The Price of Gold: A Global Required Yield Theory*

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Abstract

We construct a gold valuation theory based on viewing gold as a global real store of wealth. We show that the real price of gold varies inversely to the stock market P/E and thus is a direct function of a global yield required to achieve a constant real after-tax return equal to long-term global real GDP per-capita growth. We introduce a new exchange rate parity rule based on the equalization of inverse stock market P/Es (required yields) across nations. Foreign exchange affects the price of gold to the extent that required yields and Purchasing Power Parity equalizations do *not* take place across nations in the short run. A quarterly valuation model is constructed using concurrent economic data that is within 12% mean percentage tracking error from real U.S. gold prices from 1979– 2002. Several major world events have had a large but fleeting impact on gold prices.

Keywords: Gold Price, Stock Market, Required yield, Forward Earnings yield, Foreign Exchange, P/E, Price-Earnings Ratio.

Assessing the fair value of gold largely remains a mystery in Finance. While in some instances the existing literature has found empirical relationships between gold prices and macroeconomic variables such as inflation and exchange rates, little evidence has been offered for connections between gold and other asset classes. To date, there is no comprehensive theory of gold valuation showing how inflation, exchange rates and other asset classes may together affect gold pricing; or how gold and other asset classes may be affected by common underlying factors.

In this paper, we offer a gold asset pricing theory that treats gold as a store of wealth. We demonstrate a theoretical and empirical link between gold price, inflation, and foreign exchange rates and the general valuation of the stock market. Our approach is based on a generalization of Required Yield Theory (Faugere-Van Erlach [2003]). Required Yield Theory explains the valuation of financial assets via investors' general requirement to earn a minimum expected after-tax real return equal to long-term GDP/capita growth.

We hold that since gold fulfills the unique function of a global store of value, its yield must vary *inversely* to the yield required by any financial asset class, thus providing a hedge in the case where such assets are losing value. Our theory explains about 88% of actual $USD gold prices and 92% of actual gold returns on a quarterly basis, including the peak prices of gold, over the 1979-2002 period.

The extant literature has well documented empirical relationships between gold price and global macroeconomic variables such as inflation and currency exchange rates. For example, Sjaastad and Scacciavillani [1996] show that after excluding the sharp rise in gold prices in the early 1980's, about half of the variance in $USD gold prices during the period 1982-1990 appears to be accounted for by fluctuation in real exchange rates. Ghosh, Levin et al. [2002] find that gold is mostly an inflation hedge in the long run. They further attempt to justify short-term gold

price volatility by appealing for example to changes in the real interest rate and $USD vs. rest of the world exchange rates fluctuations.

On the other hand, the empirical record weighs heavily on the side that gold pricing apparently is related neither to GDP growth nor to other asset classes. Lawrence [2003] concludes that there is no statistically significant correlation between real returns on gold and changes in macroeconomic variables such as GDP, inflation and interest rates, and that the return on gold is less correlated with returns on equity and bond indices than are the returns of other commodities. Standing in contrast to the above findings, Coyne [1976] focuses primarily on gold as a hedging instrument and finds that for periods in which the gold market was free to fluctuate, gold tended to move in a direction opposite to the price of other financial assets.

Sherman [1983] makes a noted theoretical attempt at demystifying the pricing of gold. He uses a linear regression model to estimate elasticities of demand for gold. The key explanatory factors are exchange rates and unanticipated inflation proxies. While several useful relationships are studied, these relationships are assumed a-priori and not theoretically derived.

Barsky and Summers [1988] focus on the Gold Standard period and develop a general gold valuation model that views gold as a non-monetary durable good providing a stream of "consumption" services over time, like Jewelry or objects of art. They theoretically show a relationship between the inverse of the log of gold price and the real interest rate, which seems to hold empirically over the period 1974-1984. In their model, gold is a non-depreciable asset earning a yield equal to a government bond yield.

However, by rooting their model in the Gold Standard era, and extending their approach to the current era, they are not addressing the nature of Gold as a store of value, that is, a hedging instrument against inflation and the collapse of the value of other asset classes. In this paper, on the other hand, we undertake the analysis of gold along this exact line.

GOLD MARKET HIGHLIGHTS AND GOLD'S EMPIRICAL LINK TO STOCKS

The total aboveground value of gold in the world is currently around $1.9 trillion at $380/Troy oz. ($380/Toz. x 32,150.7 Toz./Metric ton x 155,000 Mtons) compared with the approximately $15 trillion value of the US stock market and $22.4 trillion for US non-financial debt outstanding. Gold mining is a $31 billion per year industry with gold prices at $380/Toz. Given that the price volatility of gold is around 10% per year, it is easy to see why production companies heavily engage in hedging their future production. Ibbotson, Siegel and Love [1985] estimated that gold bullion represented 5% of total investable world wealth. Today, total world gold bullion represents 5.1% of the combined US stock and bond capitalization of $37.4 trillion; and a thus a much smaller proportion of total world wealth than the Ibbotson et al. study.

The rate of growth of gold extraction has essentially matched world population growth over the past 30 years. IMF data show that the more developed nations' population grew a compounded 1.45% from 1972 – 2002, while total world population grew a compounded 1.89%. The global accumulated stock of gold grew from an estimated 98,000 tons in 1974 to 145,000 tons by mid-2001; implying a 1.46% growth rate. Thus, the world stock of gold per-capita has remained relatively stable over this period.

A preliminary empirical investigation of the price of gold reveals a non-trivial connection between real gold prices and the US stock market. Figure 1 below shows that gold's real price varies inversely to the S&P 500 P/E, and thus with the earnings-to-price ratio. Figure 1 shows the high correlation between *indexed* USD real gold prices, inverse S&P 500 forward P/E ratio and 10-year T-Bond, over the period 1979-2002.[1]

FIGURE 1 about here

The theory we develop below predicts and explains this high level of correlation based on viewing gold as a global store of value.

REQUIRED YIELD THEORY APPLIED TO GOLD VALUATION

Throughout the history of civilization, gold has been the single most important global store of value. To this day, it fulfills this unique function. For the purpose of extending Required Yield Theory to gold pricing, we postulate the following: 1) The *global* real price of gold essentially is a real P/E ratio for gold, where "earnings" represent purchasing power or a global price index. 2) The global real price of gold must vary inversely to all other main financial asset classes' real P/E to preserve the real value of any investor's capital against adverse movements in the values of financial asset classes.[2] 3) Law of One Price: exchange rate fluctuations must impact local currency-denominated gold prices to eliminate potential international gold arbitrages. 4) Mining supply must be stable in relation to supply movements in the aboveground stock and the worldwide stock of gold per capita should not increase in the long run.

Condition 1) recognizes that even though gold does not produce actual earnings, its primary purpose is to provide a stream of services by maintaining real purchasing power over time. The same unit of gold can serve to purchase a representative basket of economic goods repeatedly. We define the forward P/E for gold as the price of gold divided by expected next period's GDP deflator. It is easy to check that the real price of gold is the same as the real forward gold P/E ratio.[3] Condition 2) insures that gold behaves as a store of value, that is: capital flows to gold are dictated by changes in the minimum expected return achievable by *other* asset classes. It is important to emphasize that gold per-se does not require the *same* yield as other assets, as it

stands outside of the conventional realm of investment goals, and acts mostly as a global hedging tool against financial downturns, and inflation.

Hence, our theory postulates that movements in the global real price occur because of the precautionary demand for gold, which largely depends on changes in the inverse real P/E (or required yield) of other assets classes combined. A consequence of this postulate is that a decline in the value of the stock market index does not necessarily entail flight to gold when, for example, expected stock earnings are also falling to maintain a constant real P/E ratio. On the other hand, flight to gold will happen when stock market prices are dropping faster than expected earnings due to acceleration of inflation for example.

In addition, since gold is a global homogenous durable commodity its price must be equalized across countries after currency conversion, which is stated in condition 3). Finally, condition 4) states that the supply of gold must be stable so that investors' precautionary motive is fulfilled without major price movements driven by supply shocks. Indeed this condition seems to be characteristic of the precious metal mining industry. Later on, we provide a formal argument that shows that this must be the case under our theory.

A Model for the Global Real Price of Gold

We assume that the main alternate investment asset class is a stock market index. Since gold is a global hedging tool, Condition 2) above states that the global real price of gold must vary inversely with the global stock market forward P/E. Our gold valuation theory is constructed by making use of this postulate in addition to connecting a global stock market earnings yield to a global nominal return. In order to establish this connection, we introduce the concept of required yield from Faugere-Van Erlach [2003]. In that paper, the authors theoretically show that at any

point in time, a country's after-tax stock market forward earnings yield can be viewed as a minimum expected return. Furthermore, this forward yield must equal a required yield given by the sum of the country's GDP/capita long-term growth rate and its current expected inflation rate.[4] Faugere-Van Erlach [2003] term this approach Required Yield Theory (RYT).

Since this result is applicable to any country, it must also apply to a GDP-weighted average of any given set of countries' earnings yields.[5] In other words, when applied to a broad representative set of countries to approximate the global economy, a global stock index after-tax forward earnings yield must equal a global required yield that is given by:

$$RY_{wt+1} = g_w + \pi_{wt+1} \tag{1}$$

Where g_w stands for the global GDP/capita long-term growth rate and π_{wt+1} is the expected global inflation rate, each variable being determined as respective GDP weighted averages of a representative set of countries' long-term growth rates and current expected inflation rates.

FIGURE 2 about here

We use a value of 1.5% for g_w based on an estimate of the average long-term growth rate for a group of OECD countries by Pritchett (1997). Figure 2 above visibly shows a relationship between these two constructs for a group of OECD countries when earnings yield ratios (inverse P/Es) are weighted by their relative GDP weights.[6]

Furthermore, since Condition 2) states that the real global price of gold must be inversely related to the stock index real P/E, it then must co-vary directly with the global real required yield, as implied by Required Yield Theory (RYT). Let the global gold price be denoted by P_{wt}

and let $\widetilde{P}_{wt}$ represent gold's real global price, that is the price of gold divided by the current global price index. This implies that on an after-tax basis we must have:[7]

$$\widetilde{P}_{wt} = C \times \frac{RY_{wt+1}}{(1+\pi_{wt+1})} = C \times \frac{g_w + \pi_{wt+1}}{(1+\pi_{wt+1})} \qquad (2)$$

So that gold's real price equals a linear function of the after-tax required yield, up to a multiplicative constant *C*. In Appendix B, we show that the constant *C* can be interpreted as the real value of a perpetuity that would pay an ounce of gold every year forever, when inflation expectations and long-term productivity remain unchanged. According to expression (2) above, the real price of gold will always increase when global inflation accelerates and/or if the long-term global economic productivity is raised. In other words, the real price of gold is constant as long as inflation and productivity remain constant. In that case, the nominal price of gold rises with the general level of prices. This first result theoretically grounds the relationship of the real price of gold with inflation, long-term GDP growth, and the stock market P/E (via the required yield). Figure 3 below, shows the relationship between the global real price of gold and the global required yield.

FIGURE 3 about here

Figure 3 uses the same normalization based on the first quarter of 1998 as done before in Figure 1. Note that between 1971 and 1979, following the Gold Standard era, our model fails to account for the behavior of the global real gold price. In fact, up until 1973, the gold price was still traded at the official quotation and later up until 1978, there were strong selling activities by central banks. On the other hand, from 1979 up until the third quarter of 2002 our theory matches

the actual real global price of gold very closely, even during the spike of 1980. The mean percentage tracking absolute error is 21% covering the period January 1980 to April 2003.[8]

Local Gold Prices and Foreign Exchange Effects

In order to extend the above result to local currency denominated gold prices, we must take into account the relative importance of each country in the global economy as well as the effect of currency exchange rates. To simplify the analysis we divide the global economy into two blocs: the home country and rest of the world defined as the bloc of countries X. Let us denote by θ_t the relative nominal GDP weight of the domestic economy in the global economy, and let us denote by $\tilde{s}_t$ the real exchange rate (1 real unit of foreign basket equals $\tilde{s}_t$ real unit of domestic basket).

As shown in Appendix A, we can express the real domestic price of gold as a function of the global required yield as follows:

$$\tilde{P}_t = \frac{1}{(1+\pi_{wt+1})} \times \frac{C \times RY_{wt+1}}{\theta_t + (1-\theta_t) \times 1/\tilde{s}_t} \tag{3}$$

Expression (3) entails that the real domestic price of gold is an increasing function of global inflation. It is an increasing function of the GDP weight of the home country as long as the inflation rate abroad is relatively high and the foreign basket of currencies is weak. The lower the GDP weight of the home country is, the more dependent the real domestic gold price is on the foreign exchange rate and inflation rate. In general, relationship (3) shows that as the domestic currency depreciates, the real domestic gold price appreciates ceteris-paribus.

Figure 4 below shows the result of our formula for the U.S. Figure 4 summarizes the quarterly relationship between the nominal gold prices vs. the fair value of gold using equation

(3). Our mean percentage tracking error is 12%. We use a value of $8929 for the constant *C*, which is obtained as the intercept of the regression of the ratio of actual real price over the right hand side of equation (3) on a time trend, on a quarterly basis over 1979-2002. The regression has an adjusted R-square of 5%.[9]

FIGURE 4 about here

Figure 4-1 below illustrates formula (3) when we assume that global expected inflation match U.S. expectations. The mean percentage absolute error then drops to 9.8%. We observe the following: first it is crucial to note that the actual price of gold departs from our model, mostly for reasons that have to do with extraordinary world events impacting gold prices outside of the key factors we formulated in expression (3). The Iran hostage crisis began on November 4, 1979 and was the most covered event by all media. This was shortly followed by the Hunt silver crisis. Both events tend to increase the perceived value of gold. During the year 1980, gold peaked at $666/oz., only to converge to the fair value next quarter. The average price during 1980 was $584/oz. and was 6% above the values predicted by RYT. In 1983 gold rose 22% above the RYT based value, perhaps motivated by fears during the Mexican Debt Crisis; then converged to RYT valued gold and stayed close to the actual price from 1983:QIII – 1986:QII. Then through 1988:QI gold was about 11% above RYT value with only minor changes in the required yield itself.

FIGURE 4.1 about here

From then until 1999:QII gold and RYT value remained in extremely close in accord with the foreign exchange rates remaining unchanged from point to point while the required yield declined; clearly driving gold valuation. From 1999:QIII – 2001:QIII, gold was below RYT value by a 5% gap. Since the required yield remained stable, while the exchange rate rose against the dollars, this accounts for the low gold vs. RYT value relationship. The foreign exchange rate has since declined and the gold/RYT value relationship is again aligned while the required yield has remained stable, leading to an average gap of about 13% until 2002:QIII. In all cases, gold has converged to the RYT predicted value.

Domestic Real Gold Price, Required Yield and a New Exchange Rate Parity Rule

Due to local currencies being legal tenders, investors seek to hedge against adverse market movements in the value of *domestic* assets using *domestic currency* to buy gold. In this section, we show that under a set of reasonable circumstances, the value of gold in terms of domestic currency will only be affected by domestic-currency denominated asset values. Specifically, we show that the domestic price of gold will solely be determined by the domestic required yield when Purchasing Power Parity (PPP) is satisfied in conjunction with a new parity condition. In fact, we introduce here a new exchange rate determination rule that is based on the postulate of parity of required yields across countries.

Our starting point is to define the global required yield in relation to each bloc's required yield. We define this relationship as follows:

$$RY_{Wt+1} = \theta_t \frac{(1+\pi_{wt+1})}{(1+\pi_{t+1})} RY_{t+1} + (1-\theta_t) \frac{(1+\pi_{wt+1})}{(1+\pi_{Xt+1})} RY_{Xt+1} \qquad (4)$$

The global required yield is a weighted average of each bloc's required yield, where the weights are modified nominal GDP weights in the sense that they depend on *expected* price deflators rather than current deflators.[10]

Since the concept of required yield is tied to the concept of forward earnings yield, it is important to note that we are also able to put forth a new exchange rate parity condition that depends on comparing stock market P/E ratios across countries. Let us denote by s_t the domestic spot exchange rate (1 unit foreign basket equals s_t domestic), s^e_{t+1} the expected following period spot exchange rate, and $\widetilde{s}^e_{t+1}$ the real expected spot rate. Assuming that there is no risk premium associated with investing in foreign assets, we have:

$$RY_{Xt+1} = \frac{s_t}{s^e_{t+1}} \times RY_{t+1} \tag{5}$$

Expression (5) is a new parity rule based on required yields. This rule is derived by considering that stock indexes are homogenous commodities and consequently that index values and forward earnings must equate across nations, after currency exchange. While Required Yield parity and PPP (as it applies to general price indexes) are directionally consistent, the magnitude of exchange rate fluctuations implied by this new rule is far greater than that of PPP especially over the short-term. These two rules will have similar effects only if inflation is stable and low in both the domestic country and rest of the world.

Using expressions (A1), (3), (4) and (5), after several manipulations we get:

$$\widetilde{P}_t = \frac{C \times RY_{t+1}}{(1+\pi_{t+1})} \times \frac{\theta_t + (1-\theta_t) \times \widetilde{s}_t \big/ \widetilde{s}^e_{t+1}}{\theta_t + (1-\theta_t) \times 1 \big/ \widetilde{s}_t} \tag{6}$$

Relationship (6) gives a comprehensive account of the key factors that affect domestic real gold prices: domestic GDP/capita growth, domestic inflation, exchange rates and relative GDP

weight. Thus, the domestic price is now a direct increasing function of the domestic required yield and domestic inflation. The real domestic price rises when the domestic currency depreciates, or is expected to appreciate in the future.[11] It increases as well when the home country's GDP weight increases, as long as both its weight is less than 50% and the domestic currency is expected to appreciate strongly.

The second ratio on the RHS of expression (6) may in general be close to one. This is true, for example, when the home country GDP weight is large. This is also true when PPP is close to being satisfied. In the case where PPP is fully satisfied at any point in time, then the second ratio on the RHS does equal one, and expression (6) simply becomes:

$$\widetilde{P}_t = \frac{C \times RY_{t+1}}{(1+\pi_{t+1})} \tag{7}$$

This last equation (7) embodies the fact that when financial markets are well integrated, then the real domestic price of gold responds solely to domestic required yield and related asset value movements. Interestingly, in that case the real price of gold is independent of the weight of the domestic economy in the global economy. It is important to keep in mind that this property is true when it is assumed that exchange rates fully integrate gold and stock market valuation differentials. Although not as realistic, the other case when this result is true as well is a diametrically opposite situation where the domestic economy would be isolated or a one-world economy. In that case, the real domestic price of gold and the domestic required yield are independent from the foreign price of gold and required yield, and thus the result in (7) follows directly from equations (2), (4) and (A1).

The Local Gold Return

In this section, we focus on modeling gold's relative price change, or return. First, note that the gold price in equation (3) depends on the value for the constant *C*. A potential hurdle is that the value of *C* itself is not determined by an outside relationship. In fact, as previously mentioned, we use equation (3) and the actual gold price history to determine its empirical value, which can lead to some minor uncertainty in our estimate of the absolute price of gold. The advantage of determining gold's return over time is that we avoid the problem of estimating this constant.

Let us denote by π^{u}_{t+2} the unconditional expected inflation rate for the domestic country, based on the information set available at time t, and θ^{u}_{t+1} is the GDP weight based on that same inflation expectation.[12] A relationship about the behavior of the expected real gold return in the domestic country is obtained by taking the ratio of two consecutive periods using equation (3), the following way:

$$\frac{E(\widetilde{P}_{t+1})}{\widetilde{P}_t} = \frac{RY^{u}_{wt+2}}{RY_{wt+1}} \times \frac{(1+\pi_{wt+1})}{(1+\pi^{u}_{wt+2})} \times \frac{\theta_t + (1-\theta_t) \times 1/\widetilde{s}_t}{\theta^{u}_{t+1} + (1-\theta^{u}_{t+1}) \times 1/\widetilde{s}^{e}_{t+1}} \qquad (8)$$

Figure 5 below illustrates this result using U.S. data. Figure 5 summarizes the quarterly relationship between the change in the *real* return on gold as predicted by RYT from equation (8) and actual gold real returns. We use the spot and ex-post future spot rates for the $USD against the basket of foreign currencies.[13] Once the required yield and foreign exchange effects are taken into account, the RYT valuation model exhibits an 8% mean percentage tracking error from actual real gold returns over the 1979-2002 period.[14]

FIGURE 5 about here

The cost Approach to Pricing Gold

We now turn to explain the absolute price of gold bullion via the average production cost. We assume that the gold mining industry as a whole has achieved its long-run maximum efficient capacity, and since the industry has large barriers to entry, the *real* profit margin is not shrinking to zero, but rather converges to its long-run achievable profit given by the long-run global GDP/capita growth rate of 1.5%. The reason why the long-run GDP/capita growth rate determines the industry's profit margin is that the long run average industry margin must be equal to the long-term average corporate profit margin[15] otherwise the quantity supplied would change to bring this relationship into line. If the margin were greater, mining activity would pick up, and vice versa. Let ATC_{wt} stands for the global average total cost per ounce, and τ_{wt} is the corporate tax rate in the global economy. In Appendix C, we show that the following relationship must hold for the real world price of gold:

$$P_{wt} = \frac{ATC_{wt}}{1 - \frac{RY_{wt+1}}{(1-\tau_{wt+1})}} \tag{9}$$

The pricing formula (9) states that the price is related to the average cost and required yield via a cost-plus-margin relationship. Formula (9) is compared to actual nominal gold price for the U.S. Figure 6 below illustrates the comparison. The cost-based estimate of the gold price is plotted against the actual gold price in Figure 6. Our mean percentage tracking absolute error is 16%. Clearly, the cost-based model does not account for changes in required yield from the investor's perspective and does not generate the same volatility as our the first model did based on formulas (2) and (3).

FIGURE 6 about here

Precautionary Motives and Gibson's Paradox

Barsky and Summers [1988] provide a direct competing theory to the model developed in this article. Contrary to their result, our model does not resolve Gibson's paradox. Gibson's paradox is the puzzling observation during the Gold Standard era that interest rates were co-varying with the general level of prices but not the inflation rate as standard economic theory would predict (Fisher [1930]).

Barsky and Summers argue that the paradox took place because in effect, the Gold Standard links the real gold price inversely to the general price index. They theoretically prove that the gold price is itself inversely related to the real return. Thus the price index and real return are positively related. The reason our framework is unable to tackle the Gibson paradox phenomenon is that during the Gold Standard, outside of periods of severe inflation, the fixed convertibility of dollars into gold made it unnecessary for investors to hold gold as a precautionary motive. On the other hand, it must be remembered that high inflation rates at the beginning of the 1970s were responsible for the large depletion of the stock of US government gold reserves and the eventual collapse of the Gold Standard.

In Barsky and Summers the "consumption" motive is sufficient to explain the paradox. However, even though their model seems to explain the movement of gold prices over the period 1979-1984 just as we do, this does not refute our approach and conclusions. Recall that our theory states that the real price of gold moves in conjunction with the expected inflation rate and the real return as measured by long-term productivity. Over the period considered, nominal rates on bonds were relatively flat, thus fluctuations in expected inflation were inversely affecting real returns. Consequently, Barsky and Summers do document an inverse relationship between real

return and real price of gold, which is the same as a *positive* relationship between the real price of gold and inflation in our case.

RYT states that rational investors seek a real positive constant return. Even though this did not take place on the bond market at the time mostly due to lags in expected inflation, RYT makes provision for this requirement by stating that investors would then turn to the equity market to fulfill that condition. On the other hand, one of Barsky and Summers' key conclusions is that the real price of gold does *not* change when the real interest rate remains constant. In our case, we predict that even small movements in inflation rates will affect the real price of gold even when the real rate is unaffected.

Ultimately, Barsky and Summers' [1988] model breaks down after 1995, as the record shows that the real return on the 30-year treasury was relatively constant over 1995-2000 and fell thereafter while the price of gold steadily dropped from 1995-2001, which we explain by a slowly declining inflation rate.

IMPLICATIONS FOR THE GOLD MARKET AND ITS VALUE

In the long run, the gold mining industry's real profit margin is constant and equals the real per capita productivity. The price of gold, on average, must be the average production cost plus a constant mark-up. Furthermore, in order for the *real* value of gold to be maintained on a *per investor* basis, the stock of gold has to grow at a rate that can be no greater than population growth in the long-term. If the supply of gold grew at a lesser rate than population growth for reasons other than depletion of the exhaustible ore, gold price would grow faster than inflation and the quantity demanded for gold would drop. Eventually the supply of mined gold will dwindle, which will drive prices up unless world population experiences zero growth in the

foreseeable future. In that circumstance, far off in the future, a substitute medium of storing value may be discovered and used.

Another prediction of our theory of gold pricing is that the decrease in proportion of gold total value as compared to world wealth is explained by RYT in the fact that relative to financial assets, the long-term nominal value of gold must increase at the inflation rate, whereas the value of other assets rise with inflation plus real productivity. Thus, the proportion of investable wealth declines at an annual rate equal to real per share earnings growth or GDP/capita growth.

CONCLUSIONS

We have extended the Required Yield Theory (RYT) developed by Faugere-Van Erlach [2003] to value gold and to determine its return. RYT states that since global assets are priced to yield a global constant real return, and since gold is a global store of value, its price will vary directly with the global required yield and the global inflation rate. In the course of developing this asset valuation model we introduced a new exchange rule parity based on required yields comparisons across countries.

Specific predictions include: 1) the real price of gold varies proportionately to the change in long-term economic productivity as measured by GDP/capita growth. 2) Real gold prices vary proportionately to changes in the foreign exchange rate (direct quotation) when the domestic required yield is constant. 3) When the foreign exchange rate is constant and there are no major geopolitical or natural crises, real domestic gold price increases with domestic inflation. 4) When our new exchange rate parity rule holds, then effectively the real domestic price of gold is mostly determined by the domestic required yield. This entails that foreign exchange effects will impact the domestic real gold price to the extent that equalization of required yields is not taking place worldwide and/or that PPP is violated as well. 5) In the long-term, the gold per-capita supply

remains constant. 6) The average long-term absolute price of gold is marked-up cost where the profit margin is given by the global average long-term per-capita rate of GDP growth.

While we suspect that central bank activities, hedging activities, supply/demand fluctuations, global real GDP growth changes or changes in global income and capital gains tax rates, affect gold prices as well, the valuation approach developed here performs very well absent these factors, with over 92% accuracy in predicting US Gold returns over a 23-year period. We leave an investigation of the role of these other factors for future research.

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Figure 1: Gold and Yields Indexes. U.S. Comparisons 1979-2002. Quarterly Observations.

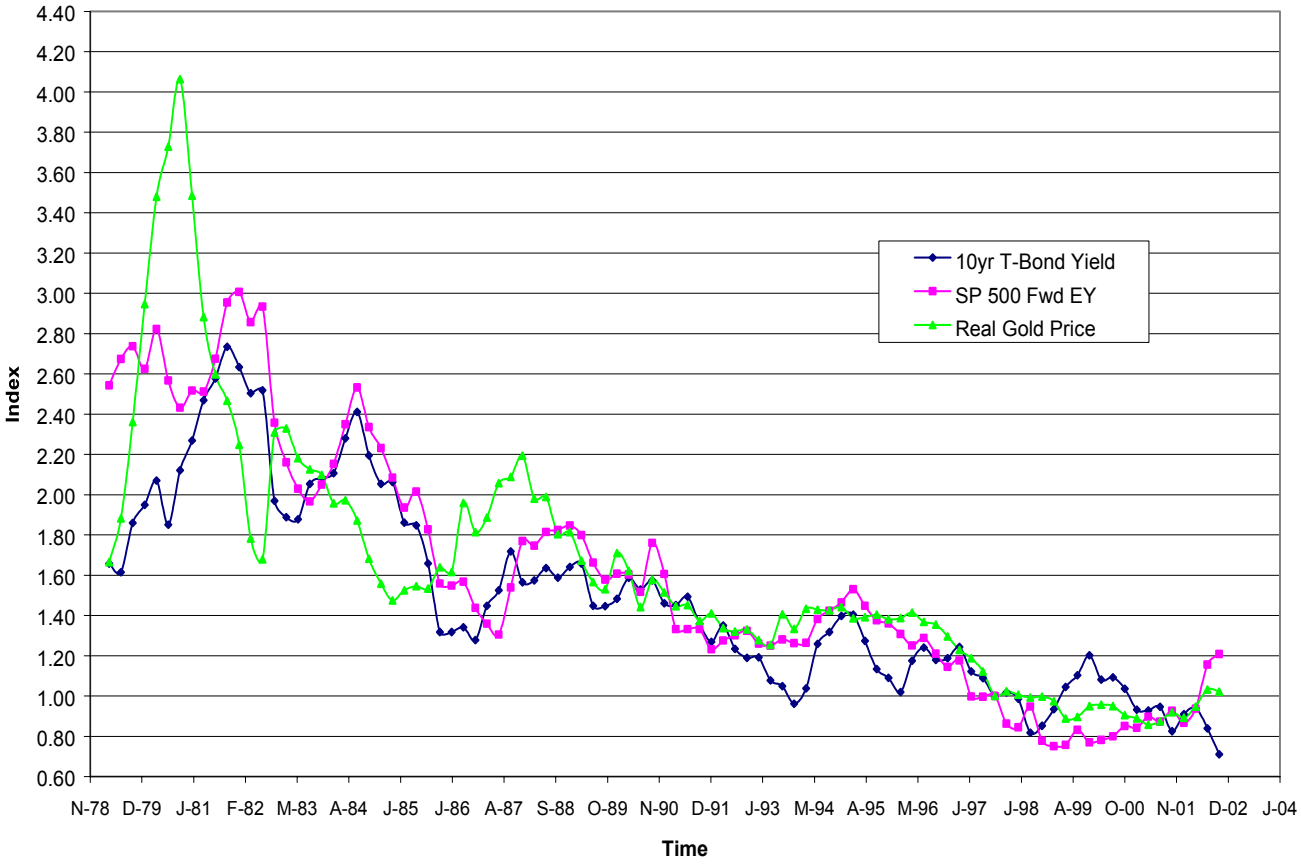


Figure 2: Annual Global Stock Market Earnings Yield vs. Global Required Yield. 1990-2001

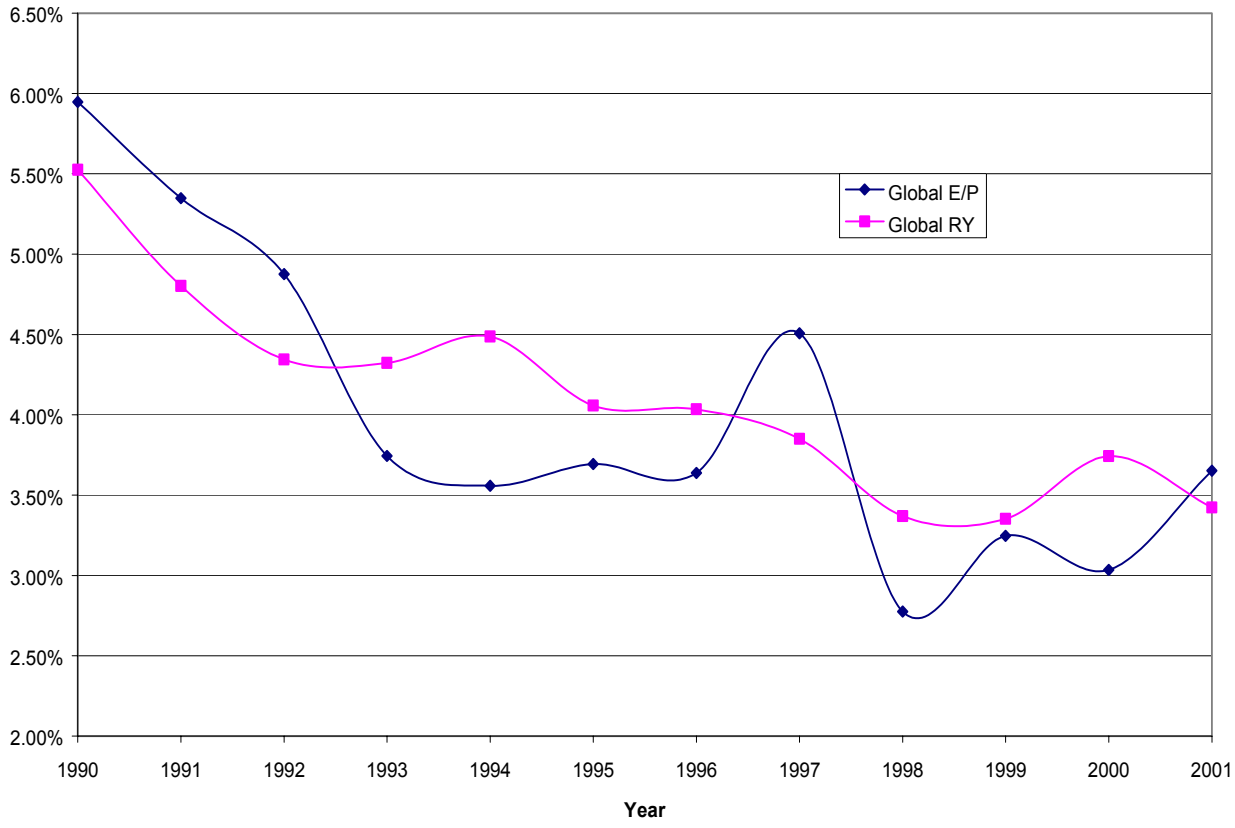


Figure 3: Actual vs. RYT-Based Global Real Price of Gold. 1971-2003. Quarterly Observations.

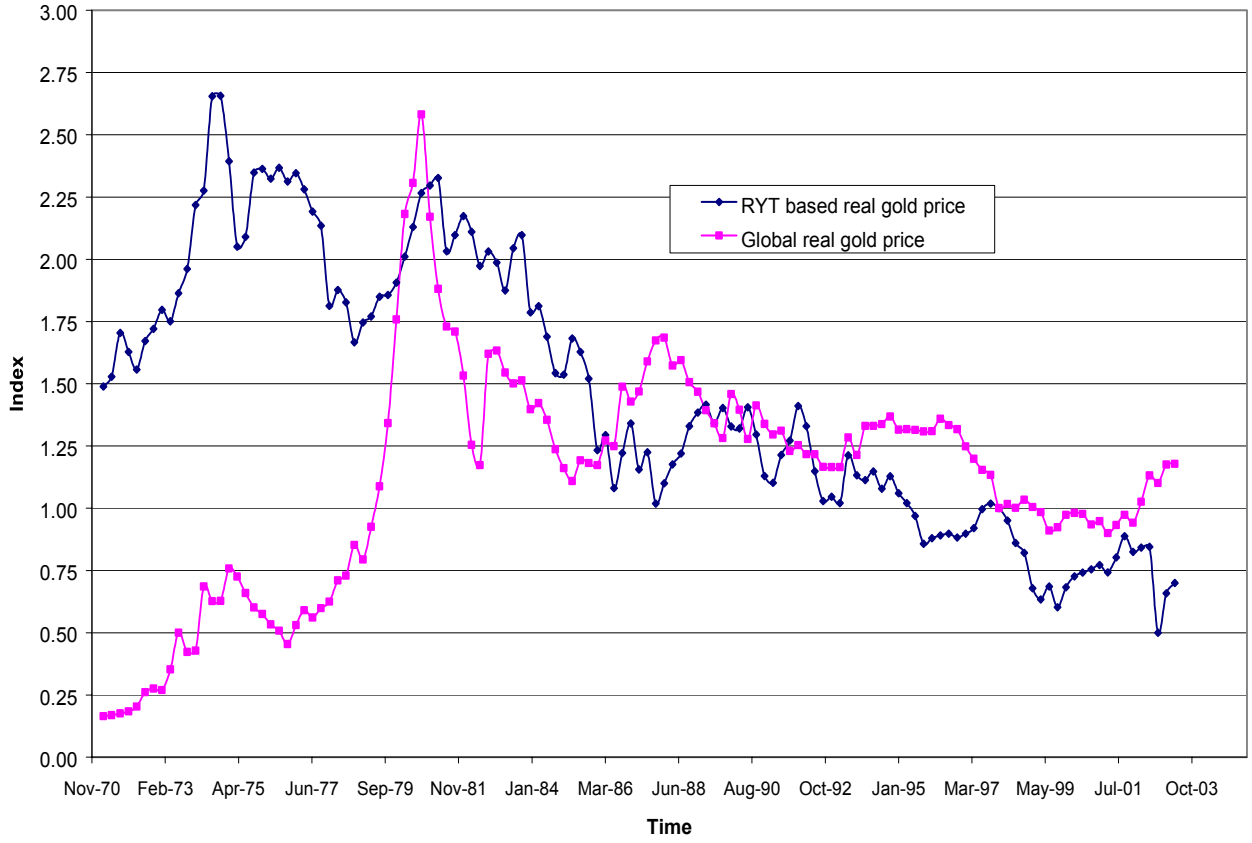


Figure 4: Nominal USD$ Gold Price vs. RYT Pricing Formula 1979-2002. Quarterly Observations

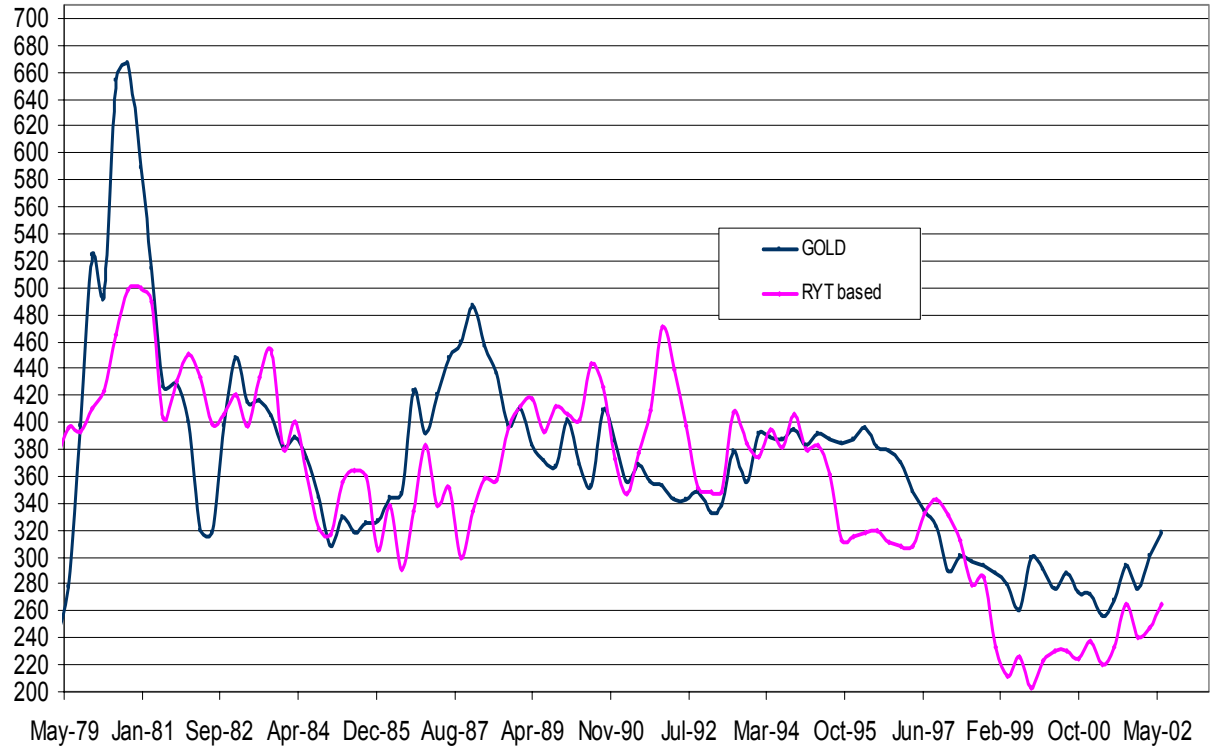


Figure 4-1: Nominal USD$ Gold Price vs. RYT Pricing Formula 1979-2002.
Quarterly Observations. U.S. inflation expectations

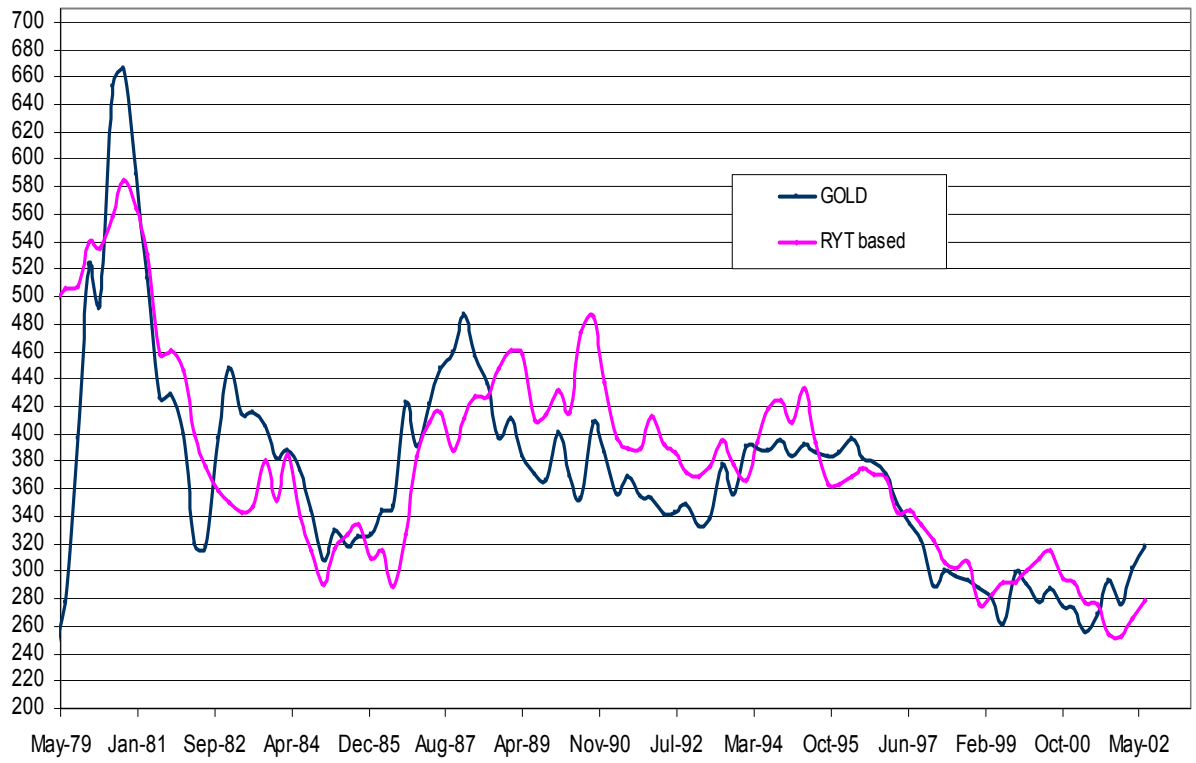


Figure 5: Gold Return: Actual vs. RYT based 1979-2002. Quarterly Observations

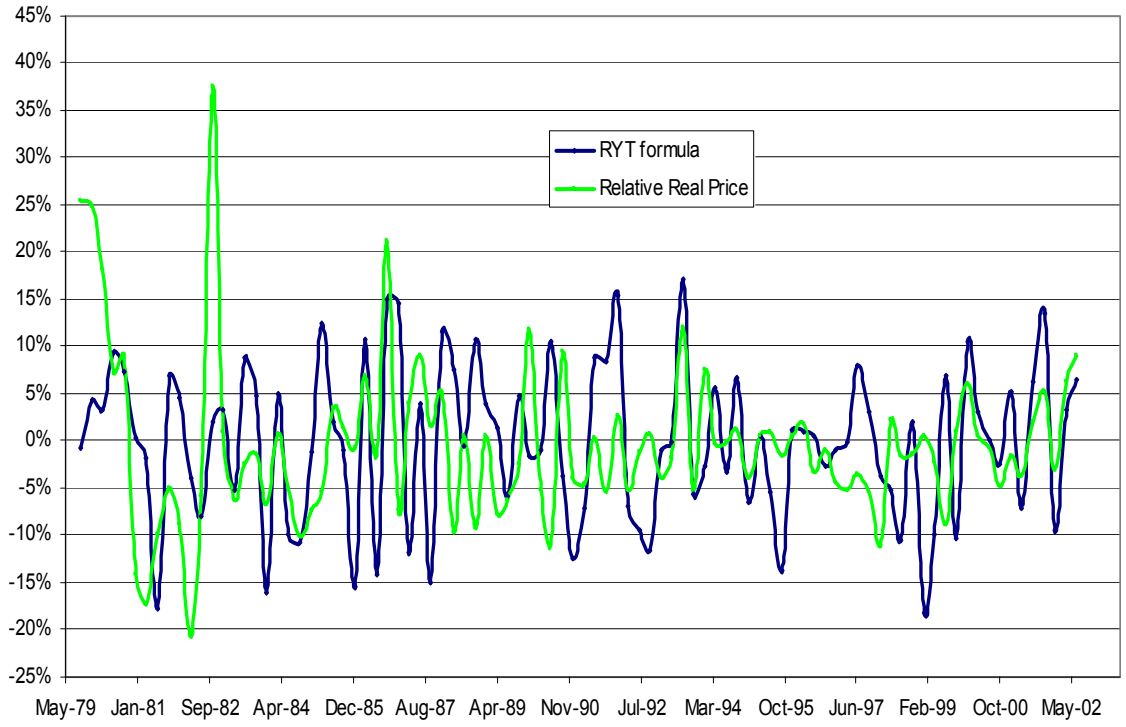
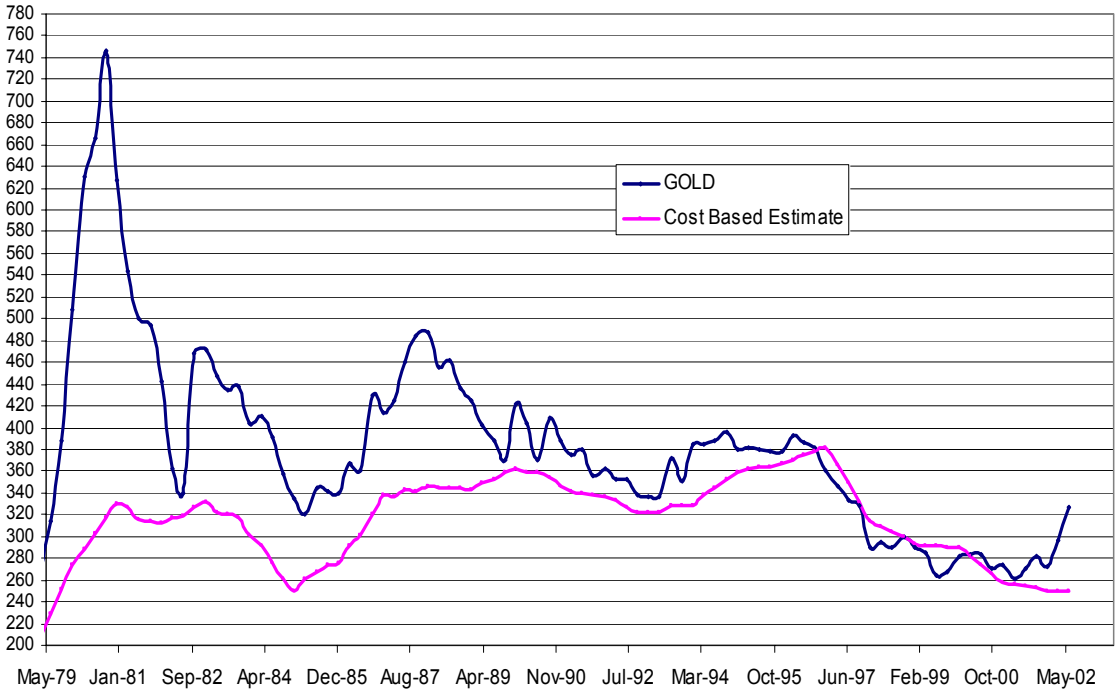


Figure 6: Nominal USD$ Global Gold Price vs. Cost-Based RYT Formula 1979-2002. Quarterly Observations.



APPENDIX A

Let $\widetilde{P}_t$ and $\widetilde{P}_{Xt}$ respectively denote the domestic and rest of the world real gold prices at time t; i.e. deflated by each respective region's GDP deflator. Let π_{t+1} and π_{Xt+1} respectively denote the domestic and rest of the world expected inflation rates. The global real price of gold is defined as follows:

$$\widetilde{P}_{Wt} = \theta_t \widetilde{P}_t + (1-\theta_t)\widetilde{P}_{Xt} \tag{A1}$$

The global real price of gold is a weighted average of home and rest of the world real prices weighted by each bloc's nominal GDP relative weight in the global economy. Given the Law of One Price given in Condition 3) we must have:

$$\widetilde{P}_{Xt} = \frac{\widetilde{P}_t}{\widetilde{s}_t} \tag{A2}$$

Obviously when Purchasing Power Parity (PPP) holds $\widetilde{s}_t$=1, and combining (A1) and (A2) leads to:

$$\widetilde{P}_{Xt} = \widetilde{P}_t = \widetilde{P}_{wt} \tag{A3}$$

On the other hand, if we choose *not* to assume PPP, and after combining equations (2), (A1) and (A2) together, the real domestic price of gold becomes a function of the global required yield as such:

$$\widetilde{P}_t = \frac{1}{(1+\pi_{wt+1})} \times \frac{C \times RY_{wt+1}}{\theta_t + (1-\theta_t) \times 1/\widetilde{s}_t} \tag{A4}$$

APPENDIX B

An interesting interpretation for the meaning of the constant *C* in equation (2) arises if we consider the following argument. Imagine a perpetuity that pays the holder one ounce of gold every year forever. The expected value of an ounce of gold is given by $E(P_{wt+1})$. Let the general price deflator be denoted as Def_{wt}. Let tilde variables denote real variables. For example, $\widetilde{P}_{wt}$ represents the price P_{wt} divided by the global GDP deflator at time t.

It is straightforward to see that the inverse of the required yield is a required forward P/E ratio, and thus that forward (gold) annuity payments $E(P_{wt+1})$ times a forward P/E equals the value of the above-described perpetuity. Using equation (2) to express the real value $\widetilde{V}_{wt}$ of this perpetuity we get:

$$\widetilde{V}_{wt} = \frac{E(P_{wt+1})}{Def_{wt}} \times \left(\frac{P}{e}\right)_{wt+1} = E(\widetilde{P}_{wt+1}) \times \frac{(1+\pi_{wt+1})}{RY_{wt+1}} = C \times \frac{E(\widetilde{P}_{wt+1})}{\widetilde{P}_{wt}} \qquad \text{(B1)}$$

Equation (B1) states that the real value of this perpetuity increases with the appreciation of the real price of gold. Whenever inflation and the real long-term productivity are stable, that value turns out to be our constant *C*. In other words, the real price of gold is determined in relation to the required P/E in order to conserve the value of this perpetuity. Equation (B1) also allows us to determine the constant long-run real price of gold by multiplying the constant *C* equal \$8929 by the real required yield of 1.5% plus LT global inflation at 4.2% divided by (1 +4.2%) to get us a 5.47% yield. Based on our sample we get an estimate of the long-term real global gold price of \$488 per ounce.

APPENDIX C

Consider global mining profits. By definition, actual ex-post earnings per ounce of gold are given by:

$$e_{wt+1} = \left[P_{wt} - ATC_{wt}\right] \times (1 - \tau_{wt+1}) \tag{C1}$$

Where ATC_{wt} stands for the global average total cost per ounce, and τ_{wt} is the corporate tax rate in the global economy. Relationship (C1) is just the standard definition of corporate net income applied to the mining industry. Dividing both sides by the price and rearranging terms, we obtain:

$$\frac{\Pi_{wt+1}}{(1-\tau_{wt+1})} = \frac{e_{wt+1}}{(1-\tau_{wt+1})P_{wt}} = \left[1 - \frac{ATC_{wt}}{P_{wt}}\right] \tag{C2}$$

The variable Π_{wt+1} represents the ex-post profit margin. Rearranging terms, we get the global price of gold as:

$$P_{wt} = \frac{ATC_{wt}}{1 - \dfrac{\Pi_{wt+1}}{(1-\tau_{wt+1})}} \tag{C3}$$

We postulate that RYT holds for the components of the profit margin, so that the profit margin must equal the required yield:

$$\Pi_{wt+1} = \frac{e_{wt+1}}{P_{wt}} = RY_{wt+1} \tag{C4}$$

Equation (C4) incorporates RYT in the sense that the "earnings" yield (earnings over lagged price) is assumed equal to the required yield (ex-post). Thus, finally combining (C3) and (C4) gives us:

$$P_{wt} = \frac{ATC_{wt}}{1 - \dfrac{RY_{wt+1}}{(1-\tau_{wt+1})}} \tag{C5}$$

APPENDIX D

DATA DESCRIPTION

Unless otherwise specified, our data covers a 33-year period on a quarterly basis from 1979 QIII until QII 2002. Nominal and real (CPI-adjusted) spot direct exchange rates are obtained from the US department of Agriculture and compiled from the International Financial Statistics of the International Monetary Fund and Financial Statistics of the Federal Reserve Board. These are used to determine the global real and nominal dollars gold prices from 1971-2003.

The representative group of non-U.S. countries is chosen to be: Euro zone, UK, Australia and Japan.

U.S. Inflation is USGDP deflator (quarterly/annualized) and USGDP deflator one-year ahead forecasts from the BEA available only since 1979. Prior to 1979, actual GDP inflation rates are used instead of GDP deflator forecasts. Global expected inflation estimates are constructed by backing out country-specific inflation rates from the ratio of nominal vs. real exchange rate period to period change. Then, we assume that each country's expected inflation rate occurs in the same proportion as the ratio of actual inflation rates to U.S. inflation (CPI-based).

The GDP weighted average expected inflation rate for the basket of foreign currencies is derived by multiplying each period-wise proportionality coefficient by the corresponding period U.S. expected inflation. Finally, the global inflation rate is obtained as a GDP weighted average of U.S. vs. non-U.S. expected inflation rates.

Gold prices are London pm Fix available monthly since 1971, quoted in each national currency: Euro, $USD, Yen, Australian $, and Pound Sterling.

For each national currency, the GDP weights are 1980 real GDP weights applied from QIII 1979 until QIV 1989. The 1997 real GDP weights are applied from QI 1990 until QIII 2002. The GDP weights data is from the CIA's Handbook of International Economic Statistics. *Real* GDP

weights are used instead of nominal GDP weights in the implementation of our pricing formulas, since they do not widely differ from nominal GDP weights for these two years.

The average total mining $USD cost is from Brook Hunt, which compiles annual dollars-converted cost information for a set of mining companies constituting between 75% to 91% of the total western world production. The weights used are relative production weights, which for the sake of our analysis, we assume proxy GDP weights. Quarterly average total costs are interpolated between years. U.S. historical top marginal rates from the IRS website, serve as proxies for the global corporate tax rates.

ENDNOTES

[*] The authors thank Brook Hunt and Mark Fellows for their assistance with providing aggregate costs data for the gold mining industry.

[1] These indexes are constructed so that their value is normalized to 1 at the beginning of the first quarter of 1998.

[2] In this paper, we will focus solely on stocks as the alternate asset class. Faugere and Van Erlach [2003] show that T-Bonds yields are essentially commensurate with the stock market earnings yield, and thus subject to a common yield requirement.

[3] To show this, take the forward P/E ratio and divide it by the ratio of current to expected GDP deflator.

[4] The condition is slightly different in Faugere-Van Erlach [2003], since it stipulates that the forward earnings yield equals the greater between the required yield as shown and the after-tax Treasury bond rate. Here we abstract from the treatment of T-bonds as another asset class.

[5] GDP weights are used in our model, since a global inflation rate must be given by an average of countries specific inflation rates where the dominant economies in terms of GDP carry more weight.

[6] The P/E ratios are obtained from the World Federation of Exchanges. Stock markets represented are from thirteen EU exchanges, London, Australian, Tokyo and the SP500 index. Data was only available since 1990.

[7] We assume that gold's real price is related to the after-individual-income-and-capital-gains-taxes forward market earnings yield. In that context, formula (2) is consistent with the result of Faugere-Van Erlach [2003].

[8] It is computed as the mean of the ratio of the absolute difference of actual minus model-based price over the actual price. Since inflation expectations are not historically available for most industrialized countries, we derive estimates using the method described in Appendix D. If we assume that global inflation expectations are identical to US expectations, the mean percentage error drops to 13.5%. We should note that these results are dependent of the choice of the base period for the normalization. We argue though that a small tracking error will result when the normalization is done for a base period characterized by 'normal' market conditions, i.e. in the absence of bubbles or market crashes. Note that all data description is contained in Appendix D.

[9] The t-statistics on the slope has a value of 2.5, signifying that the slope is not very flat. On the other hand, when using U.S. inflation expectations we get an R-square of 0.006% and a t-statistics for the slope equal to 0.07.

[10] This definition is consistent with the required yield being an *expected* nominal interest rate.
[11] Note though that the only possible way expected future spot rates may increase is if inflation expectations rise. In that case, a country may pass along the effects of its own inflation by inducing a rise in the foreign price of gold, as long as these expectations are not reflected to a full measure in the current spot rate.
[12] This is because the expectation is determined two periods ahead unconditional on next period's information set about the new state of the economy.
[13] See Appendix D for a description of the currency basket.
[14] Again, if we are assuming that global inflation expectations are well approximated by U.S. expectations this leads to a smaller mean percentage absolute error of 6%.
[15] Furthermore, the long-term average global profit margin must be the same as the long-term productivity growth per capita otherwise returns in excess of GDP could be earned on a compound basis, which is impossible.