A General Theory of Stock Market Valuation and Return

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Abstract

We show that the long-term total market and average investor's compounded stock returns are determined by GDP growth and are much less than believed because of the infeasible assumption that dividends can be fully reinvested. The long-term stock return closely approximates the return on risk-free debt, thus yielding a zero premium on a compounded per-capita basis. We demonstrate that the market earnings yield ratio (inverse P/E) is akin to a minimum nominal expected return and a direct function of inflation and a real required yield equal to long-term real GDP per capita growth, with marginal regard to risk. Our derived valuation formula is tested against the S&P 500 index and produces a 21% mean percentage tracking error, compared to 32% for the "Fed Model" over the period 1954 – 2002.

Keywords: Required yield, Earnings yield, Equity Premium, S&P 500 Valuation, Fed Model.

JEL: G12

The finance field generally holds that long-term total compounded stock returns have greatly exceeded GDP growth (Siegel (1999, 2002), Ibbotson and Chen (2003)). Risk is believed to be the main source of the equity premium over T-bonds, although the current economic models do not fully explain the observed premia (Mehra-Prescott (1985), Mehra (2003), Asness (2000)). Stock market valuation usually involves estimates of this risk premium in addition to a base risk-free return. Hence, estimated stock market returns and "fair value" measures are dependent upon views about the size of the current and future risk premia (Fama and French (2002), Arnott and Bernstein (2002)).

In this article, we provide an integrated theoretical framework that derives the stock market return from macroeconomic growth and solves the apparent mathematical paradox of equity returns that compound in excess of GDP growth. Our Required Yield Theory leads to a formula that describes the fair value of a broad stock index (S&P 500) and historically tracks the index more accurately than the "Fed Model". This work is divided into six sections. The first section covers a theory of aggregate compounded return. In section 2, we turn our attention to the average investor's return. Section 3 presents a theoretical valuation model, based on the determination of the market earnings yield ratio. We show that the equity premium compared to risk-free bonds is empirically and theoretically zero on a compounded return basis. Section 4 introduces empirical tests of our valuation model that is compared to the S&P 500, and the "Fed Model". Implications of the theory are studied in section 5. Our conclusions appear in section 6.

1. The Stock Market Aggregate Return is determined by GDP Growth

Imagine one share of stock that represents ownership of the entire economy (present and future). The return on all possible investments and reinvestments (ignoring foreign investments) must be realized within the bounds of the economy. All dividends paid must reflect dividends on reinvested dividends and so on. Total market value must include all reinvested stock dividends. This situation is similar to a savings account where all "interests" received are compounded interest, with no outside reinvestment options. Thus, year-to-year, the basis for computing the stock market return must include cumulated past dividends and capital gains/losses received. In other words, an annual compounded aggregate nominal market return can be formulated as:

$$1 + R_{t+1,0} = \frac{P_{t+1} + D_{t+1,0}}{P_t + D_{t,0}} \tag{1}$$

Where $R_{t+1,0}$ stands for the yearly compound rate between period t and t+1 for an investor who *bought* the share in period 0. With $D_{t,0} = \sum_{j=0}^{t} d_{j,0}$, and $d_{j,0}$ representing the sum of all dividends received over the single time period j by an initial investor at time 0. Thus $D_{t,0}$ represents the sum of all dividends cumulated up to the end of period t. Let us assume that for any period t, $D_{t,t}$ = 0; so that if an investor enters the market at time t, no past dividends have so far been received. Note that a buyer of this share comes in the market at time t and thus her first year return is:

$$1 + R_{t+1,t} = \frac{P_{t+1} + d_{t+1}}{P_t} \tag{2}$$

In this case $R_{t+1,t}$ represents the discount rate, which once applied, determines the value of the stock market at time t, as represented by the present value of future dividends. In other words, at any point in time the value of market equity is determined by initial investors in a manner consistent with the present value approach. To keep our notations lighter, we will work from the standpoint of an initial investor at time zero and drop the extra zeros in our notations above. That is, $R_{t+1,0}$ becomes R_{t+1} and $D_{t,0}$ becomes D_t. To prove the key result of this section, consider expression (1) that can be rewritten as follows:

$$1+R_{t+1}=\frac{P_{t+1}}{P_t}\left[\frac{P_{t+1}+D_{t+1}}{P_{t+1}}\times\frac{P_t}{P_t+D_t}\right]=\frac{P_{t+1}}{P_t}\left[\frac{1+D_{t+1}/P_{t+1}}{1+D_t/P_t}\right] \tag{3}$$

Let e_j denote earnings and b_j denote the payout ratio, then on a *per-share* basis the book value of market equity is $B_t=\sum_{j=0}^{t}(1-b_j)e_j+B_0$. Without loss of generality in our proof and result below, we assume that $B_0 = 0$. Let us assume a constant payout ratio b. Since all dividends (even reinvested dividends) are paid out of total earnings, it is easy to show that: [1]

$$B_t=\sum_{j=0}^{t}(1-b_j)e_j=(1-b)\sum_{j=0}^{t}e_j=\frac{(1-b)}{b}\sum_{j=0}^{t}d_j=\frac{(1-b)}{b}D_t \tag{4}$$

[1] Our result in this section for the aggregate economy would not be affected if we assumed that the number of shares in the economy was growing, our book value would then incorporate net new stock issues. We can alternatively assume that the payout ratio b converges in the long run to the weighted average of yearly payout ratios so as to satisfy: b = $\sum_{j=1}^{\infty}b_je_j \Big/ \sum_{j=1}^{\infty}e_j$.

Moreover, in a long run steady-state, it must be true that market value P_t converges to the book value B_t. In other words, from expression (4) above, the ratio D_t/P_t converges to the value $\frac{b}{(1-b)}$. Finally, this implies that expression (3) becomes: $1+R_{t+1}=\frac{P_{t+1}}{P_t}$ in the long run. That is, the stock market *compounded* return must equal the capital gains rate. Furthermore, since the capital gains rate must equal nominal GDP growth in the long run, the stock market return will equate GDP growth. Henceforth, our first key result is proven. It is important to point out that our argument does not assume away dividends reinvestment. On the contrary, essentially our result follows from the fact that (reinvested) dividends cannot accumulate faster than GDP, and that the basis for year-to-year compounding grows over time, thus pinning the return down to GDP growth.

Examining the Fed Funds Flow (FFF) data available for 1946 – 2002 enables us to demonstrate this result empirically. Exhibit 1 shows the total equity market average compounded return obtained by summing together total market value and total cumulated dividends paid. A comparison is made with the S&P 500 over the same period. The total compounded return (CR) using cumulated dividends for the total market is 9.13% vs. 12.56% according to the accepted (standard) method. The S&P 500 shows a compounded return of 11.12% by the accepted method and 7.78% by our method.

Exhibit 1: Total Stock Return Analysis 1946 - 2002

(All values are in $millions)

	Total Market (FFF)		S&P 500	
		w. cum. div.		w. cum. div.
Value of equities 2002	11,735		909	
Value of equities 1946	118		17	
Cum. Div. Paid	5,421		332	
Accum. Div. prior to 1946		220		32
Avg. Div. Yield	3.84%		3.66%	
Cap. Gains (CGR)	8.40%		7.20%	
Standard Method CR	12.56%		11.12%	
Standard CR per capita	11.33%			
Total Equity CR	9.13%	7.13%	7.78%	5.81%
CR per capita	7.87%	5.87%		

Computing total returns by adding prior accumulated dividends to the 1946 market or index value is even more revealing. In this case, the calculation is from the standpoint of a first time investor in 1871, whose basis for compounding in 1946 includes all past dividends ever paid. We estimate accumulated dividends paid on the total market by taking the ratio of the S&P 500 accumulated dividends per share (from 1871 through 1946) divided by the 2002 S&P 500 index value, and applying this ratio to the total market. In the case of both the aggregate market and the S&P 500, the total compound return very closely approximates GDP growth over the period. This will be true for the S&P 500 when population growth is added to S&P 500 total return to account for net new share growth (that is, 5.81%+1.23% = 7.04%). Next, we turn to examining the average investor's stock return.

2. The Average Investor's Stock Market Return

2.1 The Growth of Equity Shares Must Equal Population Growth

Over the period 1926-2000, earnings per share (EPS) grew at the rate of 5.05% while GDP grew at 6.44%. Since the ratio of corporate profit to GDP must be constant in the long run, net new share growth is obtained as the difference between GDP growth and earnings per share growth. Over the period, net share growth was 1.39% or about equal to the 1.23% population growth. For dividends to be fully reinvested in stocks, shares should have grown at a rate equal to the historic average dividend yield of 4.2%. Based on our computation of share growth, this did not happen. Therefore, the maximum reinvestment rate is only a fraction of the historic average dividend yield.

It is also logically sensible that net new share growth should equal population growth. In order for new shares to be purchased by individual investors (net of asset substitution), the price per share cannot grow faster than wage per capita in the long run. Otherwise, shares would eventually become unaffordable. Since total wages and total market value both grow at the rate of GDP, this entails that share growth must at least be population growth.[2] On the other hand, share growth permanently in excess of population growth would shrink earnings per share. It would depress current and future stock prices thus penalizing shareholders, which would be a socially unacceptable outcome.

[2] The fractionalization of shares via owning mutual fund shares seems at first glance to be a way around this argument. However, it is the *combined* growth rate of corporate and mutual fund shares which must grow at least at the rate of population. Furthermore, mutual fund share growth that would be faster than corporate share growth would mean that new investors get an ever shrinking share of the stock market, which cannot be a long run equilibrium.

2.2 The Average Investor's Return

In this section, we demonstrate that the compounded equity return per capita, assuming the proportion of stock investors in the population is constant, is given by GDP/capita growth in the long run. The currently accepted total return calculation assumes 100% dividend reinvestment. According to the calculations of Ibbotson and Chen (2003) and Siegel (2002), the compounded long-term equity total return is about 10.9% for 1926 – 2000. This figure is greatly in excess of GDP growth and is comprised of a capital gain of 6.42% and dividend and reinvestment return of 4.46%. In order for 100% dividend reinvestment to be possible at the market level, the rate of net new shares issued must be at least equal to the average dividend yield (4.19%). However, as shown above, net new share growth cannot outpace population growth. The accepted total return calculation requires that total profits grew at a rate equal to at least the sum of EPS growth and dividend yield or 9.24% compared to 6.44% GDP growth; resulting in a 6.9 – fold increase in the proportion of corporate profits to GDP. This could not have happened. On the other hand, a group of investors able to reinvest 100% of dividends would obtain a higher return than GDP growth, but only via increasing their market share of the entire market until they would virtually own the entire market.[3] They would then be unable to reinvest at the dividend yield rate of 4.2% given the slower pace of actual share growth. In the final instance, they would still obtain a total compound return equal to GDP growth. Additionally, the absence of tax-deferred compensation plans prior to the 1950s meant that dividends were taxed at very high income tax rates, making it impossible to achieve high proportions of reinvestment. This obviously remains valid for today's dividend-paying stocks outside tax-deferred plans.

Our recomputed total equity return uses a terminal value for the index of 1,641.8 that includes \$310.75 of cumulated dividends paid per share; realizing a compounded 5.43% on an index base of 12.5 and accumulated dividends of \$18.80 beginning from 1871 until 1926. Thus per share total equity return nearly matches nominal GDP/capita growth of 5.27% over 1926-2000.

2.3 Implications for the Equity Premium

The source and the size of the equity premium is one of the most debated issues in Finance. Faugere-Van Erlach (2003) for example, propose an explanation for the ex-ante premium based on GDP growth. The authors find that the ex-post arithmetic average premium over 1926-2001 is fully explained by historical GDP growth and marginal tax rates. By contrast, in this paper, our analysis of the Equity Premium (EP) hinges upon making return comparisons on a compoundable and per-capita basis. A compounded per capita equity return can be directly compared to a nominal bond return since debt instruments, contrary to equity, are available to individual investors for 100% reinvestment. Furthermore, the nominal yield or return on long-term risk-free debt is bounded by GDP per capita; otherwise this return would compound faster than GPD per capita which is impossible.

[3] A simple calculation shows that if this group of investors owned 10% of the market initially, it would take 80 years for them to own the entire market, based on a 1.23% population growth and a dividend yield of 4.2%.

By contrast to the accepted 10.9% compounded total equity return from 1926-2000, we demonstrated in the previous section a total return per share of 5.43%. On the other hand, the historical average 10-year T-Note return is 5.28%. This implies a nearly non-existent equity premium of 0.15%! Thus, the premium vanishes on a per-capita basis.[4] As we demonstrate in section 5, if an equity risk premium exists, the addition of such a premium should enhance the correlation of a Fed 'type' Model to actual stock market values by depressing relative prices. Arnott and Bernstein (2002) make several arguments that parallel our findings that the equity premium is actually far smaller than thought over 1926-2000. They claim that valuation levels are unlikely to advance as fast in the future as they did in the past. They note that real per share EPS and dividends tend to advance between 0.9% and 1.4% on a compounded annual basis, slower than real per-capita GDP ranging between 1.6% and 2%. They conclude that a normal or rational future equity premium should be in the range of 2.4%. Their premium estimate is clearly much closer to what we have computed above, but it is not expressed on a per-capita basis where the premium disappears.

We recognize that accepted theory holds that there must be an equity risk premium above a risk free rate. In the next section, we theoretically demonstrate that this is not so on a compounded basis in a general equity valuation model.

[4] A similar analysis for 1946-2002 using Fed Fund Flows shows an analogous result. This result holds on an after-tax basis as well, since we can show in the long run, given that tax rates converge to constant rates, that the long run compounded returns on stocks and bonds are unaffected by tax rates. Moreover, the impact of taxes is the same on both instruments if these are deferred.

3. A New Theory of Equity Valuation

In this section, we articulate a theory of valuation based on a new understanding of the market earnings yield ratio (inverse of P/E). Most valuation models are derivatives of the dividend discount model. A crucial underlying assumption of this model is that equity must be priced in relation to a risk premium added to a current or expected risk-free yield. For the aggregate stock market, however, this approach reduces to asserting that the expected market return is determined by the historical market return (since the risk premium is tautologically measured as the difference between the historical average market return and risk free rate). The "Fed Model" (Lander, Orphanides and al. (1997)) takes a different road by equating current or forward EPS yield with the yield of 10-year risk-free Treasuries; resulting in one of the best known correlations with actual S&P 500 stock index values. Our model theoretically shows the relationship between the expected forward earnings yield and the risk free-rate as well as a long-term real return based on GDP/capita growth. An added risk premium appears unnecessary either on a theoretical or empirical basis to maximize the explanatory power of our model.

3.1 A Minimum Expected Real Required Equity Yield

In section 1, we showed the behavior of the compounded stock return over time. Given that reinvestment can only happen at the rate of population growth, this requires either that incremental (new) members of the population cannot buy any shares; and that existing shareholders own all existing stock and buy all net new shares, or alternatively, that only new members of the population buy new shares. Eventually the latter case must come true. In the long run, dividends cannot be reinvested as new shares are all issued to new generations of investors. Therefore using our notations from section 1, and assuming now that all variables are on a per-share basis, the expected compounded return for such investor at a given period t+1 is:[5]

$$E(R_{t+1}) = \frac{E(P_{t+1}) + E(D_{t+1})}{P_t + D_t} - 1 \tag{5}$$

$$= \frac{E(P_{t+1}) + D_t + E(d_{t+1})}{P_t + D_t} - 1 = \frac{E(P_{t+1}) - P_t + E(d_{t+1})}{P_t} \times \frac{1}{1 + D_t / P_t} \tag{6}$$

We can rewrite (6) as:

$$= \left[\frac{E(d_{t+1})}{P_t} + E(g_{ct+1}) \right] \times \frac{1}{1 + D_t / P_t} \tag{7}$$

The expression $E(g_{ct+1})$ represents the expected rate of capital gains per share. A simple algebraic manipulation allows us to rewrite the above expression as a function of the expected forward earnings yield $\frac{E(e_{t+1})}{P_t}$ as follows:

$$E(R_{t+1}) = \left[\frac{E(e_{t+1})}{P_t} + E(g_{ct+1}) - g_{ct+1} \right] \times \frac{1}{1 + D_t / P_t} \tag{8}$$

Where b_{t+1} is the deterministic payout ratio. The variable $g_{ct+1} = (1 - b_{t+1}) \frac{E(e_{t+1})}{P_t}$ represents the expected retained earnings as a fraction of the current stock price.[6] Next,

[5] We ignore the impact of dividend and capital gains taxes for now. As shown in the Appendix, the main result of this section holds true on an after-tax basis, when for example, tax rates (dividend income, interest income and capital gains) all converge to their cumulative past-weighted averages.
[6] This variable is akin to a sustainable growth rate based on market price rather than book value per share.

we show that in general, a rational investor will base his capital gains expectations about the market (S&P 500) so that $E(g_{ct+1}) - g_{ct+1} \geq 0$. That is, there is indeed a minimum expected capital gains rate at the market level, which is equal to the expected retained earnings yield g_{ct+1}. In other words, the present value of *expected* growth opportunities cannot be negative at the market level. Let us argue by contradiction. If $E(g_{ct+1}) - g_{ct+1} \leq 0$ were true, an investor would anticipate that the dollars value of next period's capital gains is lower than the value of earnings to be reinvested, or that the *expected* return on reinvested earnings is negative. From a rational investor standpoint, it is better to gain from share buybacks or to receive more dividends rather than to reinvest earnings. Since corporate profits and earnings exhibit a mean reversion property on average for the economy (Fama and French (2000))[7], a potential strategy at the *market level* would be to buy a controlling majority of index shares and adopt a 100% dividend payout ratio financed by increased debt against future earnings. In other words, an increase in expected dividends would raise the value of shareholders' equity, which is impossible under Miller-Modigliani's (1961) irrelevance of dividend policy theorem. Consequently, the condition $E(g_{ct+1}) \geq g_{ct+1}$ must hold under conditions that allow for the previous strategy.[8] Thus, from an investor's standpoint there is a *minimum* expected required yield $E(R_{\min t+1})$ based on the best use of retained earnings *ex-ante*, in every period. Mathematically, we see that:

[7] Fama and French (2000) test the mean reversion of profits and earnings of a large sample of firms recorded in Compustat over 1964-1996.

[8] This is assuming that the increase in leverage does not significantly raise the cost of debt, in other words that there is no credit-crunch. In addition, to close the argument we must take into account the differential tax treatment between capital gains and dividend income.

$$E(R_{\min t+1}) = \frac{E(e_{t+1})}{P_t} \times \frac{1}{1+D_t/P_t} \tag{9}$$

From expression (9), we conclude that the minimum expected required yield on equity is a direct function of expected forward earnings yield. This has two important implications[9]: 1) From the standpoint of a first time investor the minimum expected required yield is equal to the forward earnings yield as D_0 equals zero and 2) as seen in the next section, in the long run, the minimum expected required yield must be equal to GDP/capita growth.

3.2 The Minimum Expected Required Equity Yield and Long run GDP/capita Growth

Imagine an investor holding an index share for the long-term, after having bought it in period zero. Consider expression (9) in the long run. We know from previous computation in section 1 that the ratio $\frac{1}{1+D_t/P_t}$ converges to a constant retention ratio (1-b). On the other hand, we know that in the long run, the earnings per share sustainable growth rate is given by $g_y = (1-b) \times ROE$, with g_y representing nominal GDP/capita growth. Moreover, in a steady-state the price per share P_t converges to book value B_t per share. Thus, the forward earnings yield $\frac{E(e_{t+1})}{P_t}$ converges to the ROE and

[9] Philips (1999) relates the earnings yield to expected total return and the replacement cost of capital. Another implication of this analysis is that dividend payouts or share buybacks may rise in periods when actual capital gains turn out to be lower than the expected minimum capital gains as measured by the retained forward earnings the year prior.

consequently to the value $\frac{g_y}{(1-b)}$. Plugging the two above values in expression (9) gets us that in the long run $\lim_{t\to\infty} E(R_{\min t+1}) = g_y$. Hence, long term investors obtain a nominal (*real)* minimum compounded return that is equal to nominal (*real)* GDP/capita growth. Since it is also true that the forward earnings yield converges to the total return, thus long term-investors will earn real GDP/capita growth as their compounded rate. The long-term average real GDP per capita growth rate for developed countries found by Pritchett (1997) is about 1.5%, just lower than the actual U.S. rate of 2.07% (using a 1.33% population/share growth rate) for the period 1926-2001.

3.3 A Required Yield Theory of Stock Market Valuation

Consider a new investor at time 0 who has a long-term investment horizon. Her minimum expected required return is $E(R_{\min 1}) = \frac{E(e_1)}{P_0}$ in the first year. Assume she wishes to maximize her total cumulated *after-tax real* return over her investment horizon. Thus, she will bid the index price up so that on an after-tax basis, she obtains a minimum *real* return at least equal to the long run minimum (and maximum) compounded total return given by the real per capita GDP growth rate. If at any point in time, there is perfect competition amongst long-term and short-term oriented investors, the index price will be bid up to the point where the long run real compounded return will be matched instantaneously on an *after-tax* basis. Furthermore, a simple arbitrage argument shows that this minimum expected yield must be commensurate with the minimum expected real bond rate obtainable (which turns out to be the maximum expected real risk free rate

r_{bt+1}) over the long-term investment horizon. Since nominal returns are taxed, we express the after tax version of this arbitrage condition in nominal terms as follows:[10]

$$\frac{E(e_{t+1})}{P_t} = \frac{Max\left(g_y + \pi_{t+1}; (1-\tau_{it+1})R_{bt+1} + RP_{t+1}\right)}{1 - \tau_{dt+1}b_{t+1} - \tau_{ct+1}(1-b_{t+1})} \tag{10}$$

Where π_{t+1} is the expected inflation rate, R_{bt+1} is the nominal T-bond rate, τ_{it+1} and τ_{dt+1} respectively represent the average marginal tax rates for interest income and dividend income. The top marginal capital gains tax rate is τ_{ct+1}. Expression (10) says that the after tax (capital gains and dividend) expected nominal earnings yield equals the maximum between the long term real GDP/capita growth rate indexed for inflation and the after-tax long-term T-Bond rate. The variable RP_{t+1} represents a premium that is positive due to short-term earnings downside volatility. If stock prices are determined at the margin by long-term investors, then the risk premium cannot be a function of short-term volatility, but rather of the long-term differential in compounded returns between stocks and bonds. Nonetheless, we previously showed that the equity premium as measured as the difference between S&P 500 and 10-year Treasuries long-term returns is almost zero. In the context of a long-term investor, this should be true as uncertainty is mostly resolved in the long run, and both instruments should yield the same *compounded* return. A similar argument was put forth by Glassman and Hassett (1999). However in our case, the argument rests on the natural laws of economic growth and compounding with very different implications regarding the fair valuation of a stock index. In that context, we adduce that the risk premium RP_{t+1} is zero. Finally, we have:

[10] The actual proof is in the appendix.

$$\frac{E(e_{t+1})}{P_t} = \frac{Max\left(g_y + \pi_{t+1}; (1-\tau_{it+1})R_{bt+1}\right)}{1-\tau_{dt+1}b_{t+1} - \tau_{ct+1}(1-b_{t+1})} \tag{11}$$

Our Required Yield Theory (RYT) shows that equity must be priced to yield an after tax and inflation *minimum* expected real return equal to the long-term real GDP growth rate. This mechanism provides the crucial link between equity return and GDP growth, is instantaneous, and practiced by the marginal (long-term) investor. Siegel in his book *Stocks for the Long Run* (2002) pp. 119-120 hints at a similar approach: "If we assume that investors bid stock prices up or down in response to changing taxes and inflation to obtain the same *after-tax* real return, we can calculate how shifts in these variables affect the P-E ratio."[11]

Our theory sets a foundation for why the Fed Model (Lander, Orphanides and al. (1997)) is sound from an economic standpoint. Here, we are generalizing the Fed Model by accounting for the impact of taxes as well as inflation and the requirement to yield a real return pegged to the long run real GDP/capita growth rate. Ritter (2001) notes that the Fed Model works better empirically than other models, but should not work well theoretically if most of the variation in nominal rates and thus stock yields comes from changes in expected inflation rather than changes in real rates. The logic is that for the earnings yield to move one-for-one with the nominal bond yield, as the Fed Model has it, one has to assume that the nominal yield on bonds equals the real return on stocks, since the earnings yield is believed to be a real return. Thus, the empirical success of the Fed Model appears to be inconsistent with rational valuation according to current theory. The

[11] Siegel cites McGrattan and Prescott (2000). However, neither Siegel (2002) nor McGrattan and Prescott (2000) derive a full-blown theory in that direction. Empirically, Reilly, Griggs and Wong (1983) showed

explanation according to RYT is that in fact the forward earnings yield is a nominal return and that investors are requiring a constant, not variable, minimum expected real return on stocks at all times. [12]

According to RYT, the forward earnings yield ratio or E/P provides an essentially constant minimum after-tax real return equal to the greater between the long-term real per-capita productivity rate and a real long-term bond rate.

3.4 Conditions and Assumptions Necessary for RYT

We recognize that a number of conditions are present, at a minimum, under which we have developed and tested the RYT. These include:

1. Absence of significant deflation.
2. Absence of sustained large fluctuations of, or negative real productivity per capita.
3. Small proportion of total earnings represented by earnings from foreign operations and small portion of total market capitalization from securities trading in multiple countries.

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using S&P 400 data that over 1962-1980, that the market earnings yield was positively related to inflation and risk-free yield.

[12] Using S&P 500 historical data from Shiller's (2002) website, we check that the historical average forward earnings yield has a value of 7.92% over the period 1926-2001. Our estimate using our above formula (11) is 7.84%. We apply formula (11) given an historical average S&P 500 payout ratio of 55.5%, a real GDP/capita growth rate of 2.07%, and an average inflation rate of 3.14%. We use average marginal tax estimates on dividend income from Estrella and Fuhrer (1983) for the period 1954-1979 and from the NBER TAXSIM model for the period 1980-1999. Because our marginal income tax data is limited to 1954-1999, we extrapolate the 1999 taxes rates for the years 2000 and 2001. The average marginal dividend tax rate we use equals 39.95% and the average marginal capital gains tax is 25.9% (obtained from IRS website), which leads to a blended tax rate of 33.6%. Note that using the standard sustainable growth formula for estimating the forward earnings yield leads to a value of 11.72%. This discrepancy with historical estimates is explained mostly by the fact that the S&P 500 Price-to-Book ratio was much greater than 1 over the period.

4. Absence of a short-term risk premium, such as may occur during periods of statistically abnormal negative equity market price volatility. However, any such risk premium must in the long term, contain a mechanism that nets to zero so that compound return equals GDP growth.
5. Mean reversion of expected capital gains at the market level.
6. Liquid debt markets with no sustained credit crunch periods.
7. The long-term investor is the marginal investor.
8. Financial markets are dominated by domestic investors

4. Empirical Tests of the Required Yield Theory

4.1 Mean Reversion of Capital Gains

In this section, we show that capital gains exhibit a mean reversion property at the market level. This is an essential piece of our theory, since for our equity valuation formula to hold we assume that the minimum expected capital gains matches the retained forward earnings yield. Let us define our variables: Let $A_{t+1} = \frac{P_{t+1} - P_t}{P_t} - (1 - b_{t+1}) \frac{E_{t+1}}{P_t}$ denote the difference between ex-post capital gains and our minimum expected capital gains based on the ex-post forward retained earnings yield. Let $D_{t+1} = A_{t+1} - A_t$. We run the following partial adjustment regression:

$$D_{t+1} = a + bD_t + \varepsilon_{t+1} \tag{12}$$

The regression is run using S&P 500 yearly data on price, earnings and dividends over 1926-2001. The resulting estimates are –0.0006 for the intercept and –0.38 for the slope, with an adjusted R-square of 14%. The intercept is statistically non-significant and the slope has a t-statistics of –3.57. These results indicate that actual capital gains tend to converge to the ex-post measure of the expected minimum capital gains, and that the rate of mean reversion is about 38% per year. Interestingly, the same value is found in Fama and French (2000), where they test the mean reversion of accounting profits for a sample of firms obtained from Compustat over 1964-1996. Moreover, paralleling Fama and French's (2000) further observation, we find that the rate of mean reversion is faster when the values for the differences A_{t+1} are negative[13], that is, when actual capital gains are lower than expected minimum capital gains. Although expectations cannot be directly tested, our findings are consistent with the notion that capital gains expectations are adjusted so that they exceed the minimum expected gains based on the retained portion of the earnings yield.

4.2 Test of the RYT Valuation Formula and Comparison with the Fed Model

In this section, we test the RYT valuation formula (11). Our testing period is January 1954- September 2002 for monthly data and Q1-1970 to Q3-2002 for quarterly data. Historical trailing earnings per share, dividend per share and prices are for the S&P 500. We use forward earnings per share monthly estimates from Thomson Financial over the 1979-2002 period. Prior to 1979, we use current earnings per share as an estimate of expected earnings. Expected inflation estimates are captured on a quarterly basis by the Survey of Professional Forecasters available from the Federal Reserve Bank of

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[13] In that case, the adjusted R^2 is 7.3%, the reversion speed is -0.32 (t= -2.61) compared to an adjusted R^2 of

Philadelphia website for the period 1970-2002. When measured on a monthly basis we apply the quarterly estimate to each of the three months within the quarter. Prior to 1970, we use the trailing 12 months CPI based inflation rate as our proxy for expected inflation.[14]

In Exhibit 2 we compare the performance of the RYT implied formula (11) versus the Fed Model in tracking the S&P 500 index. For each observation, we compute the percentage tracking error as $ABS\left(FairP_t - P_t\right)/P_t$, where $FairP_t$ represents the estimated S&P 500 index value using either the Fed Model or RYT formulas. We also report the mean percentage tracking error for each model, and various periods.

In general, the RYT formula leads to a smaller tracking error. Over the entire period on a monthly basis, the mean percentage error drops by 35% compared to the Fed Model. On a quarterly basis, the mean percentage error decreases by about 52%. Note that overall the RYT performs better than the Fed Model except when the market period chosen is 1990-2002, during which the Tech Bubble occurred. In that case, both models predict that the market was severely overvalued from 1998 until 2000, and they both perform relatively badly in terms of tracking error. Note though, that when an average of one year and two year forward expected earnings is used after 1990 as a proxy to forward earnings (earnings expectations rose dramatically during this period), the descriptive power of RYT is significantly increased. The mean percentage error becomes 11.9% for the RYT

3.1%, and a reversion speed of –0.19 (t= -1.83) for positive differences.

[14] Whenever our results are based on quarterly data, we use the GDP deflator index, which is only available on a quarterly basis and is consistent with the Survey of Professional Forecasters. Furthermore, after 1970, in each quarter, we reconstruct annual earnings by summing the available last four quarterly earnings.

vs. 13.2% for the Fed Model over 1990-2002 on a quarterly basis (assuming a 2.07% GDP/capita growth rate).

In Exhibit 2, we also show two types of results for the RYT depending on our assumption about the long-term real GDP/capita growth rate. The first value we use is the 2.07% US growth rate over 1926-2001. In contrast, we also use 1.5%, which is the estimated average real GDP/capita growth rate for a group of OECD members (Pritchett (1997)). This sensitivity analysis shows that the results are overall quite comparable when using either value as our estimate of long-term growth.[15]

Exhibit 2: Benchmark Comparison of RYT and FED Valuation Formulas

Tracking percentage error is computed for the RYT under two alternative long term real GDP/capita growth rates (in parentheses).

Frequency	Period	Mean % Error Compared to S&P 500 Index		
		RYT (2.07%)	RYT (1.5%)	FED Model
Monthly	1954-2002	21.4%	22.0%	32.9%
	1970-2002	16.1%	14.4%	15.1%
	1979-2002	9.9%	9.6%	12.2%
	1990-2002	11.0%	11.3%	12.5%
Quarterly	1954-2002	17.5%	17.3%	36.8%
	1970-2002	12.9%	10.9%	20.2%
	1979-2002	10.9%	10.1%	12.4%
	1990-2002	13.1%	12.6%	12.4%

[15]The fact that a slightly better fit is obtained on a quarterly basis using the global developed nations productivity rate, may point to the possibility of a global productivity-based arbitrage. Investors in lower productivity countries may bid up assets in higher productivity countries to meet a global real required return.

In order to further assess the relative performance of RYT, we examine its ability to predict price movements based on the simple valuation rule that the market is undervalued when the actual S&P price falls below the fair value according to the RYT (or alternatively the Fed Model FED). We run OLS regressions of the differential between estimated fair value and actual S&P index, against the differential of the index value at a future date (alternatively one, two and up to fourteen months ahead) minus the current index. We test the performance of RYT and the Fed Model prior to 1997 since during the 1997-2000 Tech Bubble the delay in market correction is not reflective of 'normal' periods market price adjustments. However, this does not diminish the power of either model as they both correctly predicted overvaluation during the 1997-2000 period.

Exhibit 3 below shows our results using a 2.07% GDP/capita growth rate assumption. Based on the adjusted R-square and t-statistics, we find that RYT is able to predict stock market price movements about two to three times more accurately than the Fed Model, over the period 1970-1997 and especially when the lead-time is between four and five months.

Over the 1954-1997 period, the results are more mitigated, as both models seem to perform somewhat equally with an optimal lead-time of thirteen to fourteen months. Figure 1 and Figure 2 illustrate respectively the tracking of the S&P 500 index and P/E ratios using the RYT valuation formula vs. the Fed Model over 1954-2002, using quarterly data. We use forward expected earnings from Thomson Financial after 1979 and an average of 1 yr forward and 2-year forward S&P estimates after 1990.

Exhibit 3: Predictability of S&P 500 Price Movements Based on RYT and FED Models

OLS regressions: $P_{t+j} - P_t = \alpha + \beta \times (FairP_t - P_t)$ are run using monthly observations.

T-statistics are shown in parentheses.

Lead	Period 1: 1954-1970				Lead	Period 2: 1970-1997			
	Adjusted R 2		Slope β			Adjusted R 2		Slope β	
	RYT	FED	RYT	FED		RYT	FED	RYT	FED
j=1	2.04%	1.88%	0.01 (2.23)	0.01 (2.16)	j=1	6.81%	4.06%	0.08 (4.96)	0.05 (3.83)
j=2	3.92%	3.90%	0.03 (2.96)	0.03 (2.96)	j=2	10.76%	5.26%	0.17 (6.32)	0.10 (4.35)
j=3	5.30%	5.74%	0.04 (3.42)	0.04 (3.44)	j=3	13.80%	6.08%	0.25 (7.26)	0.14 (4.68)
j=4	6.54%	7.58%	0.05 (3.79)	0.05 (4.08)	j=4	15.93%	6.38%	0.33 (7.89)	0.17 (4.80)
j=5	8.58%	9.89%	0.06 (4.35)	0.07 (4.69)	j=5	15.65%	5.35%	0.38 (7.81)	0.18 (4.39)
j=6	10.10%	11.60%	0.08 (4.74)	0.08 (5.11)	j=6	15.01%	4.22%	0.42 (7.62)	0.19 (3.90)
j=7	11.51%	13.08%	0.09 (5.08)	0.10 (5.45)	j=7	13.89%	3.09%	0.46 (7.29)	0.19 (3.36)
						Total Period: 1954-1997			
j=8	12.98%	14.50%	0.11 (5.43)	0.11 (5.78)	j=1	2.94%	1.90%	0.04 (4.07)	0.03 (3.31)
j=9	14.04%	15.56%	0.12 (5.67)	0.12 (6.02)	j=2	4.62%	2.38%	0.09 (4.09)	0.06 (3.68)
j=10	15.12%	16.24%	0.13 (5.92)	0.13 (6.17)	j=3	5.81%	2.66%	0.13 (5.72)	0.08 (3.88)
j=11	16.15%	16.66%	0.14 (6.15)	0.14 (6.26)	j=4	6.58%	2.69%	0.17 (6.11)	0.10 (3.90)
j=12	16.36%	16.69%	0.14 (6.14)	0.14 (6.26)	j=5	6.23%	2.05%	0.19 (5.93)	0.10 (3.43)
j=13	16.60%	16.54%	0.15 (6.25)	0.15 (6.23)	j=6	5.72%	1.41%	0.21 (5.68)	0.10 (2.89)
j=14	17.05%	16.20%	0.15 (6.34)	0.15 (6.16)	j=7	5.06%	0.84%	0.22 (5.33)	0.09 (2.31)

It is interesting to note that RYT seems to track the S&P 500 much more accurately than the Fed Model prior to the 1970s. It is well documented that the 10-year T-Bond was unable of tracking the S&P 500 earnings yield prior to 1970 but moved closely with the earnings yield after that.

The historical change in relative volatility between the two instruments is a suggested explanation in Asness (2000). Here, we observe that RYT predicts values on par with the S&P 500 P/E ratio and the index value from about 1955 until 1970 without appealing to changes in relative volatility measures.

Figure 1: Tracking the S&P 500: RYT vs. FED model

Quarterly data 1954-2002. RYT and FED Model estimated prices are plotted against S&P 500 Index

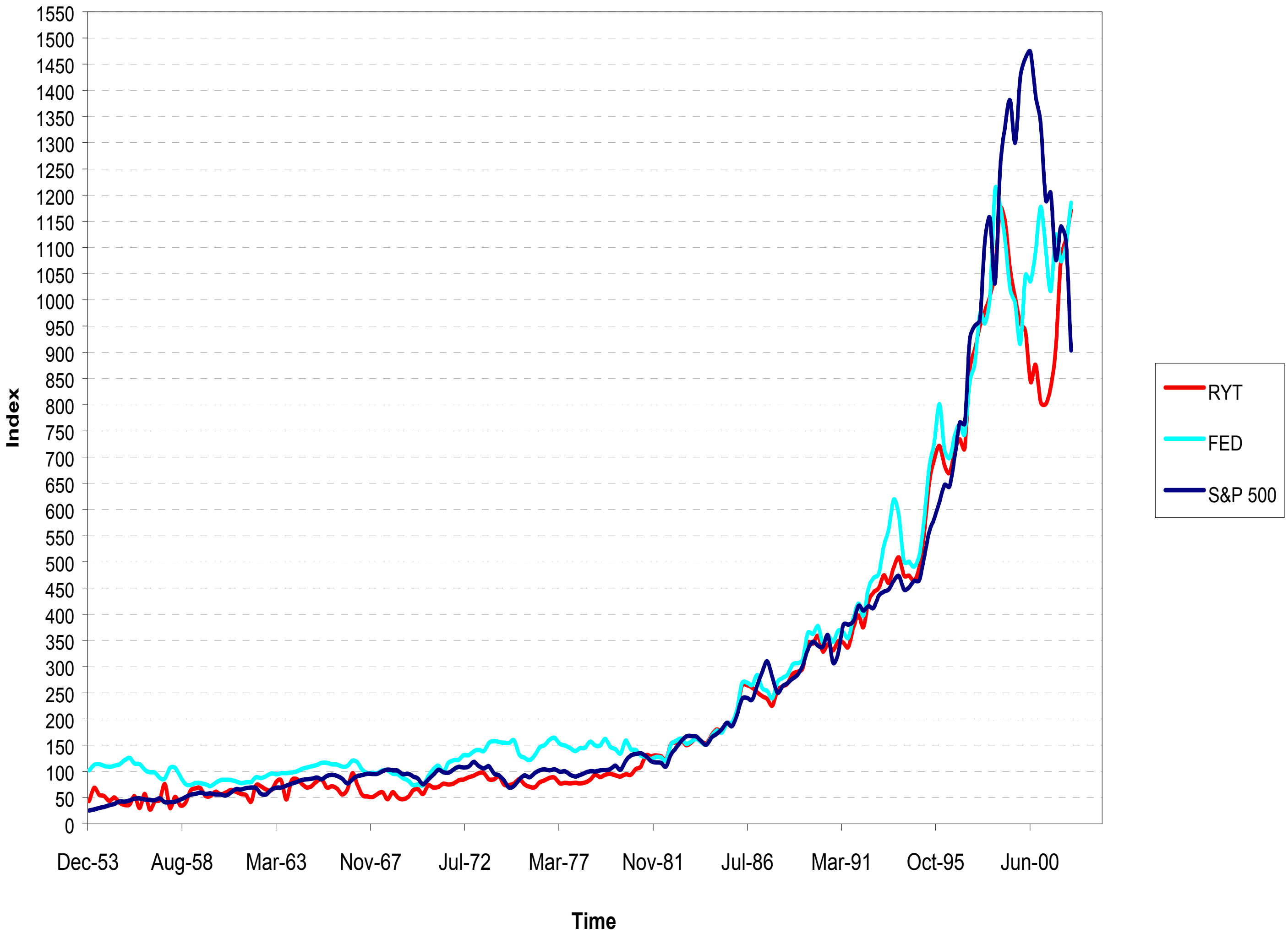

Figure 2: Comparison of P/E ratios: RYT vs. Fed Model

Quarterly data 1954-2002. RYT and FED Model estimated P/Es are plotted against S&P 500 P/E

5. Implications of the Required Yield Theory

5.1 The Fisher Hypothesis

The effect of inflation on stock returns has been the object of extensive research. Although the negative correlation of equity valuation with inflation is well documented, it has not yet been explained theoretically or empirically. Irving Fisher (1930) considered the possibility of a required real interest rate, but did not apply his concept to the earnings yield ratio nor did he derive the valuation implications of such a required yield. Sharpe (1999, 2001) finds that high expected inflation predicts low stock returns (and high dividend yields) and has a strong negative correlation to the P/E. He goes on to conclude that this effect coincides with both lower expected real earnings growth and higher required real returns. While he states that a one percentage point increase in expected inflation raises required long run real equity returns about three-quarters of a percentage point (this is not true at a wide range of inflation rates), Sharpe does not adduce a deterministic relationship between inflation and the market P/E.

In their analysis of this phenomenon, Modigliani and Cohn (1979), suggest that investors "are plagued by a form of money illusion…investors capitalize equity earnings at a rate that parallels the nominal interest rate on bonds, rather than the economically correct real rate…" Ritter and Warr (2002) are in the same camp by concluding that equities were undervalued into the early 1980s because of "cognitive valuation errors of levered stocks in the presence of inflation and mistakes in the use of nominal and real capitalization rates." They credit the subsequent Bull market of 1982 to 1999 in part due to a falling risk premium. On the other hand, RYT states that it is not the long run real return that is affected by expected or actual inflation, but an immediate minimum expected nominal yield that preserves a constant real return, with no evidence of severe cognitive valuation mistakes at monthly and quarterly frequencies.

5.2 The Equity Premium Revisited

If a premium for risk was inherent in equity valuation, then a Fed 'type' Model to which a risk premium is added should more accurately correlate to actual prices. Just as a junk bond yield includes a default rate premium in addition to a term-adjusted risk-free rate; a required stock yield must incorporate a default risk premium greater than the debt grade for the same risk class since equity comes after debt in recovery. Thus, while at the aggregate, the stock market does not default; the mix of risk-adjusted companies at any given time in the economy may affect the required yield. Our RYT formula predicts that in the long-run, the expected S&P 500 index capital gains is equal to the expected EPS growth rate, under the condition that the inflation and tax rates are stable. Furthermore, from a long-term investor perspective on a compounded basis stocks and T-bonds must return the same in real terms and after tax. Moreover, the addition of any *positive* risk premium should perform at least as well than if none were included, if risk is present in equity valuation relative to a risk-free benchmark. Nevertheless, any added risk premium of the magnitude proposed in the literature, would substantially shift estimated valuation levels below that predicted by the RYT formula and thus would raise the overall average percentage tracking error and result in a compounded stock return that far exceeds GDP growth, which is impossible.

5.3 Stock Market Volatility

How is risk or volatility incorporated in our theory? Since traditional valuation models discount dividend streams to arrive at fair equity value, and that both the growth of dividends and the discount rate are stable in the long-term, no significant fluctuations in equity prices can be predicted due to short-term earnings or interest rate changes. This problem has been well documented by Shiller (1981). Our RYT predicts and explains low frequency (monthly and quarterly) volatility in terms of the changing expectations about earnings, inflation and taxes. The "Fed Model" and related approaches also predict volatility but do so without theoretically or empirically dealing with the risk premium, dividend and earnings growth, and how the equity yield is related to a risk-free rate and tax differences between tax on interest, dividends and capital gains.

6. Conclusions

We have shown that GDP growth determines long-term total compounded equity return. The total compounded equity return equals GDP growth in the long term because dividends cannot be reinvested in the aggregate. Required Yield Theory (RYT) demonstrates that a broad index (S&P 500) value is determined by an immediate nominal expected earnings yield that results in an after-tax minimum real required yield equal to the long-term real productivity per-capita rate. The risk-free rate does not significantly affect equity prices, except on occasions when it results in a real after-tax return in excess of the real long-term productivity per capita rate. According to RYT, changes in the P/E are inversely affected by the inflation rate; and in the long term, inversely related to permanent changes in the real per capita productivity rate. Since the best results for tracking the S&P 500 index are obtained using a global developed nations productivity rate, this may point to a global productivity-based arbitrage argument. This hypothesis will be explored in future research.

We also demonstrated the effective absence of an equity premium of any sort; due to risk or otherwise in the long run, as interest on debt is fully compoundable on a per capita basis and yields a total return equal to GDP per capita. Future research will examine the potential generalization of RYT to bond pricing, gold pricing and other asset classes. The mechanism described by RYT leaves little room for the effects of psychological factors such as greed and fear, but does allow for volatility based on uncertainty as to economic growth, profitability and inflation as may be caused by political, economic or other instability. Finally, investors and fund administrators must recognize that stock market returns in the very long-term cannot exceed GDP growth. Individual investors are further limited for they can only earn GDP/capita in the stock and bond markets.

APPENDIX

In this appendix, we prove the general Required Yield Theory formula (11) that incorporates taxes. Our first step is to show that in the long run on an after tax basis the minimum real expected return converges to the real GDP/capita growth rate. We start out from a definition of the compounded after-tax (long-term capital gains and dividend) expected stock return:

$$E(R_{t+1}) = \frac{(1-\tau_{dt+1})E(d_{t+1}) + E(P_{t+1}) - \tau_{ct+1}(E(P_{t+1}) - P_t) + (1-\bar{\tau}_{dt})D_t}{(1-\bar{\tau}_{dt})D_t + P_t - \bar{\tau}_{ct}(P_t - P_0)} - 1 \qquad \text{(A1)}$$

Where $D_t = \sum_{j=0}^{t} d_j$ and d_j represent the sum of all dividends paid over the single time period j. Thus, D_t represents the sum of all dividends cumulated up to the end of period t. The tax rates are the average marginal tax rates for dividend income τ_{dt+1}, and the top marginal capital gains tax rate τ_{ct+1}. The marginal tax rates $\bar{\tau}_{dt}$ and $\bar{\tau}_{ct}$ are respectively weighted averages of past tax-rates, defined as follows:

$$\bar{\tau}_{dt} D_t = \sum_{j=0}^{t} \tau_{dj} d_j \text{ and } \bar{\tau}_{ct}(P_t - P_0) = \sum_{j=0}^{t-1} \tau_{cj+1}(P_{j+1} - P_j) \qquad \text{(A2)}$$

Expression (A1) can be rewritten as:

$$E(R_{t+1}) = \frac{\left[(1-\tau_{dt+1})b_{t+1} + (1-\tau_{ct+1})(1-b_{t+1})\right] E(e_{t+1})/P_t + (1-\tau_{ct+1})(E(g_{ct+1}) - g_{ct+1})}{(1-\bar{\tau}_{dt}) D_t/P_t + (1-\bar{\tau}_{ct}) - P_0/P_t} \qquad \text{(A3)}$$

Where $g_{ct+1} = (1-b_{t+1})\dfrac{E(e_{t+1})}{P_t}$ and $E(g_{ct+1}) = \dfrac{E(P_{t+1}) - P_t}{P_t}$.

As shown previously, in the long run, the ratio D_t / P_t converges to the value $\frac{b}{(1-b)}$ and the price P_t goes to infinity. Assume that the tax rates τ_{dt}, $\bar{\tau}_{dt}$, τ_{ct} and $\bar{\tau}_{ct}$ converge to respective constants $\bar{\tau}_d$ and $\bar{\tau}_c$, and assume that the payout ratio b_{t+1} converges to a constant b. This implies that for a large horizon t, expression (A3) becomes:

$$E(R_{t+1}) = \frac{\left[1-\bar{\tau}_d b-\bar{\tau}_c(1-b)\right]E(e_{t+1}) / P_t + (1-\bar{\tau}_c)(E(g_{ct+1})-g_{ct+1})}{(1-\bar{\tau}_d)\, b / (1-b) + (1-\bar{\tau}_c)} \tag{A4}$$

It follows from (A4) that the minimum expected stock return is obtained when $E(g_{ct+1}) - g_{ct+1} = 0$, in other words:

$$E(R_{\min t+1}) = \frac{(1-b)E(e_{t+1})}{P_t} \tag{A5}$$

Since the forward earnings yield must satisfy $\lim_{t\to\infty} \frac{E(e_{t+1})}{P_t} = \frac{g_y}{(1-b)}$, hence we have:

$$\lim_{t\to\infty} E(R_{\min t+1}) = \lim_{t\to\infty} \frac{(1-b)E(e_{t+1})}{P_t} = g_y \tag{A6}$$

The second step is to express the minimum expected return for a first time investor. Using expression (A1) again and assuming that no prior dividends and no taxes were paid, we get:

$$E(R_1) = \frac{(1-\tau_{d1})E(d_1) + E(P_1) - \tau_{c1}(E(P_1) - P_0)}{P_0} - 1 \tag{A7}$$

In other words, following the same steps as above we get:

$$E(R_{\min 1}) = \left[1-\tau_{d1}b_1 - (1-b_1)\tau_{c1}\right]\frac{E(e_1)}{P_0} \tag{A8}$$

A simple arbitrage argument is that long-term investors will bid the highest price until the expected minimum return equals real long-term GDP/capita growth indexed for expected inflation and taxes. On the other hand, long-term T-bonds also compete to provide a minimum expected return. Thus, on an after tax basis, and in every period where long-term investors join the market we must have:

$$\frac{E(e_{t+1})}{P_t} = \frac{Max\left(g_y + \pi_{t+1}; (1-\tau_{it+1})R_{bt+1} + RP_{t+1}\right)}{1-\tau_{dt+1}b_{t+1} - \tau_{ct+1}(1-b_{t+1})} \tag{A9}$$

Where π_{t+1} is the expected inflation rate, R_{bt+1} is the nominal T-bond rate, τ_{it+1} represents the average marginal tax rate for interest income. Finally, our argument in the main text of the paper is that we can safely assume that the premium RP_{t+1} is zero since long-term investors are insensitive to short term stock price fluctuations. Henceforth, we finally obtain:

$$\frac{E(e_{t+1})}{P_t} = \frac{Max\left(g_y + \pi_{t+1}; (1-\tau_{it+1})R_{bt+1}\right)}{1-\tau_{dt+1}b_{t+1} - \tau_{ct+1}(1-b_{t+1})} \tag{A10}$$

QED

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