Log-Periodicity in High Frequency Financial Series

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Abstract

We assess the log-periodicity hypothesis for financial series behavior [3-6]. We put forward a three-harmonic log-periodic formula to fit both daily and intraday data. And we take the exchange rate between the Brazilian *real* and the US dollar to illustrate our case.

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1. Introduction

A familiar way to generate randomness is to break a solid object. The line along which fracture occurs seems random, and this is almost universal to all kinds of materials. It can then be argued that whatever basic mechanism is causing randomness, it cannot depend on the details of particular materials [1].

So there is room for a unified theoretical model for predicting rupture in materials in general [2]. The breakdown can be modeled as a network of growing and interacting microcracks that finally result in rupture. It can then be found that the rate at which these cracks release energy is correlated with the time left before the material suffered failure. Thus the microcracks inform us a countdown to rupture.

The microcracks can be plotted against the log of time to failure. They then appear to repeat at perfectly regular, log-periodic intervals. By plotting three or more microcracks one could use the intervals between them to predict how long it would be before the material cracked.

From observation of actual experiments, Didier Sornette [3, 4] thinks that *discrete* scale invariance is the right way to make it possible to spot a breakdown coming in many systems under stress. The distribution of crack lengths in *ordinary* scale invariance shows plots of the length of a crack against the number of cracks of that length with a smooth

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curve in which tiny cracks are most common and big cracks least common. Under discrete scale invariance some crack lengths will occur more frequently than for an ordinary scale invariance distribution. The details will be different for each system, but the data will always be hit with signatures that reveal when the whole system will go critical. And these signatures can be log-periodic.

Financial systems under stress that ultimately crash might then exhibit log-periodic signals [3, 4]. Sornette and Johansen [5] make the sanguine claim of having picked out the signals prior to the Wall Street crashes of 1929, 1962, and 1987, as well as the 1997 crash on the Hong Kong stock exchange. They also claim to have forecasted the Nasdaq high-tech bubble burst in April 2000 and correctly predicted the sudden upturn of the Japanese Nikkei index in January 1999.

A bull market (bubble) occurs when optimism spreads, pushing the market value artificially high. The bubble may then burst in a crash. If not, a slow period of downward adjustment (bear market or anti-bubble) will follow. The equations found in the study of failure mechanisms in materials could be used to capture financial crashes. This is because the way that cracks develop and cause damage is similar to the way that information seeps through the market and changes opinion.

Bubbles and anti-bubbles are traits of herding, imitative behavior. Coherent large scale collective behavior presents structure that results from the repeated nonlinear interactions. And here the whole turns out to be greater than the sum of its parts. Such characteristics are usually present in complex systems, which are not amenable to mathematical analysis [1]. A complex system is computationally irreducible, which means that the only way to decide about its behavior over time is to let it actually evolve in time. Thus it is inherently unpredictable.

But predicting the detailed evolution of a complex system is pointless; what really matters is to predict the extreme events that result from the slow build-up of long-range correlations leading to a global cooperative behavior [4]. The slow build-up of stress eventually pushes the system to a critical time interval. In particular, crashes may be caused by local self-reinforcing imitation between traders, which leads to a bubble. After a threshold known as the critical point, many traders may place the same order (i.e. sell) at the same time, thereby provoking the crash [4].

So crashes are in a sense outliers with properties that are statistically distinct from the rest of the population. Imitation makes a financial system non-stochastic on the eve of a crash. Crashes are thus deterministic and governed by log-periodic formulas [5, 6]. As a result they could in theory be predictable.

The log-periodicity hypothesis departs from the study of extreme events of conventional statistics. If log-periodicity is present at certain times in financial data then this is suggestive that these periods of time present scale invariance in their time evolution. (And this scaling is not the one related to the power law tails of returns [7, 8].)

The literature on log-periodicity usually employs daily data and tries out fits using one-harmonic and two-harmonic log-periodic equations. By taking intraday (and also daily) financial data into account, this Letter puts forward that a three-harmonic log-periodic formula adjusts better to the data. The data sets are for the intraday and daily exchange rate between the Brazilian *real* and the US dollar.

Section 2 of this Letter presents data and adjusts the log-periodic formulas to the series. And Section 3 concludes.

2. Data and analysis

The daily data set ranges from 2 January 1995 to 31 December 2003. The set has 2259 data points obtained from the Federal Reserve website. The 15-minute set comprises 9327 data points from 9:30AM of 19 July 2001 to 4:30PM of 14 January 2003. Figures 1a and 1c display raw data of the two sets, and Figures 1b and 1d show their single-period returns.

Sornette and Johansen's [6] one-harmonic log-periodic equation is

$$\ln Z(\tau) = A + B\tau^{\lambda} + C\tau^{\lambda}\cos[\theta\ln(\tau) + \phi_1], \quad (1)$$

where τ is the time starting with the onset of an anti-bubble. We set $\tau = t - t_c > 0$, where t_c is the critical time. Parameter θ is angular log-frequency, C is amplitude, and ϕ_1 is the phase. Term $A + B\tau^{\lambda}$ is the trend across time, and A, B, and λ give its shape.

Their two-harmonic log-periodic function [6] is given by

$$\ln Z(\tau) = A + B\tau^{\lambda} + C\tau^{\lambda}\cos[\theta\ln(\tau) + \phi_1] + D\tau^{\lambda}\cos[2\theta\ln(\tau) + \phi_2]. \quad (2)$$

This version has two extra parameters, the amplitude D and phase ϕ_2 of the second harmonic.

Our suggested three-harmonic log-periodic formula is

$$\ln Z(\tau) = A + B\tau^{\lambda} + C\tau^{\lambda}\cos[\theta\ln(\tau) + \phi_1] + D\tau^{\lambda}\cos[2\theta\ln(\tau) + \phi_2] + E\tau\lambda\cos[3\theta\ln(\tau) + \phi_3] \quad (3)$$

This equation adds the third harmonic with E and its phase ϕ_3. The parameter values in equations (1)-(3) were estimated by nonlinear least squares using SAS.

Log-periodic cycles are generally described by a sum of log-periodic harmonics (*LP*), i.e.,

$$LP(\tau) = \sum_{j=1}^{J} C_j \tau^{\lambda_j}\cos[j\theta_j\ln(\tau) + \phi_j]. \quad (4)$$

But here we consider $\lambda_1 = ... = \lambda_J$ and $\theta_1 = ... = \theta_J$.

Figures 2a, 2b, and 2c display the log of the daily *real*-dollar rate from 28 August 2000 to 26 September 2003 together with its one-harmonic, two-harmonic, and three-harmonic log-periodic fit respectively. The three-harmonic formula adjusts better. Yet the adjustment fails for the entire series.

Thus an anti-bubble started at the critical time t_c = 28 August 2000. This finding is consistent with the anti-bubble that started around August 2000 in US stockmarkets [6]. Indeed as reported by the Brazilian central bank (www.bacen.gov.br), 'after two significant valuations the Brazilian currency devalued against the US dollar in the third quarter of 2002 in response to woes in US stockmarkets and Argentina'.

Figures 3a, 3b, and 3c show the fit for the intraday data using one harmonics, two harmonics, and our suggested three harmonics. As can be seen, the three-harmonic log-periodic formula adjusts better to the data either. Here we take the starting time at 1:00PM of 31 May 2002. Parameter values for the fits are presented in Tables 1 and 2.

An anti-bubble in the intraday rate thus started at the critical time t_c = 31 May 2002. This makes sense as market participants were anticipating the left-wing presidential victory of November 2002.

3. Conclusion

Although complex financial systems are not arguably amenable to mathematical analysis, imitative behavior in a bull market renders a system periodic on the eve of a crash. The log-periodicity hypothesis usually takes low frequency data to capture the long-range correlations that build up to eventually provoke the crash. The novelty of this Letter is to suggest that the log-periodicity also applies to high frequency, intraday data.

In practice one-harmonic and two-harmonic log-periodic formulas have been employed to fit daily data. This Letter puts forward a three-harmonic log-periodic formula that seems to adjust better to both daily and intraday data. We take the *real*-dollar exchange rate to illustrate our case.

Our result is thus suggestive that the signatures of coming crashes can also be detected at short time scales.

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Table 1a

Parameter	Estimate	Standard Error	95% Confidence Limits	
A	0.5991	0.00840	0.5827	0.6156
B	0.00241	0.000460	0.00151	0.00331
C	-0.00079	0.000138	-0.00106	-0.00052
θ	-8.8204	0.0573	-8.9329	-8.7079
ϕ_1	65.3479	0.3551	64.6509	66.0449
λ	0.8430	0.0280	0.7881	0.8979

Table 1b

Parameter	Estimate	Standard Error	95% Confidence Limits	
A	0.5942	0.00825	0.5780	0.6104
B	0.00298	0.000527	0.00195	0.00402
C	0.000977	0.000156	0.000670	0.00128
D	0.000170	0.000034	0.000103	0.000237
θ	-8.5215	0.0507	-8.6211	-8.4218
ϕ_1	60.3717	0.3139	59.7554	60.9879
ϕ_2	1.5558	0.6485	0.2827	2.8290
λ	0.8091	0.0257	0.7587	0.8595

Table 1c

Parameter	Estimate	Standard Error	95% Confidence Limits	
A	0.5923	0.00819	0.5762	0.6084
B	0.00325	0.000555	0.00216	0.00434
C	0.00107	0.000165	0.000744	0.00139
D	0.000187	0.000037	0.000114	0.000260
E	0.000109	0.000026	0.000059	0.000159
θ	-8.3940	0.0466	-8.4854	-8.3026
ϕ_1	59.5841	0.2880	59.0187	60.1495
ϕ_2	-0.2023	0.5954	-1.3710	0.9665
ϕ_3	-50.9140	0.8964	-52.6737	-49.1544
λ	0.7955	0.0248	0.7468	0.8441

Log-periodicity in the daily *real*-dollar rate.
Results for the one- (Table 1a), two- (Table 1b), and three- (Table 1c) harmonic log-periodic equation.

Table 2a

Parameter	Estimate	Standard Error	95% Confidence Limits	
A	0.9451	0.00449	0.9363	0.9539
B	0.000609	0.000147	0.000322	0.000897
C	0.000175	0.000040	0.000096	0.000255
θ	4.8615	0.0629	4.7382	4.9849
ϕ_1	44.8125	0.4643	43.9021	45.7228
λ	0.7952	0.0299	0.7366	0.8538

Table 2b

Parameter	Estimate	Standard Error	95% Confidence Limits	
A	0.9601	0.00254	0.9551	0.9650
B	0.000109	0.000017	0.000076	0.000142
C	0.000028	3.959E-6	0.000020	0.000036
D	-0.00002	2.492E-6	-0.00002	-0.00001
θ	9.4812	0.0337	9.4150	9.5474
ϕ_1	-3.3802	0.2525	-3.8754	-2.8851
ϕ_2	-58.5288	0.5126	-59.5339	-57.5236
λ	1.0268	0.0191	0.9893	1.0644

Table 2c

Parameter	Estimate	Standard Error	95% Confidence Limits	
A	0.9625	0.00222	0.9581	0.9669
B	0.000083	0.000014	0.000054	0.000111
C	0.000015	2.563E-6	9.716E-6	0.000020
D	-0.00002	3.019E-6	-0.00002	-0.00001
E	0.000017	2.77E-6	0.000011	0.000022
θ	5.4020	0.0203	5.3621	5.4419
ϕ_1	28.5583	0.1543	28.2558	28.8609
ϕ_2	1.7746	0.2951	1.1960	2.3532
ϕ_3	3371.3	0.4534	3370.4	3372.2
λ	1.0590	0.0222	1.0154	1.1026

Log-periodicity in the intraday *real*-dollar rate.
Results for the one- (Table 2a), two- (Table 2b), and three- (Table 2c) harmonic log-periodic equation.

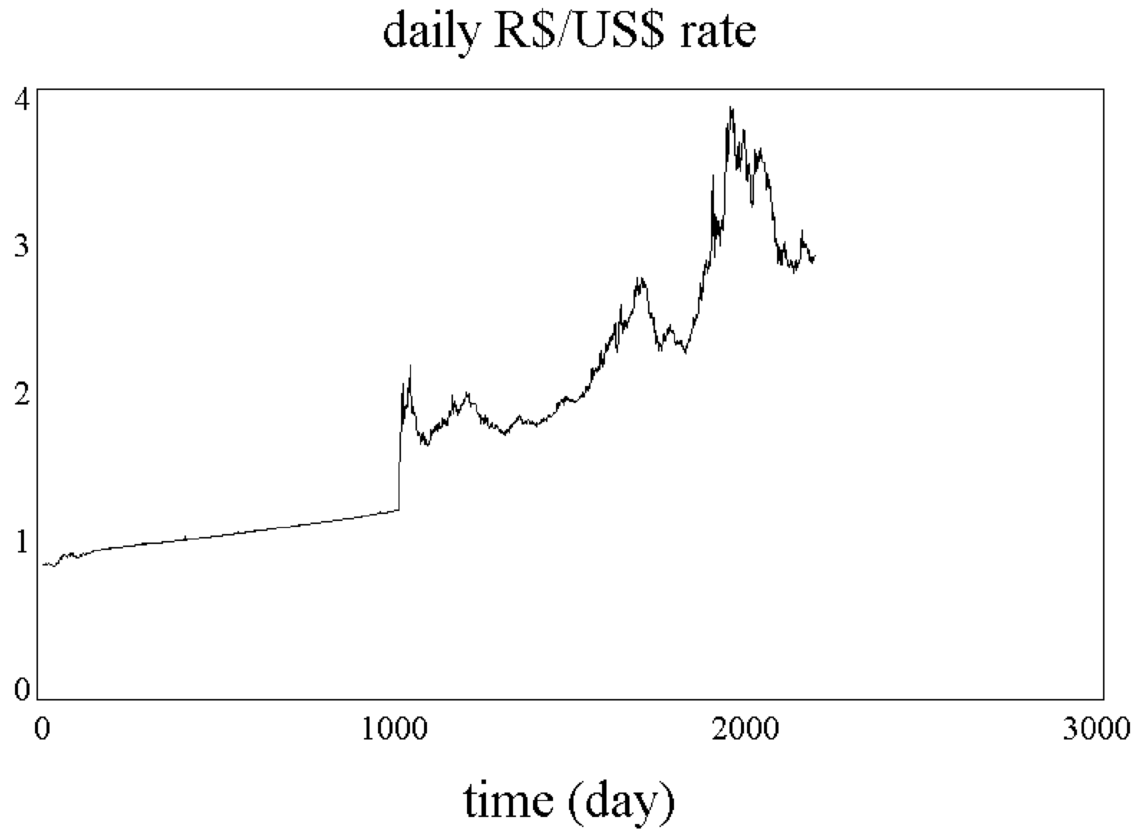

Figure 1a. Daily *real*-dollar rate from 2 January 1995 to 31 December 2003.

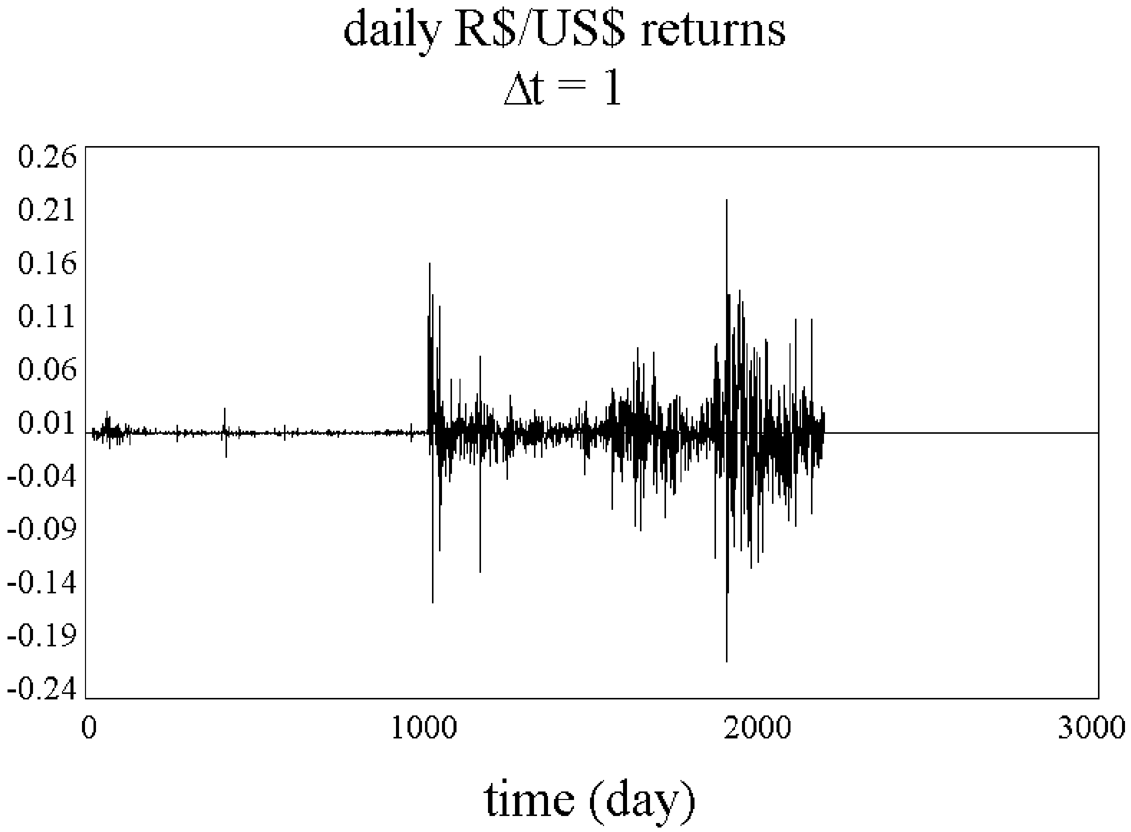

Figure 1b. Daily *real*-dollar single-period returns from 2 January 1995 to 31 December 2003.

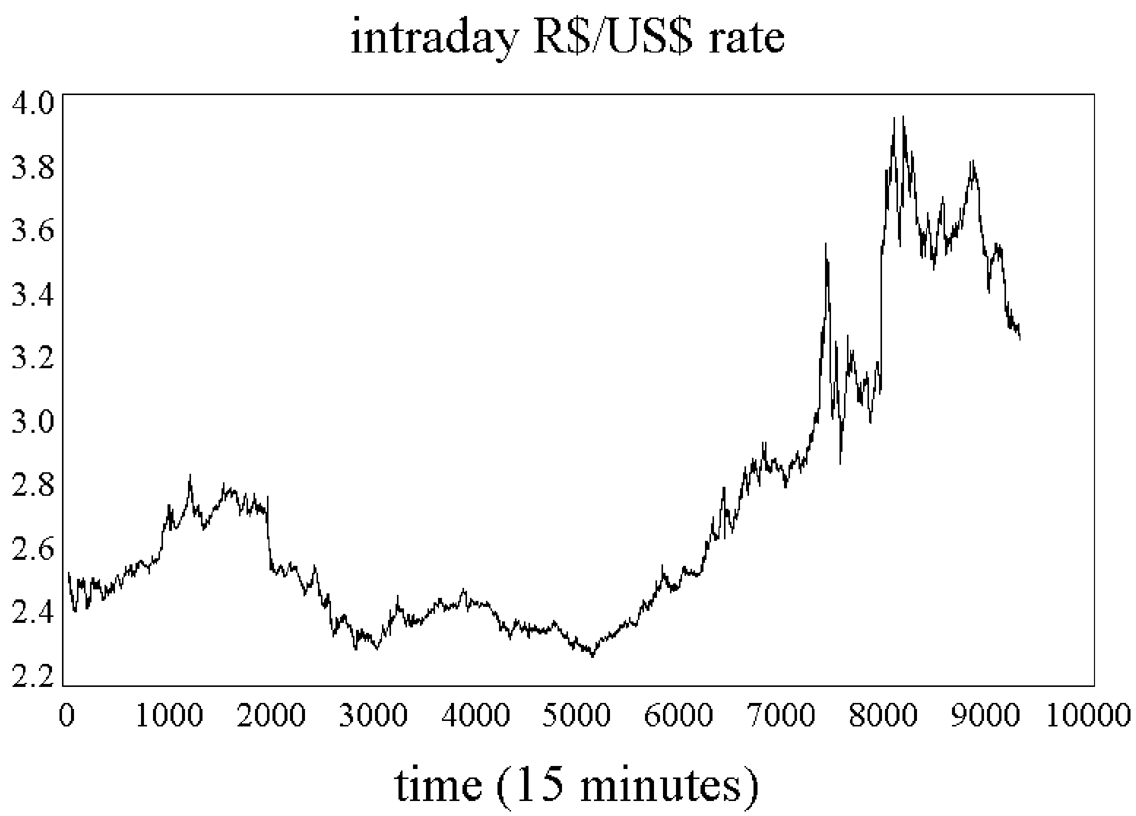

Figure 1c. Fifteen-minute *real*-dollar rate from 9:30 AM of 19 July 2001 to 4:30 PM of 14 January 2003.

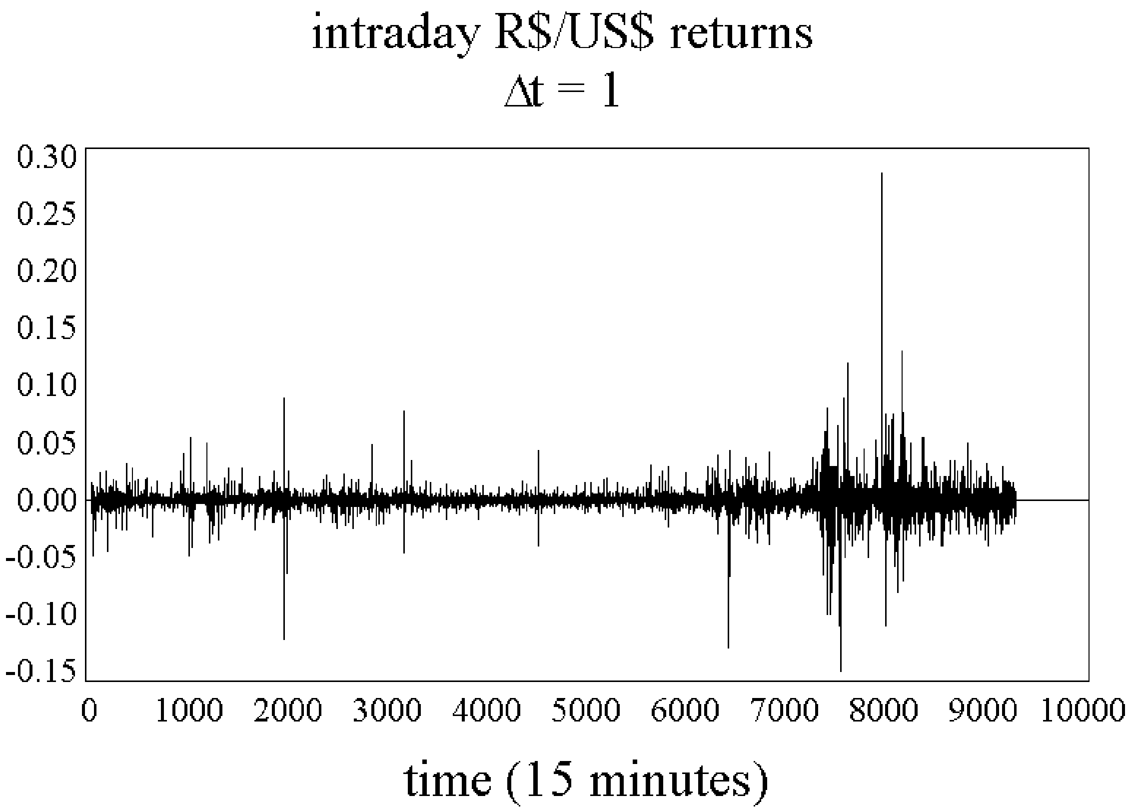

Figure 1d. Fifteen-minute *real*-dollar single-period returns from 9:30 AM of 19 July 2001 to 4:30 PM of 14 January 2003.

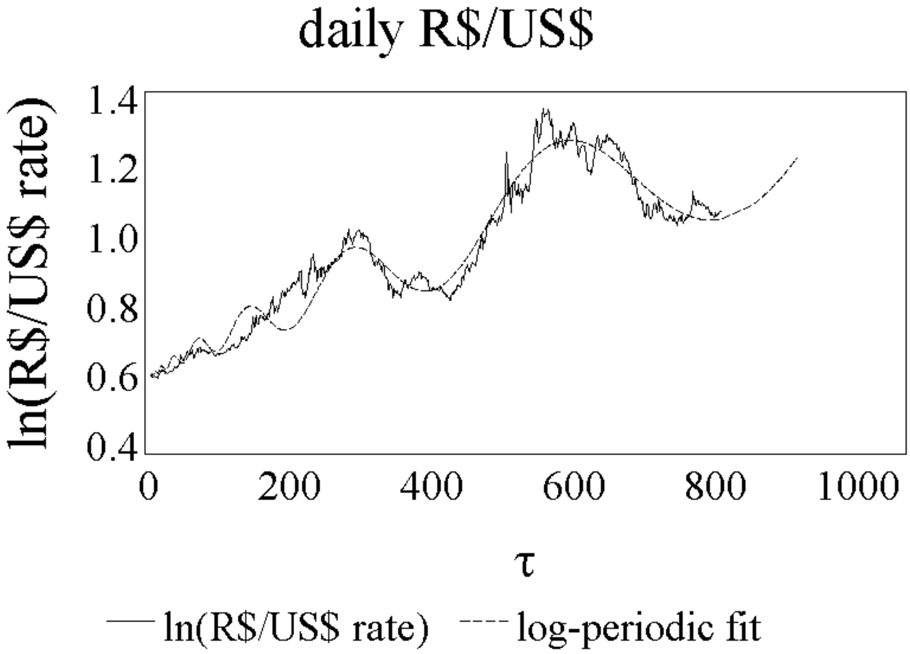

Figure 2a. Log of the daily *real*-dollar rate from 28 August 2000 to 26 September 2003 together with its one-harmonic log-periodic fit (dashed line).

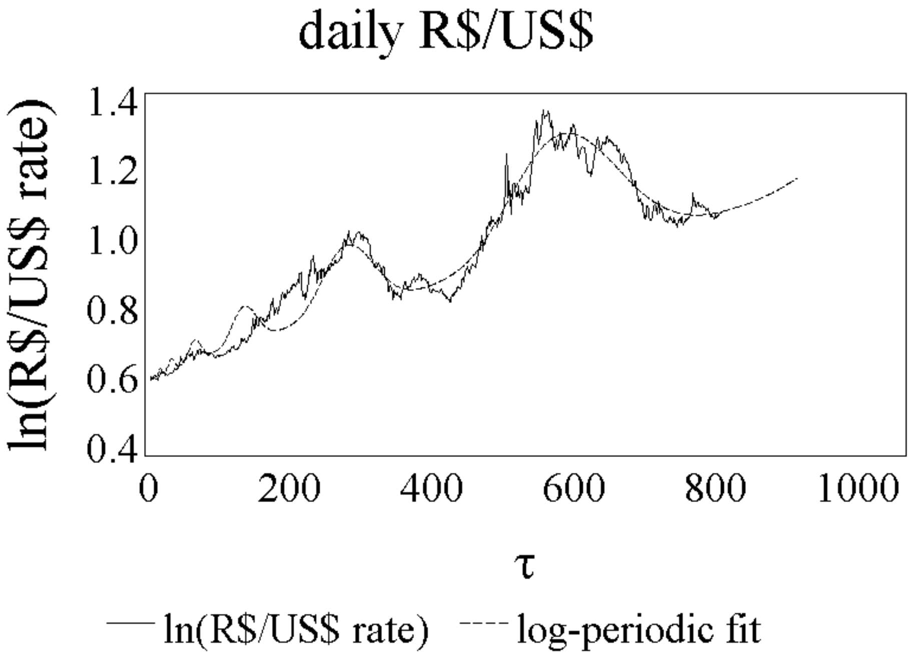

Figure 2b. Log of the daily *real*-dollar rate from 28 August 2000 to 26 September 2003 together with its two-harmonic log-periodic fit (dashed line).

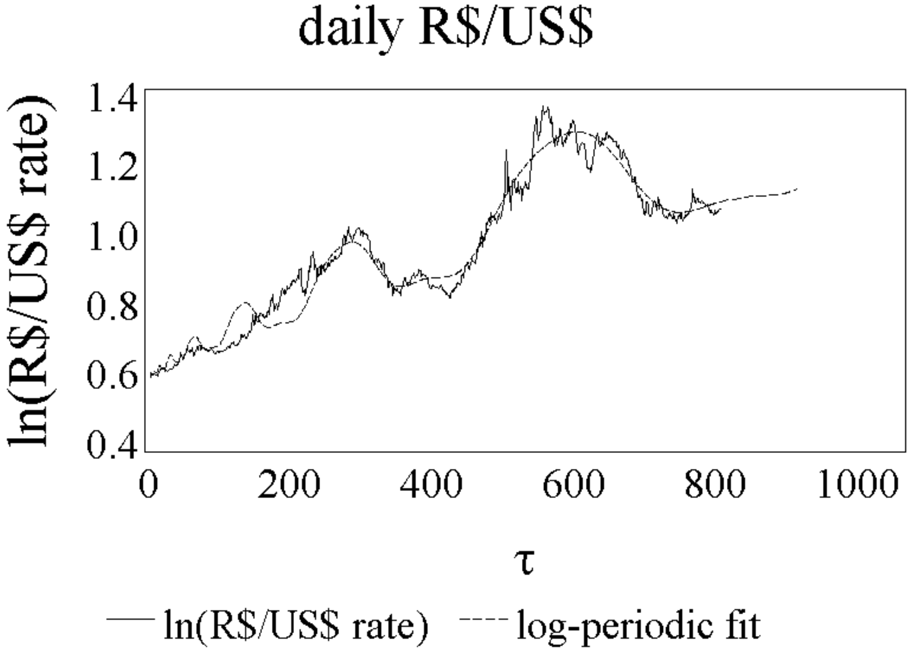

Figure 2c. Log of the daily *real*-dollar rate from 28 August 2000 to 26 September 2003 together with its three-harmonic log-periodic fit (dashed line). The three-harmonic log-periodic formula adjusts better than the previous cases. See Table 1.

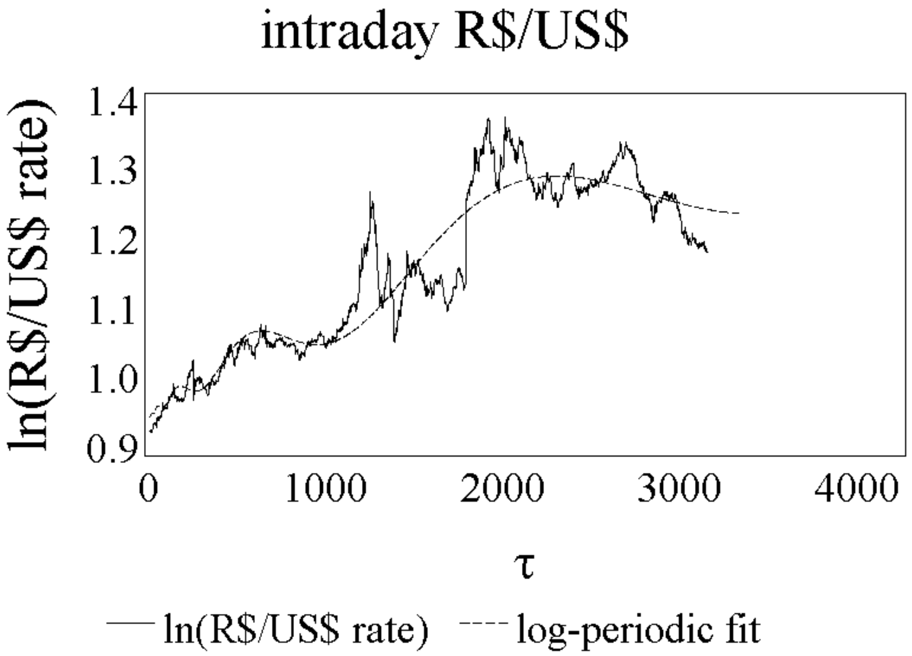

Figure 3a. One-harmonic log-periodic fit (dashed line) for the intraday *real*-dollar returns from 1:00PM of 31 May 2002 to 4:30PM of 14 January 2003.

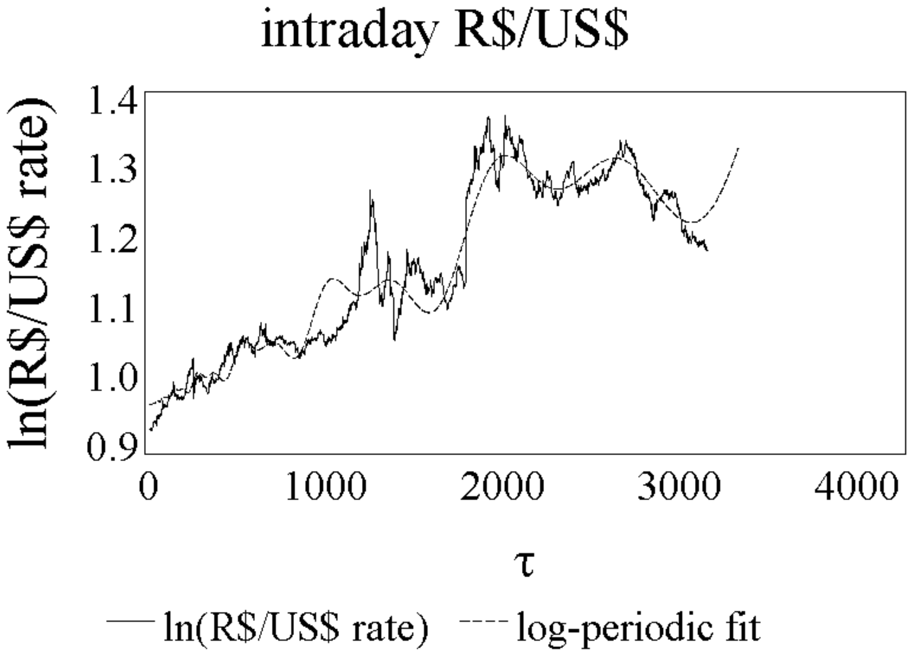

Figure 3b. Two-harmonic log-periodic fit (dashed line) for the intraday *real*-dollar returns from 1:00PM of 31 May 2002 to 4:30PM of 14 January 2003.

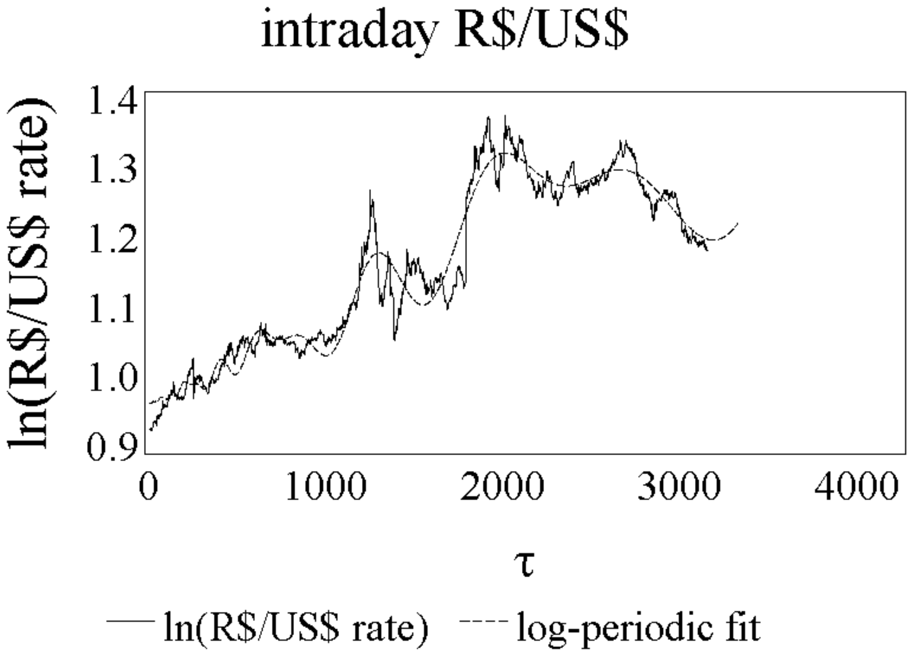

Figure 3c. Three-harmonic log-periodic fit (dashed line) for the intraday *real*-dollar returns from 1:00PM of 31 May 2002 to 4:30PM of 14 January 2003. As can be seen, the three-harmonic log-periodic formula adjusts better to the data. See Table 2.