OPTIMAL ASIAN MULTI-CURRENCY STRATEGY PORTFOLIOS WITH EXACT RISK ATTRIBUTION

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ABSTRACT. In an earlier paper [11], the exact and complete return attribution framework of Singer and Karnosky [16] was extended to include market risk measurements for n countries. Exploiting a selection matrix based on the cash accounting identities, the resulting degenerate portfolio choice problem is solved as a lower dimensional, non-degenerate problem of fundamental investment choices between stock market premiums and currency swap returns. The original n^2 multi-currency strategic investment allocations are uniquely retrieved from the resulting $2n$ optimal fundamental choice allocations. This new *optimal return-risk attribution accounting framework* is applied to monthly return data of Hong Kong, Indonesia, Japan, Malaysia, the Philippines, Singapore, Thailand, the USA and Germany from June 1992 through December 1997. This includes the illustrative period of the Asian currency crisis of July - December 1997. The USA and Germany are included as alternative low risk strategic investment allocations in the Asian portfolio for further diversification. Throughout this five and a half year period, Asian risk levels, as measured by the GMV standard deviations of return, were about five times the corresponding average returns. The evidence shows that most of the strategic investment risk in Asian countries is attributable to the risk amplitudes of the stock markets, followed by those of the currency markets and, least, the cash markets. The Thai stock market was the most volatile market to invest in throughout the period. The currency swaps facilitated the spreading contagion. For a US dollar based Asian investor, GMV portfolio risk could have been reduced by half and his return could have been doubled, when the USA would have been included in his portfolio. In contrast, diversification to Gemany (Europe) would only marginally have contributed to portfolio risk reduction. Risk management in Asia was hazardous. Spectral analysis of the covariance risks shows that during the investigated period the Asian portfolio efficiency frontier was non-stationary. The Asian diversi cation risk and the systematic risk in a few specific countries changed during this period.

1. INTRODUCTION

The deep Asian currency crisis, which erupted in July 1997,[1], has considerably increased interest in *optimal* multi-currency investment management in Southeast

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[1]A small, but timely and effective study by Montes examines the economic basis for the Asian currency crisis and characterizes the nature of the policy weaknesses that led to the crisis.[14]

Asia, i.e., financial management that seeks not only to maximize returns on multi-currency investments, but takes explicit account of quantifiable investment *risks.*[5]

As was demonstrated in the preceding methodological paper of [11], such an investment approach requires an exact, unified accounting framework for the optimization of the performance of complex, which involve various market assets, currencies and cross-currency hedging swaps or currency overlays. The resulting *exact portfolio optimization cum cash accounting framework* is an extension of the continuously compounding cash accounting framework of Singer and Karnosky and allows for a breakdown by countries of the asset allocations and currency overlays.[16],[6],[15].

Earlier, Lee discussed asset and currency allocation in the context of international fixed income markets based on a review of the historical returns and risk in the period 1971 - 1985.[8] Lee found local asset returns to be relatively independent of currency movements and suggested to analyze them independently from each other. He recommended active currency management to enhance portfolio return. Allen further developed on Lee s framework, suggested seven specific factors in evaluating portfolio returns, and measured the marginal impact of both policy and active management decisions on the portfolio returns.[2] However, his framework focused on the total portfolio hedged return and therefore didn t allow for country by country attribution analysis, in contrast to the current paper.

By using simple Kronecker product algebra, [11] combines the Singer and Karnovsky s exact accounting framework with Markowitz mean-variance optimization of global multi-currency investment portfolios.[12], [13], [3] In the process an interesting rank problem is solved, which has stymied many international portfolio managers.

The current paper extends the limited empirical analysis of that paper in three ways. First, the portfolios of strategic investments are expanded from three to seven Asian countries - Hong Kong, Indonesia, Japan, Malaysia, the Philippines, Singapore and Thailand. In addition, comparison is made between the return/risk profiles of such regional portfolios with more diversified portfolios including investments in the United States and in Germany. Such portfolios are representative for investors in the ASEAN region. In contrast to [8], substantial interaction between the asset, cash and currency markets was found in Asia.

Second, the original five year period of analysis is extended by half a year - from June 1992 through December 1997 - to include the period of Asian currency turmoil in the second half of 1997.

Third, the notorious non-stationarity of investment risks in Asia is analyzed, although the dynamization of the current framework, by using information filters, must await another paper. Since the Asian countries have experienced significant economic and financial upheavals, both the general level of portfolio risk, as well as possible structural changes in the Asian investment risk profile are analyzed using spectral decomposition of the data covariance matrices.

All data series were obtained from Datastream, Dow Jones Telerate, and their definitions are given in Appendix A.

2. EXACT ATTRIBUTION

2.1. Exact Investment Strategy Return Attribution. At time t an investor has three possible investment instruments: (1) investment in an *asset in country* i, e.g., a stock or a bond, with rate of return $r_i(t)$, (2) a *cash swap* with rate of return $c_j(t) - c_i(t)$, with $c_j(t)$ the cash rate in country j into which the nominal is swapped, and $c_i(t)$ the cash rate in country i out of which the nominal is swapped and (3) the *foreign currency* (foreign currency) *appreciation* rate $\varepsilon_j(t)$ of country j. Thus one particular bilateral investment strategy at time t is represented by the strategic rate of return

$$s_{ij}(t) = r_i(t) + [c_j(t) - c_i(t)] + \varepsilon_j(t)$$

Such a strategy return is equivalent to the sum of a risk premium and a cash return, i.e., the local market i *risk premium* $[r_i(t) - c_i(t)]$ and the *cash return* on currency j, $[c_j(t) + \varepsilon_j(t)]$:

$$s_{ij}(t) = [r_i(t) - c_i(t)] + [c_j(t) + \varepsilon_j(t)]$$

This is also equivalent to the sum of a local market i return, the return on a *currency forward cross hedge* and the foreign currency j appreciation rate, since

$$\begin{aligned} s_{ij}(t) &= r_i(t) + [\{c_1(t) - c_i(t)\} - \{c_1(t) - c_j(t)\}] + \varepsilon_j(t) \\ &= r_i(t) + [f_i(t) - f_j(t)] + \varepsilon_j(t) \\ &= r_i(t) + f_{ij}(t) + \varepsilon_j(t) \end{aligned}$$

The *return on a currency forward* is $f_i(t) = c_1(t) - c_i(t)$, with $c_1(t)$ the cash return of the base currency. The return on a *currency forward cross hedge* $f_{ij}(t)$ consists of the difference between the return on the long domestic forward $f_i(t)$ and the return on the short foreign forward $f_j(t)$. An $n \times n$ non-symmetric *strategy matrix* at time t is a matrix containing all n^2 bilateral investment strategies

$$\mathbf{S}(t) = \{s_{ij}(t); i, j = 1, ..., n\}$$

For example, for $i, j = 1, 2, 3$ we have the 3×3 non-symmetric strategy matrix at time t

$$\begin{aligned} \mathbf{S}(t) &= \begin{bmatrix} s_{11}(t) & s_{12}(t) & s_{13}(t) \\ s_{21}(t) & s_{22}(t) & s_{23}(t) \\ s_{31}(t) & s_{32}(t) & s_{33}(t) \end{bmatrix} \\ &= \left[\begin{bmatrix} r_1(t) \\ r_2(t) \\ r_3(t) \end{bmatrix} - \begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix} \right] \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \\ &\quad + \left\{ \left[\begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \\ \varepsilon_3(t) \end{bmatrix} \right] \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \right\}' \end{aligned}$$

$$= \begin{bmatrix} r_1(t) + \varepsilon_1(t) & r_1(t) - c_1(t) + c_2(t) + \varepsilon_2(t) & r_1(t) - c_1(t) + c_3(t) + \varepsilon_3(t) \\ r_2(t) - c_2(t) + c_1(t) + \varepsilon_1(t) & r_2(t) + \varepsilon_2(t) & r_2(t) - c_2(t) + c_3(t) + \varepsilon_3(t) \\ r_3(t) - c_3(t) + c_1(t) + \varepsilon_1(t) & r_3(t) - c_3(t) + c_2(t) + \varepsilon_2(t) & r_3(t) + \varepsilon_3(t) \end{bmatrix}$$

Table 1 provides the three data series for June 1997: the annualized monthly rates of return on their respective stock markets indices of Hong Kong, Indonesia, Japan, Malaysia, the Philippines, Singapore and Thailand as the seven Asian countries of interest, plus those of the United States and Germany for comparison, the respective 30-day bank lending rates and the respective rates of return on the currencies. From those rates the risk premia $[r_i(t) - c_i(t)], i = 1, 2, ..., 9$ and the cash returns $[c_j(t) + \varepsilon_j(t)], j = 1, 2, ..., 9$ are also calculated. While the annualized risk premia in

TABLE 1: ANNUALIZED RATES OF RETURN (%) IN JUNE 1997

	Stocks (1)	Cash (2)	FX (3)	Risk Premium (1)-(2)	Cash Returns (2)+(3)
Hong Kong	35.2	6.0	0.2	29.1	6.3
Indonesia	48.2	14.5	0.2	33.7	14.7
Japan	31.6	0.6	16.9	31.0	17.6
Malaysia	-30.3	7.4	-5.6	-37.6	1.7
Philippines	-0.1	11.0	0.0	-11.1	11.0
Singapore	-45.9	3.6	-0.9	-49.5	2.6
Thailand	-74.8	16.8	4.4	-91.5	21.1
Germany	75.5	3.1	-24.0	72.4	-21.0
USA	51.0	5.7	0.0	45.4	5.7

June 1997 were positive for Hong Kong, Indonesia, Japan, Germany and the USA, they were negative for Malaysia, the Philippines, Singapore and, in particular, Thailand. All the June 1997 cash returns were positive, with the exception of Germany.

Table 2 presents the corresponding matrix of strategic multi-currency investment returns $\mathbf{S}(t)$ for June 1997. Notice that on the diagonal of this investment strategy matrix we find all results for the simple buy-and-hold strategy of investing in country i, where the return is the combination of what is earned in the stock market and what is earned on the currency. Thus, in June 1997 Hong Kong earned 35.4%/yr at an annualized monthly rate on domestic investments, namely 35.2%/yr in the stock market and 0.2%/yr on cash, while Singapore lost 46.8%/yr on domestic investments, resulting from a 45.9%/yr loss in the stock market, combined with a 0.9%/yr loss on the Singapore dollar. [2]

In contrast, the off-diagonal elements contain the strategies of buying stock in country i and swapping into the cash of country j. What is earned in such a strategy is a combination of stock market, swap earnings and exchange rate earnings, i.e., the sum of risk premium and foreign cash earnings. Thus in June 1997 an investor could earn an annualized 75.5%/yr in the German stock market, but by swapping from the Deutschemark into Thailand s baht, he could earn an extra $16.8\% - 3.1\% = 13.7\%$/yr on the rate differential in cash lending and 4.4%/yr on the baht. Consequently, the rate of return on his stock investment-cum-multi-currency-swap strategy would have been a whopping 93.5%/yr at an annualized rate, i.e., 18.1 percentage points more, on an annualized basis, than what was earned by staying fully invested in Germany. Of course, such an investment strategy would ignore the risks involved in international currency swaps, but since Thailand s baht had been stable for a fairly long time that was exactly what transpired. Two days into July 1997 this preferred cash management strategy of swapping into Thai baht started to rapidly disintegrate.

[2] Rounding errors prevent exactness of some summations in the digit behind the decimal point.

TABLE 2: STRATEGIC INVESTMENT RETURNS (%) IN JUNE 1997

	CURRENCIES								
MARKETS	Hong Kong	Indonesia	Japan	Malaysia	Philippines	Singapore	Thailand	Germany	USA
Hong Kong	35.4	43.9	46.7	30.9	40.1	31.8	50.2	8.2	34.8
Indonesia	40.0	48.5	51.3	35.4	44.7	36.4	54.8	12.8	39.4
Japan	37.3	45.8	48.6	32.7	42.0	33.6	52.1	10.1	36.7
Malaysia	-31.4	-22.9	-20.1	-35.9	-26.7	-35.0	-16.5	-58.6	-31.9
Philippines	-4.8	3.7	6.5	-9.3	-0.1	-8.4	10.0	-32.0	-5.4
Singapore	-43.2	-34.7	-31.9	-47.7	-38.5	-46.8	-28.4	-70.4	-43.8
Thailand	-85.3	-76.8	-74.0	-89.8	-80.6	-88.9	-70.4	-112.5	-85.8
Germany	78.6	87.1	89.9	74.1	83.3	75.0	**93.5**	51.4	78.1
USA	51.6	60.1	62.9	47.1	56.3	48.0	66.5	24.4	51.0

It is easy to translate this rate of return story into investment dollars, since, by first order approximation [3], an initial investment $P(0)$, e.g., of \$100mln, invested in a strategy earning $s_{ij}(t)$, grows in one period t as follows

$$P_{ij}(t) = P(0)(1 + s_{ij}(t)) = P(0)e^{s_{ij}(t)}$$

the value of a strategy investment portfolio at time t can be represented by the weighted average

$$\begin{aligned} P(t) &= \sum_{i,j} w_{ij} P_{ij}(t) \\ &= P(0)[1 + \sum_{i,j}^{n} w_{ij} s_{ij}(t)] \\ &= P(0)e^{\sum_{i,j}^{n} w_{ij} s_{ij}(t)} \\ &= P(0)e^{s_p(t)} \end{aligned}$$

where w_{ij} is the share of the investment capital allocated to strategy $s_{ij}(t)$ with $\sum_{i,j}^{n} w_{ij} = 1$ and $s_p(t) = \sum_{i,j}^{n} w_{ij} s_{ij}(t)$ the average portfolio rate of return in period t. Thus the terminal value of such portfolio of multi-currency investment strategies is

$$P(T) = \sum_{i,j} w_{ij} P_{ij}(T) = P(0) \prod_{t}^{T} e^{s_p(t)}$$

Returning to the rates of return, the strategy matrix $S(t)$ at time t can now be generalized, by using matrix algebra, as follows

$$\begin{aligned} \mathbf{S}(t) &= [\mathbf{r}(t) - \mathbf{c}(t)]\boldsymbol{\iota}_n' + \{[\mathbf{c}(t) + \boldsymbol{\varepsilon}(t)]\boldsymbol{\iota}_n'\}' \\ &= [\mathbf{r}(t) - \mathbf{c}(t)]\boldsymbol{\iota}_n' + \boldsymbol{\iota}_n[\mathbf{c}(t) + \boldsymbol{\varepsilon}(t)]', \end{aligned}$$

[3] The approximation is more accurate when the periods $t = 1, 2, ..., T$ and thus the periodic rates $s_{ij}(t)$ are smaller. For example, daily valuations are more accurate than weekly valuations, which are more accurate than monthly, which are more accurate than quarterly valuations, etc. An increasing number of global financial institutions is able to compute daily valuations of their portfolios, thanks to electronic accounting systems, and as strongly recommended by the Association for Investment Management and Research [1], p. 66.

where $\mathbf{r}(t)$, $\mathbf{c}(t)$ and $\boldsymbol{\varepsilon}(t)$ are $n \times 1$ data vectors of asset rates, cash rates and foreign currency appreciation rates at time t, respectively, and $\boldsymbol{\iota}'_n$ is a $1 \times n$ unit vector, i.e., a vector consisting of n units, $\boldsymbol{\iota}'_n = [1, 1, ..., 1]$.

Next, the 3-dimensional $n \times n \times T$ historical investment strategy *array* $\mathbf{S}$ is the *sequence* of such strategy matrices $\mathbf{S} = \{S(t); t = 1, ..., T\}$. However, covariance risk analysis with three-dimensional arrays is very difficult and it is easier when these arrays are translated into simpler two-dimensional arrays by vectorization. The proper vectorization. of this strategy array is [4]

$$\begin{aligned} \mathbf{VEC(S)} &= [vec(\mathbf{S}(1)), vec(\mathbf{S}(2)), ..., vec(\mathbf{S}(T))] \\ &= [\boldsymbol{\iota}_n \otimes \mathbf{I}_n][\mathbf{r} - \mathbf{c}] + [\mathbf{I}_n \otimes \boldsymbol{\iota}_n][\mathbf{c} + \boldsymbol{\varepsilon}] \\ &= \mathbf{H} \begin{bmatrix} [\mathbf{r} - \mathbf{c}] \\ [\mathbf{c} + \boldsymbol{\varepsilon}] \end{bmatrix} \end{aligned}$$

where $\mathbf{r}$ is the $n \times T$ matrix of T observations on the rates of return of n country assets, $\mathbf{c}$ is the $n \times T$ matrix of observations on the n cash rates, and $\boldsymbol{\varepsilon}$ is the $n \times T$ matrix of T observations on the n currency appreciation rates, all with $T > 2n$. Consequently, $[\mathbf{r} - \mathbf{c}]$ is the $n \times T$ matrix of T observations on the n country risk premia and $[\mathbf{c} + \boldsymbol{\varepsilon}]$ is the $n \times T$ matrix of observations on the n country cash earning rates. $\mathbf{H} = \left[\; [\boldsymbol{\iota}_n \otimes \mathbf{I}_n] \quad [\mathbf{I}_n \otimes \boldsymbol{\iota}_n] \; \right]$ is the crucial $n^2 \times 2n$ *selector matrix*, which embodies the exact cash accounting identities. It represents, in a nutshell, the exact cash accounting framework of [16].

2.2. **Exact Investment Strategy Risk Attribution.** In this section the ground is prepared for the mean-variance analysis. The holding horizon averages of each of the n strategies are given by the $n^2 \times 1$ vector

$$\overline{\mathbf{VEC(S)}} = \frac{\mathbf{VEC(S)}.\boldsymbol{\iota}_T}{T}$$

where $\boldsymbol{\iota}_T$ is the $T \times 1$ unit vector. Thus

$$\overline{\mathbf{VEC(S)}} = \mathbf{H} \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\boldsymbol{\varepsilon}}] \end{bmatrix}$$

In the case of the seven-plus Asian countries , the $5\frac{1}{2}$-year ($T = 66$) averages for the annualized stock market premia are given in Table 3. Thus in the period July 1992 through December 1997, on average, the stock market gained most in the USA and Germany - 15.7%/yr and 14.2%/yr, respectively, and not in Asia. The best average result was obtained in Hong Kong: 10.2%/yr. Notice that these average figures are much more subdued than the monthly figures of June 1997. That is already a first indication that the monthly returns must be very volatile.

Similarly we have the averages for the domestic cash lending earnings, also in Table 3. *A substantial part of the global multi-currency strategy returns was earned in the Asian cash overlay markets*. On average, the international investor could earn on cash 14.0%/yr .in Indonesia, 12.7% in the Philippines and 11.1%/yr in Thailand. The advanced countries, Germany, the USA, Japan, and also Singapore, provided much lower average cash returns, mostly reflecting lower domestic inflation rates and more liquid cash markets. Thus, disregarding exchange rate risk, a natural incentive was created for cash to ow from those countries into Asia. But a closer

[4] When A is an $m \times n$ matrix and B a $p \times q$ matrix, then the *Kronecker product* of these two matrices is de ned by $A \otimes B = [a_{ij}B]$ and $vec(AB) = (B' \otimes I)vec(A) = (I \otimes A)vec(B)$. For proof cf. [4], pp. 519. For a compilation of useful Kronecker product results, cf. [9], pp. 543-548.

TABLE 3: AVERAGE ANNUALIZED RATES OF RETURN (%)
July 1992 - December 1997

	Stocks (1)	Cash (2)	FX (3)	Risk Premium (1)-(2)	Cash Returns (2)+(3)
Hong Kong	10.2	5.1	0.0	5.1	5.1
Indonesia	4.5	14.0	-14.0	-9.5	0.1
Japan	-0.8	1.8	-0.7	-2.6	1.1
Malaysia	0.1	6.7	-8.0	-6.7	-1.3
Philippines	3.3	12.7	-8.6	-9.4	4.1
Singapore	0.6	2.9	-0.7	-2.4	2.3
Thailand	-9.3	11.1	-11.7	-20.3	-0.6
Germany	14.2	5.2	-3.0	9.0	2.2
USA	15.7	4.8	0.0	10.9	4.8

look at Table 3 shows that most of the Asian cash rates were just compensating for the average exchange rate losses. For example, in Indonesia the domestic cash lending rate was 14%/yr, and this exactly compensated for the 14%/yr average annual loss on the rupiah, while in Thailand the 11.1%/yr cash rate did not even fully compensate the 11.7%/yr average currency depreciation. Only the Philippines cash rate of 12.7%/yr appears to have added real value, since the peso only depreciated an average 8.6%/yr. Of course, the Hong Kong dollar stayed pegged to the US dollar and its appreciation was 0%.

In Table 4 these average return strategy vectors are reconstituted into the better recognizable matrix of average strategy returns $\overline{\mathbf{S}}$. In the period July 1992 through December 1997, Singer and Karnosky s *maximizing average return strategy* would have been to invest in the USA to earn a 15.7%/yr and then, perhaps, to swap into the Hong Kong dollar to pick up an extra $5.1\% - 4.8\% = 0.3\%$, or 30 basispoints per year. After the hand-over of Hong Kong to the People s Republic of China, such a sure swap has now lost its attraction too.

Interestingly, Table 4 also shows that investing in Thailand s stock market would have been unprofitable on average (but await the final verdict until Section 4, where we analyze the several sub-periods), because Thailand s row in Table 4 presents all negative returns, no matter what currency swaps are implemented. Similar results hold true for the stock markets of Indonesia, Malaysia and the Philippines. These countries were attractive for the global investors from the USA, Germany (and, because of the currency peg, from Hong Kong), not for their stock markets, but for their currency returns.

We can now compute the $n^2 \times n^2 = 81 \times 81$ *strategy risk matrix* Σ of the $n^2 = 81$ investment strategies and exactly decompose it into its various stock market and

TABLE 4: AVERAGE STRATEGIC INVESTMENT RETURNS (%)
July 1992 - December 1997

	CURRENCIES								
MARKETS	**Hong Kong**	**Indonesia**	**Japan**	**Malaysia**	**Philippines**	**Singapore**	**Thailand**	**Germany**	**USA**
Hong Kong	10.2	5.2	6.2	3.8	9.2	7.4	4.4	7.3	9.9
Indonesia	-4.4	-9.5	-8.4	-10.8	-5.4	-7.2	-10.2	-7.3	-4.7
Japan	2.5	-2.5	-1.5	-3.9	1.5	-0.3	-3.2	-0.4	2.2
Malaysia	-1.6	-6.6	-5.5	-7.9	-2.6	-4.4	-7.3	-4.4	-1.8
Philippines	-4.3	-9.3	-8.3	-10.7	-5.3	-7.1	-10.1	-7.2	-4.6
Singapore	2.7	-2.3	-1.2	-3.6	1.7	-0.1	-3.0	-0.1	2.5
Thailand	-15.2	-20.3	-19.2	-21.6	-16.2	-18.0	-21.0	-18.1	-15.5
Germany	14.1	9.1	10.1	7.7	13.1	11.3	8.3	11.2	13.8
USA	**16.0**	11.0	12.0	9.6	15.0	13.2	10.3	13.1	15.7

cash return risks

$$\begin{aligned}
\Sigma &= \frac{\mathbf{DEV}(\mathbf{S}).\mathbf{DEV}(\mathbf{S})'}{T} \\
&= \frac{\left\{\mathbf{H} \begin{bmatrix} [\mathbf{r}-\mathbf{c}] \\ [\mathbf{c}+\varepsilon] \end{bmatrix} [\mathbf{I}_T - \frac{\iota_T\iota_T'}{T}]\right\}\left\{\mathbf{H} \begin{bmatrix} [\mathbf{r}-\mathbf{c}] \\ [\mathbf{c}+\varepsilon] \end{bmatrix} [\mathbf{I}_T - \frac{\iota_T\iota_T'}{T}]\right\}'}{T} \\
&= \mathbf{H}\Phi\mathbf{H}' \\
&= \boldsymbol{\iota}_n\boldsymbol{\iota}_n' \quad [\Sigma_{rr} + \Sigma_{cc} - \Sigma_{rc} - \Sigma_{rc}'] + [\Sigma_{cc} + \Sigma_{\varepsilon\varepsilon} + \Sigma_{c\varepsilon} + \Sigma_{c\varepsilon}'] \quad \boldsymbol{\iota}_n\boldsymbol{\iota}_n' \\
&\quad + \boldsymbol{\iota}_n \quad [\Sigma_{rc} + \Sigma_{r\varepsilon} - \Sigma_{cc} - \Sigma_{c\varepsilon}] \quad \boldsymbol{\iota}_n' + \boldsymbol{\iota}_n' \quad [\Sigma_{rc} + \Sigma_{r\varepsilon} - \Sigma_{cc} - \Sigma_{c\varepsilon}]' \quad \boldsymbol{\iota}_n
\end{aligned}$$

where Φ is the $2n \times 2n = 18 \times 18$ risk premium - cash return covariance matrix. All the sub-matrices of the strategy risk matrix Σ are given in Table 5, with the Σ_{rr}, Σ_{cc} and Σ_{ee} covariance submatrices along the diagonal of the $(3n \times 3n) = 27 \times 27$ data covariance matrix . Notice that the stock market returns exhibit the highest level of covariance risk, followed by the currencies, while the cash returns show the lowest level of covariance risk. While the Hong Kong dollar is pegged, it clearly is not convertible into the US dollar at a fixed rate. Otherwise the Hong Kong covariances would be also zero, like the ones of the US dollar base currency.

The, for mean-variance analysis relevant, covariance matrix Φ is given in Table 6 for the period July 1992 - December 1997. The Asian risk premia showed five to twenty times more risk than the cash returns, but most of the cash returns still showed very high levels of volatility risk. Cash in Asia has definitely not been risk free. As Table 6 shows, one of the reasons for the cash return risks is the correlation between the risk premia earned in the Asian stock markets and the cash returns earned in the cash lending and foreign exchange markets.

The $n \times n = 9 \times 9$ covariance matrix of the risk premia $[\Sigma_{rr} + \Sigma_{cc} - \Sigma_{rc} - \Sigma_{rc}']$ consists of the covariance matrices of the stock markets Σ_{rr}, of the cash markets Σ_{cc} and of the interaction between the stock and cash markets Σ_{rc}. The matrix Σ_{rr} measures all covariances between the $n = 9$ stock markets. The matrix Σ_{cc} measures all covariances between the $n = 9$ cash markets. Considering that covariance matrices are positive definite, this expression shows that the interaction between

TABLE 5: NUMERICAL DATA COVARIANCE MATRIX Ω (27 x 27)

Data Series: July 1992 - December 1997

	Stock market rates of return: r								
	10590.5	6678.6	1710.8	6808.6	7184.7	4969.6	8599.5	3628.7	1963.7
	6678.6	9969.0	1232.1	6370.4	6842.0	3478.5	8233.8	2907.8	1717.9
	1710.8	1232.1	5588.4	740.4	1253.0	1673.4	1169.6	881.6	759.8
	6808.6	6370.4	740.4	9030.3	7389.4	4171.5	7570.5	2361.5	777.5
r	7184.7	6842.0	1253.0	7389.4	10950.3	4269.5	8890.9	3385.0	917.4
	4969.6	3478.5	1673.4	4171.5	4269.5	4337.1	4900.0	1957.6	985.2
	8599.5	8233.8	1169.6	7570.5	8890.9	4900.0	**16967.0**	4256.7	1633.0
	3628.7	2907.8	881.6	2361.5	3385.0	1957.6	4256.7	3953.9	1136.7
	1963.7	1717.9	759.8	777.5	917.4	985.2	1633.0	1136.7	1256.4
	(72.5)	(64.4)	(32.9)	(79.4)	(64.7)	(50.4)	(91.4)	(11.1)	2.7
	(71.1)	(62.4)	(23.6)	(93.1)	(74.3)	(47.5)	(106.9)	(2.4)	25.7
	2.4	9.6	6.6	24.3	14.1	16.1	42.6	(13.5)	(8.1)
	1.4	(6.0)	(2.9)	(9.8)	(5.9)	(3.5)	(17.4)	12.9	5.9
c	(35.2)	(77.8)	(32.6)	(28.9)	(15.0)	(19.1)	(9.8)	(27.8)	(14.3)
	(29.1)	(42.5)	(10.7)	(35.3)	(27.2)	(21.3)	(54.3)	1.6	(3.0)
	(34.6)	(87.4)	(41.7)	(79.7)	(70.7)	(42.8)	(118.9)	10.3	22.7
	6.1	18.1	19.4	36.8	23.6	27.4	66.9	(21.8)	(12.6)
	(15.0)	(21.6)	(12.0)	(28.5)	(29.6)	(18.5)	(42.6)	3.8	8.7
	20.4	18.8	2.3	5.5	4.6	12.0	25.6	(3.2)	3.1
	899.6	966.0	780.4	772.5	884.8	749.3	879.9	(26.8)	47.3
	(123.7)	58.6	(78.2)	427.6	(73.0)	(65.5)	(797.5)	(672.2)	(152.3)
	158.0	937.0	453.2	469.2	579.5	155.6	437.5	(143.8)	(0.1)
ε	(499.1)	(88.0)	258.2	(357.2)	(348.5)	(80.1)	(966.7)	(804.2)	(379.1)
	263.4	252.5	325.1	230.8	253.6	283.2	128.4	(199.1)	27.8
	799.9	817.3	831.1	962.0	947.7	942.5	(575.1)	(641.9)	(154.2)
	(661.7)	(667.8)	(142.3)	(251.1)	(405.1)	(323.1)	(1196.8)	(1093.0)	(289.6)
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

	Cash rates of return: c								
	(72.5)	(71.1)	2.4	1.4	(35.2)	(29.1)	(34.6)	6.1	(15.0)
	(64.4)	(62.4)	9.6	(6.0)	(77.8)	(42.5)	(87.4)	18.1	(21.6)
	(32.9)	(23.6)	6.6	(2.9)	(32.6)	(10.7)	(41.7)	19.4	(12.0)
	(79.4)	(93.1)	24.3	(9.8)	(28.9)	(35.3)	(79.7)	36.8	(28.5)
r	(64.7)	(74.3)	14.1	(5.9)	(15.0)	(27.2)	(70.7)	23.6	(29.6)
	(50.4)	(47.5)	16.1	(3.5)	(19.1)	(21.3)	(42.8)	27.4	(18.5)
	(91.4)	(106.9)	42.6	(17.4)	(9.8)	(54.3)	(118.9)	66.9	(42.6)
	(11.1)	(2.4)	(13.5)	12.9	(27.8)	1.6	10.3	(21.8)	3.8
	2.7	25.7	(8.1)	5.9	(14.3)	(3.0)	22.7	(12.6)	8.7
	2.8	3.9	(1.4)	0.3	0.4	1.0	3.3	(2.3)	1.4
	3.9	**11.9**	(1.4)	2.2	1.4	1.4	7.5	(1.8)	1.8
	(1.4)	(1.4)	1.5	(0.1)	0.5	(0.5)	(1.4)	2.4	(1.1)
	0.3	2.2	(0.1)	1.3	0.8	0.2	1.5	0.2	(0.2)
c	0.4	1.4	0.5	0.8	5.5	0.4	2.2	1.1	(0.7)
	1.0	1.4	(0.5)	0.2	0.4	1.1	1.7	(1.0)	0.4
	3.3	7.5	(1.4)	1.5	2.2	1.7	8.7	(2.1)	1.5
	(2.3)	(1.8)	2.4	0.2	1.1	(1.0)	(2.1)	4.1	(1.9)
	1.4	1.8	(1.1)	(0.2)	(0.7)	0.4	1.5	(1.9)	1.2
	(0.0)	0.0	(0.0)	(0.0)	(0.1)	(0.2)	(0.3)	0.0	(0.1)
	(34.5)	(66.6)	11.6	(16.0)	(40.5)	(25.5)	(78.2)	19.5	(9.3)
	(14.9)	(26.7)	15.0	(5.8)	6.9	(6.9)	(1.8)	19.3	(9.6)
	(23.6)	(50.2)	7.9	(14.7)	(34.3)	(18.8)	(55.8)	11.5	(4.7)
ε	(15.6)	(47.6)	6.4	(17.2)	(14.6)	(13.9)	(62.6)	11.4	(4.6)
	(12.1)	(24.6)	4.6	(8.7)	(11.0)	(6.9)	(21.9)	5.8	(2.9)
	(37.8)	(58.0)	13.6	(21.0)	(38.6)	(25.4)	(69.0)	21.8	(9.9)
	1.7	(3.1)	3.4	(11.0)	7.4	2.8	2.6	2.3	0.7
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

	FX rates of appreciation: ε								
	20.4	899.6	(123.7)	158.0	(499.1)	263.4	799.9	(661.7)	0.0
	18.8	966.0	58.6	937.0	(88.0)	252.5	817.3	(667.8)	0.0
	2.3	780.4	(78.2)	453.2	258.2	325.1	831.1	(142.3)	0.0
	5.5	772.5	427.6	469.2	(357.2)	230.8	962.0	(251.1)	0.0
r	4.6	884.8	(73.0)	579.5	(348.5)	253.6	947.7	(405.1)	0.0
	12.0	749.3	(65.5)	155.6	(80.1)	283.2	942.5	(323.1)	0.0
	25.6	879.9	(797.5)	437.5	(966.7)	128.4	(575.1)	(1196.8)	0.0
	(3.2)	(26.8)	(672.2)	(143.8)	(804.2)	(199.1)	(641.9)	(1093.0)	0.0
	3.1	47.3	(152.3)	(0.1)	(379.1)	27.8	(154.2)	(289.6)	0.0
	(0.0)	(34.5)	(14.9)	(23.6)	(15.6)	(12.1)	(37.8)	1.7	0.0
	0.0	(66.6)	(26.7)	(50.2)	(47.6)	(24.6)	(58.0)	(3.1)	0.0
	(0.0)	11.6	15.0	7.9	6.4	4.6	13.6	3.4	0.0
	(0.0)	(16.0)	(5.8)	(14.7)	(17.2)	(8.7)	(21.0)	(11.0)	0.0
c	(0.1)	(40.5)	6.9	(34.3)	(14.6)	(11.0)	(38.6)	7.4	0.0
	(0.2)	(25.5)	(6.9)	(18.8)	(13.9)	(6.9)	(25.4)	2.8	0.0
	(0.3)	(78.2)	(1.8)	(55.8)	(62.6)	(21.9)	(69.0)	2.6	0.0
	0.0	19.5	19.3	11.5	11.4	5.8	21.8	2.3	0.0
	(0.1)	(9.3)	(9.6)	(4.7)	(4.6)	(2.9)	(9.9)	0.7	0.0
	1.0	4.7	3.1	1.8	(0.1)	3.6	1.9	13.5	0.0
	4.7	1433.8	180.4	982.9	1057.5	399.5	1423.5	(33.5)	0.0
	3.1	180.4	1513.5	263.8	54.6	264.6	534.6	652.0	0.0
	1.8	982.9	263.8	968.5	678.3	316.1	921.4	68.7	0.0
ε	(0.1)	1057.5	54.6	678.3	1743.3	278.0	1289.7	96.3	0.0
	3.6	399.5	264.6	316.1	278.0	208.2	507.1	204.2	0.0
	1.9	1423.5	534.6	921.4	1289.7	507.1	**2424.0**	326.2	0.0
	13.5	(33.5)	652.0	68.7	96.3	204.2	326.2	1206.4	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

the stock and cash markets, as measured by Σ_{rc}, reduces the overall market risk $\Sigma_{rr} + \Sigma_{cc}$ emanating from both types of markets.

Notice from both Tables 5 and 6 that (1) *most of the risk resides in the Asian stock markets* and that the German and, in particular,US stock markets show much less risk, and (2) the stock and cash *interaction risk* matrix, Σ_{rc}, is asymmetric! The most volatile stock markets were Thailand, the Philippines and Indonesia.

The $n \times n = 9 \times 9$ covariance matrix of the cash earnings $[\Sigma_{cc} + \Sigma_{\varepsilon\varepsilon} + \Sigma_{c\varepsilon} + \Sigma'_{c\varepsilon}]$ contains the covariance matrices of the cash markets Σ_{cc}, of the foreign currency markets $\Sigma_{\varepsilon\varepsilon}$ and of the interaction between the cash and foreign currency markets. This expression shows that the interaction between the cash and foreign currency markets as measured by $\Sigma_{c\varepsilon}$ increases the overall market risk $\Sigma_{cc} + \Sigma_{\varepsilon\varepsilon}$ emanating from both types of markets.

According to the numbers in Table 5 *the currency markets exhibited more risk than the local cash markets.* In the period July 1992 through December 1997, the Thai baht was the most volatile currency, followed by the Philippine peso, the Japanese Yen and the Indonesian rupiah. The volatility of the Japanese Yen is, perhaps, the most surprising observation, coming from a mature industrialized country. But the period July 1992 - December 1997 covers Japan s painful and

TABLE 6: RISK PREMIUM - CASH RETURN COVARIANCE MATRIX Φ (18 x 18)
July 1992 - December 1997

	r-c									**c+ε**								
	10738.2	6818.0	1740.0	6886.8	7285.1	5050.1	8728.8	3631.4	1977.5	(54.8)	859.1	(105.1)	182.8	(519.1)	245.4	799.7	(655.0)	(16.5)
	6818.0	10105.7	1244.8	6471.7	6995.5	3569.9	8435.6	2890.4	1715.7	(49.5)	958.3	96.3	979.0	(119.6)	233.2	780.4	(644.8)	(23.5)
	1740.0	1244.8	5576.7	718.9	1272.1	1667.5	1167.3	878.0	778.8	(29.3)	746.5	(88.1)	442.5	218.7	310.4	777.1	(128.7)	(10.9)
	6886.8	6471.7	718.9	9051.2	7425.1	4210.5	7669.1	2311.9	800.0	(74.2)	693.1	457.9	472.8	(369.8)	204.0	901.7	(203.4)	(28.3)
r-c	7285.1	6995.5	1272.1	7425.1	10985.8	4316.2	8973.6	3390.3	960.6	(60.4)	849.6	(66.4)	607.1	(354.4)	237.0	913.4	(390.0)	(28.9)
	5050.1	3569.9	1667.5	4210.5	4316.2	4380.7	4998.8	1927.6	1007.1	(39.3)	725.9	(42.1)	170.7	(85.7)	267.7	923.4	(297.5)	(18.9)
	8728.8	8435.6	1167.3	7669.1	8973.6	4998.8	**17213.6**	4177.3	1654.4	(68.8)	843.7	(751.7)	474.4	(916.1)	94.2	(633.8)	(1130.2)	(44.1)
	3631.4	2890.4	878.0	2311.9	3390.3	1927.6	4177.3	4001.5	1143.7	(12.1)	(46.9)	(707.4)	(142.5)	(844.4)	(202.4)	(651.3)	(1121.1)	5.7
	1977.5	1715.7	778.8	800.0	960.6	1007.1	1654.4	1143.7	1240.3	4.5	80.4	(149.8)	10.7	(388.2)	27.3	(123.1)	(301.1)	7.4
	(54.8)	(49.5)	(29.3)	(74.2)	(60.4)	(39.3)	(68.8)	(12.1)	4.5	3.7	(25.9)	(13.2)	(21.5)	(15.4)	(7.7)	(32.8)	12.9	1.3
	859.1	958.3	746.5	693.1	849.6	725.9	843.7	(46.9)	80.4	(25.9)	1312.5	164.0	918.9	970.8	350.8	1294.9	(18.9)	(7.5)
	(105.1)	96.3	(88.1)	457.9	(66.4)	(42.1)	(751.7)	(707.4)	(149.8)	(13.2)	164.0	1544.9	265.8	68.4	261.8	545.1	677.1	(10.7)
	182.8	979.0	442.5	472.8	607.1	170.7	474.4	(142.5)	10.7	(21.5)	918.9	265.8	940.3	627.7	288.8	846.1	69.4	(4.9)
c+ε	(519.1)	(119.6)	218.7	(369.8)	(354.4)	(85.7)	(916.1)	(844.4)	(388.2)	(15.4)	970.8	68.4	627.7	1719.6	253.5	1190.7	116.2	(5.2)
	245.4	233.2	310.4	204.0	237.0	267.7	94.2	(202.4)	27.3	(7.7)	350.8	261.8	288.8	253.5	195.4	461.5	211.8	(2.5)
	799.7	780.4	777.1	901.7	913.4	923.4	(633.8)	(651.3)	(123.1)	(32.8)	1294.9	545.1	846.1	1190.7	461.5	**2294.8**	348.4	(8.4)
	(655.0)	(644.8)	(128.7)	(203.4)	(390.0)	(297.5)	(1130.2)	(1121.1)	(301.1)	12.9	(18.9)	677.1	69.4	116.2	211.8	348.4	1215.0	(1.1)
	(16.5)	(23.5)	(10.9)	(28.3)	(28.9)	(18.9)	(44.1)	5.7	7.4	1.3	(7.5)	(10.7)	(4.9)	(5.2)	(2.5)	(8.4)	(1.1)	1.2

Determinant = 4.4E+49 **Product of Variances =** 1.8E+56 **18D Noise/Data Ratio =** 2.4E-07 **18D Signal/Noise Ratio =** 4.2E+06

too long adjustment period, which has led in global investors circles to a lot of disillusionment with Japan.

Third, the two cross terms involving the risk premia and the cash earnings contain $[\Sigma_{rc} + \Sigma_{r\varepsilon} - \Sigma_{cc} - \Sigma_{c\varepsilon}]$. In both cases the cross risk, emanating from the interaction between the stock and the cash and the stock and foreign currency markets, $\Sigma_{rc} + \Sigma_{r\varepsilon}$, is reduced by the interaction between the cash markets, Σ_{cc}. and between the cash and the foreign currency markets, $\Sigma_{c\varepsilon}$.

Clearly, the risk of the investment strategies, as measured by the strategy risk matrix Σ, is a convoluted expression of the risks of the stock, cash and foreign currency markets and their inter-correlations. It makes sense to compute this risk matrix Σ and analyze it for a fixed number of observations T only when each of its six constituent market risk matrices - Σ_{rr}, Σ_{rc}, Σ_{cc}, $\Sigma_{c\varepsilon}$, $\Sigma_{\varepsilon\varepsilon}$, $\Sigma_{r\varepsilon}$ - is constant, i.e., when the financial market pricing processes are *stationary.* When such stationarity prevails, mean-variance portfolio optimization as in the next Section could be valuable. However , if such direct stationarity does not prevail in the financial markets, a complete projective search should be conducted to determine what systematic forces change these financial market pricing processes.[5] More discussion of this

[5]Stationarity tests are insufficiently applied to this kind of research. All too often stationarity is presumed and not checked for in financial analysis and the design of trading strategies.

issue of the (lack of) risk stationarity in Asia, and therefore of the difficulty of the risk assessment in Asia, follows in Section 4.

3. MEAN-VARIANCE OPTIMIZATION

For strategic global portfolio management the fundamental question is: are there combinations of investment strategies that either lead to lower overall risk for a comparable level of average return, or, *vice versa*, to a higher return for a comparable level of risk?[3] Markowitz conventional mean-variance optimization of portfolios, which answers this question, requires that the central risk matrix is positive de - nite, i.e., nonsingular.[12][13][6] But the exact strategy risk matrix Σ is singular. and we have to adapt Markowitz procedure, not by fudging or avoiding, but by using the exact accounting identities.

3.1. Singularity of Strategy Risk Matrix.

It is easy to proof that *the strategy risk matrix is singular* and this fact has stymied a lot of portfolio managers. It is singular, since

$$rank(\Sigma) = 2n - 1 < n^2, \text{ for integer } n > 1$$

This is proved by determining the rank of the strategy deviations $\mathbf{DEV}(\mathbf{S})$, which is the same as the rank of the strategy risk matrix Σ.[7]

$$\begin{aligned} rank\left\{\mathbf{DEV}(\mathbf{S})\right\} &= rank\left\{\mathbf{H}\begin{bmatrix}[\mathbf{r}-\mathbf{c}]\\ [\mathbf{c}+\boldsymbol{\varepsilon}]\end{bmatrix}[\mathbf{I}_T - \frac{\boldsymbol{\iota}_T\boldsymbol{\iota}_T'}{T}]\right\} \\ &\leq Min\left\{rank\left(\mathbf{H}\right), rank\begin{bmatrix}[\mathbf{r}-\mathbf{c}]\\ [\mathbf{c}+\boldsymbol{\varepsilon}]\end{bmatrix}, rank\left[\mathbf{I}_T - \frac{\boldsymbol{\iota}_T\boldsymbol{\iota}_T'}{T}\right]\right\} \\ &= Min\left\{2n-1, 2n, T-1\right\} = 2n-1 < n^2 \end{aligned}$$

since $\mathbf{H}$ is the $n^2 \times 2n$ selector matrix of rank $2n-1$, $\begin{bmatrix}[\mathbf{r}-\mathbf{c}]\\ [\mathbf{c}+\boldsymbol{\varepsilon}]\end{bmatrix}$ is a risk-premium and cash return $2n \times T$ matrix of rank $2n$, and $\left[\mathbf{I}_T - \frac{\boldsymbol{\iota}_T\boldsymbol{\iota}_T'}{T}\right]$ is the deviations producing $T \times T$ matrix of rank $T-1$, where it is required that $T > 2n$. This result can be illustrated by computing the rank of the selector matrix $\mathbf{H}$ for our nine countries. Since $\mathbf{H}$ consists of zeros and ones only, this is easily done.[11] For our empirical example

$$rank(\mathbf{H}) = 2n - 1 = 17 < 81 = n^2$$

Remark. It should be emphasized that when one would numerically compute the rank of the strategy risk matrix in Table 5, the numerically computed result would deceptively show a *full* $rank(\Sigma) = 81$, because of the imprecision of the numerical computation caused by the limited registers of a computer. Thus the strategy risk matrix *appears to be of full rank*.[8] However, the exact algebraic $rank(\Sigma) = 17 < 81$

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[6] One may question the relevance of symmetric mean-variance optimization, since there are observable asymmetries in the regional risk distributions. An asymmetrical optimization would require the computation of additional, higher order moments than the first and second moments used here and the procedure would quickly become very complex, without elucidating the issue of combining portfolio optimization with complete and exact attribution.

[7] Cf. Proposition 7 of [4], p. 437.

[8] As is, indeed, the case in RiskMetrics^{TM}, as Professor Mark Garman confirmed when he revealed the instability and unreliability of RiskMetrics^{TM} s information matrix Σ^{-1}in a research seminar on March 7, 1996 at the Nanyang Technological University and was again confirmed by e-mail on April 23, 1998 by Peter Zangari of J.P. Morgan

and thus the strategy risk matrix *must logically be singular.* The imprecision of the computed result is expressed by the determinant of Σ as the product of the resulting eigenvalues $|\Sigma| = \prod_{i=1}^{n^2} \lambda_i$. Although in our example $81 - 17 = 64$ eigenvalues are exactly zero, and consequently the determinant $|\Sigma| = 0$, the (finite) computer shows these 64 eigenvalues to be very close, but unequal, to zero at any available level of numerical computing precision of the data covariance matrix .

3.2. **Extended Markowitz Procedure.** Let s now see how we can exploit the accounting identities to get around the problem of the singular strategy risk matrix. The *mean portfolio rate of return* $\overline{s}_p$ for the holding period T is the allocated linear combination of the average strategy rates of return of the strategies, $\overline{\mathbf{VEC}(\mathbf{S})}$, contained in the portfolio

$$\overline{s}_p = \mathbf{w}'\mathbf{VEC}(\mathbf{S}) = \mathbf{w}'\mathbf{H} \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\boldsymbol{\varepsilon}}] \end{bmatrix} = \mathbf{v}' \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\boldsymbol{\varepsilon}}] \end{bmatrix}$$

; $\mathbf{w}$ is a $n^2 \times 1 = 81 \times 1$ vector of *portfolio allocations*, such that the sum of the allocations equals unity, $\mathbf{w}'\boldsymbol{\iota}_{n^2} = 1$, where $\boldsymbol{\iota}_{n^2}$ is the $n^2 \times 1 = 81 \times 1$ unit vector; $\mathbf{v}$ is the $2n \times 1 = 18 \times 1$ vector of the combined portfolio allocations $\mathbf{v} = \mathbf{H}'\mathbf{w}$, such that $\mathbf{v}'\boldsymbol{\iota}_{2n} = 2$, where $\boldsymbol{\iota}_{2n}$ is the $2n \times 1 = 18 \times 1$ unit vector. The overall investment strategy *portfolio risk*, σ_{pp}, is the variance of the portfolio rates of return,

$$\sigma_{pp} = \mathbf{w}'\Sigma\mathbf{w} = \mathbf{w}'\mathbf{H}\Phi\mathbf{H}'\mathbf{w} = \mathbf{v}'\Phi\mathbf{v}$$

Notice that the first $n = 9$ combined allocations $v_{i.}, i = 1, 2, ...9$, refer to the strategy choice of how much of the capital to invest in which stock market to earn a risk premium, while the second $n = 9$ allocations $v_{.j}, j = 1, 2, ..., 9$, refer to the strategy choice of how much of the capital to invest in which currency to earn a cash return. The allocations $\mathbf{v}$ exhaust the capital allocation based on these two fundamental choices of investments in stock markets and in currencies, because of the accounting identities,

$$\begin{aligned} \mathbf{v}' &= \mathbf{v}' \left[\begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} \right] \text{ with} \\ \mathbf{v}' \begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} &= \mathbf{w}'\mathbf{H} \begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} = \mathbf{w}'\boldsymbol{\iota}_{n^2} = 1 \text{ and} \\ \mathbf{v}' \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} &= \mathbf{w}'\mathbf{H} \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} = \mathbf{w}'\boldsymbol{\iota}_{n^2} = 1, \end{aligned}$$

Now the procedure has once again become similar to Markowitz nonsingular case, which we solve using the familiar Kuhn-Tucker Theorem for constraint optimization.[7] First, form the Lagrangian with the three accounting constraints:

$$L(\mathbf{v}, \lambda_1, \lambda_2, \lambda_3) = \mathbf{v}'\Phi\mathbf{v} + \lambda_1 \left[1 - \mathbf{v}' \begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} \right] + \lambda_2 \left[1 - \mathbf{v}' \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} \right] + \lambda_3 \left[\overline{s}_p - \mathbf{v}' \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\boldsymbol{\varepsilon}}] \end{bmatrix} \right]$$

Next, to find the optimum of this Lagrangian, set the $2n+3$ partial first derivatives equal to zero, i.e., the derivatives with respect to the $2n$ elements of the allocation vector $\mathbf{v}$ and to the two Lagrangian multipliers λ_1, λ_2 and λ_3.

In [11] it is found that the optimal Lagrangian multipliers λ_1^{opt}, λ_2^{opt} and λ_3^{opt} are given by

$$\begin{bmatrix} \lambda_1^{opt} \\ \lambda_2^{opt} \\ \lambda_3^{opt} \end{bmatrix} = 2.\Delta^{-1}. \begin{bmatrix} 1 \\ 1 \\ \overline{s}_p \end{bmatrix}$$

where the 3×3 symmetric and positive de nite matrix Δ is such that

$$\Delta = \begin{bmatrix} \begin{bmatrix} \boldsymbol{\iota}_n' & \mathbf{0} \end{bmatrix} \Phi^{-1} \begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} & \begin{bmatrix} \mathbf{0} & \boldsymbol{\iota}_n' \end{bmatrix} \Phi^{-1} \begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} & \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}]' & [\overline{\mathbf{c}} + \overline{\varepsilon}]' \end{bmatrix} \Phi^{-1} \begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} \\ \begin{bmatrix} \boldsymbol{\iota}_n' & \mathbf{0} \end{bmatrix} \Phi^{-1} \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} & \begin{bmatrix} \mathbf{0} & \boldsymbol{\iota}_n' \end{bmatrix} \Phi^{-1} \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} & \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}]' & [\overline{\mathbf{c}} + \overline{\varepsilon}]' \end{bmatrix} \Phi^{-1} \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} \\ \begin{bmatrix} \boldsymbol{\iota}_n' & \mathbf{0} \end{bmatrix} \Phi^{-1} \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\varepsilon}] \end{bmatrix} & \begin{bmatrix} \mathbf{0} & \boldsymbol{\iota}_n' \end{bmatrix} \Phi^{-1} \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\varepsilon}] \end{bmatrix} & \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}]' & [\overline{\mathbf{c}} + \overline{\varepsilon}]' \end{bmatrix} \Phi^{-1} \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\varepsilon}] \end{bmatrix} \end{bmatrix}$$

So that by substitution, the optimal fundamental choice allocations are determined to be

$$\mathbf{v}^{opt} = \frac{\Phi^{-1} \left[\lambda_1^{opt} \begin{bmatrix} \boldsymbol{\iota}_n \\ \mathbf{0} \end{bmatrix} + \lambda_2^{opt} \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\iota}_n \end{bmatrix} + \lambda_3^{opt} \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\varepsilon}] \end{bmatrix} \right]}{2}$$

Remark. The $(2n + 3) \times (2n + 3)$ matrix of partial second derivatives is positive definite, since the full rank $2n \times 2n$ covariance matrix $\Phi > 0$, so that the optimum is, indeed, a - constrained - *minimum.*

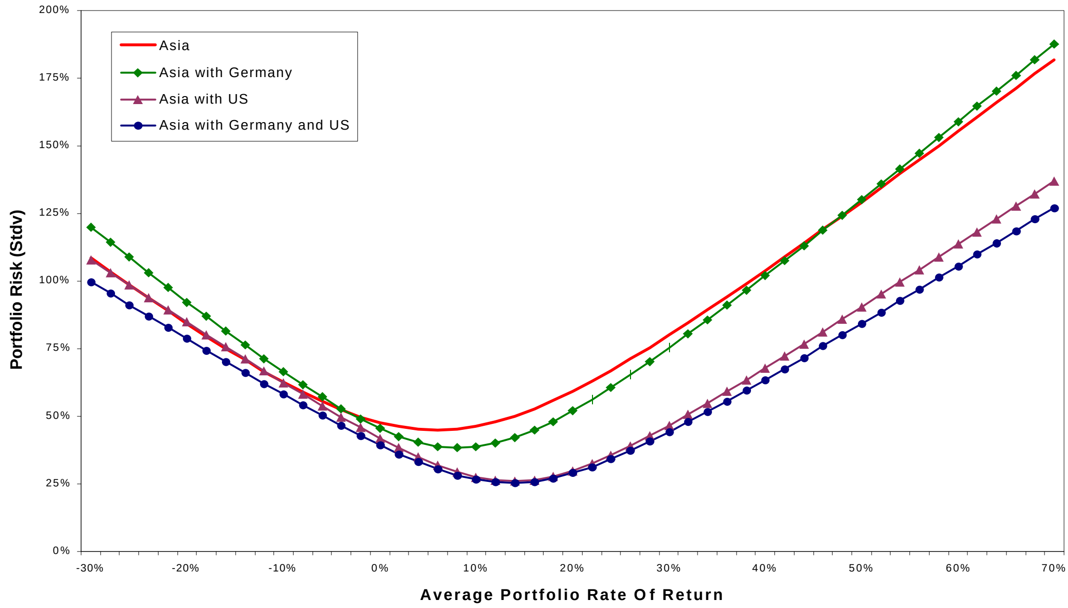
We have now the two equations of Markowitz Efficient Portfolio Frontier for $2n$ strategy investment choices, which can be plotted in a two-dimensional graph. For every portfolio strategy return $\overline{s}_p^{opt}$ there is a corresponding portfolio strategy risk σ_p^{opt}:

$$\begin{aligned} \overline{s}_p^{opt} &= \mathbf{v}^{opt\prime} \begin{bmatrix} [\overline{\mathbf{r}} - \overline{\mathbf{c}}] \\ [\overline{\mathbf{c}} + \overline{\varepsilon}] \end{bmatrix} \\ \sigma_p^{opt} &= \sqrt{(\mathbf{v}^{opt})\Phi' \mathbf{v}^{opt}} \end{aligned}$$

Figure 1. provides the efficiency frontier of the strategic multi-currency investment portfolios with exact risk attribution for the period July 1992 - December 1997 for all $n = 9$ countries, given by the bottom parabola. We compare this with the parabolic efficiency frontiers of the 7 Asian countries by themselves, given by the top parabola, and with the parabolic efficiency frontiers, when either Germany or the US alone is included in the Asian portfolio. The optimal portfolio risk σ_p^{opt} is measured on the vertical axis, while the average portfolio return is measured on the horizontal axis.

3.3. **Retrieval of the Optimal Strategy Allocations.** The original portfolio investment strategy portfolio allocations $\mathbf{w}$ can be *uniquely* retrieved from the fundamental market choice portfolio allocations $\mathbf{v}$, via the two accounting identities imposed by the geometry of the original strategy matrix $\mathbf{S}(t)$, for example, when

FIGURE 1: Efficient Portfolio Frontiers of Strategic Multi-Currency Investments in Asia, July 1992 - December 1997



$n = 3$,

$$\begin{bmatrix} \mathbf{I}_n & \mathbf{0} \end{bmatrix} \mathbf{v}\mathbf{v}' \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_n \end{bmatrix} = \begin{bmatrix} v_1v_4 & v_1v_5 & v_1v_6 \\ v_2v_4 & v_2v_5 & v_2v_6 \\ v_3v_4 & v_3v_5 & v_3v_6 \end{bmatrix} =$$

$$\begin{bmatrix} \mathbf{I}_n & \mathbf{0} \end{bmatrix} \mathbf{H}'\mathbf{w}\mathbf{w}'\mathbf{H} \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_n \end{bmatrix} = \mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix}$$

Thus the investment strategy portfolio allocations $\mathbf{w}$ are simply the *products* of the fundamental market choice portfolio allocations. Table 7 provides the optimal portfolio allocations for all nine countries combined based on the total investment of a five and a half year investment horizon (July 1992 - December 1997) for the General Minimum-Variance (GMV) allocation, i.e., for the absolute minimum risk point of the lowest parabolic efficiency frontier in Figure 1, where the average optimal portfolio return is $\overline{s}_p^{opt} = 13.7\%/\text{yr}$ and the average portfolio risk $\sigma_p^{opt}(GMV) = 25.3\%$ within one year.

At the left side we find the column of risk premium weights $v_{i.}$, $i = 1, 2, ..., 9$ and in the top row the cash return weights $v_{.j}$, $j = 1, 2, ..., 9$. In the surrounded rectangular we find the matrix $\mathbf{W}$ of investment strategy weights, which corresponds with the $n\times\, n = 9\times 9$ investment strategy matrix $\mathbf{S}(t)$. The $\mathbf{W}$ matrix contains the

TABLE 7: **OPTIMAL GMV STRATEGIC ALLOCATIONS FOR ASIAN COUNTRIES, TOGETHER WITH USA AND GERMANY**
July 1992 - December 1997

Average portfolio return = 13.7%
Average portfolio risk (stdev) = 25.3%

			Hong Kong	Indonesia	Japan	Malaysia	Philippines	Singapore	Thailand	Germany	USA	
	cash return weights		$v_{.1}$	$v_{.2}$	$v_{.3}$	$v_{.4}$	$v_{.5}$	$v_{.6}$	$v_{.7}$	$v_{.8}$	$v_{.9}$	
	risk premium weights		**110.5%**	**-48.7%**	**11.4%**	**41.4%**	**41.6%**	**-99.2%**	**11.8%**	**25.4%**	**5.9%**	**100.0%**
Hong Kong	$v_{1.}$	**-13.7%**	-15.2%	6.7%	-1.6%	-5.7%	-5.7%	13.6%	-1.6%	-3.5%	-0.8%	**-13.7%**
Indonesia	$v_{2.}$	**-14.1%**	-15.6%	6.9%	-1.6%	-5.8%	-5.9%	14.0%	-1.7%	-3.6%	-0.8%	**-14.1%**
Japan	$v_{3.}$	**6.2%**	6.9%	-3.0%	0.7%	2.6%	2.6%	-6.2%	0.7%	1.6%	0.4%	**6.2%**
Malaysia	$v_{4.}$	**11.4%**	12.6%	-5.5%	1.3%	4.7%	4.7%	-11.3%	1.3%	2.9%	0.7%	**11.4%**
Philippines	$v_{5.}$	**2.7%**	3.0%	-1.3%	0.3%	1.1%	1.1%	-2.7%	0.3%	0.7%	0.2%	**2.7%**
Singapore	$v_{6.}$	**11.6%**	12.8%	-5.6%	1.3%	4.8%	4.8%	-11.5%	1.4%	2.9%	0.7%	**11.6%**
Thailand	$v_{7.}$	**3.7%**	4.1%	-1.8%	0.4%	1.5%	1.5%	-3.7%	0.4%	0.9%	0.2%	**3.7%**
Germany	$v_{8.}$	**11.9%**	13.2%	-5.8%	1.4%	4.9%	5.0%	-11.8%	1.4%	3.0%	0.7%	**11.9%**
USA	$v_{9.}$	**80.3%**	88.7%	-39.1%	9.1%	33.2%	33.4%	-79.7%	9.4%	20.4%	4.7%	**80.3%**
		100.0%	**110.5%**	**-48.7%**	**11.4%**	**41.4%**	**41.6%**	**-99.2%**	**11.8%**	**25.4%**	**5.9%**	**200.0%**

recipe for optimal portfolio allocations for strategic multi-currency investments and can be computed for each point along the portfolio efficiency frontier.[9]

Notice that in Table 7 the exact accounting identities are maintained. Positive numbers for elements of **v** indicate buying or taking long positions; negative numbers indicate short selling or unwinding of long positions. [10] Table 7 shows that to achieve the GMV return and risk combination in the period , 80.3% of the initial capital $P(0)$ should have been invested in the US stock market, with 11.9% in Germany, 11.6% in Singapore, 11.4% in Malaysia, and a smattering in Japan, Thailand, and the Philippines, with extensive short positions in the stock markets of 14.1% in Indonesia and 13.7% in Hong Kong. Such positions could have been achieved by buying and selling futures on the stock markets (if and when they had existed!).

The net step would have been to arrange cash swaps so that 110.5% of the capital value would have been swapped into the Hong Kong dollar and 41.4% and 41.6% would have been swapped into the Yen and the Ringgit, respectively, and 25.4% into the Deutschemark. The short currency positions would be 99.2% of the capital value against the Singapore dollar, 48.7% against the Rupiah, while only 5.9% would remain in the US dollar. This would, of course, be the accumulated swap positions *resulting* from the 81 strategic investment allocations![11]

The rationale for such complex allocations is found in the covariance risk matrices of Tables 5 and 6. Such complex investment strategy allocations reduce the overall portfolio risk. Notice though, that, in each individual market, sometimes considerable gross leveraging occurs for some strategies, with allocations > 1 in absolute value, while on a net basis such leveraging cancels out.

[9]Which optimal combination of portfolio return and risk is acceptable, depends on the risk aversion characteristics of the portfolio manager and his investment clients.

[10]Until recently, Malaysia, like many Asian countries, didn t allow shorting of stocks, thereby introducing an unnecessary market inefficiency. This inefficiency was removed in October 1997.

[11]Anybody who thinks that the taking of 81 strategic investment allocations for one portfolio is excessive has not recently been on the trading floor of a modern advanced investment firm, where much higher dimensional strategy matrices can be observed on the screens of the traders.

On average, the portfolio approach to strategy investment does reduce portfolio risk. But the Asian risk levels remain high. The GMV risk is two times larger than the average return! In [11] we found for the three country portfolio of Indonesia, Singapore and Malaysia that the risk level was six times higher than the return. Thus diversification considerably reduces the overall GMV risk, by including more international stock markets and currencies. The fundamental reason is that all three Asian stock markets exhibited very high levels of risk in the five and a half year period. Because the stock and currency markets are interrelated via the capital flows, this stock market risk spills over into the currency markets.

From Figure 1 it is also clear that any investor restricting his investment strategies only to Asia would not have been able to avoid the overall high level of GMV risk of close to 50% and a return of only 2.5%. By diversifying to the stable and regularly high earning US stock market the GMV risk could have been halved and the return increased by five-fold. Only by more extensive globalization of the strategic Asian investment portfolios could the overall portfolio risk level of Asian investors have been reduced to more acceptable levels. This can be achieved by moving more capital to the more mature and stable markets of North America and Europe, which exhibited lower overall risk levels over the same time period.

4. NONSTATIONARITY OF ASIAN INVESTMENT RETURNS

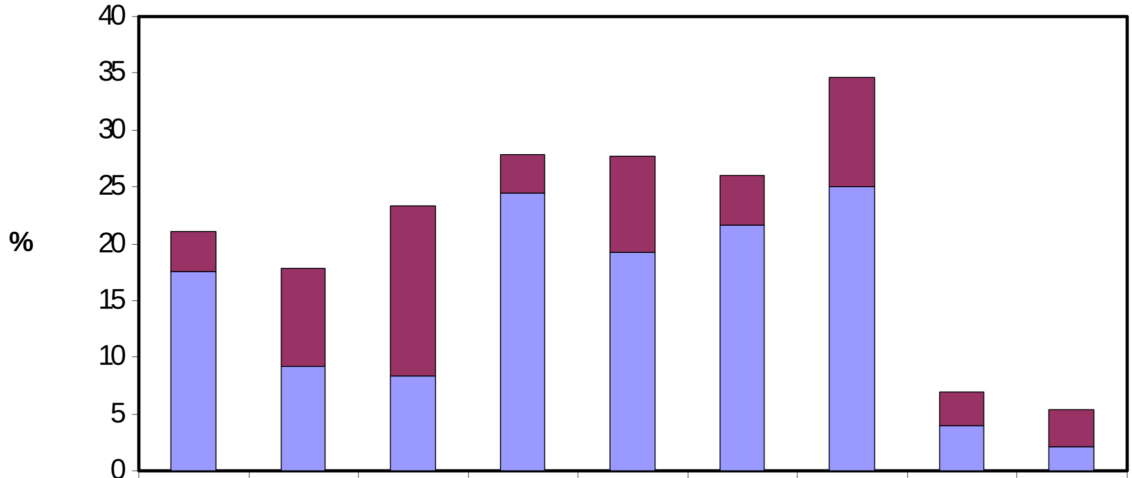
Conventionally, Markowitz mean-variance analysis assumes that the return data series are stationary in their first and second moments, i.e., that the average return rates and the covariance matrices don t change significantly over time. Since recently the Asian countries underwent significant economic and financial upheavals, it is a relevant research question to analyze if this assumption holds true.

In this section we analyze the changes in the average rates of return, changes in the general level of portfolio risk, as well as possible structural changes in the Asian investment risk profile in the period July 1992 - December 1997. We do this by windowing the empirical data set. Because of the minimal mathematical requirements for our mean-variance cum exact attribution analysis, $T > 2n = 18$, the complete data set of 66 monthly observations was, somewhat arbitrarily, divided into three equally sized windows, or data subsets, of each 22 observations. These three subsets cover the periods July 1992 - April 1994, May 1994 - February 1996 and March 1996 - December 1997.

4.1. **Level of Average Asian Strategic Investment Returns Over Time.** It appears that the level of the Asian strategic investment returns went through major changes over time and was far from stationary.

4.1.1. *July 1992 - April 1994.* Chart 1 shows that in this period all of the cash returns and risk premia were positive. They ranged from substantial to large in Asia, but were very small in Germany and the USA. This period was dominated by very large risk premia of $20 - 25\%$/yr in the stock markets in Thailand, Malaysia, the Philippines and Singapore, closely followed by 17.5% in Hong Kong. The cash returns in Indonesia and Japan were slightly less than half that size and in Germany and the USA five to ten times smaller. It is clear that in this period fortunes were made in the stock markets of Asia, while Germany (Europe) and the USA were much less attractive places for equity investments.

CHART 1: Risk Premia and Cash Returns, July 1992 - April 1994



	Hong Kong	Indonesia	Japan	Malaysia	Philippines	Singapore	Thailand	Germany	USA
Cash Return	3.5	8.7	14.9	3.4	8.6	4.5	9.6	3.0	3.4
Risk Premium	17.5	9.1	8.4	24.4	19.2	21.6	25.0	3.9	2.1

Japan exhibited a very substantial average cash return of 14.9%/yr, followed by Thailand with 9.6%/yr and the Philippines with 8.6%/yr. Thus, under the assumption of unchanging FX regimes, a strategic multi-currency investor could reap a whopping 39.9%/yr by investing in the stock market in Thailand, earning a 25% risk premium, and swapping into Japanese Yen for an extra 14.9% cash return, as is shown in Table 8.

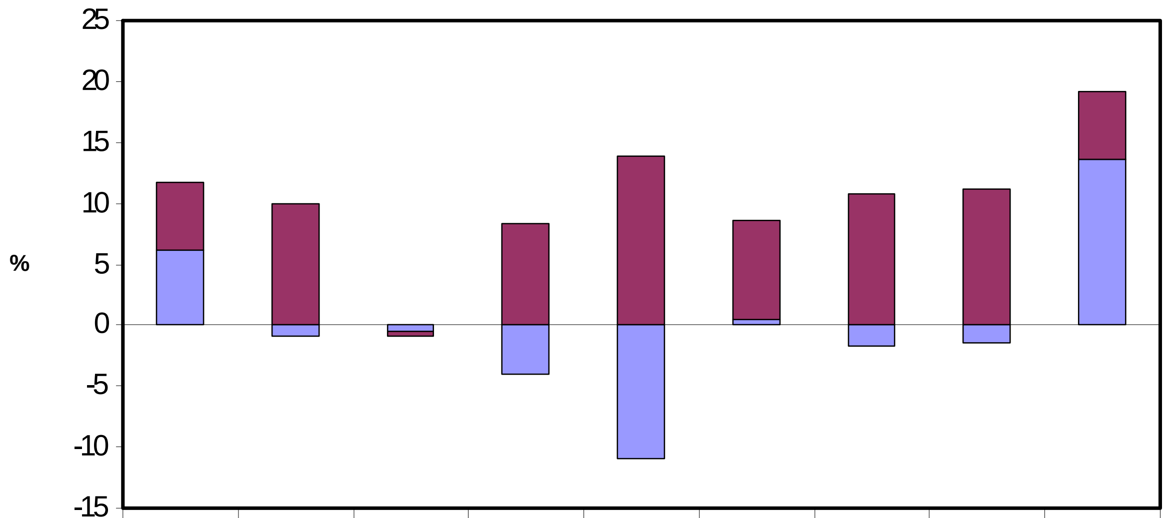
An equally profitable maximizing return strategy would have been to invest in the Malaysian stock market for the 24.4%/yr average risk premium, and then to swap into Yen, for a total average return of 39.3%/yr. The USA and Germany could never reach these return levels, even not with the currency overlays. Investing in the German stock market and swapping into Yen would have earned 18.9%/yr, a respectable return, but less than half of what was achievable in Asia. Since capital was attracted to Asia by its stock markets, cash returns could overall stay relatively low in Asia

4.1.2. *May 1994 - February 1996.* Less than two years later the situation had already drastically changed. Chart 2 shows that all the stock markets in Asia delivered negative, or close to zero, risk premia, with the exception of Hong Kong. The stock market in the Philippines was losing on average 10.9%/yr. In contrast, the stock market in the USA was already showing a very healthy rebound en produced average risk premia of 13.5%/yr.

TABLE 8: AVERAGE STRATEGIC INVESTMENT RETURNS (%)
July 1992 - April 1994

	CURRENCIES								
MARKETS	Hong Kong	Indonesia	Japan	Malaysia	Philippines	Singapore	Thailand	Germany	USA
Hong Kong	21.0	26.2	32.4	20.9	26.1	22.0	27.1	20.5	20.8
Indonesia	12.6	17.8	24.0	12.5	17.7	13.6	18.7	12.1	12.5
Japan	11.9	17.1	23.3	11.8	17.0	12.9	18.0	11.3	11.7
Malaysia	27.9	33.1	39.3	27.8	33.0	28.9	34.0	27.4	27.8
Philippines	22.7	27.8	34.1	22.5	27.8	23.7	28.8	22.1	22.5
Singapore	25.1	30.2	36.5	24.9	30.2	26.1	31.2	24.5	24.9
Thailand	28.5	33.7	**39.9**	28.4	33.6	29.5	34.6	28.0	28.4
Germany	7.5	12.6	18.9	7.3	12.5	8.4	13.5	6.9	7.3
USA	5.6	10.7	17.0	5.5	10.7	6.6	11.7	5.0	5.4

CHART 2: Risk Premia and Cash Returns, May 1994 - February 1996



	Hong Kong	Indonesia	Japan	Malaysia	Philippines	Singapore	Thailand	Germany	USA
Cash Return	5.5	10.0	-0.4	8.3	13.8	8.1	10.8	11.2	5.6
Risk Premium	6.2	-0.9	-0.5	-4.0	-10.9	0.5	-1.7	-1.4	13.5

Since capital was now induced to return to the USA, the Asian countries had to raise their cash lending rates considerably. This made it attractive for money managers to swap into Asian bonds and to lend directly to governments and the private sector alike. The highest cash returns in Asia could be earned in the Philippines, Thailand and Indonesia, 13.8%, 10.8% and 10.0% per year, respectively. At

TABLE 9: AVERAGE STRATEGIC INVESTMENT RETURNS (%)
May 1994 - February 1996

	CURRENCIES								
MARKETS	**Hong Kong**	**Indonesia**	**Japan**	**Malaysia**	**Philippines**	**Singapore**	**Thailand**	**Germany**	**USA**
Hong Kong	11.7	16.2	5.8	14.5	20.0	14.3	17.0	17.4	11.8
Indonesia	4.7	9.1	-1.3	7.4	13.0	7.3	9.9	10.3	4.7
Japan	5.1	9.5	-0.9	7.8	13.4	7.6	10.3	10.7	5.1
Malaysia	1.5	5.9	-4.5	4.2	9.8	4.1	6.8	7.2	1.6
Philippines	-5.4	-0.9	-11.3	-2.6	2.9	-2.8	-0.1	0.3	-5.3
Singapore	6.0	10.4	0.0	8.7	14.3	8.6	11.3	11.7	6.1
Thailand	3.8	8.2	-2.2	6.5	12.1	6.4	9.1	9.5	3.8
Germany	4.1	8.6	-1.8	6.9	12.4	6.7	9.4	9.8	4.2
USA	19.1	23.5	13.1	21.8	**27.4**	21.7	24.3	24.8	19.1

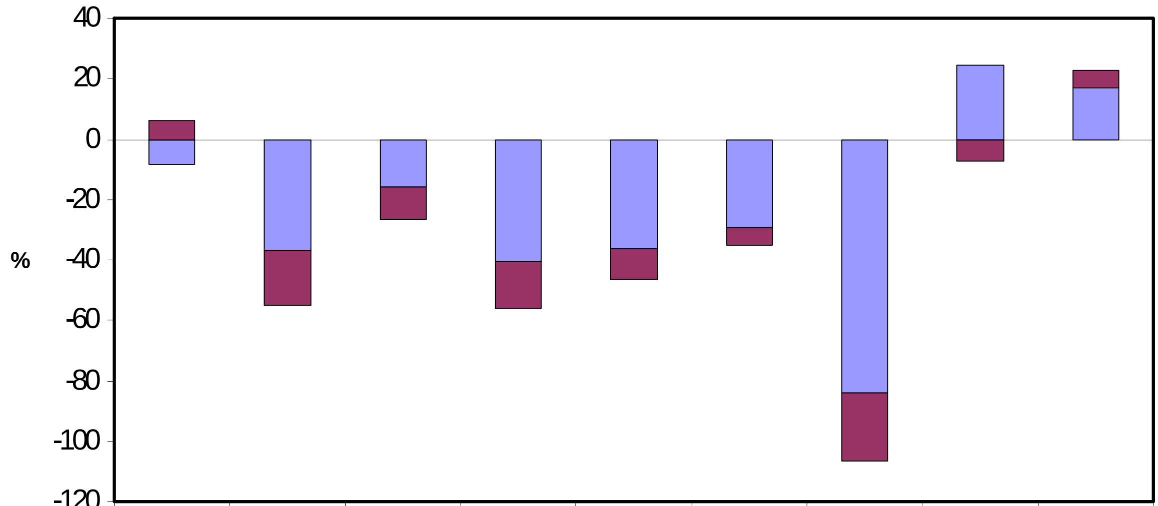
that time Germany was stepping on the inflationary brakes under the requirement of the Maastricht Treaty to prepare Europe for the Monetary Union. The German cash returns were 11.2%/yr, comparable to the Asian cash returns.

Because of the rebound in the stock market in the USA, the overall investment strategy, which clearly disregarded covariance risk, was to invest in the stock market in the USA to earn a 13.5%/yr risk premium and then to swap into an Asian currency or into Deutschemark. Table 9 shows that the total returns thus earned, were all above 20%/yr, with the highest strategic investment return of 27.4%/yr coming from investing in the USA and swapping into the Philippines peso. Direct investments into the Philippines had become decidedly unattractive.

4.1.3. *March 1996 - December 1997.* Chart 3 shows that in this third window of time the situation in Asia turned for the worse. Only six months of the 22 months in this window cover the currency turmoil and contagion, but, as a very detailed data analysis proves, the problems started actually in the Asian stock markets, spilled over into the cash lending markets and from there into the FX markets. Many of the direct investments and the financed government and private lending projects didn t deliver the returns expected. Most often, because hype combined with greed and lack of proper due diligence, many capitalized projects in Asia exhibited less than the marginally required returns. The risk premia in all Asian countries, including Japan, turned negative in this period, and so, with the exception of Hong Kong (because of the peg), did all the Asian cash returns.

In Thailand the risk premium was an average negative 84.2%/yr and the cash return, because of the collapse of the Thai baht, a negative 22.3%, so that capital was destroyed in Thailand at an average rate of 106.6%/yr, i.e., a complete annihilation of capital! Looking at the row for Thailand in Table 10, it is clear that no currency swap could have overcome the capital destruction in Thailand s stock market, since all the rates are about a negative $80 - 100\%$. Other hard hit countries were Malaysia, Indonesia and the Philippines, with domestic total returns of -55.9%/yr, -55.3%/yr and -46.6%/yr, respectively.

CHART 3: Risk Premia and Cash Returns, March 1996 - December 1997



	Hong Kong	Indonesia	Japan	Malaysia	Philippines	Singapore	Thailand	Germany	USA
Cash Return	6.3	-18.4	-11.1	-15.5	-10.1	-5.8	-22.3	-7.5	5.6
Risk Premium	-8.4	-36.8	-15.7	-40.4	-36.5	-29.1	-84.2	24.4	17.1

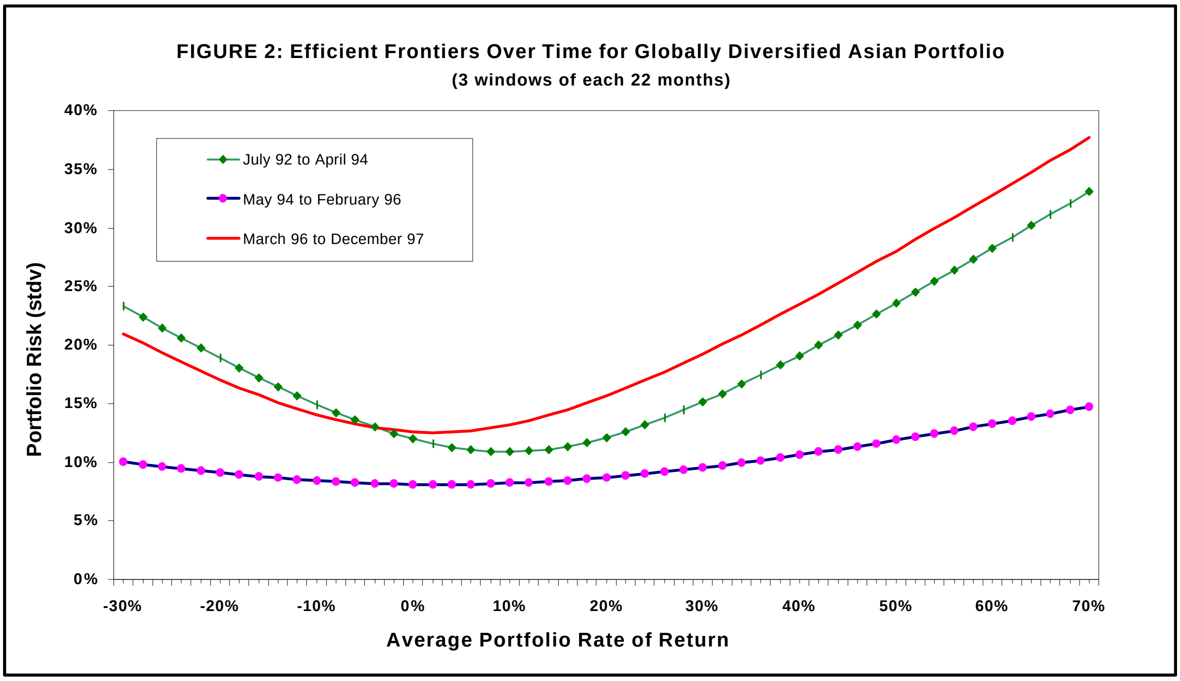
TABLE 10: AVERAGE STRATEGIC INVESTMENT RETURNS (%)
March 1996 - December 1997

	CURRENCIES								
MARKETS	Hong Kong	Indonesia	Japan	Malaysia	Philippines	Singapore	Thailand	Germany	USA
Hong Kong	-2.1	-26.8	-19.5	-23.9	-18.5	-14.2	-30.7	-15.9	-2.8
Indonesia	-30.6	-55.3	-48.0	-52.3	-47.0	-42.6	-59.2	-44.3	-31.3
Japan	-9.4	-34.1	-26.8	-31.2	-25.8	-21.5	-38.0	-23.2	-10.1
Malaysia	-34.1	-58.8	-51.5	-55.9	-50.5	-46.2	-62.7	-47.9	-34.8
Philippines	-30.2	-54.9	-47.6	-52.0	-46.6	-42.3	-58.8	-44.0	-30.9
Singapore	-22.8	-47.5	-40.2	-44.6	-39.2	-34.9	-51.4	-36.6	-23.5
Thailand	-78.0	-102.7	-95.4	-99.8	-94.4	-90.1	-106.6	-91.8	-78.7
Germany	**30.7**	6.0	13.3	8.9	14.3	18.6	2.1	16.9	30.0
USA	23.4	-1.3	6.0	1.6	7.0	11.3	-5.2	9.6	22.7

As Table 10 shows, in this period of overall capital destruction, the best total returns could be earned by investing in the German stock market to earn a very healthy risk premium of 24.4%/yr, in advance of the European Union, and by swapping into the US or Hong Kong dollars to earn an extra 5.6%/yr, respectively

6.3%/yr cash return. As long as the Hong Kong peg holds, the swap in the Hong Kong dollar appears to be as attractive as the swap into the US dollar. But the swap into the US dollar eliminates all FX risk for the US dollar based investor and is therefore probably more attractive.

4.2. **Level of Asian Investment Risk Over Time.** In the preceding subsection we discussed the level of total returns in three consecutive windows and found that it changed drastically over time. In this section we look closely at the level of covariance risk in the various markets in Asia. The general level of portfolio risk can be measured by the GMV risk of the portfolio efficiency frontiers in the respective data windows. The first window ($h = 1$) ranges from July 1992 through April 1994, the second window ($h = 2$) from May 1994 through February 1996 and the third window ($h = 3$) from March 1996 through December 1997. For each window set the average returns were computed as in Charts 1, 2 and 3, and the corresponding $2n \times 2n$ risk-premium and cash return covariance matrix $\Phi_h, h = 1, 2, 3$ (available in Appendix B) From these windowed data sets three respective efficiency frontiers traced in the risk return space in Figure 2, using the preceding procedures.



In the first window period, the highest risk premium volatility resided already in Thailand, followed by the Philippines and Hong Kong, while the highest cash return risk was in the Philippines. In the second window period, the order of the risk premium volatility remained unchanged, but Japan showed the highest cash return risk, resulting from high Yen volatility. In the third window period, the highest risk premium volatility remained in Thailand, followed by Indonesia, while the highest cash return volatility shifted now to Thailand, when the Thai baht collapsed.

Figure 2 shows that the GMV portfolio risk level declined from the first window period to the second and then sharply increased again moving from the second to the third period. At the same time the overall average strategic investment returns shifted down from the early to the middle and late 1990s. This conclusion is confirmed by the GMV return risk profiles in Table 11, which also provides the commensurate GMV portfolio fundamental choice allocations.

Most of that rate decline, from 10.0% to3.0%, occurred from the first to the second window, when the risk level declined by 270 basis points from 10.8% to 8.1%, due to the fact that investors switched from direct investments in Asian stock markets to investments in Asian cash. But the transition from the second to the third window was clearly different; the GMV portfolio risk level increased by 440 basis points from 8.1% to 12.5%, without a commensurate increase in GMV portfolio return. Instead, the GMV portfolio average return declined further from 3.0% to 2.0%. In contradiction to static finance theory, there was a pure, uncompensated, increase in risk: sharp unpredictable declines in stock markets gradually, combined in the second half of 1997 with sharp unpredictable currency depreciations.

TABLE 11:		**GMV**	**PORTFOLIO**	**PROFILES**
		Jul 92-Apr 94	**May 94-Feb 96**	**Mar 96-Dec 97**
Average return	$\overline{s}_p$	**10.0%**	**3.0%**	**2.0%**
Average **risk (stdv)**	σ_p	**10.8%**	**8.1%**	**12.5%**
Asset allocations	$v_{1.}$	-0.29	-0.02	0.11
	$v_{2.}$	0.02	-0.19	0.15
	$v_{3.}$	0.21	-0.09	0.08
	$v_{4.}$	-0.04	0.17	-0.09
	$v_{5.}$	0.30	-0.48	-0.12
	$v_{6.}$	0.00	0.84	0.52
	$v_{7.}$	0.05	0.02	0.01
	$v_{8.}$	-0.11	0.19	0.07
	$v_{9.}$	0.85	0.60	0.28
Currency allocations	$v_{.1}$	15.70	-4.02	-4.40
	$v_{.2}$	-0.57	-1.61	-0.56
	$v_{.3}$	0.09	0.05	-0.01
	$v_{.4}$	1.23	1.93	0.58
	$v_{.5}$	0.49	0.50	1.40
	$v_{.6}$	0.08	-0.72	-3.20
	$v_{.7}$	-0.21	3.57	-0.36
	$v_{.8}$	-0.72	0.50	1.31
	$v_{.9}$	-14.09	7.95	6.25

Table 11 also reveals other remarkable differences between the periods. Notice, for example, that the GMV portfolios required a positive allocation of 30% of capital to the stock market in the Philippines, but pulled such a capital allocation out in the second and third periods. The heavy and leveraged Hong Kong dollar based GMV strategy (+15.7) combined with the heavy shorting of the US dollar (-14.09) of the first period, was totally reversed in the second and third periods, when we had strategy combinations ($-4.02, +7.95$) and ($-4.40, +6.25$) respectively. Most likely, this was related to the global portfolio cautiousness in advance of the Hong Kong hand-over.

4.3. **Structural Changes in Asian Investment Risk.** In Figure 2 it is obvious that between July 1992 and December 1997 not only the level of portfolio risk changed in Asia, but also its shape: the three windowed portfolio frontiers have different shapes. In particular, the parabolic shape of the portfolio frontier in the second window for the period May 1994 to February 1997 shows much less curvature than the parabolic frontiers of the first window period from July 1992 to April 1994 and of the third window period from March 1996 to December 1997. The efficiency frontiers of the first and second window have remarkably similar curvatures.

A possible reason is that, in both the first and second window periods, the Asian stock market premia were dominant; in the first window period they were positive and in the third window period they were negative. But in the second period, from May 1994 to February 1996, cash returns were dominant. One can discern the considerable difference between the risk premium $(\mathbf{r} - \mathbf{c})$ blocks of the $2n \times 2n$ covariance matrices Φ_h (in Appendix B) and the cash return $(\mathbf{c} + \varepsilon)$ blocks of the same Φ_h matrices. First, the cash return blocks are bordered by the very low and mostly negative covariances of the Hong Kong and US cash returns. Second, we can discern the flip-flopping changes from positive to negative covariances and vice versa, going from one window to the next, again suggesting the instability of the covariance risks.

A more formal analysis is possible. The $2n = 18$-dimensional covariance risk ellipsoids, which are implied by the covariance matrices $\Phi_h, h = 1, 2, 3$, since $\mathbf{w}_h^{'}\Phi_h\mathbf{w}_h = c$ (= arbitrary constant), can be analyzed to see if there were significant structural changes in the Asian investment portfolio (= diversification) risk profile. For this analysis each covariance matrix is spectrally decomposed, $\Phi_h = \mathbf{V}_h\Lambda_h\mathbf{V}_h'$, where Λ_h is the diagonal matrix with the 18 eigenvalues $\lambda_{hi}, h = 1, 2, 3; i = 1, 2, ..., 18$, on the diagonal and $\mathbf{V}_h$ the corresponding matrix of eigenvectors, providing the rotational information with respect to the data frame of reference. The squared (all positive) eigenvalues λ_{hi} provide the amplitudes for the 18-dimensional scatter plots of the data vectors $[(\mathbf{r} - \mathbf{c}), (\mathbf{c} + \varepsilon)]$ from which the shape of each 18-dimensional covariance ellipsoid is derived. Since the matrix of eigenvectors is orthonormal, we can, by plotting simultaneously the three scree plots of eigenvalues, observe if significant structural changes have taken place in the shape of each covariance ellipsoid, and thus in the unsystematic covariance risk.

In Figure 3 these amplitudes are standardized, i.e., the maximum amplitude along the main axis of the covariance risk ellipsoid in each window is set equal to 100% and is not plotted in Figure 3. Only the second, third, etc. up to the 18th amplitude are plotted, since all but the first (maximum) amplitude represent the diversification risk (= volume of the risk ellipsoid). Notice that the second amplitude is only 31.2% of the maximum amplitude in the first window, (July 1992 - April 1994), indicating that there is a strong, unifying global market movement in the risk premia and cash returns. However, the second amplitude drops sharply to 20.2% in the second window (May 1994 - February 1996) and returns to 24.5% in the third window (March 1996 - December 1997). This represents, respectively, the lower curvature of the efficiency frontier in the second window compared with the one in the first window, and the partial return to a higher curvature in the third window. The diversification risk in Asia was clearly not stable. The Asian diversfication

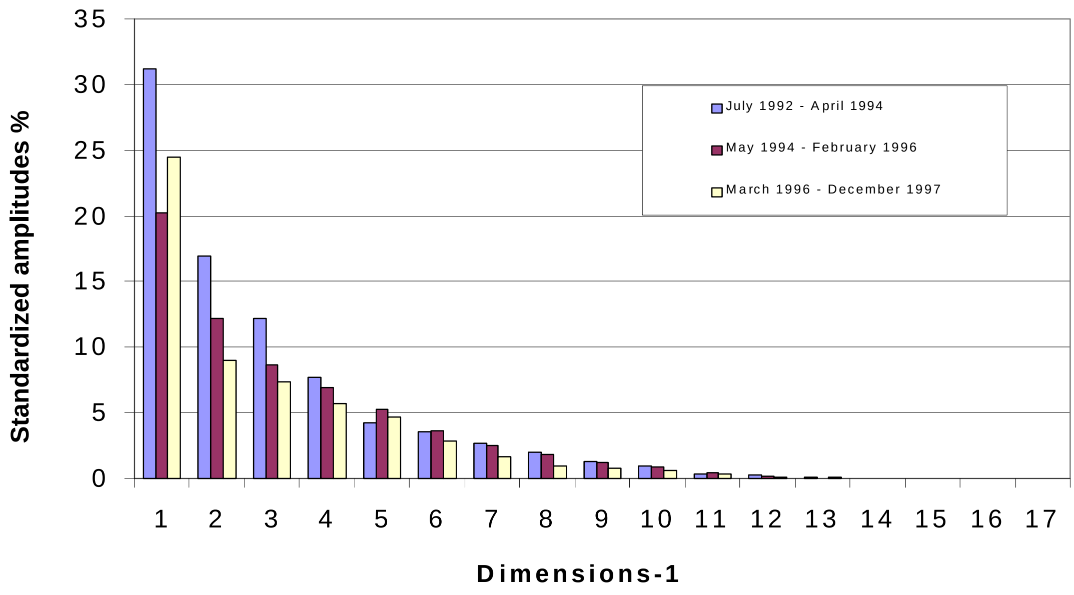


FIGURE 3: STANDARDIZED AMPLITUDES OF 18D COVARIANCE RISK ELLIPSOID

risk was very extensive in the mid-1990s and this probably caused considerable allocation problems for strategic portfolio managers investing in Asia.

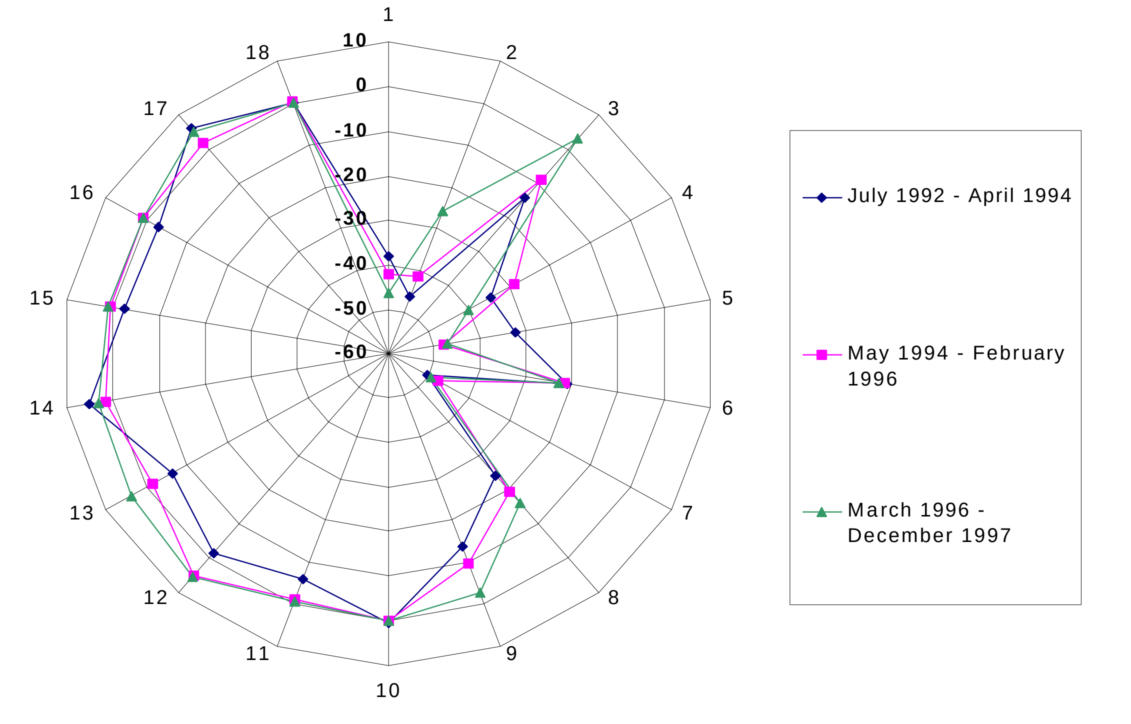
What about the changes in systematic risk? Since the mapping from one covariance risk ellipsoid into the next can be described as follows: $\Phi_h \longrightarrow \Phi_k$ is equivalent to $\mathbf{V}_h\Lambda_h\mathbf{V}'_h \longrightarrow \mathbf{V}_k\Lambda_k\mathbf{V}'_k$, there occurred also simultaneous rotation in the direction of the systematic covariance risk, as represented by the change in the matrices of eigenvectors $\mathbf{V}_h, h = 1, 2, 3$ to $\mathbf{V}_k, k = 1, 2, 3$, respectively. While the matrices of eigenvectors are orthonormal, per definition - within each data window $\mathbf{V}_h\mathbf{V}'_h = \mathbf{I} = \mathbf{V}'_h\mathbf{V}_h$ - they are usually not orthonormal between windows: $\mathbf{V}_h\mathbf{V}'_k \neq \mathbf{I} \neq \mathbf{V}'_h\mathbf{V}_k$ for $h \neq k$.[12] Because of the orthonormality within each window, it is sufficient to focus on the 18-dimensional direction of the maximum axis of the covariance risk ellipsoid, since the other seventeen axes are orthogonal to the maximum axis. A change in the direction of the maximum axis represents a change in the systematic risk and implies similar direction changes in the other seventeen axes.

In Figure 4 a radar diagram shows the $2n = 18$ direction coefficients of the eigenvector of the maximum axis of the Asian covariance risk ellipsoid for the three consecutive windows periods. Each direction coefficient is measured from the center

[12] Although, interestingly, it is true that $(\mathbf{V}_h\mathbf{V}'_k)(\mathbf{V}_h\mathbf{V}'_k)' = \mathbf{I}$, clearly demonstrating the rotational nature of these orthonormalized matrices of eigenvectors.

FIGURE 4: 18D ANALYSIS OF CHANGES IN SYSTEMATIC RISK

(Comparison of direction coefficients of maximum axes of risk ellipsoids)



(which is at -60% of the unit length of the eigenvector). The first nine axes $(1-9)$ correspond with the data axes of the risk premia, i.e., the stock markets. The second nine $(10-18)$ correspond with the data axes of the cash returns, i.e. the currency swap markets.

First, there is an initial impression of general similarity of each of the coefficients between the windows and thus of overall stability of systematic risk. Second, the systematic risk is mostly related to the risk premia, i.e., the stock markets, since the first nine coefficients in each of the three consecutive periods are closest to the center of the diagram. Hong Kong, the Philippines, Thailand and Malaysia show most stock market related systematic risk: their coefficients are closest to the center. There is much less systematic risk related to the currency swap markets: the currency return coefficients are close to zero and they shift for five of the seven Asian countries from negative to positive.

Third, there are specific country related changes discernible in the systematic risk. The largest decrease in systematic risk occurred in the stock market of Indonesia between the second window (May 1994 - February 1996) and the third window

(March 1996 - December 1997). During the same transition there occurred also an decrease in the systematic stock market risk of Japan. Interestingly, for both Indonesia and Japan the systematic currency swap risk switched signs between the early and the later 1990s. A substantial increase in systematic risk of the stock market in the Philippines also occurred between the first and second window period. In the period July 1992 - December 1997, Germany and the USA saw a gradual decrease in systematic stock market risk

From the point of view of our Asian+ portfolio, Singapore and Thailand surprisingly exhibited most stability in systematic risk in its stock market dimension, while Hong Kong and the USA, as expected, exhibited most stability in systematic risk in their respective currency return dimensions. Neither the USA nor Hong Kong showed systematic currency swap risk in any of the three periods: their systematic cash return coefficients remained zero.

5. CONCLUSIONS

Singer & Karnosky s [16], [6] complete and exact cash growth accounting framework produces a singular strategy risk matrix based on simple growth strategy investment allocations, which prevents mean-variance portfolio optimization. With the help of simple tensor algebra, and by exploiting the accounting identities, the exact accounting framework of Singer & Karnosky is reformulated so that the fundamental choice covariance matrix is nonsingular. and Markowitz mean-variance portfolio optimization can be implemented with complete and exact attribution. This implies that finally commercial procedures, such as RiskMetrics^{TM} and CreditMetrics^{TM} can properly implement portfolio optimization combined with exact, cash flow based, return attribution. Thus far, their optimization procedures have been based on singular risk matrices, which only appear nonsingular, because of errors introduced by finite computer registers.

This new optimal return-risk attribution accounting framework has been implemented to the stock market, cash market and FX return data for seven Asian countries, plus the USA and Germany, in the period July 1992 through December 1997. In this five and a half year investment horizon, out of 81 possible bilateral investment strategies, the optimal investment strategy would have been to invest 80.3% of the investable capital in the US stock market, with smaller allocations of 11.5% into Malaysia, Singapore and Germany and with some short selling of around 14% in Hong Kong and Indonesia. This stock market allocation should have been combined with currency swaps, with 110% in the Hong Kong dollar, around 41.5% in Malaysian Ringgit and Philippines peso and 25.4% in German Deutschemark, with almost 100% short selling of the Singapore dollar. The result? An annual average portfolio return of 13.7% with average portfolio risk (measured by one standard deviation of 25.3%. Was the total GMV portfolio return enough to compensate for the GMV portfolio risk? The historical facts tell us no! Even diversification into the stable and liquid markets of the USA and Germany would have been insufficient for an investor bent on investing in Asia.

There is ample evidence that many global investors disregarded normal portfolio risk considerations and that they based their strategic investment allocations strictly on policies of maximizing returns, but not on minimizing risks. Perhaps, because Asian diversi cation risk was non-stationary and thus essentially unpredictable, risk was overall ignored as a serious strategic investment consideration.

Stock market volatility made Asia a high investment risk area[13] Thailand showed the highest risk premium volatility throughout the period, and the highest cash return volatility in the early period of July 1992 to April 1994 and again in the period March 1996 to December 1997. This stock market volatility was caused by non-existent, unreliable, or non-transparent fundamental stock valuations. Consequently, the Asian stock markets operated more as speculative casinos than as mechanisms for reliable equity valuation and arbitrage.

In addition, there was latent currency risk, because of mercantilistic trade policies. The cash returns in Japan were the most volatile in the second window period, because of Yen volatility. Their mercantilistic trade policies forced most Asian countries to *artificially* peg their currencies to the US dollar. The enormous surpluses of accumulated foreign reserves in several of the Asian countries clearly indicate that these currencies should have appreciated, but were prevented from doing so by continued US dollar buying by their central banks. This caused enormous pressures on the Asian FX rates. These pressured broke into the open, when in 1997 the Asian stock market risks spilled over into and were then amplified by the Asian currency crises.[14]

By a detailed analysis of the nonstationarity of the portfolio return-risk profiles, we discern three distinct, although somewhat arbitrarily timed, window periods within this five and a half year period. In the first period, July 1992 - April 1994, the Asian stock markets delivered high risk premia, combined with moderate cash returns. Simultaneously the risk premia and cash returns in the USA and Germany were low. This situation induced a capital flow to Asia.

In the second period, May 1994-February 1996, the Asian stock markets produced zero risk premia on average, but liquidity crunches combined with the continuing capital in flows raised the cash returns to very high levels. However, the USA stock market started to produce substantial risk premia and many portfolio managers invested in the USA stock market, but swapped their cash into Asian currencies, for public and private lending.

In the third period, the Asian stock markets collapsed and sharply produced negative risk premia. This was combined with negative cash returns, when the promised cash returns on bank lending projects failed to materialize. Foreign capital started then to pull out of the region and Asian currencies began to depreciate at fast rates. The multi-lateral currency swaps transmitted the volatility fast throughout the ASEAN countries and thus the Asian financial crises resembled viral contagion.

While from the first to the second period the GMV return decline was accompanied by a decline in risk, when capital was shifted from the stock markets into cash, in the second transition this was no longer true. In the second transition from the second to the third period, GMV portfolio risk increased sharply together with a decline in total GMV return, when all capital was pulled out of Asia.

[13]Elsewhere we have shown that for Singapore, Malaysia and to a somewhat lesser extent Indonesia, the volatility of the stock markets is closely related to the instability of the local economies and not to the risks of the local cash markets.[10] In fact, we showed that the local cash markets operate almost independently from the local stock markets and the local economies.

[14]The Asian currency crisis resembles very much the breakdown of the artificial European Monetary System in September 1992, when the Pound Sterling came under attack of currency arbitrageurs. In Asia in 1997, the mercantilistic monetary system of artificially pegged exchange rates broke down for a similar reason. The artificially created currency disequilibria, expressed by the enormous foreign exchange surpluses, formed an inviting opportunity for major currency arbitrageurs to force an Asian FX equilibrium.

Spectral analysis of the fundamental covariance risks between the (mostly Asian) risk premia and the cash returns, demonstrate that the diversification risks within the Asian portfolio were sharply changing between the window periods. The Asian portfolio diversification risks were largest in the mid-1990s. Such non-stationarity changes the shape of the portfolio efficiency frontier. Hong Kong, Thailand, the Philippines and Malaysia exhibited the largest systematic risks.

But these systematic risks were changing too. The Philippines systematic risk increased in the early 1990s, between the first and second window periods. The systematic risks of Indonesia and Japan both were dramatically reduced between the second and third window periods in the mid-1990s. While the systematic risk in Thailand and Singapore remained unchanged throughout. Only intensive dynamic monitoring by information filtering (= using moving windows) may, perhaps, make it possible for a portfolio manager to project, perhaps, one or two months ahead for dynamic portfolio allocation, when the data would be timely and accurate. However, because the portfolio would have to be adjusted continuously, the transaction costs may become prohibitive for such an exercise.

References

[1] *Performance Presentation Standards Handbook.* Association for Investment Management and Research, Charlottesville, VA, 2nd edition, 1997.

[2] G. C. Allen. Performance attribution for global equity portfolios. *Journal of Portfolio Management*, 17(1):59 - 65, Fall 1991.

[3] F. Black and R. Litterman. Global portfolio optimization. *Financial Analysts Journal*, 48(5):28 - 43, September-October 1992.

[4] P. Dhrymes. *Introductory Econometrics.* Springer-Verlag, New York, NY, 1978.

[5] G. W. Gardner and T. Wuiloloud. Currency risk in international portfolios: How satisfying is optimal hedging? *The Journal of Portfolio Management*, 21(3):59 - 67, Spring 1995.

[6] D. S. Karnosky and B. D. Singer. *Global Asset Management and Performance Attribution.* The Research Foundation of the Institute of Chartered Financial Analysts, Charlottesville, VA, 1994.

[7] H. W. Kuhn and A. W. Tucker. Nonlinear programming. In J. Neyman, editor, *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, pages 481 - 492, Berkeley, CA, 1951. University of California Press.

[8] A. F. Lee. International asset and currency allocation. *Journal of Portfolio Management*, 14(1):68 - 73, Fall 1987.

[9] C. A. Los. *Econometrics of Models with Evolutionary Parameter Structures.* PhD thesis, Columbia University, 1984.

[10] C. A. Los. Valuation of six asian stock markets. In *Econometric Society Australasian Meeting*, volume 4, pages 589 - 621, Melbourne, 2-4 July 1997.

[11] C. A. Los. Optimal multi-currency investment strategies with exact attribution in three asian countries. *Journal of Multinational Financial Management*, 26 pages, Forthcoming: July 1998.

[12] H. M. Markowitz. Portfolio selection. *The Journal of Finance*, 7(1):77 - 91, March 1952.

[13] H. M. Markowitz. *Mean - Variance Analysis in Portfolio Choice and Capital Markets.* Blackwell, Oxford, 1987.

[14] M. F. Montes. *The Currency Crisis in Southeast Asia.* Institute of Southeast Asian Studies, Singapore, updated edition, 1998.

[15] B. Singer. Evaluation of portfolio performance: Attribution analysis. *The Journal of Performance Measurement*, 1(2):45 - 55, Winter 1996.

[16] B. D. Singer and D. S. Karnosky. The general framework for global investment management and performance attribution. *The Journal of Portfolio Management*, 21(2):84 - 92, Winter 1995.

6. APPENDIX A: LIST OF DATA SERIES

All series were obtained from Datastream, Dow Jones Telerate, for the period July 1992 - December 1997.

6.1. **Stock Market Indices.**

Hong Kong: Hang Seng - Price Index
Indonesia: Jakarta SE - Price Index
Japan: Nikkei 225 Stock Average - Price Index
Malaysia: Kuala Lumpur Composite - Price Index
Philippines: Philippines SE Composite - Price Index
Singapore: Straits Times Industrial Index
Thailand: Thailand DS Market - Price Index
Germany: DAX KURS - Price Index
United States: S&P 500 Composite - Price Index

6.2. **Interest Rates.**

Hong Kong: HK Inter-bank 1 Month (Middle Rate)
Indonesia: Indonesia Deposit 1 Month (Middle Rate)
Japan: Japan Inter-bank 1 Month (Middle Rate)
Malaysia: Malaysia Inter-bank 1 Month (Middle Rate)
Philippines: Philippines T-Bill 91 Days (Middle Rate)
Singapore: Singapore Inter-bank 1 Month (Middle Rate)
Thailand: Average of 3 banks Offered Rates

1)Thailand Inter-bank 1 Month (BB) - Offered Rate
2)Thailand Inter-bank 1 Month (SC) - Offered Rate
3)Thailand Inter-bank 1 Month (TF) - Offered Rate

Germany: Germany Inter-bank 1 Month (LDN:BBA - Offered Rate)
United States: U.S Inter-bank 1 Month (LDN:BBA- Offered Rate)

6.3. **Exchange Rates.**

Hong Kong: Hong Kong $ to US $ - Exchange Rate
Indonesia: Indonesian Rupiah to US $ - Exchange Rate
Japan: Japanese Yen to US $ - Exchange Rate
Malaysia: Malaysian Ringitt to US $ - Exchange Rate
Philippines: Philippine Peso to US $ - Exchange Rate
Singapore: Singapore $ to US $ - Exchange Rate
Thailand: Thai Bhat to US $ - Exchange Rate
Germany: German Mark DM to US $ - Exchange Rate

7. APPENDIX B: RISK PREMIUM - CASH RETURN MATRICES $\Phi_h, h = 1, 2, 3$

TABLE B1: RISK PREMIUM - CASH RETURN COVARIANCE MATRIX Φ_1 (18 x 18)

July 1992 - April 1994

	r-c									c+ε								
r-c	12287.4	6953.5	941.8	8412.6	9769.8	5406.9	9896.0	3449.0	971.9	18.1	221.7	(1023.0)	(888.2)	(525.1)	(277.1)	(99.6)	(1075.2)	(14.1)
	6953.5	6933.5	(1242.1)	4019.9	4606.1	2743.9	5220.1	2109.2	934.3	8.2	84.0	(598.6)	(380.7)	(95.3)	(195.0)	(71.4)	(603.6)	(13.2)
	941.8	(1242.1)	7466.8	(172.7)	(325.2)	969.7	(2486.7)	(386.5)	412.0	(6.0)	2.9	746.3	(194.0)	(373.0)	355.1	214.6	1196.3	(1.9)
	8412.6	4019.9	(172.7)	10108.3	9212.2	5406.7	8642.0	2410.8	452.4	5.7	82.7	(1047.5)	(608.2)	(474.8)	(173.6)	(92.3)	(672.6)	(3.6)
	9769.8	4606.1	(325.2)	9212.2	13295.5	4766.0	10338.8	3903.7	(12.0)	13.9	125.5	(747.8)	(810.3)	(1139.0)	(38.2)	(112.7)	(557.9)	(11.0)
	5406.9	2743.9	969.7	5406.7	4766.0	4876.0	3904.0	2009.6	801.9	20.2	7.8	(539.2)	(532.0)	(493.5)	(63.5)	(27.1)	(192.3)	0.0
	9896.0	5220.1	(2486.7)	8642.0	10338.8	3904.0	**15497.3**	3520.2	390.5	8.9	258.1	(1906.7)	(984.6)	(692.5)	(514.7)	(233.1)	(1893.6)	(11.2)
	3449.0	2109.2	(386.5)	2410.8	3903.7	2009.6	3520.2	3314.3	236.0	(4.9)	(33.7)	(438.5)	(316.2)	(238.2)	(206.1)	(72.7)	(897.3)	0.7
	971.9	934.3	412.0	452.4	(12.0)	801.9	390.5	236.0	693.2	(0.1)	8.1	(189.1)	(186.6)	(244.7)	(63.9)	(50.3)	(116.4)	(0.8)
c+ε	18.1	8.2	(6.0)	5.7	13.9	20.2	8.9	(4.9)	(0.1)	1.1	(0.5)	(3.9)	(0.8)	(0.0)	4.6	(0.2)	21.7	0.0
	221.7	84.0	2.9	82.7	125.5	7.8	258.1	(33.7)	8.1	(0.5)	39.7	(27.0)	(14.9)	53.7	(12.6)	2.6	(26.1)	(0.7)
	(1023.0)	(598.6)	746.3	(1047.5)	(747.8)	(539.2)	(1906.7)	(438.5)	(189.1)	(3.9)	(27.0)	753.5	204.5	(109.2)	146.5	80.6	367.3	(0.8)
	(888.2)	(380.7)	(194.0)	(608.2)	(810.3)	(532.0)	(984.6)	(316.2)	(186.6)	(0.8)	(14.9)	204.5	355.7	(150.1)	67.9	37.4	248.3	0.5
	(525.1)	(95.3)	(373.0)	(474.8)	(1139.0)	(493.5)	(692.5)	(238.2)	(244.7)	(0.0)	53.7	(109.2)	(150.1)	**1726.7**	(70.8)	53.8	129.0	2.3
	(277.1)	(195.0)	355.1	(173.6)	(38.2)	(63.5)	(514.7)	(206.1)	(63.9)	4.6	(12.6)	146.5	67.9	(70.8)	106.0	25.2	326.3	(0.2)
	(99.6)	(71.4)	214.6	(92.3)	(112.7)	(27.1)	(233.1)	(72.7)	(50.3)	(0.2)	2.6	80.6	37.4	53.8	25.2	30.4	97.3	(0.3)
	(1075.2)	(603.6)	1196.3	(672.6)	(557.9)	(192.3)	(1893.6)	(897.3)	(116.4)	21.7	(26.1)	367.3	248.3	129.0	326.3	97.3	1483.0	0.3
	(14.1)	(13.2)	(1.9)	(3.6)	(11.0)	0.0	(11.2)	0.7	(0.8)	0.0	(0.7)	(0.8)	0.5	2.3	(0.2)	(0.3)	0.3	0.1

Determinant = 2.6E+42 **Product of Variances =** 1.2E+50 **18D Noise/Data Ratio =** 2.2E-08 **18D Signal/Noise Ratio =** 4.6E+07

TABLE B2: RISK PREMIUM - CASH RETURN COVARIANCE MATRIX Φ_2 (18 x 18)

May 1994 - February 1996

	r-c									c+ε								
r-c	6671.5	4796.3	286.9	3789.2	5464.7	2560.2	5965.3	1992.9	1494.8	15.0	197.7	(283.2)	170.6	(355.6)	(24.6)	8.0	208.3	(5.7)
	4796.3	5917.1	918.3	3284.5	5992.9	2649.0	5902.2	2005.5	1068.1	(1.5)	177.1	(983.6)	286.2	3.8	(129.3)	(139.0)	(382.6)	(8.7)
	286.9	918.3	4900.4	(505.9)	1912.3	1046.6	624.0	1563.2	305.1	(16.5)	(77.1)	(1710.6)	(19.2)	(74.8)	(167.1)	(213.7)	(1001.6)	(6.3)
	3789.2	3284.5	(505.9)	4285.2	3754.5	2167.1	3702.6	1442.0	782.4	2.7	129.5	589.6	(66.0)	(498.6)	(40.6)	82.0	320.3	(4.9)
	5464.7	5992.9	1912.3	3754.5	7486.9	3087.9	6656.3	2974.5	1445.4	(8.5)	178.5	(600.5)	408.6	(230.7)	(42.7)	(101.5)	(336.8)	(8.9)
	2560.2	2649.0	1046.6	2167.1	3087.9	1849.9	2849.1	1240.4	492.3	0.2	77.0	(326.6)	(40.2)	(170.3)	(80.6)	(53.5)	(236.0)	(3.6)
	5965.3	5902.2	624.0	3702.6	6656.3	2849.1	**7833.5**	2242.8	1208.3	(2.8)	240.2	(747.6)	331.5	76.2	(46.8)	(123.0)	(113.7)	(18.2)
	1992.9	2005.5	1563.2	1442.0	2974.5	1240.4	2242.8	3209.6	772.3	4.7	108.3	(1057.7)	(104.7)	(352.9)	(232.9)	(174.7)	(906.1)	4.3
	1494.8	1068.1	305.1	782.4	1445.4	492.3	1208.3	772.3	723.2	11.1	43.2	3.6	87.1	(254.9)	(0.0)	21.1	84.6	2.9
c+ε	15.0	(1.5)	(16.5)	2.7	(8.5)	0.2	(2.8)	4.7	11.1	0.9	0.7	(6.7)	(5.3)	(8.9)	(3.5)	0.0	1.2	0.3
	197.7	177.1	(77.1)	129.5	178.5	77.0	240.2	108.3	43.2	0.7	27.5	15.4	7.9	20.4	3.4	(1.2)	8.4	0.3
	(283.2)	(983.6)	(1710.6)	589.6	(600.5)	(326.6)	(747.6)	(1057.7)	3.6	(6.7)	15.4	**2247.9**	158.4	(84.4)	348.5	279.9	1169.7	(0.4)
	170.6	286.2	(19.2)	(66.0)	408.6	(40.2)	331.5	(104.7)	87.1	(5.3)	7.9	158.4	182.7	72.0	78.1	28.1	71.4	(1.8)
	(355.6)	3.8	(74.8)	(498.6)	(230.7)	(170.3)	76.2	(352.9)	(254.9)	(8.9)	20.4	(84.4)	72.0	531.5	62.5	(24.1)	(132.9)	(6.3)
	(24.6)	(129.3)	(167.1)	(40.6)	(42.7)	(80.6)	(46.8)	(232.9)	(0.0)	(3.5)	3.4	348.5	78.1	62.5	103.4	44.6	189.1	(1.6)
	8.0	(139.0)	(213.7)	82.0	(101.5)	(53.5)	(123.0)	(174.7)	21.1	0.0	(1.2)	279.9	28.1	(24.1)	44.6	44.0	167.3	0.1
	208.3	(382.6)	(1001.6)	320.3	(336.8)	(236.0)	(113.7)	(906.1)	84.6	1.2	8.4	1169.7	71.4	(132.9)	189.1	167.3	1149.9	(2.4)
	(5.7)	(8.7)	(6.3)	(4.9)	(8.9)	(3.6)	(18.2)	4.3	2.9	0.3	0.3	(0.4)	(1.8)	(6.3)	(1.6)	0.1	(2.4)	0.3

Determinant = 3.6E+37 **Product of Variances =** 2.0E+48 **18D Noise/Data Ratio =** 1.8E-11 **18D Signal/Noise Ratio =** 5.6E+10

TABLE B3: RISK PREMIUM - CASH RETURN COVARIANCE MATRIX Φ_3 (18 x 18)

March 1996 - December 1997

	r-c									c+ε								
	12918.9	8095.3	3676.3	7616.0	5902.8	6522.7	8880.3	5734.7	3655.8	(162.9)	1790.7	656.1	1005.6	(935.2)	895.0	2058.8	(1248.5)	(2.2)
	8095.3	16298.7	3477.5	10588.9	9128.2	4116.1	11433.1	5162.3	3455.5	(97.8)	1862.5	1292.6	2473.5	(824.1)	711.3	1669.9	(1305.5)	(7.6)
	3676.3	3477.5	4066.2	2047.2	1563.7	2367.3	3990.0	1737.2	1791.9	(33.7)	1955.9	391.3	1282.9	847.4	600.5	1909.8	(735.6)	(0.2)
	7616.0	10588.9	2047.2	10652.4	7511.7	3405.8	7050.7	3787.4	1641.1	(143.9)	950.4	993.8	1443.3	(781.1)	469.6	1636.9	(632.6)	(7.9)
r-c	5902.8	9128.2	1563.7	7511.7	10622.4	3690.9	6925.8	3839.7	1872.3	(109.5)	1512.3	420.7	1718.4	(193.4)	518.2	2091.6	(546.9)	(3.8)
	6522.7	4116.1	2367.3	3405.8	3690.9	5120.7	5396.3	3096.1	2096.4	(70.6)	1364.3	86.1	565.1	(109.2)	661.3	1993.6	(767.8)	(0.3)
	8880.3	11433.1	3990.0	7050.7	6925.8	5396.3	**21821.4**	8178.4	4111.7	(74.9)	275.6	(981.5)	777.8	(3426.9)	118.0	(3608.4)	(2206.3)	(3.1)
	5734.7	5162.3	1737.2	3787.4	3839.7	3096.1	8178.4	5108.5	2310.1	(57.5)	216.9	(383.0)	336.0	(1598.3)	29.0	(1200.7)	(1312.4)	1.1
	3655.8	3455.5	1791.9	1641.1	1872.3	2096.4	4111.7	2310.1	2181.2	(19.8)	354.4	(61.9)	235.8	(562.6)	200.0	(145.5)	(828.4)	0.6
	(162.9)	(97.8)	(33.7)	(143.9)	(109.5)	(70.6)	(74.9)	(57.5)	(19.8)	5.0	(47.2)	7.7	(38.8)	(17.9)	(13.9)	(62.1)	24.3	0.1
	1790.7	1862.5	1955.9	950.4	1512.3	1364.3	275.6	216.9	354.4	(47.2)	3355.8	173.3	2365.7	2439.7	834.9	3280.5	(314.5)	(3.2)
	656.1	1292.6	391.3	993.8	420.7	86.1	(981.5)	(383.0)	(61.9)	7.7	173.3	1290.2	210.5	177.2	170.0	885.3	378.6	(0.4)
	1005.6	2473.5	1282.9	1443.3	1718.4	565.1	777.8	336.0	235.8	(38.8)	2365.7	210.5	1967.2	1644.8	539.3	2007.4	(339.3)	(3.2)
c+ε	(935.2)	(824.1)	847.4	(781.1)	(193.4)	(109.2)	(3426.9)	(1598.3)	(562.6)	(17.9)	2439.7	177.2	1644.8	2583.0	586.7	3076.2	123.1	(2.0)
	895.0	711.3	600.5	469.6	518.2	661.3	118.0	29.0	200.0	(13.9)	834.9	170.0	539.3	586.7	272.3	1049.7	(13.1)	(0.8)
	2058.8	1669.9	1909.8	1636.9	2091.6	1993.6	(3608.4)	(1200.7)	(145.5)	(62.1)	3280.5	885.3	2007.4	3076.2	1049.7	**6104.1**	459.1	(2.6)
	(1248.5)	(1305.5)	(735.6)	(632.6)	(546.9)	(767.8)	(2206.3)	(1312.4)	(828.4)	24.3	(314.5)	378.6	(339.3)	123.1	(13.1)	459.1	836.1	0.2
	(2.2)	(7.6)	(0.2)	(7.9)	(3.8)	(0.3)	(3.1)	1.1	0.6	0.1	(3.2)	(0.4)	(3.2)	(2.0)	(0.8)	(2.6)	0.2	0.0

Determinant = 6.3E+43 **Product of Variances =** 3.7E+56 **18D Noise/Data Ratio =** 1.7E-13 **18D Signal/Noise Ratio =** 5.9E+12

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