How Does Systematic Risk Impact Stocks? A Study On the French Financial Market

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Abstract

From the CAC40 French stock index, we induce the implied market factor's level through the inversion of a closed form pricing formula for European calls on the CAC40. For this purpose, we assume that the CAC40 index is a disturbed observation of the actual market factor, the market factor's diffusion following a geometric Brownian motion. All the assumptions prevailing in a Black & Scholes (1973) world are assumed to hold. Based on daily data, the results show that the level of the implied market factor and its instantaneous return's volatility are leptokurtic distributed.

Having a proxy for the systematic risk, we also study the impact of the implied market factor on a basket of French assets. First, we compute correlations of assets' returns with the return of the implied market factor, and realize as well a VAR study and a Granger causality test. Second, we estimate regressions of French assets' returns on the return of the implied market factor. Then, we characterize the prevailing relationship between the weekly rolling volatility of the return of the implied market factor and weekly rolling volatilities of the French assets returns. These two studies lead to mitigated results.

Keywords : Call pricing, Granger causality, implied volatility, leptokurtic, systematic risk.
JEL codes : C32, G12, G13.

1 Introduction

It is well known that systematic risk affects the prices of financial assets traded in the market (Stulz [1999a, b, c]). Indeed, the Capital Asset Pricing Model (i.e., CAPM) theory argues that each financial asset is exposed to an undiversifiable risk, named systematic or market risk as introduced by Sharpe (1963, 1964, 1970) and Treynor (1961) among others.[1] Such a risk could be proxied through a well-diversified portfolio so far as such a portfolio presents an idiosyncratic risk as low as possible (i.e., a global variance as low as possible, or equivalently, minimum[2]). Recent literature focuses mainly on the sound assessment of the influence of systematic risk on financial assets along with the beta coefficient in a CAPM framework. Koutmos & Knif (2002) estimate the influence of systematic risk while employing time-varying distributions (i.e., conditional distributions depending on past innovations). Using market stock indices of the financial markets under consideration, they find that financial assets' betas are stationary mean-reverting processes with an average degree of persistence equal to four days. Differently, Gençay, Selçuk & Whitcher (2003) use wavelet techniques to assess the influence of systematic risk on any asset, or equivalently, to compute its beta in a CAPM model. Those authors use the S&P 500 index as a market portfolio (i.e., systematic risk benchmark). Therefore, common practice tells to use available stock indices as proxies for a well-diversified portfolio (i.e., market portfolio) and pays little attention to the sound assessment of systematic risk itself.[3] However, a recent study of Campbell *et al.* (2001) shows that the number of stocks of such an index has to be high enough to offset the idiosyncratic risk. They underline the fact that the number of assets required to create a well-diversified portfolio has grown through time. Therefore, using market indices with an insufficient number of stocks may be inaccurate and even wrong as a benchmark for systematic risk. Specifically, small stock indices are inaccurate proxies for market risk. However, Campbell *et al.* (2001) show that market volatility (i.e., that part of the global volatility due to the market factor) tends to drive global volatility. Therefore, in this paper we address the question of how to find a proxy for the market factor (i.e., the systematic risk factor) in markets where only small stock indices are available and where options on such indices are traded. This task is hard given that the undiversifiable risk is not directly observable and can only be proxied.

Our paper is organized as follows. In section 2, we introduce the assumptions and the theoretical framework proposed to find a proxy for the systematic risk factor. In section 3, we employ an empirical application of such a framework. We focus on the French financial market and on its CAC40 stock index. In

[1] Improved versions of such a model are also given by Mossin (1966), Lintner (1965, 1969) and Black (1972). Dynamic versions of the CAPM are also proposed along with intertemporal models like Merton (1974) for example.

[2] See French & Poterba (1991) for example.

[3] A recent work of Milevsky (2002) studies the two dimensions of diversification, namely the number of stocks composing a portfolio and the time horizon for investment. The author discusses the benefits of the number of stocks diversification versus the time horizon diversification.

section 4, we study the impact of the implied market factor on a pool of French stocks. The impact of the systematic risk is analyzed through a two steps methodology, namely a correlation study and a Granger causality test. For further investigation, section 5 attempts to test for a non-linear relationship between the implied market factor and French financial assets. This study is realized in two stages: a linear regression analysis and a volatility analysis. Finally, we end our study with concluding remarks and suggestions for future research.

2 Theoretical framework

In this section, we introduce our assumptions and the related theoretical framework allowing the induction of the market factor.

2.1 Valuation setting

Our study assumes that each small stock index is a non-perfect proxy of the systematic risk factor. Specifically, we suppose that any small stock index represents a disturbed observation of the market factor.

Assumptions:

Each small stock index I_t, at current time t, depends on the market factor X_t in the following way:

$$I_t = \Lambda_t \, X_t \tag{1}$$

where Λ_t represents a (strictly) positive determinist scale factor whose value is time-varying and bounded on $\mathbb{R}_+^*$. Moreover, Λ_t is supposed to be a continuous and derivable function of time. This parameter could encompass many effects or factors such as short-term shocks due to announcement effects or due to some specific events occurring in the financial market.

We further suppose that all the assumptions related to the option valuation framework of Black & Scholes (1973) hold. To sum up, tradings are continuous, there are no dividend payments, no transaction costs and no taxes. Moreover, there is no arbitrage opportunity and a constant spot risk free interest rate r prevails in the market that is complete.[4]

We also assume that the market factor follows a geometric Brownian motion such as:

$$\frac{dX_t}{X_t} = \mu \, dt + \sigma \, dW_t \tag{2}$$

where

[4] Completeness means that the price of each financial asset can be reached. This is equivalent to assume that each variable is observable in the market or can be proxied.

t represents the current date;

μ and σ are constant drift and volatility parameters of the systematic factor's instantaneous rate of return;[5]

W_t is a standard Brownian motion under the historical probability.

Such assumptions have some implications when considering the evolution of the small stock index.

Dynamic of the stock index:

Applying Ito's lemma in the risk neutral universe and on the time subset $[t, T]$, we find the following expression[6] for the dynamic of the stock index:

$$d \ln (I_t) = \left[\left(\frac{1}{\Lambda_t} \frac{\partial \Lambda_t}{\partial t} \right) + \left(r - \frac{\sigma^2}{2} \right) \right] dt + \sigma \, dW_t^* \tag{3}$$

which leads to the following relation:

$$I_T = I_t \frac{\Lambda_T}{\Lambda_t} \exp \left[\left(r - \frac{\sigma^2}{2} \right) (T - t) + \sigma \, W_{T-t}^* \right] \tag{4}$$

where (W_t^*) represents a standard Brownian motion under the risk neutral probability. Of course, Λ_t and Λ_T could be estimated through the building of a well-diversified portfolio. Such a portfolio would represent a good proxy of the market factor so that the market is complete. However, we address here the question of how to proxy the market factor when starting from a small stock index, which is a non-perfect proxy of the market factor. To achieve this goal, we consider the pricing of options on such an index. Indeed, observations of index prices and call market prices will give information about both the scale factor and the market factor.

2.2 Option pricing

In this part, we induce a call pricing formula to evaluate European calls written on the stock index previously introduced.

Call's dynamic in a no dividend framework:

We consider a European call on the stock index I whose strike price and expiring date are respectively K and T. At maturity, the value of such a call is $C(T, I_T) = \max(0, I_T - K) = (I_T - K)^+$. Analogously to Black & Scholes (1973), we apply the no opportunity arbitrage valuation principle, which states that the current value of any contingent claim is equal to the discount expected

[5] This setting implies that the drift and volatility parameters satisfy the Lipschitz conditions. Such conditions warranty the existence and the unicity of the solution to the stochastic differential equation satisfied by the market factor's dynamic (given a starting value).

[6] The reader is invited to consult the appendix for explanations.

value of its future cash flows under the risk neutral probability. Following this principle, we could then state the next relation for the European call valuation:

$$C\left(t, I_t\right) = E_t^Q\left[e^{-r(T-t)}\left(I_T - K\right)^+\right]$$

where $E_t^Q[.]$ represents the expectation under the risk neutral probability Q conditional on the information set available at current date t. Therefore, given dynamics (1) and (3) of the stock index, the formula[7] for a European call on the stock index I at current date t reads:

$$C\left(t, I_t\right) \equiv C\left(T-t, K, I_t, r, \Lambda_t, \Lambda_T, \sigma\right) = \frac{\Lambda_T}{\Lambda_t} I_t N(d_1) - K e^{-r(T-t)} N(d_2) \quad (5)$$

where

$N(.)$ represents the cumulative distribution function related to the standard normal law;

$$d_1 = \frac{\ln\left(\frac{\Lambda_T}{\Lambda_t}\right) + \ln\left(\frac{I_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \ ;$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = \frac{\ln\left(\frac{\Lambda_T}{\Lambda_t}\right) + \ln\left(\frac{I_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \ .$$

Notice that, if we assume that the small stock index is a perfect proxy of the market factor, we get the classical Black & Scholes option pricing formula since we have $\Lambda_t = \Lambda_T = 1$ for each date $t < T$. Therefore, the introduction of a disturbance in our setting modifies the classical Black & Scholes formula through the presence of the ratio $\left(\frac{\Lambda_T}{\Lambda_t}\right)$ in our analytical formula.

However, assuming a Black & Scholes framework to value a call on a stock index is not appropriate insofar as the no dividend assumption is not realistic. That's why we adapt the previous formula to take into account a stock index encompassing dividend-paying equities.

Call's dynamic in a dividend framework:

We adapt here the previous framework to take into account dividend-paying stocks. Since most of the stocks that constitute financial indices pay dividends, we assume that the index I pays a dividend at a continuous (annualized) rate q (see Merton [1973] and Black [1975] for explanations). Therefore, given the Black & Scholes world and dividend-paying assumptions, one has to replace the current price of the underlying I_t with $I_t\, e^{-q(T-t)}$. This consideration allows us to adapt the European call pricing formula (5) to a dividend-paying framework, so that we get the next expression for a European call on the dividend-paying stock index I:

$$C\left(T-t, K, I_t, r, \Lambda_t, \Lambda_T, \sigma\right) = \frac{\Lambda_T}{\Lambda_t} I_t\, e^{-q(T-t)}\, N(d_1) - K\, e^{-r(T-t)} N(d_2) \quad (6)$$

where

[7] Computation details are given in the appendix.

$N(.)$ represents the cumulative distribution function related to the standard normal law;

$$d_1 = \frac{\ln\left(\frac{\Lambda_T}{\Lambda_t}\right) + \ln\left(\frac{I_t\, e^{-q(T-t)}}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \;;$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = \frac{\ln\left(\frac{\Lambda_T}{\Lambda_t}\right) + \ln\left(\frac{I_t\, e^{-q(T-t)}}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \;.$$

In our European call formula (5) or (6), all the parameters are known except the scale parameter at instants t and T (i.e., Λ_t and Λ_T), and the volatility parameter σ. In the following of the paper, we will use our knowledge about observed index prices and market prices of European index calls to extract some information about the scale parameter Λ and the volatility parameter σ. Such a process will help us to obtain information about the market factor itself.

3 Empirical study

In this section, we apply our European call pricing formula to the French stock market and its CAC40 stock index.

3.1 Data

In this subsection, we describe our data and the computation of the risk free interest rate.

We use daily data going from 2 January 2002 to 19 March 2002, namely 55 observations by series. The data are obtained from Bloomberg. We observe one month r_{1M}, three months r_{3M} and six months r_{6M} risk free interest rates. We consider the market prices of the CAC40 French stock index. This index[8] is composed of the forty most liquid stocks listed on the French market and pays a continuous annualized dividend rate q. We also obtain closing prices of three European calls[9] on the CAC40. These calls are traded on the French options market called MONEP (i.e., Marché des Options Négociables de Paris). Let q, nb_K and *spread* be respectively the dividend rate, the number of different strike prices of the CAC40 Index calls and the variation bounds of the index value (i.e., lowest-highest in euros) during the studied time period, such that we have:

[8] The CAC40 Index is a weighted stock index whose weights are proportional to each of its 40 stocks' capitalization. The 40 selected stocks are supposed to be the most important and representative of the French financial market.

[9] We consider option contracts of the continuous listing class.

Table 1 : *Index information*

Index	$q(\%)$	nb_K	*spread*
CAC40	$2,2650$	3	$4238.99 - 4682.79$

European calls on the CAC40 Index, whose maturity is 28 March 2002, present the following features:

Table 2 : *CAC40 index calls features*

Call name	Strike price (euros)
CAC 3/02 C4000	4000
CAC 3/02 C4500	4500
CAC 3/02 C5000	5000

Notice that, at the beginning of the time period, the time to maturity of our calls is 85 calendar days, and at the end of our time horizon, the remaining time to maturity falls to 9 calendar days, namely 7 working days. Some of these data will help us to compute the risk free interest rate, which has to be defined.

Given our European call pricing formula, we compute the risk free rate as a function of time to maturity. We choose a quadratic interpolation method to induce our short-term risk free rate from the one, two and three months term risk free rates. Let $r(t,T)$ be the risk free rate at current time t for the time horizon T. This rate is then described by the following relation:

$$r(t,T) = a\,(T-t)^2 + b\,(T-t) + c \tag{7}$$

with

$$\begin{cases} a = 72\left[r_{3M}(t) - 2r_{2M}(t) + r_{1M}(t)\right] \\ b = 12\left[r_{2M}(t) - r_{1M}(t) - \frac{a}{48}\right] \\ c = r_{1M}(t) - \frac{a}{144} - \frac{b}{12} \end{cases} \tag{8}$$

This method leads to a good proxy for the risk free rate given that our European calls' time to maturity (i.e., $(T-t)$) is at most three calendar months.

3.2 Systematic risk's induction

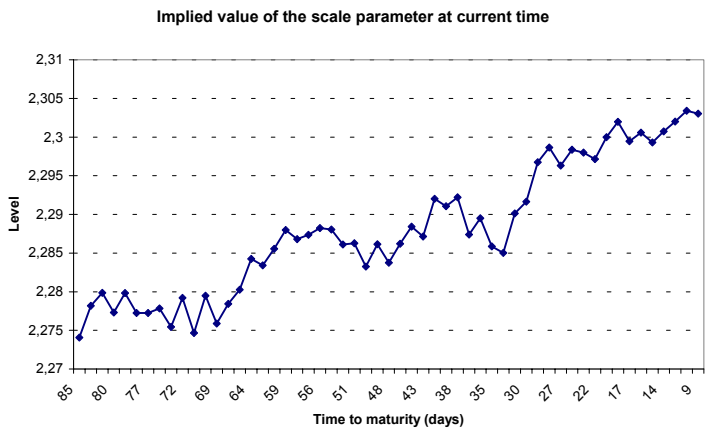
In this part, we explain how to estimate the level of the market factor when starting from market prices of a small stock index and closing prices of European calls on such an index.

According to formula (6), the estimation of the level of the market factor requires the estimation of the scale parameter at instants t and T (i.e., Λ_t and Λ_T), and the volatility parameter σ (i.e., the volatility of the instantaneous rate

of return of the market factor). Since we can observe market prices of the CAC40 index (i.e., small stock index) and closing prices of related European calls, one solution consists of inverting our formula (6) relative to the scale parameter at times t and T, and the volatility parameter. We estimate these implied parameters while minimizing the sum of squared valuation errors at each fixed date t as follows:[10]

$$\underset{\Lambda_t,\ \Lambda_T,\ \sigma}{Min} \left\{ \sum_{j=1}^{nb_K} \left[C^{Obs}\left(T-t, K_j, I_t\right) - C\left(T-t, K_j, I_t, r, \Lambda_t, \Lambda_T, \sigma\right) \right]^2 \right\} \tag{9}$$

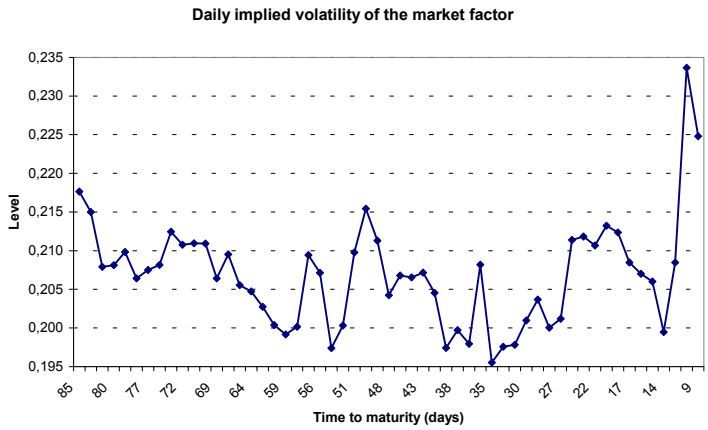
where $K_j \in \{4000, 4500, 5000\}$ and $C^{Obs}\left(T-t, K_j, I_t\right)$ corresponds to the European call's market price. This non-linear minimization problem is solved numerically while applying a quasi-Newton method with a Davidon-Fletcher-Powell type algorithm. First, we get[11] $\Lambda_T = 2.3050$ and therefore $X_T = 2033.8482$. Second, the results obtained allow us to plot the implied values of Λ_t and σ against the time to maturity[12] of related calls:



[10] Of course, we could have used other French stock index such as the SBF250 to proxy Λ_t. Recall that this index is a weighted index composed of the 40 stocks of the CAC40, 80 other most liquid stocks and 130 stocks selected for their importance and their sector representativity. There is no traded option on such an index in the MONEP. We tried to do so and then to realize our minimization program relative to Λ_T and σ. The results we obtained led us to European call pricing errors more than ten times higher than the pricing errors we get in our current study. We could then deduce that the market factor should be proxied by a French stock index composed of more than 250 stocks. Moreover, valuation errors are increasing functions of time to maturity.

[11] In fact, we get different values going from 2.2950 to 2.3150 for Λ_T. We notice that the related estimations are stable over time (since they slightly vary around a mean level). Since this value parameter should be a constant over time, we use the estimated values' arithmetic mean of Λ_T over the studied time horizon. Such a time variation may be explained by the fact that, at each time t, expectations about the index final value and therefore the final value of both the scale parameter and the implied market factor, change from day to day.

[12] Recall that Whaley (1982) argue that valuation errors do not necessarily depend on options' moneyness. Following this remark, it seems more coherent to realize plots according to time rather than according to moneyness.



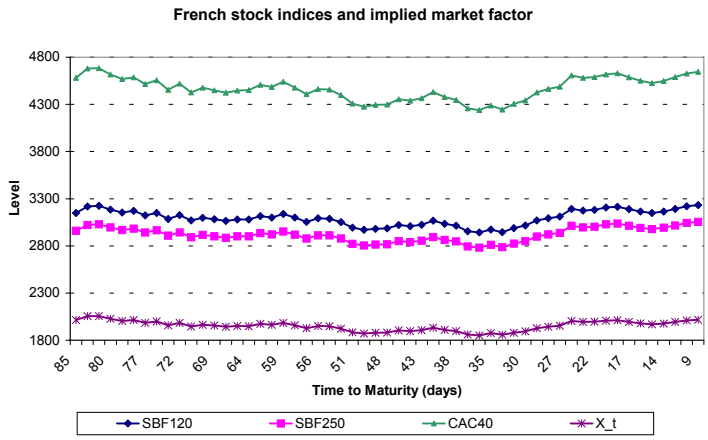
Contrary to our assumption, we notice a time-varying implied volatility parameter σ with a quadratic trend (i.e., a 'smirk' type trend). Moreover, the two implied time series of Λ_t and σ exhibit the following statistical profiles:

Table 3 : *Descriptive statistics*

	Λ_t	σ	X_t
Mean	2.2881	0.2069	1952.2699
Standard deviation	0.0086	0.0069	52.0441
Skewness	0.2144	1.1208	-0.1669
Excess Kurtosis	-1.0728	2.9231	-0.8446
Jarque-Bera Statistic	3.0589	31.0976	1.8901

We then observe a non-normal behavior for Λ_t and σ, that is leptokurtic distributions. Specifically, the volatility of the systematic risk factor should be modeled by a non-normal stochastic process or time-varying series. This stylized fact is commonly known as the Black & Scholes volatility bias characterizing non-normal observed market returns for assets.

To have a global view, we plot the levels of all available French stock indices and the implied systematic risk factor.



'X_t' represents the implied market factor X_t over time, and therefore is representative of the market's trend. And, the SBF120 is a weighted stock index composed of the 40 values of CAC40 Index and other 80 most liquid French stocks. There is no traded option on this index in the MONEP.

Knowing the market trend, we can now characterize the impact of the systematic risk on the French financial market.

4 Systematic risk's impact

Given our estimation of the market factor, we try to quantify its impact on the prices of the French stocks. Our primary econometric study is composed of a correlation study and a Granger causality test.

4.1 Correlation

We study the correlations between the implied market factor and, on one hand, French stock indices (i.e., CAC40, SBF120 and SBF250), and on the other hand, 10 stocks of the French market, namely Air liquide (Ai), Danone (Bn), L'Oréal (Or), Renault (Rno), Schneider (Su), Société générale (Gle), Thomson (Tmm), Totalfina Elf (Fp), Valéo (Fr) and Vivendi (Ex). We consider daily closing prices going from 2 January 2002 to 19 March 2002.

Such a setting requires to pay attention to the following concern. Most of commonly used descriptive statistics are valid only under the strong assumption of an elliptical distribution such as the normal law, which is a spherical distribution. When this is not the case, obtained results are false. Indeed, this feature fits some of the current questions considered by the Basel Committee. Szego (2002) and Artzner *et al.* (1999, 2000), among others, highlight the coherency problem of risk measures such as linear correlation or covariance. Such risk measures are valid only for, at least, stationary distributions when not

elliptical.[13]

Following this concern, we compute the correlation between the return of the implied market factor and returns of French stocks. The returns of both series are stationary[14] over the time period studied (i.e., 54 observations per series). We then study the link between the variation of the return of the systematic risk factor and the evolution of the returns of French stocks. Results are displayed in the next table:[15]

Table 4 : *Correlation of assets with the implied market factor*

Stock or index return	Correlation coefficient
SBF120	0.9959
SBF250	0.9952
CAC40	0.9967
ai	0.4667
bn	0.2002
or	0.6775
rno	0.5365
su	0.4736
gle	0.7078
tmm	0.5329
fp	0.6569
fr	0.5288
ex	0.7982

The average correlation of our three stock indices is 0.9959. Roughly speaking, the implied market factor seems to be highly correlated with stocks whose correlation coefficients range from 0.2002 for *bn* to 0.7982 for *ex*. In the rest of the paper, we are going to study the dependency between systematic risk and French stocks.

4.2 Causality

Any causality study needs a VAR (i.e., Vector AutoRegressive) specification before.

[13] Specifically, leptokurtic distributions do not satisfy one of the main properties accounting for risk measures coherency, namely the sub-additivity principle.

[14] Statistics are available upon request. The results show that indices (implied market factor included) and stocks are stationary when considering their first differences only. However, their returns from one day to another are stationary.

[15] The results are extremely close to the correlation coefficients computed for the first differences of both indices and stocks' prices.

4.2.1 VAR specification

Our goal is to observe the link between the implied market risk's return R_{X_t} and some French stocks or index returns R_{S_t}. Therefore, we will consider bivariate VAR representations linking R_{X_t} and R_{S_t} with $S_t \in$ {SBF120, SBF250, CAC40, ai, bn, or , rno, su, gle, tmm, fp, fr, ex}. When we consider a VAR model, we test for a statistical relation between variables. Moreover, the parameters of any VAR process have to be estimated for stationary time series like our assets' returns.

The related bidimensional VAR with p lags, called VAR(p), has the following representation:

$$Y_t = A_0 + A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \varepsilon_t \tag{10}$$

where

$Y_t = \begin{bmatrix} R_{X_t} \\ R_{S_t} \end{bmatrix}$ is the vector of variables;

$A_0 = \begin{bmatrix} a_1^0 \\ a_2^0 \end{bmatrix}$ is a vector of constant parameters;

$A_p = \begin{bmatrix} a_{1p}^1 & a_{1p}^2 \\ a_{2p}^1 & a_{2p}^2 \end{bmatrix}$ is the coefficients matrix for lag p;

$\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$ is the vector of innovations[16] assumed to follow a normal law.

Notice that when disturbances (ε_t) are correlated, the variation of one component of these errors has an impact on the other components. Namely, the studied variables have a synchrone influence on each other. A causality analysis allows then to study the kind of influence variables have on each other. Moreover, the optimal lag is determined while minimizing the Akaike and Schwarz information criteria. The maximum likelihood method leads to an optimal lag p equal to one.[17] We also get the results listed in the table underneath:

[16] In practice, the disturbances may be contemporaneously correlated with each other, without being correlated with, on one hand, their own lagged values, and on the other hand, all the lagged values of the variables.

[17] We investigated optimal lags from one to five, looking for a weekly influence at most.

Table 5 : *VAR results for stock indices*

Index return	Coefficients			$R^2(\%)$
CAC40	-0.0546	-1.1382	1.0206	
	(-0.3488)	(-0.6794)	(0.6428)	1.2644
	-0.0219	-0.6581	0.5539	0.7644
	(-0.1327)	(-0.3721)	(0.3305)	
SBF120	-0.0487	-0.4722	0.4102	
	(-0.2888)	(-0.3079)	(0.2666)	0.5899
	-0.0053	-0.4560	0.4037	0.4677
	(-0.0317)	(-0.2995)	(0.2643)	
SBF250	-0.0609	-0.5750	0.5389	
	(-0.3473)	(-0.4061)	(0.3619)	0.7086
	-0.0030	-0.5013	0.4852	0.4455
	(-0.0178)	(-0.3738)	(0.3441)	

In each table, the coefficients of the returns are displayed as follows:

$$\begin{bmatrix} a_1^0 & a_{11}^1 & a_{11}^2 \\ a_2^0 & a_{21}^1 & a_{21}^2 \end{bmatrix}$$

with their related Student statistics indicated in brackets (just under each coefficient). Moreover, when we give the R^2 statistic (i.e., explanatory power) related to the estimation of each univariate relation, this one is displayed in percent in the following way:

$$\begin{bmatrix} R^2\left(R_{X_t}\right) \\ R^2\left(R_{S_t}\right) \end{bmatrix}$$

Recall that we have the next bivariate representation for our VAR(1) specification:

$$\begin{bmatrix} R_{X_t} \\ R_{S_t} \end{bmatrix} = \begin{bmatrix} a_1^0 \\ a_2^0 \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{11}^2 \\ a_{21}^1 & a_{21}^2 \end{bmatrix} \begin{bmatrix} R_{X_{t-1}} \\ R_{S_{t-1}} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \tag{11}$$

Our VAR(1) specification does not exhibit any influence between the return of the implied market factor and the returns of French indices. As a rough guide, we also compute the statistics and coefficients related to our ten stocks' VAR(1) specification.

Table 6 : *VAR results for stocks*

Stock return	Coefficients			$R^2(\%)$
ai	-0.0588	-0.2090	0.2525	
	(-0.4002)	(-1.4016)	(2.0454)	8.1354
	0.1104	-0.2699	0.0547	4.1368
	(0.5906)	(-1.4223)	(0.3481)	
bn	-0.0352	-0.1071	0.2532	
	(-0.2366)	(-0.7840)	(1.5238)	4.8662
	0.0329	0.1050	-0.1050	2.2082
	(0.2584)	(0.8981)	(-0.7379)	
ex	-0.0078	-0.1317	0.0367	
	(-0.0477)	(-0.5784)	(0.3665)	0.7152
	-0.5975	0.0990	0.0317	0.5276
	(-1.5866)	(0.1881)	(0.1371)	
fp	-0.0326	-0.0787	0.0208	
	(-0.2106)	(-0.4343)	(0.1160)	0.4753
	0.1860	-0.0572	-0.1560	3.9099
	(1.1717)	(-0.3077)	(-0.8488)	
fr	-0.0281	-0.0611	-0.0047	
	(-0.1817)	(-0.3786)	(-0.0455)	0.4527
	0.2474	0.0482	0.0080	0.1308
	(1.0045)	(0.1875)	(0.0485)	
gle	-0.0808	-0.3013	0.2120	
	(-0.5361)	(-1.5920)	(1.7529)	6.2121
	0.1979	0.1241	-0.0068	0.5678
	(0.7984)	(0.3985)	(-0.0341)	
or	-0.0032	0.0924	-0.1809	
	(-0.0209)	(0.5057)	(-1.2728)	3.5726
	0.1839	0.0766	-0.5131	24.0139
	(1.0659)	(0.3671)	(-3.1637)	
rno	-0.1423	-0.2426	0.1884	
	(-0.9186)	(-1.5658)	(2.1443)	8.8326
	0.4518	0.0960	0.1453	3.5828
	(1.6360)	(0.3477)	(0.9278)	
su	-0.0233	-0.0384	-0.0278	
	(-0.1521)	(-0.2468)	(-0.3571)	0.7017
	0.1484	0.4611	-0.1657	4.8022
	(0.5015)	(1.5319)	(-1.1000)	
tmm	-0.0295	-0.0671	0.0017	
	(-0.1934)	(-0.4149)	(0.0250)	0.4498
	0.0393	0.0559	-0.0438	0.1454
	(0.1082)	(0.1451)	(-0.2698)	

At a five percent level of test for the Student statistics, Air liquide (ai) and Renault (rno) stocks have some influence on the implied systematic risk factor[18] while Société générale (gle) stock influences the implied systematic risk factor at a ten percent level. We will now further investigate these results through a causality test.

4.2.2 Granger Causality test

One natural application of VAR modeling consists of a causality test. Granger (1969) defines the causality[19] notion as follows: R_{X_t} is said to be the cause of R_{S_t} when taking into account the information set associated to R_{X_t} helps to improve predictions of R_{S_t}.

Analyzing the causality of R_{X_t} towards R_{S_t} is equivalent to realize a test encompassing constraints on the coefficients of R_{X_t} in its VAR representation (11) (that is, to consider a restricted VAR specification for R_{X_t}, also called RVAR). Specifically, consider the assumption[20] $H_0 : a_{21}^1 = a_{21}^2 = 0$. If we accept H_0, then R_{X_t} does not cause R_{S_t}. To test assumption H_0, we compare the unrestricted VAR (UVAR, which is relation [11]) with the VAR specification restricted to H_0 (RVAR). The related test statistic is the next likelihood ratio:

$$L = (n - c) \ln \left\{ \frac{|\Sigma_{RVAR}|}{|\Sigma_{UVAR}|} \right\} \tag{12}$$

where

n is the number of observations;

c is the number of coefficients estimated in each univariate relation of the unrestricted VAR model;

Σ_{RVAR}, Σ_{UVAR} are the covariance matrices of the restricted and unrestricted VAR models respectively;

$|A|$ represents the determinant of matrix A.

In this case, L is assumed to follow a Chi square law with two degrees of freedom (i.e., $\chi^2(2)$). Therefore, we reject H_0 assumption for a given test level α if L is greater than the critical value of the $\chi^2(2)$ law for level α (i.e., $L > \chi^2_{critical}(2)$, refer to Hamilton [1994] for details).

[18] We get the same kind of results when applying a VAR specification to the first differences of the closing prices of indices and stocks. In this case, we also find an optimal lag equal to one.

[19] Sims (1977) also defines a slightly different notion of linear causality.

[20] We could also test the reverse assumption $H_0^* : a_{11}^1 = a_{11}^2 = 0$. In such a case, if we accept H_0^*, then R_{S_t} does not cause R_{X_t}.

To study the relationships between the return of the implied systematic risk factor and returns of French stocks, we tested two assumptions, namely:

$$H_0 \quad : \quad R_{X_t} \text{ does not Granger cause } R_{S_t}.$$
$$H_0^* \quad : \quad R_{S_t} \text{ does not Granger cause } R_{X_t}.$$

and we obtained the results listed in the table below:

Table 7 : *Granger statistics for stock indices*

Index return	L	Probability	Stock return	L	Probability
CAC40	0.1385	0.7114	ai	2.0229	0.1612
	0.4132	0.5233		**4.1838**	0.0461
SBF120	0.0897	0.7658	bn	0.8066	0.3734
	0.0711	0.7909		2.3218	0.1339
SBF250	0.1397	0.7101	ex	0.0354	0.8515
	0.1310	0.7190		0.1343	0.7156

In each table, results are displayed as follows: for each asset, the first line corresponds to results related to the H_0 assumption whereas the second line corresponds to H_0^* assumption's results.

Table 8 : *Granger statistics for stocks*

Stock return	L	Probability	Stock return	L	Probability
fp	0.0947	0.7596	rno	0.1209	0.7296
	0.0135	0.9081		**4.5982**	0.0369
fr	0.0352	0.8520	su	2.3468	0.1318
	0.0021	0.9639		0.1275	0.7226
gle	0.1588	0.6920	tmm	0.0211	0.8852
	3.0727	0.0858		0.0006	0.9801
or	0.1348	0.7151			
	1.6199	0.2090			

At a fifteen percent level, both Air liquide's return (ai) and Renault's return[21] (rno) cause the return (R_{X_t}) of the implied market factor. If we enlarge our test level to forty percent, we get that Société générale's return (gle) also causes the return (R_{X_t}) of the implied market factor.[22]

[21] In fact, a ten percent level is sufficient for *rno*'s return.

[22] We get the same results for the *CAC40* index but with larger test levels since we find that *rno* causes *CAC40* at a 15% level, *ai* causes *CAC40* at a 30% level, and *gle* causes CAC40 at a 50% level. Such results are therefore less evident than for the implied market factor return's case. We would like to further underline that the same results are obtained for the returns of the SBF120 and SBF250 index for test levels larger than those of the implied market factor's return and smaller than those of the *CAC40* return. Moreover, we also get the same kind of results when considering the first differences of the closing prices of daily stocks and indices.

Our causality study leads therefore to a smaller impact of the implied market factor on French assets than what is expected. The weakness of the results may come from the small sample size used. For further investigation, we attempt to find contemporaneous links between our variables (i.e., without lag consideration). Specifically, we would like to test for a non-linear influence of the price of the implied market factor on the prices of the French stocks and indices.

5 Further investigation

In this section, we try to exhibit non-linear relations and 'quadratic' causality, or more generally, non-linear dependence between the implied market factor and French stocks. The non-linearity is captured through the study of returns. We proceed in two steps: a regression analysis of assets' returns and a volatility analysis of these daily returns.

5.1 Simple regression

Focusing on a non-linear link between the price of the implied market factor and the price of an asset is equivalent to regressing this asset's return on the return of the implied market factor. Specifically, we look for the following kind of relationship:

$$S_t = \beta\, X_t^{\alpha} \tag{13}$$

with α and β constant terms, and $S_t \in \{$SBF120, SBF250, CAC40, ai, bn, or , rno, su, gle, tmm, fp, fr, ex$\}$. Moreover, we assume (i.e., approximation) that:

$$R_{S_t} = \frac{S_t - S_{t-1}}{S_{t-1}} \approx \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{14}$$

for each time t ranging from 2 to 55, and we write relation (13) in terms of logarithm under the new form:

$$\ln(S_t) = \ln(\beta) + \alpha \ln(X_t) \tag{15}$$

Considering a variation between times $(t-1)$ and t, we then have:

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = \alpha \ln\left(\frac{X_t}{X_{t-1}}\right) \tag{16}$$

and, therefore:

$$R_{S_t} = \alpha R_{X_t} \tag{17}$$

for each time $t \in \{2, \cdots, 55\}$. Consequently, the non-linear link between X_t and S_t is equivalent to a linear regression of S_t return (R_{S_t}) on X_t return (R_{X_t}). Such a study is practical given that returns are stationary variables here.

The results related to the regressions of the returns of each index and each stock on the return of the implied market factor are displayed in the table underneath:[23]

Table 9 : *Regression results for stock indices*

Index return	α	Student t	$R^2(\%)$
CAC40	1.0514	86.3097	99.2930
SBF120	0.9925	73.1893	99.0179
SBF250	0.9460	64.8178	98.7496

Table 10 : *Regression results for stocks*

Stock return	α	Student t	$R^2(\%)$
ai	0.5650	3.8326	21.2243
bn	0.1645	1.4883	3.9693
ex	1.8302	9.0541	58.9820
fp	0.6694	6.2260	40.7725
fr	0.8384	4.4587	25.0821
gle	1.1088	7.1866	48.4465
or	0.8737	6.6437	44.7481
rno	0.9527	4.3560	18.8525
su	0.9480	3.9014	21.6706
tmm	1.2628	4.5825	28.2329

All the regressions of French financial assets' returns on the return of the implied market factor are significant at the 1% level, except for Danone (bn) stock's regression. Among available French stock indices, the highest explanatory power is achieved for the CAC40 (i.e., $R^2\,(CAC40) = 99.2930\%$) whereas the highest explanatory power among French stocks is achieved for Vivendi stock (i.e., $R^2\,(ex) = 58.8820\%$). Therefore, the implied market factor has an important influence,[24] in terms of explaining daily returns,[25] on all the financial assets previously introduced except for Danone stock (bn). We also tested[26] for assumption '$\mathrm{H}_0 : \alpha = 1$' in relation (17) for stocks. We found that α coefficient is significantly equal to one only for *fr*, *gle*, *or*, *rno*, *su* and *tmm* stocks. Therefore, these six assets are clearly driven by the market trend as proxied through the implied market factor. Such a finding is coherent with the work of Campbell *et al.* (2001).

[23] We have a number of observations equal to 54.

[24] Our results here are more significant and conclusive (when considering the number of influenced assets and the importance of this influence) than those obtained for the CAC40 index (as a proxy of the market risk factor), and those obtained for the study of regressions of the first differences of financial assets prices on the first differences of the implied market factor's price.

[25] Notice that Brailsford & Faff (1997) find a poor support to the CAPM when studying Australian daily stock returns. Moreover, in a daily stock returns setting, Koutmos & Knif (2002) show that the simple regression model works well for a systematic risk measure purpose (i.e., beta estimation). However, a dynamic model with time-varying parameters is better for forecasts purpose (i.e., to forecast efficient conditional beta estimation).

[26] Results are given in the appendix.

5.2 Volatility impact

In this subsection, we study the influence of the implied market factor on the volatility of the French financial assets. We consider weekly rolling volatilities of all assets to test for a quadratic dependence between returns.

Since one financial week represents five working days, the weekly rolling volatility of return R_{S_t} at date t writes:

$$\sigma\left(R_{S_t}\right) = \sqrt{\frac{1}{5}\sum_{i=t-4}^{t}\left(R_{S_i} - \overline{R}_{S_t}\right)^2} \tag{18}$$

with $\overline{R}_{S_t} = \frac{1}{5}\sum_{i=t-4}^{t}\left(R_{S_i}\right)$ for each date $t \in \{6, \cdots, 55\}$. We therefore have 50 observations for each return series. We analyze the impact of the volatility of the implied market factor while considering the following first differences regressions:[27]

$$\Delta\sigma\left(R_{S_t}\right) = a\,\Delta\sigma\left(R_{X_t}\right) + \eta_t \tag{19}$$

where

$\forall t \in \{7, \cdots, 55\}, \forall X_t, \Delta\sigma\left(R_{X_t}\right) = \sigma\left(R_{X_t}\right) - \sigma\left(R_{X_{t-1}}\right);$

a is a constant coefficient;

η_t is a disturbance following a normal law;

$S_t \in$ {SBF120, SBF250, CAC40, ai, bn, or , rno, su, gle, tmm, fp, fr, ex}.

Results associated to the regressions (19) for the first differences of the weekly rolling volatilities of French financial assets on those of the weekly rolling volatility of the implied market factor are listed in the tables underneath.

Table 11 : *Volatility regression results for stock indices*

Index return	a	Student t	$R^2(\%)$
CAC40	1.0329	40.2860	97.1143
SBF120	0.9614	39.3774	96.9823
SBF250	0.9349	40.6838	97.1677

[27] Notice that the weekly rolling volatilities we compute are not stationary in level but they become stationary variables when considering their differences of first order. Statistical results remain available upon request.

Table 12 : *Volatility regression results for stocks*

Stock return	a	Student t	$R^2(\%)$
ai	0.1634	1.1522	2.2470
bn	0.0402	0.2888	0.0270
ex	1.7898	*7.2466*	52.1625
fp	0.3720	*2.8797*	14.7152
fr	0.6732	*2.7991*	14.0162
gle	0.8547	*2.8667*	14.4792
or	0.4608	*4.0223*	23.6650
rno	0.8075	*2.7771*	13.8271
su	-0.0938	-0.3519	0.6453
tmm	-0.5603	-1.3889	3.7211

When considering first differences regressions of weekly rolling volatilities (19), only those concerning assets CAC40, Vivendi (ex), Totalfina Elf (fp), Valéo (fr), Société générale (gle), l'Oréal (Or), Renault (rno), SBF120 and SBF250 are significant at the 1% level. Among French indices, SBF250 presents the highest explanatory power (i.e., $R^2\left(SBF250\right) = 97.1677\%$) whereas, among stocks, Vivendi presents the highest explanatory power (i.e., $R^2\left(ex\right) = 52.1625\%$). The results[28] suggest that the implied market factor has a strong influence on the weekly rolling volatilities of CAC40, l'Oréal, Renault, SBF120, SBF250, Société générale, Totalfina Elf, Valéo and finally Vivendi assets. However, the implied market factor does not succeed in explaining the whole evolution of financial assets when considering the explanatory power of regressions.

6 Conclusion

Following the wide literature about systematic risk initiated by Sharpe (1963), we address the problem of finding a good proxy for market risk when considering a small stock index with traded options on such an index. We proceed in four steps: a theoretical framework, an empirical application of this setting and two empirical studies attempting to assess the implied systematic risk's impact on French financial assets.

First, our theoretical setting assumes that the small stock index is a disturbed observation of the actual market factor. This stock index depends on the market factor through a scale parameter supposed to be a continuous function of time. We further assume that the dynamic of the market factor follows a geometric Brownian motion, which has some implications for the stock index diffusion. We induce an analytical formula pricing European calls on the stock index given the market factor's diffusion. In our closed form formula, all the parameters are known except the scale parameter at times t and T, and the volatility of the market factor.

[28] These results are more significant than those we get when estimating regressions of weekly rolling volatilities of assets on those of the CAC40 index (always for first differences).

Second, inverting our European call pricing formula given observed market prices of European calls on the index, we get the scale parameter's values at dates t and T, and the volatility parameter. These estimations allow us to compute the level of the implied market factor when considering the stock index prices. We realize this empirical study on the French financial market while considering its CAC40 stock index. Results show that the implied volatility parameter is time-varying, and the distributions of both the volatility parameter and the market factor are leptokurtic.

Third, since we know the level of the implied market factor, we attempt to assess its impact on a basket of French stocks and indices. We study correlations between the return of the implied market factor and the returns of the French assets. Our results are poor insofar as our VAR study as well as the Granger causality test only show the strong influence of daily returns of stocks Air liquide and Renault on the return of the implied market factor.

Finally, we investigate a non-linear relationship between French assets' prices and the level of the implied market factor. This leads to a study of linear regressions of the returns of the French assets on the return of the implied market factor. The results obtained are fruitful in that the return of the implied market factor appears to have a strong influence on the returns of the French assets, except for Danone stock. Indeed, regressions exhibit high significativity and explanatory power. To go further, we also estimate first differences regressions of French assets weekly rolling volatilities on the weekly rolling volatility of the implied market factor. The implied market factor exhibits a strong link with assets CAC40, L'Oréal, Renault, SBF120, SBF250, Société générale, Totalfina Elf, Valéo and finally Vivendi. However, it fails in explaining the whole evolution of those assets. Maybe that idiosyncratic risk plays an important role in our case. Indeed, such a risk factor may explain that part of assets' evolution which remains unexplained by the systematic risk factor.

Suggested improvements for the study are first the lengthening of the time period. A larger sample could give stronger and more significant results. Second, building a diversified portfolio (i.e., replicating the market factor with accuracy) could give a benchmark for systematic risk, which could be compared with our implied market factor. Prior to this, we should address what the optimal number of stocks and the optimal composition of a well-diversified portfolio should be to achieve a sound and standardized assessment of the level of systematic risk.

7 Appendix

This section presents and explains some computation details about our European call pricing formula.

7.1 Dynamic of the stock index

We deduce here the expression of the dynamic of the stock index given the market factor's diffusion (2).

In this part, all assumptions and conditions ensuring the existence and unicity of a strong solution to the stochastic differential equation associated to the implied market factor's dynamic are assumed to hold. For technical details and theory, the reader could refer to books of Rogers & Williams (1994a,b). Then, we set:

$$\ln(I_t) = \ln(\Lambda_t) + \ln(X_t) = F(t, X_t) \tag{20}$$

which implies that:

$$\frac{\partial F(t,X_t)}{\partial t} = \frac{\partial \ln(\Lambda_t)}{\partial t} = \frac{1}{\Lambda_t}\frac{\partial \Lambda_t}{\partial t} \qquad \frac{\partial F(t,X_t)}{\partial X_t} = \frac{1}{X_t} \qquad \frac{\partial^2 F(t,X_t)}{\partial X_t^2} = -\frac{1}{X_t^2} \tag{21}$$

Applying Ito's lemma in the risk neutral universe leads to the next relation:

$$\begin{aligned} d\ln(I_t) &= \left[\frac{\partial F(t,X_t)}{\partial t} + rX_t\frac{\partial F(t,X_t)}{\partial X_t} + \frac{\sigma^2 X_t^2}{2}\frac{\partial^2 F(t,X_t)}{\partial X_t^2}\right]dt \\ &\quad +\sigma X_t \frac{\partial F(t,X_t)}{\partial X_t} dW^* \end{aligned} \tag{22}$$

which reads:

$$d\ln(I_t) = \left[\frac{1}{\Lambda_t}\frac{\partial \Lambda_t}{\partial t} + \left(r - \frac{\sigma^2}{2}\right)\right]dt + \sigma dW^* \tag{23}$$

with (W^*) a standard Brownian motion under the risk neutral probability. If we integrate this equation on the time subset $[t, T]$, we get the following relation:

$$\ln\left(\frac{I_T}{I_t}\right) = \ln\left(\frac{\Lambda_T}{\Lambda_t}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t) + \sigma W^*_{T-t}$$

or equivalently,

$$\frac{I_T}{I_t} = \frac{\Lambda_T}{\Lambda_t}\exp\left[\left(r - \frac{\sigma^2}{2}\right)(T-t) + \sigma W^*_{T-t}\right] \tag{24}$$

which, given relation (1), also implies that:

$$X_T = X_t \exp\left[\left(r - \frac{\sigma^2}{2}\right)(T-t) + \sigma W^*_{T-t}\right] \tag{25}$$

Therefore, conditional on the information set F_t available at time t, $\ln\left(\frac{I_T}{I_t}\right)$ follows a normal distribution with an expectation parameter (EXP) and a volatility parameter (VOL) respectively equal to $EXP = \ln\left(\frac{\Lambda_T}{\Lambda_t}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)$ and $VOL = \sigma\sqrt{(T-t)}$, namely $\ln\left(\frac{I_T}{I_t}\right)\Big|_{F_t} \backsim N(EXP, VOL)$.

7.2 Dynamic of the European call on the index

Given specifications (1) and (3) for the dynamic of the stock index, we show the computations leading to the European call pricing formula (5) for a call on the stock index.

Following the no arbitrage principle, we have:

$$\begin{aligned} C\left(t, I_t\right) &= E_t^Q\left[e^{-r(T-t)}\left(I_T-K\right)^+\right] = e^{-r(T-t)} E_t^Q\left[\left(\Lambda_T X_T-K\right)^+\right] \quad (26) \\ &= e^{-r(T-t)} E_t^Q\left[\left(\Lambda_T X_T-K\right) \mathbf{1}_{\{\Lambda_T X_T-K \geq 0\}}\right] \end{aligned}$$

where $\mathbf{1}_{\{\Lambda_T X_T-K \geq 0\}}$ represents the dummy function on set $\{\Lambda_T X_T-K \geq 0\}$. We set $U_T=\ln\left(\frac{X_T}{X_t}\right)$ whose distribution follows a normal law with expectation and volatility parameters respectively equal to $\left(r-\frac{\sigma^2}{2}\right)(T-t)$ and $\sigma \sqrt{(T-t)}$. This specification implies that $X_T=X_t e^{U_T}$. For further simplification, let $Y_T=\frac{U_T-\left(r-\frac{\sigma^2}{2}\right)(T-t)}{\sigma \sqrt{(T-t)}}$ which follows a standard normal law given F_t such that its distribution function is $f(Y_T)=\frac{e^{-\frac{Y_T^2}{2}}}{\sqrt{2 \pi}}$, and we have $U_T=Y_T \sigma \sqrt{(T-t)}+\left(r-\frac{\sigma^2}{2}\right)(T-t)$. Consequently, relation (26) reads:

$$C\left(t, I_t\right)=e^{-r(T-t)} \int_{\mathbb{R}}\left[\begin{array}{c} \left(\Lambda_T X_t e^{Y_T \sigma \sqrt{(T-t)}+\left(r-\frac{\sigma^2}{2}\right)(T-t)}-K\right) \\ \mathbf{1}_{\left\{\Lambda_T X_t e^{Y_T \sigma \sqrt{(T-t)}+\left(r-\frac{\sigma^2}{2}\right)(T-t)}-K \geq 0\right\}} f(Y_T) \end{array}\right] d Y_T \quad (27)$$

Notice that we can write:

$$\begin{aligned} \{I_T-K \geq 0\} &= \{\Lambda_T X_T-K \geq 0\} \\ &= \left\{\Lambda_T X_t e^{\left[Y_T \sigma \sqrt{(T-t)}+\left(r-\frac{\sigma^2}{2}\right)(T-t)\right]}-K \geq 0\right\} \\ &= \left\{Y_T \geq \frac{\ln\left(\frac{K \Lambda_t}{I_t \Lambda_T}\right)-\left(r-\frac{\sigma^2}{2}\right)(T-t)}{\sigma \sqrt{(T-t)}}\right\} \quad (28) \end{aligned}$$

If we set $d_2=-\tilde{d}_1=\frac{\ln\left(\frac{\Lambda_T}{\Lambda_t}\right)+\ln\left(\frac{I_t}{K}\right)+\left(r-\frac{\sigma^2}{2}\right)(T-t)}{\sigma \sqrt{(T-t)}}$, we have $\{I_T-K \geq 0\}=\left\{Y_T \geq \tilde{d}_1\right\}$ and relation (27) takes the new form:

$$\begin{aligned} C\left(t, I_t\right) &= e^{-r(T-t)} \int_{\tilde{d}_1}^{\infty}\left[\left(\Lambda_T X_t e^{Y_T \sigma \sqrt{(T-t)}+\left(r-\frac{\sigma^2}{2}\right)(T-t)}-K\right) f(Y_T)\right] d Y_T \\ &= Int_1-Int_2 \quad (29) \end{aligned}$$

with

$$\begin{aligned} Int_1 &= e^{-r(T-t)} \int_{\tilde{d}_1}^{\infty} \left[\Lambda_T \, X_t \, e^{\left[Y_T \, \sigma \sqrt{(T-t)} + \left(r - \frac{\sigma^2}{2} \right) (T-t) \right]} \, f(Y_T) \right] dY_T \quad (30) \\ Int_2 &= K e^{-r(T-t)} \int_{\tilde{d}_1}^{\infty} f(Y_T) dY_T \end{aligned}$$

Integrating the first member:

The first member could take the following form:

$$\begin{aligned} Int_1 &= \Lambda_T \, X_t \int_{\tilde{d}_1}^{\infty} \left[e^{\left\{ Y_T \, \sigma \sqrt{(T-t)} - \frac{\sigma^2}{2} (T-t) \right\}} \, f(Y_T) \right] dY_T \quad (31) \\ &= \Lambda_T \, X_t \int_{\tilde{d}_1}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\left\{ -\frac{1}{2} \left(Y_T - \sigma \sqrt{(T-t)} \right)^2 \right\}} dY_T \end{aligned}$$

We realize the change of variable $Z_T = Y_T - \sigma \sqrt{(T-t)}$ and we set $d_1 = -\tilde{d}_2 = \frac{\ln\left(\frac{\Lambda_T}{\Lambda_t}\right) + \ln\left(\frac{I_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma \sqrt{(T-t)}}$ which allows us to write:

$$Int_1 = \Lambda_T \, X_t \int_{\tilde{d}_2}^{\infty} f\left(Z_T\right) \, dZ_T = \Lambda_T \, X_t \left[1 - N\left(\tilde{d}_2\right) \right] = \Lambda_T \, X_t N\left(-\tilde{d}_2\right) \quad (32)$$

such that we get:

$$Int_1 = \Lambda_T \, X_t N\left(d_1\right) = \Lambda_T \, \frac{I_t}{\Lambda_t} N\left(d_1\right) \quad (33)$$

since we have relation (1).

Integrating the second member:

Since $N\left(.\right)$ represents the cumulative distribution function of the standard normal law, we could write:

$$Int_2 = K e^{-r(T-t)} \left[1 - N\left(\tilde{d}_1\right) \right] = K e^{-r(T-t)} N\left(-\tilde{d}_1\right) = K e^{-r(T-t)} N\left(d_2\right) \quad (34)$$

Finally, we find the next pricing formula for a European call on an index I with respective strike and maturity equal to K and T:

$$C\left(t, I_t\right) = \Lambda_T \, \frac{I_t}{\Lambda_t} N\left(d_1\right) - K e^{-r(T-t)} N\left(d_2\right) \quad (35)$$

with

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{\Lambda_T}{\Lambda_t}\right) + \ln\left(\frac{I_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma \sqrt{(T-t)}} \quad (36) \\ d_2 &= \frac{\ln\left(\frac{\Lambda_T}{\Lambda_t}\right) + \ln\left(\frac{I_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma \sqrt{(T-t)}} \end{aligned}$$

7.3 Testing for the market trend

In this subsection, we present our methodology to test for assumption '$\alpha = 1$' (i.e., to test for the market trend) in relation (17). Recall that we have:

$$R_{S_t} = \alpha R_{X_t} \tag{37}$$

which also writes:

$$R_{S_t} - R_{X_t} = (\alpha - 1) R_{X_t} \tag{38}$$

or equivalently,

$$R_t = \lambda R_{X_t} \tag{39}$$

with $R_t = R_{S_t} - R_{X_t}$ and $\lambda = \alpha - 1$. Therefore, testing for $\alpha = 1$ is equivalent to test for $\lambda = 0$. We display in the table underneath the results related to regression (39):

Table 13 : *Regression results for stocks*

Stock return	λ	Student t	$R^2(\%)$
ai	-0.4350	-2.9511	13.6310
bn	-0.8355	-7.5589	51.8750
ex	0.8302	4.1070	17.0789
fp	-0.3306	-3.0751	12.2387
fr	-0.1616	**-0.8595**	2.4743
gle	0.1088	**0.7054**	2.3363
or	-0.1263	**-0.9604**	0.3124
rno	-0.0473	**-0.2165**	13.8139
su	-0.0520	**-0.2140**	0.8861
tmm	0.2628	**0.9536**	1.4517

At a five percent level for the Student statistic, λ coefficient is significantly equal to zero for *fr*, *gle*, *or*, *rno*, *su* and *tmm* stocks. Such a test means that returns of *fr*, *gle*, *or*, *rno*, *su* and *tmm* assets are fully driven by the return of the implied market factor.

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References

Artzner P., Delbaen F., Eber J. M. & D. Heath, 2000, *Risk Management and Capital Allocation with Coherent Measures of Risk*, ETH Zentrum Working Paper.

———, 1999, *Coherent Measures of Risk*, Mathematical Finance, vol. 9, p. 203 - 228.

Black F., 1975, *Fact and Fantasy in the Use of Options*, Financial Analysts Journal, vol. 31, p. 36 - 72.

———, 1972, *Capital Market Equilibrium with Restricted Borrowing*, Journal of Business, vol. 7, p. 444 - 455.

Black F. & M. Scholes, 1973, *The Pricing of Options and Corporate Liabilities*, Journal of Political Economy, vol. 83, p. 637 - 653.

Brailsford T. J. & R. W. Faff, 1997, *Testing the Conditional CAPM and the Effect of Intervaling: A Note*, Pacific-Basin Finance Journal, vol. 5, p. 527 - 537.

Campbell J. Y., Lettau M., Malkiel B. G. & Y. Xu, 2001, *Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk*, Journal of Finance, vol. 56, p. 1 - 43.

French K. R. & J. M. Poterba, 1991, *International Diversification and International Equity Markets*, American Economic Review, vol. 81, n° 2, p. 222 - 226.

Granger C. W. J., 1969, *Investigating Causal Relations by Econometric Models and Cross Spectral Methods*, Econometrica, vol. 37, p. 424 - 438.

Hamilton J. D., 1994, *Time Series Analysis*, Princeton University Press.

Lintner J., 1969, *The Aggregation of Investor's Diverse Judgments and Preferences in Purely Competitive Security Markets*, Journal of Financial and Quantitative Analysis, vol. 4, p. 347 - 400.

———, 1965, *The Valuation of Risky Assets and The Selection of Risky Investments in Stock Portfolios and Capital Budgets*, Review of Economics and Statistics, vol. 47, p. 13 - 37.

Merton R. C., 1974, *On the Pricing of Corporate Debt: The Risk Structure of Interest Rates*, Journal of Finance, vol. 29, p. 449 - 470.

———, 1973, *The Theory of Rational Option Pricing*, Bell Journal of Economics & Management Science, vol. 4, p. 141 - 183.

Milevsky M. A., Winter 2002/2003, *Space-Time Diversification: Which Dimension is Better?*, Journal of Risk, vol. 5, n° 2, p. 45 - 71.

Mossin J., 1966, *Equilibrium in a Capital Asset Market*, Econometrica, vol. 34, p. 768 - 783.

Rogers L. C. G. & D. Williams, 1994*b*, *Diffusions, Markov Processes and Martingales: Itô Calculus*, Cambridge University Press (Volume 2).

———, 1994*a*, *Diffusions, Markov Processes and Martingales: Foundations*, Cambridge University Press (Volume 1).

Sharpe W. F., 1970, *Portfolio Theory and Capital Markets*, Mc Graw-Hill, New-York.

———, 1964, *Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk*, Journal of Finance, vol. 19, p. 425 - 442.

———, 1963, *A Simplified Model For Portfolio Analysis*, Management Science, vol. 9, p. 499 - 510.

Sims C., 1977, *Exogeneity and Causal Ordering in Macroeconomic Models*, New Methods in Business Cycle Research, Proceedings from a Conference, Federal Reserve Bank of Minneapolis.

Stulz R. M., 1999a, *International Portfolio Flows and Security Markets*, in Martin Feldstein, ed. International Capital Flows. Chicago: University of Chicago Press.

———, 1999b, *Globalization, Corporate Finance, and the Cost of Capital*, Journal of Applied Corporate Finance, vol. 12, p. 8 - 25.

———, 1999c, *Globalization of Equity Markets and the Cost of Capital*, Dice Center, Ohio State University, Working Paper.

Szego G., 2002, *Measures of Risk*, Journal of Banking and Finance, vol. 26, p. 1253 - 1272.

Treynor J., 1961, *Toward a Theory of the Market Value of Risky Assets*, Unpublished Manuscript.

Whaley R., 1982, *Valuation of American Call Options on dividend-paying Stocks*, Journal of Financial Economics, vol. 10, p. 29 - 58.