DF STRUCTURE MODELS FOR OPTIONS PRICING

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Abstract. Based on the Partial Distribution[16], [17], we presents the concepts and expressions of *DF* process and *DF* structure and put forward the *DF* structure models of pricing options on a non-dividend-paying underlying for the first time. The *DF* structure models are able to price the call and put options exercised at any time, so it is applicable to pricing the American and European options. Finally, examples are given to compare the options priced by *DF* formulas and by *Black-Scholes* formulas, they show, as a whole, that the *DF'* prices of options are closer to the trading prices than *Black-Scholes'* prices in many cases.

Index Terms. Partial Distribution; ***DF*** structure; options pricing; analytic formula; non-dividend-paying

1. Introduction

From the point of view of theoretical studies of world economics and finance, option pricing is a kernel and important problem to which economists pay due attention. In practice, exact option pricing is very important to monetary derivative markets and world economic market and plays an important part in the stabilization of the markets. In the studies of option pricing, there have been many significant results (Black and Scholes 1973, Merton 1976, Sharpe 1978, Whaley 1981, Gesk and Roll 1984), and approximation methods for American put option (MacMillan 1986, Stapleton and Subrahmanyam 1997). But, Up to now, no exact analytic formula has ever been produced for the value of an American put option on a non-dividend-paying stock (Hull 2000). The Black-Scholes models and its extensions assume that the probability distribution of the stock price at any given future time is lognormal. This assumption is not perfect (Hull 2000). Here, we will try to present the new pricing formulas for options based on another probability distribution—the Partial Distribution and *DF* process (F. Dai 2000).

This paper proves, by empirical research, that the Partial Distribution can preferably describe the structure properties of prices of stocks and stock indices. Based on the Partial Distribution and Partial Process, we present here, for the first time, the concepts and expressions of *DF* process and *DF* structure, and put forward a new kind of analytic method of pricing options on a non-dividend-paying underlying. The *DF* structure models are able to price the call and put options exercised at any time, so it is applicable to pricing the American and European options, especially the American put options. The option's underlying assets discussed in this paper are the stock or stock indices on a non-dividend- paying.

Here, we also offer the method of estimating the parameters in partial distribution. Finally, examples are given to compare the options priced by *DF* formulas and by *Black-Scholes* formulas. The examples show, as a whole, that the *DF'* prices of options are closer to the trading prices of the United States options market than *Black-Scholes'* prices.

2. Basic Assumptions and *DF* Structure

2.1 The basic assumptions of prices of assets

The basic assumptions we use to define an underlying (stock and stock indices) price, regarded as the basis of the discussion in this paper are as follows:

(ⅰ) There are prices (the cost price and the market price) to an underlying assert. The cost price means the average of all the prices paid by the market traders to buy an underlying asset and the market price is the current exchange price of an underlying asset.

(ii) The prices (cost price and market price) have been fluctuating with time. Any price and the fluctuation spread (i.e. the variance) of price are non-negative.

(iii) Both the cost price and the fluctuation of cost price of an underlying are the basic elements of determining the market prices of the underlying; The market prices come into being on the market exchange.

(iv) The possibilities that the market price of underlying is much lower than the cost price, or is much higher than the cost price, will be very small.

(v) When the market price drifts gradually apart from the cost price, the possibility that the price of making a trade steps down gradually.

(vi) All securities are perfectly divisible.

(vii)There are no transaction costs or taxes.

We shall prefer referring to stock instead of asset, at the same time, the stock can also be replaced by stock indices, and we mention no more in the following discussions.

2.2. Partial distribution and stock price

Definition 2.1(The Partial Distribution)**.** Let S be a non-negative stochastic variable, and it follows the distribution of density

$$f(x)=\begin{cases} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx & x\geq 0 \\ 0 & x<0 \end{cases} \tag{1}$$

then S is said to have a Partial Distribution, and denotes $S\in P(\mu, \sigma^2)$. The partial distribution is a kind of truncated normal distribution.

Definition 2.2(The Partial Process)**.** If stochastic variable S is related to time, i.e. $\forall\, t\in[0,\infty)$, we have $S(t)\in P(\mu(t),\sigma^2(t))$, then the $\{S(t),\, t\in[0,\infty)\}$ is called a partial process.

In general, the stock price varies with time, therefore we have

Assumption 2.1. Let $\mu(t)$ be the cost price of stock at the time t, and $\sigma^2(t)$ be the variance of cost price at the time t. If the market prices of stock satisfy the basic assumptions in 2.1, thus suppose that $S(t)$, the market price variable, follows the partial distribution at time t, and denotes $S(t)\in P(\mu(t), \sigma^2(t))$.

$S(t)\in P(\mu(t), \sigma^2(t))$ can be a stock or the market price of the stock. From [16], we have the following theorem 2.1 and theorem 2.2:

Theorem 2.1. Let S, the market price variable of a stock, follow the partial distribution $P(\mu, \sigma^2)$, thus

(a) The expected value $E(S)$ of S, means the average price on market exchange, is as follows

$$E(S) = \mu + \sigma^2 e^{-\frac{\mu^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{\sigma^2}} dx \tag{2}$$

where, $R(\xi) = \sigma^2 e^{-\frac{\mu^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$ is the average trading profit.

(b) The variance, $D(S)$, of the market price variable S, which means the risk of the market price, is as follows

$$D(S) = \sigma^2 + E(S)[\mu - E(S)] \tag{3}$$

Theorem 2.2. For any $x\in[0,\infty]$, the following equations are correct approximately:

(a) $\int_0^x e^{-\frac{t^2}{2}}dt = \sqrt{\frac{\pi}{2}(1-e^{-\frac{2}{\pi}x^2})}$

(b) $\int_0^x e^{-\frac{(u-\mu)^2}{2\sigma^2}}du = \sqrt{\frac{\pi}{2}}\sigma\times(\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}} + +\operatorname{sgn}(x-\mu)\sqrt{1-e^{-\frac{2}{\pi}(\frac{x-\mu}{\sigma})^2}})$

where, $\operatorname{sgn}(x)=\begin{cases}1 & x>0\\ 0 & x=0\\ -1 & x<0\end{cases}$;

Essentially, the partial distribution describes stock prices in its distribution construction.

2.3. *DF* process and *DF* structure

Definition 2.3(*DF* process)**.** If $\{\xi(t),t\in[0,\infty)\}$ is a stochastic process, and $\forall\ t\in[0,\infty)$

$$\xi(t)\in P(\mu(t),\ \sigma^2(t)t)$$

then $\{\xi(t),t\in[0,\infty)\}$ is called a ***DF*** process.

Definition 2.4. Let a and b be non-negative constants, If $a>0,b=0$, we define:

$$e^{-\frac{a}{b}} = \lim_{z\to 0^+} e^{-\frac{a}{z}}=0.$$

Definition 2.5(***DF*** structure)**.** Let X be the value of an asset related to stock $S(t)\in P(\mu(t),\ \sigma^2(t))$, if $\forall\ t\in[0,\infty)$ and $T>t$, $X_S(t,T)\in P(X,\ D[S(t)](T-t))$, then we call $X_S(t,T)$ the ***DF*** stochastic structure of X on $S(t)$. $X_S(t,T)$ is called a ***DF*** structure of X for short.

When $t=T$, $X_S(t,T)=X$. So the actual meaning of the ***DF*** structure, $X_S(t,T)$, is a stochastic value which is equal to that of an cash asset X in the future time T under-taking no discount of the interest rate.

Although the stock $S(t)$ has certain connections with ***DF*** structure $X_S(t,T)$ in variance, their stochastic movements may have no inevitable relation, so we have

Assumption 2.2. Let $X_S(t,T)$ be the ***DF*** structure of X on $S(t)$, thus $X_S(t,T)$ and $S(t)$ are independent of each other.

3. The *DF S*tructure Pricing Models of Options on A Non-Dividend-Paying Stock

3.1 Assumptions and notations

We need the following assumptions and notation before establishing the ***DF*** structure pricing models of options on a non-dividend -paying stock.

Assumption 3.1.

(i) The basic assumptions in 2.1, the assumption 2.1 and assumption 2.2 all come into existence.

(ii) There are no dividends during the life of the derivative.

(iii) The risk-free rate of interest, r, is constant.

(iv)There are no riskless arbitrage opportunities.

(v) Security trading is continuous.

All the following discussions are under Assumption 3.1. We will use the following notation:

t—current time.

$S(t)$—market price of the stock at t.

X—strike price of option on $S(t)$.

T—time of expiration of option.

r—risk-free rate of interest to maturity *T*.

$S(t)e^{r(T\text{-}t)}$—forward value of $S(t)$($\hat{E}(S(T))$,the expected value in a risk-neutral world).

$X_S(t,T)$—***DF*** stochastic structure of *X* on $S(t)e^{r(T\text{-}t)}$.

C_S—value of call option to buy one share.

P_S—value of put option to sell one share.

If $S(t)\in P(\mu(t),\sigma^2(t))$ and $X_S(t,T)\in P(X,\ D[S(t)e^{r(T\text{-}t)}](T\text{-}t))$, we have the ***DF*** structure models of options pricing (***DF*** model for short) as follows:

3.2. *DF* structure models of call options pricing

From Definition 2.1, Definition 2.5 and Theorem 2.2, we have

3.2.1. The price of call option at time *t* is

$$C_S(t)=e^{-r(T-t)}E[\max(S(t)e^{r(T-t)}-X_S(t,T),0)]$$

$$=e^{-r(T-t)}\int_0^{S(t)e^{r(T-t)}}[S(t)e^{r(T-t)}-x]f_{X_S}(x)dx$$

$$=(S(t)-Xe^{-r(T-t)})\times$$

$$\left[\frac{\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}+\operatorname{sgn}(S(t)e^{r(T-t)}-X)\sqrt{1-e^{-\frac{2}{\pi}\frac{(S(t)-Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}}\right]+\sqrt{\frac{2D[S(t)](T-t)}{\pi}}\left[\frac{e^{-\frac{(S(t)-Xe^{-r(T-t)})^2}{2D[S(t)](T-t)}}-e^{-\frac{(Xe^{-r(T-t)})^2}{2D[S(t)](T-t)}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}}\right] \tag{4}$$

When the call option is brought forward to execute at any time $\tau\in[t,T]$, the price of underlying stock, $S(\tau)$, becomes a constant to the option contract, thus $D[S(\tau)]=0$. According to (4) and definition 2.4, the current value of the option is

$C_S(\tau)=S(\tau)\text{-}Xe^{-r(T-\tau)}$, if $S(\tau)>Xe^{-r(T-\tau)}$;

$C_S(\tau)=0$, if $S(\tau)\leq Xe^{-r(T-\tau)}$;

i.e. $C_S(\tau)=max\{S(\tau)\text{-}Xe^{-r(T-\tau)},\ 0\}$. At this time, the intrinsic value of the call option is $max\ \{S(\tau)\text{-}X,\ 0\}$, thus

$$C_S(\tau)\geq max\{S(\tau)\text{-}X,\ 0\} \tag{5}$$

3.2.2. The price of put option at time *t* is

$$P_S(t)=e^{-r(T-t)}E[\max(X_S(t,T)-S(t)e^{r(T-t)},0)]$$

$$=e^{-r(T-t)}\int_{S(t)e^{r(T-t)}}^{\infty}[x-S(t)e^{r(T-t)}]f_{X_S}(x)dx$$

$$=(Xe^{-r(T-t)}-S(t))\times$$

$$\times\left[\frac{1-\operatorname{sgn}[S(t)e^{r(T-t)}-X]\sqrt{1-e^{-\frac{2}{\pi}\frac{(S(t)-Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}}\right]+\sqrt{\frac{2D[S(t)](T-t)}{\pi}}\left[\frac{e^{-\frac{(S(t)-Xe^{-r(T-t)})^2}{2D[S(t)](T-t)}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}}\right] \tag{6}$$

when the put option is brought forward to execute at any time $\tau\in[t,\ T]$, the price of underlying stock, $S(\tau)$, becomes a constant to the option contract, thus $D[S(\tau)]=0$. According to (6) and definition 2.4, the current value of the option is

$P_S(\tau)=Xe^{-r(T-\tau)}-S(\tau)$, if $S(\tau)<Xe^{-r(T-\tau)}$

$P_S(\tau)=0$, if $S(\tau)\geq Xe^{-r(T-\tau)}$

i.e. $P_S(\tau)= max\{Xe^{-r(T-\tau)}-S(\tau), 0\}$. At this time, the intrinsic value of the put option is $max\{X-S(\tau), 0\}$, thus,

$$P_S(\tau)\leq max\{X-S(\tau), 0\} \tag{7}$$

From (4) and (6), we could see that the price of option is made up of two parts. One is the market price of stock, and the other is the fluctuation of stock price.

According to (5), we know call option should not be executed before the time to expiration. Otherwise, the current option value will be encashed by the intrinsic value, and the intrinsic value may be lower at the same time. And from (7), put option should be executed before the time to expiration since the current option value will be encashed by the intrinsic value, and the intrinsic value may be higher at the same time.

When $Xe^{-r(T-t)}-S(t)>\alpha$, $t\leq T$, thus the put option should be executed at the same time, where α is, including the transactions costs and tax, all the expenses of trade options.

Noting:

$$d_1=\frac{S(t)-Xe^{-r(T-t)}}{\sqrt{D[S(t)](T-t)}},\ d_2=\frac{Xe^{-r(T-t)}}{\sqrt{D[S(t)](T-t)}},\ \varphi(x)=\int_0^x e^{-\frac{t^2}{2}}dt \text{, thus,}$$

the expression (4) can, for short, be written as:

$$C_S(t)=(S(t)-Xe^{-r(T-t)})\times\frac{\varphi(d_1)+\varphi(d_2)}{\varphi(\infty)+\varphi(d_2)}+\sqrt{D[S(t)](T-t)}\ \frac{e^{-\frac{d_1^2}{2}}-e^{-\frac{d_2^2}{2}}}{\varphi(\infty)+\varphi(d_2)} \tag{8}$$

the expression (6) can be written as:

$$P_S(t)=(Xe^{-r(T-t)}-S(t))\times\frac{\varphi(\infty)-\varphi(d_1)}{\varphi(\infty)+\varphi(d_2)}+\sqrt{D[S(t)](T-t)}\ \frac{e^{-\frac{d_1^2}{2}}}{\varphi(\infty)+\varphi(d_2)} \tag{9}$$

4. Estimation and Test of the Parameters in Partial Distribution

4.1. The modified maximum likelihood estimation

If the fields of samples of stock price are made in the following partition:

$(c_0,c_1),(c_1,c_2),\ldots,(c_{m-1},c_m)$, i=1…, m-1, Specially, $c_0=0$, $c_m=\infty$;

and the emergence number of price samples in each field: $l_1, l_2, \ldots, l_m$ ($l_i>0$, i=1, …, m);

denoting: $n=\sum_{i=1}^{m} l_i$, $x_i=(c_{i-1}+c_i)/2$, $y_i=l_i/n$, $\Delta=c_i-c_{i-1}=5\times(z_{max}-z_{min})/n$, i=1, …, m

where z_{max} and z_{min} are separately the highest price and the lowest price of all samples. By (1) we have approximatively:

$$y_i=f(x_i)=\Delta e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}\Big/\int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx \text{, } i=1,\ldots,m \text{, at this time}$$

$$\ln\frac{y_{i+1}}{y_i}=-\frac{(x_{i+1}-x_i)(x_{i+1}+x_i-2\mu)}{2\sigma^2}$$

and $\mu=\dfrac{x_{i+1}+x_i}{2}+\sigma^2\dfrac{\ln(y_{i+1})-\ln(y_i)}{x_{i+1}-x_i}$,

$$\mu = A + B\sigma^2 \tag{10}$$

where, $A = \frac{1}{m-1}\sum_{i=1}^{m-1}\frac{x_{i+1}+x_i}{2}$, $B = \frac{1}{m-1}\sum_{i=1}^{m-1}\frac{\ln(y_{i+1})-\ln(y_i)}{x_{i+1}-x_i}$

According to the method of maximum likelihood estimation, we have

$$L = ln\prod_{i=1}^{n} f(x_i),$$

$$\frac{\partial L}{\partial \mu} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu) - 2nW, \frac{\partial L}{\partial \sigma^2} = \frac{1}{2\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2 - \frac{n}{2\sigma^2} + \frac{\mu n}{\sigma^2}W$$

where, $W = \dfrac{\frac{\mu}{\pi\sigma^2}e^{-\frac{2\mu^2}{\pi\sigma^2}}}{\sqrt{1-e^{-\frac{2\mu^2}{\sigma^2}}}(1+\sqrt{1-e^{-\frac{2\mu^2}{\pi\sigma^2}}})}$

Let $\frac{\partial L}{\partial \mu} = 0$, $\frac{\partial L}{\partial \sigma^2} = 0$, then

$$A = \frac{1}{2n\sigma^2}\sum_{i=1}^{n}(x_i-\mu) = \frac{1}{2\mu}(1-\frac{1}{n\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2)$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}x_i^2 - \mu\bar{x} \tag{11}$$

combining (9) with (8), We have the following estimate formulas

$$\hat{\mu} = \frac{A+CB}{1+B\bar{x}} \tag{12}$$

$$\hat{\sigma}^2 = \frac{C-A\bar{x}}{1+B\bar{x}} \tag{13}$$

where, $C = \frac{1}{n}\sum_{i=1}^{n}x_i^2$.

In practice, $\hat{\mu}$ and $\hat{\sigma}^2$ are positive.

4.2. The fiducial test of partial distribution

Designing the *Pearson*s statistic as following:

$$\chi^2 = \sum_{i=1}^{m}\frac{[y_i - n\hat{f}(x_i)]^2}{n\hat{f}(x_i)} \tag{14}$$

where, $\hat{f}(x_i) = \int_{c_{i-1}}^{c_i} e^{-\frac{(x-\hat{\mu})^2}{2\hat{\sigma}^2}}dx \Big/ \int_0^{\infty} e^{-\frac{(x-\hat{\mu})^2}{2\hat{\sigma}^2}}dx$, i=1,···, m.

There are two parameters to be estimated, μ and σ^2, so $\chi^2 \sim \chi^2(m-2-1)$.

If there is nothing special to mention, we shall use (12) and (13) to estimate parameters underside, and use (14) as test statistic. We adopt the significance level α=0.025, i.e. the fiducial level 1-α=0.975 in the following statistic test. And the relevant statistic test results of lognormal distribution, χ_l^2, are given for the comparison with χ^2. The parameters in lognormal distribution are estimated by the maximum likelihood.

5. The Fitness Analysis of Partial Distribution

5.1. The fitness analysis for stock index and stock of American market

Here, we primarily make the fitness and statistic test with the actual trading data of the points of stock index, *DJX* (*1/100DJ INDU*), and prices of stock, *MSFT* (MICROSOFT CP).

5.1.1 The fitness *of DJX*. We take the close points of ***1/100DJ INDU*** as sample data.

Time: Jun. 19, 2002 -Dec. 24, 2002.

Trading days: n=132.

Length of each field: Δ=0.862121 2120.

Number of fields: m=25.

The estimated results of parameters are as follows:

(a) The estimated values of parameters in partial distribution $P(\mu, \sigma^2)$:

$\hat{\mu}$ =84.84577713; $\hat{\sigma}^2$ =28.65615031

(b) The estimating values of parameters in lognormal distribution $Ln(\mu_l, \sigma_l^2)$:

$\hat{\mu}_l$ =4.440767790, $\hat{\sigma}_l^2$ =0.002746603980;

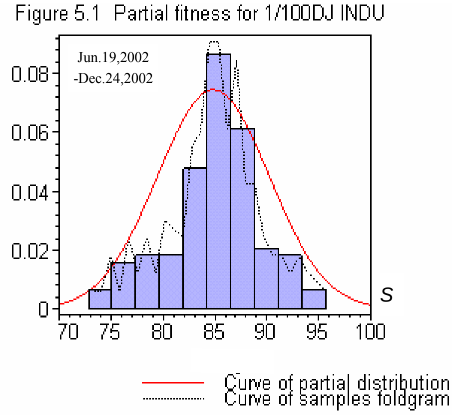
(c) The fiducial test:

χ^2 =24.90088870< $\chi^2_{0.025}$ (22)=36.781;

χ_l^2 =30.43529668< $\chi^2_{0.025}$ (22)=36.781.

The corresponding histogram, samples foldgram and fitting curve of Partial Distribution are shown in the figure 5.1.

Figure 5.1 Partial fitness for 1/100DJ INDU



5.1.2. The fitness *of MSFT*. We take the close prices of ***MICROSOFT CP*** as sample data.

Time: Jan. 29, 2002 -Dec. 24, 2002.

Trading days: n=230.

Length of each field: Δ=0.467609(US $).

Number of fields: m=46.

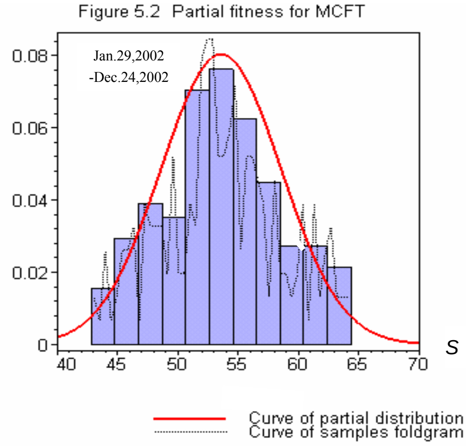
The estimated results of parameters are as follows:

(a) The estimated values of parameters in partial distribution $P(\mu, \sigma^2)$:

$\hat{\mu}$ =53.58500013; $\hat{\sigma}^2$ = 24.62632700;

(b) The estimated values of parameters in lognormal

Figure 5.2 Partial fitness for MCFT



distribution $Ln(\mu_l, \sigma_l^2)$:

$\hat{\mu}_l$=3.976827406; $\hat{\sigma}_l^2$=0.008770254161;

3) The fiducial test:

χ^2=58.08095341< $\chi^2_{0.025}$(43)=62.990;

χ_l^2=58.02536391< $\chi^2_{0.025}$(43)=62.990.

The corresponding histogram, samples foldgram and fitting curve of Partial Distribution are shown in the figure 5.2.

5.2. The fitness analysis for stock index and stock of Chinese market

Here, we primarily make the fitness and statistic test with the actual trading data of the points of stock index, the synthesis index of *Shanghai* stocks market (***SISH***), and prices of stock, *Shenzhen Development Ltd*.(***SDL***).

5.2.1. The fitness for *SISH*. We take the close points of *SISH* as sample data.

Time: Nov.11, 2001-Jul.23, 2002.

Trading days: n=152.

Length of each field: Δ=13.51940790.

Number of fields: m=28.

The estimated results of parameters are as follows:

(a) The estimated values of parameters in partial distribution $P(\mu, \sigma^2)$:

$\hat{\mu}$=1623.037149; $\hat{\sigma}^2$=14982.02741;

(b) The estimated values of parameters in lognormal distribution $Ln(\mu_l, \sigma_l^2)$:

$\hat{\mu}_l$=7.381044703; $\hat{\sigma}_l^2$=0.003609184920;

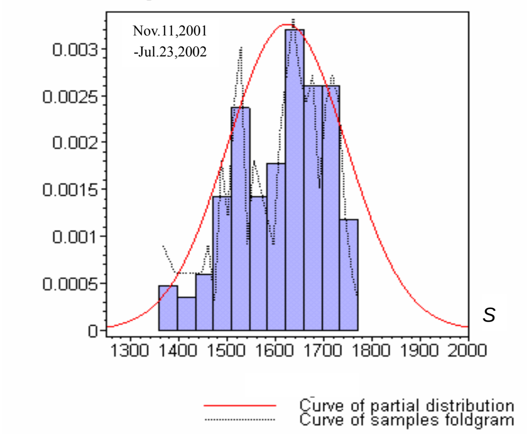
(c) The fiducial test:

χ^2=38.81864807< $\chi^2_{0.025}$(29)=40.646;

χ_l^2=72.29035611> $\chi^2_{0.025}$(29)=40.646.

The corresponding histogram, samples foldgram and fitting curve of Partial Distribution are shown in the figure 5.3.

Figure5.3 Partial fitness for SISH



5.2.2. The fitness *of SDL*. We take the close prices of *SDL* as sample data.

Time:Aug. 25, 2000—Jul. 24, 2001.

Trading days: n=215.

Length of each field: Δ=0.20 (R.M.B.).

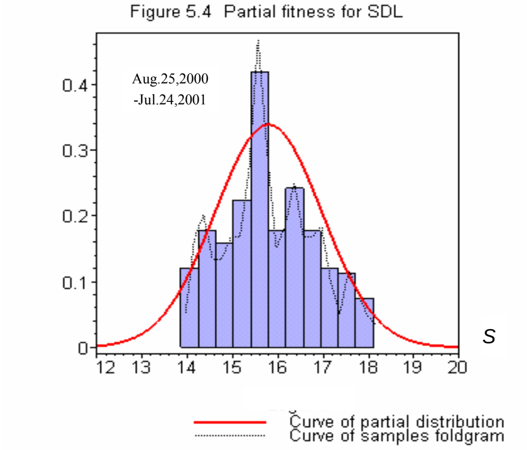
Number of fields: m=22.

The estimated results of parameters are as follows:

(a) The estimated values of parameters in partial distribution $P(\mu, \sigma^2)$:

$\hat{\mu}$=15.79142884; $\hat{\sigma}^2$=1.390993710;

Figure 5.4 Partial fitness for SDL



(b) The estimated values of parameters in lognormal distribution $Ln(\mu_l, \sigma_l^2)$:

$\hat{\mu}_l$=2.758404582; $\hat{\sigma}_l^2$ =0.004032994141;

(c) The fiducial test:

χ^2 =30.02812679< $\chi^2_{0.025}$ (19)=32.852; χ_l^2 =35.36316684> $\chi^2_{0.025}$ (19)= 32.852.

The corresponding histogram, samples foldgram and fitting curve of Partial Distribution are shown in the figure 5.4.

6. Comparison Research Between *DF* Structure Pricing and *B-S* Pricing

Suppose *r* (risk-free rate of interest) =0.07. Here we compare results from *DF* formulas with those of *B-S* (*Black- Scholes*) formulas in options pricing.

6.1. The comparative analysis for *DJX*

Time: Dec. 24, 2002

Breed: The option contract on *DJX*

Maturity: Expires After: Fri 19-Dec-03.

Underlying: Close point of *DJX* at current date, 84.48.

In table 1, there are the prices on Dec. 24, 2002, which were the closing prices traded actually in the United States option market (TP), the call and put options prices calculated by *DF* structure formulas, (*DF*), and the call and put options prices calculated by *B-S* formulas, (*B-S*). From table 1, we see the *DF* prices are closer to the actual trading prices than the *B-S* prices. If taking the strike price, *X*=88, and *T*=199 for example, the variety of call and put option prices calculated by *DF* formulas and *B-S* formulas are respectively shown in figure 6.1(a) and 6.1(b).

Table 1. Contrast of options prices of ***DJX***

Strike prices	Call options prices			Put options prices		
	TP	*DF*	*B-S*	*TP*	*DF*	*B-S*
76.0	13.60	11.34	11.33	5.50	.0108	.0001
80.0	11.00	7.542	7.484	7.20	.0668	.0086
84.0	8.70	4.015	3.831	9.00	.3900	.2053
88.0	7.10	1.457	1.197	11.00	1.682	1.421
92.0	5.70	.3180	.1812	13.00	4.393	4.256

Figure 6.1(a) Comparison of DJX call option prices

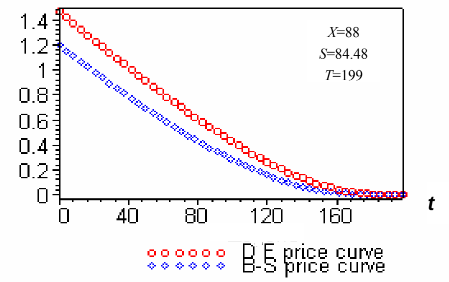
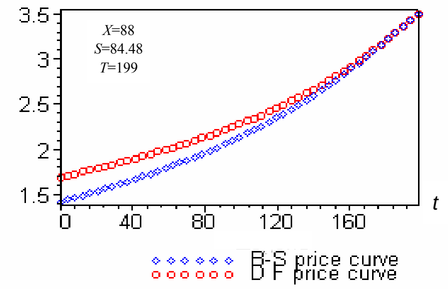


Figure 6.1(b) Comparison of DJX put option prices



6.2. The comparative analysis for *MSFT*

Time: Dec. 25, 2002.

Breed: The option contract on *MSFT*.

Maturity: Expires After: Fri,16-Jan-04.

Underlying: Close price of *MSFT* at current date, 53.39$.

In table 2, there are the prices on Dec.24, 2002, which were the closing prices traded actually in the United States option market (TP), the call and put options prices calculated by *DF* structure formulas, (*DF*) , and the call and put options prices calculated by *B-S* formulas, (*B-S*).

According to the data from table 2, it is difficult to know whether the *DF* formula is better than *B-S* formula or not, we should do further empirical research. Taking the strike price, X=60, and *T*=212 for example, the variety of call and put option prices calculated by *DF* formulas and B-*S* formulas are respectively shown in figure 6.2(a) and 6.2(b).

6.3. The comparing analysis for *SDL*

Because there is no stock option in the Chinese security market, we will make a hypothetic contrastive analysis of *SDL*.

Let Maturity=199, Strike price of option: *X*=15. Current price of stock: *S*(*t*)=16.5.

Table 2. Contrast of options prices of **MSFT**

Strike prices	Call options prices			Put options prices		
	TP	*DF*	*B-S*	*TP*	*DF*	*B-S*
50.0	11.40	5.406	5.369	7.70	.1399	.1028
55.0	9.20	1.702	1.706	9.90	1.248	1.252
60.0	6.80	.2213	.2466	12.50	4.580	4.605
65.0	5.10	.0212	.0151	15.90	9.192	9.186
70.0	3.80	.0007	.0004	19.50	13.98	13.98

Figure 6.2(a) Comparison of MSFT call option prices

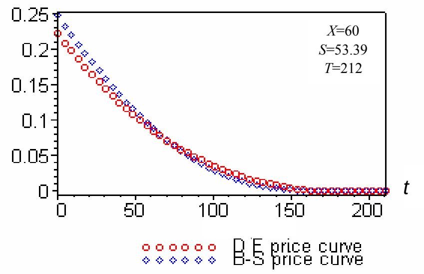
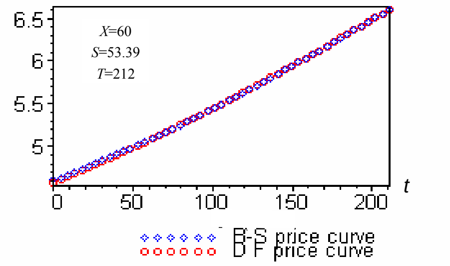


Figure 6.2(b) Comparison of MSFT put option prices



thus, the variety of call and put option prices calculated by *DF* formula and *B-S* formula are respectively shown in figure 6.3(a) and 6.3(b).

Figure 6.3(a) Comparison of SDL call option prices

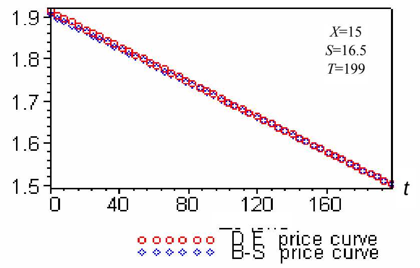
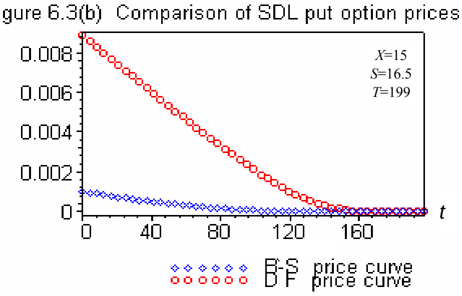


Figure 6.3(b) Comparison of SDL put option prices



Again if

Strike price of option: *X*=16.5, Current price of stock: *S*(*t*)=15.

thus, the variety of call and put option prices calculated by *DF* formula and *B-S* formula are respectively shown in figure 6.4(a) and 6.4(b).

Figure 6.4(a) Comparison of SDL call option prices

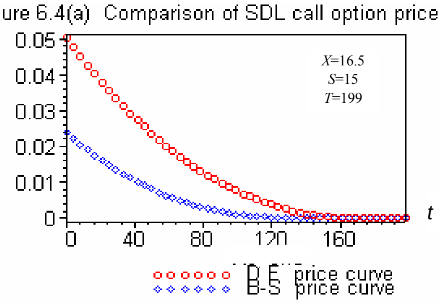
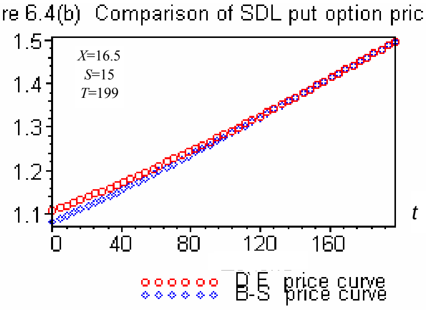


Figure 6.4(b) Comparison of SDL put option prices



7 Conclusions

Salvatore Micciche et al. (2002) investigated the historical volatility of the 100 most capitalized stocks traded in US equity markets. An empirical probability density function (pdf) of volatility is obtained and compared with the theoretical predictions of a lognormal model and of the Hull and White model. The lognormal model well describes the pdf in the region of low values of volatility whereas the Hull and White model better approximates the empirical pdf for large values of volatility. Both models fail in describing the empirical pdf over a moderately large volatility range.

Since 90's of 20th century, the U.S. stock market soar first and slump later, and the fluctuation is violent, and the volatility is large. At the same time, we can use the Levy model. But, this does not mean that the U.S. stock market or other market will be in the violent fluctuation forever. Even if the stock market is generally in the violent fluctuation, the stock price is also in the low values of volatility at some periods of time. In fact, we need to use the most proper model to analyze the probability distribution of stocks price in case the stock price behavior is in the region of low values of volatility. In this case, people accustom to the use the lognormal model. However, when a company collapses, the price of its stock will be the zero. The lognormal model can't describe the possibility of zero price of a stock. The partial distribution $P(\mu, \sigma^2)$ can do this. So the partial distribution should be applied to describe the price distribution of commodities and stocks at the low values of volatility. When value of μ is lower, partial distribution have a sharper peak than lognormal distribution.

In addition, Levy distribution and the truncated Levy distribution is usually applied to describe the price behavior in symmetry. Because of the price is non-negative, the distribution of price is generally non-symmetry. The non-symmetry can be reflected in lognormal or partial distribution.

In this paper, the models of options pricing is not studied in the current mode. In fact, the new models of option pricing are presented based on partial process and *DF* structure —*DF* structure models. Because the partial process can nicely describe the properties of construction of stock prices, the *DF* structure method of option pricing is practical, reasonable and authentic. This can be best explained from the results in the table 6-1 and table 6-2, which are calculated by *DF* structure formulas and are closer to the actual trading price of options on the whole.

As for the DF structure models of option pricing, we still have the following elucidations:

● With the time, the distribution of stock prices will continuously change, and the *DF* structure models can be vary with the price distribution of underlying stock at any time, so it may price option more accurately than *B-S* models.

● The DF structure models can estimate the prices of stock options at the expiration date or any time before the expiration date. So the models can be used for pricing European option, and American option as well, particularly for pricing American put option. Now the exact formula for pricing American put option has not been obtained.

● By means of the optimal pricing method [20], we shall get the model of calculating optimal execute price, and know the optimal opportunity for any options.

● This paper has only discussed the pricing of the options on stock and stock index. If the underlying, such as spots, futures, and foreign exchange, follow the partial distribution, and have no dividends, the textual conclusion still holds.[1]

The textual data source is in the websites:

http://finance.yahoo.com

http://www.stockstar.com.cn

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