Relative Performance Evaluation Contracts and Asset Market Equilibrium*

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Abstract

We analyse the equilibrium consequences of performance-based contracts for fund managers. Managerial remuneration is tied to a fund's absolute performance and its performance relative to rival funds. Investors choose whether or not to delegate their investment to better-informed fund managers; if they delegate they choose the parameters of the optimal contract subject to the fund manager's participation constraint. We find that the impact of relative performance evaluation on equilibrium equity premium and on portfolio herding critically depends on whether the participation constraint is binding. Simple numerical examples suggest that the increased importance of delegation and performance evaluation may lower the equity premium.

Keywords: portfolio delegation, relative performance evaluation, equity premium

JEL Classification: G11, G12, G23.

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1. **Introduction**

The explosive growth of the asset management industry during the 1990s[1] was accompanied by a growing trend towards performance-based remuneration for fund managers. Given that stock markets performed rather well over this period, the absolute return on a managed fund was not a reliable measure of managerial ability. In this environment, remunerating fund managers on the basis of their relative performance became increasingly attractive. Other things being equal, a fund manager should be paid more if he 'beats the market' or performs better than his peers. Contracts based on relative performance evaluation (RPE) provide incentives for managers to perform well, while stripping away the uncertainty common to all investment funds.

While there is a substantial literature on the impact of performance-based contracts on portfolio choice,[2] their implication for asset market equilibrium is poorly understood. In this paper we aim to analyze the equilibrium consequences of performance-based contracts in a simple model of portfolio choice. We consider a two-period model in which investors allocate their wealth across two assets: riskless bonds and risky equity shares. An investor can invest directly in these assets or delegate the portfolio choice to a professional fund manager. Delegation incurs fees, so is rational only if its benefits justify the costs. In our model, fund managers have access to better information about the relative returns of the two assets. If investors opt to delegate, they choose the optimal performance-based fee structures to remunerate fund managers. We allow managerial remuneration to be a linear function of their absolute and relative performances, and to include a fixed component that is independent of performance. Both classes of agents – investors and fund managers – are assumed to be risk-averse. Investors choose their investment strategy to maximise the expected utility of their returns net of any delegation fees. Fund managers choose portfolios to maximise the expected utility of their remuneration. Our interest lies in analyzing the equilibrium outcome, where asset

[1] Mamaysky and Spiegel (2001) report that the number of equity funds registered in the US rose from 785 in 1990 to 11,882 by 2000, while total net assets under management in equity funds grew from $296 billion to $5.81 trillion by 2000, an almost twenty-fold increase. By comparison, over the same period, the number of equities listed on the NYSE, AMEX and NASDAQ grew from 6,635 to 8,435, an increase of 27%.

[2] See, e.g., Bhattacharya and Pfleiderer (1985), Grinblatt and Titman (1989), Das and Sundaram (2002), Maug and Naik (1996), Admati and Pfleiderer (1997), and Bhattacharya (1999).

prices are determined through market clearing.

We find that fund managers' portfolio choices typically undo the incentive effects of relative performance evaluation in linear contracts. If so, does relative performance evaluation matter? In our model delegating investors choose delegation contracts to provide the right incentives to fund managers, subject to a standard participation constraint. If delegating investors can choose the parameters of the linear contract optimally, relative performance evaluation serves a limited purpose. While the use of RPE contracts is not sub-optimal for investors it does not necessarily improve on outcomes obtained through other contracts based on absolute performance alone. However, in reality the set of feasible contracts may be somewhat restricted. Consider, for instance, a plausible requirement that the fixed component of managerial remuneration cannot be negative. When this restriction poses a binding constraint, so that the chosen contract is only constrained-optimal, relative performance evaluation matters. With constrained-optimal contracts, the weight placed on RPE affects the demand of risky assets in delegated portfolios and hence the equilibrium equity premium. These effects are driven by equilibrium conditions and could not be uncovered outside the type of model we analyse here.

We also find that, even with 'fully-optimal' linear contracts, delegated portfolios are likely to have larger demand for the risky asset than if investors were to invest directly. There are two reasons for this. One, performance-based delegation contracts entail risk sharing between investors and fund managers: to the extent delegating investors bear only a part of the risk associated with a portfolio holding, they are willing to let their delegated portfolios carry higher levels of the risky asset than if they were investing directly. Two, if fund managers are better informed than direct investors, their informational advantage lowers the risk associated with any given level of holdings of the risky asset. If delegation results in greater willingness to hold risky assets, it is quite plausible that greater reliance on delegated investment will lower the required equilibrium risk premium. Empirical evidence has suggested that the equity risk premium has declined in recent years:[3] the processes described in this paper offer channels of contributory influence. We present illustrative examples quantifying some of these effects in our model.

The paper is organized as follows. We begin with a brief survey of the related literature. Sections 2 to 4 describe our model and our principal findings regarding

[3] See, for example, Claus and Thomas (2001), Graham and Harvey (2001) and Welch (2000).

portfolio choices and the delegation decision. Section 5 studies the resulting equilibria, including the implication for the equity premium. Section 6 concludes. All proofs are collected in the Appendix.

1.1. *Related literature*

Relative performance evaluation has long been an aspect of contractual relations. Even when it is not explicitly written into a contract, RPE may be a part of the implicit agreements that guide long-term remuneration. Gibbons and Murphy (1990) found that upward revision of CEO salaries tends to be positively related to firms' performance, but negatively related to industry or market performance as a whole. Lakonishok, Shleifer and Vishny (1992) found positive correlation between the relative performance of funds (as indicated by their rank in published league tables) and inflow of new investment funds. Similarly, Chevalier and Ellison (1997) and Sirri and Tufano (1998) found a positive, if nonlinear, relationship between performance and inflow of new funds to mutual funds. Given that management fees are an increasing function of the size of managed funds, outperforming the market leads to higher rewards in the future.

Holmstrom (1982) was among the first to argue that relative performance evaluation (RPE) is valuable if agents face some common uncertainty. To be precise, RPE is useful if other agents' performance reveals information about an agent's unobservable choices that cannot be inferred from his own measured performance. Of course, RPE-based contracts do not always work in the interest of the principals. Within organizations, basing reward on relative performance creates incentives to sabotage the measured performance of co-workers, to collude with co-workers, or to self-select into a pool of low ability workers. Dye (1992) pointed out such contracts may distort choice by persuading managers to select projects where their relative talent, rather than their absolute talent, is the greatest. Aggarwal and Samwick (1999) show that when firms compete in product markets, use of high-powered incentives may result in excessive competition: the need to soften the intensity of competition may induce principals to dilute incentives. And, even when the net benefit of RPE contracts is positive, they may be difficult to implement, say, if individual performance (as opposed to team performance) is hard to measure.

Bhattacharya and Pfleiderer's (1985) seminal paper on delegated portfolio management has been followed by an extensive literature on the impact of the delegation fee structure on portfolio choice. Grinblatt and Titman (1989) and Das and Sundaram (2002) focus on the differences between symmetric, 'fulcrum' contracts (which penalise under-performance just as they reward out-performance), and asymmetric, 'incentive' contracts (which reward out-performance without penalising under-performance). Our model focuses on symmetric contracts. Das and Sundaram (1998) point out that symmetric contracts have long been mandatory for US mutual funds, though regulatory exemptions have diluted this requirement to some extent. Indeed, in our model managerial remuneration is a linear function of the performance measures. While linear contracts are commonly observed in the fund management industry, they may not always be the optimal class of contracts. See Diamond (1998), among others, for a discussion of this issue.

Brennan (1993) provides an early attempt to study the general equilibrium implications of contracts that reward managers according to their performance relative to a benchmark portfolio. In that spirit, Cuoco and Kaniel (2001) examine the impact of such RPE contracts on equilibrium prices. As in our model, they have three classes of agents ('active investors', 'fund investors' and 'fund managers'), but the proportions of the three classes are fixed exogenously. Their primary purpose is to compare the impact of symmetric versus asymmetric RPE contracts: they find that symmetric contracts tilt portfolio choice towards stocks that are part of the benchmark, while asymmetric contracts lead fund managers to choose portfolios that maximise the variance of their excess return over the benchmark. These papers do not consider the choice of optimal contract parameters.

Admati and Pfleiderer (1997) do look at the issue of optimal contract parameters in such contexts. They question the usefulness of benchmark-adjusted compensation: they find that such schemes are generally inconsistent with optimal risk sharing or with the goal of obtaining the optimal portfolio for the delegating investor. Our model differs from theirs in some crucial, and significant, respects. In their model, the decision to delegate is taken as given. Further, the expected return to assets is given exogenously (i.e., they do not allow for the possibility that investment choices made by fund managers affect the equilibrium return distribution). Three, in their model, relative performance is measured relative to a "passive" benchmark, such as a stock market index. Indeed, Admati and Pfleider themselves highlight these limitations of their model, and make the case for a model along the lines we present here. In our model, the benchmark is the average return of active fund managers, and thus is endogenous. We consider the equilibrium outcome, where relative returns are determined endogenously. We find that relative performance evaluation has a more benign effect, in that it is not incompatible with optimal portfolio selection.

2. The Model

2.1. *Preferences and delegation*

To isolate the effects of performance-based contracts on the asset market equilibrium, we study a simple two-period model of portfolio choice. Time is denoted by $t = 0, 1$. There are N investors, each with initial wealth of one unit. An investor can invest his wealth directly or delegate the investment decision to a fund manager. The delegation decision is endogenous. Suppose $n \leq N$ investors choose to delegate their investment (we denote these as $i = 1, 2, \ldots, n$), while the remaining $N - n$ investors invest directly. We assume, for simplicity, that each delegating investor is matched with exactly one fund manager, so that there are as many fund managers as there are delegating investors. We also assume that managers have no investible resources of their own, nor can they borrow to invest.

All agents – investors and fund managers – are risk averse and make choices in order to maximise the expected utility of their returns. In our model the structure of asset returns and payoffs are such that individual returns are normally distributed. We assume that all agents have utility functions with constant absolute risk aversion, possibly with different degrees of risk aversion. Under these assumptions, expected utility depends on the mean and variance of an agent's payoff. Given random payoff $\tilde{w}$, agent j's utility is given as

$$V_j(\tilde{w}) = E(\tilde{w}) - \frac{\rho_j}{2} Var(\tilde{w}), \tag{1}$$

where $\rho_j > 0$ is the individual's coefficient of absolute risk aversion.

Agents allocate their wealth across two assets, namely risk-free bonds and risky equity shares. There is an unlimited supply of bonds, with risk-free rate of return $r > 0$. The aggregate supply of equity shares is fixed at $Q > 0$. The return on equity depends on its final price $\tilde{P}_1$, which is normally distributed, and its initial price P_0, which is determined endogenously in our model.

Consider an arbitrary portfolio that allocates one unit of wealth across equity and bonds. If it holds λ shares acquired at price P_0 per share and invests the rest in bonds, its value in the final period is $\lambda\tilde{P}_1 + (1-\lambda P_0)r$. It simplifies the analysis if we express the value of the portfolio as a function of the excess return of equities over bonds, $\tilde{K}(P_0) \equiv \tilde{P}_1 - P_0 r$. The value of the portfolio can then be written as

$$\tilde{W} = \lambda\tilde{K} + r. \tag{2}$$

Agents' payoffs depend on portfolio choices. Fund managers are remunerated on the basis of their absolute performance and their performance relative to other active fund managers. Let $\tilde{W}_i$ be the final value of investor i's holdings, whether direct or delegated. Define $\overline{W} = \frac{1}{n}\sum_{i=1}^{n}\tilde{W}_i$ to be the average final value of all professionally-managed portfolios. The $i-$th fund manager's remuneration is linear (or, to be precise, affine)

$$\tilde{R}_{m(i)} = I_i + a_i\tilde{W}_i + b_i(\tilde{W}_i - \overline{W}). \tag{3}$$

Here $I_i \geq 0$ is a fixed component, independent of the fund's performance. The coefficient $a_i \geq 0$ ties remuneration to the absolute performance of the fund and $b_i \geq 0$ ties it to its relative performance. Note that relative performance is measured in relation to the performance of active fund managers, rather than to the market as a whole or to any other pre-specified benchmark. Using the average performance of active fund managers as the benchmark creates the possibility of strategic interaction in fund managers' choice.

The return to delegating investor i is the value of the delegated portfolio net of the manager's remuneration

$$\tilde{R}_{d(i)} = \tilde{W}_i - \tilde{R}_{m(i)}. \tag{4}$$

The contract parameters, (I_i, a_i, b_i), determine the division of the final portfolio value between fund managers and delegating investors. In our model delegating investors choose these parameters to align the interests of their fund manager with their own objectives. Delegation contracts are subject to a participation constraint: fund managers will accept a delegation contract only if the expected utility of the contract is no less than their reservation utility. For simplicity, we assume that all fund managers have the same reservation utility, $\phi_m \geq 0$; this is easily relaxed. Thus, incentive compatibility and participation constraints will jointly affect the choice of I_i, a_i and b_i.

Investors who invest directly on their own account obtain the full value of their portfolio

$$\tilde{R}_{o(i)} = \tilde{W}_i. \tag{5}$$

Investors may yet prefer costly delegation if they expect that fund managers can make better-informed choices on their behalf. We describe this next.

2.2. *Information Structure*

All agents have a common prior distribution over the final price of the risky asset. Prior to making the portfolio choice, but after entering any delegation contract, each agent receives a signal. We assume that obtaining the signal incurs no cost or effort: this allows us to abstract from any moral hazard in the problem. Fund managers receive signals that are more informative than those received by investors. An investor will choose to delegate if the informational advantage of fund managers is strong enough to compensate for the cost of delegation. We develop this idea in an environment in which all fund managers receive identical signals. Investors receive signals that are less informative than those of fund managers, and their precision varies across investors. It is natural to expect that investors with relatively imprecise information will be more likely to delegate.

To formalise this, we assume that the prior distribution of the price of equity in the final period is known by all to be

$$\tilde{P}_1 = \bar{P}_1 + \tilde{\varepsilon}, \quad \text{where } \ \tilde{\varepsilon} \sim N(0, \sigma_\varepsilon^2).$$

Before making their portfolio choices, fund managers observe a common signal $\tilde{s}$

$$\tilde{s} = \tilde{\varepsilon} + \tilde{u}, \quad \text{where } \tilde{u} \sim N(0, \sigma_m^2), \text{ and } E(\tilde{\varepsilon}\tilde{u}) = 0.$$

Define $\alpha_m \equiv \frac{\sigma_m^2}{\sigma_\varepsilon^2 + \sigma_m^2}$; this reflects the noise or imprecision of the signal. Its value lies between 0 and 1, with lower values indicating a more informative set of signals. Together, α_m and $\tilde{S}$ specify the common information structure of all fund managers. It is straightforward to show that, conditional on receiving a signal $\tilde{s}$, the posterior distribution of $\tilde{P}_1$ has mean and variance

$$E[\tilde{P}_1|\tilde{s}] = \bar{P}_1 + (1 - \alpha_m)\tilde{s}, \tag{6}$$

$$Var(\tilde{P}_1|\tilde{s}) = \alpha_m \sigma_\varepsilon^2. \tag{7}$$

Investors have heterogeneous information structures. Investor i gets a signal $\tilde{z}_i$

$$\tilde{z}_i = \tilde{\varepsilon} + \tilde{v}_i, \quad \text{where } \tilde{v}_i \sim N(0, \sigma_i^2) \text{ and } E(\tilde{\varepsilon}\tilde{v}_i) = 0.$$

Define $\alpha_i = \frac{\sigma_i^2}{\sigma_\varepsilon^2 + \sigma_i^2}$ to reflect the imprecision of investor i's signal. Together with the set of signals $\tilde{Z}_i$, it defines the information structure for investor i. Conditional on signal $\tilde{z}_i$, the posterior distribution has mean and variance

$$E[\tilde{P}_1 | \tilde{z}_i] = \bar{P}_1 + (1 - \alpha_i)\tilde{z}_i, \quad (8)$$

$$Var(\tilde{P}_1 | \tilde{z}_i) = \alpha_i \sigma_\varepsilon^2. \quad (9)$$

We assume that $\sigma_m^2 < \sigma_i^2$ for all i. It follows directly that $\alpha_m < \alpha_i$. This assumption captures the reasonable idea that professional managers are better informed than individual investors. Without this assumption there would be no role for active fund management in our model.

2.3. *Equilibrium*

Given this structure, an asset market equilibrium can be defined in the usual fashion. We assume that investors know the distributional properties of fund managers' risk preferences and information. Investors choose whether or not to delegate, and if they delegate, the parameters of their delegation contract. Fund managers choose portfolios that maximise the expected utility of their remuneration. Direct investors choose their portfolios to maximise expected utility.

Let $\boldsymbol{\lambda} \equiv (\lambda_1, \lambda_2, \ldots, \lambda_N)'$ be the vector of demand for equity, direct or via delegated portfolios, for the N investors. Demand depends on the initial price P_0. Given the aggregate demand for equity shares and their fixed aggregate supply Q, the price, P_0, is determined through market clearing:

$$\sum_{i=1}^{N} \lambda_i(P_0) = Q. \quad (10)$$

The equilibrium outcome is subject to the familiar problem of information revelation: investors may be able to infer information received by fund managers from the equilibrium price. This problem can be addressed by allowing Q to be random with a sufficiently large variance to make inference from prices very difficult. Such randomness in Q might reflect the impact of liquidity traders. Ignoring the issue here simplifies the algebra without significantly affecting our results.

To analyse the model, we first examine the investment choices of direct investors and fund managers. We then consider the design of optimal remuneration contracts and optimal delegation. Finally we study the equilibria in some sample economies.

3. Direct Investment

We begin by examining the portfolio choices of investors who invest on their own account. The return to direct investment is given by

$$\tilde{R}_{o(i)} = \lambda \tilde{K} + r. \tag{11}$$

For any P_0, let $\bar{K}(P_0) \equiv E[\tilde{K}(P_0)] = \bar{P}_1 - P_0 r$ be the mean value of excess returns, or the equity risk premium. We have the following result:

Proposition 1 *Consider an investor i with coefficient of absolute risk aversion ρ_i and information structure $(\alpha_i, \tilde{Z}_i)$. If this investor chooses to invest directly, the optimal portfolio demand conditional on receiving signal $\tilde{z}_i$ is*

$$\lambda^*_{o(i)} = \frac{\bar{K} + (1-\alpha_i)\tilde{z}_i}{\rho_i \alpha_i \sigma_\varepsilon^2}. \tag{12}$$

The ex-ante expected utility of direct investment is

$$V_{o(i)} = \frac{\bar{K}^2 + (1-\alpha_i)\sigma_\varepsilon^2}{2\rho_i \alpha_i \sigma_\varepsilon^2} + r. \tag{13}$$

The demand for equity is standard for the assumed mean-variance structure of preferences. Equity holding is increasing in $\bar{K} + (1-\alpha_i)\tilde{z}_i$, which is the expected value of $\tilde{K}$ conditional on signal $\tilde{z}_i$. Demand is decreasing in the risk aversion parameter ρ_i and in the conditional variance $\alpha_i \sigma_\varepsilon^2$. Note that we have not ruled out short sales as these do not affect our results in any significant way. The expression for ex-ante expected utility of direct investment obtains by computing the expected utility for each signal and then aggregating across $\tilde{Z}_i$, the set of signals.

4. Delegation

We analyse delegation in three steps. First, we consider a fund manager's portfolio choice for an arbitrary remuneration contract. Next we compute the value of a delegation contract to the delegating investor, allowing us to address the choice of optimal contract parameters. We can then consider the delegation decision by comparing the value of the optimally-chosen delegation contract with the value of direct investment. For tractability we assume that all fund managers have the same degree of risk aversion, ρ_m.

4.1. *Manager's choice conditional on signal s*

Given a contract (I_i, a_i, b_i), a fund manager chooses the portfolio to maximise expected utility of remuneration, $\tilde{R}_{m(i)}$. Relative performance evaluation makes each manager's remuneration sensitive to contracts of rival fund managers. To capture this dependence, we define $C = \sum_{j=1}^{n} \frac{1}{(a_j+b_j)}$, and $D = \sum_{j=1}^{n} \frac{a_j}{(a_j+b_j)}$.[4] We have the following result:

Lemma 1 *Consider a fund manager with risk aversion ρ_m, information structure $(\alpha_m, \tilde{S})$, and remuneration contract (I_i, a_i, b_i). Conditional on receiving a signal $\tilde{s}$, his optimal portfolio demand is*

$$\lambda_{m(i)} = \left[\frac{D + b_i C}{D(a_i + b_i)}\right] \left[\frac{\bar{K} + (1-\alpha_m)\tilde{s}}{\rho_m \alpha_m \sigma_\varepsilon^2}\right]. \tag{14}$$

The ex-ante expected utility of a delegation contract to the fund manager is

$$V_{m(i)} = I_i + a_i r + \frac{\bar{K}^2 + (1-\alpha_m)\sigma_\varepsilon^2}{2\rho_m \alpha_m \sigma_\varepsilon^2}. \tag{15}$$

As with direct investment, the fund manager's equity holding is increasing in the conditional mean of $\tilde{K}$, and is decreasing in its conditional variance and in ρ_m. Further, demand for equity differs across fund managers according to differences in the (relative) weights on relative versus absolute performance in their contracts (i.e., as b_i/a_i differs). Note that if b_i/a_i is relatively large for all fund managers, their portfolio holdings will tend to be similar. In other words, emphasis on relative performance evaluation creates a tendency towards herding. If we define $\overline{\lambda}_m =$

[4] The arguments that follow assume that $a_j > 0$ for at least one delegating investor. This ensures that $D > 0$. In the absence of this assumption, it can be shown that the equilibrium risk premium is necessarily zero; if so, costly delegation is not rational.

$\frac{1}{n}\sum_{j=1}^{n}\lambda_{m(j)}$ as the average equity holding in delegated portfolios, we have

$$\overline{\lambda}_m^*(\tilde{s}) = \frac{C}{D}\left[\frac{\bar{K}+(1-\alpha_m)\tilde{s}}{\rho_m\alpha_m\sigma_\varepsilon^2}\right]. \tag{16}$$

Lemma 1 also computes the value of the contract to the fund manager by aggregating the expected utility of signal-contingent choices. As we might expect, the fund manager's expected utility is increasing in a_i and I_i. Quite remarkably, the value of the linear contract to the fund manager does not depend directly on the relative performance parameter b_i.[5] To understand this, note that while fund managers' portfolio choices are sensitive to RPE, the incentive effects of changing b_i are undone by the changes in the portfolio chosen by the fund manager. This conclusion echoes similar findings in Stoughton (1993) and Admati and Pfleiderer (1997). Indeed, while Lemma 1 establishes this for the mean-variance utility function entertained here, the result is valid for any concave utility function that fund managers might have.

4.2. *The return to delegated investment and optimal delegation contracts*

The return to delegated investment is the value of the portfolio net of the manager's remuneration: $\tilde{R}_{d(i)} = \tilde{W}_i - \tilde{R}_{m(i)}$. It depends on the remuneration contract parameters and the associated portfolio choices made by the fund manager. As the latter may depend on rival fund managers' contracts, so would the net return from delegation. The value of a delegation contract to the delegating investor is given by the following Lemma. For ease of notation, we define $M_i = \frac{D(1-a_i-b_i)+b_iC}{(a_i+b_i)D}$.

Lemma 2 *Consider an investor with risk aversion ρ_i who delegates investment to a fund manager with risk aversion ρ_m using a contract (I_i, a_i, b_i). The ex-ante expected utility of the net return to the delegating investor is*

$$V_{d(i)}(I_i,a_i,b_i) = (1-a_i)r - I_i + \left[\frac{\bar{K}^2+(1-\alpha_m)\sigma_\varepsilon^2}{\rho_m\alpha_m\sigma_\varepsilon^2}\right]\left[1-\frac{\rho_i}{2\rho_m}M_i\right]M_i. \tag{17}$$

Each delegating investor chooses the contract parameters to maximise $V_{d(i)}$. Of course, a fund manager will willingly accept a remuneration contract only if the

[5]The parameter b_i, along with the contract parameters of rival fund managers, may affect the fund manager's utility through the equilibrium value of $\bar{K}$, but this effect is indirect.

expected value of the contract, $V_{m(i)}$, exceeds his reservation utility ϕ_m. Thus, each delegating investor must choose (a_i, b_i, I_i) to maximise $V_{d(i)}$, subject to the following participation constraint

$$I_i + a_i r + \frac{\bar{K}^2 + (1-\alpha_m)\sigma_\varepsilon^2}{2\rho_m \alpha_m \sigma_\varepsilon^2} \geq \phi_m, \tag{18}$$

and the conditions that $a_i \geq 0$, $b_i \geq 0$, and $I_i \geq 0$.

Note that the objective function, $V_{d(i)}$, depends on the contract parameters I_i and a_i directly, and on b_i through the term M_i. The participation constraint depends only on I_i and a_i. The existence of a lower bound on I_i creates the possibility that the participation constraint may not bind, say, for ϕ_m small enough. Indeed, since I_i has no influence on portfolio choice, optimal contracts will assign it the lowest possible value when the participation constraint does not bind. The following Lemma describes the structure of the optimal contract.

Lemma 3 *Consider an investor with risk aversion ρ_i choosing a contract (I_i, a_i, b_i) to delegate the investment decision to a fund manager with risk aversion ρ_m:*
(i) If the participation constraint binds, the optimal contract chooses a_i and b_i so that $M_i = \frac{\rho_m}{\rho_i}$ and I_i is set so that the participation constraint just binds.
(ii) If the participation constraint does not bind, the optimal contract sets $I_i = 0$, $a_i = D/C$, and b_i satisfies

$$\frac{\bar{K}^2 + (1-\alpha_m)\sigma_\varepsilon^2}{\rho_m \alpha_m \sigma_\varepsilon^2} \left(\frac{\rho_i}{\rho_m} \frac{1-a_i}{a_i} - 1 \right) \left(\frac{1}{a_i} \right) \left(\frac{1}{a_i + b_i} \right) = r. \tag{19}$$

When the participation constraint binds, the optimal M_i aligns the fund manager's choices to the risk preferences of the delegating investor – specifically, it corrects for any divergence between ρ_i and ρ_m – while the choice of $I_i > 0$ ensures that the participation constraint is satisfied. Since M_i depends on both a_i and b_i, the optimality condition does not determine these parameters uniquely. The relationship between optimal a_i and b_i is complicated.[6] As we shall see, for this case, relative performance evaluation does not serve any essential purpose: any outcome that positive values of b_i achieve can be replicated by a suitable choice of a_i.

[6] It can be shown that, if the participation constraint binds for all delegating investors, the optimal a_i is increasing (decreasing) in b_i for investors whose risk aversion is above (below) the average for all delegating investors.

In the latter case, the restriction that I_i be non-negative imposes a binding constraint on the contract. The unconstrained optimum would have chosen a negative value for I_i, but the non-negativity constraint makes that choice infeasible. The participation constraint does not bind here because the constraint $I_i \geq 0$ does. To understand the properties of this constrained-optimum, let $(\hat{a}_i, \hat{b}_i)$ denote a solution to equation (19) for a given $\bar{K}$. The requirement that $\hat{a}_i = D/C$ implies that the optimal weight on absolute performance is the same for all delegation contracts that are constrained-optimal. Any heterogeneity in delegating investors' risk preferences ρ_i must then be accommodated through differences in the choice of $\hat{b}_i$. Also, while D/C (and hence, $\hat{a}_i$) may be fixed from a single investor's perspective, the restrictions on the optimal contract are compatible with multiple solutions $(\hat{a}_i, \hat{b}_i)$, corresponding to different values for D/C. Lastly, it follows from equation (19), that $\hat{b}_i$ is decreasing in $\hat{a}_i$: optimal contracts that place greater emphasis on absolute performance, place less weight on relative performance.[7]

Proposition 2 examines the implications of these contract structures for fund managers' portfolio choices.

Proposition 2 *Consider an investor with risk aversion ρ_i who delegates the investment to a fund manager with risk aversion ρ_m using optimally-chosen contract parameters. The optimal portfolio choice of the fund manager is*

$$\lambda^*_{m(i)} = \left(\frac{1}{\rho_i} + \frac{1}{\rho_m}\right)\left[\frac{\bar{K} + (1-\alpha_m)\tilde{s}}{\alpha_m \sigma_\varepsilon^2}\right] \tag{20}$$

if the participation constraint binds, and

$$\lambda^*_{m(i)} = \frac{1}{\hat{a}_i}\frac{1}{\rho_m}\left[\frac{\bar{K} + (1-\alpha_m)\tilde{s}}{\alpha_m \sigma_\varepsilon^2}\right] = \overline{\lambda}^*_m \tag{21}$$

if the participation constraint does not bind.

This Proposition shows that for optimal linear contracts (i.e. those with binding participation constraints), a fund manager's demand for equity depends, ultimately, on the risk aversion of the delegating investor. This result obtains because demand depends on M_i, rather than a_i and b_i directly. Hence, all combinations of a_i and

[7] To see why, note that for (19) to hold at $r > 0$, $a_i > 0$, and $b_i \geq 0$, we must have $\frac{\rho_i}{\rho_m}\frac{1-a_i}{a_i} - 1 > 0$. Evaluating $\frac{\partial b_i}{\partial a_i}$ for this range of values proves the claim.

b_i that are consistent with $M_i = \frac{\rho_m}{\rho_i}$ lead to the same level of demand. Note that as linear contracts entail risk sharing between the investor and the fund manager, demand also varies with the fund manager's risk aversion.

For linear contracts that are only constrained-optimal, varying the performance parameters does affect the demand for equity. Here, increased weight on relative performance (i.e. a higher value of $\hat{b}_i$), implies a lower weight on absolute performance (i.e., $\hat{a}_i$ must fall to maintain constrained-optimality). Equation (21) shows demand for equity to be decreasing in $\hat{a}_i$; thus, demand increases as the weight on relative performance increases. This, as we see later, has marked implications for the equilibrium equity premium.[8]

The two cases also differ in the pattern of equity holdings across investors. With optimal linear contracts, heterogeneity in delegating investors' risk aversion will lead to heterogeneity in portfolio holdings. While RPE creates a general tendency to herd, the optimal choice of contract parameters re-aligns fund managers' choices to investors' preferences, mitigating the tendency. In contrast, constrained-optimal contracts display identical $\hat{a}_i$ inducing fund managers to herd: with similar risk aversion and information, they hold identical portfolios.

The tendency to herd in the presence of RPE-based contracts has been noted extensively in the literature, both empirical and theoretical. Empirical evidence reported by Thomas and Tonks (2000) suggests that UK pension funds are "closet" trackers. They found similar patterns of returns in a large sample of more than 2000 segregated UK pension funds. At the theoretical level, Maug and Naik (1996) model a situation in which RPE contracts can induce fund managers to ignore their own superior information. Herding may also be the consequence of strategic interaction (Eichberger et al (1999)), to protect loss of reputation (Scharfstein and Stein (1990)), or due to free-riding in the information acquisition process. Our model abstracts from heterogeneity in information among fund managers. In our setting, herding is a consequence of potential constraints in optimal contract design.

[8] Note also that as $\frac{\rho_i}{\rho_m}\frac{1-\hat{a}_i}{\hat{a}_i} - 1$ must be positive to solve (19) for $r > 0$, $\hat{a}_i > 0$ and $\hat{b}_i \geq 0$, it follows that $\frac{1}{\hat{a}_i}\frac{1}{\rho_m} > \frac{1}{\rho_i} + \frac{1}{\rho_m}$. Comparing (20) and (21) shows that the constrained-optimal contract leads to higher demand than the optimal contract.

4.3. *The delegation decision*

Delegation is rational for an investor if and only if utility from the optimal delegation contract exceeds the value of direct investment. To assess this, we begin by evaluating the utility of the optimal delegation contract for delegating investors.

Proposition 3 *Consider an investor with coefficient of risk aversion ρ_i who delegates the investment to a fund manager with risk aversion ρ_m and reservation utility ϕ_m. If the participation constraint binds, the ex-ante expected utility of return to delegated investment equals*

$$V_{d(i)} = \left(\frac{1}{\rho_i} + \frac{1}{\rho_m}\right)\left[\frac{\bar{K}^2 + (1-\alpha_m)\sigma_\varepsilon^2}{2\alpha_m\sigma_\varepsilon^2}\right] + r - \phi_m. \tag{22}$$

If the participation constraint does not bind the ex-ante expected utility is

$$V_{d(i)} = \frac{1}{\rho_m}\frac{1-\hat{a}_i}{\hat{a}_i}\left[1 - \frac{\rho_i}{2\rho_m}\frac{1-\hat{a}_i}{\hat{a}_i}\right]\left[\frac{\bar{K}^2 + (1-\alpha_m)\sigma_\varepsilon^2}{\alpha_m\sigma_\varepsilon^2}\right] + (1-\hat{a}_i)r. \tag{23}$$

Propositions 1 and 3 allow us to describe the condition for rational delegation, by comparing $V_{d(i)}$ with $V_{o(i)}$. It aids intuition to express the condition in terms of 'risk tolerances' (i.e., the inverse of the coefficients of risk aversion). Define $\tau_i = \frac{1}{\rho_i}$ and $\tau_m = \frac{1}{\rho_m}$. Comparing (13) and (22), for the case where the participation constraint binds, rational delegation requires

$$(\tau_i + \tau_m)\left[\frac{\bar{K}^2 + (1-\alpha_m)\sigma_\varepsilon^2}{2\alpha_m\sigma_\varepsilon^2}\right] + r - \phi_m \geq \tau_i\left[\frac{\bar{K}^2 + (1-\alpha_i)\sigma_\varepsilon^2}{2\alpha_i\sigma_\varepsilon^2}\right] + r\ ,$$

or equivalently

$$\frac{\bar{K}^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2}\left[\frac{\tau_i}{\alpha_m} - \frac{\tau_i}{\alpha_i} + \frac{\tau_m}{\alpha_m}\right] \geq \phi_m + \frac{\tau_m}{2}. \tag{24}$$

Since $\alpha_m < \alpha_i$, the left hand side is positive, so delegation is rational if ϕ_m is not too large. Further, the gain from delegation is higher for investors with less informative signals (i.e., those with greater α_i) and those with relatively high risk tolerance (those with higher τ_i). Lastly, the gain from delegation is increasing in

$\bar{K}$: other things being the same, higher values of the equilibrium risk premium will support greater delegation.[9] We turn next to the determination of this premium.[10]

5. **Equilibrium**

Asset market equilibrium requires that aggregate demand for equity equal the supply, Q. Aggregate demand has two components: the demand from direct investors and that from fund managers investing on behalf of delegating investors. The equity premium $\bar{K}(P_0)$ affects the level of demand from each agent. It also affects the extent of delegation: given that the number of delegating investors is denoted by n, we have $n = n(\bar{K})$. The market clearing condition is

$$\sum_{i=1}^{n(\overline{K})} \lambda^*_{m(i)}(\bar{K}) + \sum_{i=n(\overline{K})+1}^{N} \lambda^*_{o(i)}(\bar{K}) = Q. \tag{25}$$

Demand is also sensitive to the signals received by investors and fund managers. It is possible that the market does not clear for very extreme realizations of the signals. We discuss the issue of existence for the case where signals take values that are not too extreme.

Note that each category of demand – direct or delegated – is increasing and continuous in $\bar{K}$. Aggregate demand may not be as well-behaved. The level of demand differs for direct and delegated investment. Recall, from Propositions 1

[9] Similarly, we could compare (13) and (23) to obtain a condition for rational delegation when the participation constraint does not bind. The delegation condition simplifies to

$$\frac{\tau_m}{\alpha_m}\left[\frac{\bar{K}^2+\sigma_\varepsilon^2}{2\sigma_\varepsilon^2}-\frac{\alpha_m}{2}\right]\left[2-\frac{\tau_m}{\tau_i}\frac{1-\hat{a}_i}{\hat{a}_i}\right]\frac{1-\hat{a}_i}{\hat{a}_i} \geq \frac{\tau_i}{\alpha_i}\left[\frac{\bar{K}^2+\sigma_\varepsilon^2}{2\sigma_\varepsilon^2}-\frac{\alpha_i}{2}\right] + \hat{a}_i r.$$

Once again the incentive to delegate is higher for investors with less informative signals and relatively high risk tolerance.

[10] Our model ignores the possibility of partial delegation. When binding non-negativity constraints restrict delegating investors to choosing constrained-optimal contracts, delegating only part of their wealth may allow them to circumvent the binding non-negativity constraint, at least for some parameter configurations. However, the gain from moving to fully optimal contracts for the delegated part of the investment must be traded against the inefficiency of investing the rest directly, with inferior information, so that it will not in general be optimal to circumvent the non-negativity constraint entirely. Our model can be extended to incorporate this, losing some simplicity in the process, and without affecting the qualitative arguments. See also the related discussion on 'coordination' in Admati and Pfleiderer (1997).

and 2, that directly invested portfolios hold

$$\lambda^*_{o(i)} = \frac{1}{\rho_i}\left[\frac{\bar{K} + (1-\alpha_i)\tilde{z}_i}{\alpha_i \sigma^2_\varepsilon}\right],$$

while optimally delegated portfolios hold

$$\lambda^*_{m(i)} = \left(\frac{1}{\rho_i} + \frac{1}{\rho_m}\right)\left[\frac{\bar{K} + (1-\alpha_m)\tilde{s}}{\alpha_m \sigma^2_\varepsilon}\right],$$

when the participation constraint binds. Since $\alpha_m < \alpha_i$ and $\rho_m > 0$, it follows that as long as the conditional equity premium is positive, delegated portfolios hold more equity than the corresponding direct investment portfolios for similar signals (i.e., for $\tilde{s} \approx \tilde{z}_i$). (Recall that with constrained-optimal contracts, demand for equity holdings is larger still). Thus, for values of $\bar{K}$ at which an investor is indifferent between direct and delegated investment, the demand for equity has two distinct solutions: we have a demand correspondence rather than a demand function. Nonetheless, standard arguments suggest that aggregate demand may be almost continuous if the population of investors is large and if individual preferences are sufficiently dispersed. Aggregate demand is clearly increasing in $\bar{K}$: each category of demand is increasing in $\bar{K}$, and the extent of delegation $n(\bar{K})$ is increasing in $\bar{K}$, so that higher values of $\bar{K}$ place greater weight on higher levels of demand.[11]

If aggregate demand is monotone and almost continuous, an equilibrium will exist as long as demand varies sufficiently along the set of feasible prices. If aggregate demand is less than Q when $\bar{K} = 0$, and larger than Q when $\bar{K}$ is very large, an equilibrium exists. For sufficiently low values of $\bar{K}$, aggregate demand for the risky asset is arbitrarily small, at least for signals close to the average. The largest value $\bar{K} = \overline{P}_1 - P_0 r$ can take (assuming P_0 is non-negative) is $\overline{P}_1$. We assume that aggregate demand for the risky asset exceeds its supply Q at this price. Thus, the usual fixed point arguments establish the existence of a unique equilibrium under broad regularity conditions. We assume that these conditions hold for the sample economies we study below.

[11] When the participation constraint does not bind, we may need additional conditions on the model parameters to ensure that aggregate demand is increasing in $\bar{K}$. Likewise, if $s \ll z_i$, delegated portfolios may hold less than direct investment portfolios, so that greater delegation at higher $\bar{K}$ could potentially lower aggregate demand.

5.1. *Implications for the equity risk premium*

The finding that delegated portfolios have higher holdings of the risky asset have direct implications for the equity risk premium. Parameter changes that affect the extent of delegation will alter the equilibrium premium. For instance, an improvement in the precision of fund managers' signals relative to that of investors' signals increases the incentive to delegate. Given that delegated portfolios have comparatively higher demand for equity, this change will be associated with a lower equity risk premium at the equilibrium. Example 1 below illustrates this effect.

Apart from the effect through changing delegation levels, the equity premium may depend on the structure of remuneration contracts. When investors can choose the contract optimally, demand for equity, and hence the equilibrium equity premium, does not depend on the contract parameters. However, there is a real possibility that non-negativity constraints on I_i may restrict the feasible set of contracts. The prevalence of actual contracts based purely on performance (i.e., those with no performance-independent component) lend some plausibility to this possibility. When contracts are only constrained-optimal, the choice of contract parameters matters. Lemma 3 tells us that for this case the problem of designing optimal delegation contracts admits multiple solutions $(\hat{a}_i, \hat{b}_i)$. Further, $\hat{a}_i$ is decreasing in $\hat{b}_i$ and demand for equity is decreasing in $\hat{a}_i$. Thus, optimal contracts that place greater emphasis on relative performance evaluation (and correspondingly less on absolute performance) lead to greater demand for equity. For a fixed supply, this greater emphasis on RPE will lead to lower equity premia at the equilibrium. Example 2 demonstrates this for a simple case.

The preceding arguments can be summarised thus:

Proposition 4 *Consider the equity market equilibrium given by equation (25). An increase in the weight on relative performance evaluation does not affect the equity premium when investors can choose the linear delegation contract optimally. However, with constrained-optimal contracts, an increase in relative performance evaluation tends to reduce the equilibrium equity premium.*

Our model suggests that higher levels of delegation may result in a decline in the equilibrium risk premium. Recent empirical evidence (see Claus and Thomas (2001), Fama and French (2002), and the surveys by Welch (2000), Graham and Harvey (2001)) have discussed the possibility that the equity risk premium has declined in recent years. Our model offers a tentative and partial explanation of such a tendency.

5.2. *Two examples*

We describe two illustrative equilibria, using a special case of the information structure. These examples are meant to demonstrate qualitative the mechanisms operating in our model and not to suggest their likely magnitude. We assume the signals observed by investors are noisier versions of the signal received by fund managers, i.e.,

$$\begin{aligned} \tilde{z}_i &= \tilde{\varepsilon} + \tilde{s} + \tilde{x}_i, \\ \text{where } \tilde{x}_i &\sim N(0, \sigma_{x_i}^2) \text{ and } E(\tilde{\varepsilon}\tilde{x}_i) = E(\tilde{s}\tilde{x}_i) = 0. \end{aligned}$$

For this case, $\alpha_i = \frac{\sigma_m^2 + \sigma_{x_i}^2}{\sigma_\varepsilon^2 + \sigma_m^2 + \sigma_{x_i}^2}$. Note that, with this structure, fund manager' signals are more precise than those of investors as long as $\sigma_{x_i}^2 > 0$. We compute the equilibria assuming each agent receives the average signal, i.e., $\tilde{s} = 0$, $\tilde{z}_i = 0$.

5.2.1. **Example 1**

Consider, first, an example in which the equilibrium outcome involves binding participation constraints for fund managers. Here investor i's demand for equity is

$$\lambda_i = \begin{cases} \frac{\tau_i + \tau_m}{\alpha_m} \left(\frac{\overline{K}}{\sigma_\varepsilon^2} \right) & \text{under delegated investment} \\ \frac{\tau_i}{\alpha_i} \left(\frac{\overline{K}}{\sigma_\varepsilon^2} \right) & \text{under direct investment} \end{cases}.$$

Delegation requires that (24) must hold at the equilibrium $\bar{K}$. We assume that all investors and fund managers have the same constant absolute risk aversion of 3.3 (i.e., $\tau_i = 0.3$ for all i, and $\tau_m = 0.3$) but differ in the precision of their signals. Investors with relatively imprecise information are more likely to delegate. Consider the following numerical example. Set $\sigma_\varepsilon^2 = 0.04$, corresponding to market volatility of 20%. Let the variance of the noise in the fund manager's signal be $\sigma_m^2 = 0.2$. Assume that half the population of investors have relatively precise information given by $\sigma_{x1}^2 = 0.2$, while the rest have $\sigma_{x2}^2 = 0.6$. (Note that these values ensure that both α_1 and α_2 are larger than α_m). Without loss of generality, we set the average number of shares per investor at 1. If the reservation utility of fund managers ϕ_m is set at 0.075 in these units, type-1 investors choose to invest directly, type-2 investors delegate and the equilibrium equity premium is $\bar{K} = 0.076$, or 7.6%. At this premium, direct investors hold 0.56 units of equity while delegated portfolios hold 1.44 units.

It is easy to check that an increase in the extent of delegation would lower this premium. If the fraction of investors with relatively imprecise information rises to two-thirds, the equilibrium equity premium declines to 6.8%. This decline is clearly a consequence of greater delegation: in a model without any delegation, an increase in the average imprecision of information would *raise* the equity premium.

5.2.2. **Example 2**

Our second example considers a case in which the parameter values are such that the equilibrium outcome involves delegation by all investors and fund managers' participation constraints do not bind at the equilibrium. As before, set $\sigma_\varepsilon^2 = .04$. Let $\sigma_m^2 = 0.12$ and $\sigma_{xi}^2 = 0.6$ for all investors. Let $\tau_i = \tau_m = 0.2$, let the interest rate r be 5%, and set $\phi_m = 0.05$. When participation constraints do not bind, the choice of optimal contract parameters is given by equation (19). For any chosen value of b_i, this equation along with the market clearing equation can be solved for $\hat{a}_i$ and $\bar{K}$. We check, of course, that the participation constraint is non-binding and that delegation is optimal at this outcome.

If we set $b_i = 0$, the equilibrium equity premium is 7.1%. Increasing the weight on relative performance to $b_i = 0.5$ reduces the equity premium to 6.8%; raising it further to $b_i = 0.9$ reduces the equity premium to 6.5%. Introducing some heterogeneity among investors, so that not all delegate at the equilibrium, can demonstrate larger reductions in the equity premium.

6. **Conclusions**

In this paper, we aim to explore the equilibrium consequences of performance-based contracts for fund managers. We consider an extremely simple model, with two time periods and two assets. Investors can invest directly or delegate their portfolio choice to better-informed fund managers. We examine linear remuneration contracts, allowing fund managers' remuneration to depend on the absolute performance of funds and their performance relative to other actively-managed funds.

The structure of managers' remuneration contracts is endogenously determined, albeit within the restricted class of contracts that are linear in the performance measures. At the equilibrium, the extent of investment delegation and the equity premium are jointly determined. Characterizing the equilibrium in a model with endogenous contracts is generally very complicated. Specializing the analysis to the case where all agents have CARA utility functions allows us to solve explicitly for the equilibrium and to investigate the dependence of the equilibrium risk premium on the parameters of the remuneration contracts.

We find that delegation in and of itself has an effect on asset market equilibrium: given that fund managers are better informed than investors, delegated portfolios hold more risky assets than direct investment portfolios. As Example 1 illustrates, an increase in the extent of delegation tends to lower the equilibrium equity premium. Separately from this, the structure of remuneration contracts – in particular the relative emphasis they place on absolute versus relative performance – may affect the outcome. Whether or not it does critically depends on whether the chosen linear contract is fully optimal or only constrained optimal. With fully optimal contracts, portfolio choices are independent of how the reward for performance is distributed between absolute and relative performance. However, when the set of feasible contracts is restricted – specifically, if the choice of the performance-independent component faces a binding non-negativity constraint, so that the chosen contract is only constrained-optimal – relative performance evaluation matters. One, it creates a tendency to herd. Two, greater weight on relative performance implies a lower weight on absolute performance, and as Example 2 illustrates, a lower equity premium. Our findings suggest that more widespread use of delegation contracts, and possibly greater reliance on relative performance evaluation, could have contributed to the recently observed decline in equilibrium equity premia.

Our model is quite simple, especially in how we model the agency relationship between investors and fund managers. We focus the agency problem purely on portfolio choice. The problem of designing optimal contracts could be augmented to address issues of screening managers according to their innate ability, and providing incentives for them to exert effort to improve their information. We could embellish the model by considering multiple risky assets. A more realistic model than ours would allow richer possibilities for matching investors to fund managers, including the possibility that a manager may handle multiple funds, or that investors may use multiple managers. Realistic concerns would also allow for an alternative specification where fund managers, rather than investors, choose the contract structure, subject to investors' participation. Manager-designed fund structures could be concerned with the long-term rewards including those based on dynamics and character of future investment flows (see, for instance, Nanda et al (2000)).

More importantly, despite the appeal of symmetric contracts, it may be worthwhile to examine contracts other than linear ones. Das and Sundaram (2002) describe a model in which asymmetric contracts may sometimes be superior from the investors' perspective. In a related context, Palomino and Prat (2003) find that, in the presence of limited liability for fund managers, the optimal contract may be a bonus contract. Lastly, there are puzzles that our model does not aim to address: for instance, why investors choose costly delegation despite strong empirical evidence that the average mutual fund underperforms passive investment. A model addressing this and related questions would need to account for transaction costs for direct and pooled investments which goes well beyond the scope of this paper.

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Appendix

Proof of Proposition 1: Direct investor i chooses λ to maximise expected utility

$$E[\tilde{R}_{o(i)}] - \frac{\rho_i}{2} Var[\tilde{R}_{o(i)}] = \lambda E[\tilde{K}] + r - \frac{\rho_i}{2}\lambda^2 Var[\tilde{K}].$$

The optimal choice conditional on signal $\tilde{z}_i$ is, using (8) and (9),

$$\lambda^*_{o(i)}(z_i) = \frac{E[\tilde{K}|\tilde{z}_i]}{\rho_i Var[\tilde{K}|\tilde{z}_i]} = \frac{\bar{K} + (1-\alpha_i)\tilde{z}_i}{\rho_i \alpha_i \sigma^2_\varepsilon}.$$

Evaluating expected utility at this optimal portfolio, we get

$$\lambda^*_{o(i)} E[\tilde{K}|\tilde{z}_i] + r - \frac{\rho_i}{2}\lambda^{*2}_{o(i)} Var[\tilde{K}|\tilde{z}_i = \frac{1}{2}\frac{(\bar{K} + (1-\alpha_i)\tilde{z}_i)^2}{\rho_i \alpha_i \sigma^2_\varepsilon} + r.$$

Aggregating this across the set of signals $\tilde{Z}_i$ and using the relation

$$\begin{aligned} E[(K + (1-\alpha_i)\tilde{z}_i)^2] &= \bar{K}^2 + (1-\alpha_i)^2 Var[\tilde{z}_i] \\ &= \bar{K}^2 + (1-\alpha_i)\sigma^2_\varepsilon \end{aligned}$$

the ex-ante value of direct investment is

$$V_{o(i)} = \frac{1}{2}\frac{\bar{K}^2 + (1-\alpha_i)\sigma^2_\varepsilon}{\rho_i \alpha_i \sigma^2_\varepsilon} + r\,. \qquad \blacksquare$$

Proof of Lemma 1: The manager of fund i maximizes $E[\tilde{R}_{m(i)}] - \frac{\rho_m}{2} Var[\tilde{R}_{m(i)}]$. Define $\overline{\lambda}_m = \frac{1}{n}\sum_{j=1}^n \lambda_{m(j)}$ as the average equity holding in delegated portfolios. We can then write

$$\tilde{R}_{m(i)} = I_i + a_i r + [(a_i + b_i)\lambda - b_i \overline{\lambda}_m]\tilde{K}.$$

This has mean and variance

$$\begin{aligned} E[\tilde{R}_{m(i)}] &= I_i + a_i r + [(a_i + b_i)\lambda_m - b_i \overline{\lambda}_m)] E[\tilde{K}], \\ Var[\tilde{R}_{m(i)}] &= [(a_i + b_i)\lambda_m - b_i \overline{\lambda})]^2 Var[\tilde{K}]. \end{aligned}$$

Fund manager i's demand for equity conditional on signal $\tilde{s}$ is, using (6) and (7),

$$\begin{aligned} \lambda^*_{m(i)}(\tilde{s}) &= \left(\frac{1}{a_i + b_i}\right)\frac{E[\tilde{K}|\tilde{s}]}{\rho_m Var[\tilde{K}|\tilde{s}]} + \left(\frac{b_i}{a_i + b_i}\right)\overline{\lambda}_m \\ &= \left(\frac{1}{a_i + b_i}\right)\left[\frac{\bar{K} + (1-\alpha_m)\tilde{s}}{\rho_m \alpha_m \sigma^2_\varepsilon}\right] + \left(\frac{b_i}{a_i + b_i}\right)\overline{\lambda}_m. \end{aligned}$$

Aggregating demand across fund managers, we have

$$n\overline{\lambda}^*_m(\tilde{s}) = \left[\frac{\bar{K} + (1-\alpha_m)\tilde{s}}{\rho_m \alpha_m \sigma^2_\varepsilon}\right] \sum_{j=1}^{n} \frac{1}{(a_j + b_j)} + \overline{\lambda}^*_m(\tilde{s}) \sum_{j=1}^{n} \frac{b_j}{(a_j + b_j)}.$$

Simplifying, and using the defined notation $C = \sum_{j=1}^{n} \frac{1}{(a_j+b_j)}$, and $D = \sum_{j=1}^{n} \frac{a_j}{(a_j+b_j)}$, average holdings in delegated portfolios are

$$\overline{\lambda}^*_m(\tilde{s}) = \left[\frac{\bar{K} + (1-\alpha_m)\tilde{s}}{\rho_m \alpha_m \sigma^2_\varepsilon}\right] \frac{C}{D}.$$

Substituting this in the expression for the optimal portfolio, we have

$$\lambda^*_{m(i)}(s) = \left[\frac{D + b_i C}{D(a_i + b_i)}\right] \left[\frac{\bar{K} + (1-\alpha_m)\tilde{s}}{\rho_m \alpha_m \sigma^2_\varepsilon}\right].$$

The conditional mean and variance of the manager's remuneration at this optimal portfolio are

$$\begin{aligned} E\left[\tilde{R}_m(\lambda^*_{m(i)}(\tilde{s}))\right] &= I_i + a_i r + [(a_i + b_i)\lambda^*_{m(i)}(\tilde{s}) - b_i \overline{\lambda}^*_m(\tilde{s})] E[\tilde{K}|\tilde{s}] \\ &= I_i + a_i r + \frac{[\bar{K} + (1-\alpha_m)\tilde{s}]^2}{\rho_m \alpha_m \sigma^2_\varepsilon}, \\ Var\left[\tilde{R}(\lambda^*_{m(i)}(\tilde{s}))\right] &= \left[(a_i + b_i)\lambda^*_{m(i)}(\tilde{s}) - b_i \overline{\lambda}_m(\tilde{s})\right]^2 Var[\tilde{K}|\tilde{s}] \\ &= \frac{[\bar{K} + (1-\alpha_m)\tilde{s}]^2}{\rho^2_m \alpha_m \sigma^2_\varepsilon}, \end{aligned}$$

so that expected utility conditional on signal $\tilde{s}$ is

$$E\left[\tilde{R}_m(\lambda^*_{m(i)}(\tilde{s}))\right] - \frac{\rho_m}{2} Var\left[\tilde{R}(\lambda^*_{m(i)}(\tilde{s}))\right] = I_i + a_i r + \frac{1}{2\rho_m} \frac{(\bar{K} + (1-\alpha_m)\tilde{s})^2}{\alpha_m \sigma^2_\varepsilon}.$$

Aggregating this across $\tilde{S}$, the ex-ante expected utility of the delegation contract is

$$\begin{aligned} V_{m(i)} &= I_i + a_i r + \frac{1}{2\rho_m \alpha_m \sigma^2_\varepsilon} E_s[(\bar{K} + (1-\alpha_m)\tilde{s})^2] \\ &= I_i + a_i r + \frac{\bar{K}^2 + (1-\alpha_m)\sigma^2_\varepsilon}{2\rho_m \alpha_m \sigma^2_\varepsilon}. \qquad \blacksquare \end{aligned}$$

Proof of Lemma 2: If fund manager i chooses $\lambda_m^*(\tilde{s})$ in response to signal $\tilde{s}$, the delegating investor's net return is

$$\tilde{R}_{d(i)} = \tilde{W}_i - \tilde{R}_{m(i)} = \left[(1-a_i-b_i)\lambda_m^*(\tilde{s}) + b_i\overline{\lambda}^*(\tilde{s})\right]\tilde{K} + (1-a_i)r - I_i.$$

Using (14) and (16), and the notation $M_i = \frac{D(1-a_i-b_i)+b_iC}{(a_i+b_i)D}$, we have

$$(1-a_i-b_i)\lambda_m^*(\tilde{s}) + b_i\overline{\lambda}^*(\tilde{s}) = M_i\left[\frac{\bar{K}+(1-\alpha_m)\tilde{s}}{\rho_m\alpha_m\sigma_\varepsilon^2}\right],$$

so that the net return for the delegating investor is

$$\tilde{R}_{d(i)} = M_i\left[\frac{\bar{K}+(1-\alpha_m)\tilde{s}}{\rho_m\alpha_m\sigma_\varepsilon^2}\right]\tilde{K} + (1-a_i)r - I_i,$$

with mean and variance

$$\begin{aligned} E[\tilde{R}_{d(i)}|\tilde{s}] &= M_i\frac{[\bar{K}+(1-\alpha_m)\tilde{s}]^2}{\rho_m\alpha_m\sigma_\varepsilon^2} + (1-a_i)r - I_i, \\ Var\left[\tilde{R}_{d(i)}|\tilde{s}\right] &= \frac{M_i^2[\bar{K}+(1-\alpha_m)\tilde{s}]^2}{\rho_m^2\alpha_m\sigma_\varepsilon^2}. \end{aligned}$$

The conditional expected utility equals

$$E[\tilde{R}_{d(i)}|\tilde{s}] - \frac{\rho_i}{2}Var\left[\tilde{R}_{d(i)}|\tilde{s}\right] = (1-a_i)r - I_i + \frac{[\bar{K}+(1-\alpha_m)\tilde{s}]^2}{\rho_m^2\alpha_m\sigma_\varepsilon^2}\left[1-\frac{\rho_i}{2\rho_m}M_i\right]M_i.$$

Taking expectations across the set of signals, we obtain the ex-ante expected utility of the contract

$$V_{d(i)}(I_i,a_i,b_i) = (1-a_i)r - I_i + \left[\frac{\bar{K}^2+(1-\alpha_m)\sigma_\varepsilon^2}{\rho_m\alpha_m\sigma_\varepsilon^2}\right]\left[1-\frac{\rho_i}{2\rho_m}M_i\right]M_i. \quad \blacksquare$$

Proof of Lemma 3: The delegating investor chooses I_i, a_i and b_i to maximise

$$(1-a_i)r - I_i + \left[\frac{\bar{K}^2+(1-\alpha_m)\sigma_\varepsilon^2}{\rho_m\alpha_m\sigma_\varepsilon^2}\right]\left[1-\frac{\rho_i}{2\rho_m}M_i\right]M_i$$

subject to the participation constraint

$$I_i + a_i r + \frac{\bar{K}^2+(1-\alpha_m)\sigma_\varepsilon^2}{2\rho_m\alpha_m\sigma_\varepsilon^2} \geq \phi_m$$

and the constraints that $a_i \geq 0$, $b_i \geq 0$ and $I_i \geq 0$. Let L be the associated Lagrangean and θ be the Lagrangean multiplier associated with the participation constraint. The first-order conditions for the maximum are

$$\begin{aligned}
\frac{\partial L}{\partial a_i} &= \frac{\bar{K}^2 + (1-\alpha_m)\sigma_\varepsilon^2}{\rho_m \alpha_m \sigma_\varepsilon^2}\left(1 - \frac{\rho_i}{\rho_m} M_i\right)\frac{\partial M_i}{\partial a_i} - r + \theta r \leq 0 \\
\frac{\partial L}{\partial b_i} &= \frac{\bar{K}^2 + (1-\alpha_m)\sigma_\varepsilon^2}{\rho_m \alpha_m \sigma_\varepsilon^2}\left(1 - \frac{\rho_i}{\rho_m} M_i\right)\frac{\partial M_i}{\partial b_i} \leq 0 \\
\frac{\partial L}{\partial I_i} &= -1 + \theta \leq 0 \\
\frac{\partial L}{\partial \theta} &= I_i + a_i r + \frac{\bar{K}^2 + (1-\alpha_m)\sigma_\varepsilon^2}{2\rho_m \alpha_m \sigma_\varepsilon^2} - \phi_m \geq 0
\end{aligned}$$

with the caveat that, due to complementary slackness, an inequality holds as an equality if the relevant variable a_i, b_i, I_i or θ is strictly positive. (The second-order conditions have been verified but are tedious to report here).

For strictly positive I_i, the third relation holds as an equality. But then $\theta = 1$, which is strictly positive so that the participation constraint binds. Also, with strictly positive a_i, the first relationship holds as an equality, so that $\theta = 1$ and $\frac{\partial M_i}{\partial a_i} = \frac{-(b_i C + D)}{(a_i + b_i)^2 D} < 0$ ensure that the optimal contract must have $M_i^* = \frac{\rho_m}{\rho_i}$. The same outcome obtains for any configuration in which b_i is strictly positive.

If the participation constraint does not bind, we have $\theta = 0$, and so $I_i = 0$. For outcomes in which both a_i and b_i are strictly positive and $r \neq 0$, a solution exists only if $\frac{\partial M_i}{\partial b_i} = \frac{a_i C - D}{(a_i + b_i)^2 D} = 0$, so $a_i = D/C$, and consequently $M_i = (1 - a_i)/a_i$. Using this in the first-order condition, we can solve for the relationship between optimal b_i and a_i :

$$\frac{\bar{K}^2 + (1-\alpha_m)\sigma_\varepsilon^2}{\rho_m \alpha_m \sigma_\varepsilon^2}\left(1 - \frac{\rho_i}{\rho_m}\frac{1-a_i}{a_i}\right)\left(\frac{-1}{a_i}\right)\left(\frac{1}{a_i + b_i}\right) = r.$$

For outcomes in which only a_i is positive, this relation can be solved for the optimal a_i, setting $b_i = 0$. ■

Proof of Proposition 2: From Lemma 1, fund manager i's equity holdings are

$$\lambda_{m(i)}^*(\tilde{s}) = \left[\frac{D + b_i C}{D(a_i + b_i)}\right]\frac{1}{\rho_m}\left(\frac{\bar{K} + (1-\alpha_m)\tilde{s}}{\alpha_m \sigma_\varepsilon^2}\right).$$

If the participation constraint binds, the optimal contract has $M_i = \frac{D + b_i C}{D(a_i + b_i)} - 1 =$

$\frac{\rho_m}{\rho_i}$, so that

$$\lambda^*_{m(i)}(\tilde{s}) = \left(\frac{1}{\rho_i} + \frac{1}{\rho_m}\right)\left(\frac{\bar{K} + (1-\alpha_m)\tilde{s}}{\alpha_m \sigma^2_\varepsilon}\right).$$

If the participation constraint does not bind, $a_i = \frac{D}{C}$ and $\frac{D+b_iC}{D(a_i+b_i)} = \frac{1}{a_i} = \frac{C}{D}$, so

$$\lambda^*_{m(i)}(\tilde{s}) = \left[\frac{C}{D}\right]\left[\frac{\bar{K} + (1-\alpha_m)\tilde{s}}{\rho_m \alpha_m \sigma^2_\varepsilon}\right] = \overline{\lambda}^*_m(\tilde{s}) \qquad ■$$

Proof of Proposition 3: From Lemma 2, the expected utility of delegating investors is

$$V_{d(i)}(I_i, a_i, b_i) = (1-a_i)r - I_i + \left[\frac{\bar{K}^2 + (1-\alpha_m)\sigma^2_\varepsilon}{\rho_m \alpha_m \sigma^2_\varepsilon}\right]\left[1 - \frac{\rho_i}{2\rho_m} M_i\right] M_i.$$

When the participation constraint binds, $M_i = \frac{\rho_m}{\rho_i}$. Using this, and substituting from the participation constraint, we get

$$V_{d(i)} = \frac{1}{2}\left(\frac{1}{\rho_i} + \frac{1}{\rho_m}\right)\left(\frac{K^2 + (1-\alpha_m)\sigma^2_\varepsilon}{\alpha_m \sigma^2_\varepsilon}\right) + r - \phi_m.$$

When the participation constraint does not bind, we have $I_i = 0$ and $M_i = \frac{1-\hat{a}_i}{\hat{a}_i}$. Evaluating the above expression at these values yields the result ■

Proof of Proposition 4: Follows directly from the arguments in the text. ■