Proxying for Expected Returns with Price Earnings Ratios*

Charlotte S. Hansen†
Zicklin School of Business
Baruch College, New York, NY

Bjorn E. Tuypens‡
Oak Hill Platinum Partners, L.L.C.
Rye Brook, NY

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Abstract

Long-run regression models using the trailing earnings over price ratio to predict future returns suggested by Campbell and Shiller (1988, 2001) work quite well. However, in this note we show that this variable might result in a downward biased proxy for expected future returns. Instead we suggest using a moving average of the log of 1 plus the earnings price ratio when forecasting long-run returns. The empirical results for the S&P 500 show the superiority of our approach to existing ones.

Keywords: Earnings yield, Stock Return, Forecasting
JEL classification: C52, G12, G14

1. Introduction

There is a vast body of literature on variables that predict expected stock returns; we will refer to Ferson, Sarkissian, and Simin (2003) for a recent overview of this literature. In this note we discuss the difference between using a moving average of earnings yields

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† Corresponding author: Charlotte S. Hansen, Mailing address: Department of Economics and Finance, Zicklin School of Business, Baruch College, One Baruch Way, Box B10-225, New York, NY 10010, Tel: (646) 312-3519, Fax: (646) 312-3451, Email: charlotte_hansen@baruch.cuny.edu.
‡ The views, thoughts and opinions expressed in this paper are those of the authors in their individual capacity and should not in any way be attributed to Oak Hill Platinum Partners L.L.C. or to Bjorn Tuypens as a representative, officer, or employee of Oak Hill Platinum Partners L.L.C.

and a ratio of a moving average of earnings relative to the current price to predict long-run stock returns. The latter ratio involves averaging earnings over a period from 4 to 30 years to smooth business cycle fluctuations. This smoothed earnings series is then divided by the most recent price level, which is assumed to contain all relevant information to predict future returns.

We provide two arguments for why one should use a moving average of the earnings yield. First, we present a simple model to interpret the earnings yield as a proxy for expected returns, and argue that a moving average of earnings over the current price results in a biased estimation of expected returns. Second, the q-period moving average of earnings yields can be interpreted as an expectation of future returns formed over q periods, where q goes to infinity as the sample size increases. Under the usual ergodicity assumption this quantity converges to the required return. Finally, we present empirical results for the S&P 500 index returns over the period 1920-2003. Using the Fair and Shiller (1990) approach we show that the moving average of earnings yields subsumes all information contained in the smoothed earnings over the current price ratio.

2. The model

We consider a Gordon (1959) constant growth firm, with its stock price P_t given by

$$P_t = \frac{D_t(1+g)}{R-g},$$

where D_t is the dividend per share, g is the growth rate of the dividends, and R is the required rate of return.

Let E_t denote earnings per share. From Modigliani and Miller (1958), we can show that, if the internal rate of return equals the required rate of return, the earnings price ratio equals the required return, i.e.

$$\frac{E_t}{P_t} = R, \qquad (1)$$

with $g = (1-\pi)R,$ where π is the payout ratio defined by $D_{t+1} = \pi E_t$. This is the basis for using the earnings yield to estimate or proxy for future returns. Accordingly, the logarithm of 1 plus the earning price ratio, ln(1+E/P), is a proxy for the log return $r = \ln(1+R)$.[1] The question in the literature on predicting stock returns is how to form an estimator for future returns. In the next section we consider two methods, currently used in practice, to proxy for expected returns using earnings price ratios.

[1] We will use the term log earnings yield for the term ln(1+E/P) throughout this note.

3. Two proxies for expected returns

3.1 Trailing earnings over price

The first proxy, $F_t^{Th} = \ln\left(1 + \left(\frac{1}{h}\sum_{j=1}^{h} E_{t-h+j}\right) / P_t\right)$, involves an average of the past h earnings divided by the most recent price. This corresponds to forming the expectation based on only one observation, namely the smoothed earnings over the current price. The argument for this proxy is that earnings fluctuate over the business cycle and therefore need to be smoothed, but only information in the current price is relevant for future returns.

We now show that the proxy, F_t^{Th}, is a downward biased estimate of expected returns. From Gordon's growth model it follows that prices grow at rate g,

$$P_{t+1} = \frac{(1+g)D_{t+1}}{R-g} = (1+g)P_t,$$

and by (1), earnings also grow at a rate g since

$$E_{t+1} = P_{t+1}R = (1+g)P_t R = (1+g)E_t .$$

Consequently the ratio of the h-period moving average of earnings and the current price

$$\begin{aligned}
\frac{h^{-1}\sum_{j=1}^{h} E_{t+1-j}}{P_t} &= \frac{E_{t+1-h}h^{-1}\sum_{j=1}^{h}(1+g)^j}{P_{t+1-h}(1+g)^h} \\
&= Rh^{-1}\sum_{j=1}^{h}\frac{1}{(1+g)^{h-j}} \\
&= Rh^{-1}\frac{(1+g)}{g}\left(1-\frac{1}{(1+g)^h}\right) \\
&\rightarrow 0 \text{ as } h \rightarrow \infty . \qquad (2)
\end{aligned}$$

Slutsky's Theorem implies that $F^{Th} = \ln\left(1 + \frac{h^{-1}\sum_{j=1}^{h} E_{t+1-j}}{P_t}\right) \rightarrow 0$ as $h \rightarrow \infty$. Hence this is a downward biased estimator of the required return.

To assess the importance of this result, assume that companies pay out 50% of their earnings as dividends. The historical real rate of return on the S&P 500 is 7%. Then the growth rate, g, equals

$$g = (1-\pi)R = 0.5 \cdot 0.07 = 0.35$$

or a growth rate of 3.5%. Taking a moving average of earnings over 30 years, but not of prices, leads to an earnings yield (or expected return) of

$$\begin{aligned}\frac{h^{-1}\sum_{j=1}^{h} E_{t+1-j}}{P_t} &= Rh^{-1}\frac{(1+g)}{g}\left(1-\frac{1}{(1+g)^h}\right) \\ &= 0.07 \cdot 30^{-1} \cdot \frac{(1+0.035)}{0.035}\left(1-\frac{1}{(1+0.035)^{30}}\right) = 0.04417\end{aligned}$$

or only 4.417% as opposed to the historical 7%.

3.2 Moving average of the log earnings yield

The second proxy, F_t^{MA}, is constructed as the average of the past q log earnings price ratios, $\frac{1}{q}\sum_{j=1}^{q}\ln\left(1+\frac{E_{t-q+j}}{P_{t-q+j}}\right)$. In effect, this is a time-series average of the log earnings yield over the past q periods, which under ergodicity converges to the expected value of the log earnings yield as $q \to \infty$. The more observations we use to form the expectation the closer it comes to its true value, so this should give an advantage to the proxy F^{MA}. Note that averaging the log earnings yield does not give rise to a downward biased estimate.

3.3 Comparison

The realized log gross return on a portfolio held for one period is $r_t = \ln\left(\frac{P_t+D_t}{P_{t-1}}\right)$ and the realized gross return over a period of q years is given by $r_{t+q}(q) = \sum_{j=1}^{q} r_{t+j}$. Figure 1 depicts the average realized real return measured over 10 periods, $\frac{1}{q}r_{t+q}(q)$, and the three proxies: F^{MA}, F^{T10}, and F^{T30} for the S&P 500 index from 1920-2003.[2] All variables are adjusted for inflation. It turns out that F^{T30} is smaller than F^{T10} for almost every period from 1920 through 2003 with the exception of the periods 1935-1939 and 1948-1951, where there is hardly any difference between the two forecasts. This is what we would expect given result (2) from the previous section saying that F^{Th} decreases as h increases. However, it is not clear-cut from the graphs which variable works better when proxying for expected returns. Further analysis is needed.

[2] Data is from Prof. Shiller's web page: www.robertshiller.com.

[Insert figure 1 about here]

4. Forecasting long-run returns

We use annual data for the S&P 500 index covering the period 1920-2003. Table 1 reports the descriptive statistics of the realized return and the different proxies.

Table 1: Descriptive Statistics

	$\bar{r}(q)$	F^{MA}	F^{T10}	F^{T30}
Mean	0.074	0.077	0.070	0.062
Median	0.076	0.077	0.064	0.060
Std. Dev.	0.058	0.019	0.031	0.028
ρ	0.847	0.944	0.720	0.747

Descriptive statistics of annual realized real $\bar{r}(q) = \frac{1}{q} r(q)$ for q=10 and the proxies F^{MA}, F^{T10}, and F^{T30}. All variables are adjusted for inflation. We let ρ denote the first order autocorrelation coefficient. The sample period for the S&P 500 index covers 1920 to 2003.

The proxies have a smaller standard deviation than the realized return, which is consistent with the notion that the proxies are expectations. Now consider the means of the proxies. The average value for F^{MA} is 7.7%, which is slightly larger than the average realized return of 7.4%. However, the median of F^{MA} and the realized return come quite close with values of 7.7% and 7.6%, respectively. Both of the proxies F^{T10} and F^{T30} are smaller on average than the realized return, and even more so when earnings are smoothed over a longer period, confirming the result in equation (2).

Table 2: Correlation Matrix

	$\bar{r}(q)$	F^{MA}	F^{T10}	F^{T30}
$\bar{r}(q)$	1	0.761	0.609	0.599
F^{MA}		1	0.493	0.457
F^{T10}			1	0.798
F^{T30}				1

Correlation matrix between the annual realized real $\bar{r}(q) = \frac{1}{q} r(q)$ for q=10 and the proxies F^{MA}, F^{T10}, and F^{T30}. All variables are adjusted for inflation. The sample period for the S&P 500 index covers 1920 to 2003.

The correlation matrix between the realized return and the proxies is reported in Table 2. The proxy F^{MA} has the highest correlation with the future realized return with a correlation of 0.76. Each of the proxies F^{T10} and F^{T30} has a correlation of about 0.6 with the future realized return. So, from this preliminary analysis F^{MA} seems to be a better forecast of future realized return.

We can evaluate the forecasts for expected return along the lines of Fair and Shiller (1990) by regressing the actual real return on the three forecasts:

$$\overline{r_{t+q}}(q) = \alpha + \beta_{MA} F_t^{MA} + \beta_{T10} F_t^{T10} + \beta_{T30} F_t^{T30} + u_{t+q}. \tag{3}$$

If neither of the forecasts contain any information relevant for the q period realized return then the estimates of β_{MA}, β_{T10}, and β_{T30} should be zero. If each of the forecasts contain independent information then β_{MA}, β_{T10}, and β_{T30} should all be non-zero. If all forecasts contain information but the information in one of the forecasts, say F^{T10}, is completely contained in the other forecasts, F^{MA} and F^{T30}, and these other forecasts contain additional information, then β_{MA} and β_{T30} are non-zero, and β_{T10} is zero.

We apply the Fair and Shiller (1990) approach using partial regressions; from the Frisch-Waugh theorem both approaches are equivalent. The reason for using partial regressions is twofold. Firstly, the forecasts are highly correlated with correlations ranging between 0.5 and 0.8, so the multivariate regression (3) is likely to be contaminated by multicollinearity. Secondly, we want to use the inference methodology for long-run regressions developed in Hansen and Tuypens (2004). While the inference methodology works for multivariate regression models with long-run or short-run explanatory variables, it does not work for a mixture of short-run and long-run explanatory variables. Therefore, we run univariate regressions instead, where we regress the actual return on each of the three forecasts separately:

$$\begin{aligned}
\overline{r_{t+q}}(q) &= \alpha_{MA} + \beta_{MA} F_t^{MA} + u_{t+q}^{MA}, \\
\overline{r_{t+q}}(q) &= \alpha_{T10} + \beta_{T10} F_t^{T10} + u_{t+q}^{T10}, \\
\overline{r_{t+q}}(q) &= \alpha_{T30} + \beta_{T30} F_t^{T30} + u_{t+q}^{T30}.
\end{aligned}$$

We can test the exact same hypotheses as above using univariate regressions. The hypotheses can be formulated as follows: If neither of the forecasts contain any information relevant for the q period realized return then the estimates of β_{MA}, β_{T10}, and β_{T30} should be zero. If each of the forecasts contain independent information then β_{MA}, β_{T10}, and β_{T30} should all be non-zero. In addition, if we regress the residual from each of the univariate regressions on the remaining forecasts, they should enter with non-zero coefficients as well. For example, if all forecasts contain information, but the information in one of the forecasts, say F^{T10}, is completely contained in the forecast F^{MA}, then β_{T10} should be non-zero in the regression:[3]

$$\hat{u}^{MA}_{t+q} = \gamma + \beta_{T10} F^{T10}_t + \varepsilon^{MA}_{t+q}.$$

We report the regression results for the regressions of demeaned realized return on the demeaned proxies for expected future return in Table 3. Inference is carried out using the methods developed in Hansen and Tuypens (2004) for long-run regressions.

From Table 3 we see that each of the forecasts contain information about expected future returns; each of them enters with a significantly positive coefficient with *t*-statistics larger than 1.96. In line two and three we examine whether the forecasts F^{T10} or F^{T30} contain information beyond that in F^{MA}. It turns out that neither F^{T10} nor F^{T30} enters with coefficients significantly different from zero, indicating that neither of these forecasts contains information that is not already contained in F^{MA}.

Table 3: Fair-Shiller test results

Dependent variable	$\hat{\beta}_{MA}$	*t*-stat	$\hat{\beta}_{T10}$	*t*-stat	$\hat{\beta}_{T30}$	*t*-stat	R^2
$\bar{r}(q)$	2.787*	(2.207)					58.0%
resid^{MA}			0.432	(1.473)			12.9%
resid^{MA}					0.501	(1.512)	13.4%
$\bar{r}(q)$			1.130*	(2.097)			37.0%
resid^{T10}	1.688*	(2.504)					33.8%
resid^{T10}					0.149	(0.325)	0.8%
$\bar{r}(q)$					1.385*	(2.300)	43.1%
resid^{T30}	1.463*	(2.191)					28.1%
resid^{T30}			-0.045	(-0.121)			0.1%

Regression estimates of 10-year real S&P 500 returns on various proxies, and regression estimates of residuals on each of the alternative proxies for the expected future return. The *t*-statistics (numbers in brackets) are calculated using the methods from Hansen and Tuypens (2004); * indicates significance at the 2.5% levels. The sample period for the S&P 500 index covers 1920 to 2003.

As a check of robustness we also regress the residuals from each of the other univariate regressions, and we cannot reject that F^{MA} contains information beyond that in F^{T10} and F^{T30}.

[3] Note that since we use estimated residuals, the *t*-statistics are likely to be biased upwards, implying that the test is more likely to reject the null hypothesis of β =0 in the residual regressions. Since we are not able to reject the null in any of the relevant cases, we can conclude that these results are robust.

[Insert figure 2 about here]

In addition, we can compare the *R*-square from each of the univariate regressions. The *R*-squared is greater for F^{MA} than for either F^{T10} or F^{T30}, indicating that using a moving average of the log earnings yield explains a greater portion of the total variation in future realized returns. Figure 2 exhibits the realized return and the returns predicted using each of the three proxies. The figure reinforces the conclusion that a moving average of the log earnings yield is a better proxy for the expected future return.

5. Conclusion

This note shows, using the Gordon growth model, that the trailing earnings over current price ratio is a downward biased estimate for expected stock returns. Instead we suggest using a moving average of 1 plus the earnings price ratio. The empirical results for the S&P 500 index support the theoretical finding that trailing earnings over price ratio results in a biased estimator and shows that a moving average of the log of 1 plus the earnings price ratio subsumes all information contained in the trailing earnings over price ratio.

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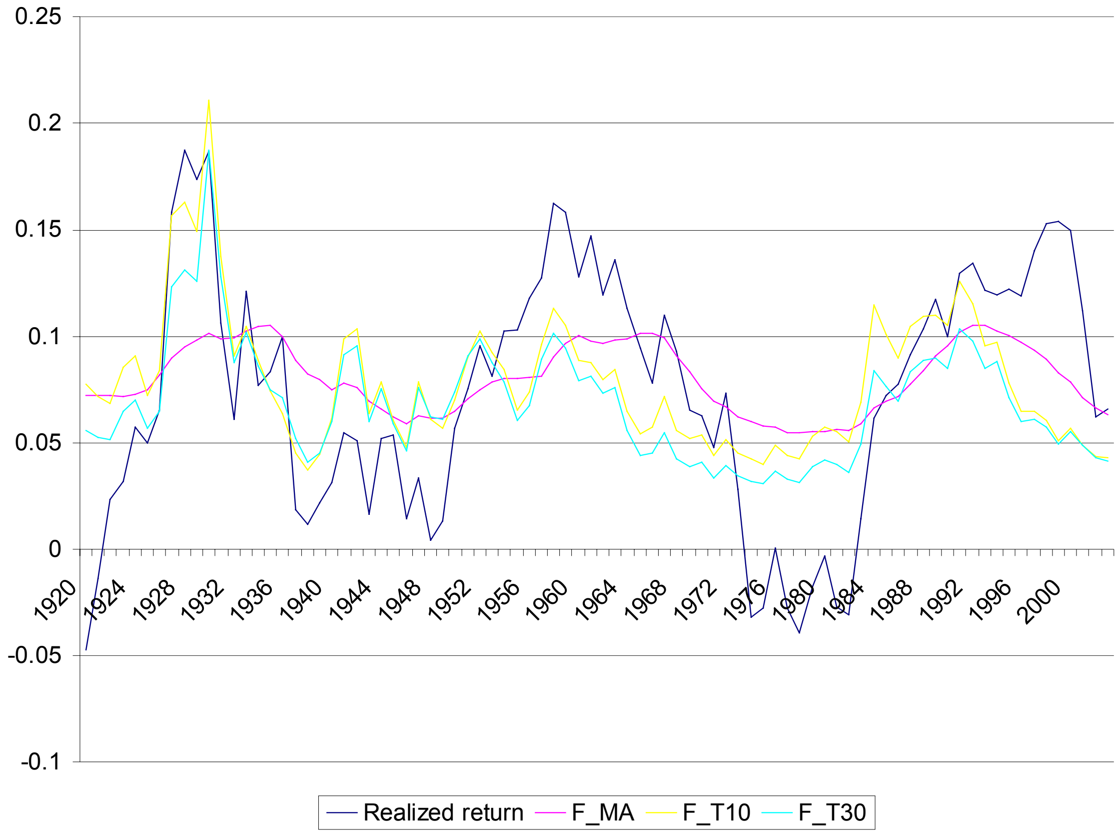
Figure 1:



0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
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1924
1928
1932
1936
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1948
1952
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1960
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1972
1976
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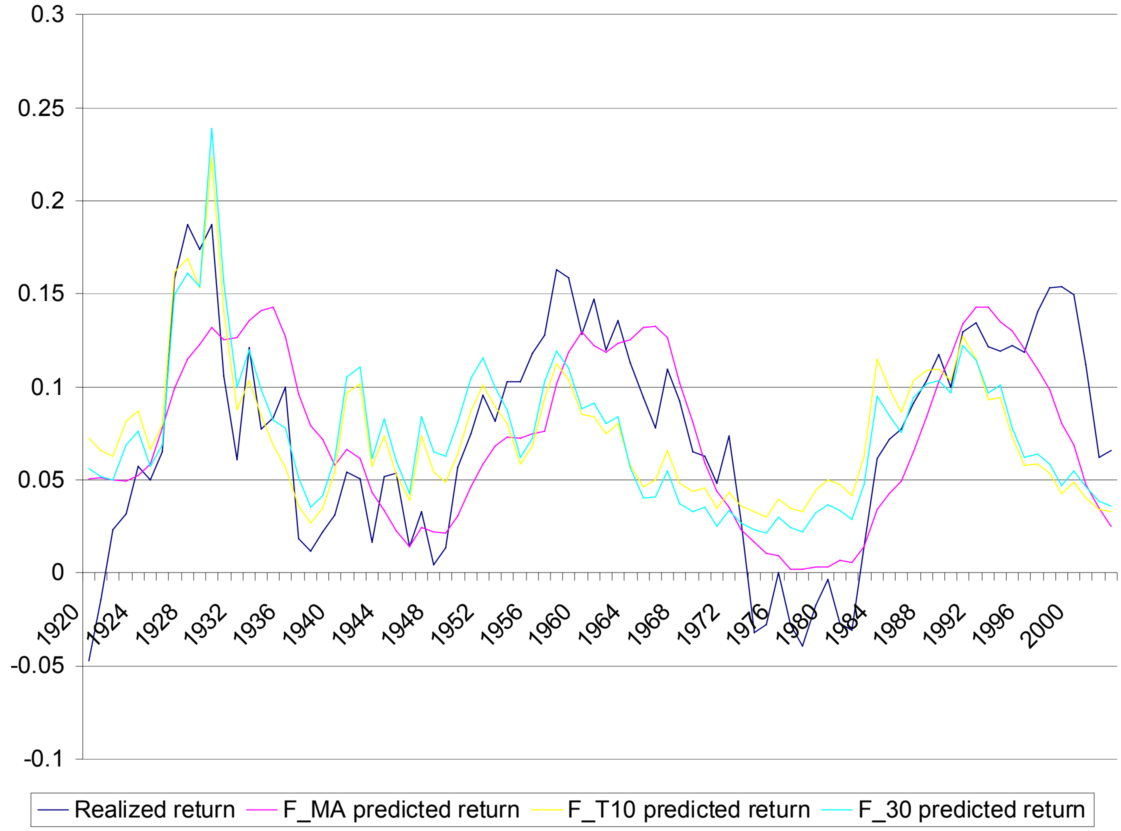
1984
1988
1992
1996
2000
Realized return
F_MA
F_T10
F_T30

Figure 2:


0.3
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Realized return
F_MA predicted return
F_T10 predicted return
F_30 predicted return