Explaining the Beta, Size and Value Effects

Under the Relative Value Theory

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ABSTRACT

The RVT predicts equilibrium prices in a world where investors ignore variance and only care about cumulative returns. Such prices determine intrinsic returns that satisfy the CAPM equation. This paper shows that assets that pay a constant (or constantly increasing) dividend but face each year the possibility of going bankrupt will exhibit asset specific cumulative returns. These returns depend not only on β but also on the probability of survival and the growth rate. The derived equations explain the β, size and value effects previously documented by several authors. Surprisingly, the RVT predicts slightly higher discount rates than the CAPM. Empirical evidence supporting the CAPM cannot reject the RVT at a significant level of confidence. The RVT explains cumulative returns better than alternative models, using a lighter set of assumptions.

Latane [1967] shows that maximizing the geometric expected return is the investing strategy that will certainly outperform any other strategy in terms of long-term cumulative returns. Alb [2001] shows that in a world where time value of money is unique, and investors are risk-indifferent in the sense they only care about maximizing cumulative returns, the market invariably evolves to an equilibrium characterized by all investors holding the market, and equilibrium prices being:

$$P_A = \left(\sum_{i=1}^{n} p_i \frac{A_i}{M_i} \right) \cdot GM \tag{1}$$

where:

P_A is the market value of asset A at equilibrium (equilibrium price)

n is the number of possible scenarios (ways in which the future can unfold)

p_i is the probability of scenario i

A_i is the intrinsic value of asset A under scenario i (present value of A's future dividends discounted at the unique time value of money rate T)

M_i is the market's intrinsic value under scenario i (obviously $M_i = A_i+B_i+Ci+\ldots$)

$GM = \prod_{i=1}^{n} M_i^{p_i}$ is the geometric mean of the market (obviously $GM = P_A+P_B+P_C+\ldots$)

Alb [2004] shows that these equilibrium prices determine intrinsic expected returns (from price P_A to intrinsic values A_i) that satisfy the CAPM equation, but stops short of explaining asset specific cumulative returns. RVT equilibrium prices were all expected to grow at the unique time value of money rate T.

The present paper shows that, under the RVT, assets that are expected to pay constant (or constantly increasing) dividends but face each year a probability of going bankrupt will generate asset specific market returns.

The key factor in explaining asset specific cumulative returns under the RVT is the constant flow of new information. In the simple case mentioned above, at the beginning of any given year investors do not know whether the company will survive or not but by the end of the year this new piece of information will become apparent. Equilibrium prices adjusting to this constant stream of new information generates asset specific cumulative returns.

The paper proceeds as follows: In section I asset specific cumulative returns are derived for three distinct categories of assets. Section II discusses RVT's differences and similarities with the CAPM. Section III considers a numerical example and plots RVT returns against CAPM's security market line. Section IV includes additional comments about the model's implications.

I. Asset specific cumulative returns

Let's consider a risky asset A, which is expected to pay a constant dividend D_A until it goes bankrupt. Asset A is also characterized by a survival rate p, representing the probability of not going bankrupt in any given year. It can be shown that such asset will generate a cumulative return equal to (see Appendix A for details):

$$\hat{R}_A = 1 + (\frac{T}{p} - 1) \cdot E(R_A) \qquad (2)$$

where:

$\hat{R}_A$ is the market return (gross) of asset A

T is the unique time value of money rate (gross)

p is the survival rate of asset A

$E(R_A) = \sum_{i=1}^{n} p_i \frac{A_i}{P_A}$ is the intrinsic expected return (gross) of asset A (intrinsic returns are returns from equilibrium price P_A to underlying intrinsic values A_i)

Obviously $\hat{R}_A$ is specific to asset A because it depends on both the survival rate p and the intrinsic expected return $E(R_A)$, which does satisfy the CAPM equation and consequently depends on the asset specific α coefficient[2].

Let's now consider a risky asset B that is expected to pay a constantly increasing dividend and is characterized by a survival rate p and growth rate g. It can be similarly shown that asset B will generate a market return equal to (see Appendix B for details):

$$\hat{R}_B = g + (\frac{T}{p} - g) \cdot E(R_B) \qquad (3)$$

where:

$\hat{R}_B$ is the market return (gross) of asset B

g is the annual growth rate of dividends (gross)

p is the survival rate of asset B

$E(R_B)$ is the intrinsic expected return (gross) of asset B

Let’s also consider a risky asset D, which is expected not to pay any dividend for n+1 years, and then to start paying a constant (or constantly increasing) dividend until it goes bankrupt. It can be shown that, in the period before dividends, asset B will generate a market return equal to (see Appendix C for details):

$$\hat{R}_D = \frac{T}{p} \tag{4}$$

where:

$\hat{R}_D$ is the market return (gross) of asset D

p is the survival rate of asset D in the period before dividends

Obviously, as asset D enters the dividend-paying phase its market return will switch to the appropriate market return, described in either equation 2 or 3. Equation 4 still applies if, after n+1 years, asset D instead of starting to pay dividends will generate a one-time payoff from a given probability distribution. Obviously, if asset D does not face the possibility of going bankrupt during the n+1 years period (in other words if p equals 1 in that period), then it will generate a market return equal to the unique time value of money rate T.

Equations 2, 3 and 4 describe asset specific market returns for three distinct categories of risky assets. These equations indicate that cumulative returns depend not only on β but also on the survival rate p and the growth rate g. The above models are sufficient to explain asset specific market returns under the RVT framework.

Shares of most public companies can be reasonably described using one of the simple models described above. More complicated models are possible, for instance assets with constantly increasing dividends facing each year a probability of growth ceasing for good, or assets with dividends being correlated to total market dividends. Alternative models are unlikely to dramatically change the above results.

II. Returns predicted by the RVT versus returns predicted by the CAPM

Alb [2004] showed that under the RVT intrinsic returns satisfy the CAPM equation (actually an equation that is slightly different, using α instead of β, with α closely approximating β). By replacing the intrinsic expected return of asset A in equation 3 with its "CAPM" expression and looking, for simplicity, at the net return of asset A, we get:

$$\hat{R}_A = g - 1 + (\frac{T}{p} - g) \cdot \left[R_f + \alpha \cdot \left(E(R_M) - R_f\right)\right] \qquad (5)$$

where:

$\alpha = \frac{Cov(R_A, \frac{1}{R_M})}{Cov(R_M, \frac{1}{R_M})}$ is the α coefficient, defined using intrinsic returns

$E(R_M)$ is the expected intrinsic return of the market

R_f is the intrinsic return of the risk-free asset

Alb [2004] shows that scenario probabilities randomly fluctuating around a central set of values would generate price movements that in turn would determine β coefficients that closely approximate the α coefficients. The situation is slightly different now because the above models bring dividends and bankruptcy into the picture. However, β should still decently approximate α (see Appendix D for a discussion):

$$\beta = \frac{Cov(\hat{R}_A, \hat{R}_M)}{Cov(\hat{R}_M, \hat{R}_M)} \approx \frac{Cov(R_A, {}^{1}\!/_{R_M})}{Cov(R_M, {}^{1}\!/_{R_M})} = \alpha \tag{6}$$

where:

R_A and R_M are intrinsic returns for asset A and, respectively, the market

$\hat{R}_A$ and $\hat{R}_M$ are total cumulative returns (price returns plus dividend yields)

Since under the RVT the market is priced at the geometric mean, the long-term cumulative return for the market portfolio will equal T:

$$\hat{R}_M = T - 1 \tag{7}$$

Also, from equation 2, the risk-free rate will be:

$$\hat{R}_f = (T - 1) \cdot R_f \tag{8}$$

Considering equations 5, 6, 7 and 8 it is now obvious that market returns predicted by the RVT are different but quite similar to those predicted by the CAPM. Equations 7 and 8 also explain the equity "premium" since, as Alb [2004] showed, the intrinsic return of the risk-free asset R_f is a ratio of harmonic and geometric means and is consequently smaller than 1, which implies $\hat{R}_f$ is smaller than $\hat{R}_M$. It is worth noting that the RVT does not need to assume investors care about variance in order to explain the market premium.

III. A numerical example and graph

A picture is worth a thousand words, so let's consider a simple numerical example and draw a graph. This exercise will highlight not only the surprising consistency of RVT's predictions with empirical data but also the practical simplicity of the model[4].

We assume the unique time value of money rate T equals 1.1 (10% a year). After considering GDP data and other relevant factors investors roughly agree on the probability distribution of market payoffs described in Table I. Five scenarios seem possible each having an associated probability and payoff. Market payoffs are nothing but the present value (discounted at T) of the market's stream of dividends for each scenario.

Table I Probability distribution of market payoffs

The table describes a simple probability distribution of market payoffs. Five possible scenarios are considered. Each row corresponds to one scenario and indicates its probability and market payoff (present value of future dividends discounted at the unique time value of money T).

Scenarios \ Payoffs	Market payoff (B)	Probabilty
Scenario 1	$100	0.15
Scenario 2	$500	0.20
Scenario 3	$800	0.30
Scenario 4	$900	0.25
Scenario 5	$1,600	0.10

The data in Table I is sufficient to compute the total market capitalization (geometric mean), the intrinsic return of the risk-free asset (harmonic over geometric mean), the expected intrinsic return of the market (arithmetic over geometric mean), and the net cumulative return of the risk-free asset (equation 8). Obviously the net cumulative return of the market will equal T (equation 7):

$$GM = \prod M_i^{\,p_i} \approx \$588.4$$

$$R_f = \frac{HM}{GM} = \frac{1}{(\sum \frac{p_i}{M_i}) \cdot \prod M_i^{\,p_i}} \approx 0.6498$$

$$E(R_M) = \frac{AM}{GM} = \frac{\sum p_i M_i}{\prod M_i^{\,p_i}} \approx 1.2576$$

$$\hat{R}_f = (T-1) \cdot R_f \approx .0650$$

$$\hat{R}_M = (T-1) \approx 0.1$$

RVT equilibrium prices are given by equation 1, which can be also written:

$$P_A = \left[Corr(A, \frac{1}{M}) \cdot \sigma_A \cdot \sigma_{\frac{1}{M}} + E(A) \cdot E(\frac{1}{M}) \right] \cdot GM \qquad (9)$$

where:

σ_A is the standard deviation of payoffs for asset A (see equation A12)

$\sigma_{\frac{1}{M}}$ is the standard deviation of $\frac{1}{M}$ and can be computed using Table I

$E(A)$ is the expected payoff of asset A (see equation A10)

$E(\frac{1}{M})$ is the expected value of $\frac{1}{M}$ and can be computed using Table I

The equilibrium prices for any given asset can be easily computed by simply estimating the correlation between the asset's payoffs and the inverse of the market's payoffs. I believe that a rational estimation of such correlation is possible by looking at the nature of the business, and past GDP and financial statement data. Computing the equilibrium price for asset A will determine immediately its market return and dividend yield.

Alternatively, if it is reasonable to assume investors can directly estimate α, the total market return and dividend yield can be computed by simply plugging α, the survival rate p and the growth rate g into equation 5.

Let's plot RVT returns for a variety of assets. For every integer α between –1 and 3 we consider four different assets: one with survival rate p=0.99 and growth rate g=1, one with p=0.97 and g=1, one with p=0.99 and g=1.04, and one with p=0.97 and g=1.04.

Table II contains total market returns (price return + dividend yield) for each of the above assets plus the risk free asset (p=1, g=1, and α=0). Figure 1 plots cumulative returns as a function of β, and the security market line predicted by the CAPM (based on the assumption $\alpha \approx \beta$).

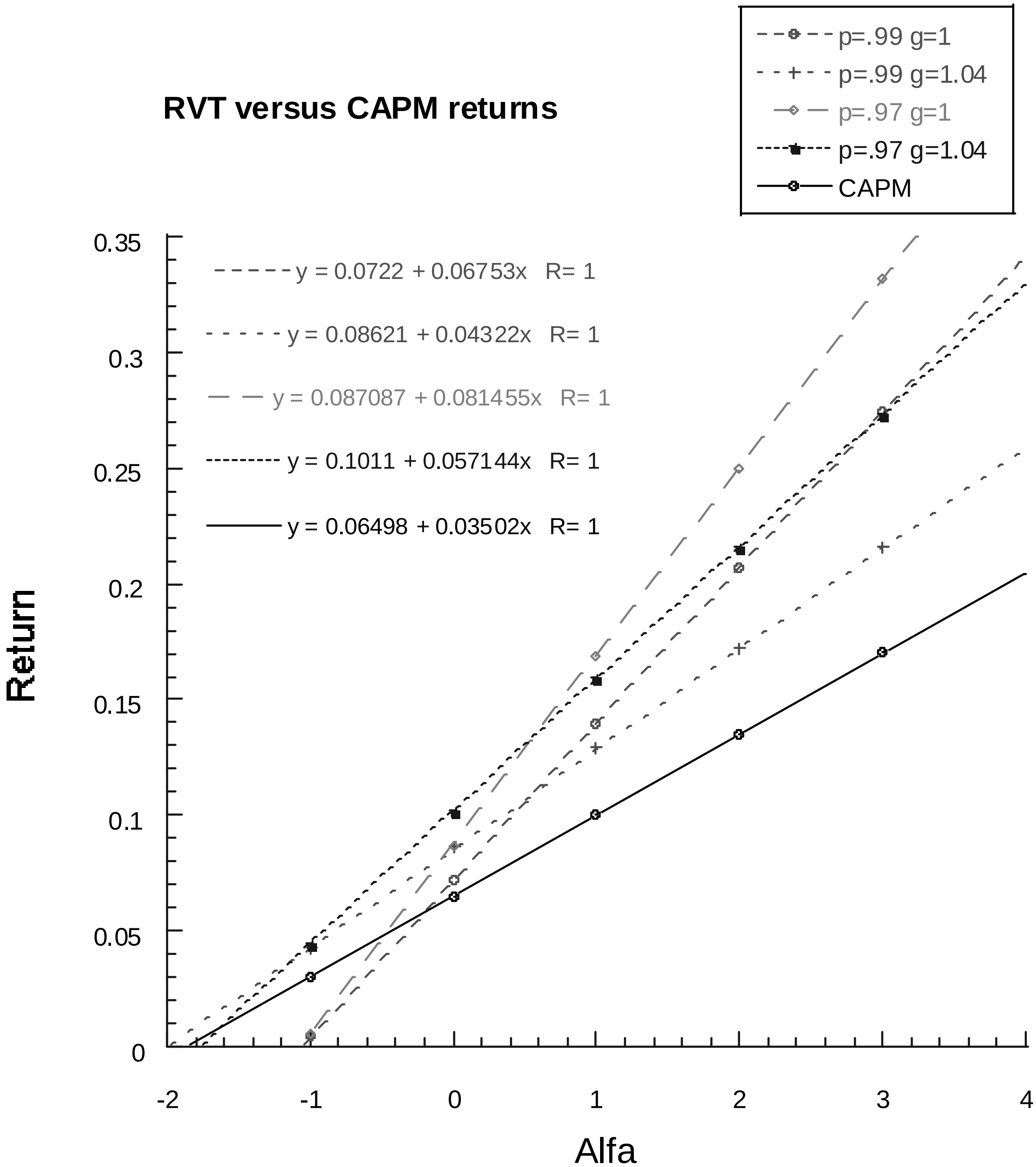
Table II RVT versus CAPM cumulative returns

The table includes total market returns (price returns plus dividend yields) predicted by the RVT for assets with different α coefficients, surviving rates p, and growth rates g. Each row corresponds to one asset. Market returns predicted by the CAPM are included in the last column.

Returns / Assets	Alfa coefficient	Survival rate (p)	Growth rate (g)	**RVT total returns**	**CAPM returns**
Asset 1	-1	0.99	1.00	**0.00467**	**0.02996**
Asset 2	-1	0.99	1.04	**0.04299**	**0.02996**
Asset 3	-1	0.97	1.00	**0.00563**	**0.02996**
Asset 4	-1	0.97	1.04	**0.04395**	**0.02996**
Risk-free asset	***0***	***1***	***1***	***0.06498***	***0.06498***
Asset 5	0	0.99	1.00	**0.07220**	**0.06498**
Asset 6	0	0.99	1.04	**0.08621**	**0.06498**
Asset 7	0	0.97	1.00	**0.08709**	**0.06498**
Asset 8	0	0.97	1.04	**0.10110**	**0.06498**
Asset 9	1	0.99	1.00	**0.13973**	**0.10000**
Asset 10	1	0.99	1.04	**0.12943**	**0.10000**
Asset 11	1	0.97	1.00	**0.16854**	**0.10000**
Asset 12	1	0.97	1.04	**0.15824**	**0.10000**
Asset 13	2	0.99	1.00	**0.20726**	**0.13502**
Asset 14	2	0.99	1.04	**0.17265**	**0.13502**
Asset 15	2	0.97	1.00	**0.25000**	**0.13502**
Asset 16	2	0.97	1.04	**0.21538**	**0.13502**
Asset 17	3	0.99	1.00	**0.27479**	**0.17004**
Asset 18	3	0.99	1.04	**0.21587**	**0.17004**
Asset 19	3	0.97	1.00	**0.33145**	**0.17004**
Asset 20	3	0.97	1.04	**0.27253**	**0.17004**

Figure 1 **RVT returns versus the CAPM security market line**

The figure plots RVT cumulative returns as a function of α. Four different categories of assets are considered, each having a given survival rate p and growth rate g. The CAPM security market line is also depicted (based on the assumption β ≈ α). The graph uses data from Table II. Both models predict linear relationships between returns and β, but the RVT predicts that returns also depend on p and g. Surprisingly, the RVT predicts higher "discount rates" than the CAPM.



It is now obvious the RVT predicts market returns that are different but similar to those predicted by the CAPM. In light of these surprising similarities, it appears that empirical evidence supporting the CAPM cannot be used to reject the RVT at a significant level of confidence.

A surprising implication of the model is that investors who only care about cumulative returns will discount risky assets more than investors who are interested in mean variance optimization. Consequently, observing higher returns for higher βs does not prove that investors try to minimize variance.

Equation 5 implies that market price returns depend not only on α but also on p and g (the probability of survival and the growth rate). These parameters might be related to α. A positive relation between p and α, or a negative relation between g and α would strengthen the similarity between RVT and CAPM predictions.

IV. Additional comments

The RVT models described above provide a theoretical explanation for the size effect first reported by Banz [1981] and further investigated by Fama and French [1992]. Equations 2, 3 and 4 all predict higher returns for companies with lower survival rates p. Consequently, assuming small companies are more likely to go bankrupt than big companies (with the same β) allows the RVT to explain the size effect. Assuming small companies also have higher growth rates should diminish but not cancel the size effect because the impact of survival rates appears to dominate the impact of growth rates (see Figure 1).

The RVT model also provides a theoretical explanation for the value effect documented by several authors (Basu [1983], Rosenberg, Reid, and Lanstein [1985], Fama and French [1992]). Equation 3 indicates that in general[5] companies with higher growth rates will generate lower returns. Consequently, the RVT model can explain value stocks outperforming growth stocks (with the same β). The impact of survival rates on returns adds to this value effect.

The cumulative returns described in equations 2, 3, and 4 assumed no change in the time value of money rate T, and no new information influencing expectations about α, p, or g. Should these parameters change in time the price would adjust accordingly and cumulative returns would temporarily stray from those described above.

Under the RVT, the fluctuation (noise) of market prices is partly explained by the influence the fortunes of a given asset exert on the market price of all other assets. Any bankruptcy (or newly created company) will affect the market prices of all other assets thus generating volatility. Such effects should be negligible in large, well-diversified markets unless entire industries are affected.

Equations 2 and 3 suggest that assets that are correlated with the market will experience a cumulative return higher than T (the market itself will experience a higher return than T in years when no assets go bankrupt). However, these higher returns won't allow a risk loving investor to beat the market over the long-term. All risky assets will eventually go bankrupt, and new ones will emerge starting another "cycle". Since the market is always priced at the geometric mean, after a sufficiently large number of cycles its long-term cumulative return will tend to T.

A meaningful empirical test could be performed if the government synthetically creates the assets described above. Observing their cumulative returns as functions of α, p and g would allow testing equations 2, 3, and 4. Designing a synthetic asset with a specific α is tricky but a solution should be possible. However, testing the RVT makes little sense because investors seem to be unaware of the geometric mean maximization strategy (this strategy is not taught in MBA programs).

Conclusions

Latane [1967] shows that maximizing the geometric expected return would certainly outperform any other strategy in terms of cumulative return. Alb [2001] assumes time value of money is unique and investors only care about maximizing cumulative returns, and shows that equilibrium can only be reached when all investors own the market and prices are given by equation 1.

Alb [2004] shows that RVT equilibrium prices determine intrinsic returns that satisfy the CAPM equation, that asset specific β can be explained by simply assuming scenario probabilities randomly fluctuate around a central set of values, and that market return βs are approximately equal to intrinsic return αs.

This paper introduces simple models to explain asset specific cumulative returns under the RVT framework. Assets that are expected to pay constant (or constantly increasing) dividends but face each year the possibility of going bankrupt will exhibit asset specific cumulative returns.

The cumulative returns predicted by the RVT are similar to CAPM returns because they are linearly related to intrinsic returns, which do satisfy the CAPM equation. In light of this similarity, empirical evidence supporting the CAPM cannot be used to reject the RVT at a significant level of confidence.

The cumulative returns predicted by the RVT depend not only on β, but also on the probability of going bankrupt and the growth rate. The proposed models provide a theoretical explanation (within the RVT framework) for the size and value effects. The RVT also explains the market premium without making the assumption investors care about variance.

A numerical example was considered to highlight the consistency of RVT's predictions with empirical data, and RVT's practical simplicity for pricing securities. Surprisingly, the RVT model (which assumes investors only care about cumulative returns) predicts higher "discount rates" for risky assets than the CAPM (which assumes investors are mean-variance optimizers).

A meaningful empirical test for the RVT can be performed if the government synthetically constructs assets according to the above models and puts them on the market. Studying prices, returns and their relation to β, the survival rate p, and the growth rate g, would allow for a conclusive test. However, no test can be meaningful unless investors are aware of the geometric mean maximization strategy (which doesn't seem to be the case).

Unlike the CAPM, the RVT does not rely on observing the market (to measure β or the market premium) in order to calculate asset prices. The RVT does not resort to the utility function concept and is robust to risk preference assumptions, unlike the CAPM, which critically relies on risk-aversion. But most importantly the RVT can now explain cumulative returns quite well using a light, credible set of assumptions.

Appendix A

Investors do not know when the asset will go bankrupt so they foresee an infinite number of possible scenarios. Under scenario 0 the risky asset would go bankrupt before paying any dividend. Under scenario 1 the asset would pay one dividend and then go bankrupt, under scenario 2 the asset would pay 2 dividends and then go bankrupt, and so forth. It is easy to show that the probability of scenario i is:

$$p_i = (1 - p) \cdot p^i \qquad \text{(A1)}$$

where:

p_i is the probability of scenario i

p is the survival rate of asset A

Also the intrinsic value of asset A under scenario i is given by:

$$A_i = D_A \cdot \frac{T^i - 1}{T^i \cdot (T - 1)} \qquad \text{(A2)}$$

where:

A_i is the intrinsic value of asset A under scenario i (the present value of the i dividends paid before bankruptcy)

D_A is the constant annual dividend paid by asset A

T is the unique time value of money rate

Since every year the future prospects of asset A are the same, its equilibrium price will not change[1] and consequently the total market return of asset A will be given by its dividend yield:

$$\hat{R}_A = 1 + \frac{D_A}{P_A} \tag{A3}$$

where:

$\hat{R}_A$ is the cumulative market return of asset A

P_A is the equilibrium price of asset A

The intrinsic expected return of asset A can be written as:

$$E(R_A) = \frac{E(A)}{P_A} \tag{A4}$$

where:

$E(R_A)$ is the intrinsic expected return (intrinsic returns are returns from equilibrium price to underlying intrinsic values)

$E(A)$ is the expected intrinsic value (expected payoff) of asset A

Extracting P_A from A4 and plugging into A3 we get:

$$\hat{R}_A = 1 + \frac{D_A}{E(A)} \cdot E(R_A) \tag{A5}$$

The expected payoff $E(A)$ is given by:

$$E(A) = \sum_{i=0}^{\infty} p_i * A_i \tag{A6}$$

Plugging p_i and A_i from equations A1 and A2 into equation A6 and rearranging yields:

$$E(A) = D_A \frac{(1-p)}{(T-1)} \cdot \left[\sum_{i=0}^{\infty} p^i - \sum_{i=0}^{\infty} \left(\frac{p}{T} \right)^i \right] \quad \text{(A7)}$$

Writing the two series as limits when i goes to infinity and solving them yields:

$$\sum_{i=0}^{\infty} p^i = \lim_{i \to \infty} \frac{p^{n+1} - 1}{p - 1} = \frac{1}{1-p} \quad \text{(A8)}$$

$$\sum_{i=0}^{\infty} \left(\frac{p}{T} \right)^i = \frac{1}{1 - \frac{p}{T}} \quad \text{(A9)}$$

Replacing A8 and A9 into A7 and carrying out the appropriate simplifications yields:

$$E(A) = \frac{D_A}{\frac{T}{p} - 1} \quad \text{(A10)}$$

After plugging A10 into A5 and simplifying D_A we get the cumulative return of asset A:

$$\hat{R}_A = 1 + (\frac{T}{p} - 1) \cdot E(R_A) \quad \text{(A11)}$$

A similar calculation for the variance of the payoffs would yield:

$$\sigma_A{}^2 = \frac{D_A{}^2 T^2 (1-p) p}{(T^2 - p)(T - p)^2} \quad \text{(A12)}$$

Appendix B

For an asset B which is expected to pay a constantly growing dividend until it goes bankrupt the scenario payoffs are given by:

$$B_i = D_B \cdot \frac{t^i - 1}{t^i \cdot (t-1)} \tag{B1}$$

where now:

$t = \frac{T}{g}$, g being the annual growth rate of dividends (gross).

D_B is the last dividend paid (so the next dividend will be $D_B \cdot g$)

Considering that equations B1 and A2 point to similar expressions for A_i and B_i, we can simply substitute t for T in equation A10 and obtain the expected payoff for asset B as:

$$E(B) = \frac{D_B \cdot g}{\frac{T}{p} - g} \tag{B2}$$

The equilibrium price of asset B can be written as the ratio between the expected payoff and the intrinsic expected return:

$$P_B = \frac{E(B)}{E(R_B)} \tag{B3}$$

Replacing the expected payoff from equation B2 into equation B3 and considering that the first dividend is equal to $D_B \cdot g$ we can write the dividend yield of asset B as:

$$Div\_yield_B = (\frac{T}{p} - g) \cdot E(R_B) \tag{B4}$$

The market price of asset B will not be constant so we have to take it into account. The intrinsic expected return satisfies the CAPM equation so it only depends on α, which should not change as the dividend increases. If the intrinsic expected return is constant then, from equation B3, the increase in price must equal the increase in expected payoff $E(B)$, which is obviously equal to g, the dividend growth rate.

Combining the dividend yield and the market price increase, the total market price return of asset B can be written as:

$$\hat{R}_B = g + (\frac{T}{p} - g) \cdot E(R_B) \tag{B5}$$

Appendix C

Asset D will not pay any dividend for n+1 years, and then will start paying a constant (or constantly increasing) dividend until it goes bankrupt. Just as in the previous models we assume that p (the probability of survival) is constant[3]. For such an asset the scenario probabilities become:

$$p_i = (1 - p) \cdot p^{i+n-1} \tag{C1}$$

Equation C1 does not apply to the case i = 0, but we do not need the probability of scenario 0 (asset D going bankrupt before paying any dividend) for the purpose at hand.

Similarly the scenario payoffs become:

$$D_i = D_D \cdot \frac{T^i - 1}{T^{i+n-1} \cdot (T-1)} \tag{C2}$$

The equilibrium price of asset D can be written as the ratio between the expected payoff and the intrinsic expected return:

$$P_D = \frac{E(D)}{E(R_D)} \tag{C3}$$

Since the intrinsic expected return $E(R_D)$ is constant as previously discussed, any increase in market price must equal the increase in expected payoff $E(D)$.

Equations C1 and C2 show that, during the period in which asset D doesn't pay dividends, all scenario probabilities increase at a constant annual rate $\frac{1}{p}$ (dividends became more likely), and all scenario payoffs increase at a constant annual rate T (dividends are less far into the future). The expected payoff therefore increases at a constant annual rate of $\frac{T}{p}$, and consequently the market price return of asset D is equal to:

$$\hat{R}_D = \frac{T}{p} \tag{C4}$$

Appendix D

Alb [2004] showed that price movements generated by a randomly fluctuating set of scenario probabilities determine β coefficients that closely approximate αs. Two aspects are different in the current problem: bankruptcies and dividends. Although bankruptcies have the potential to permanently alter the probability distribution of market payoffs this should not happen because newly created assets replace those that go bankrupt. However, unlike the original problem, the market is now negatively impacted by bankruptcies without being positively impacted by new assets being created. This would determine the market in the current problem to "lag" behind the market in the original problem. I feel the above considerations do not have the power to dramatically alter the original result so I will use it here:

$$\beta = \frac{Cov(\hat{R}_A, \hat{R}_M)}{Cov(\hat{R}_M, \hat{R}_M)} \approx \frac{Cov(R_A, {}^{1}\!/_{R_M})}{Cov(R_M, {}^{1}\!/_{R_M})} = \alpha \qquad \text{(D1)}$$

The models described bring dividends into the picture, which in turn introduce a dividend yield component in total returns. Developing the covariance we can write:

$$Cov(\tilde{R}_A, \tilde{R}_M) = Cov(\hat{R}_A, \hat{R}_M) + Cov(\frac{D_A}{P_A}, \hat{R}_M) +$$

$$+ Cov(\hat{R}_A, \frac{D_M}{GM}) + Cov(\frac{D_A}{P_A}, \frac{D_M}{GM}) \qquad \text{(D2)}$$

where:

$\hat{R}_A$ is the market price return of asset A and $\frac{D_A}{P_A}$ is the dividend yield of asset A

$\tilde{R}_A = \hat{R}_A + \frac{D_A}{P_A}$ is the total return of asset A (market price return plus dividend yield)

$\hat{R}_M$ is the market price return of the market and $\frac{D_M}{GM}$ is the dividend yield of the market

$\tilde{R}_M = \hat{R}_M + \frac{D_M}{GM}$ is the total return of the market (market price return plus dividend yield)

The last three terms in equation D2 all include dividend yields. Since dividend yields are likely to be relatively small, their impact on the covariance of total returns should be marginal. In most cases we should have that:

$$Cov(\tilde{R}_A, \tilde{R}_M) \approx Cov(\hat{R}_A, \hat{R}_M) \qquad \text{(D3)}$$

Similarly we have that:

$$Cov(\tilde{R}_M, \tilde{R}_M) = Cov(\hat{R}_M, \hat{R}_M) + 2Cov(\frac{D_M}{GM}, \hat{R}_M) + Cov(\frac{D_M}{GM}, \frac{D_M}{GM}) \approx$$

$$\approx Cov(\hat{R}_M, \hat{R}_M) \qquad \text{(D4)}$$

Using equations D1, D3 and D4 we can conclude the following approximation holds:

$$\beta = \frac{Cov(\tilde{R}_A, \tilde{R}_M)}{Cov(\tilde{R}_M, \tilde{R}_M)} \approx \frac{Cov(R_A, \,^1\!/_{R_M})}{Cov(R_M, \,^1\!/_{R_M})} = \alpha \qquad \text{(D5)}$$

REFERENCES

Alb, Silviu I. “The Relative Value Theory.” Economics Working Paper Archive at WUSTL, June 2001, http://econpapers.hhs.se/paper/wpawuwpfi/0106003.htm

Alb, Silviu I. “Maximizing Long-Term Cumulative Returns.” Journal of Wealth Management, Vol. 6, No. 4, Spring 2004, pp. 29-32

Alb, Silviu I. “Would the CAPM Hold in a Risk-Indifferent World?” Economics Working Paper Archive at WUSTL, February 2004, http://econwpa.wustl.edu/eprints/fin/papers/0402/0402020.abs

Banz, Rolf W. “The Relation between Return and Market Value of Common Stocks.”, Journal of Financial Economics No. 9, 1981, pp. 3-18

Basu, Sanjoy, “The Relationship between Earnings Yield, Market Value, and Return for NYSE Common Stocks: Further Evidence.”, Journal of Financial Economics No. 12, 1983, pp. 129-156

Fama, Eugene F., and Kenneth R. French, “The Cross-Section of Expected Stock Returns.”, Journal of Finance Vol. 47, 1992, pp. 427-465

Latane, Henry A. “Criteria for Choice among Risky Ventures.” Journal of Political Economy, Vol. 67, No. 2 (April 1967), pp. 144-155

Rosenberg, Barr, Kenneth Reid, and Ronald Lanstein, “Persuasive Evidence of Market Inefficiency.”, Journal of Portfolio Management, Vol. 11, pp. 9-17

FOOTNOTES

[1] Except for price fluctuations briefly mentioned in Section III, which tend to be negligible and cancel out over the long term.

[2] Asset specific αs can be explained for the above models by bankruptcies being correlated events across various assets. Asset specific αs could also be explained considering more complex models in which the dividends themselves are correlated with total market dividends.

[3] The probability of going bankrupt could change as the asset starts paying dividends but here, for simplicity, we assume this is not the case.

[4] The fundamental RVT assumption is that investors maximize the cumulative returns of their entire wealth, which includes human capital (the present value of future labor income). Here, for simplicity, we assume dividends are the only source of income.

[5] Other things being equal, a higher growth rate will determine a lower return as long as the intrinsic return is higher than 1, which is true for most companies.