Banking Crises and the Lender of Last Resort:
How crucial is the role of information?

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Abstract. This article develops a model of bank runs and crises and analyses how the presence of a lender of last resort (LOLR) affects the solvency of the banking system. We obtain a one to one mapping from the depositors' equilibrium strategy to an optimal contract prevailing in the economy. The study finds that the difference between a perfectly informed and an imperfectly informed LOLR can be crucial. Our results indicate that a perfectly informed LOLR is a Pareto improvement. However, if the supervisory process of the LOLR is subject to noise, then the gains from ex post efficiency may be outweighed by ex ante inefficiency induced by moral hazard which is conducive to lower lending rates in the economy.

1. Introduction

Banks are an integral part of the economy as they provide an important channel through which funds are transferred from investors to the entrepreneurial sector. However, history has shown that banks are subject to runs and panics. A bank run occurs when depositors fearing that the bank will be unable to fulfill its obligations, attempt to withdraw their funds immediately. If a bank run is severe enough, then even healthy banks can ultimately become insolvent or even bankrupt. Such banking crises can seriously disrupt economic activity.[1] Because of the central position of financial intermediaries in the economy, the adverse impact of banking crises on economic activity cannot be overemphasised.

Since banks hold only a fraction of their deposits as reserves, they are vulnerable to liquidity shocks which might hit the economy as such shocks might induce panic and may affect the behaviour of the depositors. The role of the central bank as a lender of last resort was thus a natural response to the fractional reserve system. Some economists claim that the LOLR is not necessary in a well developed financial system as the interbank market can provide liquidity to solvent banks facing liquidity problems.[2] However, as argued by Goodhart and Huang (2003), the interbank market cannot provide liquidity in two instances. First, the interbank market might not suffice in case of a market failure, for instance, when a large amount, which is too much for a single bank, is needed to bail out

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[1]Bernanke (1983) claims that a substantial part of the decline in real output during the Great Depression was a consequence of the breakdown of economic institutions and the subsequent collapse of credit rather than the decline in the quantity of money.

[2]See, for example, Goodfriend and King (1988).

a solvent institution.[3] Second, the market mechanism cannot provide insurance against liquidity shocks which affect the whole economy.

Since Diamond and Dybvig (1983) there has been a growing interest in models of bank runs. However the problem with Diamond-Dybvig type models is that runs take place because of self-fulfilling equilibria subsequent to liquidity shocks experienced by depositors and hence are random events. The Diamond Dybvig model exhibits multiple equilibria and the good or the bad equilibrium might prevail irrespective of the underlying fundamentals. In practice, however, bank runs take place when the depositors doubt the solvency of the bank given their beliefs regarding the underlying fundamentals. Thus the bad equilibrium is more likely to prevail if fundamentals are weak and vice versa. Evidence by Gorton (1988) supports this view. He finds that during the US National Banking Era (1865-1914), panics were triggered when the leading indicator of recession reached a threshold level. His results therefore reject the sunspot theories of panics.

Our approach is based on the 'global games' methodology first introduced by Carlsson and van Damme (1993) and later modified by Morris and Shin (1998). As discussed in more detail later in this paper, it is not straightforward to apply this approach to banking crises because it is based on the assumption that an agents incentive to take a particular action increases as more and more agents take that action. In general, however, bank run models do not satisfy this assumption of 'full strategic complementarities' because if the bank is already bankrupt then an agents payoff from withdrawing decreases when more and more depositors run. Nevertheless, Dasgupta (2002) and Goldstein and Pauzner (2002) get round this problem and show that a unique equilibrium can still be obtained in bank run models. The advantage of using global games analysis is that it enables us to link the probability of crises to the real economy.

Our paper also provides a methodological contribution to the global games literature. We show that for any equilibrium strategy of depositors, there exists a corresponding optimal lending rate in the economy. Thus by using global games we are not only able to identify a unique equilibrium in the depositors' strategy but are also able to pin down and study the unique optimal contract.

As mentioned by Goodhart and Huang (2003), there have been few formal models analysing the role of the LOLR. Goodhart and Huang study the trade off faced by the LOLR between contagion and moral hazard effects. They show that even in the presence of moral hazard, providing LOLR facilities is justified given the cost of contagion. Freixas (1999) considers the optimal bail out policy of the LOLR. However, Freixas restricts attention to the bail out (or liquidation) of insolvent banks.[4] He justifies the 'too big to fail' argument by assuming that the cost of bank liquidation increases with size and hence concludes that it might be rational to bail out an insolvent bank. In contrast, we focus on the bail out of solvent but illiquid banks, in the presence of both perfect and imperfect information.

Perhaps the paper that comes closest to ours is the one by Rochet and Vives (2002). Rochet and Vives analyse the role of the LOLR in the presence of coordination failure among depositors. They study a LOLR whose objective is to bail out solvent banks facing liquidity problems and they show that for an intermediate range of fundamentals,

[3] For example, on November 21st 1985, the Bank of New York required a bail-out because of a computer bug in its T-Bills clearing system which denied any incoming payments. The Fed then had to provide an emergency loan of $22.6 billion which was too much for a single bank and because of coordination problems could not be provided by the market as a whole.

[4] Freixas assumes that solvent banks will be bailed out by the interbank market. However as discussed before this need not be the case.

there may exist solvent but illiquid banks. These features of their model are similar to ours. Rochet and Vives show that the LOLR is a Pareto improvement as it can avoid the cost of inefficient liquidation. They thus conclude that Bagehot was right after all in claiming that there exists a role for the LOLR in lending to illiquid solvent financial institutions. However throughout their analysis they assume that the LOLR has perfect information about the bank's fundamentals. We show in our work that the results can change dramatically if a small amount of noise is introduced in the supervisory process of the LOLR. We show that if the LOLR only imperfectly observes the bank's fundamentals then the moral hazard problem sets in as the bank realises that it might be bailed out even when it is insolvent. It is important to study the imperfect information scenario as it is realistic given the difficulty often faced by policymakers in distinguishing between solvency and liquidity problems.

There are three main objectives of our model. First, we intend to show clearly how shocks are transmitted within sectors via the banking system. This can be done by endogenising the entrepreneurial sector in a bank runs model. Most of the existing literature on banking crises takes the asset side of the bank activities as given and assumes that the bank's returns are determined by an exogenously given production function. However, a general equilibrium setting gives a clear picture of where the bank's return comes from and it is then possible to see clearly how a liquidity shock is transmitted from the entrepreneurial sector to the depositors via the banking system, and conversely how the depositors' equilibrium behaviour affects the behaviour of the entrepreneurs. More importantly, such a setting enables us to characterise the optimal contract between the banks and the entrepreneurs, and thus the lending rate is determined endogenously in the model. To the best of our knowledge, our model is the first one which analyses if and how the presence of the LOLR has any affect on the lending rate and hence on entrepreneurial investment.

Second, an important objective of the model is to study how the presence of the LOLR affects the solvency of the banking system. Many economists have argued that the presence of the LOLR is conducive to moral hazard.[5] However, these arguments have tended to be informal and have thus failed to show under what circumstances the presence of the LOLR will have an adverse affect on the solvency of the banking system. Because of this, precise policy recommendations have been difficult to justify. We clearly show *when* and *how* the presence of the LOLR will cause a moral hazard problem and how this can be mitigated.

Lastly, but not least important, we analyse implications for the transparency of the banking system. Since rational agents base their decisions on all available information, it is crucial to study how more or less transparency of the banking system has an affect on the evolution of crises. We show that the difference between common knowledge and almost common knowledge is non trivial. We thus carry out a comprehensive study of how a banking crisis occurs in the presence of both perfect and imperfect information, with and without the presence of the LOLR.

The rest of the paper is organised as follows. Section 2 introduces the basic setup and the main players in the model. Section 3 incorporates a macroeconomic shock in the basic setup. Section 4 considers the second best contract which will prevail in the absence of any bank runs. Section 5 then introduces bank runs and derives solvency and failure thresholds of banks. Section 6 studies the perfect information benchmark of the model. Section 7 introduces asymmetric information between banks and the depositors

[5]See, for example, Calomiris (1998) and Krugman (1998).

and studies the equilibrium behaviour of the depositors in this imperfect information setting. Section 8 analyses how the presence of an imperfectly informed LOLR affects the economy. Section 9 provides a discussion of the model and finally section 10 gives a summary of the main results.

2. The Basic Setup and the Players

Consider an economy with three periods, $t = 0, 1, 2$. There exists a single divisible consumption good in each period. There are three types of agents in the economy: *depositors*, financial intermediaries or *banks*, and *entrepreneurs*. Later on we will introduce a fourth agent, the central bank or the *LOLR*. The model can also be applied to an international setting, where the depositors can be interpreted as international investors and the central bank can be thought of as the international lender of last resort, like the IMF.

All lenders (depositors and banks) have access to a risk free storage technology such that one unit of a good at $t = 0$ becomes $(1 + r)$ units at $t = 2$. Thus $(1 + r)$ can be thought of as the opportunity cost of funds between $t = 0$ and $t = 2$. Further, credit markets are competitive, i.e. there are more agents who wish to invest in the risky assets than there are investment opportunities available. Thus the number of depositors or investors are large relative to the available entrepreneurial projects. Finally, all agents are risk neutral. We now give a brief description of the three agents in our economy.

2.1. Entrepreneurs. The economy is populated with a total of T entrepreneurs, each of which has access to a perfectly divisible risky technology. The entrepreneurs have zero wealth and therefore require funding for their projects. The risky technology converts 1 unit of the consumption good at $t = 0$ to X units at $t = 2$ with probability p and 0 with probability $1-p$.[6] The entrepreneurs are heterogeneous and thus the probability of success varies for each entrepreneur depending on his or her skills. The probability of success p_j of entrepreneur j can be thought of as a measure of the skill of entrepreneur j or alternatively as a measure of the quality of the project which the entrepreneur has access to. The probability of success p is a random variable which is independent across entrepreneurs. Let $f(p)$ be the density function of p on $[0, 1]$ and $F(p)$ the corresponding cumulative distribution. The entrepreneur's skill, p_j, is private information and is observed neither by the intermediary nor the depositors. Further, p is realised at $t = 1$ and hence in the interim period the entrepreneurs know whether their projects have succeeded or failed.

The reservation utility of the entrepreneurs is b units of the consumption good. b can be interpreted as the wage income of the entrepreneurs if they decide not to take up the risky projects. Hence b represents the value of the entrepreneurs' outside option.

2.2. Depositors. There are D investors or depositors each endowed with 1 unit of the consumption good for investment purposes. As in Diamond and Dybvig (1983) there are two types of depositors: patient depositors and impatient depositors. The patient agents prefer to consume at $t = 2$, while impatient agents can only consume at $t = 1$. A proportion θ of the depositors are impatient while a proportion $1 - \theta$ are patient. At $t = 0$, the depositors do not know whether they are of the impatient or patient types. This information is revealed to the depositors at $t = 1$. The depositors' type is *iid* and is their private information. Thus a patient depositor can claim to be of the impatient type at $t = 1$.

[6] In section 3 we will introduce an exogenous shock which will adversely affect the success probability of projects.

2.3. Banks. The banks just act as intermediaries between the depositors and the entrepreneurs and they channel funds from the investors to the entrepreneurial sector. The banks exist in this model primarily because of two reasons. First, the banks can perfectly and costlessly observe whether the entrepreneurs' projects succeeded or not. There is therefore no moral hazard problem between the entrepreneurs and the banks. Second, as in Diamond (1984), banks can rely on the strong law of large numbers (SLLN) and hence they can diversify out any idiosyncratic risk. This allows us to focus on systemic risk.[7]

The banking sector is perfectly competitive. Since banks make zero profits, they offer the same contract to entrepreneurs as the one that would be offered by a single bank maximising the welfare of the agents in the economy. It would thus be simple to think of the homogenous group of banks as one single bank.

The bank offers deposit contracts the structure of which is as follows: for every unit of endowment deposited at $t = 0$, the face value of the deposit at $t = 2$ is $(1 + h) > 1$. Given risk neutrality the face value of the deposit at $t = 2$ is determined in a manner such that depositors on average get the opportunity cost of funds $(1 + r)$. Thus a bank failure occurs whenever the return of the bank is less than the opportunity cost of funds. Further the depositor can demand early withdrawal at $t = 1$, in which case the bank promises to pay $(1 + r)$ units, which is the reservation utility of the depositors.[8]

The bank can allocate its endowments to three possible alternatives. It can invest its endowments in the risky projects of the entrepreneurs; it can invest the funds in the riskless storage technology; and finally it can retain a fraction of its endowments as reserves to meet the demand of the impatient investors. 'Reserves' can be interpreted as a short term storage technology such that one unit retained as reserves at $t = 0$ gives one unit at $t = 1$. Thus, reserves have a zero net rate of return. Henceforth, the short term storage technology will be referred to as reserves and the long term storage technology will just be referred to as the storage technology.

Given competitive credit markets, we assume that the bank always has enough funds to finance all of the entrepreneurs willing to initiate their risky projects. Clearly, holding reserves is costly, and the bank would like to hold as low a level of reserves as possible. This is because, the bank faces a positive opportunity cost to holding reserves as any consumption good not retained as reserves can be stored in the risk free storage technology. Thus the opportunity cost of retaining one unit of the endowment in reserves is $(1 + r)$, which is the return from the riskless storage technology. Later on, we will see that ultimately this opportunity cost will be borne by the entrepreneurs, and thus in competitive credit markets, it will not be in the interest of any bank to hold more reserves than is necessary. The reason why banks hold any reserves at all is that liquidating the entrepreneurial projects/storage technology in the interim period is more costly relative to holding reserves. This will be discussed in more detail in section 5.

Let I represent the bank's investment portfolio comprising of investment in the risky projects and the risk free storage technology. Then

$$I = I_p + I_s \tag{1}$$

where I_p and I_s denote the total investments in the entrepreneurial projects and storage technology respectively. Let $\omega_p = \frac{I_p}{I}$ denote the fraction of investment funds invested in

[7] Note that in the absence of any systemic risk there will be no uncertaintly in the model as long as the SLLN holds. Uncertainty will be introduced in section 3 of the paper.

[8] See footnote 15

the entrepreneurial projects.

3. Macroeconomic Shock

We now introduce uncertainty in the basic setup of our model. In the interim period, $t = 1$, a macroeconomic shock, $\tilde{\phi}$, hits the entrepreneurial sector and subsequently the return of the bank.

We model the macroeconomic shock as a multiplicative shock that affects the proportion of projects that succeed. In the absence of any systemic risk the proportion of successful projects will be $\bar{p}$ if the SLLN holds, where $\bar{p}$ is the average skill level of the *active* entrepreneurs.[9]. However when the macroeconomic shock hits the economy, the proportion of successful projects are scaled by $\tilde{\phi}$, where $\tilde{\phi} \sim U\left[\underline{\phi}, 1\right]$. Thus in the presence of systemic risk the proportion of successful projects is $\bar{p} \cdot \tilde{\phi}$. This will subsequently affect the bank's return since the return of the bank is a function of the proportion of successful projects.[10] Note that since the shock is systemic in nature, it cannot be insured by the interbank market.

Further, this will also affect the ex ante success probability of entrepreneurs. This is because the macroeconomic shock hits all the projects that have not yet failed at $t = 1$. Assuming that the shock can randomly hit any entrepreneur whose project has not yet failed at $t = 1$, the ex ante success probability of a project is given by $\pi_j = p_j \cdot \hat{\phi}$, where $\hat{\phi} \equiv E(\phi) = \frac{1+\underline{\phi}}{2}$. Thus in order for a project to succeed, the project must be 'good' *and* it must survive the random shock $\tilde{\phi}$, the expected value of which is given by $\hat{\phi}$. This formulation implies that the success probability depends not only on entrepreneurial skill but also on a common risk factor.[11] Thus in the presence of an exogenous shock the probability of success across projects is no longer *iid* but is affected by a common factor ϕ.

The information structure of the model is such that the banks perfectly observe the realisation of the shock. The investors, however, may perfectly or imperfectly observe the realisation of $\tilde{\phi}$. (We analyse these two cases in later sections.) The ex ante distribution of the shock is public information and is thus known by all the agents. The shock, ϕ , can also be interpreted as a measure of the fundamentals of the banks. Thus, banks are aware of their fundamentals but the investors or the regulator may or may not have perfect information regarding the bank fundamentals. As we will see this will have interesting implications for the behaviour of the depositors and the entrepreneurs in the economy.

4. The Optimal Contract with no Bank Runs

We now derive the optimal contract in the presence of uncertainty with no bank runs taking the interim deposit contract as given. The 'no runs' optimal contract will provide a benchmark against which we can later measure the welfare effects of introducing a LOLR in the presence of bank runs.

To specify the optimal contract between the entrepreneurs and a competitive intermediary we first examine the entrepreneurs' decisions more closely. As a first step note that

[9] See footnote 12.

[10] We will derive an explicit expression of the bank's return in the following section.

[11] To avoid confusion we will henceforth refer to p_j as the skill level of entrepreneur j as distinct from π_j which is the actual success probability.

not all entrepreneurs will be willing to undertake their projects. We know that in the presence of systemic risk, entrepreneur j's project will succeed with probability π_j and fail with probability $1-\pi_j$, where $\pi_j = p_j \cdot \overset{\wedge}{\phi}$. Thus, entrepreneur j will invest if and only if the expected benefit of doing so is greater than the opportunity cost as measured by the value of the outside option, b. Therefore only the entrepreneurs who are good enough in terms of having a high likelihood of succeeding will undertake their risky projects. The measure of entrepreneurial skill is p_j and thus if p_j is high enough, entrepreneur j will take up his project. Let p^* be the reservation skill level, i.e. entrepreneurs with $p_j \geq p^*$ will undertake their projects whilst the others will consume their outside option. Then $F(p^*)$ is the fraction of projects that are rejected and $1-F(p^*)$ is the fraction that are accepted. Then,

$$\bar{p} = E(p|p \geq p^*) = \frac{\int_{p^*}^{1} p dF(p)}{1-F(p^*)}.$$

To give more structure to the problem, suppose that p is uniformly distributed on $[0,1]$. Assuming that entrepreneurs are evenly spaced on $[0,1]$, the total number of active entrepreneurs who accept their projects is $N = T(1-p^*)$, while the rest of the entrepreneurs, $T-N$, consume their outside option. Then given that $p \sim U[0,1]$ the average skill level of the active entrepreneurs is $\bar{p} = \frac{1+p^*}{2}$. Assuming for now that there are no exogenous shocks on the economy, then the actual proportion of the projects which succeed is $\bar{p}$, given the strong law of large numbers.[12]

We can now deduce the bank's rate of return at $t=2$. Let R denote the bank's rate of return. Suppose that $(1+\rho)$ is the lending rate charged by the banks to the entrepreneurs. Then the bank's rate of return from its investment portfolio is a weighted average of the return from the entrepreneurial projects and the return from the storage technology. Note that the rate of return from the entrepreneurial projects is $(1+\rho) \times$ (proportion of projects succeeded). Hence, in the absence of an exogenous shock, the bank's return per unit of investment is given by

$$R = \omega_p \left[(1+\rho)\bar{p}\right] + (1-\omega_p)(1+r).$$

If at $t=1$, a macroeconomic shock, $\tilde{\phi}$, hits the returns of the banks, then the bank's rate of return on its investment portfolio, I, at $t=2$ is given by

$$\tilde{R} = \omega_p \left[(1+\rho)\bar{p}\tilde{\phi}\right] + (1-\omega_p)(1+r). \qquad (2)$$

Having derived an expression for the bank's rate of return we can now characterise the optimal contract. Suppose only the truly impatient depositors withdraw their funds

[12] More formally, if there are no exogenous shocks on the economy, then the actual successes or failures of the projects are independent and identically distributed so that they form a sequence of Bernoulli trials with success parameter $\bar{p}$. Then the number of successful projects, $\acute{s}$, will have a binomial distribution with mean $N\bar{p}$ and variance $N\bar{p}\left(1-\bar{p}\right)$. Thus $\acute{s} \sim B\left(N\bar{p}, N\bar{p}\left(1-\bar{p}\right)\right)$. The distribution of the proportion of successful projects follows from this and hence if λ is the proportion of projects which succeed, then $\lambda \sim B\left(\frac{1+p^*}{2}, \frac{1-p^{*2}}{4N}\right)$. Then note that as $N \to \infty$, $\lambda \to \bar{p}$. Therefore, in the limit if the SLLN holds, then the actual proportion of projects that succeed will be exactly $\bar{p}$, abstracting from any exogenous shocks on the economy.

in the interim period so that there are no bank runs in the economy. Let re denote the level of the bank's reserves which is the amount of funds invested in the riskfree storage technology and let $\hat{\pi} = \bar{p}\hat{\phi}$ be the average success probability. Then the optimal contract with no bank runs solves the following problem:[13]

$$\max_{I,re,1+\rho,1+h} (1 - F(p^*)) \left[\hat{\pi} (X - (1 + \rho))\right] + F(p^*)b \tag{3}$$

subject to

$$\theta (1 + r) + (1 - \theta) E_{\tilde{R}} [1 + h] \geq 1 + r \tag{4}$$

$$re \geq \theta D (1 + r) \tag{5}$$

$$I \left\{ \omega_p (1 + \rho) \bar{p}\hat{\phi} + (1 - \omega_p) (1 + r) \right\} \geq (1 - \theta) DE_{\tilde{R}} [1 + h] \tag{6}$$

$$I + re \equiv D \tag{7}$$

$$p^* = \arg\max (1 - F(p^*)) [\pi (X - (1 + \rho))] + F(p^*)b. \tag{8}$$

Expression (3) is the entrepreneurs' expected utility which is a weighted average of the expected net profit from a typical project and the value of the outside option. Since credit markets are competitive, the optimal contract maximises the expected utility of the entrepreneurs subject to constraints (4), (5), (6), (7) and (8).

Constraint (4) is the investor rationality constraint of the depositors. At time $t = 0$, the depositors do not know whether they are patient or impatient, and thus they will invest if and only if their expected ex ante return is at least equal to the opportunity cost of their funds. Since the depositors get $(1 + r)$ if they turn out to be impatient, thus the constraint can also be rewritten as

$$E_{\tilde{R}} [1 + h] \geq 1 + r. \tag{9}$$

Thus the investor rationality constraint requires that the face value of the deposit constraint at $t = 2$ be such that the depositors on average at least receive the opportunity cost of their endowments.

Constraint (5) is the reserve constraint of the bank, which is the requirement that the level of the bank's reserves be at least sufficient to satisfy the impatient depositors who withdraw their funds in the interim period $t = 1$. Constraint (6) is the requirement that the total expected return of the bank on its investment portfolio, I, be at least equal to the total reservation utility of the patient depositors. It can be thus be thought of as the bank's investor rationality constraint.

Constraint (7) is actually a balance sheet identity which states that the total assets of the bank be equal to its total liabilities. Thus the investment in the risky portfolio, I, plus the reserves, re, cannot exceed the total amount of deposits in the bank. Hence constraint (7) can be thought of as the budget constraint of the bank.

[13] The optimal contract here bears some resemblance to the optimal financial contract studied by Bernanke and Gertler (1990) when borrower type is unobservable. However, Bernanke and Gertler consider a two period model and hence they do not model reserves.

Finally constraint (8) is the incentive compatibility condition of the entrepreneurs which states that the reservation skill level, p^*, chosen by the entrepreneur is such that it maximises his expected utility subject to the terms of the financial contract. Taking the derivative of the objective function (3) with respect to p^* yields the following first order condition:

$$\left(p^*\overset{\wedge}{\phi}\right)[X-(1+\rho)]=b. \tag{10}$$

According to equation (10), p^* is the threshold skill level such that entrepreneurs are just indifferent between proceeding with their risky project and consuming their outside option. Hence entrepreneurs with a skill level $p_j \geq p^*$ proceed with their risky projects as the expected return net of the payment to the intermediary exceeds their opportunity cost.

We assume that $X-(1+\rho)-b>0$ so that the projects are feasible in the sense that if the project is successful than the return exceeds the interest payment to the bank and the reservation utility of the entrepreneurs. This also ensures that the second order condition $X>(1+\rho)$ is satisfied.

Thus, the optimal contract maximises the expected profits of the entrepreneurs subject to the investor rationality constraint of the depositors, the investor rationality constraint of the bank, the reserve constraint, the budget constraint of the bank and the incentive compatibility condition of the entrepreneurs.[14]

Proposition 1. *The optimal contract in an economy with no bank runs is characterised by the following:*

$$1+\rho=\left(\frac{1+r}{\bar{p}\overset{\wedge}{\phi}}\right)\left(1+\frac{\theta Dr}{N}\right) \tag{11}$$

$$1+h=\frac{1+r}{\overset{\wedge}{\phi}} \tag{12}$$

$$re=\theta D(1+r) \tag{13}$$

$$I=D[1-\theta(1+r)] \tag{14}$$

$$p^*=\frac{b}{\overset{\wedge}{\phi}[X-(1+\rho)]}. \tag{15}$$

Proof. Since $X>(1+\rho)+b$, we have a problem with five constraints (4), (5), (6), (7), (8) and a concave objective function (3). To solve the problem we should first establish that all the constraints must bind. Note first that the objective is monotonically decreasing

[14] Krasa and Villamil (1992) also consider competitive credit markets and the optimal contract in their setup maximises the entrepreneurs' profits subject to investor rationality constraints. However, they do not model the entrepreneurs and hence they do not have an incentive compatibility condition. Further, they have a two period model. However in a three period model, we also need a reserve constraint as investors can withdraw in the interim period. This reserve constraint is crucial and we will see later, how the possibility of bank runs affects the reserve constraint and hence the optimal contract.

in $(1+\rho)$. Thus constraint (6) must be binding; otherwise given a competitive banking sector, a bank could lower the lending rate and attract all the entrepreneurs. Thus in a competitive market, constraint (6) will be binding and will actually be the zero profit condition of the bank. Since constraint (6) is binding, this implies that constraint (4) which is equivalent to (9) must also be binding. Thus we can rewrite constraint (6) as the following zero profit condition

$$I\left\{\omega_p\left(1+\rho\right)\bar{p}\hat{\phi}+\left(1-\omega_p\right)\left(1+r\right)\right\}=\left(1-\theta\right)D\left(1+r\right). \tag{16}$$

Next note that the zero profit condition implies that the lending rate $(1+\rho)$ is decreasing monotonically in I. Given the budget constraint (7), $(1+\rho)$ is therefore monotonically increasing in re. Hence, constraint (5) must also bind. Having established that all the constraints must be binding, we have four linear equations (4), (5), (6), (7) in four unknowns $1+h$, re, $1+\rho$, I. Solving these gives us the solution in (11), (12), (13), and (14). As for p^*, we have already shown that constraint (8) is equivalent to equation (10). This gives us the threshold p^* in (15). ■

The interpretation of the optimal contract is simple. The level of reserves held by the bank will be as low as possible since reserves have a positive opportunity cost. Thus, reserves will be just sufficient to payoff the impatient depositors in the interim period. The total demand of the impatient depositors in the interim period is $\theta D\left(1+r\right)$ and hence this will be the level of the bank's reserves. The rest of the bank's resources $(D-re)$ will then be devoted to investment purposes.

Next note, that for every depositor who withdraws in the interim period, the bank has to hold $(1+r)$ units as reserves instead of one unit, given that the reserves have a zero net rate of return. This extra cost of r units per depositor who withdraws in the interim period, ultimately has to be recovered from the entrepreneurial projects. The total cost is θDr units since the total number of impatient depositors is θD. This cost will be recovered from the entrepreneurial sector and will be divided over the total number of active entrepreneurs, N. This explains why the lending rate charged by the bank to each entrepreneur, as given in equation (11), is equal to the risk free rate scaled by average project risk and then adjusted to retrieve the extra cost incurred by the bank to service withdrawals in the interim period. This lending rate ensures that all the investors on average receive their reservation utility.

The face value of the deposit contract, $1+h$, is simply equal to the risk free rate scaled by the average risk in the economy.

Finally, the threshold skill level, p^*, as expected is increasing in the value of the outside option but decreasing in the expected return from the project. The threshold p^* determines the amount of investment in the entrepreneurial projects since $I_p = T\left(1-p^*\right)$. Naturally, the higher the skill threshold, the lower would be the investment level and vice versa. Also observe that the reserve level has an affect on p^* through the lending rate. An increase in the level of reserves, increases p^* and thus reduces the amount of investment in the economy.

The optimal contract just derived is the second best contract since entrepreneurial type is unobvservable. The intermediary therefore faces a problem of adverse selection whereby it cannot distinguish the different probabilities of success. Thus we have a pooling equilibrium where the intermediary offers the same contract to all the entrepreneurs. In a world, with observable types, however, the first best contract will prevail, and in such a world, the intermediary would offer different contracts to each entrepreneur. Thus, in the first best world, we would have a separating equilibrium and the bank would charge each entrepreneur an interest rate commensurate with his probability of success. As suggested by Bernanke and Gertler (1990), when an intermediary faces a problem of adverse selection, there is naturally a cross subsidisation from the good types to the bad types. This cross subsidisation also exists in our setup since the lending rate set by the bank is based on *average* project risk and thus below average entrepreneurs gain at the expense of above average entrepreneurs.

5. Bank Runs

We now introduce bank runs in our model, so that patient depositors may also withdraw their deposits in the interim period. Let n be the actual proportion of depositors who withdraw their money in the intermediate period, $t = 1$. We may now have a situation where the bank's reserves are insufficient to meet the demands of the depositors. If in the intermediate period the bank is unable to meet the demands of its depositors then the bank will have to inefficiently liquidate some of its investments. The bank will cover the shortfall between the demands of the investors and the reserves by inefficiently liquidating a fraction ξ of its investment portfolio I.

It would be simple to think of the bank's investment portfolio as a mutual fund (comprising of both investment in the risky project and investment in the riskless asset). Thus, when the bank's reserves are insufficient, the bank sells a portion of its fund in the market. For the sake of tractability and simplicity we therefore do not either assume that the bank liquidates its risky investments first or its riskless investments. The bank just liquidates a certain proportion of its mutual fund to meet the demands of its depositors. This is without loss of generality and simplifies our exposition.

Let R_l denote the rate of return per unit of liquidation of the bank's portfolio. The bank does not obtain the full value of the assets from the sale and the difference between the actual portfolio value and the liquidation value can be interpreted as the cost of premature liquidation.[15] We impose the following restrictions on R_l:

$$0 < R_l < 1.$$

$R_l > 0$ since the average value of the bank's portfolio is greater than zero. Thus the sale of a fraction of the bank's portfolio should fetch a positive value. However $R_l < 1$ as otherwise it would never be in the interest of any bank to hold positive reserves. The restriction is only sensible and implies that liquidation is costly enough to induce banks to hold some reserves. Hence premature liquidation is costly and inefficient.

5.1. The optimal contract with bank runs. In the presence of bank runs, the optimal contract derived in section (4) will no longer hold as the reserve constraint (5)

[15] One explanation of why premature liquidation is costly is that the secondary market is charecterised by a problem of adverse selection as in Flannery (1996) and Rochet and Vives (2002). The investments of the bank consist of a continuum of assets and agents in the secondary market are infinitesimal and cannot observe the type of asset being sold to them. Each agent fears that he might end up with the worst quality of asset and hence because of the problem of adverse selection, the bank's assets are sold at a discount to the their face value.

Note that if the depositors had invested their endowments in the riskless storage technology instead of the bank, then all impatient depositors would have liquidated their investments at the face value of $1+r$. Thus the problem of asymmetric information does not arise when the impatient investor in isolation liquidates the storage technology as he holds only one type of asset. This also justifies why the bank's interim deposit contract specifies a payment of $1 + r$ if funds are withdrawn at $t = 1$.

will be invalid if the patient depositors can also withdraw their funds in the interim period. To solve for the optimal contract with bank runs, we will first specify a generic reserve level. We will then solve for the optimal contract implied by the generic reserve level. Later on, we will specify what the optimal reserve level will be in the presence of bank runs.

Suppose that the optimal reserve level of a bank is given by

$$re = \hat{n} D (1 + r) \tag{17}$$

where $\hat{n} \geq \theta$, is the bank's ex ante estimate of the proportion of depositors who will come to the bank at $t = 1$. Given this generic reserve level, it is then straightforward to show that the optimal contract is characterised by the following lending rate and investment portfolio:[16]

$$1 + \rho = \left(\frac{1 + r}{\bar{p} \hat{\phi}} \right) \left(1 + \frac{\hat{n} D r}{N} \right) \tag{18}$$

$$I = D \left[1 - \hat{n} (1 + r) \right] \tag{19}$$

Denote the second best contract specified in section (4) by the subscript SB. Then observe that $1 + \rho \geq (1 + \rho)_{SB}$ and $I \geq I_{SB}$. Thus in the presence of bank runs, the size of the bank's investment portfolio is smaller but more importantly, the lending rate is higher. Thus fewer entrepreneurs, relative to the second best, will now participate and thus there will be underinvestment in the economy.

Note that the optimal contract is completely defined by $\hat{n}$ and that every level of $\hat{n}$ corresponds to a different optimal contract. We can restate this in the following remark.

Remark 1. *There exists a one to one mapping from the depositors' equilibrium strategy to the optimal contract offered by the banks to the entrepreneurs.*

What remains therefore is the specification of $\hat{n}$. We leave this to later when we have characterised the depositors' equilibrium strategy.

5.2. Solvency and failure thresholds of banks. Having introduced bank runs, we can now determine the solvency and failure thresholds of banks. To do so, however, we need to adopt a definition of bank failure. We define bank failure as follows: A bank failure occurs when the bank is unable to pay the opportunity cost of funds to the investors. Hence a bank will fail when its depositors do not receive their reservation utility. Note that according to this definition a bank will not be classified as a failure if it covers the opportunity cost of funds but is unable to pay the face value of deposits, $1 + h$, at $t = 2$. Since depositors are risk neutral, they will be satisfied as long as they receive at least their reservation utility. This is why ex ante $1 + h$ is chosen such that depositors on average receive $1 + r$. Thus a bank will not fail as long as it pays at least $1 + r$ to its depositors.[17]

[16] The proof is exactly on the same lines as that to Proposition 1.

[17] However, if the bank were required to pay on average more than $1 + r$ to its depositors for different reasons as risk aversion, bank credibility, etc., then ex ante the lending rate would be such that on average the bank did pay this higher amount to the depositors. However, we keep our setup as simple as possible and the depositors in the model will be satisifed as long as ex ante they receieve on average the opportunity cost of their endowments.

We assume that there exists a $\phi_U < 1$ such that if $\phi \in [\phi_U, 1]$, then the bank never fails. We refer to the range $[\phi_U, 1]$ as the *upper dominance region*. In other words, we are assuming that if the bank fundamentals are high enough, such that systemic risk is very limited, then the bank never fails. Dasgupta (2002) and Goldstein and Pauzner (2002) also make a similar assumption. They assume that if the fundamentals driving the bank's return are very strong, then the bank does not fail. Nevertheless this assumption is more plausible in our general equilibrium setting. This is because we have shown in section 3 that what we refer to as 'bank fundamentals' is essentially a measure of systemic risk. In the absence of any systemic risk ($\phi = 1$), the banks never fail. Thus if systemic risk is very limited (such that ϕ is arbitrarily close to 1) then the banking system will not collapse.[18] Hence, it is only plausible to assume that there exists an upper dominance region, such that if $\widetilde{\phi}$ lies in that range then the banks will not fail.

We now consider three different cases that banks may encounter. This will enable us to specify the solvency, failure and bankruptcy thresholds of banks. Let re_2 denote the level of reserves, if any, the bank has at $t = 2$.

Case 1. *If $n = \theta$, so that only impatient depositors withdraw at $t = 1$, then no liquidation will occur at $t = 1$, and the bank will fail at $t = 2$ if and only if*

$$\widetilde{R}I + re_2 < (1 - \theta) D (1 + r)$$

where $re_2 = \left(\hat{n} - \theta\right) D (1 + r)$. Substituting the values of re_2, $\widetilde{R}$ and I the above condition implies that the bank will fail at $t = 2$ if and only if

$$\widetilde{\phi} < \phi_L$$

where $\phi_L = \overset{\wedge}{\phi}$. Thus the bank will be insolvent if it fails even if only the impatient depositors withdraw in the interim period. The insolvency point is therefore ϕ_L and the bank's solvency condition is: $\phi \geq \phi_L$. We refer to the range $\left[\underline{\phi}, \phi_L\right)$ as the lower dominance region, since the bank always fails in this region irrespective of the proportion of depositors who run.

Case 2. *If $n > \hat{n}$ then the bank will need to liquidate some of its investments. The shortage in the bank's reserves will be $\left(n - \hat{n}\right) D (1 + r)$. Inefficient liquidation would fetch the bank $R_l (\xi I)$. The maximum amount that the bank can receive from liquidation is $R_l I$. Hence if $re < nD (1 + r) \leq re + R_l I$ than there will be partial liquidation at $t = 1$. Failure will now occur at $t = 2$ if and only if*

$$\widetilde{R} (1 - \xi) I < (1 - n) D (1 + r)$$

where $\xi = \frac{\left(n - \hat{n}\right) D(1+r)}{R_l I}$. Substituting the values of $\widetilde{R}$ and I the above condition implies that the bank will fail at $t = 2$ if and only if

$$\widetilde{\phi} < \phi_f$$

[18] I would like to thank Jean-Charles Rochet for this observation.

where $\phi_f = \frac{\hat{\phi}\left\{(1-n)D-(1-\xi)D\left[1-\hat{n}(1+r)\right]-N\right\}}{(1-\xi)[\theta Dr+N]}$. *The failure threshold of the bank will now increase relative to case 1 (i.e.* $\phi_f > \phi_L$*) as a larger proportion of the depositors now withdraw in the interim period and because premature liquidation is costly* ($R_l < 1$)*.*

Case 3. *Finally if* $nD(1+r) > re + R_l I$*, then the bank will be closed down in the intermediate period* $t = 1$*. Thus the bank will be bankrupt and closed down at* $t = 1$ *whenever* $n > n_B$*, where*

$$n_B = \frac{\hat{n}(1+r)(1-R_l)+R_l}{1+r}. \tag{20}$$

To summarise the discussion so far, the bank always fails if $\phi < \phi_L$; the bank never fails if $\phi \geq \phi_U$; and in the range $\phi \in [\phi_L, \phi_U)$, failure depends on the proportion of depositors who withdraw their funds in the intermediate period $t = 1$. Thus if fundamentals are in the range $\phi \in [\phi_L, \phi_U)$, then we might have solvent but illiquid banks.

5.3. Investor payoffs. We next turn to the determination of the investor payoffs. Assume that the bank's returns are equally divided among investors. Then the payoffs to the depositors are summarised in the following matrix.

	Bank does not fail	Bank fails
Run	$1+r$	$\max\left(1+r, \frac{re+R_l I}{nD}\right)$
Wait	$\frac{R(1-\xi)I}{(1-n)D} \geq 1+r$	$\frac{R(1-\xi)I}{(1-n)D} < 1+r$

(21)

Note that if the bank fails, then we will have one of two scenarios. Either the bank will fail in the final period $t = 2$, or it will go bankrupt and will be closed in the interim period $t = 1$. Thus if depositors run and the bank fails then they will get $1+r$ if the bank is not bankrupt. But if the bank is bankrupt then the proceeds from bankruptcy will be equally divided among the investors. If the bank is bankrupt, and the depositors wait till $t = 2$, then they will get zero since all the bank's investment will be liquidated in the interim period ($\xi = 1$).

It is clear from (21) that the depositors are better off by running if the bank fails and by waiting if the bank does not fail.

6. PERFECT INFORMATION BENCHMARK

Having completed the construction of the basic model in the presence of bank runs we now turn to characterising the depositors' equilibrium strategy. In this section we will assume that depositors have perfect information regarding the fundamentals of the bank, i.e. the depositors can perfectly observe the realised value of $\tilde{\phi}$ at $t = 1$. Thus the realisation of $\tilde{\phi}$ is common knowledge. In the next section we will relax this assumption and consider the more realistic case where depositors only imperfectly observe the realisation of $\tilde{\phi}$. The perfect information case will serve as a useful benchmark before we introduce imperfect information.

It is clear from (21) that if the realised value of $\tilde{\phi}$ was common knowledge at $t = 1$, then all investors will run if $\phi < \phi_L$ as in this region the bank always fails. Conversely, if $\phi \geq \phi_U$, then the dominant strategy of the patient investors will be not to run irrespective of the decision of the other depositors. However in the intermediate region $\phi_L < \phi \leq \phi_U$, there exist a multiplicity of equilibria a la Diamond and Dybvig (1983) and the payoff to each investor will now depend on the proportion of patient depositors who withdraw. If a patient depositor expects all other patient depositors not to withdraw then he will also not withdraw his funds and hence an equilibrium will exist where all patient depositors do not withdraw. On the other hand, if a patient depositor expects all the other depositors to run, then an equilibrium will exist where everyone runs.

The Pareto efficient equilibrium is for all (patient) investors to refrain from withdrawing as long as the bank is solvent. Thus as long as $\phi > \phi_L$, the (patient) depositors should not run. However, because of coordination failure the dispersed depositors might not be able to achieve the Pareto efficient outcome.

6.1. Problems with the perfect information benchmark. There are essentially three problems with the perfect information benchmark. Firstly, it is more realistic to assume a small amount of noise in the signals of the dispersed depositors. Secondly, we showed that in the region $\phi \in [\phi_L, \phi_U)$ there exist multiple equilibria. In this region there are two equilibria in pure strategies, where either everyone runs or none of the patient depositors run, and one equilibrium in mixed strategy. Hence we have three possible equilibria. The problem with having such multiple equilibria is that there will not exist a unique optimal reserve level (since there will exist three possible values for $\hat{n}$). Ex ante the bank will anticipate three possible equilibria each associated with a different optimal reserve level. Hence we will be unable to identify a unique optimal contract if we assume common knowledge on the part of the investors.[19] The third problem is just an implication of the second one. If we assume that all agents have perfect information, then policy analysis becomes very difficult because of the presence of multiple equilibria. A policymaker will be unable to attach probabilities to the different outcomes, which is precisely the reason why it becomes impossible to identify a unique optimal contract.

6.2. LOLR and perfect information. We have seen that in the region $\phi \in [\phi_L, \phi_U)$ the Pareto efficient equilibrium might not be attained because of coordination problems and the economy might end up with the bad equilibrium where everyone runs even though the banks are solvent. Thus, there exists a potential role for the lender of last resort. Suppose there exists a LOLR, that announces that as long as the bank is solvent (i.e. as long as $\phi \geq \phi_L$), it will bail out the bank if anyone runs. We assume that the LOLR has access to funds from international markets at the safe world rate of interest.[20][21] We also assume for now that like the depositors, the LOLR is also perfectly informed about the bank fundamentals and thus the bank is perfectly transparent.

If the LOLR is prepared to bail out all solvent institutions then we will no longer have inefficient liquidation as the LOLR will simply pay off any depositor who runs when

[19] We will show later how with imperfect information we can get rid of this problem using global games methodology.

[20] We assume for simplicity that at $t = 1$, the LOLR can borrow from the international markets at a zero rate of interest. Thus if the LOLR lends 1 unit of the consumption good to the bank in the interim period then it will just be paid back 1 unit in the final period, as long as the bank does not fail. Alternatively, we could have assumed that the opportunity cost of funds between $t = 0$ and $t = 1$ is $(1 + r_{0,1})$ and that between $t = 1$ and $t = 2$ is $(1 + r_{1,2})$ such that $1 + r = (1 + r_{0,1})(1 + r_{1,2})$. In such a case, the LOLR would have to be paid $(1 + r_{1,2})$ for every unit lent in the interim period. However for reasons of simplicity we work with the former assumption.

[21] This is especially true for an institution like the IMF which has access to funds at a positive but relatively low rate of interest. Banks may not have access to international markets especially in times of crises (unless a big lender is willing to bail them out as as such a bail out sends a positive signal as regards to the solvency of the banks). Even if banks do have access to international markets in times of crises, they are likely to face a high rate of interest and thus it makes sense for the LOLR rather than the banks to access world markets.

the bank is solvent. Thus *if* information is perfectly transparent and *if* the LOLR's announcement is *credible*, then the dominant strategy of the patient depositors is not to run as long as the bank is solvent. This is because in the presence of a perfectly informed credible LOLR, the investors' payoffs are independent of the proportion of agents who run, as any inefficient liquidation is avoided. We will thus no longer have a situation where the bank is solvent but illiquid.

However, even in the absence of credibility, a situation where banks are solvent but illiquid can be avoided as long as the LOLR is perfectly informed. In the absence of credibility, 'waiting' will no longer be a dominant strategy when the banks are solvent, as the depositors will not be certain if all solvent institutions will be bailed out.[22] Hence we might have panic based runs in the absence of LOLR credibility. Nevertheless as long as the LOLR has perfect information regarding the liquidity shock the cost of premature liquidation will be avoided. This is because the LOLR will simply bail out all solvent institutions. We can thus conclude that in a perfectly transparent economy, the presence of a LOLR will be a Pareto improvement.

Furthermore, we will now also be able to identify a unique optimal reserve level. Since premature liquidation will not take place as long as the bank is solvent, therefore, the reserve level will be just sufficient to accommodate the impatient depositors. Thus the reserve level will be $re = \theta D$ which is the same as the no-runs reserve level. Hence the second best optimal contract characterised in Proposition 1 will now prevail.

7. Imperfect information

We now extend our model to incorporate asymmetric information between the depositors and the banks. Suppose that depositors do not perfectly observe the liquidity shock but get precise albeit imperfect information regarding the fundamentals of the bank. In the interim period investor i receives the realisation of the private signal

$$s_i = \phi + \epsilon_i \tag{22}$$

where the noise term ϵ_i is independent across depositors and is uniformly distributed over the interval $[-\epsilon, \epsilon]$. Since the noise term is *iid*, thus the signals conditional on the fundamentals are also independently and uniformly distributed across the depositors.

Rational agents will use their noisy signals primarily in two ways. First, after observing the signals, the depositors will update their beliefs of the shock ϕ and thus each investor will update the prior distribution of ϕ with the posterior distribution. Second, the signals will allow the agents to infer the beliefs of the other agents. Thus in this environment, given the private signal, a depositor will form beliefs not only about the underlying fundamentals but also about the beliefs of other players and other players' beliefs about other player' beliefs and so on. This is because the investors will realise that their payoffs do not only depend on the economic fundamentals but also on the proportion of people who run.

Agents will now condition their actions on their private signals and will run if the expected conditional payoff from running exceeds the expected conditional payoff from waiting and vice versa. As discussed by Morris and Shin (2000), the *equilibrium* strategy of an investor will be such that it maximises his expected utility conditional on his private

[22] An example where such credibility is absent would be when it is not common knowledge that the LOLR has perfect information regarding the bank's fundamentals. We examine the case of an imperfectly informed LOLR in section 8.

information and the strategies followed by the other agents. We thus need to solve for the Bayesian Nash equilibrium of the imperfect information game.

It is useful to rewrite the payoff matrix in (21) as follows:

	$n \leq n_B$	$n > n_B$
Run	$1+r$	$\frac{re+R_l I}{nD}$
Wait	$\frac{R(1-\xi)I}{(1-n)D}$	0

(23)

where n_B is the bankruptcy threshold defined in (20). It should be noted that we need to solve for the equilibrium of a *global game*, where a global game was first defined by Carlsson and van Damme (1993) as a game of incomplete information where the actual payoff structure is determined by a random draw from a given distribution and where each player receives a noisy signal of the realisation. Carlsson and van Damme (1993) and Morris and Shin (1998) showed that if a binary action global game satisfied full strategic complementarities, i.e. an agent's incentive to take a particular action increases when more and more agents take that action, then there would exist a unique equilibrium such that all agents will take a particular action if their signal is below a threshold signal and vice versa.[23]

However, a general feature of bank run models is that they do no satisfy the property of full strategic complementarities. As is clear from the payoff matrix in (23), once the bank is already bankrupt, the payoff from running *decreases* as the number of agents who are running increases.[24] It is therefore not straightforward to show that a unique equilibrium exists in models of banking crises. Rochet and Vives (2002) get round this problem by assuming that the decision to withdraw in the interim period is delegated to fund managers who face reputation costs. The fund managers prefer not to withdraw early but their reputation suffers if they do not withdraw when the bank fails. With this assumption the payoffs to the fund managers satisfy full strategic complementarities and therefore the standard argument to show the uniqueness result can be used. Nevertheless we follow the technical approach adopted by Dasgupta (2002) and Goldstein and Pauzner (2002) to show that a unique equilibrium will exist even if the payoffs do not satisfy full strategic complementarities.

Proposition 2. ***(Existence)*** *There exists a threshold, s^*, such that patient agents who receive a signal below s^* will run and withdraw their funds at $t = 1$, while patient agents who receive a signal above s^* do not run and wait till $t = 2$.*

Proof. See Appendix. ■

Proposition 2 shows the existence of a threshold signal which defines the actions of the depositors. We next show in the Appendix that if premature liquidation is sufficiently costly, then this threshold signal is unique and thus there is a unique equilibrium in the economy.[25]

Proposition 3. ***(Uniqueness)*** *The threshold signal, s^*, is unique and hence there is a unique equilibrium in the depositors' strategy.*

[23] Carlsson and van Damme (1993) showed this result for a two player binary action game. Morris and Shin (1998) extended their result to the case where there are a continuum of agents. See Morris and Shin (2002) for a comprehensive review of the literature on global games.

[24] See lemma 2 in the Appendix for a formal proof.

[25] As discussed in the Appendix, even if the sufficient condition regarding premature liquidation (stated in the Appendix) is not satisfied, there will still exist a unique equilibrium for plausible values of R_l. Nevertheless, we assume that early liquidation is sufficiently costly so that our result holds globally.

Proof. See Appendix. ■

Having shown the existence of a unique equilibrium in the presence of imperfect information we now provide an interpretation of the cutoff signal s^*. As discussed more formally in the Appendix, the unique threshold signal s^* is such that a patient agent who receives the signal s^* will be indifferent between withdrawing early at $t = 1$ or waiting till $t = 2$. In other words, the unique s^* is implicitly defined in a manner such that it solves the following:

$$\int_{n=\theta}^{n=n_B} \left(\frac{R(\phi)(1-\xi) I}{(1-n) D} - (1+r) \right) dn(\phi, s^*) = \int_{n=n_B}^{1} \left(\frac{re + R_l I}{nD} \right) dn(\phi, s^*) .$$

where $n(\phi, s^*)$ is the proportion of depositors who run for any given ϕ and s^*. This equation says that at the threshold signal s^* the expected payoff from waiting exactly equals the expected payoff from running.

The uniqueness result enables us to find ex ante the expected proportion of depositors who will run. Since the noisy signals conditional on the realisation of the fundamentals are all *iid*, therefore the expectation of the proportion of investors who observe a signal less than s^* is exactly equal to the probability an investor assigns to any one depositor observing a signal less than s^*. Thus $E(n) = \theta + (1-\theta)\Pr(s_i < s^*)$. We show in the Appendix that $n(\phi, s^*)$ is defined by equation (37). Taking the unconditional expectation of (37) we have[26]

$$\hat{n} \equiv E(n) = \frac{1}{1-\underline{\phi}} \left[(1-\theta) s^* - \underline{\phi} + \theta (1-2\epsilon) \right] . \tag{24}$$

Thus the presence of a unique equilibrium allows the bank to identify a unique $\hat{n}$ which is the ex ante expected proportion of depositors who withdraw their funds in the interim period. From Remark 1 established in section 5.1 we know that there exists a one to one mapping from a depositors' equilibrium strategy to an optimal contract. Since we have a unique equilibrium in the presence of imperfect information it follows that there must also exist a unique optimal contract in the economy. Hence we have the following corollary to Proposition 3:

Corollary 1. *There exists a unique optimal contract in the presence of imperfect information.*

The optimal contract in the presence of bank runs and imperfectly informed depositors maximises entrepreneurs' expected profits given by (3), subject to the budget constraint (7), the incentive compatibility condition (8), and the following constraints:

$$\hat{n}(1+r) + \left(1 - \hat{n}\right) E_{\underset{\sim}{R}} [1+h] = 1 + r \tag{25}$$

$$re = \hat{n} D (1+r) \tag{26}$$

[26] Note that $E(n) = 1 \cdot \Pr(\phi \leq s^* - \epsilon) + \left[\theta + (1-\theta)\left(\frac{1}{2} + \frac{s^*-\phi}{2\epsilon}\right)\right] \Pr(s^* - \epsilon < \phi < s^* + \epsilon) + \theta \Pr(\phi \geq s^* + \epsilon)$. Calculating the probabilities and simplifying we get expression (24).

$$I\left\{\omega_p\left(1+\rho\right)\bar{p}\overset{\wedge}{\phi}+\left(1-\omega_p\right)\left(1+r\right)\right\}=\left(1-\hat{n}\right)D\left(1+r\right) \tag{27}$$

where $\hat{n}$ is as defined by (24). Equation (25) is the participation constraint of the depositors which says that in competitive credit markets depositors on average need to receive their reservation utility. Equation (26) is the reserve constraint of the bank. It should be noted that unlike the perfect information case we can now identify a unique optimal reserve level since we are able to pin down a unique $\hat{n}$. Finally equation (27) is the bank's zero profit condition.

Thus the optimal contract maximises entrepreneurial profits subject to the depositors' participation constraint, the reserve constraint, the bank's zero profit condition, the budget constraint of the bank and the entrepreneurs' incentive compatibility condition. The difference between the imperfect information optimal contract and the second best contract is that we need to adjust the constraints to accommodate for the possibility of bank runs.

7.1. LOLR and imperfect information. Now we introduce a LOLR in this imperfect information setting. Suppose for now that the LOLR has perfect information of the banking system but the depositors observe noisy signals of the economic fundamentals. Further suppose that the LOLR announces that as long as the bank is solvent in the sense that in the final period it can pay the opportunity cost of funds to the investors who did not run and the LOLR its loan then it will bail the bank out if it faces any liquidity problems in the interim period. Hence the LOLR does a debt sustainability analysis and bails the bank out if and only if

$$RI+re_2\geq\left(1-n\right)D\left(1+r\right)+\left(n-\theta\right)D\left(1+r\right)$$

or

$$RI+re_2<\left(1-\theta\right)D\left(1+r\right).$$

This is precisely the solvency condition described in case 1. Thus the LOLR bails the bank out only if it is solvent, i.e. $\phi>\phi_L$.

Thus the LOLR here clearly plays a productive role as it avoids the cost of panic runs and bails out all solvent banks who are experiencing temporary liquidity problems. The level of bank reserves will now again me at a minimum, just enough to satisfy the impatient depositors. Hence, we will now be able to reach the second best.

Remark 2. *The presence of a perfectly informed LOLR is a Pareto improvement.*

Rochet and Vives (2002) also showed that Bagehot was right after all and that the LOLR is a Pareto improvement. Nevertheless in the next section we check the robustness of this result when we drop the assumption that the LOLR is perfectly informed.

8. IMPERFECTLY INFORMED LOLR

We now consider the more interesting case where the LOLR does not have perfect information and observes a noisy signal of the bank fundamentals.[27][28] This is realistic and

[27] We will often refer to an imperfectly informed LOLR as a 'noisy' LOLR for the purpose of brevity.

[28] Since the LOLR observes the number of withdrawals, $n\left(\phi,s^{*}\right)$, in order to ensure that the LOLR cannot perfectly infer ϕ, we assume that the distribution of the depositors' signal is not known by the

important given that the banks may not be perfectly transparent to the supervisory authorities. Suppose that the LOLR observes a noisy signal, s_l, of the bank fundamentals where

$$s_l = \phi + \epsilon_l. \tag{28}$$

The noise term ϵ_l is uniformly distributed over $[-\epsilon_L, \epsilon_L]$. We assume that the distribution of ϵ_l is known by the bank. Further, the precision of the LOLR's signal is higher than that of the investors' signals, i.e. $\epsilon_L < \epsilon$. This is likely to be the case given that the LOLR generally has more information about the banks than the individual investors.

We next need to specify the LOLR's objective function. As before the LOLR aims to bail out all solvent banks facing short term liquidity problems. But if the information set of the LOLR is noisy then it might not always be easy to distinguish the solvent but illiquid banks. Indeed an imperfectly informed LOLR might not bail out a solvent bank and conversely a noisy LOLR might bail out an insolvent institution. Thus there is a possibility that an imperfectly informed LOLR may make Type I or Type II errors. We refer to the situation of not bailing out a solvent bank as a Type I error and the situation where the LOLR bails out an insolvent bank as a Type II error.

Making either a Type I or Type II error is costly and the nature of the cost will differ depending on which error is made. Let C_I and C_{II} denote the cost of a Type I and Type II error respectively. C_I will incorporate the cost of not bailing out a solvent bank. If the LOLR does not bail out a solvent bank then it will face a reputational cost apart from the immediate welfare costs that will be incurred. Furthermore as we will see the bank will have to hold additional reserves ex ante in anticipation of this error and holding more reserves than the second best level is inefficient. The level of reserves will be higher, the higher the probability of a Type I error. Thus we should expect C_I to increase as the probability of a Type I error increases. C_{II} incorporates the cost associated with bailing out an insolvent bank. We will show later in this section that bailing out an insolvent bank has moral hazard effects and in general the moral hazard effect increases as the probability of Type II error increases. Hence we should expect C_{II} to increase as the probability of a Type II error increases.

Let P_I and P_{II} denote the probabilities of Type I and Type II errors respectively. Further, let s_l^* define the bail-out strategy of the LOLR such that it bails out the bank if and only if $s_l \geq s_l^*$ and does not bail out if $s_l < s_l^*$. Then a natural objective function of the LOLR would be to choose s_l^* such that it minimises the expected costs of making errors. More formally s_l^* would be defined as follows:

$$s_l^* = \arg\min P_I \cdot C_I(P_I) + P_{II} \cdot C_{II}(P_{II}). \tag{29}$$

We do not specify the functional form of the cost functions. Instead we work with a generic s_l^* subject to two qualifications. First, since the LOLR objective is to bail out solvent banks, therefore $s_l^* \in (\phi_L - \epsilon_L, \phi_L + \epsilon_L)$. If $s_l^* > \phi_L + \epsilon_L$ then solvent banks will

LOLR (but is known by the bank since the bank is a pool of depositors). If both n and the distribution were known by the LOLR, then it will be able to calculate s^* and hence will be able to observe ϕ by inverting n.

More realistically we can also further assume that s_i is independently but not identically distributed so that a depositor is also not aware of the distribution of the signals of other agents. Even if the signals are independently but not identically distributed, it can be shown that there will exist a unique threshold s^* for each class of depositors. Thus all the qualitiative results will remain unchanged. Hence to avoid further complexity in the analysis we retain the simpler assumption of *iid* signals.

not be bailed out. Conversely, if $s_l^* < \phi_L - \epsilon_{L,}$, then insolvent banks will be bailed out. Hence it needs to be the case that $\phi_L - \epsilon_{L,} < s_l^* < \phi_L + \epsilon_L$.[29] Second, s_l^* is such that there exists a positive probability that the LOLR might make Type I and Type II errors. This is plausible and is likely to be the case in practice if the LOLR has incomplete information.

8.1. The reserve constraint and the zero profit condition in the presence of a noisy LOLR. We now consider how the presence of a noisy LOLR affects the reserve constraint and the zero profit condition of the bank. We assume that it is common knowledge that the LOLR follows a bail-out strategy around $s_l^* = \phi_L$.

In the presence of a noisy LOLR, the reserve constraint will be as follows:

$$\begin{aligned} re &= \Pr\left(s_l < s_l^* | \phi < \phi_L\right) E\left(n | \phi < \phi_L\right) D\left(1+r\right) \\ &\quad + \Pr\left(s_l < s_l^* | \phi \geq \phi_L\right) E\left(n | \phi \geq \phi_L\right) D\left(1+r\right) \\ &= \overset{\Delta}{n} D\left(1+r\right) \end{aligned} \tag{30}$$

where

$$\overset{\Delta}{n} \equiv \Pr\left(s_l < s_l^* | \phi < \phi_L\right) E\left(n | \phi < \phi_L\right) + \Pr\left(s_l < s_l^* | \phi \geq \phi_L\right) E\left(n | \phi \geq \phi_L\right). \tag{31}$$

Equation (30) says that in the presence of a noisy LOLR the reserves will be just sufficient to ensure that the bank on average has enough funds in the interim period to satisfy the depositors who run *if* the bank is not bailed out. Note that the probability of a Type I error, which is given by $P_I = \Pr\left(s_l < s_l^* | \phi \geq \phi_L\right)$, has a direct impact on the reserve constraint of the bank. The higher the probability of a Type I error, the higher will be the level of bank reserves since the bank also needs to consider the possibility that it might not be bailed out even if it is solvent. Nevertheless, it is easy to prove the following proposition.

Proposition 4. *In the imperfect information setting where the bank fundamentals are not perfectly observable to the investors, the level of bank reserves in the presence of a (noisy) LOLR will be lower relative to the case where the LOLR does not exist.*

Proof. See Appendix. ■

The reason why the level of reserves falls in the presence of a noisy LOLR is that now the bank will only hold reserves to insure against the possibility that it might not be bailed out rather than holding reserves to insure against bank runs in general. Recall that with imperfect information the bank holds more reserves, relative to the second best level, as some patient depositors may also withdraw in the interim period. Thus the fall in the level of reserves in the presence of a LOLR is an improvement as it brings us closer to the second best reserve level.

Let us now consider how the presence of a noisy LOLR affects the zero profit condition. Before doing so, however, it is instructive to reinterpret the zero profit condition stated earlier. The zero profit condition in equation (27) can be rewritten as follows:

[29] We make a technical assumption that $\epsilon_L < \frac{1+\underline{\phi}}{2}$. This ensures that $\phi_L + \epsilon_L < 1$ and $\phi_L - \epsilon_{L,} > \underline{\phi}$ and hence s_l^* is always in the range $\left(\underline{\phi}, 1\right)$.

$$I \cdot E\left(\widetilde{R}\right) = I\left(1+r\right) + \hat{n}Dr\left(1+r\right).$$

In section 4 we noted that for every investor expected to run in the interim period, the bank needs to hold $(1+r)$ units as reserves instead of one unit. Thus if a proportion $\hat{n}$ of investors are expected to run, then an extra $\hat{n}Dr$ will be tied up in reserves which have a zero net rate of return. These resources have an opportunity cost of $(1+r)$ per unit and thus the 'reserve cost' will be $\hat{n}Dr\left(1+r\right)$. Thus, the zero profit condition restated above states that the expected return from the investment portfolio must cover the opportunity cost of investment funds and the reserve cost.

In the presence of a LOLR, the expected return from investments must cover the opportunity cost of investment funds and the reserve cost. However, if the LOLR has imperfect information there will be a third term in the zero profit condition which will have an offsetting effect. A bank operating in a competitive environment will realise that there is always a probability that an imperfectly informed LOLR might bail the bank out even if it is insolvent. This asymmetry of information will act as a subsidy that will relax the zero profit condition. The zero profit condition in the presence of a noisy LOLR will be as follows:

$$\begin{aligned} \underbrace{I \cdot E\left(\widetilde{R}\right)}_{\text{Exp. Return on inv.}} &= \underbrace{I\left(1+r\right)}_{\text{Opportunity cost of inv.}} + \underbrace{\overset{\Delta}{n}Dr\left(1+r\right)}_{\text{reserve cost}} \\ &\quad \underbrace{-\Pr\left(s_l \geq s_l^* | \phi < \phi_L\right) E\left(n | \phi < \phi_L\right) D\left(1+r\right)}_{\text{LOLR subsidy}} \end{aligned}$$

which in terms of our original formulation can be written as:

$$\begin{aligned} I\left\{\omega_p\left(1+\rho\right)\overline{p}\overset{\wedge}{\phi} + \left(1-\omega_p\right)\left(1+r\right)\right\} &= \left(1-\overset{\Delta}{n}\right)D\left(1+r\right) \qquad (32) \\ &\quad - \Pr\left(s_l \geq s_l^* | \phi < \phi_L\right) E\left(n | \phi < \phi_L\right) D\left(1+r\right). \end{aligned}$$

Note that the probability of a Type II error, which is given by $P_{II} = \Pr\left(s_l \geq s_l^* | \phi < \phi_L\right)$, directly affects the zero profit condition. The higher the probability of a Type II error, the higher will be the value of the implicit subsidy provided by the LOLR to the banking system. Whereas in the reserve constraint, the distortion came from the possibility of a Type I error, the zero profit condition is distorted by the possibility of a Type II error.

Given the existence of the LOLR subsidy and given from Proposition 4 that the reserve level is lower in the presence of a noisy LOLR, it is immediately clear from the zero profit condition (32) that the lending rate will decrease. We have thus shown the following result.

Remark 3. *The lending rate, $(1+\rho)$, decreases in the presence of a (noisy) LOLR when bank fundamentals are not common knowledge.*

The reason as to why the lending rate decreases in the presence of a noisy LOLR is that if the banking system is competitive then the reduction in reserve costs and the LOLR subsidy will be passed on to the entrepreneurs.

Having worked out the reserve constraint and the zero profit condition it is now straightforward to state the optimal contract in the presence of a noisy LOLR. The optimal contract in the presence of an imperfectly informed LOLR maximises entrepreneurial utility (3), subject to investor rationality (9), reserve constraint (30), zero profit condition (32), budget constraint (7) and incentive compatibility (8). We can solve for the lending rate directly from the zero profit condition. Solving for $(1+\rho)$ in (32), after some manipulation we get the following intuitive expression for the lending rate in the presence of the LOLR:

$$(1+\rho)_{LOLR} = \left(\frac{1+r}{\bar{p}\overset{\wedge}{\phi}}\right)\left(1+\frac{\overset{\Delta}{n}Dr - SD}{N}\right) \tag{33}$$

where $S \equiv \Pr\left(s_l \geq s_l^* | \phi < \phi_L\right) E\left(n|\phi < \phi_L\right)$. Thus the lending rate is the risk free rate scaled by risk and adjusted for reserve costs and the LOLR subsidy.

8.2. LOLR and insolvency. We next examine the important and crucial question of how the presence of an imperfectly informed LOLR affects the solvency of the banking system. We show that in the presence of a noisy LOLR, banks are more likely to be adversely affected by the macroeconomic shock. This result is stated in the following proposition.

Proposition 5. *The probability of insolvency of the banking system increases in the presence of an imperfectly informed LOLR.*

Proof. We know from the insolvency criteria described in Case 1, section 5.2, that a bank is insolvent if it is unable to repay the patient depositors in the final period even if only the impatient depositors withdraw in the interim period. Thus the bank is insolvent if and only if

$$\widetilde{R}I + re_2 < (1-\theta)\,D\,(1+r)$$

where $\widetilde{R}$ is as defined by equation (2). In the presence of the LOLR, the level of reserves at $t=1$ is $re_2 = \left(\overset{\Delta}{n} - \theta\right) D\,(1+r)$ if only the θ-depositors withdraw. To make a distinction between the insolvency points with and without the LOLR, let ϕ_{L1} and ϕ_{L2} denote the insolvency points in the absence and presence of the LOLR respectively. Then at the insolvency point in the presence of the LOLR the following holds:

$$\left\{\omega_p\left[(1+\rho)\,\bar{p}\phi_{L2}\right] + (1-\omega_p)\,(1+r)\right\} I = (1-\theta)\,D\,(1+r) \tag{34}$$

where $I = D\left[1 - \overset{\Delta}{n}\,(1+r)\right]$ and $(1+\rho)$ is as defined by expression (33). Inserting the expressions for I and $(1+\rho)$ in equation (34) and noting that $\omega_p = N/I$ and $\phi_{L1} = \overset{\wedge}{\phi}$, after some simplification we get

$$\phi_{L2} = \phi_{L1}\left[\frac{\overset{\Delta}{n}Dr + N}{\overset{\Delta}{n}Dr + N - SD}\right]. \tag{35}$$

Given competitive credit markets, N will be large enough so that $N > \overset{\triangle}{n}Dr - SD$. Thus $[\frac{\overset{\triangle}{n}Dr+N}{\overset{\triangle}{n}Dr+N-SD}] > 1$, implying that $\phi_{L2} > \phi_{L1}$. Since $\phi \sim U\left[\underline{\phi}, 1\right]$, the probability of insolvency in the presence of a noisy LOLR increases by $\frac{\phi_{L2}-\phi_{L1}}{1-\underline{\phi}}$. ■

The reason why the probability of insolvency of the bank increases in the presence of the LOLR is that the bank realises that it might be bailed out even when it is actually insolvent given information asymmetry between the bank and the LOLR. Thus a competitive bank internalises the ex ante implicit subsidy provided by the LOLR in its zero profit condition. This in turn reduces the lending rate subsequently affecting the insolvency criteria.[30]

Hence as shown in the proof, in the presence of a noisy LOLR, the probability of insolvency increases by a factor of $[\frac{\overset{\triangle}{n}Dr+N}{\overset{\triangle}{n}Dr+N-SD}]$. Note also that that the result in Proposition 5 always holds irrespective of how much the lending rate falls. All that is required is the presence of an imperfectly informed LOLR which might inadvertently bail out a solvent institution.

It should be noted that the depositors' switching threshold, s^*, will not change as a result of the presence of the LOLR. The switching threshold by definition is the cutoff signal at which the expected payoff from running is equal to the expected payoff from waiting and hence it is the marginal signal at which depositors are indifferent between running and waiting. Thus the switching threshold will only change if either the investor payoffs change or the probabilities of receiving those payoffs changes. Since the investor payoffs do not change, s^* will only change if the probabilities of receiving a certain payoff changes. We have seen that in the presence of LOLR, the probability of bank failure increases. Nevertheless depositors are also aware that the LOLR might bail the bank out. The zero profit condition is such that these two effects exactly offset each other, thus leaving the switching threshold unaltered. This result is natural since the bank is just a pool of depositors and the zero profit condition changes such that the expected payoffs to the depositors remain the same *after* taking into account the LOLR subsidy.

8.3. Transparency. We next carry out comparative statics to analyse the effects of more or less transparency of the banking system. A higher degree of transparency will translate into more precise signals obtained by the investors, and in particular the LOLR. A good measure of transparency is therefore the precision of ϵ_l. Since $\epsilon_l \sim U[-\epsilon_L, \epsilon_L]$, precision can be measured by the bound ϵ_L. The higher is ϵ_L, the less precise the signal of the LOLR and hence the lower the degree of transparency of the banking system.

An interesting question that then arises is how the probability of insolvency of the banking system changes as the degree of transparency changes. We prove the following proposition.

Proposition 6. *The moral hazard problem is directly proportional to the amount of noise in the information set of the LOLR. Thus the probability of insolvency of the banking system increases as the degree of transparency worsens.*

Proof. See Appendix. ■

[30] Since the LOLR subsidy is internalised in the zero profit condition, a competitive bank on average will be able to pay off the depositors their reservation utility only after a bail out which is expected with some probability ex ante.

The intuition behind the above result is simple. As the amount of noise in the information set of the LOLR increases, the probability of the LOLR inadvertently bailing out an insolvent institution increases. A competitive bank realises this and hence adjusts its lending rate accordingly. Thus an increase in the noise, increases the moral hazard problem which in turn increases the probability of insolvency.

On the other hand, as expected, as the precision of the LOLR's signal improves, the probability of both a Type I and Type II error decreases.[31] In the limit, as we have also seen in section 7.1, the economy will reach the second best level if the LOLR is perfectly informed. This is because as noise decreases, not only does the implicit subsidy provided by the LOLR decreases, but also bank reserves approach the second best level. This is because the probability of a Type I error decreases and thus $\overset{\triangle}{n}$ approaches θ. Thus, as $\epsilon_L \downarrow 0$, $\overset{\triangle}{n} \to \theta$ and hence $(1+\rho)_{LOLR} \to (1+\rho)_{SB}$. In other words, as the amount of noise in the LOLR's information set decreases, the reserve level and the lending rate approach the second best levels as defined in Proposition 1. We further establish this result by studying the impact of noise on aggregate entrepreneurial investment.

Impact on entrepreneurial investment. We next analyse the impact of a noisy LOLR on entrepreneurial investment level. Note that the total level of entrepreneurial investment is given by $N = T(1-p^*)$. We saw in section 5.1 that in an environment with bank runs where the LOLR does not exist, there is too little investment, i.e. $N < N_{SB}$. The reason for underinvestment relative to the second best investment level is that the bank has to hold higher reserves in order to insure against the possibility of bank runs. A higher reserve level implies a lower level of entrepreneurial investment as higher reserve costs increase the lending rate. The increase in the lending rate has an affect on the incentive compatibility condition increasing the critical skill level p^* and thus crowding out investment. In this context the presence of a LOLR can be productive as we know from Proposition 4 that the level of bank reserves falls in the presence of a LOLR. Thus the level of entrepreneurial investment increases in the presence of a LOLR.

Nevertheless, there exists a possibility that there might be overinvestment in the presence of a noisy LOLR as entrepreneurs might also undertake negative NPV projects. Such overinvestment will take place if the lending rate falls below the second best level. More precisely $N > N_{SB}$ if and only if $(1+\rho) < (1+\rho)_{SB}$. This will be the case if the following condition is satisfied.

Condition 1. *A sufficient condition for $N > N_{SB}$ is that $r < S/\overset{\triangle}{n}$.*

Condition 1 follows from a straightforward comparison of expression (33) with expression (11). If $S > \overset{\triangle}{n}r$ then $(1+\rho)_{LOLR} < (1+\rho)_{SB}$ given that $\overset{\triangle}{n} > \theta > 0$. The condition states that there will be entrepreneurial overinvestment in the presence of a noisy LOLR if the risk free rate is not too high. Note that this condition is likely to be satisfied for plausible levels of the risk free rate. The intuition behind Condition 1 is simple. If the risk free rate is not too high, then the opportunity cost of holding reserves will be low. Low reserve costs coupled with a LOLR subsidy will push the lending rate below the second best level consequently causing overinvestment in the economy.

It is easy to check that $\partial(1+\rho)/\partial\epsilon_L$ is negative if Condition 1 is satisfied and positive otherwise. Thus irrespective of whether Condition 1 is satisfied or not, a reduction in the noise level will bring the economy closer to the second best level.

[31] See Proof of Proposition 6.

Remark 4. *As $\epsilon_L \downarrow 0$, the economy approaches the second best level.*

Ex ante versus ex post efficiency. Intuition then suggests that if the LOLR's noise level is 'low enough', an imperfectly informed LOLR might be a Pareto improvement. We therefore now compare the costs and benefits of the presence of a noisy LOLR and see how these vary with the noise level. Our intuition will be confirmed if we show that at low noise levels the benefits will outweigh the costs and vice versa.

In Proposition 5 we showed that the presence of a noisy LOLR increases the probability of insolvency. Thus, the presence of an imperfectly informed LOLR is ex ante inefficient. Nevertheless, ex post the LOLR can prevent inefficient liquidation. There therefore exists a trade-off between ex ante versus ex post efficiency.

To quantify this trade-off we need to compare the expected costs and benefits for a given level of noise. Suppose that the cost of insolvency is given by $\mathbb{C}$ units of the consumption good.[32] We showed in the proof to Proposition 5 that with the presence of a noisy LOLR, the probability of insolvency increases by $\frac{\phi_{L2}-\overset{\wedge}{\phi}}{1-\underset{-}{\phi}}$, where ϕ_{L2} is the insolvency point in the presence of a LOLR. Since the insolvency point, ϕ_{L2}, is a function of the amount of noise, we write this explicitly as $\phi_{L2}(\epsilon_L)$. Thus, the expected cost of insolvency is $\frac{\phi_{L2}(\epsilon_L)-\overset{\wedge}{\phi}}{1-\underset{-}{\phi}}\mathbb{C}$.

Having quantified ex ante inefficiency we next turn to the gains from ex post efficiency. We know that the optimal contract is such that ex ante no premature liquidations take place. This is because it is less costly to hold reserves relative to inefficient liquidations and thus banks adjust their reserve levels accordingly. Thus the reduction in the reserve cost with the presence of a LOLR is a measure of the ex post efficiency. The reduction in the reserve cost with the presence of a noisy LOLR is $\left(\hat{n} - \overset{\Delta}{n}(\epsilon_L)\right) Dr(1+r)$. Note that as discussed in section 8.1, the reserve level is higher, the higher is the probability of a Type I error. Thus as noise increases the expected gains from ex post efficiency decreases.

Let ϵ^* be the level of noise at which the expected gains are just equal to the expected costs of insolvency. Then ϵ^* solves the following:

$$\left(\hat{n} - \overset{\Delta}{n}(\epsilon^*)\right) Dr(1+r) - \frac{\phi_{L2}(\epsilon^*) - \overset{\wedge}{\phi}}{1-\underset{-}{\phi}}\mathbb{C} = 0. \tag{36}$$

We know that such a critical level, ϵ^*, exists because the expected gains are monotonically decreasing in ϵ_L and the expected costs are monotonically increasing in ϵ_L. Thus if $\epsilon_L < \epsilon^*$, then the expected gains from ex post efficiency outweigh the costs of ex ante inefficiency and vice versa.

Proposition 7. *The presence of an imperfectly informed LOLR will be a Pareto improvement if and only if $\epsilon_L < \epsilon^*$ where ϵ^* solves equation (36).*

It can be argued that one policy implication that emerges from the analysis is that the LOLR should choose a bail-out strategy, s_l^*, such that given the distribution of the noise term, there is a zero probability of a Type II error. This is because we know that the moral hazard problem is directly proportional to the probability of a Type II error.

[32] The cost of insolvency will include the cost of bail out, the cost of unemployment, social inefficiencies and other social costs. Endogenising these costs is outside the scope of this paper and we take $\mathbb{C}$ as given.

Nevertheless such a statement is misleading and is subject to two qualifications. First, since the probabilities of a Type I and Type II error are inversely related, decreasing the probability of a Type II error to zero will significantly increase the probability of a Type I error. The costs associated with a Type I error should not be understated. By not bailing out a solvent bank, the LOLR will not only suffer reputational and credibility costs but more importantly there are likely to be contagious costs which will spill over to the entrepreneurial sector. Thus the cost of a Type I error cannot be overemphasised.[33] The LOLR therefore needs to be very careful before deciding not to bail out a bank given that the immediate costs in case of a Type I error can be substantial. Thus the optimal bail-out strategy will be as defined in expression (29) and will give weight to the costs of both a Type I and Type II error.

Second, we have assumed that the noise term in the LOLR's signal has a bounded uniform distribution. This assumption was made for tractability. However, if the noise term followed an unbounded distribution, say normality, then there will always exist a positive probability of both a Type I and Type II error. In reality there is always likely to be a chance that the LOLR might make mistakes in both directions. Thus the sensible policy implication which follows is that the LOLR should improve the quality of bank monitoring and supervision so as to minimise the probabilities of errors.

9. Discussion

As mentioned before, the model that we have developed can be applied both to a domestic and international setting. In a domestic setting, the central bank will carry out LOLR operations. In an international setting, the depositors in our model can be interpreted as international investors and an institution like the IMF would play the role of international lender of last resort.

Our model shows the interlinkages between different sectors of the economy and how shocks are transmitted within sectors via the banking system. We showed that there exists a one to one mapping from the depositors' equilibrium strategy to an optimal contract prevailing in the economy. In the presence of imperfect information, depositors in our model run only if their signals about the underlying fundamentals are below a certain threshold. Thus banking crises are not unrelated to the real economy as long as depositors do not have perfect information. The sunspot theories are thus a special case which hold validity only if the depositors have perfect information.

In the case of perfect information, we showed that it is indeed true that there may exist multiple equilibria, where either everyone runs or no one runs. However, we showed that if depositors observe only noisy signals of the fundamentals, then in general there will exist a unique equilibrium. Thus perfect information can actually be destablising relative to a small amount of noise. This result was also obtained by Morris and Shin (2001) and Rochet and Vives (2002) who argued that a small amount of noise in the fundamentals can actually be stabilising. However, we showed in our model that this result only holds in the absence of the LOLR. We thus argue that in the presence of another institution, like the LOLR, perfect information can actually lead the economy back to the second best arrangement. Hence, under such a scenario the presence of the LOLR is clearly a Pareto improvement.

However, if the information set of the LOLR is noisy, then our earlier conclusion that the LOLR is a Pareto improvement falls into ambiguity. On the one hand, the LOLR is

[33] Goodhart and Huang (2003) show that the central bank will have an incentive to carry out LOLR operations, even in the presence of moral hazard, given the sizeable cost of contagion.

productive ex post as it can avoid inefficient liquidation. On the other hand, the presence of an imperfectly informed LOLR is conducive to the moral hazard problem and is thus ex ante inefficient. Because of this moral hazard problem, we showed that the probability of insolvency of the banking system actually increases in the presence of the LOLR.

Furthermore, the moral hazard problem aggravates the adverse selection problem faced by the banks. This is because competitive banks internalise the implicit subsidy provided by the LOLR and thus reduce the lending rates. This in turn affects the incentives of the entrepreneurs and we showed that there exists a possibility that the level of investment might increase above the second best optimal level. In other words, the presence of a LOLR might lead to overinvestment in the economy.

In our model, we had assumed throughout that the strong law of large number applies and hence there was no uncertainty with regards to idiosyncratic entrepreneurial risk. Hence the only source of uncertainty was the macroeconomic shock which hit the economy. However, had we not assumed that the SLLN holds, then our results would have been even stronger. This is because the banks would then face an additional risk regarding the realisation of the idiosyncratic entrepreneurial risk. In the presence of an imperfectly informed LOLR, a reduction in the lending rate will increase the entrepreneurial risk faced by the banks because the average success probability of projects will decline. Under such a scenario, if there is overinvestment in the economy, then the economy will be more fragile relative to the case where the LOLR does not exist.

We believe that this is what happened in the Asian crisis (1997). The East Asian economies had a competitive banking system and the government had provided implicit or explicit insurance to the financial intermediaries. The government in turn expected an IMF bail out in case of a debt crisis.[34] Subsequently, there was substantial overinvestment in East Asia. A good example is Thailand, where the finance companies were largely unregulated and there was massive overinvestment in the real estate and property sector. Thus the Asian economies were already fragile before they were hit by a speculative shock which adversely affected their currencies.

Krugman (1998) argued that the Asian crisis occurred because of the moral hazard problem. However, in the following year, Krugman (1999) asserted that he was wrong, and that the Asian crisis was in fact due a balance sheet transfer problem. Our model shows that Krugman was actually correct both times. There did exist a moral hazard problem in Asia. Subsequently, when the Asian economies were hit by a shock, the moral hazard problem translated into a balance sheet transfer problem. The adverse impact on the entrepreneurs' balance sheet was consequently transferred to investors via the banking system.

We do not recommend that a LOLR should not exist. Indeed, we have shown that the LOLR can be ex post efficient. The policy implication that stems from our model is that the banking system be as transparent as possible to the LOLR. In other words, the noise in the LOLR's information set be at a minimal. We showed that if the noise in the LOLR's information set is sufficiently low then the gains from ex post efficiency can outweigh the ex ante inefficiency. On the contrary, if the precision of the LOLR's signal is not high enough, then the net effect of the presence of a LOLR on the economy will be negative. Hence, if the LOLR is to play a productive role, it is imperative that the

[34] Note that as we showed in our model, there will exist a morzal hazard problem, even if a bail out is not guaranteed. All that is required is that the agent expect a bail out from the principal with a positive probability. Of course, the higher the probability of bailing out an insolvent institution, the more severe would be the moral hazard problem.

information set of the LOLR be as precise as possible. This can be achieved by closer monitoring of the banking system. Thus the supervisory process of the banking system should be strict and efficient. It is also vital that the supervisory process be independent and free of any political interference.

This brings into question the increasing trend to delegate supervisory authority to a separate agency as distinct from the central bank. For example, banking supervision in the UK is now under the domain of the Financial Services Authority (FSA). However, the LOLR operations are carried by the central bank. To the extent that there exist conflicts of interest between the supervisory authority and the central bank, the LOLR operations can be adversely affected. This is because the LOLR might not have access to 'relevant information'. The objective function of the supervisory authority might be different from the central bank and hence the two bodies might have different perspectives on what constitutes 'relevant information'.[35] As we demonstrated in our model, as the precision of the LOLR's signal regarding the solvency of the banks worsens, the probability of insolvency of the banking system increases. Hence, the prime objective of the supervisory authorities should be to pass as precise information as possible to the LOLR so that it can carry out its operations effectively. If the LOLR is better able to collect such information directly, then the delegation of banking supervision to another body is not justified. This would therefore largely depend on how well the two bodies can cooperate with each other.

10. Summary and conclusions

We obtain the following results with respect to the role of the LOLR in banking crises: (a) the presence of a perfectly informed LOLR can be a Pareto improvement as costly liquidations can be avoided; (b) perfect transparency between banks and depositors is good if and only if there exists a perfectly informed LOLR; (c) in the presence of an imperfectly informed LOLR, the probability of insolvency of the banking system increases; (d) the moral hazard problem is directly proportional to the noise in the LOLR's information set; (e) as the noise in the LOLR's information set increases, the economy moves further away from the second best arrangement and vice versa; (f) the presence of an imperfectly informed LOLR can be a Pareto improvement as long as the noise in its information set is sufficiently low; (g) the banking system should be as much transparent as possible to the LOLR. We have therefore provided a framework which places considerable emphasis on the role of information in the evolution of crises.

Appendix

Proof of Proposition 2: When an agent receives a signal s_i her beliefs about the distribution of the fundamentals are uniform in the interval $[s_i - \epsilon, s_i + \epsilon]$. Suppose for now that all agents follow a switching strategy around s^*, i.e. they withdraw in the interim period if and only if $s < s^*$. Following Dasgupta (2002) we shall refer to such switching strategies as *monotone strategies* and the associated equilibria as *monotone equilibria*. Let $n(\phi, s^*)$ denote the proportion of people who will run for any given ϕ and s^*. Given that a proportion θ of the depositors are impatient and will always run, therefore $n(\phi, s^*)$ is given by

$$n(\phi, s^*) = \begin{cases} 1 & \text{if } \phi \leq s^* - \epsilon \\ \theta + (1-\theta)\left(\frac{1}{2} + \frac{s^* - \phi}{2\epsilon}\right) & \text{if } \phi \in (s^* - \epsilon, s^* + \epsilon) \\ \theta & \text{if } \phi \geq s^* + \epsilon \end{cases} . \quad (37)$$

[35] For example as argued by Goodhart (2000), the supervisory body might be more concerned about the conduct of business and issues of consumer protection, rather than systemic financial crises per se.

Since $\xi = \frac{\left(n-\hat{n}\right)D(1+r)}{R_l I}$ and the optimal value of re and I is as defined in (17) and (19) respectively, we can rewrite the investor payoffs in (23) in terms of a generic optimal contract. The payoff matrix can thus be written as follows:

	$n \leq n_B$	$n > n_B$
Run	$1+r$	$g(\phi, s^*)$
Wait	$h(\phi, s^*)$	0

where

$$g(\phi, s^*) = \frac{R_l + \hat{n}(1+r)(1-R_l)}{n}$$

and

$$h(\phi, s^*) = \frac{R\left(\tilde{\phi}\right)\left[\left(1-\hat{n}(1+r)\right) - \left(n-\hat{n}\right)(1+r)/R_l\right]}{(1-n)}.$$

Note that the payoffs depend on the optimal reserve level which in turn is a function of the threshold signal s^*.Let $\pi(\phi, n)$ denote the difference in the patient agent's payoff function from waiting till $t=2$ rather than running at $t=1$. $\pi(\phi, n)$ is thus given by

$$\pi(\phi, n) = \begin{cases} -g(\phi, s^*) & \text{if } n > n_B \\ h(\phi, s^*) - (1+r) & \text{if } n \leq n_B \end{cases}.$$

Let $\Pi(s_i, s^*)$ be the difference in the expected utility of a patient depositor from waiting rather than running conditional on the observed signal s_i. The posterior distribution of fundamentals given the observed signal is uniform and is given by

$$\phi | s_i \sim U[s_i - \epsilon, s_i + \epsilon]$$

Hence the expected payoff from waiting rather than running given the signal s_i is

$$\Pi(s_i, s^*) = \int_{\phi=s_i-\epsilon}^{\phi=s_i+\epsilon} \pi(\phi, n(\phi, s^*))\, d\phi.$$

A monotone equilibrium s^* exists if $\Pi(s^*, s^*) = 0$, $\Pi(s_i, s^*) > 0$ for $s_i > s^*$ and if $\Pi(s_i, s^*) < 0$ for $s_i < s^*$. The existence of upper and lower dominance regions implies that $\Pi(s_i, s^*)$ is negative for sufficiently low s^* and positive for sufficiently high s^*. Thus by continuity, $\Pi(\cdot, \cdot)$ must cross the axis somewhere. Continuity holds because a change in s_i, given s^*, only change the limits of integration. We know that $\pi(\phi, n) < 0\ \forall \phi < s^* - \epsilon$ and $\pi(\phi, n) > 0\ \forall \phi < s^* + \epsilon$. Since by continuity, $\Pi(s^*, s^*) = 0$ at some s^*, thus the positive and negative parts of the integral exactly cancel out. Increasing s_i above s^* increases the positive part of the integral, while reducing the negative part. Hence $\Pi(s_i, s^*) > 0$ for $s_i > s^*$. Conversely, $\Pi(s_i, s^*) < 0$ for $s_i < s^*$. This proves the existence of a monotone equilibrium. Q.E.D.

Proof of Proposition 3: We break the proof of the uniqueness of equilibrium into two parts. We first show that there exists a unique equilibrium in monotone strategies. After showing the uniqueness of equilibrium in monotone strategies we then show that there are no *non-monotone* equilibria.

Let $\Pi(s^*) = \Pi(s^*, s^*)$. We need to show that $\Pi(s^*)$ is monotone in s^*. Given that beliefs are uniformly distributed over the interval $[\phi - \epsilon, \phi + \epsilon]$, note that $n_B = (1/2\epsilon)(s^* - (\phi - \epsilon))$, which implies that $\phi = s^* + \epsilon(1 - 2n_B)$. Therefore if $n \leq n_B$, then $\phi > s^* + \epsilon(1 - 2n_B)$. Conversely, $n > n_B$ implies that $\phi < s^* + \epsilon(1 - 2n_B)$. Hence $\Pi(s^*)$ can be written as

$$\Pi(s^*) = -\int_{s^*-\epsilon}^{s^*+\epsilon(1-2n_B)} g(\phi, s^*)\, d\phi + \int_{s^*+\epsilon(1-2n_B)}^{s^*+\epsilon} [h(\phi, s^*) - (1+r)]\, d\phi.$$

Differentiating $\Pi(s^*)$ with respect to s^* we have

$$\frac{d\Pi(s^*)}{ds^*} = -\frac{d}{ds^*}\int_{s^*-\epsilon}^{s^*+\epsilon(1-2n_B)} g(\phi, s^*)\, d\phi + \frac{d}{ds^*}\int_{s^*+\epsilon(1-2n_B)}^{s^*+\epsilon} [h(\phi, s^*) - (1+r)]\, d\phi.$$

The derivatives of the limits of integration are unity and thus

$$\begin{aligned}\frac{d}{ds^*}\int_{s^*-\epsilon}^{s^*+\epsilon(1-2n_B)} g(\phi, s^*)\, d\phi &= g(s^* + \epsilon(1-2n_B), s^*) - g(s^* - \epsilon, s^*) \\ &+ \int_{s^*-\epsilon}^{s^*+\epsilon(1-2n_B)} \frac{d}{ds^*} g(\phi, s^*)\, d\phi\end{aligned}$$

which can be written as

$$\frac{d}{ds^*}\int_{s^*-\epsilon}^{s^*+\epsilon(1-2n_B)} g(\phi, s^*)\, d\phi = \int_{s^*-\epsilon}^{s^*+\epsilon(1-2n_B)} \frac{d}{d\phi} g(\phi, s^*)\, d\phi + \int_{s^*-\epsilon}^{s^*+\epsilon(1-2n_B)} \frac{d}{ds^*} g(\phi, s^*)\, d\phi.$$

Similarly we get an analogous expression for the derivative of the integral under $h(\phi, s^*)$. Taking the derivatives, and noting that $\left|\frac{dn(\phi,s^*)}{d\phi}\right| = \left|\frac{dn(\phi,s^*)}{ds^*}\right|$ given that ϕ and s^* enter symmetrically in $n(\phi, s^*)$, it can then be shown that

$$\frac{d}{ds^*}\int_{s^*-\epsilon}^{s^*+\epsilon(1-2n_B)} g(\phi, s^*)\, d\phi = \int_{s^*-\epsilon}^{s^*+\epsilon(1-2n_B)} \left[\frac{(1+r)(1-R_l)}{n}\right] \frac{d\hat{n}}{ds^*} d\phi$$

and

$$\begin{aligned}\frac{d}{ds^*}\int_{s^*+\epsilon(1-2n_B)}^{s^*+\epsilon} h(\phi, s^*)\, d\phi &= \int_{s^*+\epsilon(1-2n_B)}^{s^*+\epsilon} \left[\frac{1-\hat{n}(1+r)}{1-n}\right] \frac{dR(\phi)}{d\phi} d\phi \\ &+ \int_{s^*+\epsilon(1-2n_B)}^{s^*+\epsilon} \left[\frac{1-\hat{n}(1+r)}{1-n}\right] \frac{dR(\phi)}{ds^*} d\phi \\ &+ \int_{s^*+\epsilon(1-2n_B)}^{s^*+\epsilon} \left[\frac{(1+r)/R_l - R(\phi)(1+r)}{1-n}\right] \frac{d\hat{n}}{ds^*} d\phi\end{aligned}$$

Next observe that $\frac{d\hat{n}}{ds^*} > 0$, $\frac{dR(\phi)}{d\phi} > 0$, $\frac{dR(\phi)}{ds^*} > 0$. Thus a sufficient condition for $\frac{d\Pi(s^*)}{ds^*} > 0$ is that:

$$\int_{s^*-\epsilon}^{s^*+\epsilon} \left\{ \left[\frac{(1+r)/R_l - R(\phi)(1+r)}{1-n} \right] - \left[\frac{(1+r)(1-R_l)}{n} \right] \right\} \frac{d\hat{n}}{ds^*} d\phi > 0$$

It can be shown that if premature liquidation is sufficiently costly, then this condition will be satisfied. More, precisely, a sufficient condition for a unique equilibrium in monotone strategies is that:

$$R_l \left[\left(\frac{1-\theta}{\theta} \right) (1-R_l) + \omega_p (1+\rho) + (1-\omega_p)(1+r) \right] < 1.$$

Even if this sufficient condition is not satisfied, it is more likely than not that the equilibrium in monotone strategies will be unique. Nevertheless, we impose this sufficient condition to ensure that there is always a unique equilibrium in monotone strategies.

We next need to demonstrate that there are no equilibria associated with *non-monotone* strategies, i.e. there are no non-monotone equilibria. To do this we first establish a number of lemmas which we will then use to demonstrate our uniqueness result.

Lemma 1. *Let $n(\phi)$ be an agent's arbitrary feasible belief regarding the proportion of agents demanding early withdrawal as a function of the true state ϕ. Thus, $n(\phi)$ reflects an agent's belief as to how other agents will act as a function of their signals. Then,* $\frac{dn(\phi)}{d\phi} \in \left[-\frac{(1-\theta)}{2\epsilon}, \frac{(1-\theta)}{2\epsilon} \right]$.

Proof. Let $y(s_i)$ represent the belief of agent i regarding the mass of patient agents who will run when she receives signal s_i. Then by definition we have

$$n(\phi) = \theta + \frac{1-\theta}{2\epsilon} \int_{s_i=\phi-\epsilon}^{s_i=\phi+\epsilon} y(s_i)\, ds_i.$$

Differentiating with respect to ϕ, we get

$$\frac{dn(\phi)}{d\phi} = \frac{1-\theta}{2\epsilon} \left[y(\phi+\epsilon) - y(\phi-\epsilon) \right].$$

The result then follows since $y(s_i) \in [0,1]$. ■

Thus, Lemma 1 places restrictions on the derivative of $n(\phi)$ with respect to ϕ. Lemma 2 below states that full strategic complementarities are only satisfied when the bank is not bankrupt. Thus, when the bank is not bankrupt an agent's incentive to run increases with the proportion of agents running. However, once the bank is bankrupt, agent's actions are strategic substitutes rather than complements. In this case the payoff from running decreases with the proportion of agents running.

Lemma 2. *When $n \leq n_B$, $\frac{\partial \pi(\phi,n)}{\partial \phi} > 0$ and $\frac{\partial \pi(\phi,n)}{\partial n} < 0$. However, when $n > n_B$, $\frac{\partial \pi(\phi,n)}{\partial \phi} < 0$ and $\frac{\partial \pi(\phi,n)}{\partial n} > 0$.*

Proof. When $n \leq n_B$, $\pi(\phi, n) = h(\phi, s^*) - (1+r)$. Taking the partial derivative of $h(\phi, s^*)$ with respect to n, we have

$$\frac{\partial h(\phi, s^*)}{\partial n} = \frac{R(\cdot)}{(1-n)^2}\left(1 - \hat{n}(1+r) + \frac{\hat{n}(1+r)}{R_l}\right) - \frac{R(\cdot)(1+r)}{R_l(1-n)} - \frac{R(\cdot)\,n(1+r)}{R_l(1-n)^2}.$$

Thus, $\frac{\partial h(\phi, s^*)}{\partial n} < 0$ if and only if

$$\frac{R(\cdot)(1+r)}{R_l(1-n)} + \frac{R(\cdot)\,n(1+r)}{R_l(1-n)^2} > \frac{R(\cdot)}{(1-n)^2}\left(1 - \hat{n}(1+r) + \frac{\hat{n}(1+r)}{R_l}\right)$$

which after some simplification is equivalent to

$$\frac{(1+r) - \hat{n}(1+r)}{R_l} > 1 - \hat{n}(1+r).$$

This is indeed the case since $R_l < 1$. Hence $\frac{\partial \pi(\phi,n)}{\partial n} < 0$ when $n \leq n_B$. Further, since n is decreasing in ϕ, we have $\frac{\partial \pi(\phi,n)}{\partial \phi} > 0$ when $n \leq n_B$.

Now consider the case when $n > n_B$. We now have $\pi(\phi, n) = -g(\phi, s^*)$. Taking the partial derivative of $g(\phi, s^*)$ with respect to n, we have

$$\frac{\partial g(\phi, s^*)}{\partial n} = -\frac{R_l + \hat{n}(1+r)(1-R_l)}{n^2}.$$

Thus, $\frac{\partial g(\phi, s^*)}{\partial n} < 0$ if and only if

$$\frac{R_l + \hat{n}(1+r)(1-R_l)}{n^2} > 0$$

which is indeed the case since $R_l < 1$. Hence, $\frac{\partial \pi(\phi,n)}{\partial n} > 0$ when $n > n_B$. Further, since n is decreasing in ϕ, we have $\frac{\partial \pi(\phi,n)}{\partial \phi} < 0$ when $n > n_B$. ■

Lemma 3 below shows that even though the payoffs do not satisfy full strategic complementarities, but nevertheless under some restrictions they satisfy a weak form of strategic complementarities. Lemma 3 will later be used to show the uniqueness of the switching threshold.

Lemma 3. *Suppose there exist ϕ_T, ϕ_B and $\phi'(\phi)$ such that $0 < \phi_T - \phi_B \leq 2\epsilon$ and $\phi'(\phi) \leq \phi_B$. Assume that $n(\phi) \geq \theta + (1-\theta)\frac{(\phi_T - \phi)}{2\epsilon}$ $\forall \phi \in [\phi_T, \phi_B]$. Then,*
if $\int_{\phi_B}^{\phi_T} \pi\left(\phi, \theta + (1-\theta)\frac{(\phi_T-\phi)}{2\epsilon}\right) d\phi \geq 0$, we have

$$\int_{\phi_B}^{\phi_T} \pi\left(\phi, \theta + (1-\theta)\frac{(\phi_T - \phi)}{2\epsilon}\right) d\phi > \int_{\phi_B}^{\phi_T} \pi\left(\phi'(\phi), n(\phi)\right) d\phi.$$

Proof. First suppose that $\pi\left(\phi, \theta + (1-\theta)\frac{(\phi_T-\phi)}{2\epsilon}\right) > 0$ for all $\phi \in [\phi_T, \phi_B]$. Then in this case using lemma 2 it is trivial to show that $\pi\left(\phi, \theta + (1-\theta)\frac{(\phi_T-\phi)}{2\epsilon}\right) \geq \pi\left(\phi'(\phi), n(\phi)\right)$ $\forall \phi$ and hence the result of the lemma holds.

Since $n(\phi) \geq \theta + (1-\theta)\frac{(\phi_T-\phi)}{2\epsilon}$, $\pi\left(\phi'(\phi), n(\phi)\right)$ can either fall in the strategic complements range (where the bank is not bankrupt and $n \leq n_B$) or in the strategic substitutes range (where the bank is bankrupt and $n > n_B$). If $\pi\left(\phi'(\phi), n(\phi)\right)$ falls in the strategic complements range, then from lemma 2 we know that $\pi\left(\phi, \theta + (1-\theta)\frac{(\phi_T-\phi)}{2\epsilon}\right) \geq \pi\left(\phi'(\phi), n(\phi)\right)$, as $\frac{d\pi(\cdot,\cdot)}{d\phi} > 0$ and $\frac{d\pi(\cdot,\cdot)}{dn} < 0$. If $\pi\left(\phi'(\phi), n(\phi)\right)$ falls in the strategic complements range, then $\pi\left(\overset{\wedge}{\phi}(\phi), n(\phi)\right) < 0$. This is because $\pi(\phi, n)$ is always negative in the strategic substitutes range. In fact, since $\pi(\phi, n) = -g(\phi, n)$ in this range, the maximum value that it can attain is $-\left[R_l + \hat{n}(1+r)(1-R_l)\right] < 0$. (This value will be attained when $n = 1$.) Thus if $\pi\left(\phi, \theta + (1-\theta)\frac{(\phi_T-\phi)}{2\epsilon}\right) > 0$ then the result of the lemma follows trivially, given lemma 2.

Now suppose that $\pi\left(\phi, \theta + (1-\theta)\frac{(\phi_T-\phi)}{2\epsilon}\right)$ is positive for some ϕ, but negative for some other ϕ, where $\phi \in [\phi_T, \phi_B]$. Let $m(\phi) = \theta + (1-\theta)\frac{(\phi_T-\phi)}{2\epsilon}$. Since $m(\phi)$ is monotone in ϕ, there is exactly one point, say ϕ_1, where $\pi(\phi, m(\phi)) = 0$, i.e. $\pi(\phi_1, m(\phi_1)) = 0$. Define ϕ_2 as

$$\phi_2 = \inf\left\{\phi \in [\phi_T, \phi_B] : \pi\left(\phi'(\phi), n(\phi)\right) = 0\right\}.$$

Next, we need to show that the following holds

$$\int_{\phi_B}^{\phi_1} \pi(\phi, m(\phi))\, d\phi \geq \int_{\phi_B}^{\phi_2} \pi\left(\phi'(\phi), n(\phi)\right) d\phi. \tag{38}$$

To show that the inequality in (38), holds we need to prove two claims.

Claim 1.

$$\pi\left(\phi'(\phi), n(\phi)\right) < 0\ \forall \phi \in [\phi_B, \phi_2).$$

Proof. *Suppose that $\phi < \min(\phi_1, \phi_2)$. Then since ϕ is strictly less than $\min(\phi_1, \phi_2)$, $\pi(\phi, m(\phi)) < 0$. Now we have two possibilities. First, we might have $m(\phi) > n_B$. Since $n(\phi) \geq m(\phi)$, we have $n(\phi) > n_B$. Thus $\pi\left(\phi'(\phi), n(\phi)\right) < 0$ as it is in the strategic substitutes range. Second, we might have $m(\phi) < n_B$. Now either $n(\phi) > n_B$, in which case $\pi\left(\phi'(\phi), n(\phi)\right) < 0$ or $n(\phi) \leq n_B$ in which case $\pi\left(\phi'(\phi), n(\phi)\right)$ lies in the strategic complements range. If $\pi(\cdot,\cdot)$ lies in the strategic complements range then since in this range $\frac{d\pi(\cdot,\cdot)}{d\phi} > 0$ and $\frac{d\pi(\cdot,\cdot)}{dn} < 0$, the following inequality holds: $\pi\left(\phi'(\phi), n(\phi)\right) \leq \pi(\phi, m(\phi)) < 0$ for all $\phi \in [\phi_B, \min(\phi_1, \phi_2))$. Thus, we need to verify that $\min(\phi_1, \phi_2) = \phi_2$ to show that the claim is true. Suppose that $\min(\phi_1, \phi_2) = \phi_1$. Then there exists a $\phi \in [\phi_1, \phi_2]$ such that $\pi\left(\phi'(\phi), n(\phi)\right) \geq 0$. But then by continuity there exists a point, say $\phi_3 < \phi_2$, such that $\pi\left(\phi'(\phi_3), n(\phi_3)\right) = 0$, which is a contradiction as $\phi_2 = \inf\left\{\phi \in [\phi_T, \phi_B] : \pi\left(\phi'(\phi), n(\phi)\right) = 0\right\}$. Thus, $\phi_2 < \phi_1$.* ■

Claim 2.

$$n(\phi_2) < \theta + (1-\theta)\frac{(\phi_T - \phi_1)}{2\epsilon}.$$

Proof. *At ϕ_1, $\pi\left(\phi_1, \theta + (1-\theta)\frac{(\phi_T-\phi_1)}{2\epsilon}\right) = 0$ and hence*

$$\frac{R(\phi_1)\left[\left(1-\hat{n}(1+r)\right)-\left(m(\phi_1)-\hat{n}\right)(1+r)/R_l\right]}{1-m(\phi_1)}=1+r.$$

At ϕ_2, $\pi\left(\phi'(\phi), n(\phi)\right)=0$ *and hence*

$$\frac{R\left(\phi'(\phi_2)\right)\left[\left(1-\hat{n}(1+r)\right)-\left(n(\phi_2)-\hat{n}\right)(1+r)/R_l\right]}{1-n(\phi_2)}=1+r.$$

Since $\phi'(\phi)\leq\phi_B$ *and* $\phi_1\in(\phi_B,\phi_T)$, *therefore* $\phi'(\phi_2)<\phi_1$. *But then given the above it has to be the case that* $n(\phi_2)<m(\phi_1)$. ■

Now we can use claims 1 and 2 to show that the inequality in (38) holds. By changing variables of integration we have

$$\int_{\phi_B}^{\phi_1}\pi(\phi,m(\phi))\,d\phi=\int_{m(\phi_1)}^{m(\phi_B)}\pi(\phi(m),m)\left|\frac{\partial\phi}{\partial m}\right|dm$$

and

$$\begin{aligned}\int_{\phi_B}^{\phi_2}\pi\left(\overset{\wedge}{\phi}(\phi),n(\phi)\right)d\phi &= \int_{\min(n(\phi):\phi\in[\phi_B,\phi_2])}^{\max(n(\phi):\phi\in[\phi_B,\phi_2])}\pi\left(\phi'(\phi(n)),n\right)\left|\frac{\partial\phi}{\partial n}\right|dn\\ &\quad+\int_{\phi:\frac{\partial\phi}{\partial n}=0}\pi\left(\phi'(\phi(n)),n\right)d\phi.\end{aligned}$$

Now the second integral is smaller than the first integral because it is negative in the range considered by claim 1, because it is computed over a range that is smaller by claim 2 ($n(\phi_2)<m(\phi_1)$) and because $|\partial\phi/\partial n|\geq|\partial\phi/\partial m|$ by lemma 1. This proves that the inequality in (38) holds.

It follows that

$$\int_{\phi_1}^{\phi_T}\pi(\phi,m(\phi))\,d\phi\geq\int_{\phi_2}^{\phi_T}\pi(\phi,m(\phi))\,d\phi>\int_{\phi_2}^{\phi_T}\pi\left(\phi'(\phi),n(\phi)\right)d\phi. \tag{39}$$

The first inequality holds as $\phi_2<\phi_1$ by claim 1. The second inequality follows from inequality (38) and because $m(\phi)$ declines faster than $n(\phi)$.

Combining inequalities (38) and (39) we have

$$\int_{\phi_B}^{\phi_T}\pi(\phi,m(\phi))\,d\phi>\int_{\phi_B}^{\phi_T}\pi\left(\overset{\wedge}{\phi}(\phi),n(\phi)\right)d\phi$$

which proves the lemma. ■

We can now use the result obtained from lemma (3) to show that there do not exist any equilibria in non-monotone strategies. From Proposition (2) we know that there exists a threshold signal s^* such that $\Pi(s^*, n) = 0$ and that $\Pi(s_i, n) \lesseqgtr 0 \; \forall s_i \lesseqgtr s^*$. If there is only one equilibrium then the threshold s^* will be unique and it will be a monotone equilibrium. Hence, if we can establish that there is only one point s^*, such that $\Pi(s^*, n) = 0$, then we would have proved the existence of non-monotone equilibrium. We provide a proof by contradiction.

Suppose that there exist more than one s^*, which solve $\Pi(s^*, n) = 0$. Let $\phi_H = \sup\{s_i : \Pi(s_i, n) = 0\}$, and $\phi_L = \sup\{s_i : s_i < \phi_H, \Pi(s_i, n) = 0\}$. Now $\Pi(s_i, n) < 0$ $\forall s_i \in (\phi_L, \phi_H)$. Suppose that $\phi_H - \phi_L < 2\epsilon$. Then, we have

$$\phi_L - \epsilon < \phi_L < \phi_H - \epsilon < \phi_L + \epsilon < \phi_H < \phi_H + \epsilon.$$

Now

$$\Pi(\phi_H, n) - \Pi(\phi_L, n) = \int_{\phi_H - \epsilon}^{\phi_H + \epsilon} \pi(\phi, n(\phi))\, d\phi - \int_{\phi_L - \epsilon}^{\phi_L + \epsilon} \pi(\phi, n(\phi))\, d\phi.$$

Eliminating the common parts of the two integrals, this can be rewritten as

$$\Pi(\phi_H, n) - \Pi(\phi_L, n) = \int_{\phi_L + \epsilon}^{\phi_H + \epsilon} \pi(\phi, n(\phi))\, d\phi - \int_{\phi_L - \epsilon}^{\phi_H - \epsilon} \pi(\phi, n(\phi))\, d\phi'$$

where $\phi' \leq \phi_L + \epsilon$. Consider the transformation $\phi'(\phi) = \phi_L + \phi_H - \phi$. Then by a change of variables we have

$$\Pi(\phi_H, n) - \Pi(\phi_L, n) = \int_{\phi_L + \epsilon}^{\phi_H + \epsilon} \pi(\phi, n(\phi))\, d\phi - \int_{\phi_L + \epsilon}^{\phi_H + \epsilon} \pi\left(\phi'(\phi), n(\phi)\right) d\phi.$$

We will now prove a claim which will then be used to demonstrate our Proposition.

Claim 3. *$n(\phi) = \theta + \frac{(1-\theta)}{2\epsilon}(\phi_H - \phi + \epsilon)$ for all $\phi \in [\phi_L + \epsilon, \phi_H + \epsilon]$ and $n(\phi_H - \epsilon) \geq n(\phi_L + \epsilon)$.*

Proof. Note that when $\phi \in [\phi_L + \epsilon, \infty)$, $s_i \in [\phi_L, \infty)$ and the only s_i for which $\Pi(s_i, n) < 0$ lie in the range (ϕ_L, ϕ_H). Thus for all $\phi \in [\phi_L + \epsilon, \infty)$, $n(\phi) = \Pr(s_i \leq \phi_H | \phi)$. In particular for $\phi \in [\phi_L + \epsilon, \phi_H + \epsilon]$, $n(\phi) = \Pr(s_i \leq \phi_H | \phi) = \theta + \frac{(1-\theta)}{2\epsilon}(\phi_H - \phi + \epsilon)$, which proves the first part of the claim. Similarly, $n(\phi_L + \epsilon) = \theta + \frac{(1-\theta)}{2\epsilon}(\phi_H - \phi_L)$. Note that for $\phi \in (-\infty, \phi_H - \epsilon]$, $n(\phi) \geq \Pr(s_i > \phi_L | \phi)$. This is because $\phi_L < \phi_H - \epsilon$ and while $\Pi(s_i, n)$ is definitely negative between ϕ_L and ϕ_H, it can also be negative elsewhere. Therefore, $n(\phi_H - \epsilon) \geq \Pr(s_i > \phi_L | \phi_H - \epsilon) = \theta + \frac{(1-\theta)}{2\epsilon}(\phi_H - \phi_L)$. Hence, $n(\phi_H - \epsilon) \geq \theta + \frac{(1-\theta)}{2\epsilon}(\phi_H - \phi_L) = n(\phi_L + \epsilon)$, which proves the claim. ■

Let $\phi_B = \phi_L + \epsilon$ and $\phi_T = \phi_H + \epsilon$. Then given claim 3 we have

$$\int_{\phi_B}^{\phi_T} \pi(\phi, n(\phi))\, d\phi = \int_{\phi_B}^{\phi_T} \pi\left(\phi, \theta + \frac{(1-\theta)}{2\epsilon}(\phi_T - \phi)\right) d\phi.$$

Note that $\int_{\phi_B}^{\phi_T} \pi\left(\phi, \theta + \frac{(1-\theta)}{2\epsilon}(\phi_T - \phi)\right) d\phi \geq 0$. Since $\phi'(\phi) \leq \phi_B$, then given claim 3 we have $n(\phi') \geq \theta + \frac{(1-\theta)}{2\epsilon}(\phi_T - \phi)$ for all $\phi \in [\phi_B, \phi_T]$. Now from a straightforward application of lemma 3 it follows that $\Pi(\phi_H, n) > \Pi(\phi_L, n)$. But if there exist more than one s^*, then $\Pi(\phi_H, n) - \Pi(\phi_L, n) = 0$, which is a contradiction. Q.E.D.

Proof of Proposition 4: Let $re_{LOLR} = \overset{\Delta}{n} D(1+r)$ and $re_{no\ LOLR} = \hat{n} D(1+r)$ where $\overset{\Delta}{n}$ and $\hat{n}$ are as defined in (31) and (24) respectively. We need to show that $re_{LOLR} < re_{no\ LOLR}$. This will be true if and only if $\overset{\Delta}{n} < \hat{n}$ or if and only if

$$\Pr(s_l < s_l^* | \phi < \phi_L) E(n | \phi < \phi_L) + \Pr(s_l < s_l^* | \phi \geq \phi_L) E(n | \phi \geq \phi_L) < E(n).$$

Using the law of iterated expectations, $E(n) = E_\phi [E(n|\phi)]$. Thus $re_{LOLR} < re_{no\ LOLR}$ if and only if

$$\Pr(s_l < s_l^* | \phi < \phi_L) E(n | \phi < \phi_L) + \Pr(s_l < s_l^* | \phi \geq \phi_L) E(n | \phi \geq \phi_L) <$$

$$\Pr(\phi < \phi_L) E(n | \phi < \phi_L) + \Pr(\phi \geq \phi_L) E(n | \phi \geq \phi_L)$$

or if and only if

$$[\Pr(\phi < \phi_L) - \Pr(s_l < s_l^* | \phi < \phi_L)] E(n | \phi < \phi_L) +$$

$$[\Pr(\phi \geq \phi_L) - \Pr(s_l < s_l^* | \phi \geq \phi_L)] E(n | \phi \geq \phi_L) > 0.$$

This will be the case if $\Pr(\phi < \phi_L) > \Pr(s_l < s_l^* | \phi < \phi_L)$ and

$\Pr(\phi \geq \phi_L) > \Pr(s_l < s_l^* | \phi \geq \phi_L)$. Using Baye's Theorem, $\Pr(s_l < s_l^* | \phi < \phi_L) = \frac{\Pr(s_l < s_l^* \cap \phi < \phi_L)}{\Pr(\phi < \phi_L)}$ and $\Pr(s_l < s_l^* | \phi \geq \phi_L) = \frac{\Pr(s_l < s_l^* \cap \phi \geq \phi_L)}{\Pr(\phi \geq \phi_L)}$.

Since $\Pr(s_l < s_l^* \cap \phi < \phi_L) < 1$ and $\Pr(s_l < s_l^* \cap \phi \geq \phi_L) < 1$, this proves the proposition. Q.E.D.

Proof of Proposition 6: We know from the zero profit condition (32) that the higher the LOLR subsidy, the greater would be the negative impact on the zero profit condition and hence the worse the moral hazard problem. It is also clear from equation (35) that an increase in S, will increase the insolvency point ϕ_{L2} and hence the probability of insolvency. Thus to prove the proposition, all that we need to show is that $\frac{\partial S}{\partial \epsilon_L} > 0$. Since, $S = \Pr(s_l \geq s_l^* | \phi < \phi_L) E(n | \phi < \phi_L)$, $\frac{\partial S}{\partial \epsilon_L} > 0$ if and only if $\frac{\partial \Pr(s_l \geq s_l^* | \phi < \phi_L)}{\partial \epsilon_L} > 0$.

To show that $\frac{\partial \Pr(s_l \geq s_l^* | \phi < \phi_L)}{\partial \epsilon_L} > 0$, we first need to calculate an analytical expression for $\Pr(s_l \geq s_l^* | \phi < \phi_L)$. We do this as follows:

$$\begin{aligned} P_{II} &= \Pr(s_l \geq s_l^* | \phi < \phi_L) \\ &= \Pr\left(\tilde{\phi} + \tilde{\epsilon}_l \geq s_l^* | \phi < \phi_L\right) \\ &= \frac{\Pr\left(\tilde{\epsilon}_l \geq s_l^* - \tilde{\phi}, \phi < \phi_L\right)}{\Pr(\phi < \phi_L)}. \end{aligned}$$

Given that $\phi \sim U\left[\underset{-}{\phi}, 1\right]$ and $\epsilon_l \sim U[-\epsilon_L, \epsilon_L]$, we have

$$P_{II} = \frac{\int_{\underline{\phi}}^{\phi_L} \int_{s_l^* - \widetilde{\phi}}^{\epsilon_L} \frac{1}{2\epsilon_L} \frac{1}{1-\underline{\phi}} d\widetilde{\epsilon_l} d\widetilde{\phi}}{\frac{\phi_L - \underline{\phi}}{1-\underline{\phi}}}$$

$$= \frac{\int_{\underline{\phi}}^{\phi_L} \frac{1}{2\epsilon_L} \frac{1}{1-\underline{\phi}} \left[\epsilon_L - s_l^* + \widetilde{\phi} \right] d\widetilde{\phi}}{\frac{\phi_L - \underline{\phi}}{1-\underline{\phi}}}$$

$$= \frac{\frac{1}{2\epsilon_L} \frac{1}{1-\underline{\phi}} \left(\epsilon_L - s_l^* \right) \left(\phi_L - \underline{\phi} \right) + \frac{1}{2\epsilon_L} \frac{1}{1-\underline{\phi}} \frac{\left(\phi_L^2 - \underline{\phi}^2 \right)}{2}}{\frac{\phi_L - \underline{\phi}}{1-\underline{\phi}}}.$$

Simplifying the above, we get the following expression for the probability of a Type II error:

$$P_{II} = \frac{\epsilon_L - s_l^* + \frac{1}{2} \left(\phi_L + \underline{\phi} \right)}{2\epsilon_L}. \tag{40}$$

Analagously, it can be shown that the probability of a Type I error is as follows:

$$P_I = \frac{\epsilon_L + s_l^* - \frac{1}{2} \left(1 + \phi_L \right)}{2\epsilon_L}. \tag{41}$$

Since s_l^* is such that both P_I and P_{II} are positive, therefore from (40) it follows that $P_{II} > 0$ if and only if $s_l^* < \left(\phi_L + \underline{\phi} \right) /2 + \epsilon_L$. Similarly, from (41) it follows that $P_I > 0$ if and only if $s_l^* > (1 + \phi_L)/2 - \epsilon_L$. Hence both P_I and P_{II} are positive if $(1 + \phi_L)/2 - \epsilon_L < s_l^* < \left(\phi_L + \underline{\phi} \right) /2 + \epsilon_L$. Note that for this restriction to make sense it needs to be the case that $\epsilon_L > \left(\phi_L - \underline{\phi} \right) /2$. The intuition is that if signals are very precise then it will not be possible to make both Type I and Type II errors given a bounded distribution. From footnote 29 also note that $\epsilon_L < (1 + \phi_L)/2$. Hence if P_I and P_{II} are positive it is always the case that

$$\frac{\left(\phi_L + \underline{\phi} \right)}{2} < s_l^* < \frac{(1 + \phi_L)}{2}.$$

Finally, taking the partial derivative of (40) with respect to ϵ_L we have

$$\frac{\partial P_{II}}{\partial \epsilon_L} = \frac{2\epsilon_L - 2\left[\epsilon_L - s_l^* + \frac{1}{2}\left(\phi_L + \underset{-}{\phi}\right)\right]}{4\epsilon_L^2}.$$

Thus $\frac{\partial P_{II}}{\partial \epsilon_L} > 0$ if and only if $2s_l^* - \phi_L - \underset{-}{\phi} > 0$, or if and only if $s_l^* > \left(\phi_L + \underset{-}{\phi}\right)/2$, which is the case. Using the same line of reasoning it follows that $\frac{\partial P_I}{\partial \epsilon_L} > 0$. Q.E.D.

References

[1] Bernanke, Ben, 1983, Non monetary effects of the financial crisis in propagation of the great depression, *American Economic Review* 73, 257-276.

[2] Bernanke, Ben, and Mark Gertler, 1990, Financial fragility and economic performance, *Quarterly Journal of Economics* 105, 87-114.

[3] Calomiris, Charles W., 1998, The IMF's imprudent role as lender of last resort, *Cato Journal* 17, No. 3.

[4] Carlsson, Hans, and Eric van Damme, 1993, Global games and equilibrium selection, *Econometrica* 61, 989-1018.

[5] Dasgupta, Amil, 2002, Financial contagion through capital connections: a model of the origin and spread of bank panics, FMG discussion paper 436.

[6] Diamond, Douglas, 1984, Financial intermediation and delegated monitoring, *Review of Economic Studies* 51, 393-414.

[7] Diamond, Douglas, and Philip Dybvig, 1983, Bank runs, deposit insurance and liquidity, *Journal of Political Economy* 91, 401-419.

[8] Flannery, Mark J., 1996, Financial crises, payment system problems, and discount window lending, *Journal of Money, Credit and Banking 28, 804-824.*

[9] Freixas, Xavier, 1999, Optimal bail out policy, conditionality and constructive ambiguity, FMG discussion paper 327.

[10] Freixas, Xavier, and Jean-Charles Rochet, 1997, Microeconomics of banking, Cambridge: MIT Press.

[11] Goldstein, Itay, and Ady Pauzner, 2002, Demand deposit contracts and the probability of bank runs, mimeo, Tel Aviv University.

[12] Goodfriend, Marvin, and Robert King, 1988, Financial deregulation, monetary policy and central banking, *Federal Reserve Bank of Richmond Economic Review* May/June, 3-22.

[13] Goodhart, Charles A. E., 2000, The organisational structure of banking supervision, FMG special paper 127.

[14] Goodhart, Charles A. E., and Haizhou Huang, 2003, Lender of last resort, IMF working paper.

[15] Gorton, Gary, 1988, Banking panics and business cycles, *Oxford Economic Papers* 40, 751-781.

[16] Krasa, Stefan, and Anne P. Villamil, 1992, Monitoring the monitor: an incentive structure for a financial intermediary, *Journal of Economic Theory* 57, 197-221.

[17] Krugman, Paul, 1998, What happened to Asia, mimeo, MIT.

[18] Krugman, Paul, 1999, Balance sheets, the transfer problem, and financial crises, mimeo, MIT.

[19] Morris, Stephen, and Hyun S. Shin, 1998, Unique equilibrium in a model of self-fulfilling currency attacks, *American Economic Review* 88, 587-597.

[20] Morris, Stephen, and Hyun S. Shin, 2000, Rethinking multiple equilibria in macroeconomic modelling, NBER *Macroeconomics Annual*.

[21] Morris, Stephen, and Hyun S. Shin, 2001, Coordination risk and the price of debt, forthcoming in *European Economic Review*.

[22] Morris, Stephen, and Hyun S. Shin, 2002, Global games: theory and applications, in *Advances in Economics and Econometrics, the Eight World Congress* (edited by M. Dewatripont, L. Hansen and S. Turnovsky), Cambridge University Press.

[23] Naqvi, Hassan, 2001, Foundations of intermediation, LSE term paper.

[24] Rochet, Jean-Charles, and Xavier Vives, 2002, Coordination failures and the lender of last resort: was Bagehot right after all?, FMG discussion paper 408.