OPTIMAL MULTI-CURRENCY INVESTMENT STRATEGIES WITH EXACT ATTRIBUTION IN THREE ASIAN COUNTRIES

CORNELIS A. LOS, PH.D.

Centre for Research in Financial Services, Working Paper No. 98-01

ABSTRACT. Singer and Karnosky s (1995) exact and complete return attribution framework does not account for risk, since it ignores accumulated historical information. Its implied investment strategy selection is based on simple return maximization and ignores that investment strategies are correlated via intra-and inter-market risks. Using simple tensor algebra we extend their exact accounting framework to include market risk measurements for n countries. The resulting $n^2 \times n^2$ strategy risk matrix exactly decomposes into a tensor sum of the $n \times n$ fundamental market risk matrices. Since the strategy risk matrix is singular with $rank = 2n - 1 < n^2$, the resulting portfolio choice problem is degenerate. But the portfolio constraints imposed by the exact accounting framework allow to solve the conventional Markowitz mean-variance optimization problem as a nondegenerate lower dimensional problem of fundamental investment choice between stock markets and currency overlays, with a nonsingular $2n \times 2n$ risk matrix. The original n^2 investment strategy allocations are then uniquely retrieved from the resulting $2n$ optimal investment choices. Thus we solve also the problem of the optimization of complete, exact investment strategy portfolios, like RiskMetrics^{TM} and CreditMetrics^{TM}. Our complete and exact return-risk attribution accounting framework is applied to monthly return data of Singapore, Malaysia and Indonesia from July 1992 through June1997. The average historically maximal simple and risk-adjusted investment strategy returns are compared with the efficiency frontier computed for the ve year horizon of an efficiency-seeking global investor to determine their implied minimal risk levels. Furthermore, the paper analyzes which markets exhibit most risk in these Asian countries. The evidence shows that most of the risk is attributable to the magnitudes of the risks of the stock markets, followed by those of the currency markets and the cash markets.

1. INTRODUCTION

Since the floating of the Thai baht on July 2, 1997, currency, banking and stock market crises in Thailand, the Philippines, Indonesia and Malaysia have dramatically increased interest in multi-currency investment management in Southeast

Date: December 9, 1997.

The author is Associate Professor of Banking and Finance at the Nanyang Technological University in Singapore. He thanks his students Ho Su-Ping, Jeyanthi and Lim Cheer Hwi, who collected the data and performed the computations. This paper has been presented at the 10th Annual Australasian Finance and Banking Conference, 4 & 5 December 1997, Sydney, Australia.

Asia.[1] Such investment management requires an *exact, uni ed accounting framework* for analysis of the performance of multi-currency portfolios, which involve various market assets, currencies and cross-currency hedging swaps or currency overlays.[2] Unfortunately, most Asian banks don t have such sophisticated portfolio accounting frameworks in place and this paper attempts to ll the existing theoretical lacunae in their accounting systems.

The key components of the performance of a multi-currency investment portfolio include: (1) market selection, (2) security selection, (3) currency selection and (4) hedge selection, and their cross-product effects. The investor should be able to account for the separate impact of each of these key components on the portfolio s performance, both in terms of return and risk. This paper concentrates on (1), (3) and (4) and ignores the issue of security selection, because it would unnecessarily complicate the presentation.[3]

The *cash accounting* approach presented here is based on that of Singer and Karnosky [25], [14][24], who use an extension of Sharpe s Capital Asset Pricing Model (CAPM) to complement the earlier *exact performance attribution* accounting framework of [3], [6] and [5]. The continuously compounding cash accounting framework of [25] and [14] accommodates only *return maximizing* investment strategies. But modern portfolio management requires *risk/return optimizing* investment strategies, i.e., investment strategies which seek the most desirable combination of high investment returns and low risks.[9][13][12] By using simple Kronecker product algebra we demonstrate, both theoretically and empirically, that Singer and Karnovsky s exact accounting framework can be extended to include Markowitz mean-variance optimization of a multi-currency investment portfolio. The empirical demonstration of the extended framework uses a simple portfolio of investments in the three Asian countries of Singapore, Malaysia and Indonesia, using monthly data from June 1993 to June 1997.[4]

This paper doesn t touch upon two important related issues, to be discussed in forthcoming papers. First, the demonstration in this paper remains static and backward looking, i.e., based on historical return data instead of on predicted returns. However, its framework can be further extended to a dynamic context with predictions of returns, using, for example, Kalman information lters. Secondly, in this paper we don t distinguish between systematic and unsystematic risk, as

[1] For a typical descriptive commentary on how globalization has magni ed the costs of bad policies and weak institutions in the region, in particular when regional integration and private capital ows are not managed properly, see the article by a Senior Economist with the Asian Development Bank in Manilla Pradumna B. Rana, Globalization and Currencies, *The Far Eastern Economic Review*, September 11, 1997, p.29. For an early description of the fears that the stock market and currency turmoil may cause growth stagnation in the ASEAN region, see reporter Darren McDermott s article Regional Economies May Be Hit Next, *The Asian Wall Street Journal*, September 3, 1997, p. 1.

[2] Such cross-currency hedges facilitate the (observed) risk of *currency contagion*, since they exactly connect stock and currency markets.

[3] The simple tensor algebra framework used in this paper can easily include the security selection when actual asset attributions and analysis are required, but it would only increase the dimensional complexity of the portfolio optimization problem.

[4] The current portfolio for Singapore, Malaysia and Indonesia has a narrow regional emphasis for the typical Singapore based investor. A more complete report is prepared to include a similar demonstration for a portfolio of multi-currency investments in seven Asian countries plus the United States and Japan. Such a portfolio would be more representative for a typical ASEAN investor.

Sharpe s centroid CAPM cash accounting would suggest, but take account of total nancial risk in a multi-currency investment portfolio, as suggested by Markowitz portfolio diversi cation approach.[5]

2. EXACT ATTRIBUTION

2.1. **Exact Investment Strategy Return Attribution.** According to the exact cash accounting framework of Singer and Karnosky, at time t an investor has three possible investment instruments: (1) investment in an *asset in country* i, e.g., a stock or a bond, with rate of return $r_i(t)$, (2) a *cash swap* with rate of return $c_j(t)-c_i(t)$, with $c_j(t)$ the cash rate in country j into which the nominal is swapped, and $c_i(t)$ the cash rate in country i out of which the nominal is swapped and (3) the *foreign currency* (foreign currency) *appreciation* rate $\varepsilon_j(t)$ of country j.[6] Thus one particular bilateral investment strategy at time t is represented by the strategic rate of return[7]

$$s_{ij}(t) = r_i(t) + [c_j(t) - c_i(t)] + \varepsilon_j(t)$$

Notice that such a strategy is, indeed, as Sharpe s CAPM prescribes, equivalent to the sum of a risk premium and a cash return, i.e., the local market i *risk premium* $[r_i(t) - c_i(t)]$ and the *cash return* on currency j, $[c_j(t) + \varepsilon_j(t)]$:[16][2][28]

$$s_{ij}(t) = [r_i(t) - c_i(t)] + [c_j(t) + \varepsilon_j(t)]$$

This is also equivalent to the sum of a local market i return, the return on a *currency forward cross hedge*, consisting of the difference of the returns between two currency forwards, and the foreign currency j appreciation rate, since

$$\begin{aligned} s_{ij}(t) &= r_i(t) + [\{c_1(t) - c_i(t)\} - \{c_1(t) - c_j(t)\}] + \varepsilon_j(t) \\ &= r_i(t) + [f_i(t) - f_j(t)] + \varepsilon_j(t) \\ &= r_i(t) + f_{ij}(t) + \varepsilon_j(t) \end{aligned}$$

The *return on a currency forward* is $f_i(t) = c_1(t) - c_i(t)$, with $c_1(t)$ the cash return of the base currency. The return on a *currency forward cross hedge* $f_{ij}(t)$ consists of the difference between the return on the long domestic forward $f_i(t)$ and the return on the short foreign forward $f_j(t)$.[17][23][15] Therefore, such a strategy selection can exactly account for the currency mismatches so prevalent in Southeast Asia. For

[5]The problem of decomposing risk into systematic and unsystematic risk in a portfolio context has not yet been solved for the following reason. Markowitz portfolio optimization requires the inversion of a positive de nite total return covariance matrix, while the systematic covariance matrix is singular, *by de nition*, and can t be inverted. Therefore one cannot uniquely determine the model connecting the systematic factors and the efficient portfolio frontier. Some epistemic uncertainty will remain. This is the topic of a fortcoming paper The Failure of RiskMetrics Attempted Grand Uni cation.

[6]Here asset and country selection are one and the same. Separate asset and country selection is possible within an extended framework. The US dollar is the *base* currency throughout this discussion.

[7]According to [27], a *strategy* is a rule which speci es how action at any time will depend on opportunities available at that time and information which has accumulated up to that time. While [25], [14] account for the current opportunities, they ignore the accumulated historical information in the strategy risk matrix discussed in this paper. Therefore, while the concept of a strategy used in this paper satis es von Neumann and Morgenstern s de nition, Singer and Karnosky s concept does not.

example, in dollar investment terms, an initial investment $P(0)$, e.g., of \$100mln, invested in a strategy earning $s_{ij}(t)$, grows in one period t as follows

$$P_{ij}(t) = P(0).e^{s_{ij}(t)} = P(0).e^{r_i(t)+[c_j(t)-c_i(t)]+\varepsilon_j(t)}$$

A domestic investor in country i, who borrows cash at $c_i(t)$ percent and invest in an asset to earn a total rate of return $r_i(t)$, gains the *risk premium* $r_i(t) - c_i(t)$, so that $P_{ij}(t) = P(0).e^{r_i(t)-c_i(t)}$. This particular accounting representation includes options, since any option can be represented by a combination of a risky asset and cash borrowing. The *total cash rate* earned in country j is $c_j(t) + \varepsilon_j(t)$. This accounting representation includes both domestic and foreign currency futures, $P_{ij}(t) = P(0).e^{c_j(t)+\varepsilon_j(t)}$, and cross-currency swaps, $P_{ij}(t) = P(0).e^{c_j(t)-c_i(t)}$. Thus both the spot, on balance-sheet items as well as the derivative, off-balance sheet instruments are included.[8] The terminal value of capital $P(0)$ invested over the holding horizon T implementing one particular investment strategy $s_{ij}(t)$ can now be represented by the concatenated expression

$$P_{ij}(T) = P(0)\prod_t^T e^{s_{ij}(t)} = P(0)\prod_t^T e^{r_i(t)+[c_j(t)-c_i(t)]+\varepsilon_j(t)}$$

An $n \times n$ non-symmetric *strategy matrix* at time t is a matrix containing all n^2 bilateral investment strategies

$$S(t) = \{s_{ij}(t); i,j = 1,...,n\}$$

For example, for $i,j = 1,2,3$ we have the 3×3 non-symmetric strategy matrix at time t

$$\begin{aligned} S(t) &= \begin{bmatrix} s_{11}(t) & s_{12}(t) & s_{13}(t) \\ s_{21}(t) & s_{22}(t) & s_{23}(t) \\ s_{31}(t) & s_{32}(t) & s_{33}(t) \end{bmatrix} \\ &= \left[\begin{bmatrix} r_1(t) \\ r_2(t) \\ r_3(t) \end{bmatrix} - \begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix}\right] \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \\ &\quad + \left\{\left[\begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \\ \varepsilon_3(t) \end{bmatrix}\right] \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\right\}' \end{aligned}$$

$$= \begin{bmatrix} r_1(t)+\varepsilon_1(t) & r_1(t)-c_1(t)+c_2(t)+\varepsilon_2(t) & r_1(t)-c_1(t)+c_3(t)+\varepsilon_3(t) \\ r_2(t)-c_2(t)+c_1(t)+\varepsilon_1(t) & r_2(t)+\varepsilon_2(t) & r_2(t)-c_2(t)+c_3(t)+\varepsilon_3(t) \\ r_3(t)-c_3(t)+c_1(t)+\varepsilon_1(t) & r_3(t)-c_3(t)+c_2(t)+\varepsilon_2(t) & r_3(t)+\varepsilon_3(t) \end{bmatrix}$$

Table 1 provides a real world example of such a matrix of pure return strategies for June 1997 with Singapore, Malaysia and Indonesia as the three investment countries of interest, with annualized monthly rates of return on their respective

[8]Such synthesis of hedging decisions and integration of portfolios with options and futures is desired.[11][26] Furthermore, this cash accounting framework is the core of J.P. Morgan s *RiskMetrics*^{TM}[18], as was also explained by Professor Mark Garman of Financial Engineering Associates, Inc., when he presented his seminar on Cash ow Mapping and Other Choices in Value at Risk (VaR), at the Nanyang Technological University, March 7, 1996. Furthermore, as in *RiskMetrics*^{TM}, this paper uses linear simpli cation to reduce the risk of all (contingent) assets to a linear risk representation.

stock markets indices in Singapore, Kuala Lumpur and Jakarta, the respective 30-day bank lending rates and the respective rates of return on the Singapore dollar, the ringgit and the rupiah.[9]

TABLE 1:	RETURN (%)	STRATEGIES,	IN JUNE 1997
$S(t) =$	1. Singapore	2. Malaysia	3. Indonesia
1. Singapore	44.2	34.8	45.3
2. Malaysia	48.0	38.7	49.1
3. Indonesia	87.8	78.4	88.9

$$S(t) = \begin{bmatrix} 33.4 + 10.8 & 33.4 - 3.9 + 7.9 - 2.6 & 33.4 - 3.9 + 14.5 + 1.3 \\ 41.3 - 7.9 + 3.9 + 10.8 & 41.3 - 2.6 & 41.3 - 7.9 + 14.5 + 1.3 \\ 87.6 - 14.5 + 3.9 + 10.8 & 87.6 - 14.5 + 7.9 - 2.6 & 87.6 + 1.3 \end{bmatrix}$$

Notice that on the diagonal of this investment strategy matrix we nd all strategies for the simple buy-and-hold strategy of investing in country i, where the return is the combination of what is earned in the stock market and what is earned on the currency. Thus, in June 1997 Singapore earned 44.2% on domestic investments, namely 33.4% at an annualized monthly rate in the stock market and10.8% at an annualized monthly rate on cash.[10] In contrast, the off-diagonal elements contain the strategies of buying stock in country i and swapping into the cash of country j. What is earned in such a strategy is a combination of stock market, swap earnings and exchange rate earnings, i.e., the sum of risk premium and foreign cash earnings. Thus in June 1997 an investor would earn an 33.4% in the Singapore stock market, but by swapping from the Sing dollar into the rupiah, he could earn an extra $14.5 - 3.9 = 10.6$%on the cash and 1.3% on the rupiah. Consequently, the rate of return on his strategy would have been 45.3% at an annualized rate, i.e., 113 basispoints more, on an annualized basis, than what was earned by staying fully invested in Singapore.

Table 1 shows that the dominant *maximizing* return strategy in June 1997, which disregards market risk, was to fully invest in the Indonesian stock market, since the Jakarta stock market gained 87.6% at an annual rate and the rupiah appreciated 1.3% at an annual rate. In June 1997, when invested in the stock markets Singapore or in Malaysia, an investor could have picked up some extra basis points by swapping into rupiahs. This would have been very advantageous to an investor in the Singapore stock market, who could have picked up 10.6% in the cash swap, but less so for an investor in the Kuala Lumpur stock market, who could have picked up only $14.5 - 7.9 = 6.6$%, on an annualized basis. Two days into July 1997 this preferred cash management strategy of swapping into rupiahs started to rapidly disintegrate.

[9]In Southeast Asia bank it is custom to use bank loans for margin cash, not Treasury paper. Vice versa, in Asia, bank loans are often collateralized by stock. Consequently, the interrelationships between the stock markets and cash markets is tighter than, say, in the USA or the USA, where such practices are (still) not allowed under the Glass-Steagall Act of 1933 and similar legislation. Interestingly, under the impact of the Asian, in particular Japanese practices in the 1980s, the full force of the Glass-Steagall Act has been eroded. The 1994 Mexican, the early 1990s Japanese, and the current ASEAN nancial crises may lead to a fundamental reconsideration of the prudence of such strictly separating the securities dealing and the commercial banking business.

[10]Rounding errors prevent exactness of some summations in the digit behind the decimal point.

Since, by rst order approximation[11], the portfolio value grows as follows in period t

$$P_{ij}(t) = P(0)(1 + s_{ij}(t)) = P(0)e^{s_{ij}(t)}$$

the *accumulation of a portfolio* of investment strategies in period t can be represented by

$$P(t) - P(0) = \sum_{i,j} w_{ij}[P_{ij}(t) - P(0)] = P(0)\sum_{i,j}^{n} w_{ij}s_{ij}(t)$$

where w_{ij} is the share of the investment capital allocated to strategy $s_{ij}(t)$ with $\sum_{i,j}^{n} w_{ij} = 1$. Thus the portfolio rate of return in period t is

$$s_p(t) = \sum_{i,j}^{n} w_{ij}s_{ij}(t)$$

But then the value of a strategy investment portfolio at time t is

$$P(t) = \sum_{i,j} w_{ij}P_{ij}(t) = P(0)[1 + \sum_{i,j}^{n} w_{ij}s_{ij}(t)] = P(0)e^{\sum_{i,j}^{n} w_{ij}s_{ij}(t)}$$

Combining the preceding equations we nd that, by rst order approximation, the terminal value of a portfolio of investment strategies is

$$P(T) = \sum_{i,j} w_{ij}P_{ij}(T) = P(0)\prod_{t}^{T} e^{\sum_{i,j}^{n} w_{ij}s_{ij}(t)}$$

By using simple linear algebra we will now rst recon gure the cash accounting system for easier computations for the portfolio optimizations. Using the de nition of a bilateral strategy, the strategy matrix $S(t)$ at time t can be generalized, $i, j = 1, 2, ..., n$, by

$$\begin{aligned} S(t) &= [\mathbf{r}(t) - \mathbf{c}(t)]\boldsymbol{\iota}_n' + \{[\mathbf{c}(t) + \boldsymbol{\varepsilon}(t)]\boldsymbol{\iota}_n'\}' \\ &= [\mathbf{r}(t) - \mathbf{c}(t)]\boldsymbol{\iota}_n' + \boldsymbol{\iota}_n[\mathbf{c}(t) + \boldsymbol{\varepsilon}(t)]', \end{aligned}$$

where $\mathbf{r}(t)$, $\mathbf{c}(t)$ and $\boldsymbol{\varepsilon}(t)$ are $n \times 1$ data vectors of asset rates, cash rates and foreign currency appreciation rates at time t, respectively, and $\boldsymbol{\iota}_n'$ is a $1 \times n$ unit vector, i.e., a vector consisting of n units, $\boldsymbol{\iota}_n' = [1, 1, ..., 1]$.

The 3-dimensional $n \times n \times T$ historical investment strategy *array* $\mathbf{S}$ is the *sequence* of strategy matrices $\mathbf{S} = \{S(t); t = 1, ..., T\}$. For our empirical example, $\mathbf{S}$ is a $3 \times 3 \times 60$ array. Vectorization of this investment strategy array $\mathbf{S}$ and the use of Kronecker products facilitates the analysis of the presumed stationary strategy time series. The notation $vec(S(t))$ means the $n^2 \times 1$ strategy vector whose rst n elements are the rst column of $S(t)$, $\mathbf{s}_1(t)$; the second n elements, the second

[11] The approximation is more accurate when the periods $t = 1, 2, ..., T$ and thus the periodic rates $s_{ij}(t)$ are smaller. For example, daily valuations are more accurate than weekly valuations, which are more accurate than monthly, which are more accurate than quarterly valuations, etc. An increasing number of global nancial institutions is able to compute daily valuations of their portfolios, thanks to electronic accounting systems, and as strongly recommended by the Association for Investment Management and Research [1], p. 66.

column of $S(t)$, $\mathbf{s}_2(t)$, and so on. Thus $vec(S(t)) = [\mathbf{s}_1(t)', \mathbf{s}_2(t)', ..., \mathbf{s}_n(t)]'$ and the strategy vector at time t can be written as[12]

$$vec(S(t)) = [\boldsymbol{\iota}_n \quad \mathbf{I}_n][\mathbf{r}(t) - \mathbf{c}(t)] + [\mathbf{I}_n \quad \boldsymbol{\iota}_n][\mathbf{c}(t) + \boldsymbol{\varepsilon}(t)]$$

where $\mathbf{I}_n$ is the $n \times n$ identity matrix. Thus the vectorization of the $n \times n \times T$ historical investment strategy array $\mathbf{S}$ is represented by the $n^2 \times T$ matrix

$$\begin{aligned} \mathbf{VEC(S)} &= [vec(S(1)), vec(S(2)), ..., vec(S(T))] \\ &= [\boldsymbol{\iota}_n \quad \mathbf{I}_n][\mathbf{r} - \mathbf{c}] + [\mathbf{I}_n \quad \boldsymbol{\iota}_n][\mathbf{c} + \boldsymbol{\varepsilon}] \\ &= \mathbf{H} \begin{bmatrix} [\mathbf{r} - \mathbf{c}] \\ [\mathbf{c} + \boldsymbol{\varepsilon}] \end{bmatrix} \end{aligned}$$

where $\mathbf{r}$ is the $n \times T$ matrix of T observations on the rates of return of n country assets, $\mathbf{c}$ is the $n \times T$ matrix of observations on the n cash rates, and $\boldsymbol{\varepsilon}$ is the $n \times T$ matrix of T observations on the n currency appreciation rates, all with $T > n^2$. Consequently, $[\mathbf{r} - \mathbf{c}]$ is the $n \times T$ matrix of T observations on the n country risk premia and $[\mathbf{c} + \boldsymbol{\varepsilon}]$ is the $n \times T$ matrix of observations on the n country cash earning rates. $\mathbf{H} = \begin{bmatrix} [\boldsymbol{\iota}_n \quad \mathbf{I}_n] & [\mathbf{I}_n \quad \boldsymbol{\iota}_n] \end{bmatrix}$ is the $n^2 \times 2n$ *selector matrix*, which embodies the exact accounting identities and which, as we will see, plays an important role in the computation of the strategy risks.[13]

Table 2 shows the $\mathbf{VEC(S)}$ matrix of the three Asian countries, i.e., monthly time series of strategy vectors for Singapore, Malaysia and Indonesia for the ve years from July 1993 through June 1997. There are $n^2 = 9$ strategies over $T = 60$ periods, thus $\mathbf{VEC(S)}$ is a 9×60 matrix. By arcing we have indicated which strategies were the return maximizing strategies for each month. We ll discuss the switching behavior of the maximizing strategies in the next section.

TABLE2: RATES OF RETURN OF INVESTMENT STRATEGIES

Pure Strategies	J-92	A-92	S-92	O-92	N-92	D-92	J-93	F-93	M-93	A-93	M-93	J-93	J-93	A-93	S-93	O-93	N-93	D-93
s_{11}	-9.1	-32.9	-48.9	-16.4	33.2	27.8	43.3	89.6	11.6	12.0	102.8	74.8	-79.2	17.8	132.7	19.0	46.0	-5.4
s_{21}	23.7	9.6	-52.3	48.1	68.7	-39.0	1.8	-43.6	15.1	26.5	132.7	40.1	-55.1	74.5	90.1	**67.0**	129.8	48.8
s_{31}	52.9	-2.0	-60.5	-22.5	-6.7	-112.4	-60.3	15.6	71.2	35.8	18.6	101.1	37.6	-12.6	203.4	66.7	55.4	107.1
s_{12}	9.0	-31.8	-47.8	-17.2	54.4	35.2	15.9	92.3	12.2	31.6	102.9	76.3	-68.1	21.8	135.8	7.5	50.4	6.3
s_{22}	41.8	10.6	-51.2	47.3	90.0	-31.6	-25.6	-41.0	15.7	46.1	**132.8**	41.6	-43.9	**78.5**	93.2	55.6	134.2	60.5
s_{32}	**71.0**	-1.0	-59.4	-23.2	14.6	-105.0	-87.7	18.2	71.8	**55.4**	18.7	**102.6**	48.8	-8.5	**206.5**	55.3	59.8	**118.7**
s_{13}	-3.0	-22.1	**-45.4**	-14.3	71.4	**40.2**	**58.5**	**103.2**	21.7	11.6	92.0	75.3	-60.2	15.1	129.1	8.3	59.4	6.2
s_{23}	29.9	**20.4**	-48.8	**50.2**	**106.9**	-26.5	17.0	-30.0	25.2	26.1	121.9	40.6	-36.0	71.8	86.6	56.4	**143.3**	60.4
s_{33}	59.1	8.8	-57.0	-20.3	31.5	-100.0	-45.1	29.2	**81.4**	35.4	7.8	101.6	**56.7**	-15.2	199.9	56.1	68.8	118.7
Maximum Strategy	71.0	20.4	-45.4	50.2	106.9	40.2	58.5	103.2	81.4	55.4	132.8	102.6	56.7	78.5	206.5	67.0	143.3	118.7

TABLE 3: RISK-ADJUSTED RATES OF RETURN OF INVESTMENT STRATEGIES

Risk/adjust. Strategies	J-92	A-92	S-92	O-92	N-92	D-92	J-93	F-93	M-93	A-93	M-93	J-93	J-93	A-93	S-93	O-93	N-93	D-93
s_{11}/σ_{s11}	-0.16	-0.58	-0.87	-0.29	0.59	0.49	0.77	1.59	0.21	0.21	1.82	1.32	-1.40	0.32	2.35	0.34	0.81	-0.10
s_{21}/σ_{s21}	0.30	0.12	-0.67	0.61	0.88	-0.50	0.02	-0.56	0.19	0.34	1.69	0.51	-0.70	0.95	1.15	0.85	1.65	0.62
s_{31}/σ_{s31}	0.71	-0.03	-0.81	-0.30	-0.09	-1.50	-0.80	0.21	0.95	0.48	0.25	1.35	0.50	-0.17	**2.72**	**0.89**	0.74	1.43
s_{12}/σ_{s12}	0.16	-0.58	-0.87	-0.31	0.99	0.64	0.29	1.68	0.22	0.57	**1.87**	**1.38**	-1.24	0.40	2.46	0.14	0.92	0.11
s_{22}/σ_{s22}	0.53	0.13	-0.65	0.60	1.14	-0.40	-0.32	-0.52	0.20	0.58	1.68	0.53	-0.56	**0.99**	1.18	0.70	1.70	0.77
s_{32}/σ_{s32}	**0.92**	-0.01	-0.77	-0.30	0.19	-1.36	-1.14	0.24	0.93	**0.72**	0.24	1.33	0.63	-0.11	2.68	0.72	0.78	**1.54**
s_{13}/σ_{s13}	-0.05	-0.39	-0.80	-0.25	1.25	**0.70**	**1.03**	**1.81**	0.38	0.20	1.61	1.32	-1.05	0.26	2.26	0.15	1.04	0.11
s_{23}/σ_{s23}	0.37	**0.26**	**-0.61**	**0.63**	**1.34**	-0.33	0.21	-0.38	0.32	0.33	1.53	0.51	-0.45	0.90	1.09	0.71	**1.80**	0.76
s_{33}/σ_{s33}	0.77	0.11	-0.74	-0.26	0.41	-1.30	-0.59	0.38	**1.06**	0.46	0.10	1.32	**0.74**	-0.20	2.60	0.73	0.89	1.54
Max Risk-Adj. Strategy	0.92	0.26	-0.61	0.63	1.34	0.70	1.03	1.81	1.06	0.72	1.87	1.38	0.74	0.99	2.72	0.89	1.80	1.54

Table 2A

[12]When A is an $m \times n$ matrix and B a $p \times q$ matrix, then the *Kronecker product* of these two matrices is de ned by $A \quad B = [a_{ij}B]$ and $vec(AB) = (B' \quad I)vec(A) = (I \quad A)vec(B)$. For proof cf. [8], pp. 519. For a compilation of useful Kronecker product results, cf. [19], pp. 543-548.

[13]A similar crucial use of an exact selector matrix combined with Kronecker products can be found in [19] and, more recently, in [7].

J-94	F-94	M-94	A-94	M-94	J-94	J-94	A-94	S-94	O-94	N-94	D-94	J-95	F-95	M-95	A-95	M-95	J-95	J-95	A-95	S-95	O-95	N-95	D-95
168.6	-30.8	-11.8	-118.1	**129.5**	2.1	-24.3	31.5	54.6	**19.1**	18.8	**-64.8**	8.4	**-80.4**	28.7	4.1	10.3	74.8	-64.7	28.8	-29.1	-14.5	**4.0**	16.3
274.1	-170.7	10.7	-171.2	122.8	-68.6	**31.0**	**73.6**	113.0	-17.4	-27.3	-119.2	-16.6	-110.6	124.8	**15.2**	-10.1	136.4	-60.3	41.2	-85.8	-41.4	-41.4	37.5
140.8	**13.6**	-94.4	-120.8	-69.9	93.0	-88.6	-10.1	147.7	-31.4	64.0	-104.2	-37.8	-100.2	40.3	-55.3	-27.5	173.9	-1.1	38.6	-59.9	-47.1	-3.4	-16.3
128.8	-86.7	4.2	**-103.5**	119.1	31.2	-41.6	24.2	66.1	2.5	13.9	-69.5	**8.6**	-83.6	28.6	-14.6	**28.8**	84.9	-48.4	23.3	-23.2	-16.0	-18.6	27.6
234.3	-226.6	**26.7**	-156.6	112.4	-39.6	13.7	66.2	124.4	-34.0	-32.2	-124.0	-16.4	-113.9	124.7	-3.5	8.4	146.5	-44.0	35.7	-79.9	-42.8	-64.1	**48.8**
101.0	-42.4	-78.5	-106.1	-80.3	**122.0**	-105.9	-17.4	**159.2**	-48.1	59.1	-108.9	-37.7	-103.4	40.2	-74.0	-8.9	183.9	**15.2**	33.1	-54.0	-48.6	-26.0	-5.1
177.6	-41.1	-29.8	-125.7	118.1	-12.6	-27.0	26.9	52.9	13.7	19.3	-64.9	4.0	-85.2	37.5	-27.0	11.8	92.3	-48.5	32.7	**-9.2**	**6.1**	-3.9	25.0
283.1	-181.0	-7.3	-178.8	111.5	-83.3	28.3	68.9	111.2	-22.8	-26.8	-119.3	-21.0	-115.4	**133.6**	-15.9	-8.6	153.8	-44.1	**45.1**	-65.8	-20.7	-49.4	46.2
149.8	3.3	-112.5	-128.3	-81.3	78.3	-91.3	-14.7	146.0	-36.8	**64.5**	-104.2	-42.3	-104.9	49.2	-86.3	-25.9	**191.3**	15.1	42.5	-40.0	-26.5	-11.3	-7.6
283.1	13.6	26.7	-103.5	129.5	122.0	31.0	73.6	159.2	19.1	64.5	-64.8	8.6	-80.4	133.6	15.2	28.8	191.3	15.2	45.1	-9.2	6.1	4.0	48.8

J-94	F-94	M-94	A-94	M-94	J-94	J-94	A-94	S-94	O-94	N-94	D-94	J-95	F-95	M-95	A-95	M-95	J-95	J-95	A-95	S-95	O-95	N-95	D-95
2.99	-0.55	-0.21	-2.09	**2.29**	0.04	-0.43	0.56	0.97	**0.34**	0.33	-1.15	0.15	-1.42	0.51	0.07	0.18	1.33	-1.15	0.51	-0.52	-0.26	**0.07**	0.29
3.49	-2.17	0.14	-2.18	1.57	-0.87	**0.39**	**0.94**	1.44	-0.22	-0.35	-1.52	-0.21	-1.41	1.59	**0.19**	-0.13	1.74	-0.77	0.52	-1.09	-0.53	-0.53	0.48
1.88	**0.18**	-1.26	-1.61	-0.93	1.24	-1.18	-0.13	1.97	-0.42	**0.85**	-1.39	-0.50	**-1.34**	0.54	-0.74	-0.37	2.32	-0.02	0.52	-0.80	-0.63	-0.05	-0.22
2.34	-1.57	0.08	-1.88	2.16	0.57	-0.76	0.44	1.20	0.05	0.25	-1.26	**0.16**	-1.52	0.52	-0.27	**0.52**	1.54	-0.88	0.42	-0.42	-0.29	-0.34	0.50
2.97	-2.87	**0.34**	-1.98	1.42	-0.50	0.17	0.84	1.58	-0.43	-0.41	-1.57	-0.21	-1.44	1.58	-0.04	0.11	1.86	-0.56	0.45	-1.01	-0.54	-0.81	**0.62**
1.31	-0.55	-1.02	**-1.38**	-1.04	**1.58**	-1.37	-0.23	**2.07**	-0.62	0.77	-1.41	-0.49	-1.34	0.52	-0.96	-0.12	2.39	**0.20**	0.43	-0.70	-0.63	-0.34	-0.07
3.11	-0.72	-0.52	-2.20	2.07	-0.22	-0.47	0.47	0.93	0.24	0.34	**-1.14**	0.07	-1.49	0.66	-0.47	0.21	1.62	-0.85	**0.57**	**-0.16**	**0.11**	-0.07	0.44
3.55	-2.27	-0.09	-2.24	1.40	-1.04	0.35	0.86	1.39	-0.29	-0.34	-1.50	-0.26	-1.45	**1.68**	-0.20	-0.11	1.93	-0.55	0.57	-0.83	-0.26	-0.62	0.58
1.95	0.04	-1.46	-1.67	-1.06	1.02	-1.19	-0.19	1.90	-0.48	0.84	-1.35	-0.55	-1.36	0.64	-1.12	-0.34	**2.48**	0.20	0.55	-0.52	-0.34	-0.15	-0.10
3.55	0.18	0.34	-1.38	2.29	1.58	0.39	0.94	2.07	0.34	0.85	-1.14	0.16	-1.34	1.68	0.19	0.52	2.48	0.20	0.57	-0.16	0.11	0.07	0.62

Table 2B

J-96	F-96	M-96	A-96	M-96	J-96	J-96	A-96	S-96	O-96	N-96	D-96	J-97	F-97	M-97	A-97	M-97	J-97	Pure Strategies
73.7	90.0	-8.9	-17.7	21.5	-49.4	-19.4	-96.3	11.2	23.4	-59.2	76.4	15.1	-6.7	-27.4	-78.9	-46.7	44.2	s_{11}
19.1	61.4	43.9	69.2	28.6	-67.6	-1.1	-59.4	22.2	26.2	34.5	49.9	15.5	-28.0	17.4	-85.8	-123.0	48.0	s_{21}
52.0	130.5	9.8	-10.9	71.5	-24.7	-70.0	-113.3	-13.9	62.9	-31.9	**86.7**	31.4	71.1	-5.2	-107.2	**-19.2**	87.8	s_{31}
74.0	90.7	-3.9	-11.0	43.2	-45.5	-9.3	-90.0	9.2	23.1	-66.2	75.1	16.8	21.8	-1.6	-62.5	-53.5	34.8	s_{12}
19.5	62.1	**48.9**	**75.9**	50.3	-63.7	9.0	**-53.2**	20.2	25.9	27.5	48.7	17.3	0.4	**43.2**	-69.3	-129.8	38.6	s_{22}
52.3	131.1	14.8	-4.3	**93.2**	-20.8	-59.9	-107.0	-15.9	62.6	-38.9	85.5	**33.1**	**99.6**	20.6	-90.7	-26.0	78.4	s_{32}
81.7	107.9	-13.3	-22.7	37.5	-36.1	-2.1	-91.4	17.4	44.2	-48.9	73.6	15.6	6.8	-11.1	**-58.8**	-46.8	45.3	s_{13}
27.2	79.3	39.5	64.2	44.6	-54.4	**16.3**	-54.5	**28.4**	47.0	**44.8**	47.2	16.0	-14.5	33.6	-65.7	-123.1	49.1	s_{23}
60.1	**148.3**	5.4	-15.9	87.6	**-11.4**	-52.7	-108.4	-7.7	**83.7**	-21.6	84.0	31.9	84.6	11.0	-87.0	-19.3	**88.9**	s_{33}
81.7	148.3	48.9	75.9	93.2	-11.4	16.3	-53.2	28.4	83.7	44.8	86.7	33.1	99.6	43.2	-58.8	-19.2	88.9	**Maximum Strategy**

J-96	F-96	M-96	A-96	M-96	J-96	J-96	A-96	S-96	O-96	N-96	D-96	J-97	F-97	M-97	A-97	M-97	J-97	Risk/adjust. Strategies
1.31	1.59	-0.16	-0.31	0.38	-0.87	-0.34	-1.71	0.20	0.41	-1.05	1.35	0.27	-0.12	-0.49	-1.40	-0.83	0.78	s_{11}/σ_{s11}
0.24	0.78	0.56	0.88	0.36	-0.86	-0.01	-0.76	0.28	0.33	0.44	0.64	0.20	-0.36	0.22	-1.09	-1.57	0.61	s_{21}/σ_{s21}
0.69	1.74	0.13	-0.15	0.95	-0.33	-0.93	-1.51	-0.19	0.84	-0.43	1.16	0.42	0.95	-0.07	-1.43	-0.26	**1.17**	s_{31}/σ_{s31}
1.34	1.65	-0.07	-0.20	0.78	-0.83	-0.17	-1.63	0.17	0.42	-1.20	**1.36**	0.31	0.40	-0.03	-1.13	-0.97	0.63	s_{12}/σ_{s12}
0.25	0.79	**0.62**	**0.96**	0.64	-0.81	0.11	**-0.67**	0.26	0.33	0.35	0.62	0.22	0.01	**0.55**	-0.88	-1.64	0.49	s_{22}/σ_{s22}
0.68	1.70	0.19	-0.06	**1.21**	-0.27	-0.78	-1.39	-0.21	0.81	-0.50	1.11	**0.43**	**1.29**	0.27	-1.18	-0.34	1.02	s_{32}/σ_{s32}
1.43	1.89	-0.23	-0.40	0.66	-0.63	-0.04	-1.60	0.30	0.77	-0.86	1.29	0.27	0.12	-0.19	-1.03	-0.82	0.79	s_{13}/σ_{s13}
0.34	0.99	0.50	0.81	0.56	-0.68	**0.20**	-0.68	**0.36**	0.59	**0.56**	0.59	0.20	-0.18	0.42	**-0.82**	-1.54	0.62	s_{23}/σ_{s23}
0.78	**1.93**	0.07	-0.21	1.14	**-0.15**	-0.68	-1.41	-0.10	**1.09**	-0.28	1.09	0.41	1.10	0.14	-1.13	**-0.25**	1.15	s_{33}/σ_{s33}
1.43	1.93	0.62	0.96	1.21	-0.15	0.20	-0.67	0.36	1.09	0.56	1.36	0.43	1.29	0.55	-0.82	-0.25	1.17	**Max Risk-Adj. Strategy**

Table 2C

2.2. **Exact Investment Strategy Risk Attribution.** To nd the market risks involved in the various investment strategies the data covariance matrix of all the strategies is computed from the deviations from their holding horizon averages. The averages of each of the n strategies are given by the $n^2 \times 1$ vector

$$\overline{\mathbf{VEC}(\mathbf{S})} = \frac{\mathbf{VEC}(\mathbf{S}).\boldsymbol{\iota}_T}{T}$$

where $\boldsymbol{\iota}_T$ is the $T \times 1$ unit vector. Thus

$$\overline{\mathbf{VEC}(\mathbf{S})} = \mathbf{H} \begin{bmatrix} [\bar{\mathbf{r}} - \bar{\mathbf{c}}] \\ [\bar{\mathbf{c}} + \bar{\boldsymbol{\varepsilon}}] \end{bmatrix}$$

In the case of the three Asian countries the 5-year average for the stock market premia are

$$\overline{\mathbf{r}} - \overline{\mathbf{c}} = \begin{bmatrix} 6.2 \\ 12.9 \\ 17.0 \end{bmatrix} - \begin{bmatrix} 2.7 \\ 6.6 \\ 13.3 \end{bmatrix} = \begin{bmatrix} 3.5 \\ 6.3 \\ 3.7 \end{bmatrix}$$

Thus in the period July 1992 through June 1997, on average, the stock market gained most in Indonesia, 17.0%, and least in Singapore at 6.2%. But their stock market premia did not contribute much: 3.7% and 3.5%, respectively. On average, Malaysia had the largest stock market risk premium at 6.3%. Similarly we have the 5-year average for the cash earnings

$$\overline{\mathbf{c}} + \overline{\varepsilon} = \begin{bmatrix} 2.7 \\ 6.6 \\ 13.3 \end{bmatrix} + \begin{bmatrix} 2.5 \\ 0.2 \\ -3.6 \end{bmatrix} = \begin{bmatrix} 5.2 \\ 6.8 \\ 9.7 \end{bmatrix}$$

The substance of the strategy returns was earned in the cash overlay markets. On average, the Singapore dollar appreciated, while the ringgit remained about even and the rupiah depreciated versus the US dollar. On cash international investors could gain, on average, most in Indonesia at 9.7%, followed by Malaysia at 6.8% and Singapore at 5.2%. The (transposed) vector for the average returns earned by the 9 strategies is thus in Table 4.

TABLE 4:	AVERAGE	RETURN	STRATEGIES	(%)	7/92	-	6/97		
	$\overline{s}_{11}$	$\overline{s}_{21}$	$\overline{s}_{31}$	$\overline{s}_{12}$	$\overline{s}_{22}$	$\overline{s}_{32}$	$\overline{s}_{13}$	$\overline{s}_{23}$	$\overline{s}_{33}$
$\overline{\mathbf{VEC(S)}}' =$	8.7	11.6	9.0	10.3	13.2	10.6	13.2	**16.0**	13.4

In Table 5 this average return strategy vector is reconstituted into the better recognizable matrix of (5 year) average strategy returns.

TABLE 5:	AVERAGE RETURN	STRATEGIES	(%), 7/92 - 6/97
$\overline{\mathbf{S}} =$	1. Singapore	2. Malaysia	3. Indonesia
1. Singapore	8.7	10.3	13.2
2. Malaysia	11.6	13.2	**16.0**
3. Indonesia	9.0	10.6	13.4

In the period July 1992 through June 1997, Singer and Karnosky s *maximizing average return strategy* would have been to invest in the Malaysian stock market and to swap into the rupiah. On average, this strategy would have earned a 16% return per year. Indeed, we just noticed that, on overage, Malaysia had the largest stock market risk premium, while Indonesia earned most of its returns on its cash (versus the US dollar). Next, the $n^2 \times T$ matrix of deviations from the means are given by

$$\begin{aligned} \mathbf{DEV(S)} &= \mathbf{VEC(S)} - \overline{\mathbf{VEC(S)}}.\boldsymbol{\iota}_T' \\ &= \mathbf{H} \begin{bmatrix} [\mathbf{r}-\mathbf{c}] \\ [\mathbf{c}+\varepsilon] \end{bmatrix} [\mathbf{I}_T - \frac{\boldsymbol{\iota}_T\boldsymbol{\iota}_T'}{T}] \end{aligned}$$

where $\mathbf{I}_T$ is the $T \times T$ identity matrix.[14] We can now compute the $n^2 \times n^2$ data covariance, or *strategy risk matrix* of the n^2 investment strategies

$$\Sigma = \frac{\mathbf{DEV(S).DEV(S)}'}{T}$$

and exactly decompose this expression into its various nancial market risks

$$\begin{aligned} \Sigma &= \left\{ \mathbf{H} \overset{\square}{} \begin{bmatrix} [\mathbf{r}-\mathbf{c}] \\ [\mathbf{c}+\varepsilon] \end{bmatrix} [\mathbf{I}_T - \frac{\boldsymbol{\iota}_T \boldsymbol{\iota}_T'}{T}] \right\} \left\{ \mathbf{H} \overset{\square}{} \begin{bmatrix} [\mathbf{r}-\mathbf{c}] \\ [\mathbf{c}+\varepsilon] \end{bmatrix} [\mathbf{I}_T - \frac{\boldsymbol{\iota}_T \boldsymbol{\iota}_T'}{T}] \right\}' /T \\ &= \mathbf{H \Phi H}' \end{aligned}$$

where Φ is the $2n \times 2n$ risk premium - cash return covariance matrix. First, the term involving the asset risk premia is[15]

$$\begin{aligned} & [\boldsymbol{\iota}_n \quad \mathbf{I}_n][\mathbf{r}-\mathbf{c}][\mathbf{I}_T - \frac{\boldsymbol{\iota}_T \boldsymbol{\iota}_T'}{T}]^2 [\mathbf{r}-\mathbf{c}]'[\boldsymbol{\iota}_n \quad \mathbf{I}_n]'/T \\ = \; & [\boldsymbol{\iota}_n \quad \mathbf{I}_n][\Sigma_{rr} + \Sigma_{cc} - \Sigma_{rc} - \Sigma_{rc}'][\boldsymbol{\iota}_n' \quad \mathbf{I}_n] \\ = \; & \boldsymbol{\iota}_n \boldsymbol{\iota}_n' \quad [\Sigma_{rr} + \Sigma_{cc} - \Sigma_{rc} - \Sigma_{rc}'] \end{aligned}$$

where $\boldsymbol{\iota}_n \boldsymbol{\iota}_n'$ is an $n \times n$ unit matrix, i.e., a matrix consisting of ones. The $n \times n$ covariance matrix of the risk premia $[\Sigma_{rr} + \Sigma_{cc} - \Sigma_{rc} - \Sigma_{rc}']$ consists of the covariance matrices of the stock markets Σ_{rr}, of the cash markets Σ_{cc} and of the interaction between the stock and cash markets Σ_{rc}. The matrix Σ_{rr} measures all covariances between the n stock markets. The matrix Σ_{cc} measures all covariances between the n cash markets. Considering that covariance matrices are positive de nite, this expression shows that the interaction between the stock and cash markets, as measured by Σ_{rc}, reduces the overall market risk $\Sigma_{rr} + \Sigma_{cc}$ emanating from both types of markets. For the 5-year horizon from July 1992 through June 1997 we have for the risk premia of the stock and cash markets the 3×3 summation matrix

$$\begin{aligned} & [\Sigma_{rr} + \Sigma_{cc} - \Sigma_{rc} - \Sigma_{rc}'] = \\ & \begin{bmatrix} 3151.43 & 3340.20 & 2613.38 \\ 3340.20 & 6238.26 & 3266.89 \\ 2613.38 & 3266.89 & 5662.13 \end{bmatrix} + \begin{bmatrix} 0.54 & -0.28 & -0.30 \\ -0.28 & 1.11 & 1.05 \\ -0.30 & 1.05 & 6.22 \end{bmatrix} \\ & - \begin{bmatrix} -4.74 & 5.04 & -20.91 \\ -9.54 & 9.34 & -20.65 \\ -10.07 & 12.07 & -9.95 \end{bmatrix} - \begin{bmatrix} -4.74 & -9.54 & -10.07 \\ 5.04 & 9.34 & 12.07 \\ -20.91 & -20.65 & -9.95 \end{bmatrix} \\ & = \begin{bmatrix} 3161.45 & 3344.42 & 2644.07 \\ 3344.42 & 6220.69 & 3276.52 \\ 2644.07 & 3276.52 & 5688.25 \end{bmatrix} \end{aligned}$$

---

[14]The $T \times T$ symmetric matrix $[\mathbf{I}_T - \frac{\iota_T \iota_T'}{T}]$ computes deviations from means of any $n \times T$ matrix of T observations on n variables. This singular matrix has the form

$$\begin{bmatrix} 1-\frac{1}{T} & -\frac{1}{T} & \ldots & -\frac{1}{T} \\ -\frac{1}{T} & 1-\frac{1}{T} & \ldots & -\frac{1}{T} \\ \ldots & \ldots & \ldots & \ldots \\ -\frac{1}{T} & -\frac{1}{T} & \ldots & 1-\frac{1}{T} \end{bmatrix}$$

All its eigenvalues equal unity, $\lambda_i = 1$ for $i = 1, 2, ..., T-1$, except one, which is zero and thus the determinant $\left| \mathbf{I}_T - \frac{\iota_T \iota_T'}{T} \right| = \prod_i^T \lambda_i = \lambda_T = 0$ and $rank[\mathbf{I}_T - \frac{\iota_T \iota_T'}{T}] = T - 1$.

[15]Matrix multiplication of Kronecker products proceeds as follows $(\mathbf{A}_1 \quad \mathbf{B}_1)(\mathbf{A}_2 \quad \mathbf{B}_2) = (\mathbf{A}_1\mathbf{A}_2 \quad \mathbf{B}_1\mathbf{B}_2)$ for compatible matrices $\mathbf{A}_1, \mathbf{A}_2, \mathbf{B}_1$ and $\mathbf{B}_2$.

Notice that (1) *most of the risk resides in the stock markets* of Malaysia, Indonesia and then Singapore; (2) the cash market of Indonesia had more risk than Malaysia, followed by Singapore; (3) the stock and cash *interaction risk* matrix, Σ_{rc}, is asymmetric! From this stock and cash interaction risk matrix we observe that the returns of the cash markets of Singapore and Indonesia are negatively related to the returns in the stock markets in Singapore, Malaysia and Indonesia, but that the returns of the cash market of Malaysia are positively related to the returns in these three regional stock markets. This suggests that in Singapore and Indonesia we observe the usual trade-off between equity and cash investment because of cash hedging portfolio activity. But in Malaysia it appears that the returns earned in the three regional stock markets are the ones that are paid out on the cash deposits.

Second, the term involving the domestic and foreign cash earnings is

$$\begin{aligned}
& [\mathbf{I}_n \quad \boldsymbol{\iota}_n][\mathbf{c}+\boldsymbol{\varepsilon}][\mathbf{I}_T - \frac{\boldsymbol{\iota}_T\boldsymbol{\iota}_T'}{T}]^2[\mathbf{c}+\boldsymbol{\varepsilon}]'[\mathbf{I}_n \quad \boldsymbol{\iota}_n]'/T \\
= & [\mathbf{I}_n \quad \boldsymbol{\iota}_n][\Sigma_{cc}+\Sigma_{\varepsilon\varepsilon}+\Sigma_{c\varepsilon}+\Sigma_{c\varepsilon}'][\mathbf{I}_n \quad \boldsymbol{\iota}_n'] \\
= & [\Sigma_{cc}+\Sigma_{\varepsilon\varepsilon}+\Sigma_{c\varepsilon}+\Sigma_{c\varepsilon}'] \quad \boldsymbol{\iota}_n\boldsymbol{\iota}_n'
\end{aligned}$$

The $n \times n$ covariance matrix of the cash earnings $[\Sigma_{cc}+\Sigma_{\varepsilon\varepsilon}+\Sigma_{c\varepsilon}+\Sigma_{c\varepsilon}']$ contains the covariance matrices of the cash markets Σ_{cc}, of the foreign currency markets $\Sigma_{\varepsilon\varepsilon}$ and of the interaction between the cash and foreign currency markets. This expression shows that the interaction between the cash and foreign currency markets as measured by $\Sigma_{c\varepsilon}$ increases the overall market risk $\Sigma_{cc}+\Sigma_{\varepsilon\varepsilon}$ emanating from both types of markets. For the 5-year horizon from July 1992 through June 1997 we have for the cash and the foreign currency appreciation rates the 3×3 summation matrix

$$\begin{aligned}
& [\Sigma_{cc}+\Sigma_{\varepsilon\varepsilon}+\Sigma_{c\varepsilon}+\Sigma_{c\varepsilon}'] = \\
& \begin{bmatrix} 0.54 & -0.28 & -0.30 \\ -0.28 & 1.11 & 1.05 \\ -0.30 & 1.05 & 6.22 \end{bmatrix} + \begin{bmatrix} 102.09 & 57.21 & 7.06 \\ 57.21 & 232.80 & 4.01 \\ 7.06 & 4.01 & 28.51 \end{bmatrix} \\
& + \begin{bmatrix} 1.46 & -0.37 & -0.37 \\ -3.70 & -3.30 & 0.05 \\ -5.63 & -0.14 & -0.24 \end{bmatrix} + \begin{bmatrix} 1.46 & -3.7 & -5.63 \\ -.37 & -3.30 & -0.14 \\ -.37 & 0.05 & -0.24 \end{bmatrix} \\
& = \begin{bmatrix} 105.55 & 52.86 & 0.77 \\ 52.86 & 227.30 & 4.97 \\ 0.77 & 4.97 & 34.25 \end{bmatrix}
\end{aligned}$$

Notice that *the currency markets exhibit more risk than the local cash markets* and that the ringgit was more volatile than the Singapore dollar, followed by the rupiah in the period July 1992 through June 1997. Furthermore, in Singapore the cash and foreign currency market were positively correlated - thus, on average, when the cash rate in Singapore rose the Singapore dollar appreciated, but that in Malaysia and Indonesia they were negatively correlated. Also, the Singapore cash rate was negatively related to the ringgit and rupiah. This suggests that when the cash rate in Singapore rose, money was transferred from the ringgit and rupiah into the Singapore dollar and vice versa. Also, when in Malaysia the cash rate rose, on average the Singapore dollar depreciated and the rupiah was hardly affected.

Third, the two cross terms involving the risk premia and the cash earnings are

$$\begin{aligned} & [\boldsymbol{\iota}_n \quad \mathbf{I}_n][\mathbf{r}-\mathbf{c}][\mathbf{I}_T - \frac{\boldsymbol{\iota}_T\boldsymbol{\iota}_T'}{T}]^2[\mathbf{c}+\boldsymbol{\varepsilon}]'[\mathbf{I}_n \quad \boldsymbol{\iota}_n]'/T \\ = & [\boldsymbol{\iota}_n \quad \mathbf{I}_n][\Sigma_{rc}+\Sigma_{r\varepsilon}-\Sigma_{cc}-\Sigma_{c\varepsilon}][\mathbf{I}_n \quad \boldsymbol{\iota}_n'] \\ = & \boldsymbol{\iota}_n \quad [\Sigma_{rc}+\Sigma_{r\varepsilon}-\Sigma_{cc}-\Sigma_{c\varepsilon}] \quad \boldsymbol{\iota}_n' \end{aligned}$$

and its transpose, respectively,

$$\begin{aligned} & [\mathbf{I}_n \quad \boldsymbol{\iota}_n][\Sigma_{rc}+\Sigma_{r\varepsilon}-\Sigma_{cc}-\Sigma_{c\varepsilon}][\boldsymbol{\iota}_n' \quad \mathbf{I}_n] \\ = & \boldsymbol{\iota}_n' \quad [\Sigma_{rc}+\Sigma_{r\varepsilon}-\Sigma_{cc}-\Sigma_{c\varepsilon}]' \quad \boldsymbol{\iota}_n \end{aligned}$$

In both cases the cross risk emanating from the interaction between the stock and the cash and the stock and foreign currency markets, $\Sigma_{rc}+\Sigma_{r\varepsilon}$, is reduced by the interaction between the cash markets, Σ_{cc}. and between the cash and the foreign currency markets, $\Sigma_{c\varepsilon}$. For the 5-year horizon from July 1992 through June 1997 we have for the cross terms of the risk premia and the cash earnings the 3×3 summation matrix

$$[\Sigma_{rc}+\Sigma_{r\varepsilon}-\Sigma_{cc}-\Sigma_{c\varepsilon}] =$$

$$\begin{bmatrix} -4.74 & 5.04 & -20.91 \\ -9.54 & 9.34 & -20.65 \\ -10.07 & 12.07 & -9.95 \end{bmatrix} + \begin{bmatrix} -39.93 & -177.42 & 32.47 \\ -83.27 & -107.17 & 51.62 \\ -90.94 & 10.31 & 104.35 \end{bmatrix}$$

$$- \begin{bmatrix} 0.54 & -0.28 & -0.30 \\ -0.28 & 1.11 & 1.05 \\ -0.30 & 1.05 & 6.22 \end{bmatrix} - \begin{bmatrix} 1.46 & -0.37 & -0.37 \\ -3.70 & -3.30 & 0.05 \\ -5.63 & -0.14 & -0.24 \end{bmatrix}$$

$$= \begin{bmatrix} -39.94 & -177.41 & 32.46 \\ -83.27 & -107.17 & 51.62 \\ -90.94 & 10.30 & 104.35 \end{bmatrix}$$

Combining all elements, we have the following exactly attributed $n^2 \times n^2$ strategy risk matrix for the n^2 bilateral investment strategies

$$\begin{aligned} \Sigma = & \mathbf{H}\Phi\mathbf{H}' \\ = & \boldsymbol{\iota}_n\boldsymbol{\iota}_n' \quad [\Sigma_{rr}+\Sigma_{cc}-\Sigma_{rc}-\Sigma_{rc}'] + [\Sigma_{cc}+\Sigma_{\varepsilon\varepsilon}+\Sigma_{c\varepsilon}+\Sigma_{c\varepsilon}'] \quad \boldsymbol{\iota}_n\boldsymbol{\iota}_n' \\ & +\boldsymbol{\iota}_n \quad [\Sigma_{rc}+\Sigma_{r\varepsilon}-\Sigma_{cc}-\Sigma_{c\varepsilon}] \quad \boldsymbol{\iota}_n' + \boldsymbol{\iota}_n' \quad [\Sigma_{rc}+\Sigma_{r\varepsilon}-\Sigma_{cc}-\Sigma_{c\varepsilon}]' \quad \boldsymbol{\iota}_n \end{aligned}$$

Clearly, the risk of the investment strategies, as measured by the risk matrix Σ, is a convoluted expression of the risks of the stock, cash and foreign currency markets and their inter-correlations. It makes sense to compute this risk matrix Σ and analyze it for a xed number of observations T only when each of its six constituent market risk matrices - Σ_{rr}, Σ_{rc}, Σ_{cc}, $\Sigma_{c\varepsilon}$, $\Sigma_{\varepsilon\varepsilon}$, $\Sigma_{r\varepsilon}$ - is constant, i.e., when the nancial market pricing processes are *stationary.* When such stationarity prevails, mean-variance portfolio optimization as in the next Section could be valuable. However if such direct stationarity does not prevail in the nancial markets, a complete projective search should be conducted to determine what systematic forces change these nancial market pricing processes.[16]

Remark. This expression for the strategy risk matrix accounts exactly for all the market risks involved in multi-currency portfolio management, as can be seen

[16]Stationarity tests are insufficiently applied to this kind of research. All too often stationarity is presumed and not checked for in nancial analysis and the design of trading strategies.

when we arrange these submatrices according to the data covariance matrix (Cf. Lecture 3) in Table 6. This $3n \times 3n$ data covariance matrix   with full rank $3n$ in Table 6 should not be confused with the $2n \times 2n$ risk premium - cash return covariance matrix Φ with full rank $2n$ in Table 7, or with the $n^2 \times n^2$ strategy risk matrix Σ with degenerated rank $2n - 1$ of Table 8. Each of these three data covariance matrices , Φ, and Σ provides different kinds of information and are used for different purposes, although they are based on the same data set. This is a clear demonstration of the crucial fact that data are not equivalent to information. This fact is insufficiently understood by the nancial industry, which tend to equate data with information.

TABLE 6:	DATA	COVARIANCE	MATRIX $(3n \times 3n) = (9 \times 9)$
Data series	$\mathbf{r}$	$\mathbf{c}$	$\boldsymbol{\varepsilon}$
$\mathbf{r}$	Σ_{rr}	Σ_{rc}	$\Sigma_{r\varepsilon}$
$\mathbf{c}$	Σ'_{rc}	Σ_{cc}	$\Sigma_{c\varepsilon}$
$\boldsymbol{\varepsilon}$	$\Sigma'_{r\varepsilon}$	$\Sigma'_{c\varepsilon}$	$\Sigma_{\varepsilon\varepsilon}$

TABLE 7: Risk & cash returns	RISK PREMIUM - CASH RETURN COVARIANCE MATRIX Φ $(2n \times 2n) = (6 \times 6)$					
	$\mathbf{r} - \mathbf{c}$			$\mathbf{c} + \boldsymbol{\varepsilon}$		
$\mathbf{r} - \mathbf{c}$	3161.45	3344.42	2644.07	−39.94	−177.41	32.46
	3344.42	6220.69	3276.52	−83.27	−107.17	51.62
	2644.07	3276.52	5688.25	−90.94	10.30	104.35
$\mathbf{c} + \boldsymbol{\varepsilon}$	−39.94	−83.27	−90.94	105.55	52.86	0.77
	−177.41	−107.17	10.30	52.86	227.30	4.97
	32.46	51.62	104.35	0.77	4.97	34.25

Table 6 provides this 9×9 ve year holding period strategy risk matrix Σ for Singapore, Malaysia and Indonesia (The covariance matrix is computed from the annualized rates of return over the 5-year period July 1992 - June 1997).

TABLE 8: STRATEGY RISK MATRIX Σ $(n^2 \times n^2)$ (9×9)									
Strategies	s_{11}	s_{21}	s_{31}	s_{12}	s_{22}	s_{32}	s_{13}	s_{23}	s_{33}
s_{11}	3187.1	3326.8	2618.8	2997.0	3136.6	2428.6	3154.7	3294.4	2586.4
s_{21}	3326.8	6159.7	3207.9	3250.2	6083.1	3131.3	3356.9	6189.8	3238.0
s_{31}	2618.8	3207.9	5611.9	2667.3	3256.4	5660.5	2709.3	3298.4	5702.4
s_{12}	2997.0	3250.2	2667.3	3033.9	3287.1	2704.3	3021.5	3274.7	2691.8
s_{22}	3136.6	6083.1	3256.4	3287.1	6233.7	3407.0	3223.6	6170.1	3343.4
s_{32}	2428.6	3131.3	5660.5	2704.3	3407.0	5936.2	2576.0	3278.7	5807.9
s_{13}	3154.7	3356.9	2709.3	3021.5	3223.6	2576.0	3260.6	3462.7	2815.1
s_{23}	3294.4	6189.8	3298.4	3274.7	6170.1	3278.7	3462.7	6358.2	3466.7
s_{33}	2586.4	3238.0	5702.4	2691.8	3343.4	5807.9	2815.1	3466.7	5931.2

3. COMPARISON OF MAXIMIZING RETURN AND RISK-ADJUSTED RETURN STRATEGIES

Before we tackle the issue of how to optimize a portfolio of strategic investments, let s rst compare the average return/risk pro les by close scrutiny of Tables 2 and

3. Since the square roots of the diagonal elements of the strategy risk matrix in Table 8 provide the standard measure (= standard deviation) of each strategic investment risk, we observe that over the period July 1992 through June 1997, the single *lowest risk strategy* has been strategy s_{12}: to invest in the Singapore stock market and to swap into ringgit. Its standard deviation was a very substantial $\sigma_{s_{12}} = \sqrt{3033.9}\% = 55.08\%$. Table 9 provides the average return/risk pro le of these $n^2 = 9$ strategies for both the unadulterated average strategy return $\overline{s}_{ij}$ and the risk-adjusted average strategy return $\overline{s}_{ij}/\sigma_{s_{ij}}$.

TABLE 9:	AVERAGE	RETURN	RISK	PROFILES	7/92	-	6/97		
$i,j =$	$1,1$	$2,1$	$3,1$	$1,2$	$2,2$	$3,2$	$1,3$	$2,3$	$3,3$
$\overline{s}_{ij} =$	8.7	11.6	9.0	10.3	13.2	10.6	13.2	**16.0**	13.4
$\overline{s}_{ij}/\sigma_{s_{ij}} =$	0.1541	0.1478	0.1201	0.1870	0.1672	0.1376	**0.2312**	0.2007	0.1740

Over the period July 1992 through June 1997, the maximum average return/risk strategy was strategy s_{13} with the return-risk pair $(\overline{s}_{13}, \sigma_{s_{13}}) = (13.2, 57.1)$, so that the risk-adjusted return $\overline{s}_{13}/\sigma_{s_{13}} = 0.2312$. Remarkably, none of the 9 strategies produced enough average return to compensate for even one quart of the market risk in these three Southeast Asian countries in the past ve years! In particular the average risk premiums of ca. 3.2% have been much too low for such a risk-ridden investment environment, since, on average, such a return could have been wiped out every half year. From this fact alone one can surmise that Asian investors have ignored the price of risk, when they devised investment strategies.

Comparing the maximizing return and risk-adjusted return strategies over time in Tables 2 and 3, we nd that following a maximizing return strategy (if we could have known the returns *ex ante*!) would have led to a very large number of portfolio allocation shifts. It is small consolation that maximization of the risk-adjusted return strategies would have led to a slightly smaller number of strategy shifts. In fact the relative frequency distributions of the best monthly strategies look very similar, as can be seen in Table 10. They are both bimodal distributions with the two modes on strategies $(3,2)$ and $(2,3)$, meaning that both the strategy to invest in the Jakarta stock market and to swap into ringgit and the strategy to invest in the Kuala Lumpur stock market and to swap into rupiah showed most occurrences of the highest returns in simple and in risk-adjusted form: on average one sixth of the time, i.e., every half year.

TABLE 10:	RELATIVE	FREQUENCY	OF	OCCURRENCE	7/92	-	6/97		
$i,j =$	$1,1$	$2,1$	$3,1$	$1,2$	$2,2$	$3,2$	$1,3$	$2,3$	$3,3$
$f(max\ s_{ij}) =$	$\frac{5}{60}$	$\frac{4}{60}$	$\frac{3}{60}$	$\frac{3}{60}$	$\frac{8}{60}$	$\frac{11}{60}$	$\frac{8}{60}$	$\frac{10}{60}$	$\frac{8}{60}$
$f(max\ s_{ij}/\sigma_{s_{ij}}) =$	$\frac{3}{60}$	$\frac{3}{60}$	$\frac{6}{60}$	$\frac{5}{60}$	$\frac{7}{60}$	$\frac{10}{60}$	$\frac{8}{60}$	$\frac{11}{60}$	$\frac{7}{60}$

The highest number of pure maximum return occurrences is scored by strategy $s_{3,2}$, i.e., investing in the Jakarta stockmarket and swapping into ringgit. The highest frequency of maximum risk-adjusted return occurrences is scored by the strategy s_{23}, investing in the Kuala Lumpur stock market and swapping into rupiah.

4. SINGULARITY OF STRATEGY RISK MATRIX

We turn now to the problem of the optimization and management of a portfolio of strategic investments. For a strategy risk matrix to be a direct input in the Markowitz mean-variance optimization procedure it must be nonsingular. However, *the strategy risk matrix is singular*, since

$$rank(\Sigma) = 2n - 1 < n^2, \text{ for integer } n > 1$$

This is proved by determining the rank of the strategy deviations of the preceding section, which is the same as the rank of the strategy risk matrix.[17]

$$\begin{aligned} rank\left\{\mathbf{DEV(S)}\right\} &= rank\left\{\mathbf{H}\begin{bmatrix}[\mathbf{r}-\mathbf{c}] \\ [\mathbf{c}+\boldsymbol{\varepsilon}]\end{bmatrix}[\mathbf{I}_T - \frac{\boldsymbol{\iota}_T\boldsymbol{\iota}_T'}{T}]\right\} \\ &\leq Min\left\{rank\left(\mathbf{H}\right), rank\begin{bmatrix}[\mathbf{r}-\mathbf{c}] \\ [\mathbf{c}+\boldsymbol{\varepsilon}]\end{bmatrix}, rank\left[\mathbf{I}_T - \frac{\boldsymbol{\iota}_T\boldsymbol{\iota}_T'}{T}\right]\right\} \\ &= Min\left\{2n-1, 2n, T-1\right\} < n^2 \end{aligned}$$

since $\mathbf{H}$ is the $n^2 \times 2n$ selector matrix of rank $2n-1$, $\begin{bmatrix}[\mathbf{r}-\mathbf{c}] \\ [\mathbf{c}+\boldsymbol{\varepsilon}]\end{bmatrix}$ is a risk-premium and cash return $2n \times T$ matrix of rank $2n$, and $\left[\mathbf{I}_T - \frac{\boldsymbol{\iota}_T\boldsymbol{\iota}_T'}{T}\right]$ is the deviations producing $T \times T$ matrix of rank $T-1$. This result can be illustrated by computing the rank of the selector matrix for our three Asian countries, as follows:

$$rank(\mathbf{H}) = rank\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} = 5 < 9$$

Remark. It should be emphasized that when one would numerically compute the rank of the strategy risk matrix in Table 8, the numerically computed result would deceptively show a *full* $rank(\Sigma) = 9$, because of the imprecision of the computation caused by the limited registers of a computer. Thus the strategy risk matrix *appears to be of full rank.*[18] But the exact algebraic $rank(\Sigma) = 5$ and thus the strategy risk matrix *must be singular*. The imprecision of the computed result is expressed by the determinant as the product of the resulting eigenvalues $|\Sigma| = \prod_{i=1}^{n^2} \lambda_i$. Although in our example $9 - 5 = 4$ eigenvalues are exactly zero, and consequently the determinant, the (nite) computer shows these four eigenvalues to be very close to, but unequal, zero at any level of computing precision of the fundamental data covariance matrix .

[17] According to Proposition 7 of [8], p. 437.

[18] As is, indeed, the case in CreditMetrics^{TM}, as professsor Garman con rmed when he revealed the instability and unreliability of RiskMetrics^{TM} s information matrix Σ^{-1}.

5. MEAN-VARIANCE OPTIMIZATION

5.1. **Markowitz Procedure with Singular Risk Matrix.** For strategic global portfolio management the crucial question is: are there combinations of investment strategies that either lead to lower overall risk for a comparable level of average return, or, vice versa, for to a higher return for a comparable level of risk?[4] Markowitz' conventional mean-variance optimization of portfolios, which answers this question, requires that the central risk matrix is positive definite, i.e., nonsingular.[20][21][22][19] But our exact strategy risk matrix Σ is singular. and we have to adapt Markowitz' procedure to our singular risk matrix. The *mean portfolio rate of return* $\overline{s}_p$ for the holding period T is the allocationed linear combination of the average strategy rates of return of the strategies, $\overline{\mathbf{VEC(S)}}$, contained in the portfolio

$$\overline{s}_p = \mathbf{w}'\overline{\mathbf{VEC(S)}}$$

and $\mathbf{w}$ is a $n \times 1$ vector of *portfolio allocations*, such that the sum of the allocations equals unity, $\mathbf{w}'\boldsymbol{\iota}_{n^2} = 1$, where $\boldsymbol{\iota}_{n^2}$ is the $n^2 \times 1$ unit vector. The overall investment strategy *portfolio risk*, σ_{pp}, is the variance of the portfolio rates of return

$$\sigma_{pp} = \mathbf{w}'\Sigma\mathbf{w} = \mathbf{w}'\mathbf{H}\Phi\mathbf{H}'\mathbf{w} = \mathbf{v}'\Phi\mathbf{v}$$

where the new combined portfolio allocations $\mathbf{v} = \mathbf{H}'\mathbf{w}$. For example

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \mathbf{H}'\mathbf{w} = \begin{bmatrix} w_{11} + w_{12} + w_{13} \\ w_{21} + w_{22} + w_{23} \\ w_{31} + w_{32} + w_{33} \\ w_{11} + w_{21} + w_{31} \\ w_{12} + w_{22} + w_{32} \\ w_{13} + w_{23} + w_{33} \end{bmatrix}$$

Notice that the first $n = 3$ combined allocations $v_k, k = 1, 2, 3$, refer to the fundamental strategy choice how much of the capital to invest in which stock market to earn a risk premium, while the second $n = 3$ allocations $v_k, k = 4, 5, 6$ refer to the fundamental strategy choice how much of the capital to invest in which currency to earn a cash return. The allocations $\mathbf{v}$ exhaust the capital allocation based on the two fundamental choices of investments in stock markets and in currencies, since, because of the accounting identities,

$$\begin{aligned} \mathbf{v}' &= \mathbf{v}'\left[\begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix}\right] \text{ with} \\ \mathbf{v}'\begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} &= \mathbf{w}'\mathbf{H}\begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} = \mathbf{w}'\boldsymbol{\iota}_{n^2} = 1 \text{ and} \\ \mathbf{v}'\begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} &= \mathbf{w}'\mathbf{H}\begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} = \mathbf{w}'\boldsymbol{\iota}_{n^2} = 1, \end{aligned}$$

[19] One may question the relevance of symmetric mean-variance optimization, since there are observable asymmetries in the regional risk distributions. An asymmetrical optimization would require the computation of additional, higher order moments than the first and second moments used here and the procedure would quickly become very complex, without elucidating the issue of combining portfolio optimization with complete and exact attribution.

while the average portfolio return remains

$$\overline{s}_p = \mathbf{w}'\overline{\mathbf{VEC(S)}} = \mathbf{w}'\mathbf{H}\begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\varepsilon}] \end{bmatrix} = \mathbf{v}'\begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\varepsilon}] \end{bmatrix}$$

Now the procedure has once again become similar to Markowitz nonsingular case, which we solve using the familiar Kuhn-Tucker Theorem for constraint optimization. First, form the Lagrangian with the three accounting constraints:

$$L(\mathbf{v}, \lambda_1, \lambda_2, \lambda_3) = \mathbf{v}'\Phi\mathbf{v} + \lambda_1\left(1 - \mathbf{v}'\begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix}\right) + \lambda_2\left(1 - \mathbf{v}'\begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix}\right) + \lambda_3\left(\overline{s}_p - \mathbf{v}'\begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\varepsilon}] \end{bmatrix}\right)$$

Next, to nd the optimum of this Lagrangian, set the $2n+3$ partial rst derivatives equal to zero, i.e., the derivatives with respect to the $2n$ elements of the allocation vector $\mathbf{v}$ and to the two Lagrangian multipliers λ_1, λ_2 and λ_3:

$$\begin{aligned}
\frac{\partial L}{\partial \mathbf{v}} &= 2\mathbf{v}'\Phi - \lambda_1 \begin{bmatrix} \boldsymbol{\iota}_n' & \mathbf{0} \end{bmatrix} - \lambda_2 \begin{bmatrix} \mathbf{0} & \boldsymbol{\iota}_n' \end{bmatrix} - \lambda_3 \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}]' & [\overline{\mathbf{c}} + \overline{\varepsilon}]' \end{bmatrix} = \mathbf{0} \\
\frac{\partial L}{\partial \lambda_1} &= 1 - \mathbf{v}'\begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} = 0 \\
\frac{\partial L}{\partial \lambda_2} &= 1 - \mathbf{v}'\begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} = 0 \\
\frac{\partial L}{\partial \lambda_3} &= \overline{s}_p - \mathbf{v}'\begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\varepsilon}] \end{bmatrix} = 0
\end{aligned}$$

Remark. The $(2n+3) \times (2n+3)$ matrix of partial second derivatives is positive de nite, since the full rank $2n \times 2n$ covariance matrix $\Phi > 0$, so that the optimum is, indeed, a - constrained - *minimum.*

By post-multiplying the rst $2n$ equations, resulting from taking the $2n$ derivatives wrt. $\mathbf{v}$, by the matrix Φ^{-1}, we nd that

$$\mathbf{v}' = \frac{\left[\lambda_1 \begin{bmatrix} \boldsymbol{\iota}_n' & \mathbf{0} \end{bmatrix} + \lambda_2 \begin{bmatrix} \mathbf{0} & \boldsymbol{\iota}_n' \end{bmatrix} + \lambda_2 \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}]' & [\overline{\mathbf{c}} + \overline{\varepsilon}]' \end{bmatrix}\right]\Phi^{-1}}{2}$$

Substitute this expression into the derivatives wrt. λ_1 andλ_2 to get the three equations

$$\begin{aligned}
\frac{\left[\lambda_1 \begin{bmatrix} \boldsymbol{\iota}_n' & \mathbf{0} \end{bmatrix} + \lambda_2 \begin{bmatrix} \mathbf{0} & \boldsymbol{\iota}_n' \end{bmatrix} + \lambda_3 \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}]' & [\overline{\mathbf{c}} + \overline{\varepsilon}]' \end{bmatrix}\right]\Phi^{-1}\begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix}}{2} &= 1 \\
\frac{\left[\lambda_1 \begin{bmatrix} \boldsymbol{\iota}_n' & \mathbf{0} \end{bmatrix} + \lambda_2 \begin{bmatrix} \mathbf{0} & \boldsymbol{\iota}_n' \end{bmatrix} + \lambda_3 \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}]' & [\overline{\mathbf{c}} + \overline{\varepsilon}]' \end{bmatrix}\right]\Phi^{-1}\begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix}}{2} &= 1 \\
\frac{\left[\lambda_1 \begin{bmatrix} \boldsymbol{\iota}_n' & \mathbf{0} \end{bmatrix} + \lambda_2 \begin{bmatrix} \mathbf{0} & \boldsymbol{\iota}_n' \end{bmatrix} + \lambda_2 \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}]' & [\overline{\mathbf{c}} + \overline{\varepsilon}]' \end{bmatrix}\right]\Phi^{-1}\begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\varepsilon}] \end{bmatrix}}{2} &= \overline{s}_p
\end{aligned}$$

By expanding these three equations. the optimal Lagrangian multipliers λ_1^{opt}, λ_2^{opt} and λ_3^{opt} can be found to be

$$\begin{bmatrix} \lambda_1^{opt} \\ \lambda_2^{opt} \\ \lambda_3^{opt} \end{bmatrix} = 2.\Delta^{-1}.\begin{bmatrix} 1 \\ 1 \\ \overline{s}_p \end{bmatrix}$$

where the 3×3 symmetric and positive definite matrix Δ is such that

$$\Delta = \begin{bmatrix} \begin{bmatrix} \boldsymbol{\iota}_n' & \mathbf{0} \end{bmatrix} \Phi^{-1} \begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} & \begin{bmatrix} \mathbf{0} & \boldsymbol{\iota}_n' \end{bmatrix} \Phi^{-1} \begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} & \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}]' & [\overline{\mathbf{c}} + \overline{\boldsymbol{\varepsilon}}]' \end{bmatrix} \Phi^{-1} \begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} \\ \begin{bmatrix} \boldsymbol{\iota}_n' & \mathbf{0} \end{bmatrix} \Phi^{-1} \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} & \begin{bmatrix} \mathbf{0} & \boldsymbol{\iota}_n' \end{bmatrix} \Phi^{-1} \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} & \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}]' & [\overline{\mathbf{c}} + \overline{\boldsymbol{\varepsilon}}]' \end{bmatrix} \Phi^{-1} \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} \\ \begin{bmatrix} \boldsymbol{\iota}_n' & \mathbf{0} \end{bmatrix} \Phi^{-1} \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\boldsymbol{\varepsilon}}] \end{bmatrix} & \begin{bmatrix} \mathbf{0} & \boldsymbol{\iota}_n' \end{bmatrix} \Phi^{-1} \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\boldsymbol{\varepsilon}}] \end{bmatrix} & \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}]' & [\overline{\mathbf{c}} + \overline{\boldsymbol{\varepsilon}}]' \end{bmatrix} \Phi^{-1} \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\boldsymbol{\varepsilon}}] \end{bmatrix} \end{bmatrix}$$

So that by substitution we find the optimal fundamental choice allocations

$$\mathbf{v}^{opt} = \frac{\Phi^{-1} \left(\lambda_1^{opt} \begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} + \lambda_2^{opt} \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} + \lambda_3^{opt} \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\boldsymbol{\varepsilon}}] \end{bmatrix} \right)}{2}$$

We have now the two equations of Markowitz Efficient Portfolio Frontier for $2n$ strategy investment choices, which can be plotted in a two-dimensional graph. For every portfolio strategy return $\overline{s}_p^{opt}$ there is a corresponding portfolio strategy risk σ_p^{opt}:

$$\begin{aligned} \overline{s}_p^{opt} &= \mathbf{v}^{opt\prime} \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\boldsymbol{\varepsilon}}] \end{bmatrix} \\ \sigma_p^{opt} &= \sqrt{(\mathbf{v}^{opt})\Phi' \mathbf{v}^{opt}} \end{aligned}$$

Figure 1. provides the Efficiency Frontier of the strategic investment portfolio with exact attribution computed for the three Asian countries for the period July 1992 through June 1997.

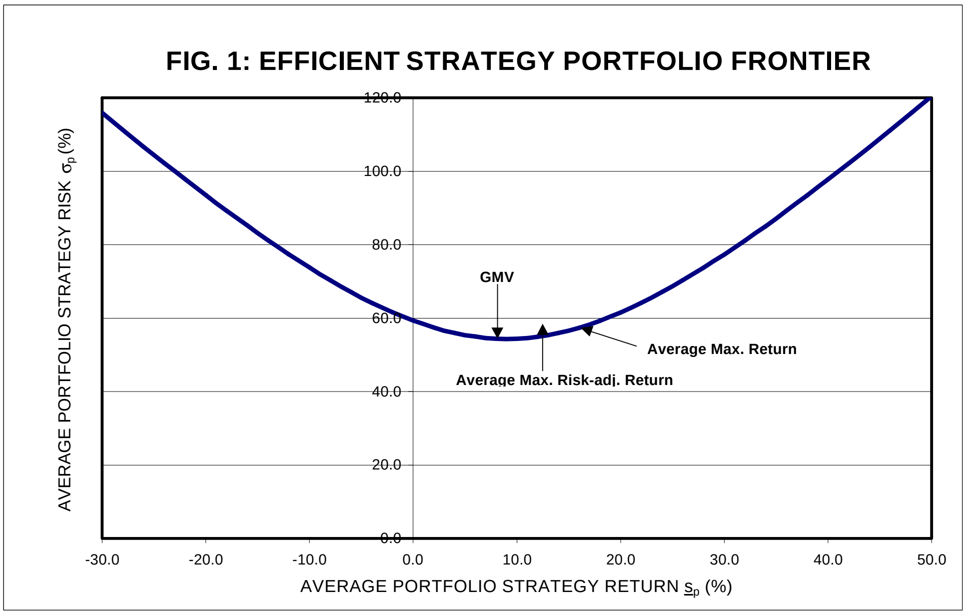

FIGURE 1

5.2. **Retrieval of the Optimal Strategy Allocations.** The next question is: can the original portfolio strategy allocations $\mathbf{w}$ be *uniquely* retrieved from the intermediate portfolio strategy choice allocations $\mathbf{v}$? Yes, since a unique correspondence exists between the $2n$ elements of $\mathbf{v}$ and the n^2 elements of $\mathbf{w}$ via the two accounting identities imposed by the geometry of the original strategy matrix S. It can be easily con rmed that

$$\begin{aligned} \begin{bmatrix} \mathbf{I}_n & \mathbf{0} \end{bmatrix} \mathbf{v}\mathbf{v}' \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_n \end{bmatrix} &= \begin{bmatrix} v_1v_4 & v_1v_5 & v_1v_6 \\ v_2v_4 & v_2v_5 & v_2v_6 \\ v_3v_4 & v_3v_5 & v_3v_6 \end{bmatrix} = \\ \begin{bmatrix} \mathbf{I}_n & \mathbf{0} \end{bmatrix} \mathbf{H}'\mathbf{w}\mathbf{w}'\mathbf{H} \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_n \end{bmatrix} &= \mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \end{aligned}$$

since, for example,

$$\begin{aligned} \begin{bmatrix} \mathbf{I}_n & \mathbf{0} \end{bmatrix} \mathbf{v}\mathbf{v}' \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_n \end{bmatrix} \boldsymbol{\iota}_n &= \begin{bmatrix} v_1v_4 & v_1v_5 & v_1v_6 \\ v_2v_4 & v_2v_5 & v_2v_6 \\ v_3v_4 & v_3v_5 & v_3v_6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} v_1(v_4+v_5+v_6) \\ v_2(v_4+v_5+v_6) \\ v_3(v_4+v_5+v_6) \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_2 \end{bmatrix} \\ &= \begin{bmatrix} w_{11}+w_{12}+w_{13} \\ w_{21}+w_{22}+w_{23} \\ w_{31}+w_{32}+w_{33} \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ &= \mathbf{W}\boldsymbol{\iota}_n \end{aligned}$$

A similar con rmation is also obtained for $\begin{bmatrix} v_4 & v_5 & v_6 \end{bmatrix}$.

6. ARE MAXIMUM RETURN OR RISK-ADJUSTED RETURN STRATEGIES EFFICIENT?

In Figure 1 the Singer and Karnovsky s pure maximizing return strategy return-risk pair and the maximizing risk-adjusted return strategy pair are compared with the computed Efficiency Frontier of strategy pairs, based on the data set of July 1992 - June 1997. The best risk-adjusted return combination of Table 9 is slightly inefficient since, by comparison with the Efficiency Frontier, it is associated with higher risk for the same return: $(\bar{s}_{13}, \sigma_{s_{13}}) = (13.2, 57.1) > (\bar{s}_{13}, \sigma_p) = (13.2, 55.5)$. The simple maximizing strategy return of Singer & Karnosky is associated with the following efficient risk level: $(\bar{s}_{23}, \sigma_p) = (16.0, 57.4)$. (Of course, there is no associated inefficient risk level, since Singer and Karnosky ignore risk).

Table 11 shows some of the resulting optimal strategy choice allocations, the corresponding investment strategy allocations and the Markowitz optimal pairs of average portfolio return and risk for our dataset. These examples all center around the General Minimum Variance (= GMV risk = overall minimum portfolio risk) investment strategy, indicated with bold numbers. The GMV strategy results in a Markowitz pair of $(\bar{s}_p, \sigma_p) = (9.0, 54.4)$ (in percent, at an annual rate), which has a lower return and lower risk than the best risk-adjusted return combination $(\bar{s}_{13}, \sigma_{s_{13}}) = (13.2, 57.1)$ of Table 9.

Notice that of the 9.0% GMV return, about one third, 3.2%, consists of risk premium and the remaining 5.8% is cash earnings. Most of the variation in the average portfolio returns around the GMV return is due to variations in the average portfolio cash returns. The stock market risk premium remains around 3.2%. Indeed, currency overlays can make a lot of difference to average portfolio returns. They can enhance them and reduce them, depending on the amount of risk an investor is willing to take. However, around the GMV portfolio return substantial variations in portfolio returns are not accompanied by much variation in the average risk level.

TABLE 11:	EXAMPLES	OF	OPT.	PORTFOLIO	ALLOCATIONS			
Average return	$\overline{s}_p$	2.00	6.00	8.00	**9.00**	10.00	12.00	16.00
Avg. risk premium	$\overline{r}_p - \overline{c}_p$	3.18	3.21	3.23	**3.24**	3.25	3.26	3.30
Avg. cash return	$\overline{c}_p + \overline{\varepsilon}_p$	-1.18	2.79	4.77	**5.76**	6.75	9.74	12.70
Average risk	σ_p	57.4	54.9	54.4	**54.4**	54.4	54.9	57.4
Choice allocations	v_1	0.95	0.97	0.97	**0.98**	0.98	0.99	1.01
	v_2	-0.13	-0.11	-0.11	**-0.10**	-0.10	-0.09	-0.08
	v_3	0.18	0.15	0.13	**0.12**	0.12	0.10	0.07
	v_4	1.92	1.03	0.59	**0.37**	0.14	-0.30	-1.19
	v_5	0.82	0.81	0.80	**0.80**	0.79	0.79	0.77
	v_6	-1.75	-0.84	-0.39	**-0.16**	0.06	0.52	1.42
Strategy allocations	w_{11}	1.83	1.00	0.57	**0.36**	0.14	-0.30	-1.20
	w_{21}	-0.24	-0.12	-0.06	**-0.04**	-0.01	0.03	0.09
	w_{31}	0.34	0.15	0.08	**0.05**	0.02	-0.03	-0.09
	w_{12}	0.78	0.78	0.78	**0.78**	0.78	0.78	0.78
	w_{22}	-0.10	-0.09	-0.09	**-0.08**	-0.08	-0.07	-0.06
	w_{32}	0.15	0.12	0.11	**0.10**	0.09	0.08	0.06
	w_{13}	-1.66	-0.81	-0.38	**-0.16**	0.06	0.51	1.43
	w_{23}	0.22	0.10	0.04	**0.02**	-0.01	-0.05	-0.11
	w_{33}	-0.31	-0.12	-0.05	**-0.02**	0.01	0.05	0.10

Notice in Table 11 that the exact accounting identities $(v_1+v_2+v_3) = 1$, $(v_4+v_5+v_6) = 1$, and $(w_{11}+w_{21}+w_{31}+w_{12}+w_{22}+w_{32}+w_{13}+w_{23}+w_{33} = 1$ are maintained. Positive numbers for $v_k, k = 1, 2, ..., 6$ and $w_{ij}, i, j = 1, 2, ..., 9$ indicate buying or taking long positions; negative numbers indicate short selling or unwinding of long positions. For example, for the optimal GMV portfolio result $(\overline{s}_p, \sigma_p) = (9.00, 54.4)$, 98% of the capital in stocks to earn the risk premium is invested in Singapore, with 12% in Jakarta and 10% shorted in Malaysia.[20] The currency overlay of the GMV portfolio is 80% in ringgit, 37% in Singapore dollars with 16% cash borrowed in rupiah. For convenience of checking the accounting identities, we have reconstituted the GMV portfolio strategy allocations $w_{ij}, i.j = 1, 2, 3$ in strategy matrix format

[20] Like many Asian countries, officially Malaysia didn t allow shorting of stocks, thereby introducing an unnecessary market inefficiency.

in Table 12.[21]

TABLE 12:	OPTIMAL	GMV	STRATEGY	ALLOCATIONS
		1. Singapore	2. Malaysia	3. Indonesia
	Fundamental choices	$v_4 = 0.37$	$v_5 = 0.80$	$v_6 = -0.16$
1. Singapore	$v_1 = 0.98$	0.36	0.78	-0.16
2. Malaysia	$v_2 = -0.10$	-0.04	-0.08	0.02
3. Indonesia	$v_3 = 0.12$	0.05	0.10	-0.02

The corresponding investment strategy allocations are that 36% of all investment capital is allocated to earn both the risk premium and the cash return in Singapore and thus remains in Singapore dollars, while 78% earns the risk premium in Singapore, but the cash return in ringgit. Interestingly, 16% earns the risk premium in Singapore, but is willing to pay the high cost of cash in Indonesia. The rationale for such an allocation is easily found in the fundamental data matrices, where we found that the Singapore stock market exhibited the lowest risk of all three regional stock markets, while the rupiah was most stable. Thus this investment strategy allocation reduces the overall portfolio risk. Furthermore, notice that sometimes considerable gross leveraging occurs for some strategies, with allocation allocations > 1 in absolute value, while on a net basis such leveraging cancels out.

On average, the portfolio approach to strategy investment clearly would have lead to somewhat lower risk, but not very much lower: only $1.1 - 2.7\%$ lower on average. The GMV risk is more than six times larger than the average return! The fundamental reason is that, in particular, all three Asian stock markets exhibited overall very high levels of risk in the ve year period, while the stock and currency markets are interrelated so that this stock market risk spills over into the currency markets. Thus any investor restricting his investment strategies only to Singapore, Malaysia and Indonesia would not have been able to avoid the overall high level of risk. Thus portfolio diversi cation within this very restricted Southeast Asian region did not help to reduce overall portfolio risk. Therefore, all the recently emerged Singapore-centered regional unit trusts show very similar risk levels and offer little alternative portfolio risk reduction. Only by more extensive globalization of the strategic investment portfolios could the overall portfolio risk level have been reduced, by moving more capital to the more mature and stable markets of North America and Europe, which exhibited lower overall risk levels over the same time period.

7. CONCLUSIONS

Singer & Karnosky s [25], [14] complete and exact growth accounting framework produces a singular strategy risk matrix based on simple growth strategy investment allocations. Because Markowitz mean-variance portfolio optimization requires a nonsingular risk matrix, portfolio optimization appears incompatible with exact attribution, i.e., with complete and exact accounting. Consequently, Singer & Karnosky discuss only return maximizing strategies. With the help of simple tensor algebra, we reformulated the exact growth accounting framework of Singer

[21] Because of rounding error the summations in Table 12 don t add up exactly in the second digit after the decimal point.

& Karnosky to reduce the dimensions of the strategy risk matrix, so that it is nonsingular and can be inverted and Markowitz mean-variance portfolio optimization can be implemented with compete and exact attribution. This implies that nally commercial procedures, such as RiskMetrics^{TM} and CreditMetrics^{TM} can properly implement portfolio optimization combined with exact, cash ow based, return attribution. Thus far their optimization procedures have been improperly based on singular risk matrices, which only appear nonsingular because of errors introduced by nite computer registers.

Over the period July 1992 through June 1997, out of nine possible bilateral investment strategies, Singer and Karnovsky s average *maximizing return strategy* would have been to invest in the Malaysian stock market and to swap into rupiah. On average, this strategy would have earned a 16.0% return per year. But in the same period the *maximum average risk-adjusted return strategy* would have been to invest in the Singapore stock market and to swap into the rupiah. However, none of the nine strategies produced enough average return to compensate for even one quart of the market risk in these three Southeast Asian countries in the past ve years! This conclusion is in accordance with ndings elsewhere in the literature.[10] Moreover, maximizing strategies would have led to a large number of radical switches in portfolio allocation. About every half year a portfolio would have been reallocated over the various investment strategies.

By using a more stable portfolio optimization allocation the average risk could have been reduced by only $1.1 - 2.7\%$ for the regionally restricted example portfolio for Singapore, Malaysia and Indonesia. The overall minimum risk return strategy would have delivered an average 9.0% annual return, which would have been better than average contemporary 5-year US Treasury bond returns, but with a risk level still **six** times larger than the 9.0%!

Thus portfolio diversi cation within this very restricted region did not help to reduce overall portfolio risk. Only by more extensive globalization of his or her strategic investment portfolio can the overall portfolio risk level be reduced, by moving more capital to the more mature and stable markets of North America and Europe, which exhibit lower overall risk levels. Therefore, we are extending now our analysis to a portfolio with seven Asian (mostly ASEAN) countries, together with Japan and the United States as investment alternatives, to see if further risk reduction is possible for the investor based in the ASEAN region.

This paper shows that most of this portfolio investment strategy risk originates in the stock markets and their inter-linkages with the cash and foreign exchange markets (Because of herd instinct? Hedging linkages? Diversi cation linkages?), only to be followed by the risks originating by the foreign exchange markets.[22] In other words, the assertion by some local political leaders that the foreign exchange markets exhibit most risk is factually erroneous. The local cash markets show the least amount of risk, presumably because the bank lending rates are administratively controlled and not free market rates. Since in addition local bank lending to investors remains very restricted, the volatility of the local stock markets is primarily linked to the volatility foreign exchange markets and cannot be sufficiently reduced by the cash markets.

[22]Elsewhere we have shown that for Singapore, Malaysia and to a somewhat lesser extent Indonesia, the volatility of the stock markets is closely related to the instability of the local economies and not to the risks of the local cash markets.[**?**] In fact, we showed that the local cash markets operate almost independently from the local stock markets and the local economies. .

References

[1] AIMR. *AIMR Performance Presenttion Standards Handbook.* Association for Investment Management and Research, Charlottesville, VA, 2nd edition, 1997.

[2] G. C. Allen. Performance attribution for global equity portfolios. *Journal of Portfolio Management*, ?(?):59 65, Fall 1991.

[3] E. M. Ankrim. Risk-adjusted performance attribution. *Financial Analysts Journal*, ?(?):?, March-April 1992.

[4] F. Black and R. Litterman. Global portfolio optimization. *Financial Analysts Journal*, ?(?):28 43, September/October 1992.

[5] B. S. Brinson, G.P. and G. Beebower. Determinants of portfolio performance ii: An update. *Financial Analysts Journal*, ?(?):40 48, May/June 1991.

[6] R. H. Brinson, G.P. and G. Beebower. Determinants of portfolio performance. *Financial Analysts Journal*, ?(?):39 44, July/August 1986.

[7] D. A. Chapman. Approximating the asset pricing kernel. *The Journal of Finance*, LII(4):1383 1410, September 1997.

[8] P. Dhrymes. *Introductory Econometrics.* Springer-Verlag, New York, NY, 1978.

[9] C. Eun and B. Resnick. Exchange rate uncertainty, forward contracts and international portfolio selection. *The Journal of Finance*, ?(?):197 215, March 1988.

[10] G. W. Gardner and T. Wuiloloud. Currency risk in international portfolios: How satisfying is optimal hedging. *The Journal of Portfolio Management*, 21(3):59 67, Spring 1995.

[11] G. L. Gastineau. The currency hedging decision: A search for synthesis in asset allocation. *Financial Analysts Journal*, 51(3):8 17, May-June 1995.

[12] J. Glen and P. Jorion. Currency hedging for international portfolios. *The Journal of Finance*, ?(?):1865 1886, December 1993.

[13] P. Jorion. Asset allocation with hedged and unhedged foreign stocks and bonds. *Journal of Portfolio Management*, ?(?):49 54, Summer 1989.

[14] D. S. Karnsoky and B. D. Singer. *Global Asset Management and Performance Attribution.* The Research Foundation of the Institute of Chartered Financial Analysts, Charlottesville, VA, 1994.

[15] Kritzman. The optimal currency hedging policy with biased forward rates. *Journal of Portfolio Management*, ?(?):94 100, Summer 1993.

[16] A. F. Lee. International asset and currency allocation. *Journal of Portfolio Management*, ?(?):68 73, Fall 1987.

[17] A. F. Lee. Currency hedging and implementation issues. In L. R. Thomas, editor, *The Currency-Hedging Debate*, page ? IFR Publishing Ltd, London, 1990.

[18] J. Longerstaey and M. Spencer. Riskmetrics(tm)-technical document. Technical report, J.P.Morgan/Reuters, New York, NY, 1996.

[19] C. A. Los. *Econometrics of Models with Evolutionary Parameter Structures.* PhD thesis, Columbia University, 1984.

[20] H. Markowitz. Portfolio selection. *The Journal of Finance*, 7(?):77 91, ? 1952.

[21] H. M. Markowitz. *Mean-Variance Analysis in Portfolio Choice and Capital Markets.* Blackwell, Oxford, 1987.

[22] H. M. Markowitz. *Mean-Variance Analysis in Portfolio Choice and Capital Markets.* Blackwell Publishers, Cambridge, MA, 1990.

[23] S. L. Nesbitt. Currency hedging rules for plan sponsors. *Financial Analysts Journal*, ?(?):73 81, March/April 1991.

[24] B. Singer. Evaluation of portfolio performance: Attribution analysis. *The Journal of Performance Measurement*, 1(2):45 55, Winter 1996.

[25] B. D. Singer and D. S. Karnosky. The general framework for global investment management and performance attribution. *The Journal of Portfolio Management*, 21(2):84 92, Winter 1995.

[26] J. C. Stannard. Measuring investment returns of portfolios containing futures and options. *The Journal of Performance Measurement*, 1(1):27 33, Fall 1996.

[27] J. Von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior.* Princeton University Press, Princeton, New Jersey, 2nd (1st ed., 1944) edition, 1947.

[28] P. Willett. How do we measure currency s impact in international equity accounts? *The Journal of Performance Attribution*, 1(3):22 37, Spring 1997.

Nanyang Technological University, Singapore 639798
E-mail address: ACALOS@ntu.edu.sg