

VALUATION OF SIX ASIAN STOCK MARKETS: FINANCIAL SYSTEM IDENTIFICATION IN NOISY ENVIRONMENTS

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ABSTRACT. The open financial economic systems of six Asian countries - Taiwan, Malaysia, Singapore, Philippines, Indonesia and Japan - over the period 1986 through 1995 are identified from empirical data to determine how their stock markets, economies and financial markets are interrelated. The objective is to find rational stock market valuations using a country's nominal GDP and a short term interest rate, based on a modified version of the Dividend Discount Model. But our empirical results contradict such conventional financial economic theory. Various methods are used to analyze the 3D data covariance ellipsoids: spectral analysis, analysis of information matrices, 2D and 3D noise/signal determination and superfluous system identification based on 3D projections. The new superfluous method provides the sharpest identification of the Grassmanian invariant q of the empirical systems and the best computation of the finite boundaries of the empirical parameter ranges. All six Asian systems are high noise environments, in which it is very difficult to separate systematic signals from noise. Because of these high noise levels, spectral analysis is not reliable. By plotting all 3D $q = 2$ Complete Least Squares projections we find that only Taiwan has a clear $q = 2$ system, i.e., Taiwan's stock market, economy and financial market are rationally coherent. In contrast, Malaysia, Singapore, Philippines and Indonesia have $q = 1$ systems, in which stock markets and economies are closely related, but unrelated to the respective domestic financial markets. Several possible economic explanations are provided. We also quantitatively establish the incoherence of Japan's financial economic system. Japan's stock market operates independently from its economy and from its financial market, which are mutually unrelated.

1. INTRODUCTION

The Asian stock markets have seen a substantial capitalization in the past fifteen years. In 1982, the 32 emerging stock markets surveyed by the International Finance Corporation had a total market capitalization of US\$67 billion, representing about 2.5% of world market capitalization. By the end of 1994, the market capitalization

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of these emerging stock markets had grown nearly 27-fold, exceeding US\$1.8 trillion, or 11.8% of world market capitalization.¹ Despite the recent moderate slowdown of the Asian countries, many global investors and foreign fund managers continue to pour substantial funds into these markets, as Asia promises to be a region that offers substantial returns for their investments.

There is little doubt that the region's Gross Domestic Product (GDP) growth is positively correlated to the growth of equity market returns. Asian countries, excluding Japan, have seen annualized real GDP growth rates of 6% - 9% over the past ten years. An average investor in these markets could have earned annualized market returns of 15% - 20% over the same period. But there is some doubt about the impact of monetary policy on the growth rates of both the economies and their stock markets. Thus the main question for global investors is: what is the systematic relationship between these countries' stock market valuations, their nominal GDPs and their (short term) interest rates?

In contrast to most researchers, who use conventional single equation regression models, we use a new system identification or super filtering methodology to answer this question.² This super filter methodology extracts the financial economic system structures from the empirical observations without undue theoretical presumptions. The six Asian countries analyzed are Taiwan, Malaysia, Singapore, the Philippines, Indonesia and Japan. Each country's major stock market index is used to track the valuation of its stock market. Its nominal GDP and short term interest rate are the two other macroeconomic variables. The data used for this analysis range from 1986 first quarter to 1995 third quarter.

For the analysis we combine some elements of the conventional Dividend Discount Model (DDM) following [15] and of modern Asset Pricing Theory (APT) following [1], [2] and [17], but, taking account of some earlier critical discussions in [4], [16], [18], we allow for greater parameter freedom and we discuss the advantages and disadvantages of principal component versus projection theory for multivariate analysis. We demonstrate how the conventional analytic schemes are encompassed and how the data can be explained more discriminatingly by the new super filter theory. In the process we gain a better understanding of the relative coherence of the financial economic systems of the six Asian countries and identify the (uncertain) valuation systems of their stock markets. Thus a new methodology is implemented to a rather unique Asian data set of three important financial economic variables.³

The paper consists of five sections. Section 2 discusses the data and the methodology. In Section 3 we first spectrally decompose the data covariance matrices and analyze the information matrices and various parameter plots. In Section 4 we take an initial look at the 2D data scatter plots and 2D noise/data and noise/signal ratios. Section 5 discusses the 3D Complete Least Squares (CLS) projection plots

¹The NYU Salomon Center Report 1996-97, p.10.

²The designation super filter for our new methodology was first used by Professor Rouchaleau of the École des Mines, Paris, at an advanced econometrics seminar at I.N.R.E.A. in Sophia Antipolis in Southern France in the Spring of 1996, according to a personal e-mail message of March 26, 1996 from Professor Kalman of the ETH, Zürich. For the theoretical foundation of the super filter, see [6], [7],[10], [11] and [13].

³For a simpler implementation of the same methodology to bivariate data sets of mutual funds using the CAPM as hypothesized model, see [14]

of each of the six Asian countries and their 3D noise/data and noise/signal ratios. Section 6 summarizes the conclusions about the relative coherence of these Asian financial economic systems and their measured parameter ranges, discusses the limitations of this research and provides some recommendations for future research. The Appendices present the data sources in more detail and the algebraic formulation of the new super filter theory.

2. DATA AND METHODOLOGY

2.1. Data Sources. The stock market indices (S) include the Nikkei 225 Stock Average (Japan), Straits Times Industrials Index (Singapore), Weighted Price Index (Taiwan), Kuala Lumpur Composite Index (Malaysia), Jakarta Composite Index (Indonesia) and Philippines SE Composite Index (Philippines). The observations on the stock market indices were collected from the respective issues of the *Asian Wall Street Journal* and *The Business Times*.⁴

Nominal GDP ($NGDP$) is used for all countries, except for Indonesia due to the non-availability of its quarterly $NGDP$ data. For Indonesia, the available Petroleum Production (PP) Index is the closest proxy we had available for its Gross Domestic Product. For the short-term interest rate (IR), the 90-days Money Market Rate is used for Taiwan, while the respective bank lending rates are used for the other five countries. These $NGDP$, PP , and IR figures were extracted from the *International Financial Statistics*, *Asia Pacific Economic Outlook*, *United Overseas Bank* and *Financial Quarterly* publications for the appropriate quarters.⁵

The overall behavior of S , $NGDP$, PP , and IR were pre-analyzed for each country. The PP series for Indonesia had a clear outlier data point in the last quarter of 1993. Since the validity of this data point is very doubtful, this period was excluded from the analysis for Indonesia. Consequently, the analysis for Indonesia has been based on one quarter less than the other five countries, i.e., 38 instead of 39 quarterly observations.

2.2. Hypothetical Model Structures. As suggested by i.a.[15], we have the following simple theoretical DDM structure for a country's system for stock market valuation⁶:

$$S_t = \frac{E\{NGDP_{t+1}\}}{IR_t}$$

The valuation of a country's stock market at time $t = 1, \dots, T$, is the infinitely discounted value of the *expected value* $E\{\dots\}$ of the one-quarter-ahead nominal GDP_{t+1} , representing the expected market value of all goods and services domestically produced during the next quarter, whereby the discounting is done by the most readily available current cost of capital, i.e., the domestic short-term interest rate IR_t . In these six Asian countries, obtaining longer term capital for stock market investment is still very difficult and this difficulty restricts us to the kind of short-term capital access that an average stock investor has.

⁴The data are available upon request on a 3.5" diskettes at cost.

⁵Cf. Appendix I for the specific sources and the discussion of the various data conversions, which were required to make the data of the six Asian countries mutually comparable.

⁶This macro approach to a country's valuation was first suggested to me in 1990 by the Director of Financial Research of Smith-Barley, Inc. in New York, who attempted to produce a comparable framework for global investment valuation.

Since we deal with historical data only, the implicit, and admittedly somewhat unrealistic, assumption of our modeling approach is that there was, historically, *perfect foresight* and the expected $E\{NGDP_{t+1}\} = \text{actual } NGDP_{t+1}$ at all times t . Thus

$$S_t = \frac{NGDP_{t+1}}{IR_t}$$

Alternatively one could, for example, have assumed a simple extrapolation, i.e., that the best expectation for next quarter's $NGDP_{t+1}$ is the current $NGDP_t$ multiplied by one plus the trend growth rate. By taking logarithms of this theoretical structure, the following single linear equation results:

$$\ln S_t = \ln NGDP_{t+1} - \ln IR_t$$

However, since the hypothesized theoretical model is rather restrictive, we introduce extra modeling flexibility to test the postulated theoretical system structure against the empirical data structure revealed by covariance analysis. We do this in two steps.

First, inspired by multivariate Arbitrage Pricing Theory (APT) [1] and the classical economists Cobb-Douglas (C-D) Production Function Theory [3], we postulate the following single equation ($q = 1$) system structure for the modified DDM

$$\ln S_t = a. \ln NGDP_{t+1} + b. \ln IR_t + \ln D$$

Effectively, we introduce two elasticities, whose (uncertain) values are to be determined from the empirical data:

$a =$ nominal GDP (expected income) elasticity, which finance theory expects to be positive; and $b =$ interest rate elasticity, which finance theory expects to be negative.

Theoretically, the added $\ln D$ term represents the deterministic value innovations in the stock market introduced by technological advances. By exponentiation the modified DDM model transforms to the flexible nonlinear structure

$$S_t = D.(NGDP_{t+1})^a.(IR_t)^b$$

Notice that this structure encompasses the original DDM structure, when the theoretical parameters $a = 1$, $b = -1$ and $D = 1$.

Second, we introduce further structure flexibility by allowing for two independent linear equations ($q = 2$), by postulating the following system:

$$\ln S_t = c. \ln NGDP_{t+1} + \ln F$$

$$\ln S_t = d. \ln IR_t + \ln G$$

where c and d are the new expected income and interest rate elasticities respectively. This two equation system encompasses the single equation system ($q = 1$) by linear combination, i.e., by taking a weighted average of the two independent equations.

While we can create a unique single equation system from this two-equation system, the reverse does *not* hold true. Thus single ($q = 1$) and two-equation ($q = 2$) systems are structurally *not* equivalent. In terms of principal component analysis, a two-equation system ($q = 2$) behaves like a one factor ($r = 1$) system, since all variables move simultaneously as a bundle in the same (or opposite) direction. In contrast, in a true single equation system ($q = 1$) there are two factors ($r = 2$), since two variables move independently from each other.

Note that $r + q = n$ where n is the number of *variables*, q is the number of *independent equations* and r is the number of *factors*. In our analysis, $n = 3$ for each of the six Asian countries. Since the $q = 2$ system encompasses the $q = 1$ system, the $q = 2$ system forms the basis for our system identification procedure to determine which of the two system structures provides the best explanation of the observed noise-contaminated empirical data covariances.

By exponentiation, this C-D model transforms to the system of two simultaneous *nonlinear* equations:

$$S_t = F.(NGDP_{t+1})^c$$

$$S_t = G.(IR_t)^d$$

The covariance analysis to determine the system structure (is $q = 1$ or 2 ?) and the finite parameter ranges for the elasticity parameters, $c_* \square c \square c^*$ and $d_* \square d \square d^*$, is executed on laterally shifted frames of data reference, as follows:

$$x_{1t} = \ln S_t - \overline{\ln S_t}$$

$$x_{2t} = \ln NGDP_{t+1} - \overline{\ln NGDP_{t+1}}$$

$$x_{3t} = \ln IR_t - \overline{\ln IR_t}$$

The averages, indicated by bars over the variables, are taken over all $T = 39$ observations ($T = 38$ for Indonesia). After the covariance analysis the (projected and non-projected) deviations are transformed into the original variables by adding back these averages. For example, $\ln S_t = x_{1t} + \overline{\ln S_t}$. By taking averages the residual technology terms, $\ln F$ and $\ln G$ are of no substantial importance for the system identification. Once the parameter ranges for the income elasticity c and the interest elasticity d are computed, the parameter ranges for the $\ln F$ and $\ln G$ terms (respectively for F and G) can immediately be determined.

2.3. Identification in High Noise Environments. The analysis follows the same recipe for each of the six Asian countries. We computed the data covariance matrices Σ (one matrix per country) of the logarithmic data. These six data covariance matrices formed the *complete data set* for this paper's noisy system identification.

We use several procedures to identify the system invariant q from this set of covariance data and to compute the finite parameter ranges $c_* \square c \square c^*$ and $d_* \square d \square d^*$. Our identification recipe consists of the following five steps:

1. *Spectral decomposition* of the data covariance matrices attempts to determine the number of factors r and thus, indirectly, the number of equations $q = n - r$ that support each country's covariance data. However, it becomes quickly clear that spectral analysis fails in high noise environments and that other, more discerning algebraic geometric methods are required.

2. *Information matrices* Σ^{-1} are used for (i) analysis, e.g., via *visual inspection of the (a, b) coefficient plots*, using the fundamental Theorem that each row of an information matrix is an elementary ($q = 1$) regression, and for (ii) computing the $q = 2$ projectors in the 3D data space.⁷

3. Three-dimensional *Complete Least Squares (CLS) analysis*, using the CLS noise projectors $\tilde{\mathbf{P}}^{CLS}$ and CLS signal projectors $\hat{\mathbf{P}}^{CLS}$ of [6] and [13], provide the

⁷These $q = 2$ projectors are derived in Appendix II.

most discerning system identification to extract the systematic shape of the 3D data covariance ellipsoid.

4. Classical *bivariate Noise/Data and Noise/Signal ratios* provide 2D measurement of the relative noise levels. We prove in Appendix III that these bivariate ratios form the constituents of the $q = 2$ 3D CLS signal projectors.

5. *Three-dimensional Noise/Data and Noise/Signal ratios* provide an overall quantitative assessment of the systems relative noise levels. To compute the 3D Noise/Data ratios, we use a simple Theorem of Linear Algebra to compute the volume of the noise covariance matrix $\hat{\Sigma}^{LS}$ relative to the volume of the data covariance matrix Σ . This provides us with a standard for comparison of the relative noise levels in all six countries.

3. COMPLETE DATA ANALYSIS

3.1. Spectral Decomposition of Data Covariance Matrices. The spectral decomposition of the (3×3) data covariance matrix $\Sigma = \mathbf{U}'\Lambda\mathbf{U}$ produces the (3×3) diagonal matrix Λ with three eigenvalues λ_1 , λ_2 and λ_3 , while the matrix \mathbf{U} contains the corresponding orthonormal eigenvectors ($\mathbf{U}'\mathbf{U} = \mathbf{I}$). Such a decomposition of a positive definite matrix like Σ is *always* possible [11].

The scree plots for each of the six Asian countries are presented in Table 1. The determinant of the data covariance matrix of each country, $|\Sigma| = \lambda_1\lambda_2\lambda_3$, is an indication of the overall level of uncertainty, or structural independence, of each of the variables. When the three variables x_1 , x_2 and x_3 are strictly linearly dependent, this determinant equals zero. The more independent the variables, the larger the determinant. But there exists no standard for absolute comparison, since the magnitudes of the eigenvalues depend on the data variances.⁸

TABLE 1.	SCREE	PLOT OF	EIGENVALUES	DETERMINANTS
	λ_1	λ_2	λ_3	$ \Sigma = \lambda_1.\lambda_2.\lambda_3$
TAIWAN	0.532	0.075	0.047	$1.90E - 03$
MALAYSIA	0.357	0.022	0.005	$3.75E - 05$
SINGAPORE	0.201	0.012	0.007	$1.63E - 05$
PHILIPPINES	0.533	0.050	0.013	$3.39E - 04$
INDONESIA	0.574	0.014	0.002	$1.91E - 05$
JAPAN	0.073	0.033	0.016	$3.98E - 05$

From Table 1 it appears that for five of the six countries - Taiwan, Malaysia, Singapore, Philippines and Indonesia - the number of principal components is one ($r = 1$), since each of these countries exhibits one dominant eigenvalue, $\lambda_1 \gg \lambda_2, \lambda_3$. For these five countries spectral decomposition suggests that there are at least $q = 2$ two linear combinations of the three variables, so that the systematic components of the three variables move simultaneously. Thus a minimal level of coherence between the respective stock markets, economies and the financial markets of these five countries exists. Japan is clearly an exception. For Japan it is not possible to distinguish between the situation of one ($r = 1$, $q = 3 - 1 = 2$) or two ($r = 2$, $q = 3 - 2 = 1$) dominant eigenvalues. There appears to be no obvious linear

⁸We don't want to use the normalization of *correlation* matrices, since such diagonal-wise normalization distorts the information content, as explained in [10], p. 1278.

combination among the three variables, ($r = 3, q = 0$), suggesting that the stock market, the economy and the financial market in Japan are disjointed.

However, caution with interpretation of spectral decompositions is required, since they contain arbitrary convolutions of noise and signal elements.⁹ A scree plot may be characteristic for a particular country, like it is for a particular chemical element, but it does not assist in the identification of the underlying system structure, since it does not provide an unambiguous decision on what part of the data variation is signal and what noise.¹⁰ Therefore, we decided to use more discerning geometric methods of analysis.[11]

3.2. Information Matrices and (a, b) Parameter Plots. A Theorem of Kalman [6] states that each row of the information matrix Σ^{-1} is an elementary regression, e.g., the first row of Σ^{-1} is the elementary regression, or Least Squares (LS) projection, of variable x_{1t} on variables x_{2t} and x_{3t} , the second row is the elementary regression of variable x_{2t} on x_{1t} and x_{3t} , etc.¹¹

The information matrices are collected in Table 2, where the elementary projections are normalized on the first variable x_{1t} , i.e., its coefficient is always one, so that the coefficients a and b are comparable:

$$\hat{x}_{1t} = a.\hat{x}_{2t} + b.\hat{x}_{3t}$$

which is equivalent to the linear combination:

$$\hat{x}_{1t} - a.\hat{x}_{2t} - b.\hat{x}_{3t} = 0 \text{ or, in matrix notation, } \mathbf{A}'\hat{\mathbf{x}}_t = \begin{bmatrix} 1 & -a & -b \end{bmatrix} \begin{bmatrix} \hat{x}_{1t} \\ \hat{x}_{2t} \\ \hat{x}_{3t} \end{bmatrix} = 0$$

The hats over the variables x_{it} denote the signals or systematic parts of the data. Other normalizations on variable x_2 (by dividing the preceding equation by a), or x_3 (by dividing the equation by b), are equivalent.¹²

⁹Since $\mathbf{U}'\Sigma\mathbf{U} = \Lambda$ diagonal, we can write $\Lambda = \lambda_1\mathbf{E}_1 + \lambda_2\mathbf{E}_2 + \lambda_3\mathbf{E}_3$ where \mathbf{E}_i is a zero matrix with unity in the i, i diagonal position. Since also $\Sigma = \mathbf{U}\Lambda\mathbf{U}'$ we can spectrally decompose $\Sigma = \lambda_1\mathbf{U}\mathbf{E}_1\mathbf{U}' + \lambda_2\mathbf{U}\mathbf{E}_2\mathbf{U}' + \lambda_3\mathbf{U}\mathbf{E}_3\mathbf{U}' = \lambda_1\mathbf{R}_1 + \lambda_2\mathbf{R}_2 + \lambda_3\mathbf{R}_3$ where $\mathbf{R}_i = \mathbf{U}\mathbf{E}_i\mathbf{U}'$ for $i = 1, 2, 3$. There is no clear conceptual separation between signal and noise. For example, does $\lambda_1\mathbf{R}_1$ constitute the signal and $\lambda_2\mathbf{R}_2 + \lambda_3\mathbf{R}_3$ the noise, or is $\lambda_2\mathbf{R}_2$ the signal and $\lambda_1\mathbf{R}_1 + \lambda_3\mathbf{R}_3$ the noise? In both cases $r = 1$ and $q = 2$. Or does $\lambda_1\mathbf{R}_1 + \lambda_2\mathbf{R}_2$ constitute the signal and $\lambda_3\mathbf{R}_3$ the noise, or is $\lambda_1\mathbf{R}_1 + \lambda_3\mathbf{R}_3$ the signal and $\lambda_2\mathbf{R}_2$ the noise? In both these last cases $r = 2$ and $q = 1$. Spectral analysis alone cannot determine what constitutes a signal and what noise and therefore it cannot unambiguously identify the system invariant q , particularly not in high noise environments.

¹⁰This is not an uncommon situation in science. Until 1953 spectral analysis could determine which chemical elements are present in the protein molecule of DNA, but it could not determine the structure of the protein. In 1953 Crick and Watson identified the helical structure of DNA by geometric methods.

¹¹This Theorem can be easily proved by elementary partitioned matrix algebra. The misleading term "regression" is Galton's. We prefer the mathematically well-defined term "LS projection".

¹² $(a, b)_{i,j}$ is the set of income (a) and interest elasticities (b) from the $q = 1$ CLS projection on variables \mathbf{x}_i and \mathbf{x}_j .

TABLE 2.	INCOME (a) AND	INTEREST RATE (b)	ELASTICITIES
From ($q = 1$) CLS Projections	$(a, b)_{2,3}$	$(a, b)_{1,3}$	$(a, b)_{1,2}$
TAIWAN	+0.9096, +0.7262	+4.5042, -0.6887	-0.8626, +3.4887
MALAYSIA	+1.4320, -0.8291	+1.5653, -0.7766	+1.3414, -2.0187
SINGAPORE	+1.0033, -0.3225	+1.1789, -0.2112	+0.6570, -4.5692
PHILIPPINES	+1.6103, -0.7515	+1.8920, -0.7494	+1.6058, -2.5332
INDONESIA	+7.3722, -0.6586	+13.3330, +0.2081	-2.3291, -25.2981
JAPAN	-0.1875, +0.3858	-17.7667, -1.2813	+0.6229, +3.8562

Notice that only the (a, b) coefficients of Singapore, Malaysia and Philippines show sign consistency: each of the three elementary projections delivers the same sign for both the expected income elasticity $a > 0$ and interest rate elasticity $b < 0$ and we conclude that for these three countries the hypothesis $q = 1$ cannot be rejected.¹³ At least two out of the three variables are independent in these three economies. For example the financial market and the economy are disjoint, but both influence the stock market, or the financial market and the stock market are disjoint, but both influence the economy. The (a, b) coefficients for Taiwan, Indonesia and Japan show sign inconsistency and we conclude that for these three countries the hypothesis of $q = 1$ is rejected, *ergo*, for these countries $q = 2$.

But there is considerable system uncertainty. In the conventional economic representation for Singapore, the measured (expected) income elasticity $0.6570 \leq a \leq 1.1789$. This range includes the unit elasticity $a = 1$. Similarly, Singapore's measured interest rate elasticity b ranges from -4.5692 to -0.2112 , which includes the unit elasticity $b = -1$. Thus the classic DDM cannot be rejected for Singapore. Similar conclusions hold for Malaysia, where the expected income elasticity a ranges from 1.3414 to 1.5653 , which does *not* include $a = 1$, and the interest rate elasticity $-0.7766 \leq b \leq 2.0187$, which does *not* include $b = -1$, and for the Philippines, where the expected income elasticity is in a narrow range $1.6058 \leq a \leq 1.8920$, which does *not* include $a = 1$, and the interest rate elasticity $-2.5332 \leq b \leq -0.7515$, which includes the unit elasticity $b = -1$. Thus for Malaysia and the Philippines the classic $q = 1$ DDM is rejected, but not our modified $q = 1$ DDM.

Note that these conventional financial economic interpretations are dependent on the particular normalization on variable x_1 . Once another normalization is chosen, e.g., on variable x_2 , such an easy economic interpretation - based on the concepts of income and interest rate elasticities, is no longer possible. Such economic interpretations are arbitrary theoretical conventions, independent from the identified mathematical system, which depend on a particular rotation of the frame of reference or viewpoint. Notice also that the conclusion of $q = 1$ for Singapore, Malaysia and Philippines *contradicts* the results of the earlier spectral analysis in Section 4.1, where the conventional interpretation of the scree plot found $q = 2$ for those countries.

The other three countries, Taiwan, Indonesia and Japan show much more system coherence than Malaysia, Singapore and the Philippines, since the elementary $q = 1$ projection coefficients show sign inconsistency. Sign inconsistency occurs when the expected income elasticity a and the interest rate elasticity b are both positive and

¹³In the Popperian sense of cannot be falsified.

negative for the same covariance data, depending on which projection direction is chosen. This is an indication that a single $q = 1$ linear equation cannot properly capture the complex system variation and at least $q = 2$ independent linear equations are required. Thus analysis of the information matrices of Taiwan, Indonesia and Japan suggests that the stock market, the economy and the financial market in these countries are dependent on each other.

This information matrix analysis based on sign consistency, or lack of it, does not reveal the relative noise levels or uncertainty in the systems, since it is based only on the adjoint of the data covariance matrix.¹⁴ Thus the information matrix analysis may produce a spurious systematic correlation, giving the appearance of system coherence where there is not any. But a graphical presentation of the information matrix analysis, i.e., of the magnitudes of the (a, b) coefficients, does provide an indication of the relative noise levels.

The theory of these (a, b) plots is explained in Appendix IV. Compare, for example, the first (a, b) plot for Taiwan in Fig. 1 with the one for the Philippines in Fig. 2. In both cases the information matrices are normalized on x_1 .¹⁵

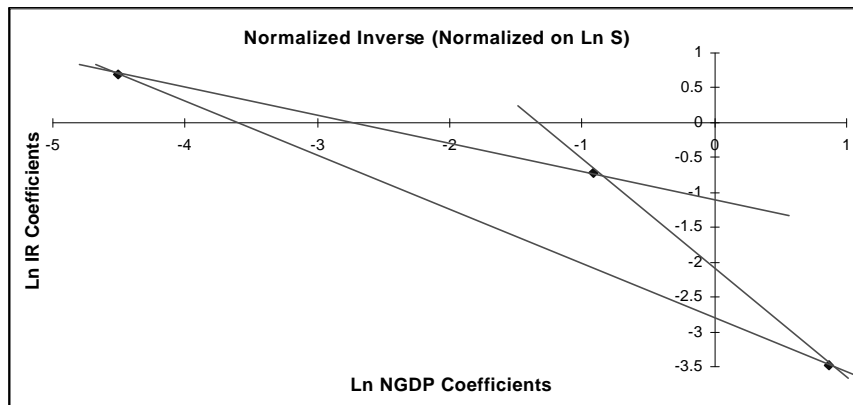
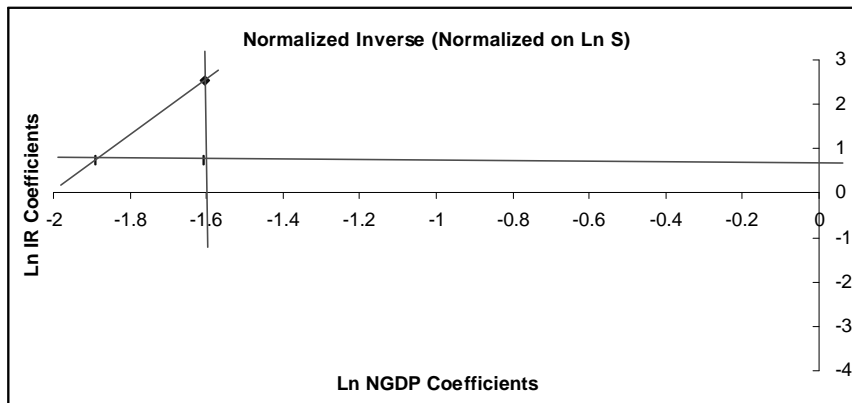


Fig. 1. Taiwan (a, b) plot: $q = 2$

¹⁴The determinant $|\Sigma|$ indicates the system's noise level. The adjoint $Adj(\Sigma)$ reflects the system's structure.

¹⁵There are three (a, b) dots per country. The normalization does affect the (a, b) dots, because of the relative noise distribution in each picture, i.e., the volume, of the triangle formed by the three dots, but not the sign (in-)consistencies. All (a, b) plots of all countries were graphed and analyzed and the results are available upon request. Lack of space prevents their complete presentation in this paper.

Fig. 2. Philippines (a, b) plot: $q = 1$

The expected income elasticity (with sign reversed!) is plotted on the horizontal axis while the interest rate elasticity (again, with sign reversed!) is plotted on the vertical axis. Sign consistency results only when the (a, b) plots all cluster in the same orthant. There are four orthants in the bivariate (a, b) space: $(+, +)$, $(+, -)$, $(-, +)$ and $(-, -)$, but the even sign combinations $(+, +)$ and $(-, -)$ are mirror-equivalent and so are the odd sign combinations $(+, -)$ and $(-, +)$.¹⁶ It is clear that Taiwan has a $q = 2$ system, since its (a, b) plots show sign inconsistency and the Philippines a $q = 1$ system, since its (a, b) plots show sign consistency.

3.3. 2D Data Scatter Plots and Noise/Signal Ratios. The conventional forms of multivariate analysis are deficient, in the sense that they deliver no unambiguous separation of data into signal and noise.[11] Therefore, we propose an alternative approach in the form of the Complete Least Squares (CLS) computations made possible by Kalman and Los ([6], [7], [10], [12], [13]).¹⁷ These CLS computations exploit all bivariate covariance information in the $n \cdot (n - 1) / 2 = 3 \cdot 2 / 2 = 3$ bivariate covariance pictures per country.

In Section 4, the country data x_i for $i = 1, 2, 3$ are plotted in 3D data space as ellipsoid scatter in the center of the 3D data frames. Each black dot represents a 3D measurement of the three respective variables - stock valuation, expected nominal GDP and interest rate (after logarithmic transformation and lateral shifts). The side panels of the 3D data frames show the orthogonal projections of these scatterplots as grey dots, forming three bivariate scatter plots. We computed the Noise/Data and Noise/Signal Ratios, collected in Table 3. for the respective bivariate (x_1, x_2) , (x_1, x_3) and (x_2, x_3) scatterplots.¹⁸

¹⁶[10] provides a complete analysis of such (a, b) coefficient plots. In particular, the geometric analysis of Fig. 1 on [10], p. 1291, which extended and refined an earlier analysis by Klepper and Leamer [9].

¹⁷Multidirectional Complete Least Squares (CLS) projections are not to be confused with the conventional *unidirectional* Generalized Least Squares (GLS) projections. CLS projections account completely for all the data covariances from all projection directions, like the goniometric projections in X-ray crystallographic structure determination. Conventional GLS projection account only for a small subset of these covariances - $(n - 1)$ out of the $n(n - 1) / 2$ available -, because it *a priori* selects only one projection direction.

¹⁸The bivariate Noise/Data Ratio $(N/D)_{ij} = (1 - \rho_{ij}^2)$ and the corresponding Noise/Signal Ratio $(N/S)_{ij} = (1 - \rho_{ij}^2) / \rho_{ij}^2$ for $i, j = 1, 2$, where ρ_{ij} is the bivariate correlation coefficient between variables \mathbf{x}_i and \mathbf{x}_j .

Table 3 shows that the noise/signal ratios of Taiwan are between $1.63\times$ and $2.96\times$, but they are evenly distributed. In other words, although Taiwan is clearly a high noise environment, some information can be obtained by looking at all three bivariate scatterplots simultaneously as a $q = 2$ system. Despite the high noise level, Taiwan functions as a coherent financial economic system: the stock market is influenced by the expected economy and the financial market, while the expected economy is also influenced by the domestic financial market and vice versa.

TABLE 3.	2D NOISE/DATA		AND NOISE/SIGNAL		RATIOS	
	$(N/D)_{12}$	$(N/S)_{12}$	$(N/D)_{13}$	$(N/S)_{13}$	$(N/D)_{23}$	$(N/S)_{23}$
TAIWAN	0.62	1.63	0.62	1.60	0.75	2.96
MALAYSIA	0.12	0.14	0.85	5.79	0.97	32.38
SINGAPORE	0.14	0.17	0.90	9.49	0.95	18.34
PHILIPPINES	0.20	0.25	0.94	15.43	1.00	54721.40
INDONESIA	0.42	0.73	0.92	11.43	0.94	16.47
JAPAN	0.98	46.47	0.89	8.12	0.98	44.15

We found earlier that Japan's data covariance structure is somewhat similar to Taiwan's ($q = 2$), except that Japan's system noise swamps its signals: Japan's noise/signal ratios are between $8.12\times$ and $46.47\times$! We cannot ascertain from the data covariances if there is any relationship between Japan's stock market and its expected economy, between its stock market and the financial market or between its expected economy and its financial market.

The noise signal ratios of Malaysia, Singapore, Philippines and Indonesia show close relationships between x_1 and x_2 and very uncertain bivariate relations between x_1 and x_3 and between x_2 and x_3 , respectively. We tentatively conclude that in those countries there is a close relationship between the stock market and the expected economy, but no relationship between the stock market and the financial market or between the expected economy and the financial market. These countries exhibit $q = 1$ systematic covariance. The financial markets in these countries appear to operate independently from their respective economies and stock markets.

3.4. 3D Complete Least Squares (CLS) Plots. We can combine this bivariate projection information in the form of 3D Complete Least Squares projections for both $q = 2$ and $q = 1$. The $q = 2$ CLS plots encompass the $q = 1$ plots. The $q = 2$ plots are *rays*, representing the projected $q = 2$ systems, while the $q = 1$ systems are *planes*, representing the projected $q = 1$ systems.

It can be shown that the three $(n, q) = (3, 2)$ CLS systematic projector matrices $\hat{\mathbf{P}}_i = \hat{\Sigma}^{CLS}\Sigma^{-1}$ with $i = 1, 2, 3$, are configured as follows.¹⁹

For the $q = 2$ projections on variable x_1 we have the systematic signal projector

$$\hat{\mathbf{P}}_1^{CLS} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\sigma_{12}}{\sigma_{11}} & 0 & 0 \\ \frac{\sigma_{13}}{\sigma_{11}} & 0 & 0 \end{bmatrix}$$

¹⁹Hats $\hat{\cdot}$ denote systematic signals and waves $\tilde{\cdot}$ denote unsystematic noise. For simple algebraic derivations of the $(3, 2)$ and $(3, 1)$ systematic projector matrices $\hat{\mathbf{P}}_i$, see Appendices II and IV.

This signal projector, $\widehat{\mathbf{P}}_i^{CLS}$ combines the bivariate covariances of the data covariance matrix Σ . When $\widehat{\mathbf{P}}_1$ is postmultiplied by the $n \times T = 3 \times 39$ data matrix $\mathbf{x}' = (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$, so that the 3×39 systematic data matrix is $\widehat{\mathbf{x}}_1' = \widehat{\mathbf{P}}_1^{CLS} \mathbf{x}'$, the first data series in $\widehat{\mathbf{x}} = (\widehat{\mathbf{x}}_1, \widehat{\mathbf{x}}_2, \widehat{\mathbf{x}}_3)$, remains non-projected, $\widehat{\mathbf{x}}_1 = \mathbf{x}_1$, since it is the series on which we project, i.e., the one which is assumed to have no noise, $\tilde{\sigma}_{11} = 0$. The other two series, $\widehat{\mathbf{x}}_2$ and $\widehat{\mathbf{x}}_3$ result from the simple bivariate orthogonal projections on \mathbf{x}_1 . Thus we have the first CLS_1 projected system

$$\begin{aligned} \widehat{\mathbf{x}}_1 &= \mathbf{x}_1 & \widehat{\mathbf{x}}_1 &= \mathbf{x}_1 \\ \widehat{\mathbf{x}}_2 &= \frac{\sigma_{12}}{\sigma_{11}} \widehat{\mathbf{x}}_1 & \widehat{\mathbf{x}}_2 &= \frac{\sigma_{11}}{\sigma_{12}} \widehat{\mathbf{x}}_2 = c \cdot \widehat{\mathbf{x}}_2 \\ \widehat{\mathbf{x}}_3 &= \frac{\sigma_{13}}{\sigma_{11}} \widehat{\mathbf{x}}_1 & \widehat{\mathbf{x}}_3 &= \frac{\sigma_{11}}{\sigma_{13}} \widehat{\mathbf{x}}_3 = d \cdot \widehat{\mathbf{x}}_3 \end{aligned}$$

which is equivalent to

Notice that this projection system consists of two planes which cut each other and form a ray through the origin of the data frame of reference. Each ray provides a bit of information about the true underlying system, but only when viewed together with the rays from the other two orthogonal projections. In fact, an infinite number of projections - a *complete projection cone* - can be computed from linear combinations from these three extreme orthogonal CLS projections.

Similarly, the other two extreme $(n, q) = (3, 2)$ systematic projectors for the CLS_2 and CLS_3 projections are given by²⁰

$$\widehat{\mathbf{P}}_2^{CLS} = \begin{bmatrix} 0 & \frac{\sigma_{12}}{\sigma_{22}} & 0 \\ 0 & 1 & 0 \\ 0 & \frac{\sigma_{23}}{\sigma_{22}} & 0 \end{bmatrix}, \quad \widehat{\mathbf{P}}_3^{CLS} = \begin{bmatrix} 0 & 0 & \frac{\sigma_{13}}{\sigma_{33}} \\ 0 & 0 & \frac{\sigma_{23}}{\sigma_{33}} \\ 0 & 0 & 1 \end{bmatrix}$$

The $(n, q) = (3, 1)$ CLS projections follow similarly (Cf. Appendix IV). In the following country sections, we use these 3D CLS projection systems to infer the structure of the true underlying financial economic valuation systems of the six Asian countries: Taiwan, Malaysia, Singapore, Philippines, Indonesia and Japan.

4. SYSTEM IDENTIFICATION BY COUNTRY

4.1. **Taiwan.** In Fig. 3. we have plotted Taiwan's measured data \mathbf{x} in the 3D data space: x_1 is measured on the first horizontal axis, x_2 on the second horizontal axis and x_3 on the vertical axis. From the 3D scatter plot it is difficult to obtain a definite conclusion although some information may be gleaned from the three 2D scatterplots in the side panels. However, when we plot the three $q = 2$ projection systems $\widehat{\mathbf{x}}_i' = \widehat{\mathbf{P}}_i^{CLS} \mathbf{x}'$, $i = 1, 2, 3$, three rays are produced in the center of the 3D scatter plot, as shown in the same frame of reference (but without the data scatter) in Fig. 4.

²⁰It is easy to check that these are all projector matrices since $(\widehat{\mathbf{P}}_i^{CLS})^2 = \widehat{\mathbf{P}}_i^{CLS}$, $i = 1, 2, 3$.

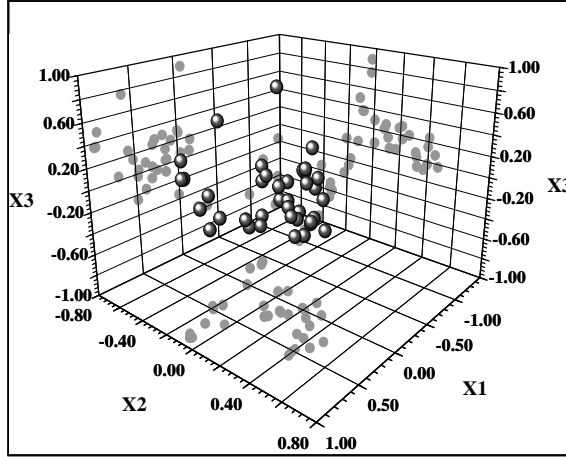


Fig. 3. Taiwan 3D Scatterplot

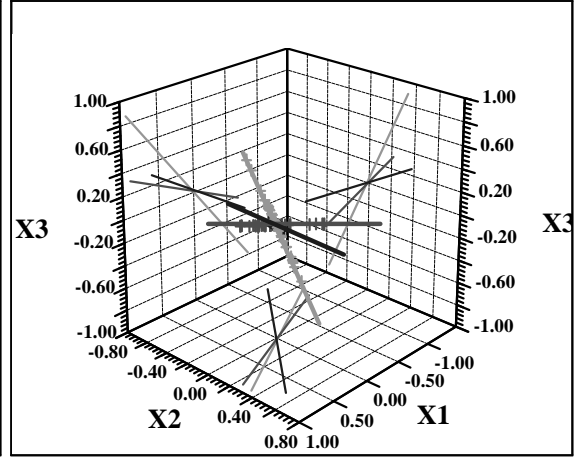


Fig. 4. Taiwan 3D CLS Plot

With appropriate visualization software²¹ we can rotate these $q = 2$ systems and we observe that the three CLS rays are lying close together in a fairly tight cone. There is a positive systematic relationship between x_1 and x_2 as observed in the bottom (x_1, x_2) grid, a positive relationship between x_1 and x_3 as observed in the (x_1, x_3) grid, and consequently, there is also a positive relationship between x_2 and x_3 as observed in the (x_2, x_3) grid. Thus we find that Taiwan has a $q = 2$ financial economic system, represented by the two simultaneous equations

$$\begin{aligned} \hat{x}_1 - c.\hat{x}_2 &= 0 & \text{or, equivalently} & \hat{x}_1 = c.\hat{x}_2 \\ \hat{x}_1 - d.\hat{x}_3 &= 0 & \text{or, equivalently} & \hat{x}_1 = d.\hat{x}_3 \end{aligned}$$

with both $c > 0$ and $d > 0$. The stock market has a positive income elasticity and a *positive* interest rate elasticity, empirically contradicting conventional *economic* theory, which presumes a negative interest rate elasticity, $d < 0$. When Taiwan's interest rate is raised, its stock market valuation increases. One possible cost-push explanation could be that when cash becomes expensive in Taiwan, the domestic banks, which are deeply invested in the Taiwanese stock market, bid up the stock market to prevent their balance sheets from deteriorating. Such an economically irrational result is possible in an illiquid stock market, where the supply of shares is limited.

The three finite boundaries of these elasticities from the corresponding bivariate elements in the $\hat{\mathbf{P}}_i^{CLS}, i = 1, 2, 3$ systematic projector matrices are given in Table 4.²²

²¹E.g., Stanford Graphics, MathCad 6.0 PLUS, Matlab, Mathematica, IBM Visual Data Explorer.

²² $(c, d)_i$ is the set of income (c) and interest elasticities (d) from the $q = 2$ CLS projection on variable \mathbf{x}_i .

TABLE 4.	INCOME (c) AND	INTEREST RATE (d)	ELASTICITIES
From ($q = 2$) CLS Projections	$(c, d)_1$	$(c, d)_2$	$(c, d)_3$
TAIWAN	+3.623, +2.817	+1.375, +2.145	+2.751, +1.084
MALAYSIA	+1.706, -9.433	+1.495, -13.1578	+3.541, -1.392
SINGAPORE	+1.205, -10.000	+1.030, -12.561	+1.513, -0.959
PHILIPPINES	+2.012, -12.500	+1.612, -537.333	+104.560, -0.763
INDONESIA	+13.210, -21.505	+7.632, -19.384	+11.902, -1.731
JAPAN	-13.158, +3.676	-0.278, +1.188	-4.253, +0.040

Notice, first, that the wide modeling uncertainty ranges reflect the high noise environments, and, second, that theoretically expected unit elasticities are mostly outside these empirical ranges, contradicting conventional financial (DDM) theory. The overall 3D noise/signal ratios are presented in Table 5. We compute the volume of the 3D noise/data box from the CLS systematic projectors $\hat{\mathbf{P}}_i^{CLS}, i = 1, 2, 3$, using a well-known Volume Theorem in linear algebra, as explained in Appendix III..

TABLE 5.	3D NOISE/DATA RATIO	3D NOISE/SIGNAL RATIO
TAIWAN	0.37	0.58
MALAYSIA	0.07	0.08
SINGAPORE	0.13	0.15
PHILIPPINES	0.14	0.16
INDONESIA	0.39	0.63
JAPAN	0.86	6.19

From Table 5, we can see that in Taiwan the noise cone spanned by the three projected rays occupies 37% of the data space. This means that 37% of the 3D variation of the Taiwanese data is unsystematic, while the remaining 63% is systematic. Note that in the case of complete certainty, i.e., when all variation is certain, the three projected rays would coincide on one ray and the 3D noise/data ratio would be zero.

Taiwan's 3D noise/data ratio of 37% implies a 3D noise/signal ratio of 58%, i.e., the unsystematic variation in its financial economy is slightly larger than half of its systematic variation. Thus Taiwan forms a fairly coherent economy. There is interaction between both its financial market and its stock market; between its financial market and its expected economy and, consequently, also between its expected economy and its stock market. But this empirically observable interdependence is neither according to conventional economic, nor according to conventional financial theory.

4.2. Malaysia. Malaysia's financial economic system is very different from Taiwan's, as is immediately clear from the $(n, q) = (3, 2)$ plots in Fig. 5. and Fig. 6. One of the $q = 2$ rays in Fig. 6 is almost orthogonal to the other two rays, as observed from their reflections in the two side grids for (x_1, x_3) and (x_2, x_3) ,

indicating that two planes are crossing almost vertically.²³ This indicates that the interest rate variable x_3 varies almost independently from the stock market index x_1 and the *NGDP* variable x_2 . At the same time the reactions in the bottom (x_1, x_2) grid are bundling closely together, suggesting a close bivariate relationship between the stock market S and *NGDP*. Malaysia's projected (c, d) coefficients do have the correct sign according to conventional economic theory, but not the correct financial magnitude, as can be seen in Table 4.

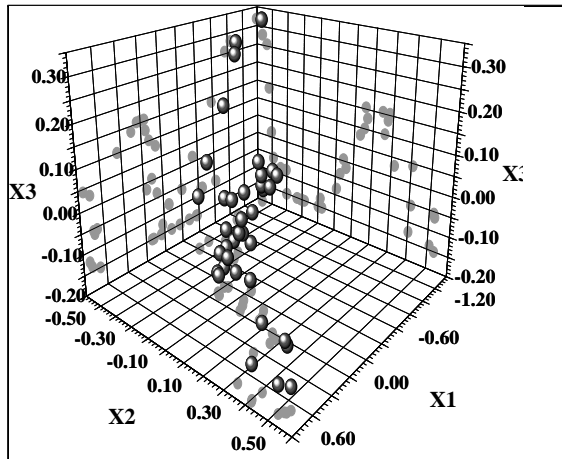


Fig. 5. Malaysia 3D Scatterplot

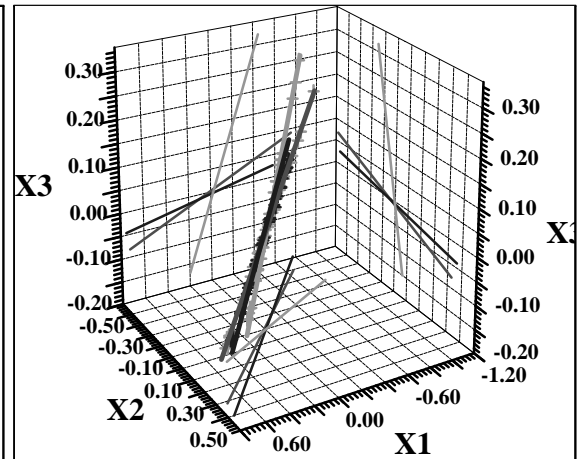


Fig. 6. Malaysia 3D CLS Plot

The reasons why Malaysia has effectively a $q = 1$ system are, possibly, the following. First, the stock market index in Kuala Lumpur varies mostly because of in flows and out flows of foreign capital. Foreign investors are attracted to invest in the Malaysia stock market based on their expectations about the Malaysian economy. When the prospects of Malaysia's economy improve, foreign investors will move more portfolio investments into listed Malaysian stocks. When these prospects deteriorate, they shift their capital out of Malaysia.

Second, at the same time, these foreign investors are not influenced by the relative scarcity of domestic liquidity. They raise their investment capital in the international capital markets, such as in the United States and Europe. They are more influenced by the monetary policies of the Federal Reserve in the United States and the Bundesbank in Germany, than the monetary policy of the Monetary Authority in Malaysia. Thus there is virtually no relationship between Malaysia's financial market and its stock market.

Malaysia, like Singapore and several similar regional economies, uses its interest rate to influence its exchange rate. This is because it wants to keep the Ringgit linked to other major anchor currencies, especially the Japanese Yen and to a lesser degree, the US Dollar. It does not use its interest rate to influence its economy, which is more directed by fiscal policy than by monetary policy.

Third, in this region of the world, debt financing is not as common as in the United States or Europe. Most of the growth in the economy is equity-financed by private, often family-owned, business enterprises. Consequently, secondary debt markets are virtually non-existent and monetary policy, in the sense of influential

²³This is corroborated by rotating the 3D plot. Cf. section 8.1 in Appendix III for the relevant projection theory.

open-market operations, does not (yet) exist. Monetary policy in South-East Asia is often exercised in the form of administrative bank lending rates, combined with a tight control over the domestic currency in circulation to prevent the currency from becoming internationalized. Thus capital flows occur mostly in the form of either direct investments in plants and equipment, or portfolio investments in equity shares. Capital inflows and outflows in the form of investments in debt paper are minimal. Even when global investors invest in debt, they usually invest in sovereign debt and not in privately generated debt.

The bivariate noise/signal ratios in Table 3 in Section 3.3 point to a similar conclusion, which should not surprise us since the $(n, q) = (3, 2)$ systematic projectors take complete account of all systematic bivariate grid information.²⁴ While the noise/signal ratio between the stock market x_1 and the expected economy x_2 is only 0.14, the noise/signal ratio for the bivariate relationship between the stock market x_1 and the financial market x_3 is 5.79 while the noise/signal ratio between the expected economy and the financial market is a very large 32.38.

However, when we look at 3D noise/data and noise/signal ratios of 7% and 8% respectively in Table 5 in Section 4.1, the financial economy of Malaysia is clearly not a high noise environment since the single $q = 1$ relationship is very clear. From a system point of view, only 7% of all observed 3D data variation is noise, but 93% is systematic! There is a very tight relationship between the expected economy and the stock market, even though Malaysia's cash market behaves independently from both.

Thus the magnitude of system noise does *not* depend on large variation of the variables. The stock and financial market and the expected economy can all show substantial variation and yet the financial economic system can still be knitted together by rational economic calculations and behave (almost) like a machine. In the case of Malaysia, the stock market and the economy appear to be in a tight rational relationship. Malaysia's financial market behaves irrationally, because it is still administratively controlled, but, fortunately, its gyrations have little impact on Malaysia's stock market and its economy.²⁵

4.3. Singapore. Singapore's financial economic system operates in a similar fashion as Malaysia's. A comparison of the $(n, q) = (3, 2)$ plots of Malaysia and Singapore in Figures 6. and 8, respectively, show an almost identical configuration. Singapore's projected (c, d) coefficients do also have the correct sign according to conventional economic theory, but not the correct financial magnitude, although the uncertainty range for the interest elasticity includes unity in absolute value (Cf. Table 4). This conclusion reinforces the impression that our new system identification method identifies similarities and dissimilarities between systems, which are not immediately obvious from the original time series data.

²⁴As proved in Appendix III.

²⁵In the light of our analysis one should not be alarmed when Malaysia Signals Caution With Curbs on Lending. Recently Malaysia's central bank moved to curb lending to property and stock purchasers. (*Asian Wall Street Journal*, March 31, 1997, p. 18 and p. 21). Our analysis suggests that these investors may not feel affected by a tightening of Malaysia's domestic lending.

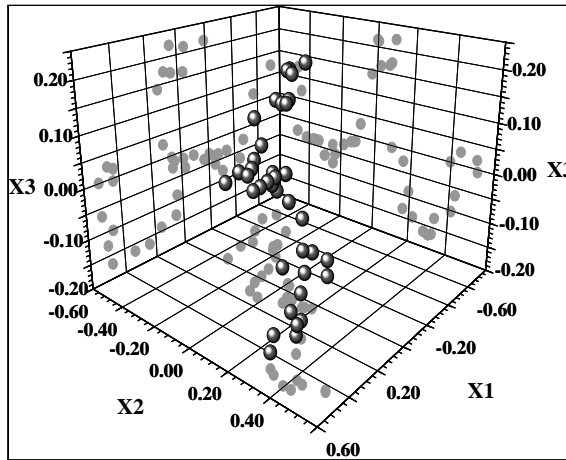


Fig. 7. Singapore 3D Scatterplot

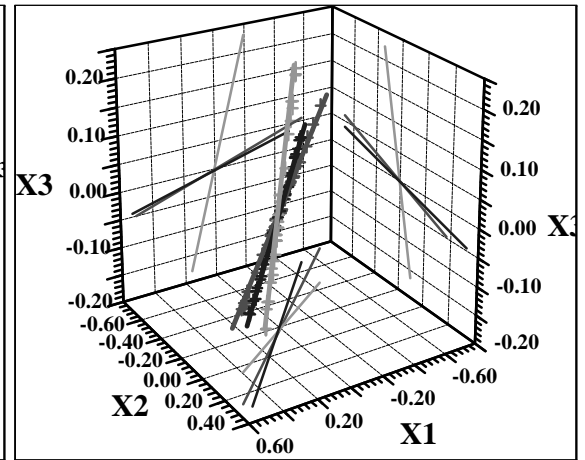


Fig. 8. Singapore 3D CLS Plot

While there is a tight relationship between the expected economy and the stock market in Singapore, its financial market operates virtually independently from these two variables, as it is administratively controlled by the Monetary Authority of Singapore (MAS). The main objective of MAS is not to base its financial calculations on the ground of rational economic behavior, but to disengage its financial market from wealth and economic growth, i.e., the interest rate is controlled for the stabilization of the Singapore dollar, in particular to maintain a stable link between Singapore and US dollars.²⁶

At the same time Singapore's stock market is closely linked to its economic prospects. The bivariate noise/signal ratio of the (x_1, x_2) relationship between the expected economy and the stock market is 0.17 in Singapore, compared to 0.14 for Malaysia, as seen in Table 2. The bivariate noise/signal ratios for the (x_1, x_3) and (x_2, x_3) relationships between the stock and financial market and the expected economy and the financial market are very substantial: 9.49 and 18.34, respectively.

But again, this does not necessarily indicate a high noise environment from a system point of view. The 3D noise/data and noise/signal ratios for the financial economic system of Singapore in Table 3 in Section 4.4 are 13% and 15% respectively. Thus only 13% of all 3D data variation combined is noise and the remaining 87% is systematic! The reason is because of the close relationship between the projected economy and the stock market, despite the fact that Singapore's domestic financial market operates virtually independently from both.

4.4. The Philippines. The $q = 1$ financial economic system of the Philippines operates also similarly to the ones of Malaysia and Singapore, as can be found by comparison of Fig. 10 with Figures 6 and 8. And, again, the projected (c, d) coefficients of the Philippines do have the correct sign according to conventional economic theory. Also, the uncertainty range of the interest rate elasticity includes again unity in absolute value (Cf. Table 4). But there are two aspects which make the Philippines somewhat different from Malaysia and Singapore.

²⁶As was explicitly confirmed by Dr. Richard Hu, Minister of Finance, who, as Guest of Honor, discoursed on Macroeconomic Policies in Singapore: Principles, Milestones and Future Prospects at the Annual Meeting of the Economic Society of Singapore in the Regent Hotel, Singapore on March 22nd, 1997. However, since his speech, the Singapore dollar has continued to depreciate versus the US dollar.

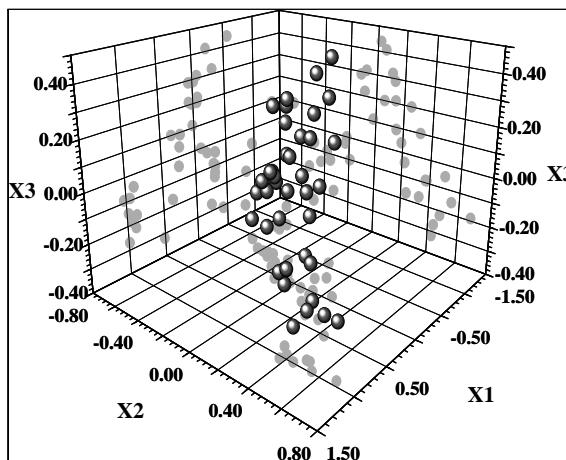


Fig. 9. Philippines 3D Scatterplot

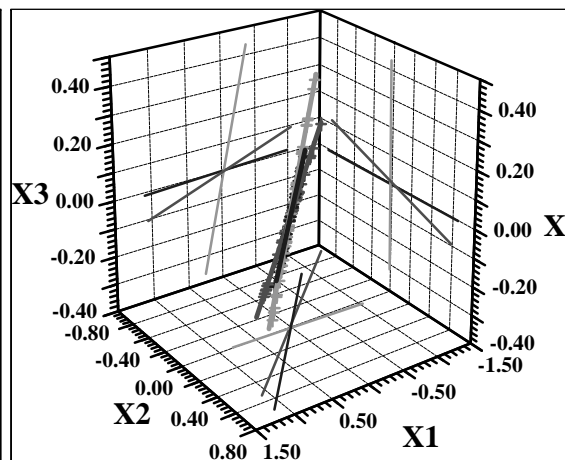


Fig. 10. Philippines 3D CLS Plot

First, the relationship between the Philippines stock market and the expected economy is somewhat looser than in Singapore. In Table 3, the bivariate (x_1, x_2) noise/signal ratio of Philippines is 0.25 instead of Singapore's 0.17 and there is absolutely no relationship between the expected economy x_2 and the financial market x_3 : the noise/signal ratio is 54721.40! Apparently, the Philippines government exclusively uses fiscal policy to guide the economy, but uses monetary policy for purely political and administrative purposes, devoid of macro financial and economic rationale.

Still, this doesn't mean that the Philippines economy is completely disjointed. Surprisingly, it is almost as coherent as Singapore's economy. The 3D noise/data and noise/signal ratios of the Philippines in Table 5 are 14% and 16%, respectively, which is very similar to Singapore's 13% and 14%, respectively. This is again due to the close relationship between the stock market valuation and the expected economy.

4.5. Indonesia. Indonesia follows the pattern discovered for Malaysia, Singapore and the Philippines. We find in Figures 11 and 12 a financial system where the expected economy and the stock market vary together, but where the financial market is disjoint from the first two entities. Indonesia's projected (c, d) coefficients do have the correct sign according to conventional economic theory, but not the correct financial magnitude (Cf. Table 4).

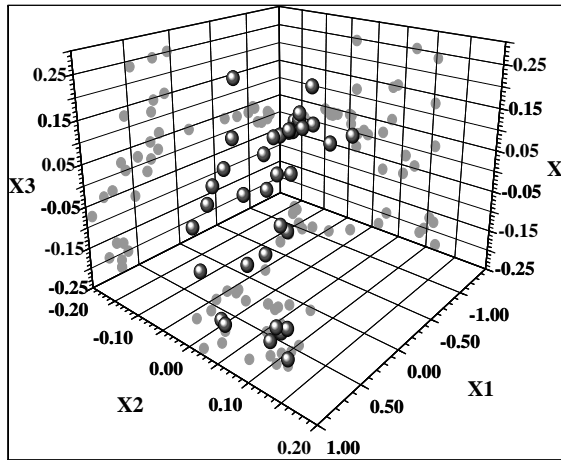


Fig. 11. Indonesia 3D Scatterplot

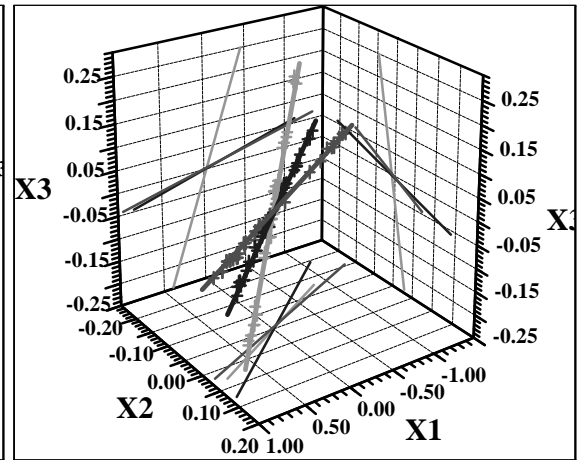


Fig. 12. Indonesia 3D CLS Plot

However, Indonesia distinguishes itself by:

(i) having a considerably looser relationship between the stock market x_1 and the expected economy (represented by petroleum production) x_2 than Malaysia, Singapore or the Philippines. According to Table 3, Indonesias bivariate (x_1, x_2) noise/signal ratio is 0.73, which is higher than Malaysia s 0.14, Singapore s 0.17 or Philippine s 0.25. Thus Indonesia s stock market valuation appears to follow an even less rational pattern than the stock markets of Malaysia, Singapore and Philippines.²⁷

(ii) having very weak relationship between its financial market and both its stock market and its expected economy, very much like Singapore. The bivariate relationship between the stock market and financial market and the expected economy and financial market, measured by (x_1, x_3) and (x_2, x_3) bivariate noise signal ratios of 11.43 and 16.47 (high noise) respectively. This is very similar to Singapores 9.49 and 18.34.

(iii) an overall more integrated economy than Malaysia, Singapore or the Philippines, and more like Taiwan s, since Indonesia s 3D noise/data and noise/signal ratios are 39% and 63%, respectively, as seen in Table 5. That is roughly three times as large as for the Philippines, four times as for Singapore and six times as for Malaysia. These figures are almost the same as those for Taiwan, which shows 3D noise/data and noise signal ratios of 37% and 58%, respectively.

The $q = 2$ system identification using two consecutive data windows and the time-series data suggest that since the first quarter of 1990 Indonesia s financial market has had some negative relationship with the expected economy, which did not exist before. Perhaps this was because before 1990 the variability of the interest rates in Indonesia was virtually nil: the deviation was no more than 5 basis points in the pre-1990 period. The interest rate was an administratively controlled rate and not a price determined by the demand and supply of the financial market. When the variability of the interest rate increased thereafter, the economy, as very crudely measured by Indonesia s petroleum production, correlated somewhat negatively with the interest rate movements. If this finding holds true, then it would be

²⁷This conclusion is subject to the observation that we use Indonesia s petroleum production as a proxy for its GDP. The additional system uncertainty may be attributable to that substitution. A strict comparison would therefore be invalid. This remark also holds for the next point (ii).

an indication that since 1990 the importance of monetary policy has increased in Indonesia, perhaps because the Rupiah was allowed to float in a limited bandwidth. Movements in the managed float of the exchange rate may affect the exports of oil and therefore oil production.

We conclude that although the financial economic system of Indonesia resembles the systems of Malaysia and Singapore, it is more disjointed and therefore behaves more like Taiwan. It is clearly a $(n, q) = (3, 1)$ system with a positive, though loose relationship between its stock market and economy. But it exhibits an overall disjointedness, or noise level, like Taiwan's financial economic system. Perhaps, one may conclude that Indonesia is on the crossroads between being like Malaysia, Singapore and the Philippines, and being like Taiwan.

4.6. Japan. Although Japan is usually assumed to be the most advanced financial economy of the six Asian economies in this paper, it exhibits the most disjointed financial economic system! In first instance we seem to encounter a financial economy similar to that of Taiwan, i.e., a $(n, q) = (3, 2)$ financial economic system, with high noise levels. However, the spectral analysis in Section 3 provides the first indication that Japan's system did not produce any discernible system signal, in contrast to Taiwan, where there was a clearly discernible signal. In Table 1 Japan shows a very gradual fall-off of the scree plot of its data covariance matrix ($r = 3, q = 0$), while Taiwan shows one dominant eigenvalue ($r = 1, q = 2$).

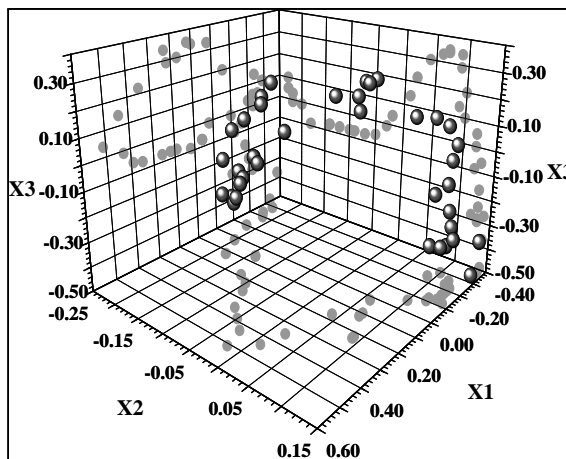


Fig. 13. Japan 3D Scatterplot

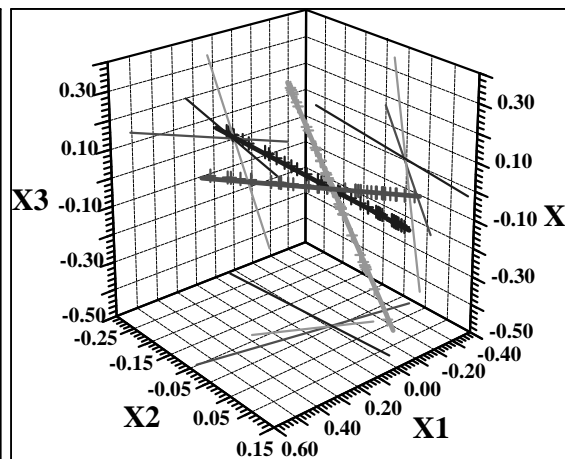


Fig. 14. Japan 3D CLS Plot

The $(n, q) = (3, 2)$ plot in Fig. 14 shows a financial economic system that is almost completely disjointed. Japan's financial market operates almost independently from its expected economy and from its stock market x_1 : the $q = 2$ CLS projection on the interest rate x_3 is virtually orthogonal to the projections on the stock market value x_1 and the expected economic earnings x_2 . Simultaneously the $q = 2$ projection on x_2 is also almost orthogonal to the $q = 2$ projection on x_1 as observed in the various side grids. In addition, the projected (c, d) elasticities show signs opposite of those expected by conventional economic and financial theory, i.e., a negative income elasticity and a positive interest rate elasticity, both larger than unity in absolute value.

The bivariate noise/signal ratios in Table 3 tell the same story. The bivariate noise/signal ratio of the (x_1, x_2) relationship between the stock market and the

expected economy is an extremely high of 46.47; the (x_1, x_3) noise/signal ratio between the stock market and the financial market is a high 8.12; and the (x_2, x_3) noise/signal ratio between the expected economy and the financial market is again extremely high 44.15. In addition, the 3D noise/data and noise/signal ratios in Table 5 does indicate almost complete disjointedness: they are a large 86% and an explosive 619%, respectively. Thus there is no rational system signal of any significance in Japan's financial economic data!

A clue for a possible economic explanation of this surprising result can be found in recent comment like those by Roche, the Chief Strategist for the London-based company Independent Strategy. Roche wrote an op-ed article "Japan's Contradictory Economy" in *The Asian Wall Street Journal* of Wednesday, March 12, 1997, in which he stated: "It is irrelevant to say that Japan is, or is not, a centrally planned economy. What is relevant is that it is not an economically rational one."

Japan's financial economic system is disjointed because its various markets are not rationally linked by economic calculations. Its stock market is rigged by consent between public administrators of the Ministry of Finance and large private securities houses like Nomura, Daiwa, Nikko, Yamaichi, etc.. Its economy is guided by the industrial policies of Japan's Ministry of Trade and Industry (MITI). Its financial market is administratively controlled by the Bank of Japan. Because of the lack of decentralized rational decision making based on financial economic price calculations, these elements of the system operate virtually independently from each other.

In consequence of this financial economic disjointedness in the past decade, Japan's economy is currently burdened by substantial financial economic adjustment problems, now that it tries to induce some economic rationale in its system. Japan's centralized government bureaucracies have to be dismantled and its macroeconomic decisions have to be decentralized to create a rationally integrated economy, more like Taiwan's.

There is a lesson for countries like Malaysia, Singapore, the Philippines and Indonesia: follow the example of Taiwan and not that of Japan! Taiwan has a rationally integrated financial economic system. It is not as rational and tightly integrated as it, perhaps, could be, but the noise levels are manageable. Taiwan's financial economic signals are not yet completely swamped by administrative irrationality. In Japan, the financial economic system has clearly broken down and thus created a huge adjustment burden for Japan's future population. Such adjustment problems can be avoided by judicious privatization and decentralization of the financial economic decision making to let the market pricing mechanisms produce the correct system signals.

5. CONCLUSION

5.1. Conclusions. In this paper we investigate the financial economic systems of six Asian countries - Taiwan, Malaysia, Singapore, Philippines, Indonesia and Japan - over the period of first quarter of 1986 through third quarter of 1995 to determine how their stock markets, expected economies and financial markets are interrelated. The purpose of this exercise is to obtain rational stock market valuations for each country using their (expected) nominal GDPs and short term interest rates.

We used various methods of analysis: spectral analysis of the data covariance matrices, analysis of the elementary regressions exhibited in the information matrices, 2D bivariate noise/signal determination, 3D noise/signal determination and 3D system identification based on $q = 1$ and $q = 2$ projections.

The last two methods are very new. These so-called 'super lter' methods provide the least ambiguous identifications the Grassmanian invariants of the structures of these national financial economic systems and the best computations of the boundaries of the empirical parameter ranges allowed by the finite data. The Grassmanian invariant of a system is the minimum number of independent linear equations required to model the systematic variation of the data. Thus for three variables $q = 1$ or 2 , i.e., a single equation system or a two-equation system.

Once we had identified the invariant of each financial economic system, we computed the observable parameter ranges for the systems (Cf. Tables 2 and 4). We also checked our conclusions against the time series data by computing the relevant time series projections and by comparing them against the actual historical data. No in-sample or out-of-sample testing of the homogeneity of the data series and the integrity of the identified systems has been done. This would require several more years of relatively scarce comparable observations. However, we find that in around 1990 the structure of Indonesia's financial economic system changed: its financial market became linked to its economy and its stock market. In formal terms: in 1990 Indonesia's system probably changed from a $q = 1$ to a $q = 2$ structure. It became more coherent.

Our identification results for the Grassmanian invariants are summarized in Table 6, where we present our conclusions about structure and relative noise levels.

TABLE 6.	IDENTIFICATION	OF SYSTEM	INVARIANT	RANK Q	
From:	Spectral Analysis of Σ	Information Matrix Σ^{-1}	Inspection of (a, b) Plots	3D CLS Plots	3D N/D Ratio
TAIWAN	$r = 1, q = 2$	2	2	2	0.37
MALAYSIA	$r = 1; q = 2$	1	1 ($\mathbf{x}_3 = \text{noise}$)	1	0.07
SINGAPORE	$r = 1; q = 2$	1	1 ($\mathbf{x}_3 = \text{noise}$)	1	0.13
PHILIPPINES	$r = 1; q = 2$	1	1 ($\mathbf{x}_3 = \text{noise}$)	1	0.14
INDONESIA	$r = 1; q = 2$	2	2 ($\mathbf{x}_3 = \text{noise}$)	1	0.39
JAPAN	$r = 3; q = 0$	2	2 ($\forall \mathbf{x}_i = \text{noise}$)	0	0.86

First, we found that all six financial economic data sets are high noise environments, in which it is difficult to discern the system signal from the noise. Because of high noise levels, spectral analysis is not very discriminatory. It suggests that all financial economic systems consist of two independent equations, except Japan where this analysis cannot identify any system structure.

Second, the analysis of the information matrix and the (a, b) plots of the elementary regressions concludes that the countries can be classified into two groups. Malaysia, Singapore and the Philippines show $q = 1$ system structures, while Taiwan, Indonesia and Japan *appear* to show $q = 2$ system structures. The (a, b) plots also confirm that both Indonesia and Japan are high noise environments.

Third, implementing our new 'super lter' theory a much sharper discrimination was made possible, by plotting the 3D $q = 2$ orthogonal Complete Least Squares (CLS) projections. This analysis revealed that only Taiwan has a clear $q = 2$ system.

Of the six Asian countries, only Taiwan has a financial economic system, where the stock market, the economy and the financial market are rationally integrated and the relevant parameter ranges are to be computed from the *two* identified equations.

In addition to Malaysia, Singapore, Philippines, Indonesia has also a $q = 1$ system: while their stock markets and expected economies are closely related and vary together, their respective domestic financial markets have virtually no relationship with their stock markets or their economies. One explanation is that their financial markets are still administratively controlled and not rationally integrated with the other two entities based on decentralized financial economic calculations and decision making. These four economies differ only in the degree of tightness of the relationship between their stock markets and respective economies. Malaysia shows the tightest relationship between these two variables, while Indonesia appears to have the loosest relationship. However, analysis of the relevant computed elasticities in Table 4 reveals that Indonesia's expected petroleum production does have a larger impact on the stock market in Indonesia, than the respective expected nominal GDPs on the stock markets in Malaysia, Singapore and the Philippines.

The biggest surprise of this research is to quantitatively discover the disjointedness of Japan's financial-economic system, which shows no rational coherence at all! Japan's stock market operates independently from its economy and its financial market, while its economy and its financial market are not correlated either.

5.2. Limitations. Due to the limited available data resources in Asia, much time and effort is spent on data collection. Indeed, Barry Hing argued in the December 19, 1996 issue of the *Far Eastern Economic Review* (p. 31) that there should be "Clear, Swift Market Data For All". He complained that "Many of Asia's emerging markets are infamous for their opaque nature and inefficiencies" and provided the examples of Malaysia and Indonesia. Most of our data were sourced and extracted from hard copies of newspapers and magazines (Cf. Appendix), and no additional quality check of the data was possible.

All countries, except Indonesia, produce quarterly nominal *GDP* figures. Indonesia does not have a policy of regularly publishing them, since it remains uninformative about its imports and exports.²⁸ Therefore, we use its quarterly petroleum production index as a crude proxy for its economic growth rate. This proxy ignores also the overall inflation component of nominal *GDP*.

Furthermore, in some instances, nominal quarterly *GDP* had to be calculated and converted from its quarterly real *GDP* to suit the purpose of our study. In doing this conversion, some loss in information may also have resulted.

To ensure compatibility and consistency, we've tried, where possible, to use comparable data in all our computations. For instance, we have chosen the lending rate to be the interest rate, representing the financial markets for all countries. However, this type of interest rate is not available for Taiwan where we used the money market rate instead.

World stock markets are usually driven by many other quantitative and qualitative factors. Using our modified version of DDM to identify the relationship between these two variables and the level of the stock market index is not sufficient to fully understand all driving forces behind Asian stock markets. We have identified *open* systems. It is likely that more factors should be included in additional

²⁸Perhaps this huge country of 16,000 islands, and a fractal, very long coastline, is fundamentally unable to monitor all its imports and exports?

system identification research, which involve more than three variables ($n > 3$). In particular, we should, perhaps, have included the countries' exchange rates to distinguish the Asian countries which peg their exchange rates to the US dollar from the ones which float theirs.²⁹ However, the complexity of such financial economic system identification increase combinatorially with the number of variables involved.

Finally, we have not taken account of dynamics, other than a one-quarter expectation lag, because, first, global investors make their asset allocation decisions faster than the available data frequency and, second, a higher data frequency for comparative data is not (yet) available.

6. APPENDIX I: DATA SOURCES AND CONVERSIONS

6.1. Data Sources. The following data sources are used for the analysis in this paper:

- *Asia Pacific Economic Outlook* (4th quarter 1992, pp. 47 - 48 and 4th quarter 1996, pp. 67, 69).
- *The Asian Wall Street Journal* (March 1986 - December 1995).
- *The Business Times* (March 1986 - December 1995).
- *International Financial Statistics* (April 1988, April 1992, April 1996).
- *United Overseas Bank Economic and Financial Quarterly* (1987 II, pp. 13 - 14; 1988 II, pp. 18 - 19; 1989 II, pp. 22 - 23; and 1990 III, pp. 14 - 15).

6.2. Data Conversions. Each country's stock market index is based on the last trading day of each quarter. As the other two variables, $NGDP$ and IR , are quoted as period averages, the stock market indices were converted to comparable period averages using geometric averaging of consecutive observations

$$S_t = [P_t \cdot P_{t-1}]^{0.5}$$

where

S_t = stock market index as quarter t average

P_t = index on the last trading day of quarter t

P_{t-1} = index on the last trading day of the preceding quarter $t - 1$.

Since only real GDP figures for Singapore and Malaysia are quoted in quarterly year-on-year percentage change, their respective quarterly $NGDP$ figures have to be estimated. No proper GDP implicit deflators are published, consequently we used their Consumer Price Indices (CPI) as proxies. First, their CPI figures were converted to a common base year 1985 using the same method of conversion as used for the Indonesian PP Index. Next, we use Fisher's Equation to estimate the quarterly $NGDP$ rate of change

$$g_t^{NGDP} = (1 + g_t^{GDP}) \cdot (1 + f_t) - 1$$

where

g_t^{NGDP} = rate of change of nominal GDP

g_t^{GDP} = rate of change of real GDP

²⁹A starting point could be Kholdy and Sohrabian's finding that the asset-market approach to exchange rate determination has not performed well empirically. They attribute that to the testing of the model by conventional ordinary (single equation) least squares econometric techniques, which they deem inappropriate if variables are nonstationary and cointegrated.[8] We concur with their assessment. Single equation least squares regression provides an incomplete and therefore prejudiced data analysis and system identification.

f_t = rate of inflation.

As the Petroleum Production PP index of Indonesia is published with different base years, it is converted to a common base year 1985 using the next two formulas. Values with base year before 1985 are converted using the rebasing formula 1 while those with base year after 1985 are converted using the concatenation formula 2.

$$\text{Formula 1: } PP_t^{1985} = \frac{PP_t}{PP_t^{1985A}} \cdot 100$$

where

PP_t^{1985} = Petroleum Production Index for quarter t with base year 1985

PP_t = Petroleum Production Index for quarter t with base year before 1985

PP_t^{1985A} = Annualized Petroleum Production Index of base year 1985

$$\text{Formula 2: } PP_t^{1985} = \frac{PP_{mt}}{100} \cdot PP_{mA}$$

where

PP_t^{1985} = Petroleum Production Index for quarter t with base year 1985

PP_{mt} = Petroleum Production Index for quarter t with base year before m ,

where m is a period after 1985

PP_{mA} = Annualized Petroleum Production Index for year m with base year 1985.

7. APPENDIX II: $(n, q) = (3, 2)$ CLS PROJECTIONS

This Appendix II derives the explicit Complete Least Squares (CLS) projectors for the $(n, q) = (3, 2)$ case implementing Kalman's seminal superlter theory [6], [13].

Proposition 7.1. *The three $(n, q) = (3, 2)$ CLS signal projectors are, respectively, for the projection on variable x_1*

$$\hat{\mathbf{P}}_1 = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\sigma_{12}}{\sigma_{11}} & 0 & 0 \\ \frac{\sigma_{13}}{\sigma_{11}} & 0 & 0 \end{bmatrix}$$

, for the projection on variable x_2

$$\hat{\mathbf{P}}_2 = \begin{bmatrix} 0 & \frac{\sigma_{12}}{\sigma_{22}} & 0 \\ 0 & 1 & 0 \\ 0 & \frac{\sigma_{23}}{\sigma_{22}} & 1 \end{bmatrix}$$

and, for the projection on variable x_3

$$\hat{\mathbf{P}}_3 = \begin{bmatrix} 0 & 0 & \frac{\sigma_{13}}{\sigma_{33}} \\ 0 & 0 & \frac{\sigma_{23}}{\sigma_{33}} \\ 0 & 0 & 1 \end{bmatrix}$$

Proof. The 3×3 data covariance matrix is

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}$$

This is all available information to identify the structure of the 3D system covariance ellipsoid. The 3D system uncertainty, or noise, in this data space of x_1 , x_2 and x_3 , is represented by the determinant of Σ

$$|\Sigma| = \sigma_{11}\sigma_{22}\sigma_{33} - \sigma_{11}\sigma_{23}^2 - \sigma_{22}\sigma_{13}^2 - \sigma_{33}\sigma_{12}^2 + 2\sigma_{12}\sigma_{13}\sigma_{23}$$

The adjoint of the data covariance matrix provides a picture of the relative connectedness of the underlying system, since it consists of the bivariate determinants:

$$\text{Adj}(\Sigma) = \begin{bmatrix} \sigma_{22}\sigma_{33} - \sigma_{23}^2 & -\sigma_{12}\sigma_{33} + \sigma_{13}\sigma_{23} & \sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22} \\ -\sigma_{12}\sigma_{33} + \sigma_{13}\sigma_{23} & \sigma_{11}\sigma_{33} - \sigma_{13}^2 & -\sigma_{11}\sigma_{23} + \sigma_{12}\sigma_{13} \\ \sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22} & -\sigma_{11}\sigma_{23} + \sigma_{12}\sigma_{13} & \sigma_{11}\sigma_{22} - \sigma_{12}^2 \end{bmatrix}$$

When $q = 2$ we have the following three combinatorial options. First, select the second and third row of the adjoint of Σ to form the $q \times n = 2 \times 3$, $\text{rank}(2)$ CLS coefficient matrix \mathbf{A}'_1 , which presumes that the first variable doesn't contain noise, i.e., that the noise variance of the first variable is $\tilde{\sigma}_{11} = 0$. Second, select the first and third row to form the coefficient matrix \mathbf{A}'_2 , which presumes that we project on the second variable, so that $\tilde{\sigma}_{22} = 0$. Finally, select the first and second row, to form the coefficient matrix \mathbf{A}'_3 , which presumes that we project on the third variable, so that $\tilde{\sigma}_{33} = 0$.

For example, project on variable x_3 , i.e., presuming that $\tilde{\sigma}_{33} = 0$. The other two CLS projections, which presume $\tilde{\sigma}_{11} = 0$ and $\tilde{\sigma}_{22} = 0$ follow similarly. First we derive the noise matrices for these orthogonal CLS projections, followed by the derivation of their noise and signal projectors. We'll construct the noise matrices in steps, to demonstrate some important relationships, like the following orthogonality relationship

$$\mathbf{A}'_3 \Sigma = \begin{bmatrix} |\Sigma| & 0 & 0 \\ 0 & |\Sigma| & 0 \end{bmatrix}$$

Notice that $\mathbf{A}'_3 \Sigma$ only delivers the complete system uncertainty $|\Sigma|$, since

$$\mathbf{A}'_3 \Sigma = \mathbf{A}'_3 \hat{\Sigma} + \mathbf{A}'_3 \tilde{\Sigma} = \mathbf{A}'_3 \tilde{\Sigma}$$

The next step is to pre-multiply the transpose of this expression by \mathbf{A}'_3 to arrive at an expression that contains the complete system uncertainty $|\Sigma|$ the subsystem inexactitudes as measured by the respective 2×2 determinants of bivariate

$$\mathbf{A}'_3 \Sigma \mathbf{A}_3 = \begin{bmatrix} (\sigma_{22}\sigma_{33} - \sigma_{23}^2) & -(\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23}) \\ -(\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23}) & (\sigma_{11}\sigma_{33} - \sigma_{13}^2) \end{bmatrix} |\Sigma|$$

The determinant of this expression is $|\mathbf{A}'_3 \Sigma \mathbf{A}_3| = \sigma_{33} |\Sigma|^2$, which is the square of the 3D system uncertainty multiplied by the measured data variance of the variable on which we project. The inverse of this expression is thus

$$(\mathbf{A}'_3 \Sigma \mathbf{A}_3)^{-1} = \begin{bmatrix} |\Sigma_{13}| & (\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23}) \\ (\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23}) & |\Sigma_{23}| \end{bmatrix} \frac{1}{\sigma_{33} |\Sigma|^2}$$

where we have substituted the determinants of the appropriate 2×2 bivariate submatrices. This expression allows us to finally assemble the CLS noise matrix³⁰

$$\tilde{\Sigma}_3^{CLS} = \Sigma \mathbf{A}_3 (\mathbf{A}'_3 \Sigma \mathbf{A}_3)^{-1} \mathbf{A}'_3 \Sigma$$

³⁰Kalman [7] proved this CLS noise matrix to be the minimal of all linear system noise matrices.

$$= \begin{bmatrix} |\Sigma_{13}| & (\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23}) & 0 \\ (\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23}) & |\Sigma_{23}| & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{1}{\sigma_{33}}$$

Notice that the noise of the third variable equals zero and that the uncertainty of the relationships between x_1 and x_3 and between x_2 and x_3 , as measured by, respectively $|\Sigma_{13}|$ and $|\Sigma_{23}|$ are measured relative to the variation of the variable x_3 on which we project. Or, amounting to the same conclusion, the uncertainty of the bivariate relations as measured by the 2×2 determinants is multiplied by the absolute measurement uncertainty in variable x_3 as measured by $\tilde{\sigma}_{33} = \frac{1}{\sigma_{33}}$. The other two *CLS* noise matrices $\tilde{\Sigma}_1^{CLS}$ and $\tilde{\Sigma}_2^{CLS}$ are derived similarly. The noise projectors are products of the respective noise covariance matrices and the information matrix and have very simple structures. These noise projectors produce the CLS noise from the data for a particular projection direction

$$\tilde{\mathbf{x}}' = \tilde{\mathbf{P}}_i^{CLS} \mathbf{x}', \quad i = 1, 2, 3$$

Thus for the orthogonal projection on variable x_3 we have

$$\begin{aligned} \tilde{\mathbf{P}}_3^{CLS} &= \tilde{\Sigma}_3^{CLS} \Sigma^{-1} = \tilde{\Sigma}_3^{CLS} \frac{Adj(\Sigma)}{|\Sigma|} \\ &= \begin{bmatrix} 1 & 0 & -\frac{\sigma_{13}}{\sigma_{33}} \\ 0 & 1 & -\frac{\sigma_{23}}{\sigma_{33}} \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

For example, this noise projector produces the following (recognizable) CLS noise series for $\tilde{x}_1 = x_1 - \frac{\sigma_{13}}{\sigma_{33}}x_3$ and $\tilde{x}_2 = x_2 - \frac{\sigma_{23}}{\sigma_{33}}x_3$ from the orthogonal projections on variable x_3 . The other two noise covariance projectors follow similarly

$$\begin{aligned} \tilde{\mathbf{P}}_2^{CLS} &= \tilde{\Sigma}_2^{CLS} \Sigma^{-1} = \begin{bmatrix} 1 & -\frac{\sigma_{12}}{\sigma_{22}} & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{\sigma_{23}}{\sigma_{22}} & 1 \end{bmatrix} \\ \tilde{\mathbf{P}}_1^{CLS} &= \tilde{\Sigma}_1^{CLS} \Sigma^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{\sigma_{12}}{\sigma_{11}} & 1 & 0 \\ -\frac{\sigma_{13}}{\sigma_{11}} & 0 & 1 \end{bmatrix} \end{aligned}$$

The corresponding multichannel signal projectors (signal extractors, or super letters), which extract the system signals from the data, depending on the projection direction i , follow immediately, since

$$\hat{\mathbf{x}}' = \hat{\mathbf{P}}_i^{CLS} \mathbf{x}' = (\mathbf{I} - \tilde{\mathbf{P}}_i^{CLS}) \mathbf{x}', \quad i = 1, 2, 3$$

Thus for the projection on variable x_3 we have the signal projector

$$\hat{\mathbf{P}}_3^{CLS} = \mathbf{I} - \tilde{\mathbf{P}}_3^{CLS} = (\Sigma - \tilde{\Sigma}_3^{CLS}) \Sigma^{-1} = \begin{bmatrix} 0 & 0 & \frac{\sigma_{13}}{\sigma_{33}} \\ 0 & 0 & \frac{\sigma_{23}}{\sigma_{33}} \\ 0 & 0 & 1 \end{bmatrix}$$

This signal projector produces the following (recognizable) CLS signal series for $\hat{x}_1 = \frac{\sigma_{13}}{\sigma_{33}}x_3$ and $\hat{x}_2 = \frac{\sigma_{23}}{\sigma_{33}}x_3$ from the orthogonal projections on variable x_3 . The other two signal projectors $\hat{\mathbf{P}}_1^{CLS}$ and $\hat{\mathbf{P}}_2^{CLS}$ follow similarly. ■

8. APPENDIX III: 3D NOISE/SIGNAL RATIO

The 3D noise/signal ratio for the $q = 2$ signal projectors is derived from the 3D noise/data ratio.

Proposition 8.1.

$$3D \text{ Noise/Data Ratio} = \frac{|\Sigma|}{\sigma_{11}\sigma_{22}\sigma_{33}} \geq 0$$

and

$$3D \text{ Noise/Signal Ratio} = \frac{|\Sigma|}{\sigma_{11}\sigma_{22}\sigma_{33} - |\Sigma|} \geq 0$$

Proof. Using a simple Theorem of linear algebra, we measure the volume of the system noise box relative to the volume of the data box. (Cf.[5], p. 304) The system noise box measures how much the projected exact signals differ from each other. The volume of the data box is given by the volume spanned by the 3×3 identity matrix \mathbf{I} , representing the orthogonal frame of reference for the three data measurements, which is unity.

$$\begin{aligned} 3D \text{ Noise/Data Ratio} &= \sqrt{|(\widehat{\mathbf{P}}_1^{CLS} + \widehat{\mathbf{P}}_2^{CLS} + \widehat{\mathbf{P}}_3^{CLS})'(\widehat{\mathbf{P}}_1^{CLS} + \widehat{\mathbf{P}}_2^{CLS} + \widehat{\mathbf{P}}_3^{CLS})|} \\ &= \begin{vmatrix} 1 & \frac{\sigma_{12}}{\sigma_{22}} & \frac{\sigma_{13}}{\sigma_{33}} \\ \frac{\sigma_{12}}{\sigma_{11}} & 1 & \frac{\sigma_{23}}{\sigma_{33}} \\ \frac{\sigma_{13}}{\sigma_{11}} & \frac{\sigma_{23}}{\sigma_{22}} & 1 \end{vmatrix} \\ &= \frac{\sigma_{11}\sigma_{22}\sigma_{33} - \sigma_{33}\sigma_{12}^2 - \sigma_{22}\sigma_{13}^2 - \sigma_{11}\sigma_{23}^2 + 2\sigma_{12}\sigma_{13}\sigma_{23}}{\sigma_{11}\sigma_{22}\sigma_{33}} \\ &= \frac{|\Sigma|}{\sigma_{11}\sigma_{22}\sigma_{33}} \geq 0 \end{aligned}$$

Thus the 3D noise/data ratio is equal to the system uncertainty as measured by the determinant $|\Sigma|$, divided by the product of the three data variances. The 3D noise/signal ratio follows immediately. ■

Remark. Normalization of the three $q = 2$ signal projectors does not change the value of the noise/data ratio. For example, when we normalize on the first elements, we also have

$$\begin{vmatrix} 1 & 1 & 1 \\ \frac{\sigma_{12}}{\sigma_{11}} & \frac{\sigma_{22}}{\sigma_{11}} & \frac{\sigma_{23}}{\sigma_{11}} \\ \frac{\sigma_{13}}{\sigma_{11}} & \frac{\sigma_{23}}{\sigma_{12}} & \frac{\sigma_{33}}{\sigma_{13}} \end{vmatrix} = \frac{|\Sigma|}{\sigma_{11}\sigma_{22}\sigma_{33}}$$

Corollary 8.2.

$$3D \text{ Noise/Data Ratio} = 1 - \rho_{12}^2 - \rho_{13}^2 - \rho_{23}^2 + 2\rho_{12}\rho_{13}\rho_{23} \geq 0$$

Proof. Write out the expression

$$\begin{aligned} 3D \text{ Noise/Data Ratio} &= \frac{|\Sigma|}{\sigma_{11}\sigma_{22}\sigma_{33}} \\ &= 1 - \frac{\sigma_{12}^2}{\sigma_{11}\sigma_{22}} - \frac{\sigma_{13}^2}{\sigma_{11}\sigma_{33}} - \frac{\sigma_{23}^2}{\sigma_{22}\sigma_{33}} + 2\frac{\sigma_{12}\sigma_{13}\sigma_{23}}{\sigma_{11}\sigma_{22}\sigma_{33}} \\ &= 1 - \rho_{12}^2 - \rho_{13}^2 - \rho_{23}^2 + 2\rho_{12}\rho_{13}\rho_{23} \geq 0 \end{aligned}$$

where $\rho_{ij}^2 = \frac{\sigma_{ij}^2}{\sigma_{ii}\sigma_{jj}}$, for $i, j = 1, 2, 3, i \neq j$ are the familiar coefficients of bivariate determination. ■

Thus the 3D noise/data ratio and the 3D noise/signal ratio, *completely* account for all bivariate information, including the systematic interaction, measured by the $\rho_{12}\rho_{13}\rho_{23}$ term.³¹

Remark. The theoretical minimal and maximal values of the 3D noise/data ratio are zero and unity respectively. The theoretical maximal value of the 3D noise/data ratio would occur when there is absolutely no systematic relationship between any of the three variables and all $\rho_{ij}^2 \rightarrow 0$ for $i, j = 1, 2, 3, i \neq j$. Then no measured (observed) system exists and all data are noise, $|\Sigma| \rightarrow \sigma_{11}\sigma_{22}\sigma_{33}$, and the 3D noise/data ratio $\rightarrow 1$.

8.1. Modeling Uncertainty, Inexactness, or Non-uniqueness. Using the 3D noise/data ratio, we have the following equivalent ways of presenting modeling certainty and uncertainty:

(1) *Trivariate modeling uncertainty*³²

(i) $|\Sigma| > 0$, the data covariance matrix is positive definite, i.e. its determinant is positive;

(ii) $0 < \rho_{ij}^2 < 1$, for some $i, j = 1, 2, 3, i \neq j$, i.e., some coefficient of bivariate determination shows less than complete explanation or inexactness;

(iii) $\hat{\mathbf{P}}_i^{CLS} \neq \hat{\mathbf{P}}_j^{CLS}$ (after normalization), for all $i, j = 1, 2, 3, i \neq j$, the CLS projectors don't coincide;

(iv) $0 < \left| \hat{\mathbf{P}}_1^{CLS} + \hat{\mathbf{P}}_2^{CLS} + \hat{\mathbf{P}}_3^{CLS} \right| < 1$, there exists an uncertainty gap within the orthant frame of data reference;

(v) 3D $N/S > 0$, the noise/signal ratio is positive, since the inexact data contain some noise together with the signal.

(2A) *Trivariate modeling certainty for $q=2$*

(i) $|\Sigma| = 0$, the data covariance matrix is singular, i.e., its determinant equals zero;

(ii) $\rho_{ij}^2 = 1$, for all $i, j = 1, 2, 3, i \neq j$, i.e., all coefficients of bivariate determination show exactness;

(iii) $\hat{\mathbf{P}}_i^{CLS} = \hat{\mathbf{P}}_j^{CLS}$ (after normalization), for all $i, j = 1, 2, 3, i \neq j$, all CLS projectors coincide;

(iv) $\left| \hat{\mathbf{P}}_1^{CLS} + \hat{\mathbf{P}}_2^{CLS} + \hat{\mathbf{P}}_3^{CLS} \right| = 0$, there exists no uncertainty gap within the orthant frame of data reference;

(v) 3D $N/S = 0$, the noise/signal ratio is zero, since the data contain only the signal.

(2B) *Trivariate modeling certainty for $q=1$*

(i) $|\Sigma| > 0$, the data covariance matrix is positive definite, i.e., its determinant is positive definite;

(ii) $\rho_{ij}^2 = 0$, for some $i, j = 1, 2, 3, i \neq j$, i.e., at least one coefficient of bivariate determination shows no explanation;

³¹The factor 2 results from the symmetry of the covariance matrix.

³²Similarly for the bivariate case, cf. [14], p. 19.

- (iii) $\hat{\mathbf{P}}_i^{CLS} \perp \hat{\mathbf{P}}_j^{CLS}$ and $\hat{\mathbf{P}}_i^{CLS} \neq \hat{\mathbf{P}}_k^{CLS} = \hat{\mathbf{P}}_j^{CLS}$ (after normalization), for some $i, j, k = 1, 2, 3, i \neq j, k$, at least one CLS projectors is orthogonal to another while the other two coincide;
- (iv) $0 < \left| \hat{\mathbf{P}}_1^{CLS} + \hat{\mathbf{P}}_2^{CLS} + \hat{\mathbf{P}}_3^{CLS} \right| \ll 1$, there exists an uncertainty gap within the orthant frame of data reference;
- (v) 3D $N/S = 0$, the 3D noise/signal ratio is positive, since the inexact data contain some noise together with the signal.

Let us explore possible values of this 3D noise/data ratio for some $q = 2$ and $q = 1$ cases.

8.2. Case $q = 2$. When the system is a ray, $q = 2$, there are two relationships between the variables, for example between variables x_1 and x_2 , and between variables x_1 and x_3 : $\rho_{12}^2 = \rho_{13}^2 = 1$, and

$$\text{3D Noise/Data Ratio} = -1 - \rho_{23}^2 + 2\rho_{23} > 0$$

When there is low noise, the 3D noise/data ratio $\rightarrow 0$, and

$$2\rho_{23} = 1 + \rho_{23}^2$$

This can only occur when $\rho_{23}^2 = 1$. Then the two *exact* relations imply the third!

8.3. Case $q = 1$. When the system is a plane, $q = 1$, there is only one relationship between the three variables, and two of the three variables are independent. For example, $\sigma_{23} = 0$ and $\rho_{23}^2 = 0$. Then

$$\text{3D Noise/Data Ratio} = 1 - \rho_{12}^2 - \rho_{13}^2 > 0$$

and there is no systematic interaction term! There are many combinations of ρ_{12}^2 and ρ_{13}^2 for which the 3D noise/data ratio $\rightarrow 0$, depending on the position of the plane in the 3D data space, even for an exact plane! For an exact plane, one can rotate the frame of data reference and find, for example, that $\rho_{12}^2 \rightarrow 1$ and $\rho_{13}^2 \rightarrow 0$.

9. APPENDIX IV: (a, b) PARAMETER ANALYSIS

Analysis of the conventional ordinary single equation, $q = 1$, or plane projectors produces the following comparison of their empirical parameter values with their theoretical ones when there is no relationship between variables x_2 and x_3 .

Proposition 9.1. *The adjoint of the data covariance matrix Σ contains all required information for the parameter values of the $q = 1$ plane projectors.*

Proof. Recall the adjoint $Adj(\Sigma)$ from Appendix II. We normalize on the first column³³

$$\text{Normalized } Adj(\Sigma) = \begin{bmatrix} 1 & \frac{\sigma_{13}\sigma_{23} - \sigma_{12}\sigma_{33}}{\sigma_{22}\sigma_{33} - \sigma_{23}^2} & \frac{\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22}}{\sigma_{22}\sigma_{33} - \sigma_{23}^2} \\ 1 & \frac{\sigma_{11}\sigma_{33} - \sigma_{13}^2}{\sigma_{13}\sigma_{23} - \sigma_{12}\sigma_{33}} & \frac{\sigma_{12}\sigma_{13} - \sigma_{11}\sigma_{23}}{\sigma_{13}\sigma_{23} - \sigma_{12}\sigma_{33}} \\ 1 & \frac{\sigma_{12}\sigma_{13} - \sigma_{11}\sigma_{23}}{\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22}} & \frac{\sigma_{11}\sigma_{22} - \sigma_{12}^2}{\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22}} \end{bmatrix}$$

The second column elements are the a -parameters and the third column elements are the b -parameters for the respective (a, b) plots. The parameters of the normalized $Adj(\Sigma)$ are the parameters of the $q = 1$ plane projectors, with their sign

³³We normalize here on the first column, but could just as well normalize on the second or third columns.

reversed, and after suitable transformation. For example, the conventional projection of variable x_1 on x_2 and x_3 has the following con guration:³⁴

$$\begin{aligned}\widehat{\mathbf{P}}_{23}^{CLS} &= \widehat{\Sigma}_{23}^{CLS} \Sigma^{-1} \\ &= \begin{bmatrix} 0 & \frac{(\sigma_{12}\sigma_{33}-\sigma_{13}\sigma_{23})}{\sigma_{22}\sigma_{33}-\sigma_{23}^2} & \frac{(\sigma_{13}\sigma_{22}-\sigma_{12}\sigma_{23})}{\sigma_{22}\sigma_{33}-\sigma_{23}^2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & \frac{\frac{\sigma_{12}}{\sigma_{22}} - \frac{\sigma_{13}\sigma_{23}}{\sigma_{33}\sigma_{22}}}{1-\rho_{23}^2} & \frac{\frac{\sigma_{13}}{\sigma_{33}} - \frac{\sigma_{12}\sigma_{23}}{\sigma_{22}\sigma_{33}}}{1-\rho_{23}^2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\end{aligned}$$

It can be immediately observed that the a and b coefficients of the first row of the normalized $Adj(\Sigma)$ are found in the (1, 2) and (1, 3) cells of the $\widehat{\mathbf{P}}_{23}^{CLS}$ projector matrix, respectively, after sign reversal. Of course, we have two additional $q = 1$ projectors, which have equivalent informative value. For the projection of x_2 on x_1 and x_3

$$\begin{aligned}\widehat{\mathbf{P}}_{13}^{CLS} &= \widehat{\Sigma}_{13}^{CLS} \Sigma^{-1} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ \frac{\frac{\sigma_{12}}{\sigma_{11}} - \frac{\sigma_{13}\sigma_{23}}{\sigma_{11}\sigma_{33}}}{1-\rho_{13}^2} & 0 & \frac{\frac{\sigma_{23}}{\sigma_{33}} - \frac{\sigma_{12}\sigma_{13}}{\sigma_{11}\sigma_{33}}}{1-\rho_{13}^2} \\ 0 & 0 & 1 \end{bmatrix}\end{aligned}$$

and for the projection of x_3 on x_1 and x_2

$$\begin{aligned}\widehat{\mathbf{P}}_{12}^{CLS} &= \widehat{\Sigma}_{12}^{CLS} \Sigma^{-1} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{\frac{\sigma_{13}}{\sigma_{11}} - \frac{\sigma_{12}\sigma_{23}}{\sigma_{11}\sigma_{22}}}{1-\rho_{12}^2} & \frac{\frac{\sigma_{23}}{\sigma_{22}} - \frac{\sigma_{12}\sigma_{13}}{\sigma_{22}\sigma_{11}}}{1-\rho_{12}^2} & 0 \end{bmatrix}\end{aligned}$$

The corresponding (a, b) parameters are found in the second row of $\widehat{\mathbf{P}}_{13}^{CLS}$ and the third row of $\widehat{\mathbf{P}}_{12}^{CLS}$ after suitable transformation. Thus for $\widehat{\mathbf{P}}_{13}^{CLS}$

$$a = -\frac{1}{\frac{\frac{\sigma_{12}}{\sigma_{11}} - \frac{\sigma_{13}\sigma_{23}}{\sigma_{11}\sigma_{33}}}{1-\rho_{13}^2}} = \frac{\sigma_{11}\sigma_{33} - \sigma_{13}^2}{\sigma_{13}\sigma_{23} - \sigma_{12}\sigma_{33}} = [Normalized\ Adj(\Sigma)]_{2,2}$$

and

$$b = -\frac{\frac{\frac{\sigma_{23}}{\sigma_{33}} - \frac{\sigma_{12}\sigma_{13}}{\sigma_{11}\sigma_{33}}}{1-\rho_{13}^2}}{\frac{\frac{\sigma_{12}}{\sigma_{11}} - \frac{\sigma_{13}\sigma_{23}}{\sigma_{11}\sigma_{33}}}{1-\rho_{13}^2}} = \frac{\sigma_{12}\sigma_{13} - \sigma_{11}\sigma_{23}}{\sigma_{13}\sigma_{23} - \sigma_{12}\sigma_{33}} = [Normalized\ Adj(\Sigma)]_{2,3}$$

Similarly for the corresponding (a, b) parameters for $\widehat{\mathbf{P}}_{12}^{CLS}$. ■

Remark. Notice how the (a, b) parameters for each $q = 1$ projector is affected by the correlation between the variables on which it projects. When this correlation is high, the (a, b) parameters are biased upwards.

³⁴This is the conventional Ordinary Least Squares projection of x_1 on x_2 and x_3 . Researchers who use such $q = 1$ OLS projections tend to forget that one can also project x_2 on x_1 and x_3 , and x_3 on x_1 and x_2 and are *a fortiori* guilty of prejudice by not presenting the *complete* set of these results of the $q = 1$ analysis. This ordinary and the equivalent additional $q = 1$ projectors are derived by selecting single rows of $Adj(\Sigma)$ as the \mathbf{A}_i Grassmanian matrices.

We analyze the theoretical $q = 1$ plane situation when there is no relationship between variables x_2 and x_3 , $\sigma_{23} = 0$ and thus $\rho_{23}^2 = 0$, which is the conventional presumption in ordinary single equation regression. First, we have

$$\text{Normalized Adj}(\Sigma) = \begin{bmatrix} 1 & -\frac{\sigma_{12}}{\sigma_{22}} & -\frac{\sigma_{13}}{\sigma_{33}} \\ 1 & \frac{\sigma_{11}\sigma_{33}-\sigma_{13}^2}{-\sigma_{12}\sigma_{33}} & -\frac{\sigma_{13}}{\sigma_{33}} \\ 1 & -\frac{\sigma_{12}}{\sigma_{22}} & \frac{\sigma_{11}\sigma_{22}-\sigma_{12}^2}{-\sigma_{13}\sigma_{22}} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{\sigma_{12}}{\sigma_{22}} & -\frac{\sigma_{13}}{\sigma_{33}} \\ 1 & \frac{1-\rho_{13}^2}{-\frac{\sigma_{12}}{\sigma_{11}}} & -\frac{\sigma_{13}}{\sigma_{33}} \\ 1 & -\frac{\sigma_{12}}{\sigma_{22}} & \frac{1-\rho_{12}^2}{\frac{\sigma_{13}}{\sigma_{11}}} \end{bmatrix}$$

Thus, if there is an exact $q = 1$ plane system, in the normalized $\text{Adj}(\Sigma)$ the (1, 2) coefficient = (3, 2) coefficient and the (1, 3) coefficient = (2, 3) coefficient. For example, cf. the (a, b) plot of the Philippines in Fig. 2.

The first corresponding plane projector is for the projection of x_1 on x_2 and x_3

$$\hat{\mathbf{P}}_{23}^{CLS} = \begin{bmatrix} 0 & \frac{\sigma_{12}}{\sigma_{22}} & \frac{\sigma_{13}}{\sigma_{33}} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Remark. In this *exceptional, purely theoretical* $\sigma_{23} = 0$ case, for this particular $\hat{\mathbf{P}}_{23}^{CLS}$ projector, the conventional t -statistics are correct, since they test $t_2 = \frac{\sigma_{12}}{\sqrt{|\Sigma|}}$ and $t_3 = \frac{\sigma_{13}}{\sqrt{|\Sigma|}}$. These are the conventional t -tests test for the statistical validity of the bivariate relationships (x_1, x_2) and (x_1, x_3) , respectively. They test a measured bivariate covariance relative to the overall system noise as measured by the determinant $|\Sigma|$.

But, in addition, in a *complete* analysis, we have also, for when $\sigma_{23} = 0$, the $q = 1$ projectors for the reverse regressions. The projector for the projection of x_2 on x_1 and x_3

$$\hat{\mathbf{P}}_{13}^{CLS} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\frac{\sigma_{12}}{\sigma_{11}}}{1-\rho_{13}^2} & 0 & \frac{-\frac{\sigma_{12}\sigma_{13}}{\sigma_{11}\sigma_{33}}}{1-\rho_{13}^2} \\ 0 & 0 & 1 \end{bmatrix}$$

and the projector for the projection of x_3 on x_1 and x_2

$$\hat{\mathbf{P}}_{12}^{CLS} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{\frac{\sigma_{13}}{\sigma_{11}}}{1-\rho_{12}^2} & \frac{-\frac{\sigma_{12}\sigma_{13}}{\sigma_{22}\sigma_{11}}}{1-\rho_{12}^2} & 0 \end{bmatrix}$$

Notice that when $q^{true} = 2$ and thus $(\rho_{13}^2)^{true} = (\rho_{12}^2)^{true} \rightarrow 1$, the parameters of $\hat{\mathbf{P}}_{12}^{CLS}$ and $\hat{\mathbf{P}}_{13}^{CLS}$ approach infinity. If, in addition, $\sigma_{13} = 0$, and thus a degenerate $q = 1$ system,

$$\text{Normalized Adj}(\Sigma) = \begin{bmatrix} 1 & -\frac{\sigma_{12}}{\sigma_{22}} & 0 \\ 1 & -\frac{\sigma_{11}}{\sigma_{22}} & 0 \\ 1 & -\frac{\sigma_{12}}{\sigma_{22}} & \infty \end{bmatrix}$$

and the three $q = 1$ projectors are, respectively

$$\hat{\mathbf{P}}_{23} = \begin{bmatrix} 0 & \frac{\sigma_{12}}{\sigma_{22}} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \hat{\mathbf{P}}_{13} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\sigma_{12}}{\sigma_{11}} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \hat{\mathbf{P}}_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

, which describes the pure bivariate case, like in Indonesia before 1990. Thus $q = 1$ projectors can directly identify only $q = 1$ systems and not $q = 2$ systems (although they can indirectly, via the graphical (a, b) plots, as demonstrated in [10]). But the $q = 2$ projectors can identify both $q = 1$ and $q = 2$ systems.

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