Association between Markov regime-switching market volatility and beta risk: Evidence from Dow Jones industrial securities

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Abstract

In this paper, the volatility of the return generating process of the market portfolio and the slope coefficient of the market model is assumed to follow a Markov switching process of order one. The results indicate very strong evidence of volatility switching behaviour in a sample of returns in the S&P500 index. In three of the thirty securities in the Dow Jones index, the estimated slope in the market model show strong switching behaviour. In these three securities the low risk state is more persistent than the high-risk state. For each security we estimate the conditional probabilities that the security is in the high (low) risk state given the market is in the high (low) volatility regime and show that this information can be used to classify securities into three distinct groups. There is no association between these groups and the securities' constant beta estimated in the market model and the Sharpe index. Some directions for further research are discussed.

Key words: Asset pricing, Markov regime-switching, market volatility, beta risk

JEL Classification: G11, G12

Introduction

The beta - the percentage change in a security price relative to percentage change of a relevant market index - is one of the most commonly used measures of security price

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movement. In empirical studies, the beta value is obtained by estimating a linear relationship between excess return on the security and excess return on a market portfolio, where excess return is the return in excess of the return on a risk-free asset. Many studies have reported that beta is unstable and the instability is evident in many different markets. See for example, Fabozzi and Francis (1977), Chen (1982), Bos and Newbold (1984), French, Schwert and Stambaugh (1984) for evidence in the US stock market, Faff, Lee and Fry (1992), Brooks, Faff and Lee (1994) and Faff and Brooks (1998) for evidence in the Australian stock market. Further, it is well documented that the variance of the market portfolio returns distribution is time varying. See for example, Bollerslev, Engle and Nelson (1994), Campbell, Lo and MacKinlay (1997) and the references therein. Two popular methods suggested in the literature to model variation in market volatility is the ARCH and GARCH processes due to Engle (1982) and Bollerslev (1986) respectively. Another approach to model financial time series is the Markov switching technique proposed by Hamilton (1989) where the parameters are viewed as the outcome of a discrete-state Markov process. Such models are known to accurately capture typical stock market patterns such as jumps and crashes. Hamilton and Susmel (1994) modelled changes in market volatility as a Markov-switching model and as ARCH models without switching and reported evidence in favour of the former.

Recently, Huang (2000) modelled beta as a first-order Markov chain where it is assumed that beta switches back and forth from one state to another. In particular, Huang (2000), using Microsoft Corporation monthly stock returns, tested whether the capital asset pricing model (CAPM[1]) is consistent with data drawn from two different regimes: a high-risk state and a low-risk state. Huang (2000) showed that the data from the low-risk state is consistent with the CAPM whereas the data from the high-risk state is not. Hess (2003) compared competing Markov regime-switching model specifications and reported that for the Swiss security market index monthly returns, the market movement is optimally tracked by time-varying first and second moments, including a memory effect. Assoe (1998), in an analysis of nine

[1] CAPM conveys the notion that securities are priced so that their expected return will compensate investors for their expected risk.

emerging market return series, shows very strong evidence of regime switching behaviour. Assoe (1998) observed that the emerging markets evolve through two regimes and concluded that the switching models, where the regimes differ only in terms of market volatility, describe the return generating process better when returns are expressed in US dollars than when expressed in local currency. Further applications of the Markov switching technique to model market returns is available in Schwert (1989) where two states of the variance is modelled, Turner, Startz and Nelson (1989) where the mean and variance separately and together are considered to differ between two regimes, and Hamilton and Susmel (1993) where regime shifts in the volatility is modelled.

A number of studies have examined the association between general changes in the market conditions and beta instability. This is usually done by classifying the market conditions into different phases based on some arbitrarily chosen threshold values of market portfolio return. The common approach is to classify the market movements as bull and bear and capture the differential effects of these two phases on the beta. A few studies considered several market phases including two bull and two bear (Gooding and O'Malley, 1977), and nine phases in terms of three levels of both the mean and standard deviation of market returns (Faff and Brooks, 1998). In general, there appears to be some evidence of differential beta risk in bull and bear markets.

The studies that considered Markov switching phenomenon to capture parameter instability in the security and market portfolio return generating processes examined the variation in the beta risk and the variation in the market volatility. However, they did not examine the association between the beta risk states and the market volatility regimes[2]. In this paper we model the market and security return generating processes as Markov switching

[2] Chu, Santoni and Liu (1996) examined the association between the variation in the market volatility and the regime shifts in the market returns. Firstly they applied switching models to the returns. Thereafter, they estimated a volatility equation given the different regimes observed in the first stage and concluded that the returns and volatility are related nonlinearly and that the relationship is asymmetric.

processes and investigate the association between the market volatility regimes and the states associated with the beta risk[3]. The aim here is to classify securities according to the likelihood of a security being in a particular state of risk, given the market is in a particular volatility regime. We believe that such information can be used as a diagnostic tool and incorporated in portfolio construction and asset allocation models.

We consider two market volatility regimes and two beta risk states and classify securities into groups based on the conditional probability of the security being in the high (low) risk state given that the market is in the high (low) volatility regime. We believe that such information is captured better with high frequency data and therefore use daily data, whereas the previous studies use monthly data[4]. In a sample of returns of the thirty Dow Jones industrial securities and with S&P500 index as a proxy for the market portfolio, we identify three distinct groups of securities: (i) the securities with high probability of being in the low risk state given the market is in the low volatility regime, (ii) the securities with high probability of being in the high risk state given the market is in the high volatility regime, and (iii) other securities. The constant beta estimated in the market model and the Sharpe ratio that are considered as measures of security performance is not associated with any of the three identified groups of securities. Therefore, portfolio managers can benefit from knowing which securities fall into what group because the groups are characterised on how the security risk is associated with the general market conditions.

[3] A number of studies have shown that the volatility of security returns varies over time. See for example, Mandelbrot (1963). The variation in security returns volatility has also been associated with regime shifts. We do not pursue this in this paper.

[4] A reason in favour of using monthly data is the likely presence of more noise at high frequencies that can hinder the isolation of cyclical variations and consequently obscure the analysis of the driving moments of switching behaviour (Hess, 2003). Non-availability of high frequency data is another reason for using monthly data. In this study we investigate the overlap of security risk states with market volatility regimes on the time line, and therefore we prefer high frequency data.

The paper is organised as follows. The switching models are specified in the next section. In the third section the data is described and the results are analysed in the fourth section. The final section summarises the findings and gives some directions for future work.

Model Specification

The return generating process of the market portfolio is postulated as:

$$r_{mt} = \mu_m + (\sigma_{m1} S_{mt} + \sigma_{m2})\varepsilon_t \qquad (1)$$

where r_{mt} is return of market portfolio in excess of the risk-free rate, $\mu_m = E(r_{mt})$, S_{mt} is an unobserved binary variable that identifies which of the two regimes the market is in at time *t* (S_{mt}=1 for the high volatility regime and S_{mt}=0 for the low volatility regime), and $\varepsilon_t | \Phi_{t-1} \sim N(0,1)$ where Φ_{t-1} is information set at time *t*-1. In model (1), we assume that the changes in regimes can only affect the volatility of the market return distribution and there is no switching in mean[5]. In other words, we assume that market returns are drawn from two distributions that differ only in their variances. In model (1), the regimes are characterised by σ_{m2} and $(\sigma_{m1} + \sigma_{m2})$ where $\sigma_{m2} < (\sigma_{m1} + \sigma_{m2})$. It is assumed further that S_{mt} follows a Markov chain of order one with constant transition probabilities where

$$P(S_{mt} = 1 | S_{m,t-1} = 1) = p_{m11}, \qquad (2)$$

$$P(S_{mt} = 0 | S_{m,t-1} = 1) = 1 - p_{m11}, \qquad (3)$$

$$P(S_{mt} = 0 | S_{m,t-1} = 0) = p_{m00} \qquad (4)$$

and

$$P(S_{mt} = 1 | S_{m,t-1} = 0) = 1 - p_{m00}. \qquad (5)$$

[5] Studies have shown that the switching behaviour in market portfolio returns can be primarily attributed to the switching in volatility (Assoe, 1998; Hess, 2003).

When the model is estimated, the probability of being in any regime which is time variant, would be estimated as $P(S_{mt}=1)=p_{mt}^1$ and $P(S_{mt}=0)=(1-p_{mt}^1)=p_{mt}^0$. When there is no switching in market volatility, model (1) reduces to the single regime model given as:

$$r_{mt}=\mu_m+(\sigma_m)\varepsilon_t \tag{6}$$

The return generating process of security *i* is assumed to take the following form:

$$r_{it}=(\alpha_{i1}S_{it}+\alpha_{i2})+(\beta_{i1}S_{it}+\beta_{i2})r_{mt}+\varepsilon_{it} \tag{7}$$

where r_{it} is the return of security *i* in excess of the risk-free return, S_{it} is an unobserved binary variable that identifies which of the two risk states the security is in at time *t*, and $\varepsilon_{it}\sim N(0,\sigma_i^2)$. In model (7), we assume that the changes in risk regimes can only affect the intercept and the slope coefficient and there is no switching in volatility. It is assumed further that the transition probabilities are time-invariant such that S_{it} switches between 1 and 0 according to a Markov chain of order one where

$$P(S_{it}=1|S_{i,t-1}=1)=p_{i11}, \tag{8}$$

$$P(S_{it}=0|S_{i,t-1}=1)=1-p_{i11}, \tag{9}$$

$$P(S_{it}=0|S_{i,t-1}=0)=p_{i00} \tag{10}$$

and

$$P(S_{it}=1|S_{i,t-1}=0)=1-p_{i00}. \tag{11}$$

When the model is estimated[6], the probability of being in any regime which is time variant, would be estimated as $P(S_{it}=1)=p_{it}^1$ and $P(S_{it}=0)=(1-p_{it}^1)=p_{it}^0$.

When there is no switching in the intercept term, model (7) reduces to

[6] Usually, to identify the events S_{it}=1 and S_{it}=0, restrictions are placed on the parameters of model (7). We do not place any restriction, and instead, we identify different states based on the sign and the magnitude of the estimates. In model (7), the regimes are classified by α_{i2} and $(\alpha_{i1}+\alpha_{i2})$ and β_{i2} and $(\beta_{i1}+\beta_{i2})$. However, if the estimated β_{i1} is negative, then $(\beta_{i1}+\beta_{i2})$ represents the smaller slope and therefore the beta in the low risk state. On the other hand, if β_{i1} is positive, β_{i2} represents the beta in the low risk state.

$$r_{it} = \alpha_i + (\beta_{i1} S_{it} + \beta_{i2}) r_{mt} + \varepsilon_{it} \tag{12}$$

and where there is no switching, model (7) reduces to the market model given as:

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it} \,. \tag{13}$$

Estimation

Let y_t denote the observed return at time *t* whose distribution is denoted as *f*, and Φ_t denote the information set at time *t* where $\Phi_t = (y_1, y_2, ... y_t)$. $y_t = r_{mt}$ in model (1) and $y_t = r_{it}$ in models (7) and (12). The distribution from which returns are drawn is determined by the state variable S_{it}. Following Hamilton's (1989) procedure for filtering, the iterative algorithm uses an input value at time *t*, $P(S_t | \Phi_t)$, which will be developed by using Bayes theorem into the output value at time *t*+1, $P(S_{t+1} | \Phi_{t+1})$. To set up the iteration, the procedure needs an initial value $P(S_1 | \Phi_1)$. This value is set equal to the unconditional probability $P(S_1)$ that has two elements given by $P(S_1 = 1) = \pi_0$ and $P(S_1 = 0) = 1 - \pi_0$, where $\pi_0 = (1 - p_{00})/(2 - p_{00} - p_{11})$ is the limiting probability of the Markov process.

The following iterative steps will be carried out.

Input: $P(S_t | \Phi_t)$

Step-I: $P(S_{t+1}, S_t | \Phi_t) = P(S_{t+1} | S_t) P(S_t | \Phi_t)$

Step-II: $P(S_{t+1} | \Phi_t) = \sum_{S_t = 0}^{1} P(S_{t+1}, S_t | \Phi_t)$

Step-III: $f(y_{t+1}, S_{t+1} | \Phi_t) = f(y_{t+1} | S_{t+1}, \Phi_t) P(S_{t+1} | \Phi_t)$

where $f\left(y_{t+1}\middle|S_{t+1},\Phi_t\right)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{\hat{e}_{t+1}^2}{2\sigma^2}}$, σ is the standard deviation of error and $\hat{e}$ is the estimated error of the model being estimated[7].

Step-IV: $f\left(y_{t+1}\middle|\Phi_t\right)=\sum_{S_{t+1}}^{1} f\left(y_{t+1},S_{t+1}\middle|\Phi_t\right)$

Step-V (output): $P\left(S_{t+1}\middle|\Phi_{t+1}\right)=\frac{f\left(y_{t+1},S_{t+1}\middle|\Phi_t\right)}{f\left(y_{t+1}\middle|\Phi_t\right)}$

Step-IV provides the conditional distributions for the calculation of the likelihood function $L=\prod_{t=1}^{T} f\left(y_{t+1}\middle|\Phi_t\right)$ where T is the sample size.

Data

We use the daily price series of the thirty securities in the Dow Jones Industrial index. The data covers the period from 2 January 1990 to 23 May 1996, and consists of 1619 observations for each security. The daily returns are calculated as the change in the logarithm of the closing prices of successive days. The return on the Standard and Poor's 500 Index (S&P500) is used to proxy the market portfolio return and the return on the US 1-month Treasury Bill (TB) is used to proxy the risk-free return.

Table 1 provides some summary statistics of the thirty securities, the US 1-month TB and the market portfolio returns. The returns vary widely across the securities, with the highest being 13.26 per cent and the lowest being –26.15 per cent. The market return, as expected, has a smaller range with the lowest and the highest returns being –3.73 per cent and 3.66 per cent respectively. The standard deviation of the market return distribution, 0.73 per cent, is much smaller compared to that of the securities, of which the lowest is 1.13 per cent and the highest

[7] When estimating model (1), $\sigma=\left(\sigma_{m1}S_{m,t+1}+\sigma_{m2}\right)$ and $\hat{e}_{t+1}=\left(R_{m,t+1}-\mu_m\right)$. When estimating model (7), $\sigma=\sigma_i$ and $\hat{e}_{t+1}=r_{i,t+1}-\left(\alpha_{i1}S_{t+1}+\alpha_{i2}\right)-\left(\beta_{i1}S_{t+1}+\beta_{i2}\right)r_{m,t+1}$, and when estimating model (12), $\sigma=\sigma_i$ and $\hat{e}_{t+1}=r_{i,t+1}-\alpha_i-\left(\beta_{i1}S_{t+1}+\beta_{i2}\right)r_{m,t+1}$.

is 2.36 per cent. The market and seven securities are negatively skewed. The excess kurtosis of one security, PM, is extremely high compared to the others. When PM is left out the excess kurtosis varies only between 6.40 and 0.96. The excess kurtosis of the market return distribution is 2.39. The US 1-month TB returns distribution is tri modal, positively skewed and has mean 0.0128 per cent and standard deviation 0.0039 per cent.

Results

First, we estimated[8] model (1) for S&P500 returns. The results are reported in Table 2. The volatility estimates show that the volatility in the high-volatility regime is slightly less than twice the volatility in the low-volatility regime. The estimates of the transition probabilities are fairly high, indicating that the two regimes are strongly persistent with the low-volatility regime being more persistent[9,10] than the high-volatility regime. See Figure 1 for the plot of the estimated probability of the market being in the high volatility regime. The expected duration in the high and low volatility regimes are 28.3 and 70.4 days respectively, and the unconditional probabilities of being in each regime are 0.2868 and 0.7132.

Thereafter, we estimated model (7) for each security in the sample. The results are reported in Table 3. We examined the time series plots of the probability that the security is in

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[8] The parameters of the models are estimated by maximising the conditional log-likelihood function evaluated using Hamilton's (1989) recursive procedure. All models are estimated using GAUSS. As stressed by Goodwin (1993) and Boldin (1996), we tested the stability of the parameter estimates by using different sets of initial values.

[9] van Norden and Schaller (1993) analysed value-weighted monthly US stock market excess returns for the period January 1929 to December 1989 and reported very strong evidence of switching behaviour in models where the returns are drawn from two distributions with different (i) means, (ii) variances and (iii) means and variances. When allowed for switching in variances, they observed both states to be persistent with the low variance state being extremely persistent.

[10] Emerging markets also seem to portray such switching behaviour. In a similar analysis, Assoe (1998) observed that in seven out of nine emerging markets the low-volatility regime is more persistent than the high-volatility regime.

the high-risk state, $P(S_{it} = 1)$, for evidence of switching behaviour. The results indicate that there is evidence of strong switching behaviour in only two securities namely, Coca Cola and PG. Further, in these two securities the low-risk state is likely to be more persistent compared to that of in the high risk state. As an illustration of strong switching behaviour, a plot of the estimated probability that the security, namely Coca Cola is in the high-risk state is displayed in Figure 2. When such a graph of a security displays sharp spikes at irregular intervals suggesting that the transition from a particular state to the other occurs for a very short period of time, we categorised that security as one with no strong switching behaviour. The graphs of $P(S_{it} = 1)$ for Walmart and JP Morgan displayed in Figures 3 and 4 respectively are examples of securities that do not portray strong switching behaviour in risk states but have contrasting patterns in the probability of being in the high-risk state. We like to point out here that the aim of this study is not to classify securities according to their strength of switching behaviour in the risky states. Our aim is to classify securities according to the overlap in the periods that are marked by shifts in the market volatility regimes and security risk states. We discuss switching behaviour mainly to highlight that the securities clearly display three different patterns of switching in the beta risk.

We also estimated the parameters of model (13) where no switching is imposed. See Table 4 for the results. In this case, with MSFT being an exception, the intercept terms in all other twenty-nine securities are not statistically significant.

Finally, we estimated model (12) for each security in the sample. The results are reported in Table 5. In this slope-switching model, the regimes are classified by β_{i2} and $(\beta_{i1} + \beta_{i2})$ as in model (7). For all securities, the constant beta[11] lies between the betas in the low- and high-risk states. When the probability of staying in the low (high) risk state is dominant, the constant beta is closer to the beta in the low (high) risk state. Examinations of the plots of the

[11] Is estimated in the security return generating process given in (13).

probability that the security is in the high-risk state however indicate strong switching behaviour in risk states only in three securities namely, Boeing, Coca Cola and PG. In the analysis with model (12), Boeing did not display strong switching behaviour. In these three securities it appears that the low risk state is more persistent than the high-risk state. For Boeing, Coca Cola and PG, the expected duration in the low risk state are 62, 158 and 208 days and in the high risk state are 25, 91 and 140 days respectively. This indicates that in our sample, those securities that display change in regimes have the characteristic of staying longer in the low-risk state compared to that in the high-risk state. The beta of Boeing, Coca Cola and PG are 1.44, 1.54 and 1.38 in the high-risk state and are 0.61, 0.99 and 0.85 in the low-risk state respectively. Although for Caterpillar, Alcoa, Honeywell, PM and Intel the beta exceeds 10.00 in the high-risk state, this high-risk state does not persist. In fact, for these securities, the expected duration in the high-risk state is only about one day. On the other hand, the low-risk state is very persistent in 22 securities with the probability of being in the low-risk state exceeding 0.95. These observations clearly indicate that switching behaviour is inconsistent across the securities and in many of them one state persists through most of the sample period.

Market volatility regimes and security risk states

Here we investigate the association between market volatility regimes and security risk states. We examine for each security, the overlap in the time periods where the market is in a volatility regime and the security is in a risky state. To distinguish the high volatility regime (risk state) from the low volatility regime (risk state), initially we set 0.5 as the threshold probability. We then count the number of days the security is in the high and low risk states when the market is in the high and low volatility regimes. Based on these counts we computed four probabilities: (i) the security is in a high risk state and the market is in the high volatility regime, (ii) the security is in a low risk state and the market is in the high volatility regime, (iii) the probability that the security is in a high risk state and the market is in the low volatility regime and (iv) the probability that the security is in a low risk state and the market is in the low volatility regime. These probability estimates obtained with models (1) and (12) are reported in Table 6. The probabilities are based on 1618 estimates of which 1207 (74.6%) correspond to the days when the market is classified as being in the low volatility regime.

For ease of interpretation, we compute the conditional probabilities that the security is in the high (low) risk state given the market is in the high (low) volatility regime. The conditional probabilities estimated with models (1) and (12) displayed in Figure 5 clearly indicate that the securities may be classified into three groups: (A) very high probability of being in a high risk state given that the market is in a high volatility regime, (B) very high probability of being in a low risk state given that the market is in a low volatility regime and (C) those that do not belong to groups A and B. A security that belongs to group A has the property that there is a substantial overlap in the periods that are marked by the high-risk state of the security and the high volatility regime of the market. On the other hand, a security that belongs to group B has the property that there is a substantial overlap in the periods that are marked by the low-risk state of the security and the low volatility regime of the market. In our analysis we arbitrarily set 0.95 as the benchmark for very high probability. Accordingly in our sample data set, five securities namely, ATT, GM, Exxon, HP and Walmart belong to group A, twenty securities belong to group B and the other five securities namely, Boeing, CITIGRP, Coca Cola, MMM and PG belong to group C. The securities that show strong evidence of switching behaviour based on the probability plots namely, Boeing, Coca Cola and PG belong to group C. The transition probability plots of the other two securities, namely CITIGRP and MMM that belong to group C display frequently changing regimes. Therefore, we do not classify them as portraying regime switching behaviour[12]. The plots of the estimated probability of being in the high risk state of a typical security in Group A and in Group B are shown in Figures 3 and 4. As evidenced in Figure 5, the constituent securities in

[12] On the other hand, the securities that belong to group C may be divided into two sub-groups: C1, the securities that display strong switching behaviour and C2, the securities that belong to group C but do not display strong switching behaviour. Of course, then there will be four classifications of securities.

the three groups do not change even if the benchmark for very high probability is changed from 0.95 to 0.90.

We repeated the above analysis with estimates obtained in models (1) and (7) as well. Here we observed ATT, CITIGRP, GM, Exxon, HP and Walmart belonging to group A, Coca Cola and PG belonging to group C and the rest of the twenty-two securities falling into group B. When the analysis is repeated with 0.6, 0.7, 0.8 and 0.9 as the threshold probabilities that distinguish the high volatility regime (risk state) from the low volatility regime (risk state) the observations we made on the results reported in Table 6 and displayed in Figure 3 remain largely unchanged.

In general, we observe a noteworthy association between market volatility regimes and security risk states. The nature of the association enables clear identification of three clusters of securities: (i) those that are highly likely to persist in the high risk state irrespective of the changes in the market volatility, (ii) those that are highly likely to persist in the low risk state irrespective of the changes in the market volatility and (iii) those that are likely to display a positive association between the risk states and market volatility regimes. In all the securities that we sampled, the pattern of switching in the beta risk is highly unlikely to be identical with the pattern of switching in the market volatility. It is also highly unlikely that the security risk states switch completely opposite to that of the market volatility regimes. Alternatively, $P(S_{mt}=1)=P(S_{it}=1)$ for all t and $P(S_{mt}=1)=1-P(S_{it}=1)$ for all t are highly improbable scenarios.

Association of groups with economic performance measures

(i) Constant beta

We investigated the constant beta[13] of securities across the three groups for any association between the beta and the groups by testing the null hypothesis of equal mean beta against the

[13] The constant beta is estimated in model (13). The results are reported in Table 4. A beta value greater (smaller) than one indicates the stock is more (less) volatile than the market index by which the beta is measured.

alternative of at least one mean is different. An F-test on H_0: mean constant beta of group A securities, mean constant beta of group B securities and mean constant beta of group C securities are equal against H_A: at least one mean is different from the others, reveals that the null hypothesis of equal means can not be rejected at the 1% level of significance (F-value =0.2709 and p-value= 0.7646). This suggests that there is no difference in the mean constant beta of the three groups identified with models (1) and (12). An F-test of the null hypothesis of equal mean constant beta across the three groups identified with models (1) and (7) against the alternative hypothesis of at least one mean is different from the other means also failed to reject the null hypothesis at the 1% level of significance (F=0.8127 and p-value=0.4542). It appears that the constant beta alone might not be able to capture the underlying characteristics of the securities that belong to the different groups.

(ii) Sharpe index

Sharpe (1966) suggested that the historical performance of a security might be calculated as the excess return earned for bearing risk per unit of total risk[14]. We conducted an investigation similar to the one carried out earlier with the Sharpe index as well. The results of an F-test on the mean Sharpe ratios of the groups identified with models (1) and (12) (F-value= 1.4825 and p-value= 0.2450) and of the groups identified with models (1) and (7) (F-value= 2.0413 and p-value= 0.1494) reveal that there is no difference in the mean Sharpe ratios across the groups at the 1% level of significance. In the sampled data set there is evidence that the Sharpe index is not associated with the groups of securities identified through Markov switching behavioural characteristics of the security beta and the market volatility.

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[14] Symbolically, the Sharpe index, S_i, is written as $S_i = \left(\overline{R}_i - \overline{R}_f\right)/\sigma_i$ where $\overline{R}_i$ is the mean security return, $\overline{R}_f$ is the mean risk-free asset return and σ_i is the standard deviation of security returns. A higher value for S_i indicates that the security delivers a higher performance for its level of total risk measured by σ_i.

Concluding remarks

This paper modelled volatility of the market portfolio return generating process and the slope coefficient of the security return generating process as Markov regime switching processes of order one. A sample of daily returns of thirty securities in the Dow Jones index reveals strong regime-switching behaviour in three securities. In these three securities the low risk state appears to be more persistent than the high-risk state. A sample of daily returns of the S&P500 index that we use as a proxy for the market portfolio reveals strong volatility switching behaviour with low-volatility regime being more persistent than the high-volatility regime.

We then estimated for each security, the probability of being in the low (high) risk state given that the market is in the low (high) volatility regime. Based on these estimates we propose classification of securities into three groups: (i) the securities with high probability of being in the low risk state given the market is in the low volatility regime, (ii) the securities with high probability of being in the high risk state given the market is in the high volatility regime, and (iii) other securities. These groups are not associated with the constant beta estimated in the market model and the Sharpe ratio. Modelling switching behaviour in the market volatility and the security beta therefore can provide useful information to the investor. Such information can be used in the construction of portfolios.

We do not consider competing models such as switching in the variance of the security return generating process and higher order Markov switching processes. Moreover, in twenty-nine of the thirty Dow Jones securities, the risk level as measured by the constant beta in the market model exceeds 0.8. Our sample therefore, does not include low beta securities. The range of beta will be wider in a larger sample of securities and more so with securities in emerging markets. Therefore, competing switching models and different markets need to be explored in future studies.

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Table 1. Some descriptive statistics of the distributions of the continuously compounded daily returns of Dow Jones industrial securities

Security	Mean	Max	Min	Standard deviation	Skewness	Excess kurtosis
Dupont	0.0437	7.0351	-6.1548	1.4405	0.1989	1.3886
Boeing	0.0474	8.0165	-11.7571	1.6119	-0.0468	3.9943
Caterpillar	0.0525	8.8147	-10.8175	1.7651	0.0510	3.5391
Alcoa	0.0316	8.1309	-8.3716	1.6607	0.2003	1.5540
Amex	0.0270	9.6100	-9.7466	1.9364	0.1239	1.9854
ATT	0.0195	10.1103	-6.4044	1.3310	0.3214	3.1347
CITIGRP	0.0914	11.2095	-10.6916	2.0009	0.0083	2.3436
Coca Cola	0.0973	7.5945	-5.7500	1.3933	0.1811	1.7486
Home Depot	0.1144	9.0151	-10.3622	1.9034	-0.0139	1.7856
GE	0.0597	5.9719	-6.3084	1.2349	0.0270	1.7511
GM	0.0176	7.1153	-8.3560	1.8797	0.0812	0.9682
Kodak	0.0494	10.5585	-12.2729	1.6496	0.0654	6.1489
Exxon	0.0349	5.6240	-4.3222	1.1266	0.1079	1.1863
Honeywell	0.0745	12.4121	-6.3918	1.6350	0.6464	4.2264
HP	0.0926	13.2552	-19.3955	2.1933	0.0526	6.3984
IBM	0.0092	11.0782	-11.3736	1.7086	0.0476	6.0508
INTL Paper	0.0252	6.7090	-8.7292	1.4437	0.0676	1.4353
JP Morgan	0.0428	6.6975	-6.0331	1.4796	0.2936	1.7619
JJ	0.0729	7.5801	-6.4568	1.5008	0.0241	1.3069
MCD	0.0647	6.3149	-8.7011	1.5088	0.0296	1.4721
MERCK	0.0555	5.3820	-6.3911	1.5369	0.0178	0.9606
MSFT	0.1548	9.9091	-8.1041	2.1203	0.1389	1.2503
MMM	0.0343	4.9461	-9.0476	1.1752	-0.3528	4.0753
PM	0.0565	6.2250	-26.1523	1.6107	-2.6417	43.6380
PG	0.0578	5.5280	-5.6041	1.3212	0.1523	1.1559
SBC	0.0271	7.2321	-5.3476	1.3374	-0.0031	1.1647
United Tec	0.0441	8.3160	-6.9054	1.4424	0.2320	2.6466
Walmart	0.0486	7.5913	-9.8961	1.7352	-0.0047	1.8396
Disney	0.0491	11.2655	-6.6880	1.5806	0.4532	2.6014
Intel	0.1301	9.009	-14.5082	2.3631	-0.3969	2.9882
US 1-month TB	0.0128	0.0218	0.0059	0.0039	0.2609	-0.9868
S&P500	0.0401	3.6642	-3.7272	0.7268	-0.1664	2.3902

Notes: The statistics are based on 1618 observations. The sample period is January 1990 through May 1996.

Table 2. Estimation results of the market return generating process

Parameter	Estimate	Standard error
μ_m	0.0370	0.0245
p_{m11}	0.9647	0.0333
p_{m00}	0.9858	0.0113
σ_{m1}	0.4940	0.0913
σ_{m2}	0.5534	0.0282

Notes: The estimates are from $r_{mt} = \mu_m + (\sigma_{m1} S_{mt} + \sigma_{m2})\varepsilon_t$ where r_{mt} is return of market portfolio in excess of the risk-free rate, $\mu_m = E(R_{mt})$, S_{mt} is an unobserved binary variable that identifies which of the two regimes the market is in at time *t* (S_{mt} =1 for the high volatility regime and S_{mt} =0 for the low volatility regime), and $\varepsilon_t | \Phi_{t-1} \sim N(0,1)$ where Φ_{t-1} is information set at time *t*-1. $p_{m11} = P(S_{mt} = 1 | S_{m,t-1} = 1)$ and $p_{m00} = P(S_{mt} = 0 | S_{m,t-1} = 0)$.

Table 3. Estimation results of the security return generating process with switching in intercept and slope

Security (i)	Intercept in high risk state	Intercept in low risk state	Beta in high risk state	Beta in low risk state	P(staying in high risk state)	P(staying in low risk state)	σ_i
Dupont	0.3125	-0.0244	2.6893	0.9641	0.2136	0.9503	1.1721
Boeing	0.2687	-0.0520	2.0592	0.6581	0.6031	0.9048	1.3474
Caterpillar	5.1605	5.2157	0.8237	-0.1828	0.9887	0.1445	1.4868
Alcoa	3.6015	-0.1132	1.2399	0.8809	0.1450	0.9746	1.3924
Amex	4.0530	-0.1176	1.7915	1.2837	0.0746	0.9784	1.5561
ATT	-0.0038	-2.5687	1.0052	-2.5217	0.9915	0.0000	1.0786
CITIGRP	0.0723	7.9232	1.4425	-0.0169	0.3179	0.9981	1.6291
Coca Cola	0.0908	0.0364	1.5229	0.9662	0.9548	0.9778	1.0724
Home Depot	5.1203	0.0215	1.8594	1.4916	0.0000	0.9921	1.4949
GE	-1.3023	0.0315	3.5422	1.0876	0.2141	0.9862	0.8835
GM	-0.0312	-0.0232	1.4291	0.0651	0.8953	0.1187	1.6048
Kodak	6.4721	6.5146	0.2864	-0.6405	0.9921	0.0757	1.3802
Exxon	3.3450	3.3696	0.3303	-0.3771	0.9917	0.0041	0.9574
Honeywell	4.4690	0.0174	7.0992	0.8841	0.1721	0.9956	1.4332
HP	9.8391	9.8443	0.6378	-0.7474	0.9952	0.0000	1.8127
IBM	6.6351	-0.0767	2.8173	0.9236	0.0918	0.9936	1.4552
INTL Paper	0.4955	-0.0413	2.4198	0.8267	0.0270	0.9444	1.2342
JP Morgan	2.1695	-0.1018	2.0475	1.0205	0.2259	0.9647	1.1382
JJ	0.4638	0.0281	6.1258	1.0603	0.0000	0.9884	1.2236
MCD	-1.1728	0.0367	3.5725	1.0406	0.7440	0.9975	1.2752
MERCK	-0.1457	0.0132	2.0603	1.0806	0.3654	0.9999	1.3203
MSFT	-1.4683	0.1388	6.1004	1.3108	0.6727	0.9881	1.7635
MMM	-6.3203	0.0189	3.2003	0.8410	0.0000	0.9963	0.9467
PM	-7.0640	0.0636	8.8267	1.0274	0.2736	0.9937	1.2158
PG	0.1121	-0.0184	1.3970	0.8728	0.9883	0.9948	1.0606
SBC	-0.3810	0.0055	3.1458	0.8957	0.9263	0.9963	1.1180
United Tec	-3.3414	0.0452	2.8559	0.8517	0.9999	0.9999	1.1963
Walmart	3.1372	-0.0725	1.2677	-0.1524	0.9825	0.2111	1.3185
Disney	4.5553	-0.0397	1.0462	-0.0156	0.9892	0.0000	1.2809
Intel	-9.6240	0.1460	1.3757	-0.0664	0.9933	0.0000	1.9469

Notes: The estimates are from $r_{it} = (\alpha_{i1}S_{it} + \alpha_{i2}) + (\beta_{i1}S_{it} + \beta_{i2})r_{mt} + \varepsilon_{it}$ where r_{it} is the return of security *i* in excess of the risk-free return, S_{it} is an unobserved binary variable that identifies which of the two risk states the security is in at time *t*, and $\varepsilon_{it} \sim N(0, \sigma_i^2)$. In thirteen securities, at least one of the transition probability estimates is at the boundary (covariance matrices are not positive definite).). In these cases it is not possible to obtain the estimates of standard errors of the parameters. Therefore, standard errors are not reported.

Table 4. Estimation results of the market model

Security (i)	α_i	β_i	Std error of α_i	Std error of β_i	σ_i
Dupont	0.0018	1.0673***	0.0302	0.0415	1.2142
Boeing	0.0065	1.0292***	0.0355	0.0489	1.4282
Caterpillar	0.0116	1.0295***	0.0398	0.0547	1.5993
Alcoa	-0.0057	0.8969***	0.0380	0.0523	1.5279
Amex	-0.0218	1.3212***	0.0418	0.0576	1.6822
ATT	-0.0202	0.9849***	0.0279	0.0384	1.1227
CITIGRP	0.0395	1.4339***	0.0425	0.0585	1.7087
Coca Cola	0.0520	1.1914***	0.0272	0.0374	1.0917
Home Depot	0.0608	1.4964***	0.0389	0.0535	1.5624
GE	0.0161	1.1290***	0.0230	0.0316	0.9232
GM	-0.0301	1.2769***	0.0407	0.0560	1.6353
Kodak	0.0107	0.9493***	0.0373	0.0513	1.4989
Exxon	0.0030	0.6989***	0.0250	0.0344	1.0059
Honeywell	0.0369	0.9108***	0.0372	0.0512	1.4956
HP	0.0412	1.4159***	0.0482	0.0663	1.9375
IBM	-0.0292	0.9376***	0.0390	0.0536	1.5672
INTL Paper	-0.0130	0.9328***	0.0317	0.0436	1.2750
JP Morgan	0.0003	1.0912***	0.0311	0.0428	1.2495
JJ	0.0300	1.1020***	0.0316	0.0434	1.2694
MCD	0.0228	1.0663***	0.0322	0.0443	1.2950
MERCK	0.0132	1.0807***	0.0329	0.0452	1.3212
MSFT	0.1032**	1.4203***	0.0461	0.0634	1.8524
MMM	-0.0012	0.8339***	0.0251	0.0345	1.0071
PM	0.0145	1.0712***	0.0351	0.0483	1.4103
PG	0.0163	1.0506***	0.0268	0.0369	1.0785
SBC	-0.0116	0.9471***	0.0285	0.0393	1.1471
United Tec	0.0072	0.8833***	0.0321	0.0442	1.2921
Walmart	-0.0027	1.4114***	0.0348	0.0479	1.3998
Disney	0.0071	1.0700***	0.0342	0.0471	1.3765
Intel	0.0768	1.4818***	0.0523	0.0720	2.1042

Notes: The estimates are from $r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it}$ where r_{it} is the return of security *i* in excess of the risk-free return and $\varepsilon_{it} \sim N\left(0, \sigma_i^2\right)$. *** Indicates significant at the 1% level and ** at the 5% level.

Table 5. Estimation results of the security return generating process with switching in slope

Security (i)	α_i	Beta in high risk state	Beta in low risk state	P(staying in high risk state)	P(staying in low risk state)	σ_i
Dupont	-0.0057	3.5928	1.0105	0.0001	0.9754	1.1815
Boeing	0.0020	1.4419	0.6139	0.9929	0.9952	1.3952
Caterpillar	0.0127	10.8727	0.9941	0.2447	0.9932	1.5397
Alcoa	-0.0079	10.297	0.8847	0.0081	0.9998	1.5011
Amex	-0.0324	4.7225	1.1768	0.3687	0.9744	1.6018
ATT	-0.0142	0.9974	-7.1304	0.9965	0.0000	1.0952
CITIGRP	0.0381	2.2147	0.9656	0.0000	0.3760	1.6500
Coca Cola	0.0537	1.5392	0.9909	0.9612	0.9839	1.0740
Home Depot	0.0565	3.0553	1.4031	0.4283	0.9721	1.5366
GE	0.0140	3.5172	1.0859	0.1639	0.9865	0.8926
GM	-0.0293	1.4143	-0.0087	0.9049	0.0884	1.6045
Kodak	0.0018	5.156	0.9137	0.0000	0.9873	1.4599
Exxon	0.0007	0.8809	-0.1989	0.8973	0.4207	0.9613
Honeywell	0.0208	14.2071	0.8921	0.0000	0.9966	1.4478
HP	0.0466	1.4812	-2.1472	0.9906	0.2185	1.9042
IBM	-0.0277	8.9036	0.8753	0.2341	0.9892	1.4802
INTL Paper	-0.0149	2.4011	0.8273	0.0063	0.9430	1.2417
JP Morgan	-0.0186	3.9362	0.9864	0.1811	0.9665	1.1835
JJ	0.0337	6.0484	1.0597	0.0000	0.9885	1.2241
MCD	0.0293	4.4239	1.0468	0.9989	0.9995	1.2810
MERCK	0.0132	2.1682	1.0805	0.4917	0.9999	1.3203
MSFT	0.1004	6.3828	1.3293	0.3443	0.9846	1.7856
MMM	-0.0010	1.074	0.6260	0.3615	0.5033	0.9933
PM	0.0256	13.008	1.0065	0.0049	0.9961	1.2555
PG	0.0222	1.3758	0.8543	0.9891	0.9937	1.0621
SBC	-0.0149	4.7734	0.9153	0.7746	0.9956	1.1179
United Tec	0.0180	2.0443	0.8862	0.9999	0.9999	1.1877
Walmart	-0.0081	1.4949	0.0383	0.9734	0.5222	1.3763
Disney	0.0107	1.9633	0.9994	0.8672	0.9907	1.3635
Intel	0.0974	18.2393	1.4382	0.0000	0.9927	2.0113

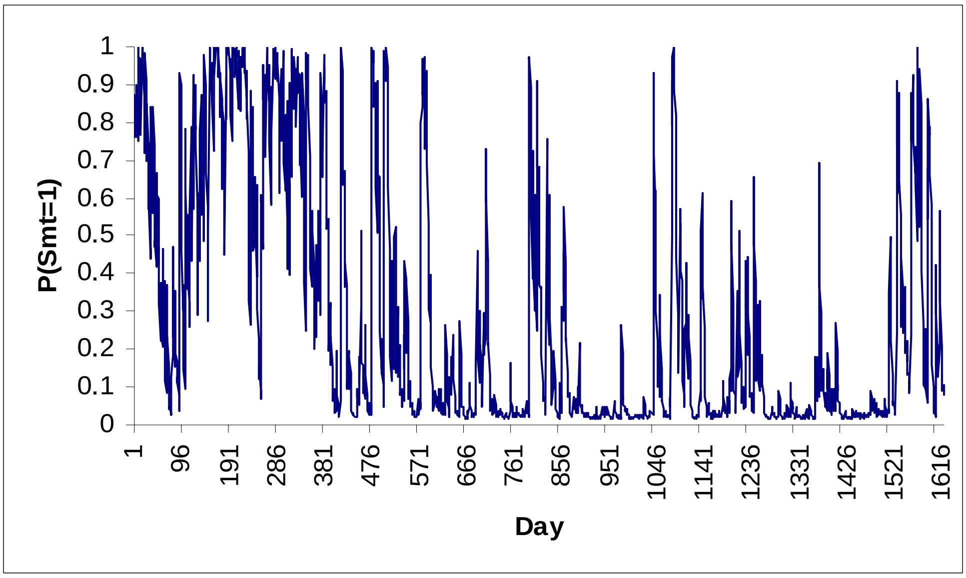
Notes: The estimates are from $r_{it} = \alpha_i + (\beta_{i1} S_{it} + \beta_{i2}) r_{mt} + \varepsilon_{it}$ where r_{it} is the return of security *i* in excess of the risk-free return, S_{it} is an unobserved binary variable that identifies which of the two risk states the security is in at time *t*, and $\varepsilon_{it} \sim N(0, \sigma_i^2)$. In twelve securities, at least one of the transition probability estimates is at the boundary (covariance matrices are not positive definite).). In these cases it is not possible to obtain the estimates of standard errors of the parameters. Therefore, standard errors are not reported.

Table 6. Probability estimates of a security being in high/low risk state and the market in high/low volatility regime

Security (*i*)	P(HRS and HVM)	P(HRS and LVM)	P(LRS and HVM)	P(LRS and LVM)
Dupont	0.0025	0.0012	0.2515	0.7447
Boeing	0.1261	0.2046	0.1279	0.5414
Caterpillar	0.0031	0.0019	0.2509	0.7441
Alcoa	0.0012	0.0000	0.2528	0.7460
Amex	0.0056	0.0056	0.2485	0.7404
ATT	0.2528	0.7460	0.0012	0.0000
CITIGRP	0.0606	0.0871	0.1934	0.6588
Coca Cola	0.0723	0.0754	0.1817	0.6706
Home Depot	0.0025	0.0012	0.2515	0.7447
GE	0.0031	0.0012	0.2509	0.7447
GM	0.2497	0.7454	0.0043	0.0006
Kodak	0.0012	0.0031	0.2528	0.7429
Exxon	0.2417	0.7398	0.0124	0.0062
Honeywell	0.0012	0.0006	0.2528	0.7454
HP	0.2528	0.7454	0.0012	0.0006
IBM	0.0031	0.0019	0.2509	0.7441
INTL Paper	0.0031	0.0006	0.2509	0.7454
JP Morgan	0.0074	0.0019	0.2466	0.7441
JJ	0.0000	0.0031	0.2540	0.7429
MCD	0.0037	0.0012	0.2503	0.7447
MERCK	0.0000	0.0000	0.2540	0.7460
MSFT	0.0025	0.0031	0.2515	0.7429
MMM	0.0414	0.0655	0.2126	0.6805
PM	0.0006	0.0012	0.2534	0.7447
PG	0.0754	0.2528	0.1786	0.4932
SBC	0.0019	0.0049	0.2522	0.7410
United Tec	0.0124	0.0068	0.2417	0.7392
Walmart	0.2497	0.7441	0.0043	0.0019
Disney	0.0068	0.0019	0.2472	0.7441
Intel	0.0012	0.0025	0.2528	0.7435

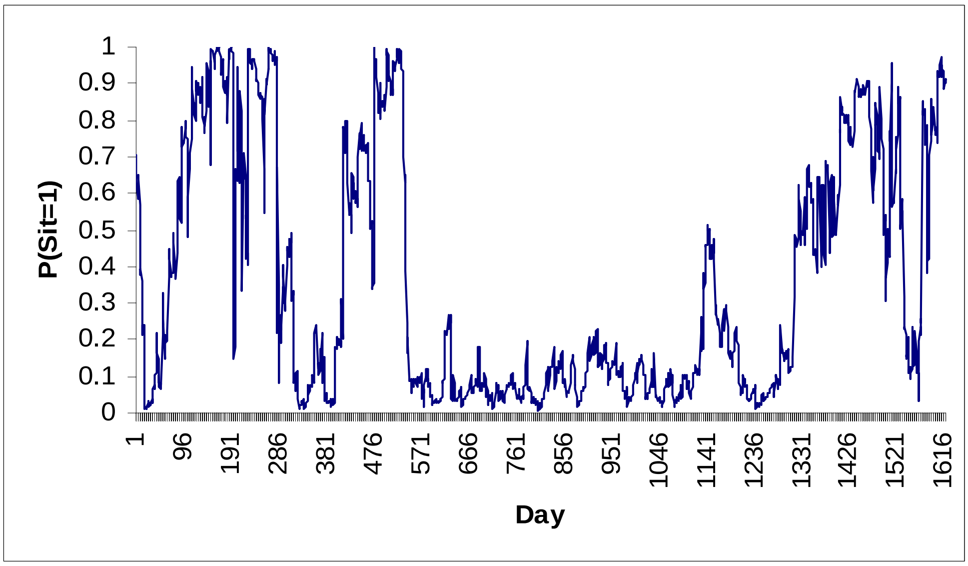
Notes: The estimates are from (i) $r_{mt} = \mu_m + (\sigma_{m1} S_{mt} + \sigma_{m2})\varepsilon_t$ where r_{mt} is return of market portfolio in excess of the risk-free rate, $\mu_m = E(R_{mt})$, S_{mt} is an unobserved binary variable that identifies which of the two regimes the market is in at time *t* (S_{mt} =1 for the high volatility regime and S_{mt} =0 for the low volatility regime), and $\varepsilon_t | \Phi_{t-1} \sim N(0,1)$ where Φ_{t-1} is information set at time *t*-1 and (ii) $r_{it} = \alpha_i + (\beta_{i1} S_{it} + \beta_{i2}) r_{mt} + \varepsilon_{it}$ where r_{it} is the return of security *i* in excess of the risk-free return, S_{it} is an unobserved binary variable that identifies which of the two risk states the security is in at time *t* (S_{it} =1 for the high-risk state and S_{it} =0 for the low-risk state), and $\varepsilon_{it} \sim N(0, \sigma_i^2)$. HRS (LRS) =high (low) security beta risk state and HVM (LVM) = high (low) market volatility regime.

Figure 1. Estimated probability of the market being in high volatility regime



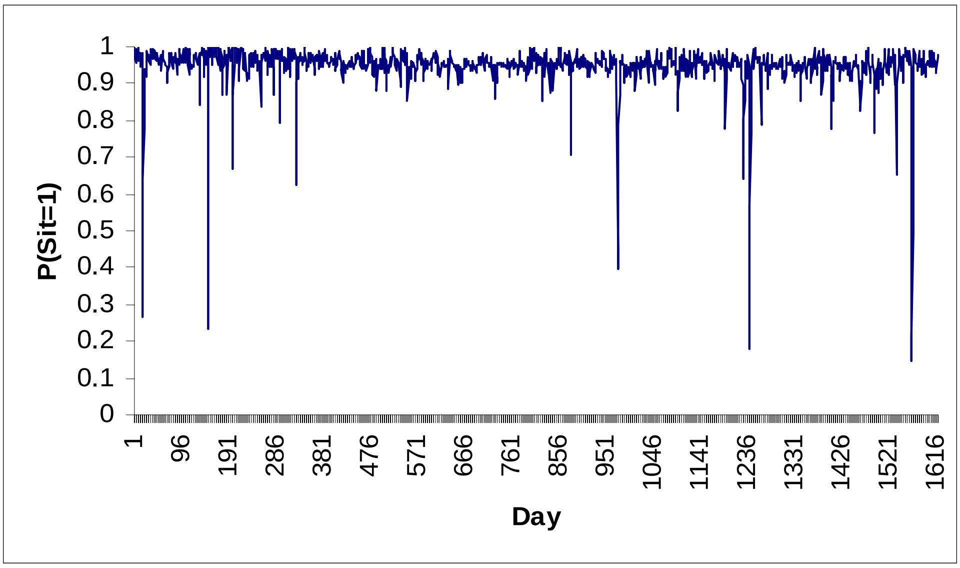
Notes: $P(S_{mt}=1)$ = P(market is in the high volatility regime in day *t*). The number of observations is 1618. The sample period is January 1990 through May 1996.

Figure 2. Estimated probability of security- Coca Cola being in the high-risk state



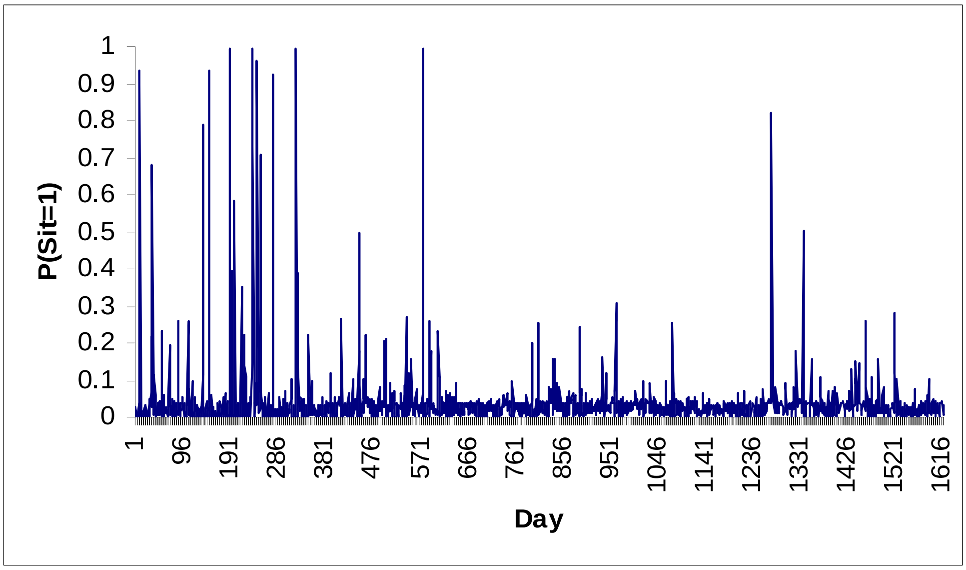
Notes: $P(S_{it}=1)$ = P(security is in the high risk state in day *t*). The number of observations is 1618. The sample period is January 1990 through May 1996.

Figure 3. Estimated probability of security- Walmart being in the high-risk state



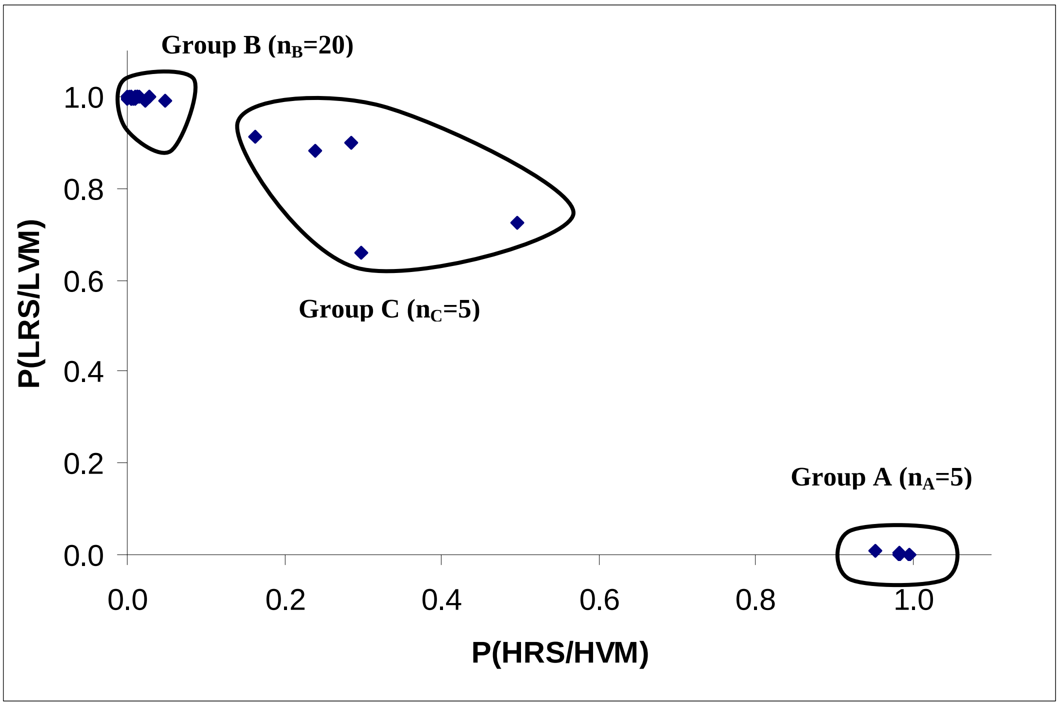
Notes: $P(S_{it}=1)$ = P(security is in the high risk state in day *t*). The number of observations is 1618. The sample period is January 1990 through May 1996.

Figure 4. Estimated probability of security- JP Morgan being in the high-risk state



Notes: $P(S_{it}=1)$ = P(security is in the high risk state in day *t*). The number of observations is 1618. The sample period is January 1990 through May 1996.

Figure 5. Estimated probability of a security being in the low-risk state given market in the low volatility regime and probability of a security being in the high-risk state given market in the high volatility regime



Notes: n_i: is the number of securities that belongs to group i where i=A,B,C. The securities that belongs to group A are: ATT, GM, Exxon, HP and Walmart. The securities that belongs to group C are: Boeing, CITIGRP, Coca Cola, MMM and PG. The estimates are from (i) $r_{mt} = \mu_m + (\sigma_{m1} S_{mt} + \sigma_{m2})\varepsilon_t$ where r_{mt} is return of market portfolio in excess of the risk-free rate, $\mu_m = E(r_{mt})$, S_{mt} is an unobserved binary variable that identifies which of the two regimes the market is in at time *t* (S_{mt} =1 for the high volatility regime and S_{mt} =0 for the low volatility regime), and $\varepsilon_t | \Phi_{t-1} \sim N(0,1)$ where Φ_{t-1} is information set at time *t*-1 and (ii) $r_{it} = \alpha_i + (\beta_{i1} S_{it} + \beta_{i2}) r_{mt} + \varepsilon_{it}$ where r_{it} is the return of security *i* in excess of the risk-free return, S_{it} is an unobserved binary variable that identifies which of the two risk states the security is in at time *t* (S_{it} =1 for the high-risk state and S_{it} =0 for the low-risk state), and $\varepsilon_{it} \sim N(0, \sigma_i^2)$. HRS (LRS) =high (low) security beta risk state and HVM (LVM) = high (low) market volatility regime.