Dynamic Risk Profile of the US Term Structure by Wavelet MRA

Sutthisit Jamdee and Cornelis A. Los, Ph.D.*

Department of Finance, Kent State University, Kent, Ohio 44242

Abstract

A careful examination of interest rate time series from different U.S. Treasury maturities by Wavelet Multiresolution Analysis (MRA) suggests that the first differences of the term structure of interest rate series are periodic or, at least, cyclic, non-stationary, long-term dependent, in particular, anti-persistent. Each nodal time series from a particular maturity has its own uniqueness and accordingly supports the Market Segmentation theory. The findings also imply that affine models are insufficient to describe the dynamics of the interest rate diffusion processes and call for more intensive research that might provide better, most likely fractal or nonlinear, term structure models for each maturity. If this is correct, empirical term structure models may describe chaotic, *i.e.*, diffusion processes with non-unique dynamic equilibria.

JEL classification: E43

Keywords: Wavelet; Interest rates; Hurst exponent; Term structure; Yield curve

---

*Cornelis A. Los, Ph.D., Associate Professor of Finance; Tel: 330-672-1207; Fax: 330-672-9806; e-mail: clos@kent.edu

1. Introduction

The standard rational expectations based term structure model of interest rates is known as a non-linear relationship between interest rates and the term to maturity of a security. Assuming that all characteristics (*i.e.*, default and liquidity risk except maturity) are the same, the term structure of interest rates enables the comparison for the interest rates on securities. The change in required interest rates as the maturity of a security changes is called the maturity premium.

The maturity premium or the difference between the required yield on short and long securities of the same characteristics except maturity can be negative, positive, or null. The most commonly reported and analyzed yield to maturity is that of U.S. Treasury securities.

The desired maturity a fixed-income security (*e.g.*, U.S. Treasury bills/bonds and corporate bonds) varies depending on the need of the investors' investment horizon. However, the most commonly reported maturities are four-week, three-month, sixth-month, and one-year for U.S. Treasury bills. It is also common to compute the Treasury Constant Yield to Maturity for different maturities. The commonly reported constant yield to maturities includes one-month, three-month, six-month, one-year, two-year, three-year, five-year, seven-year, ten-year, twenty-year, and thirty-year maturities. The "bellwether" thirty-year maturity is no longer reported because the U.S. government has discontinued this issue. The time to maturity is, of course, only one of several factors that affect nominal interest rates.

Plotting the yield to maturity against the time to maturity, one finds several shapes of yield curve in many forms over the years. Nevertheless, there are four common shapes for the yield curve; 1) upward-sloping 2) downward-sloping 3) humped, and 4) Flat.

It is not surprising to find several theories that attempt to explain the dynamic development of those various shapes. Nonetheless, explanations for the shapes fall predominantly into three categories: the unbiased expectation theory, the liquidity premium theory, and the market segmentation theory.

Perhaps, more interesting issues are the importance of the term structure of interest rate and its roles and applications in the financial world. Changing interest rate levels affect security values.

One of the simplest examples of this nonlinear price-yield relationship is a bond value. The valuation of a bond instrument employs the time value of money concept. The fair value of a bond reflects the present value of all cash flows expected or projected to be received on that bond and then discounted at the required rate of return. Similarly, the expected rate of return is the interest rate that equates the current market price of the bond with the present value of all promised cash flows to be received over the life of the bond. Thus the levels of interest rate at various maturities and the rapid changes in those levels dominate the valuation of a bond or asset pricing.

The term structure of interest rate also plays an important role in derivatives markets. It is one of major components for the valuation of many derivatives instruments, for instance of an interest rate option. In the past few decades, the derivatives markets have grown significantly, and investors have become more active in managing their portfolio risk using interest rate options. The role of interest rate levels and the changes in those levels can only neglected by any fixed income portfolio manager at great risk.

These applications and the overall usefulness of the term structure of interest rate for asset valuation challenge financial scholars to further develop more advanced interest rate models. These models started from a very basic one-factor affine model based on Geometric Brownian Motion (GBM) to more sophisticated infinite-dimensional models.

The development of modeling techniques significantly contributes to the advancement of the term structure of interest rate model research that is beneficial to all stakeholders, such as investors, hedgers, arbitrageurs, and policy planners. However, the usefulness of the standard models, such as affine models, is now doubted while new models such as non-affine models have not yet significantly proven their own validity.

Practitioners still rely on the affine interest rate diffusion models such as one- or two-factor models because of their simplicity generated by the incorporated linearity and Gaussian distribution assumptions. The lack of substantial use and test of new advanced models (term structure fitting models) either misleads or slows down the improvement of modeling techniques and does not provide sufficiently strong evidence to support the fact that future research should tilt toward the non-affine or non-linear models.

In this paper, our goal is to empirically re-examine the validity of the strongly held assumptions of affine. In other words, the classical assumptions of non-periodicity, stationarity, and independency, or, in short, of “white noise” innovations, is here closely examined by advanced signal processing techniques for publicly available daily interest rate series for different maturities. Furthermore, with considerably more empirical information of risk, in particular of local obtained by Wavelet Multiresolution Analysis (MRA), a proper theory might be identified, or reconfirmed with more accuracy.

The following sections are organized as follows. Section two briefly reviews the literature of the theories of term structure and existing term structure models. Section three discusses data and new methodologies. Section four presents the empirical results and some discussion. Finally, section five provides the conclusion of this paper.

2. Term Structure of Interest Rate Models

Prior research has proposed three main theories to explain the shapes of the term structure. Each theory has its own rationale and assumptions as follows.

(1) Unbiased Expectations Theory

According to the unbiased expectations theory of the term structure of interest rates, at a given point in time the yield curve reflects the markets' current expectation of future short-term rates. Specifically, the unbiased expectations theory assumes that current long-term interest rates are geometric averages of current and expected future short-term interest rates, a relationship can be described, in a discrete form, as follows;

$$(1+R_N)^N = (1+R_1)(1+E(r_2))...(1+E(r_N)) \tag{1}$$

therefore:

$$\overline{R}_N = [(1+\overline{R}_1)(1+E(\tilde{r}_2))...(1+E(\tilde{r}_N))]^{1/N} - 1 \tag{2}$$

where

$\overline{R}_N$ = Actual N-period rate today

N = Term to maturity

$\overline{R}_1$ = Actual 1-year rate today

$E(\tilde{r}_i)$ = Expected one-year rates for years 2, 3, 4, …, N in the future

(2) Liquidity Premium Theory

The liquidity premium theory can be viewed as an extension of the unbiased expectations theory. Its concept is that investors will hold long-term maturities only if they are offered at a premium to compensate for future uncertainty in a security's value which increases with an asset's maturity. The theory states that long-term rates are equal to geometric averages of current and expected short-term rates with additional components, liquidity risk premiums that increase with the maturity of the security.

The liquidity premium theory might be mathematically represented as:

$$\overline{R}_N = [(1+\overline{R}_1)(1+E(\tilde{r}_2)+L_2)...(1+E(\tilde{r}_N)+L_N)]^{1/N} - 1 \qquad (3)$$

where

L_t = Liquidity premium for a period t

$L_2 < L_3 < ...L_N$

(3) Market Segmentation Theory

The market segmentation theory suggests that individual and institutional investors have specific maturity preferences. To persuade them to hold securities with maturities other than their more preferred requires a higher interest rate. Accordingly, the theory does not consider securities with different maturities as perfect substitutes.

Rather, investors have preferred investment horizons controlled by the nature of the liabilities they hold. Thus, interest rates are determined by distinct supply and demand conditions within a particular maturity segment. The major assumption is that investors and borrowers are generally unwilling to shift from one maturity sector to another without sufficient compensation in the form of an interest rate premium. The Market

Segmentation Theory suggests inefficiency in the general equilibrium of the partially non-communicating interest rate markets, ranked by maturity, which is not completely eliminated by inter-market arbitrage.

The conventional parametrization of the term structure involves searching a small set of parameters that influence discount factors using error minimization or a calibration technique, while those discount factors equate an asset’s price with its related future cash flows. This approximating or fitting search generates an “average” yield curve. Los (2003) suggests that, in current common use are two types of curves for the parametrization: the dynamic affine interest rate diffusion models and these averaging term structure fitting models.

2.1 Affine Interest Rate Diffusion Models

2.1.1 Linear Term Structure Affine Models

The affine interest rate diffusion models or, in short, affine models are very popular ranging from the one-factor models with mean-reverting short rate dynamics of Vasicek (1977) and Cox, Ingersoll, and Ross (1985), the two-factor models of Heath, Jarrow, and Morton (1990 and 1992) and Hull and White (1990 and 1993), the two-factor dynamic mean model of Sorensen (1994), the three-factor model of Balduzzi et al. (1996), the stochastic volatility model of Longstaff and Schwartz (1992) to the random field of Kennedy (1994 and 1997) that is later extended by Goldstein (2000) and Santa-Clara and Sornette (2001).

Prior research suggests that the most popular and perhaps the simplest affine models are the one-factor models. Chapman and Pearson (2001) argues that the source of their popularity derives from the fact that from empirical studies using a principal component analysis, approximately 90 percent of the variation of the term structure can be attributed to the first principal component. In other words, 90 percent of the variation is in interest rates can be explained by changes in the level of the whole yield curve. In other words, any point on the term structure can be used as a proxy for the whole yield curve. Generally, the intercept of the term structure, or the instantaneous short rate of interest, is used for that one factor.

However, Los (2003) argues that the statistical pitfalls of principal component analysis might create a problem (Los, 1989). As generally accepted, the percentage of variation attributable to a particular component depends on the number of components that are retained from the covariance matrix. Thus, both sizes of the covariance matrix, or the number of maturity segments, as well as how many of these segments are considered significant, become very important since that will determine the percentage of variation decomposition. The same difficulty applies even more so to more-than-one-factor affine models.

Once those one-factor diffusion models are solved by either symbolic or numerical integration, the exponentially linear price process is obtained. For example, the very popular one-factor model of Vasicek (1977) generates a zero bond price that is exponentially linear in the short rate implying that the spot rates of all maturities are linear in the short-term rate. This simple Vasicek model appears to prove itself to fit all of the term structure shapes, but does not generate the correct third and higher order dynamic distribution moments. In addition, Bakshi and Chen (1996) and Rogers (1996) comment that with its assumed Gaussian distribution, the model erroneously produces a negative interest rate that affects the modeling of nominal interest rates and the pricing of interest rate derivatives.

Cox, Ingersoll, and Ross (1985) attempt to solve the problem by incorporate a reflecting boundary for the diffusion process. The resulting price process remains the same, an exponentially linear price process, even after their adjustment although the horizon dependent deterministic functions are different. Duffie and Kan (1994) and Dai and Singleton (2000) correctly pointed out that a price process that is exponentially linear in the short rate price process and with drift and variance components linear in the one or two factors are features of the general class of affine models. These features make these models theoretically useful, but they are incorrect from a scientific identification or modeling perspective,

Recently, Kennedy (1997), Goldstein (2000), and Santa-Clara and Sornette (2001) introduced the so-called random field or stochastic string model. This model attempts to incorporate an infinite-dimensional Gaussian shock for each point of the forward curve. In other words, each point of the yield curve has its own particular affine model. This would imply a covariance matrix of the interest rate maturity segments of infinite order. However, in practice, the observations of the yield curve are limited to a few well-defined maturities. Moreover, empirical shocks are finite in number. There is an obvious problem of reconciling the infinitely dimensional theoretical model with the finite number of empirical observations available.

2.1.2 Non-Linear Term Structure Affine Models

This class of affine models, *e.g.*, the quadratic term structure models deserves much more attention because they incorporate the potential for chaotic dynamic equilibrium components appearing in the empirical interest rate time series data. Longstaff (1989) and Beaglehole and Tenney (1992) provide nonlinear term structure models. Constantinides (1992) develops also such a nominal term structure model.

The three-factor model by Tice and Webber (1997), which can produce Lorentzian chaotic behavior, has a non-linear component (quadratic) factor and two linear factors. The three factors are the short-term interest rate, the short rate reversion level, and a feedback parameter where the non-linear term has been included. The difficulty with this model is to determine the proper empirical values of the parameters of the model's stochastic processes. James and Webber (2001) provide values of the parameters without any empirical justification.

2.2 Term Structure Fitting Models

The term structure fitting or averaging models do not require a specific dynamic interest rate diffusion model. Smoothing splines and a model by Nelson and Siegel (1985) can well serve as a linear and non-linear family of curve of fitting models, respectively. These models rely heavily on stationary price conditions and a fixed set of parameters in the models. The models' accuracy is determined by the realization of the assumption of stationarity in the time series data of the interest rates. This is only realized in very calm interest rate markets.

De Boor (1978) and Dierckx (1995) suggest spline models, where the splines are linear non-parametric interpolation methods including smoothing, cubic, exponential, and B-splines. Vasicek and Fong (1982) and Steeley (1991) apply those methods to the term structure.

Nelson and Siegel (1985) propose a nonlinear family of forward rate curves. Although their model has only four parameters, it fits U.K. government bond rates very well. However, when applying to a different set of empirical data, from another time window, the model does not fit the term structure well, and deviates particularly at the short end, *i.e.*, precisely at the intercept or instantaneous interest rate. The short term rates are relatively most volatile.

Thus, the major assumptions of the term structure models involve Gaussian distributions, stationarity, independency, and, overall, white noise. Therefore, it should be worth investigating these basic theoretical assumptions and justify whether or not the proposed models are identifying the dynamics of actual empirical term structures. This paper aims to examine the distributions and the dynamic time- and frequency dependence relationships of the term structure of interest rates. Fortunately, today advanced signal processing technology, with time-frequency or time-scale analysis by wavelet multiresolution (MRA) enables the execution of such in-depth empirical studies.

3. Data and Methodology

3.1 Data

Table 1 summarizes the interest rate series obtained from the Federal Reserve website (H.15). There are two major sets of the interest rate series. The first set is the Treasury bill rates quoted from secondary markets. The second set is the computational interest rate series called Treasury constant maturity. In this study, all available series were analyzed. However, only the results of Treasury constant maturity interest rate series with 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, and 10-year are reported in this paper.

[Please insert Table 1 about here]

The interest series could have the different lengths of observations. Some series are even discontinued while some have only recently appeared. With the number of observations constraint of an online Wavelet MRA, the period covered in this paper is shorter and aimed to focus on the 1990s, specifically the last 12 year period (January of 1991- October of 2002). The same length of period is used to calculate the homogeneous or uni-fractal Hurst exponent as well.

3.2 Methodology

In this paper, the Hurst exponents and Wavelet MRA are employed to test different characteristics of the empirical interest rate time series and their difference series from some particular maturities or nodes. We used the online interactive wavelet MRA in ION Script of Research Systems (A Kodak Company, www.ResearchSystems.com), described in Torrence and Compo (1998).

3.2.1 Measure of Persistence

Most of the term structure models, if not all, incorporate the theoretical Geometric Brownian motion (GBM) diffusion model which assumes that the series are only short-term serially dependent. Moreover, these models assume that the time series data exhibit no long-term dependence. When we find evidence of long-term dependency of the interest rate series, it suggests an essential violation of most of the term structure models.

The Hurst exponent (H) is designed to test the long-term dependence of time series. Such long-term dependence can be persistent or anti-persistent. If $0 < H < 0.5$, the series are anti-persistent. If $0.5 < H < 1$, the series are persistent. If $H = 0.5$, the series is called white noise. Integrated white noise results in brown noise, *i.e.*, described by the Brownian motion.

The Hurst exponent can be calculated using the rescaled range or R/S Analysis. The rescaled range statistic RSH is defined as

$$RS_H(T) \equiv \frac{1}{c_2^{0.5}}[\underset{1 \le t \le T}{Max} \sum_{t=1}^{t}[x(t) - m_1] - \underset{1 \le t \le T}{Min} \sum_{t=1}^{t}[x(t) - m_1]] \ge 0 \tag{4}$$

where $c_2 = \frac{1}{T} \sum_{t=1}^{T} [x(t) - m1]^2$

The Hurst exponent self is defined by

$$H = \lim_{T \longrightarrow \infty} \frac{\ln(RSH(T))}{\ln T} \tag{5}$$

3.2.2 Wavelet MRA: Logarithmic Scalegram and Scalogram

Unlike any other traditional “global” or average risk measurement, the Wavelet MRA enables one to measure “local” or instantaneous risk and thus provides extremely accurate risk information about any time series. A major attribute of the Wavelet MRA is that such analysis can be easily visualized. Scalegrams (average frequency analysis) and scalograms (localized instantaneous frequency analysis) can be constructed by computing the Wavelet resonance coefficients as follow:[1]

$$d_{j,n} = \int_{-\infty}^{+\infty} x(t) y(\cdot) dt \text{, with j, n } \in \mathrm{Z} \tag{6}$$

where the wavelet function $y(\cdot)$ in this study is Morlet wavelet which is a localized function of the form

---

[1] Los (2003) provides the mathematical and technical details

$p^{-1/4} e^{i w_0 h} e^{-h^2/2}$

The variance of the zero-mean wavelet resonance coefficients $d_{j,n}$ can now be computed as

$$Var\{d_{j,n}\} = E\{\left|d_{j,n}\right|^2\} \tag{7}$$

This last expression provides a simple scaling law, from which the Hurst exponent can be directly derived from the scaling slope.

4. Empirical Result and Discussions

4.1 Persistence Analysis of the Very Short-term (3-month) Maturity Interest rate series

Figure 1-A, Panel (a), shows the plot of time series of 2953 daily observations on three-month Treasury constant maturity interest rate for the period January 2, 1991 through October 18, 2002. The Morlet (6) wavelet used in this analysis is shown next to the right of panel (a). Panel (b) presents the localizing wavelet scalogram, MRA, or local power spectrum. It shows the power spectrum at each instance of time. Panel (c) shows the global wavelet scalegram, or the conventional average power spectrum. Vertical axes of both panel (b) and (c) indicate the frequency by the number of days in the time horizon t. High frequencies are shown at the top of both panels, while low frequencies are at the bottom.

[Please insert Figure 1 about here]

The horizontal axis of the scalegram presents the variance of the wavelet resonance coefficients: $Var\{d_{j,n}\} = E\{\left|d_{j,n}\right|^2\}$ on a decimal logarithmic scale. The scalogram in panel (b) shows the contour plot of the localized power $\left|d_{j,n}\right|^2$ that is colorized into five categories ranging from white, indicating no risk or no significant energy, to red color indicating the highest risk or significant energy. Using the same methodologies as appearing in figure 1-A, figure 1-B shows an analysis of the first differences of three-month Treasury constant maturity interest rate series. There were at least nine disturbances during a period between 1991 and 2002. The high power or risk appears in both high and low frequencies' domain, but subsides in the medium frequencies, especially in the later years (about 1999-2000) where there was almost no risk indicated for the high frequencies from about 2 days to 6 months.

The increasing power or risk of the high frequency trading appears for most time period, particularly some red color area in last quarter of 1994 (920th-1000th observations), at the beginning of 4th quarter of 1998 (1930th observation), at the beginning of August 1999 (2150th observation), in the mid of May 2000 (2350th observation), end of year 2000 (2500th observation), and about the beginning of the 4th quarter of year 2001 (2700th observation).

When considering the scalegram in the panel (c), it is obvious that the differencing of the interest rate series does not suggest white noise. The scalegram would show a completely flat spectrum if the series was white noise. Several peaks (at least 7 peaks) appear in the plot of scalegram clearly indicating the institutional periodicities at the 5-trading day or "weekly" frequency, the 10-trading day or "bi-weekly" frequency, the 21-trading day or "monthly" frequency, the 32-trading day, the 63-trading day or "quarterly" frequency, and the 189-trading day.

The Hurst exponents (H) computed from the scalegrams, which measure the degree and type of long-term dependence of the interest rate series are shown in Table 2.

[Please insert Table 2 about here]

The H = 0.16 of 3-month maturity interest rate during the 1990s indicates clear anti-persistence of the first difference noise. In fact, all H's of the studied maturities suggest anti-persistent noise or "innovations". In other words, these markets are ultra-fast mean-reverting. However, it should be noted that the H is a global or an average exponent for the whole time domain. It is possible that in different time domains or windows the H exhibits different values. Since anti-persistence implies the ultra-efficiency of markets or systems, efficiency enhancing financial turbulence can be expected to occur in these markets. It is important to emphasize that such turbulence is, indeed, efficiency enhancing, instead of the unpredictable catastrophic breaks that can occur in highly persistent markets, like stock markets or real estate markets.

4.2 Persistence Analysis across the Maturity Interest rate series

The same analysis as above can be applied for the other maturity interest series in this study. Clearly seen from figures 1-8, the localized power or risk is different for each maturity node.

[Please insert Figure 2 about here]

[Please insert Figure 3 about here]

[Please insert Figure 4 about here]

[Please insert Figure 5 about here]

[Please insert Figure 6 about here]

[Please insert Figure 7 about here]

[Please insert Figure 8 about here]

More interestingly, the longer the investment horizon, the more disturbance periods are shown in the scalograms. There are much more noticeable red areas when the analysis shifts across maturities from 3-month to 10-year maturities. The high power or risk is shown in both high and low frequencies still.[2]

Moving to the first differenced interest rate series, the scalegrams confirm that none of the interest series suggest the conventionally assumed white noise behavior. More importantly, there are clearly observable changes in the spectral periodicities for different maturities. As discussed earlier, the 3-month and other short-term maturity show the periodicities in both high and low frequencies. In other words, investors and bond traders conduct pricing transactions very often or very seldom, but not at intermediate frequencies. However, when the term to maturities becomes longer than one year, the high-frequency short-term transactions are smoothed out. The peaks in the scalegrams on the high frequencies become less prominent. Only the peaks in the low frequencies are then visible.

Although the Hurst exponents for all maturities suggest anti-persistence or ultra-efficiency in these interest rate markets, there exist obvious differences in the degree of anti-persistence. The short-term and medium maturities (3-month to 3-year) have lower H values, thus a higher degree of anti-persistence than those of long-term maturities, a result that should not be surprising. One possible explanation of that anti-persistence is that investors prefer short-term and medium-term investment horizons to the less certain long-term maturity interest rates. Another is the roll-over behavior of interest rate swaps.

---

[2] We made rather dramatic dynamic movies of some of the Figures in this paper, which suggest that real-time time-frequency analysis is possible.

As presented, the long maturities show higher localized power or risk than the short and medium maturities.

It is clear that each maturity node of the yield curve possesses its unique dynamic behavior, from both a time and frequency perspective, suggesting a unique market or demand and supply situation. Investors might use the short-term interest rate securities for their both short- and long-term investment horizon, while investors only use long-term interest rate securities for long-term investment horizon. In other words, investors might attempt to match the investment horizon with the term to maturities. The characteristics found here are thus more supportive of the Market Segmentaion Theory than the Unbiased Expectations Theory, which assumes effortless efficiency.

Moreover, the results also suggest that the assumptions of most, if not all, theoretical affine models are violated, which is consistent with prior research. One-factor and two-factor affine models assume that the price process of a zero bond is influenced by the one or two sources of random shocks, and, thus, the term structure spot rates are locally perfectly correlated with each other. The Wavelet MRA in this study provides strong evidence that localized power or risk is greatly different among nodes or maturities and that such perfect correlation does not exist. Our findings suggest that the correlation, or linear dependence, between the various maturity spot rates is not unity, and that such correlation is significantly reduced for different investment horizons. These technically better supported findings are consistent with those more crudely obtained by Chapman and Pearson (2001). Conclusively, the term structure of interest rates or the yield curve is highly segmented, and the localized risk dynamics are beyond the simple assumptions of the affine and simply fitted, simply parametrized term structure models.

The question in this paper is not whether affine models are completely falsified, but it is important to specify how one can apply those traditional term structure models to different market segments. Investors might not be able to merely use one global affine model for all maturities, as the yield curve is segmented qua time and frequency characteristics. Each maturity node might require a unique model. A complete resolution of this empirical modeling problem opens the field of term structure modeling to escape the too restrictive confines of the outdated principal component or factor models.

5. Conclusion

Like prior research, this paper uses empirical publicly available interest rate time series data to test a few crucial assumptions of most term structure of interest rate affine models, particularly those very popular one- and two-factor models. With new methodology of the Wavelet MRA, the results are much more precise and revealing than those generated by traditional methodologies. They strongly suggest that the term structure of interest rate is segmented qua dynamic time-and frequency characteristics.

Analyzing eight different major interest rate maturities, this study provides strong evidence that almost all, if not all, of the basic assumptions for the term structure models are violated. All of the eight maturities interest rate series show clear periodicities, long-term dependence, particularly anti-persistence, and, perhaps most important, each maturity node has its own localized power or risk and degree of efficiency measured by the homogeneous Hurst exponents. Unfortunately, all affine models assume that all maturities have the same risk or are perfectly correlated with each other. This assumption is clearly empirically rejected. Our results support the Market Segmentation Theory instead for U.S. Treasury securities. The demand and supply formed by investors' maturity preferences play an important role in such a maturity segmented system.

To be successful in modeling the term structure with highly segmented maturities, further research on the one model for one maturity might be an alternative, or an even better alternative would be to try a cascade-type of model, as suggested by Los (2003). Another option is to further develop and test the non-linear term-structure models like the quadratic models.

Reference:

Bakshi, G. and Z. Chen, 1996, Inflation, Asset Prices, and the Term Structure of Interest Rates, Review of Financial Studies 9, 241-276.

Balduzzi, P., S. R. Das, S. Foresi, and R. Sundaram, 1996, A simple Approach to Three Factor Affine Term Structure Models, Journal of Fixed Income 6, 43-53.

Beaglehole, D. and M. Tenney, 1992, Corrections and Additions to A Nonlinear Equilibrium Model of the Term Structure of Interest Rates, Journal of Financial Economics 32, 345-353.

Chapman, David A. and Neil D. Pearson, 2001, Recent Advances in Estimating Term Structure Models, Financial Analysts Journal 57, 77-95.

Constantinides, G., 1992, A Theory of Nominal Terms Structure of Interest Rates, Review of Financial Studies 5, 531-552.

Cox, J. C., J. E. Ingersoll, and S. A. Ross, 1985, A Theory of the Term Structure of Interest Rate, Econometrica 53, 385-407.

Dai, Q., and K. Singleton, 2000, Specification Analysis of Affine Term Structure Models, Journal of Finance 55, 385-407.

De Boor, C., 1978, A Practical Guide to Splines, (Springer – Verlag, New York).

Dierckx, P., 1995, Curve and Surface Fitting with Splines (Oxford Science Publications, London).

Duffie, D., and R. Kan, 1994, Multi – Factor Term Structure Models in: Philosophical Transactions of the Royal Society of London, A, Vol. 347, 577 – 586.

Goldstein, R., 2000, The Term Structure of Interest Rates as a Random Field, Review of Financial Studies 13, 365-384.

Heath, D., R. A. Jarrow, and A. J. Morton, 1990, Bond Pricing and the Term Structure of Interest Rates: A Discrete Time Approximation, Journal of Financial and Quantitative Analysis 25, 419-440.

Heath, D., R. A. Jarrow, and A. J. Morton, 1992, Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation, Econometrica 60, 77-105.

Hull, J. C. and A. D. White, 1990, Pricing Interest Rate Derivative Securities, Review of Financial Studies 3, 573-592.

Hull, J. C. and A. D. White, 1993, One – Factor Interest – Rate Models and the Valuation of Interest – Rate Derivative Securities, Journal of Financial and Quantitative Analysis 28, 235-254.

James, Jessica and Nick Webber, 2001, Interest Rate Modeling (John Wiley & Sons, New Jersey).

Kennedy, D. P., 1994, The Term Structure of Interest Rates as a Gaussian Random Field, Mathematical Finance 4, 247-258.

Kennedy, D. P., 1997, Characterizing Gaussian Models of the Term Structure of Interest Rates, Mathematical Finance 7, 107-118.

Longstaff, F. A. and E. S. Schwartz, 1992, Interest Rate Volatility and the Term Structure: A Two – Factor General Equilibrium Model, Journal of Finance 47, 1259-1282.

Longstaff, F. A., 1989, A Nonlinear General Equilibrium Model of the Term Structure of Interest Rates, Journal of Financial Economics 23, 195-224.

Los, Cornelis A., 1989, The Prejudices of Least Squares, Principal Components and Common Factors, Computers & Mathematics with Applications 17, 1269 - 1283.

Los, Cornelis A., 2003, Financial Market Risk: Measurement & Analysis (Routledge Studies in Money and Banking, an imprint of Taylor & Francis Books Ltd, London).

Nelson, C. R. and A. F. Siegel, 1985, Parsimonious Modeling of Yield Curves, Journal of Business 60, 473-489.

Rogers, L. C. G., 1996, Gaussian Errors, Risk 9, 42-45.

Santa – Clara, P. and D. Sornette, 2001, The Dynamics of the Forward Interest Rate Curve with Stochastic String Shocks, Review of Financial Studies 14, 149-185.

Sorensen, C., 1994, Option Pricing in a Gaussian Two – Factor Model of the Term Structure of Interest Rates, Working Paper, WP – 94 – 4, Copenhagen Business School, Copenhagen, Denmark.

Steeley, J. M., 1991, Estimating the Gilt - Edged Term Structure: Basis Splines and Confidence Intervals, Journal of Business Finance and Accounting 18, 513-530.

Tice, J. and N. J. Webber, 1997, A Non – Linear Model of the Term Structure of Interest Rates, Mathematical Finance 7, 177-209.

Torrence, C. and G. P. Compo, 1998, A Practical Guide to Wavelet Analysis, Bulletin of the American Meteorological Society 79, 61-78.

Vasicek, O. A., 1977, An Equilibrium Characterization of the Term Structure, Journal of Financial Economics 5, 177-188.

Vasicek, O. and G. Fong, 1982, Term Structure Modeling Using Exponential Splines, Journal of Finance 37, 339-348.

Table 1

The Summary of the Various Interest Rate Series

The Federal reserve website provides various interest rate series (H-15 form). Those series might have different starting dates. Some series have been discontinued. The data is last updated for this paper on October 18, 2002. The results of studies in this paper are based on the interest rates of Treasury Constant Maturities. The results from Treasury Bills, specifically for Short horizon, are quite similar.

Treasury Bills (secondary Market)	Start	End
4-W	7/31/2001	10/18/2002
3-M	1/4/1954	10/18/2002
6-M	12/9/1958	10/18/2002
1-Y	7/15/1959	8/27/2001

Treasury Constant Maturities	Start	End
1-M	7/31/2001	10/18/2002
3-M	1/4/1982	10/18/2002
6-M	1/4/1982	10/18/2002
1-Y	1/2/1962	10/18/2002
2-Y	6/1/1976	10/18/2002
3-Y	1/2/1962	10/18/2002
5-Y	1/2/1962	10/18/2002
7-Y	7/1/1969	10/18/2002
10-Y	1/2/1962	10/18/2002
20-Y (historical)	1/2/1962	12/31/1986
20-Y	10/1/1993	10/18/2002
30-Y	2/15/1977	2/15/2002

Table 2

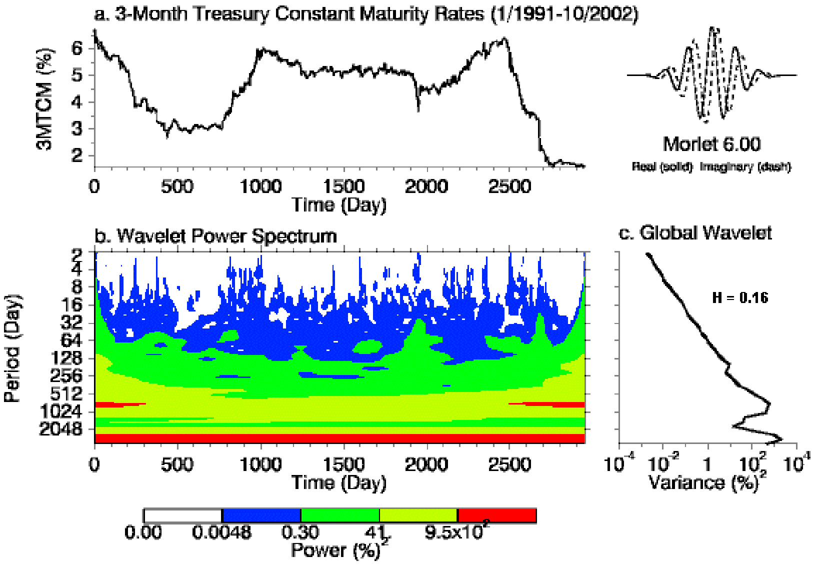
Hurst Exponents of the Term Structure of Interest rate

The covered period is from October 23, 1990 to October 18, 2002. The number of daily interest rate observations is 3,000. R/S range scale method is used to generate the Hurst Exponents (H). If H = 0.5, the term structure follows the Geometric Brownian Motion or indicates white noise. If 0 < H < 0.5, the term structure is anti-persistent implying that the system is rather efficient. If 0.5 < H <1, the system is rather inefficient. Also shown is the fractional differencing parameter d where H = d + 0.5.

Maturities	H Exponents	d	State of system
3-Month	0.157	-0.343	Anti-Persistent
6-Month	0.164	-0.336	Anti-Persistent
1-Year	0.170	-0.330	Anti-Persistent
2-Year	0.142	-0.358	Anti-Persistent
3-Year	0.140	-0.360	Anti-Persistent
5-Year	0.232	-0.268	Anti-Persistent
7-Year	0.246	-0.254	Anti-Persistent
10-Year	0.264	-0.236	Anti-Persistent

Figure 1

A) Wavelet MRA of the 3-month Treasury constant Maturity Rates.



B) Wavelet MRA of the first differences of the 3-month Treasury Constant Maturity Rates

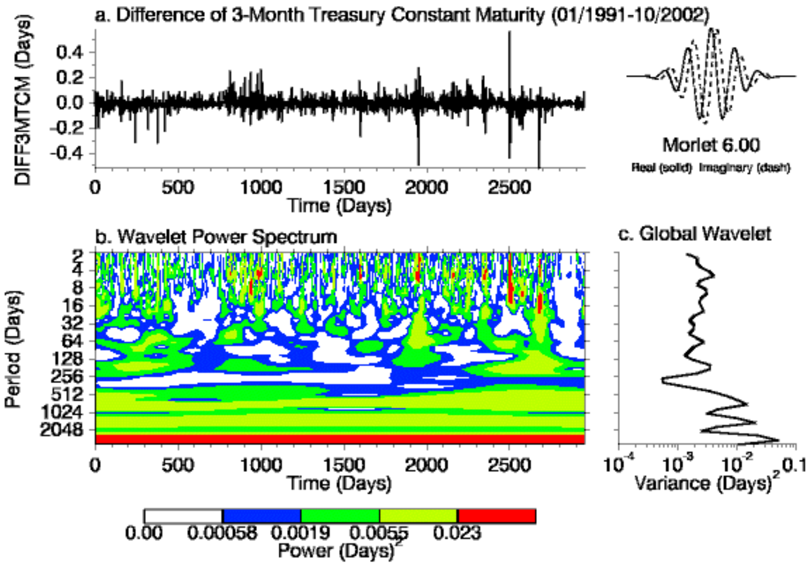
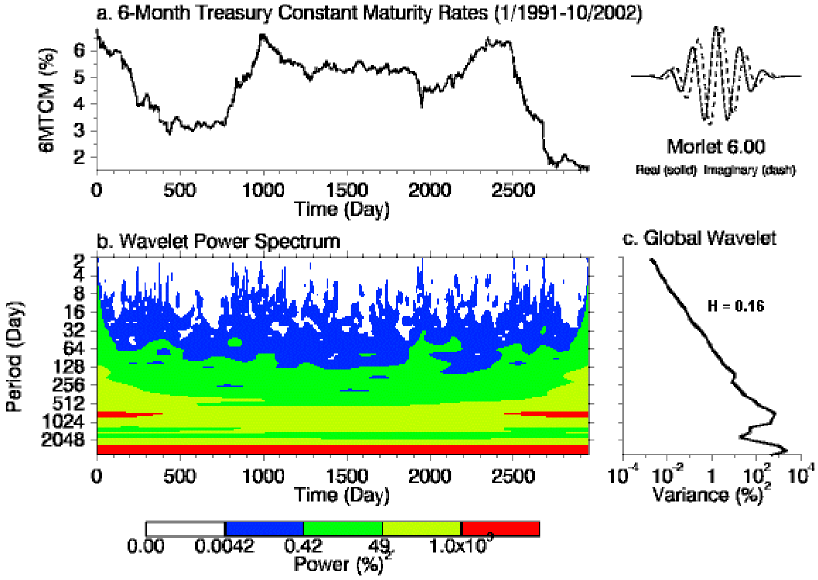


Figure 2

A) Wavelet MRA of the 6-month Treasury constant Maturity Rates.



B) Wavelet MRA of the first differences of the 6-month Treasury Constant Maturity Rates

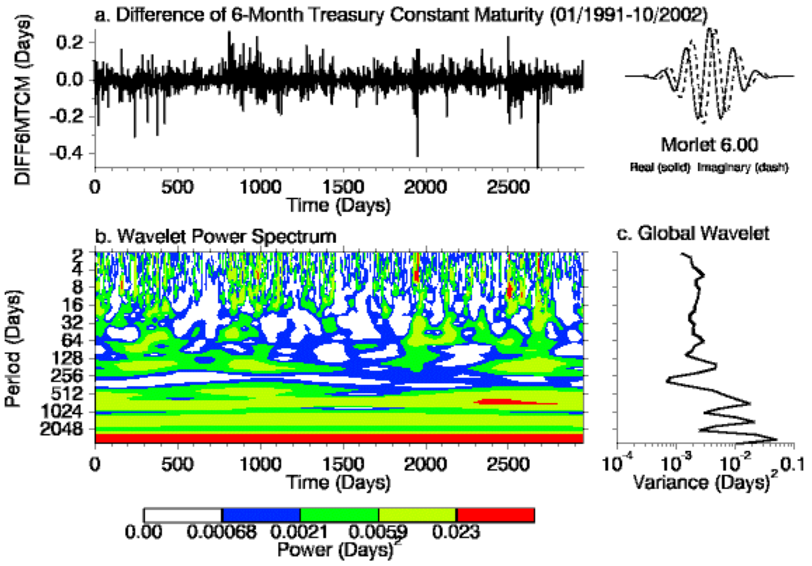
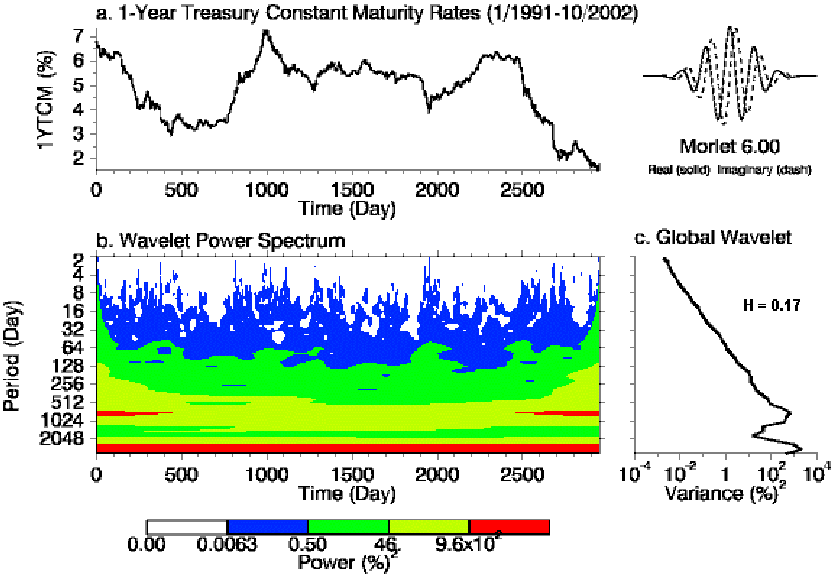


Figure 3

A) Wavelet MRA of the1-year Treasury constant Maturity Rates.



B) Wavelet MRA of the first differences of the 1-year Treasury Constant Maturity Rates

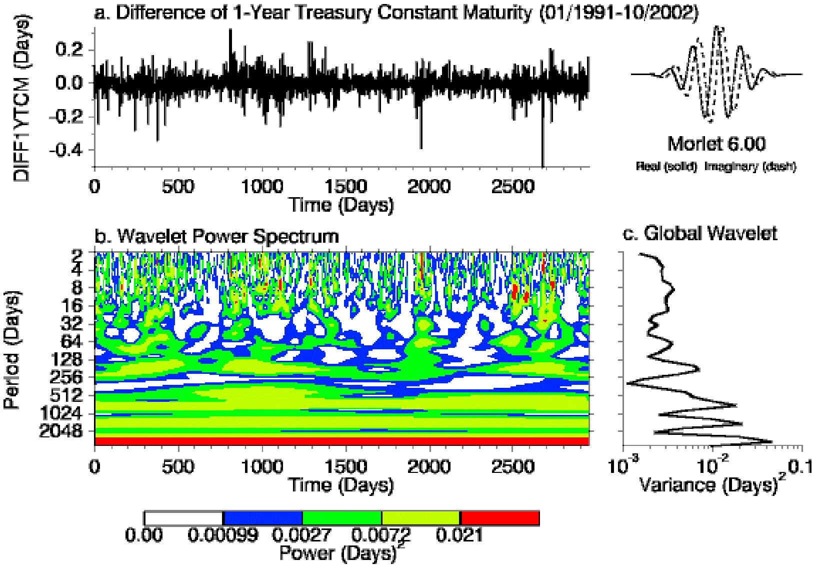
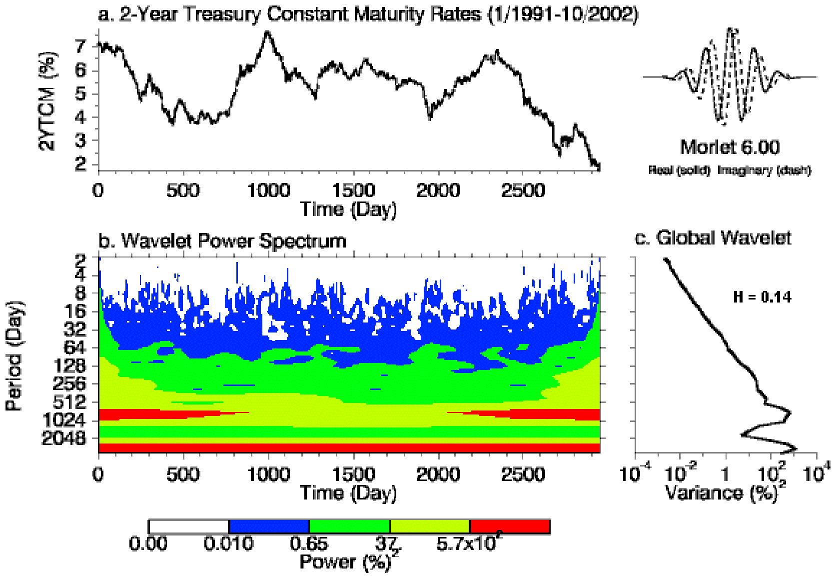


Figure 4

A) Wavelet MRA of the 2-year Treasury constant Maturity Rates.



B) Wavelet MRA of the first differences of the 2-year Treasury Constant Maturity Rates

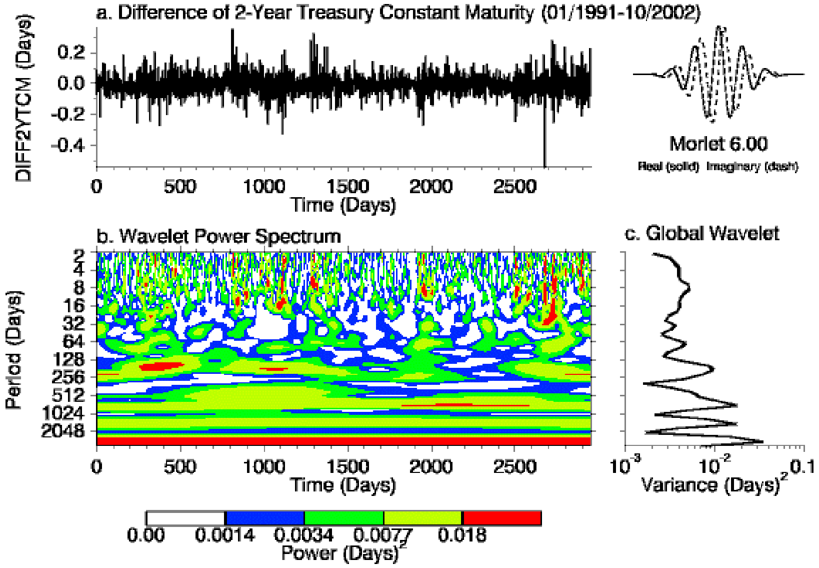
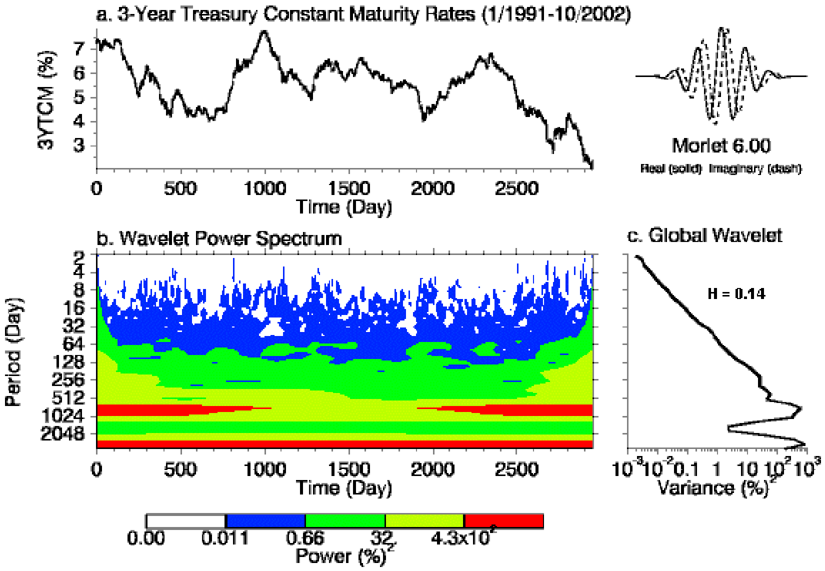


Figure 5

A) Wavelet MRA of the 3-year Treasury constant Maturity Rates.



B) Wavelet MRA of the first differences of the 3-year Treasury Constant Maturity Rates

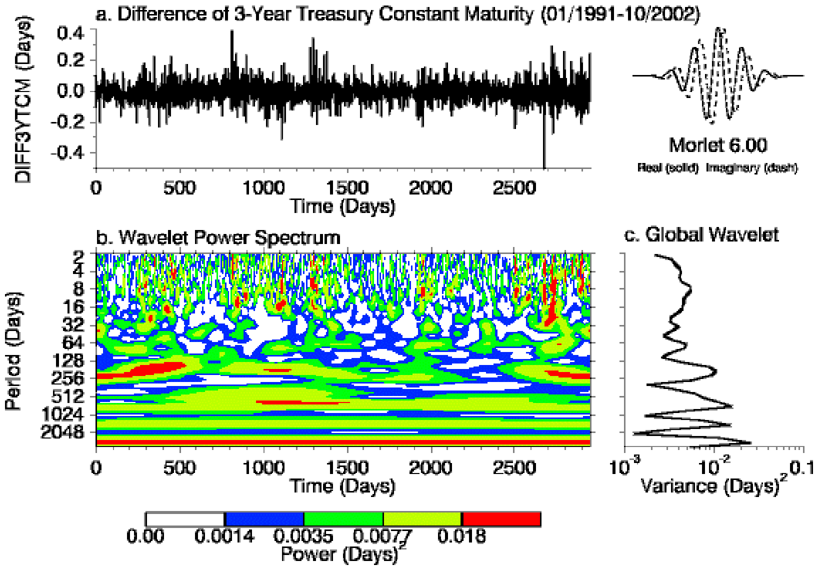
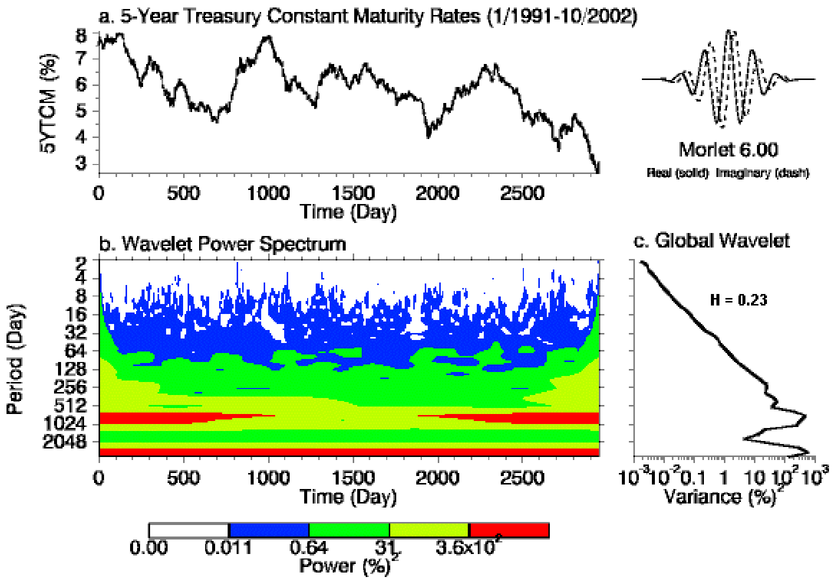


Figure 6

A) Wavelet MRA of the 5-year Treasury constant Maturity Rates.



B) Wavelet MRA of the first differences of the 5-year Treasury Constant Maturity Rates

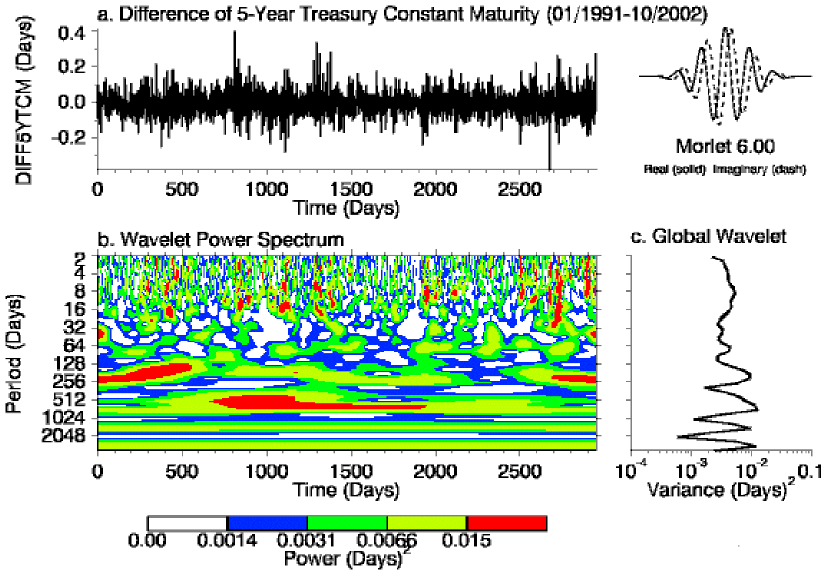
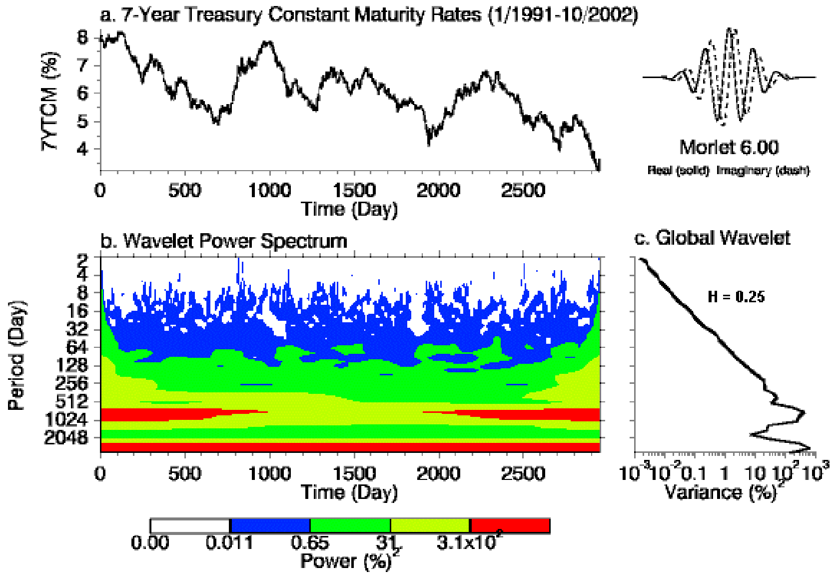


Figure 7

A) Wavelet MRA of the 7-year Treasury constant Maturity Rates.



B) Wavelet MRA of the first differences of the 7-year Treasury Constant Maturity Rates

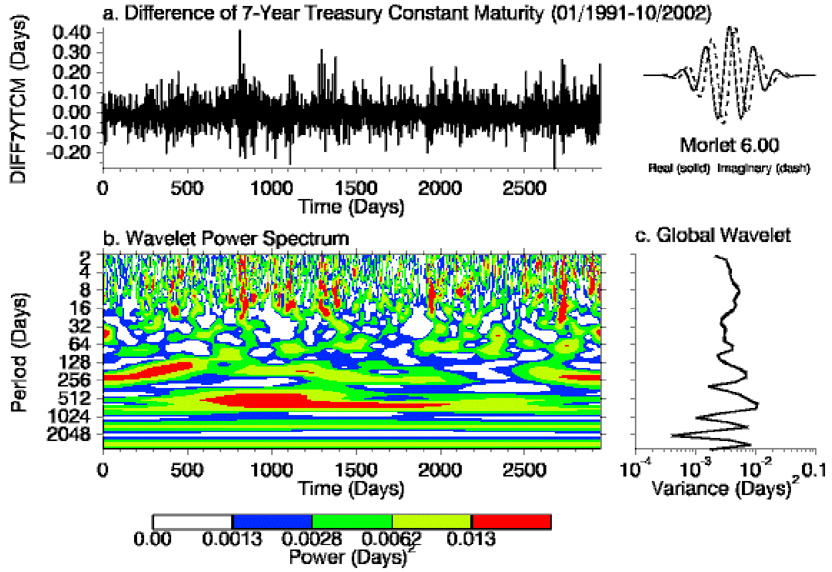
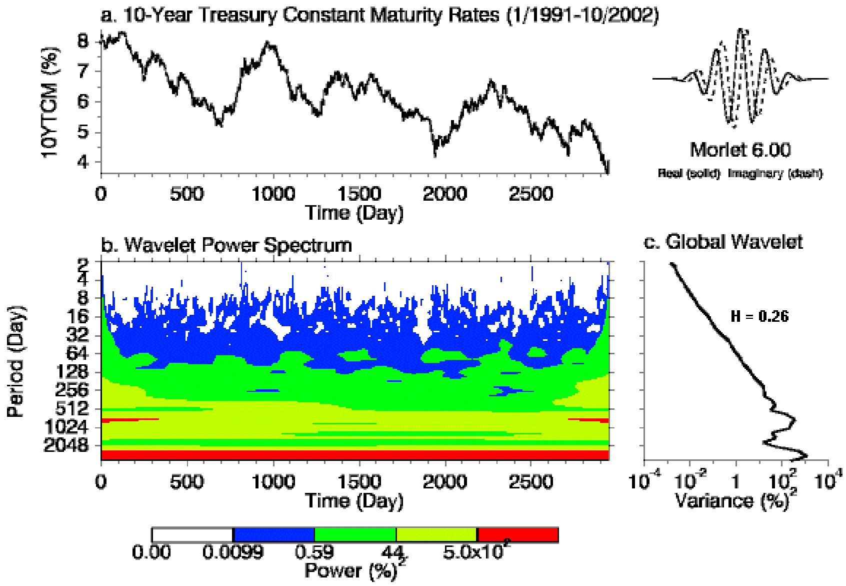


Figure 8

A) Wavelet MRA of the 10-year Treasury constant Maturity Rates.



B) Wavelet MRA of the first differences of the 10-year Treasury Constant Maturity Rates