Fortunately, for most well-behaved separating equilibria (like the one in Subsection B above) "zero insurance" will be an optimal choice only for borrowers with $n = 0$ or $n = 1$, and these extreme cases have been excluded by the assumption that cross-sectionally n lies in a subset of the *open* interval (0, 1). However, one might argue that, in general, expecting nonparticipants to pay a positive price for a product they do not consume may be an unreasonable demand to make. Of course, if $y^*(n) = 0$ for $n = 0$, there is no real problem because for such borrowers the cost of compensating insurers is (presumably) outweighed by the benefit of having their true quality identified by the market. On the other hand, if $y^*(n) = 0$ for $n = 1$, there may be some difficulty in ensuring the payment of a positive amount by these borrowers—they have nothing to lose by totally excluding themselves from the market!

Apart from imposing arbitrary restrictions on $K(n)$, perhaps the only reasonable means of avoiding this corner-solution problem is to introduce the boundary condition, $\phi(n, y = 0) = 0$. Proportional premium schedules like $\phi(n, y) \equiv y\,\Psi(n)$ will obviously work, but the use of such schedules is not generally consistent with the equilibrium concept employed thus far. To explore the characteristics of markets in which such premium schedules may have to be invoked, it is necessary to develop an alternative equilibrium concept like the one below.

In equilibrium, each borrower picks a coverage that satisfies

$$y^*(n) \in \operatorname{argmin}_{y(n)\in[0,1]} (1 + r)^{-1}(1 - n)[1 + W(y)] + y\,\Psi(n) \qquad (1'')$$

subject to

$$\sum_{n\in N} y^*(n)\Psi(n) \geq \sum_{n\in N}\{K(n) + (1 + r)^{-1}ny^*(n)[1 + W(y^*(n))]\} \qquad (2'')$$

and

$$(1 + r) = (1 - n)[1 + W(y^*(n))] + ny^*(n)[1 + W(y^*(n))] \qquad (3'')$$

where the countable set $N \equiv \{n_1, n_2, \cdots, n_J\}$ and $n_j (j = 1, \cdots, J)$ represents the default probability of the jth borrower; there are J borrowers in the market.

The difference between (2) and (2″) is that the insurers' zero-expected-profit condition no longer has to hold for every contract: it is only required that the *total* premium collected from all the borrowers is *at least* sufficient to compensate the insurer for the cumulative risk borne as well as the total cost of information production. The reason for allowing insurers to (possibly) earn excess profits is that an *equality* in (2″) will usually be incompatible with the deployment of strictly proportional schedules like $y\Psi(n)$, which do not permit any nonconvexity at the zero-insurance point.

The notion of equilibrium defined above is not necessarily inconsistent with perfect competition among insurers. If in equilibrium, (2″) holds as a strict inequality, there will be a strong incentive for information producing insurers to expand their activities as intermediaries and offer a variety of other financial services to borrowers (in separate markets which do not impinge on the information-revelation aspects of insurance) at subsidized prices. With a cost function, $K(n)$, that varies across information producers (as opposed to the "identical information production efficiency" assumption I have used), these subsidies would enable the more efficient producers to pass on to the borrowers the excess gains harvested in the insurance market. Consequently, in the long run only the most efficient information producers will survive, and under perfect competition all the excess profits earned by them in the insurance market will be *dissipated* in the noninsurance markets.[12]

One might argue that an equality in (2″) could be attained if the production cost function $K(n)$ is influenced by competition among information producers—if the cost of producing information rises with the number of firms engaging in that activity, $K(n)$ may adjust in such a way that each insurer earns a zero excess profit in equilibrium. However, this argument is weak because information is usually unlike the typical physical commodity whose production requires the purchase of inputs from suppliers who would bid up the prices of these inputs when faced by increasing demand from users. Perhaps one situation in which a rising information cost function could be plausibly conjectured in this model is one in which insurers obtain part of their information about default probabilities by soliciting it, maybe at a cost, from the borrowers themselves. In this case, as the number of insurers bidding for information increases, the cost of procuring the information could also go up. Unfortunately, such an information-transfer mechanism is fraught with an obvious and serious moral hazard problem and is clearly infeasible. If information is truly produced rather than solicited by insurers, there appears to be no meaningful link between the cost of information production and the number of information producers: the only reasonable assumption is that this cost depends on the default probability itself and perhaps the number of potential borrowers being investigated. This implies the information cost function faced by *any* information producer is invariant to changes in the number of information producers it is competing with.

Illustration of Equilibrium: Suppose there are five borrowers in the market with $n_1 = 0.10$, $n_2 = 0.20$, $n_3 = 0.30$, $n_4 = 0.40$, and $n_5 = 0.50$. Let $K(n) = 0.05 \ \forall\, n$, $1 + W(y) = (1 + r)(y^2 - 2y)^{-1}$, and $y\Psi(n) = 2ny(1 + n)^{-2}(1 - n)^{-1}$. Then in equilibrium each borrower picks an optimal insurance coverage, $y^*(n)$, which satisfies

$$y^*(n) = 1 - n,$$
$$\textstyle\sum_{n\epsilon N} y^*(n)\Psi(n) = 1.6507,$$

and

$$\textstyle\sum_{n\epsilon N} \{K(n) + ny^*(n)(1 + r)^{-1}[1 + W(y^*(n))]\} = 1.3574$$

Thus, insurers collectively earn an excess profit of 0.2933. In this illustration, it is easy to verify that given $K(n)$ and $W(y)$, it is impossible to find a function $y\Psi(n)$

[12] To a certain extent, such a phenomenon has also been observed in commercial banking. Faced with Regulation Q ceilings which have severely restricted explicit price competition for deposits, banks have resorted to nonprice competition by offering transaction services to depositors at subsidized prices.

such that each borrower picks a $y^*(n)$ consistent with the satisfaction of an equality in (2″).

It appears, then, that if premium schedules are not restricted to take a specific functional form (like strict proportionality, for example) there is little difficulty in constructing a feasible separating equilibrium. Unfortunately, much of this apparent lack of difficulty stems from the assumption that borrowers and lenders are risk neutral. I shall use an illustration in the next section to highlight the delicacy involved in obtaining separating equilibria with risk averse agents.

III. Illustrations of Equilibria: Model 2

I shall first show that a nonseparating equilibrium, identical to the one constructed for Model 1, is also consistent with Model 2, and then demonstrate that although a feasible separating equilibrium is also possible to obtain, it usually involves rather severe restrictions on certain parameters.

A. *A Nonseparating Equilibrium*

A unique nonseparating equilibrium which satisfies (2′), (3′), and (8′) is given by[13]

$$y^*(n) = 1 \ \forall\, n,$$
$$W(y^*(n)) = r,$$

and

$$\phi(n, y(n)) = K(n) + ny$$

which implies $\phi(n, y^*(n)) = K(n) + n$, and $\Omega = \{n: Z_0 \geq K(n) + n, Z_1 \geq 1 + r\}$.

It is obvious that (2′) and (3′) are satisfied. To check whether (8′) holds, note that $dy^*/dn = 0$, $\phi_n(n, y^*(n)) - K_n(n) = 1$, and $\xi = 1 + r$. As in the risk neutral case, every borrower buys complete insurance and there is no net default. However, the equilibrium here has an added intuitive appeal—*all* the risk is being transferred to risk neutral insurers, and risk averse borrowers and lenders are completely insulated from it. There is an interesting analogy between this result and a well-known fact in the optimal incentive contracts literature (see Harris and Raviv [7], for example) that if the agent is risk neutral, he bears all the risk and the optimal sharing arrangement involves a fixed compensation for the principal.

B. *A Separating Equilibrium*

Suppose $Q(x) = \sqrt{x}$, $V(x) = \sqrt{x}$ and $U(x) = \delta\sqrt{x}$, where δ can be viewed as a "single period utility discount factor." Let η be a positive scalar, less than 1.0. If $\phi(n, y) = K(n) + n(y + 4n)^{-1}$, $1 + W(y) = (1 + r)(y^2 + 4y - 4y^{3/2})^{-1}$, and cross-sectionally, the n's lie in the interval $(0, \eta)$, the following separating equilibrium

[13] As in the risk neutrality case, the unique nonseparating equilibrium here can be generated by a function of the form $W(y) = ry + M(1 - y)$, where M is large enough to guarantee $y^*(n) = 1 \ \forall\, n \in \Omega$.

satisfies (2′), (3′), (8′), and (9′) under the assumption that certain conditions (which will be established shortly) hold

$$\sqrt{y^*(n)} = (1 - n)\ \forall\ n \in (0, \eta),$$

$$1 + W(y^*(n)) = (1 + r)(1 - n^2)^{-2},$$

$$\phi(n, y^*(n)) = K(n) + n(1 + n)^{-2},$$

$$(0, \eta) \subseteq \{n{:}Z_0 \geq K(n) + n(1 + n)^{-2}, \qquad Z_1 \geq (1 + r)(1 - n^2)^{-2}\}$$

It is easy to verify that (2′) and (3′) hold. Since verification of (8′) directly is rather cumbersome, it is simpler to insert the specific functional forms assumed for $\phi(n, y)$ and $W(y)$ into the borrower's objective function and check if the first order condition is satisfied for $\sqrt{y^*(n)} = (1 - n)$. The borrower's problem is

$$\text{Max}_{y(n)}\ \delta(1 - n)\ \sqrt{Z_1 - (1 + r)(y^2 + 4y - 4y^{3/2})^{-1}} + \sqrt{Z_0 - \{K(n) + n(y + 4n)^{-1}\}}$$

and the corresponding first-order condition is

$$\delta(1 - n)(2)^{-1}[Z_1 - (1 + r)(y^2 + 4y - 4y^{3/2})^{-1}]^{-1/2} \cdot[(1 + r)(2y + 4 - 6y^{1/2})(y^2 + 4y - 4y^{3/2})^{-2}] + [Z_0 - \{K(n) + n(y + 4n)^{-1}\}]^{-1/2}(2)^{-1}[n(y + 4n)^{-2}] = 0 \qquad \text{(B.1)}$$

Substituting $\sqrt{y^*(n)} = (1 - n)$ above and simplifying yields the following relationship:

$$K(n) = Z_0 - n(1 + n)^{-2} - (1 - n)^4[Z_1(1 - n^2)^2 - (1 + \text{r})][4\delta^2(1 + \text{r})^2(1 + \text{n})^4]^{-1} \qquad \text{(B.2)}$$

The above condition represents a fairly severe restriction on the information production cost function and highlights the delicacy involved in designing a feasible separating equilibrium in this setting; this problem was not encountered with risk neutral borrowers and lenders. Also note that since $Z_1 \geq (1 + r)(1 - n^2)^{-2}\ \forall\ n < \eta$, the third term on the right-hand side of (B.2) is nonnegative and this in turn implies (by (B.2)) that a condition stronger than $Z_0 \geq K(n) + n(1 + n)^{-2}$ is needed. Thus, $(0, \eta)$ will be a *subset* of the set of n's for which the inequalities $Z_0 \geq K(n) + n(1 + n)^{-2}$ and $Z_1 \geq (1 + r)(1 - n^2)^{-2}$ are satisfied.

In checking the second-order condition, note that the first and the third terms in (9′) are obviously nonpositive. Thus, if $W_{yy}(y^*(n)) > 0$ and $\phi_{yy}(n, y^*(n)) > 0$, (9′) will clearly be satisfied. Straightforward calculations show that

$$W_{yy}(y^*(n)) = (1 + r)(1 - n)^{-6}(1 + n)^{-4}(6n^2 + n + 1) > 0 \qquad \text{(B.3)}$$

and

$$\phi_{yy}(y^*(n)) = 2n(1 + n)^{-6} > 0 \qquad \text{(B.4)}$$

In summary, then, the stipulated separating equilibrium is feasible for $n \in (0, \eta)$, where η is such that (B.2) is satisfied for all $n \in (0, \eta)$.

Discussion of properties of equilibrium

For a fixed n, both $\phi(n, y)$ and $1 + W(y)$ are decreasing convex functions of y, with the extremal values

$$\sup_{y\in(0,1)} \phi(n, y) = K(n) + 0.25, \qquad \inf_{y\in(0,1)} \phi(n, y) = K(n) + n(1 + 4n)^{-1}$$

and

$$\sup_{y\in(0,1)}[1 + W(y)] = \infty, \qquad \inf_{y\in(0,1)}[1 + W(y)] = 1 + r$$

These relationships are graphed in Figure 3. In contrast to the equilibrium illustrated in Section IIB, $\phi(n, y)$ is stated as a *declining* function of the coverage purchased: obviously, if the coverage did not act as a signal, such a premium function would be nonsensical! Figure 4 displays the relevant equilibrium relationships, with n restricted to lie in the interval $(0, \eta)$. For illustrative purposes, $K(n)$ is assumed to be $0\cdot 10 + \sqrt{n}$. As in Figure 2, $1 + W(y^*(n))$ is increasing and convex in n and $\phi(n, y^*(n))$ is increasing and concave in n; $y^*(n)$ also declines with n, but is convex rather than linear. Thus, the equilibrium behavior of the various functions here is not dramatically different from that shown in Figure 2.

Comments

With respect to the above illustration, and particularly its comparison with the illustration in Section IIB, two interesting questions are:

(1) is it likely that the restriction on $K(n)$ will be encountered *generally* in separating equilibria with risk averse borrowers and lenders, or is it merely an artifact of the functional forms assumed here? and

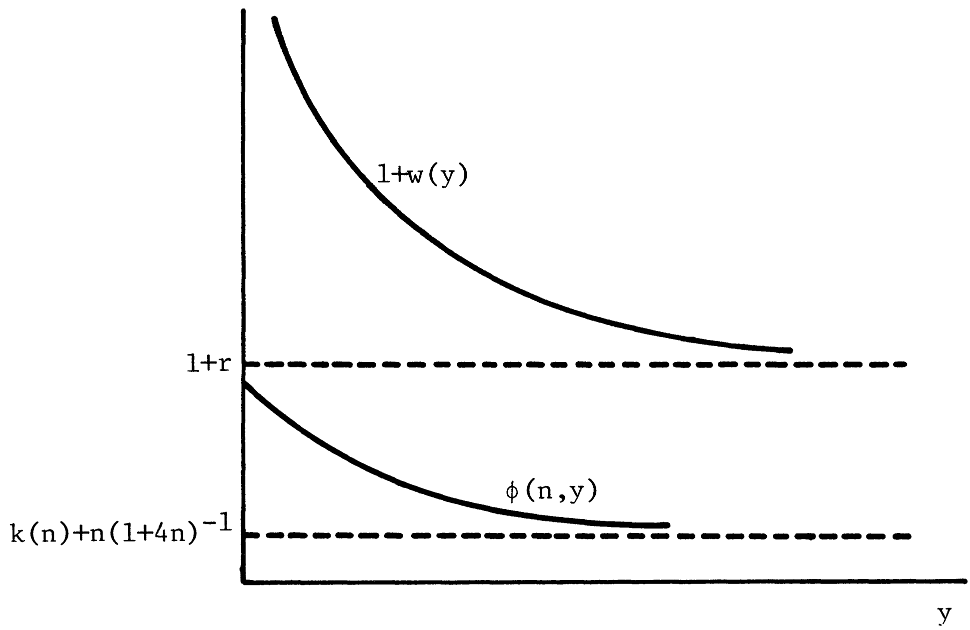


Figure 3. Interest Yield and Insurance Premium as Functions of Insurance Coverage for a Fixed n (Risk Aversion).

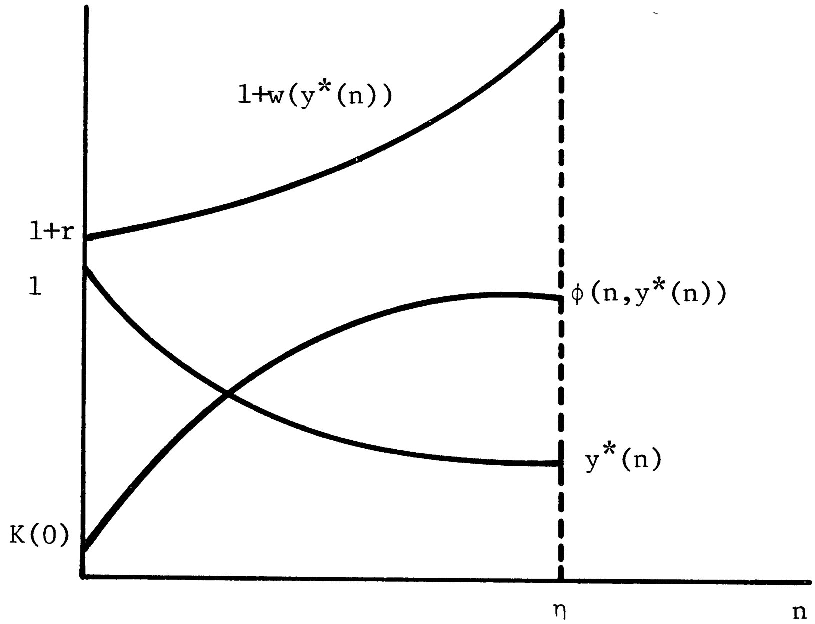


Figure 4. Interest Yield, Insurance Premium, and Insurance Coverage as Functions of n in Equilibrium (Risk Aversion).

(2) why does this problem not appear with risk neutrality?

In response to these questions, I conjecture that restrictions similar to the ones which emerged in the illustration are likely to plague all separating equilibria (of the type considered here) with risk averse agents. The intuition is simple. To satisfy (2′), $K(n)$ must somehow be embedded in $\phi(n, y)$, which is a part of the argument in the borrower's utility function—with a nonlinear utility function, it will usually be impossible to obtain a separating equilibrium condition in which $K(n)$, Z_0 and Z_1 are not linked. This can also be seen by looking at (8′). If $dy^*/dn \neq 0$ (a necessary condition for a separating equilibrium) and the relevant marginal utilities are nonzero (a consequence of nonsatiation), $K(n)$, Z_0 and Z_1 will not generally drop out of the equilibrium condition. This means that some type of arbitrary restrictions will usually have to be imposed on the information production cost function. The reason why this problem does not arise with risk neutral borrowers and lenders is that although (2) necessitates that $K(n)$ be embedded in $\phi(n, y)$, as long as there exist functions $\alpha(n)$ and $\beta(n, y)$, such that $\phi(n, y)$ can be expressed as $K(n) + \alpha(n) + \beta(n, y)$, $K(n)$ can always be *eliminated* from the equilibrium condition.

C. *Optimality of Equilibria*

To repeat the analysis of Section IIC for the equilibria in the risk aversion case, I define

$$T(n, y, W(y)) \equiv (1 - n)U(Z_1 - [1 + W(y)]) + V(Z_0 - \phi(n, y))$$

and $T_N(n, y^*(n), W(y^*(n)))$ and $T_S(n, y^*(n), W(y^*(n)))$ as the equilibrium values of $T(\cdot, \cdot, \cdot)$ for the nonseparating and separating equilibria respectively. Then,

$$T_N(n, y^*(n), W(y^*(n))) = \delta(1 - n)\sqrt{Z_1 - (1 + r)} + \sqrt{Z_0 - (K(n) + n)}$$

and

$$T_S(n, y^*(n), W(y^*(n))) = \delta(1 - n)\sqrt{Z_1 - (1 + r)(1 - n^2)^{-2}} + \sqrt{Z_0 - (K(n) + n(1 + n)^{-2})}$$

It is clear that the Pareto equivalence between the two types of equilibrium encountered in the risk neutrality case will not generally hold here.[14] Given the stringency of the conditions needed to obtain a feasible separating equilibrium in this setting, it is worth identifying the circumstances under which a nonseparating equilibrium will be Pareto dominant. Trite calculations show that if $n \in \Omega \cap (0, \eta)$, this Pareto dominance can be achieved as long as

$$\delta n^3(1 + r)[2(1 - n^2)]^{-1} < \sqrt{\theta_0\theta_1} - \sqrt{\xi_0\xi_1} \ \forall\, n \in \Omega \cap (0, \eta)$$

where

$$\theta_0 \equiv Z_0 - (K(n) + n);$$
$$\theta_1 \equiv Z_1 - (1 + r);$$
$$\xi_0 \equiv Z_0 - (K(n) + n(1 + n)^{-2});$$
$$\xi_1 \equiv Z_1 - (1 + r)(1 - n^2)^{-2};$$

and Ω and η are defined in Sections IIA and IIB respectively.

If the above condition is indeed satisfied, every borrower will enjoy a higher expected utility in the nonseparating equilibrium and thus, the separating equilibrium may never come about.

IV. Municipal Bond Insurance

The type of insurance discussed in this paper appears to be popular in the municipal bond market. For example, American Municipal Bond Assurance Corporation (AMBAC) offers insurance against default on principal and interest for most general obligation and revenue bonds. This insurance is underwritten by

[14] Note that the assumed risk aversion of borrowers and lenders endows the model with a dissipative signalling cost structure, as in Rothschild and Stiglitz [10].

MGIC Indemnity Corporation and is available to investors for existing municipal bonds held in their portfolios, and to borrowing units like state and local governments when they issue new bonds. The AMBAC insurance premium schedule is similar, though not identical, to the structure proposed in this paper—it ranges from ½% to 2½% of the principal and interest charges and is based on an assessment of the insurance risk involved. Premiums are lowest for the most creditworthy issues. The insurance premium is payable in full when the insured bonds are issued and can be paid either by the issuer or the successful bidder. The relatively low insurance premiums seem to suggest that the bonds under consideration are mostly low default risk issues.[15]

The proferred explanation for the desirability of such insurance is that it can result in substantial "interest savings" for public borrowers. In the absence of informational asymmetries, this line of reasoning implicitly asserts that the benefits of these interest savings are somehow not offset entirely by the insurance premia charged to the insured. To justify such reasoning, risk aversion and either market incompleteness or some other market imperfection has to be explicitly introduced. However, this paper argues that municipal bond insurance could be valuable even in a perfectly competitive and complete market, because in addition to serving its usual risk reduction function, it has an informational role to play: investors can observe the insurance coverages purchased by different borrowers and learn something about the true underlying default probabilities of their debt issues.

V. Concluding Remarks

In this section, I wish to tie up a few loose ends by addressing some issues germane to the preceding analysis. I hope this discussion will clarify the distinctive nature as well as the limitations of the model explored in the previous sections.

In analyzing signalling equilibria, it is useful to ponder the social welfare aspects of using the signal. What would happen if no borrower decided to buy insurance? In the absence of any other signalling mechanism or some device for resolving the informational asymmetry problem, the answer is straightforward. In a rational expectations world, investors would probably assign a default probability of $\bar{n}$ to every issue, where

$$\bar{n} = \int_0^1 n d\Theta(n)$$

and $\Theta(\cdot)$ is the cumulative distribution function investors believe[16] describes the dispersion of default probabilities across borrowers. Those familiar with Akerlof [1] will see that this could rapidly lead to a market failure in the sense that no worthwhile borrowers will be willing to participate. On the other hand, what would happen if some isolated borrower decided not to buy insurance when other borrowers do? In general, such a strategy should not be optimal, unless the

[15] For practical purposes then, confining η to be less than 1.0 may not be unduly restrictive in most cases.

[16] In general, $\bar{n}$ can be taken to be a Stieltjes integral.

borrower happens to be the "lowest member" and the equilibrium is one in which such members do not signal. For instance, in the context of all the separating equilibria illustrated in this paper, any borrower who buys zero insurance faces an infinite interest yield. Thus, a borrower's decision not to signal is equivalent to a decision to exclude its debt from the market!

The next issue worth addressing is why information producers would wish to sell insurance rather than organize themselves as intermediaries to identify and invest in undervalued bonds either on their own account or for others. To analyze this issue, recall Assumptions 5 and 6 stated in the first section. Assumption 6 implies that if an information producer incurs a cost of $K(n)$ for every n and then enters into a bidding process to acquire undervalued bonds, its actions will signal to the other investors (who have made no investment in the production of information) the true identities of all bonds, and in a *tâtonnement* market investors will respond to this new knowledge by appropriately bidding up and down the prices of undervalued and overvalued bonds respectively. Since no trades take place out of equilibrium, the information producer will incur a dead loss of $K(n)$ for every n and thus will not be willing to identify and invest in undervalued assets in this manner.[17] In the context of financial intermediation, Campbell and Kracaw [4] also make the *tâtonnement* process assumption about the determination of equilibrium prices, and then focus on side-payments that mispriced firms could make to investors to either induce them to produce information or prevail upon them to abstain from information production. They find that in such markets side-payments from firms to information producers will result in the production of information necessary to resolve the informational asymmetry problem, but the possibility of unreliable information being generated may prevent the most efficient (least cost) producer from winning the bid to produce information. Since an analysis of that nature would be rather tangential to the subject at hand, Assumption 5 is used to rule out such side-payment mechanisms.

Assumption 3 precludes *direct* signalling by borrowers, and thus in conjunction with Assumptions 4, 5, and 6, paves the way for the three party-information production-signalling scheme that is the focal point of the analysis in this paper. How the informational asymmetry problem is *actually* resolved in a given market setting will depend on which of the three mechanisms is the most productive in the aggregate (assuming they are all viable in that setting). Perhaps a more ambitious research venture could explore this issue rigorously.

Finally, in the context of the application of the model to municipal bond insurance, Assumption 4 appears unreasonable. Fortunately, it can be relaxed if one assumes that credit ratings convey noisy information and third-party signalling permits a superior discrimination between borrowers of varying risks. In other words, a weaker assumption that would suffice is that credit ratings do not completely resolve the asymmetrical information problem.

[17]Grossman [5] and Grossman and Stiglitz [6] have argued that in markets characterized by asymmetric information about assets, if market prices instantaneously and efficiently aggregate all available information about asset values, information can be labeled a public good. This phenomenon can destroy an investor's incentive to engage in costly information production for trading purposes.

APPENDIX

Derivation of Conditions for Existence of Equilibrium with Risk Averse Agents

The steps in this derivation are similar to those for Model 1. The first-order condition is (primes also denote derivatives)

$$-(1-n)\,U'(Z_1-[1+W(y^*(n))])\,W_y(y^*(n)) - V'(Z_0-\phi(n, y^*(n)))\,\phi_y(n, y^*(n)) = 0 \quad (4')$$

The "total derivative" condition corresponding to $(2')$ is the same as (5):

$$[\phi_y(n, y^*(n))(1+r) - n[1+W(y^*(n))] - ny^*(n)\,W_y(y^*(n))]\,dy^*/dn + [\{\phi_n(n, y^*(n)) - K_n(n)\}(1+r) - y^*(n)\{1+W(y^*(n))\}] = 0 \quad (5')$$

Taking the total derivative in $(3')$ gives (defining $\xi \equiv 1 + W(y^*(n))$ for notational convenience)

$$[(1-n)\,Q'(\xi)\,W_y(y^*(n)) + nQ'(y^*(n)\xi)\{\xi + y^*(n)\,W_y(y^*(n))\}]\,dy^*/dn + Q(y^*(n)\xi) - Q(\xi) = 0 \quad (6')$$

Substituting $(4')$ into $(6')$ gives

$$[-Q'(\xi)\,V'(Z_0-\phi(n, y^*(n)))\phi_y(n, y^*(n))\{U'(Z_1-\xi)\}^{-1} + nQ'(y^*(n)\xi)\{\xi + y^*(n)\,W_y(y^*(n))\}]dy^*/dn + Q(y^*(n)\xi) - Q(\xi) = 0 \quad (7')$$

Adding $(5')$ and $(7')$ produces the analog to (8):

$$[\phi_y(n, y^*(n))[(1+r) - Q'(\xi)\,V'(Z_0-\phi(n, y^*(n)))\{U'(Z_1-\xi)\}^{-1}] + n\xi[Q'(y^*(n)\xi) - 1] + ny^*(n)\,W_y(y^*(n))[Q'(y^*(n)\xi) - 1]]dy^*/dn + [\phi_n(n, y^*(n)) - K_n(n)](1+r) - y^*(n)\xi + Q(y^*(n)\xi) - Q(\xi) = 0 \quad (8')$$

This condition must be satisfied, along $y^*(n)$, for every n. For a separating equilibrium, in addition to $(8')$, the following second-order sufficiency condition should also be satisfied:

$$\begin{aligned}
&+ (1-n)\,U''(Z_1-[1+W(y^*(n))])[W_y(y^*(n))]^2 \\
&- (1-n)\,U'(Z_1-[1+W(y^*(n))])\,W_{yy}(y^*(n)) \\
&+ V''(Z_0-\phi(n, y^*(n)))[\phi_y(n, y^*(n))]^2 \\
&- V'(Z_0-\phi(n, y^*(n)))\,\phi_{yy}(n, y^*(n)) < 0 \qquad (9')
\end{aligned}$$

REFERENCES

1. George A. Akerlof. "The Market for 'Lemons': Qualitative Uncertainty and the Market Mechanism." *Quarterly Journal of Economics* 89 (August 1970), 488–500.
2. Sudipto Bhattacharya. "Imperfect Information, Dividend Policy, and the 'Bird in the Hand Fallacy.'" *The Bell Journal of Economics* 10 (Spring 1979), 259–70.
3. ———. "Nondissipative Signalling Structures and Dividend Policy." *Quarterly Journal of Economics* 95 (August 1980), 1–24.

4. Tim Campbell and William Kracaw. "Information Production, Market Signalling and the Theory of Financial Intermediation." *The Journal of Finance* 35 (September 1980), 863–82.
5. Sanford Grossman. "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information." *The Journal of Finance* 31 (May 1976), 573–85.
6. ——— and Joseph E. Stiglitz. "Information and Competitive Price Systems." *The American Economic Review* 66 (May 1976), 246–53.
7. Milton Harris and Artur Raviv. "Optimal Incentive Contracts with Imperfect Information." *Journal of Economic Theory* 20 (April 1979), 231–59.
8. Joseph Riley. "Competitive Signalling." *Journal of Economic Theory* 10 (April 1975), 174–86.
9. Stephen A. Ross. "The Determination of Financial Structure: The Incentive Signalling Approach." *The Bell Journal of Economics* 8 (Spring 1977), 23–40.
10. Michael Rothschild and Joseph Stiglitz. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." *Quarterly Journal of Economics* 90 (November 1976), 629–49.
11. Joanne Salop and Steven Salop. "Self Selection and Turnover in the Labor Market." *Quarterly Journal of Economics* 90 (November 1976), 619–28.
12. Michael Spence. "Job Market Signalling." *Quarterly Journal of Economics* 87 (August 1973), 355–74.
13. ———. "Competitive and Optimal Responses to Signals: Analysis of Efficiency and Distribution." *Journal of Economic Theory* 7 (March 1974), 296–332.
14. ———. "Consumer Misperceptions, Product Failure and Producer Liability." *Review of Economic Studies* 3 (October 1977), 561–72.