Would the CAPM Hold in a Risk-Indifferent World?

By Silviu Iulian Alb

ABSTRACT

The Relative Value Theory predicts equilibrium prices in a world in which time value of money is unique, and investors are risk-indifferent and only care about maximizing cumulative returns. This paper shows that RVT's equilibrium prices determine intrinsic expected returns that satisfy the CAPM equation. The intrinsic return of the risk-free asset is equal to the harmonic mean of the market's intrinsic returns (intrinsic returns are returns from equilibrium price to underlying intrinsic values). Asset specific βs can be explained by simply assuming scenario probabilities fluctuate in time. Market price return βs are approximately equal to intrinsic return βs. Market price expected returns do not satisfy the CAPM equation but will appear linear in the market premium, with the risk-free rate as intercept. The above results significantly strengthen RVT's ability to explain market prices' behavior. Recasting most finance theory results into an RVT framework appears possible and beneficial.

Latane [1967] shows that maximizing the geometric expected return is the investing strategy that will certainly outperform any other strategy in terms of long-term cumulative returns. Alb [2001] suggests that geometric mean maximization should therefore replace arithmetic mean maximization as rationality criterion[1]. Assuming rational investors are only interested in long-term cumulative returns is sufficient to explain most finance phenomena, without having to assume risk aversion or rely on such questionable concepts as utility functions or required rates of return. Alb [2001] shows that in a world in which time value of money is unique, and investors are risk-indifferent and only care about maximizing cumulative returns, the market invariably evolves to an equilibrium characterized by all investors holding the market, and equilibrium prices (asset valuations) being:

$$P_A = \left(\sum_{i=1}^{n} p_i \frac{A_i}{M_i} \right) \cdot GM \qquad (1)$$

where:

P_A is the market value of asset A at equilibrium (equilibrium price)

n is the number of possible scenarios (ways in which the future can unfold)

p_i is the probability of scenario i

A_i is the intrinsic value of asset A under scenario i (present value of asset A’s stream of future cash flows discounted at the unique time value of money rate T)

M_i is the market’s intrinsic value under scenario i (sum of the present values of all assets)

$GM = \prod_{i=1}^{n} M_i^{\,p_i}$ is the geometric mean of the market (obviously $GM = P_A+P_B+P_C+\ldots$)

This paper investigates the implications of the above equilibrium prices formula. The paper proceeds as follows: Section I shows that intrinsic expected returns predicted by the RVT satisfy the CAPM equation, and that the intrinsic return of the risk-free asset equals the harmonic mean of the intrinsic returns of the market. Section II shows that asset specific βs can be explained by simply assuming scenario probabilities fluctuate in time, and that market price return βs are approximately equal to the βs calculated using intrinsic returns. Section III investigates whether, under the RVT, market price returns satisfy the CAPM equation. Section IV contains additional remarks about RVT's implications for finance theory. Throughout the paper returns refer to gross returns.

I. Properties of RVT equilibrium prices

We use C as a notation for the present value of the risk-free asset (obviously C is the same in all scenarios), and P_C as a notation for the market value of the risk-free asset. Under the RVT, P_C will be different than C. Applying equation 1 to the risk-free asset will result in:

$$R_f = \frac{1}{\sum_{i=1}^{n} \frac{p_i}{R_{M_i}}} \tag{2}$$

where:

$R_f = \frac{C}{P_C}$ is the risk-free asset intrinsic return (from market value P_c to intrinsic value C)

$R_{M_i} = \frac{M_i}{GM}$ is the market intrinsic return under scenario i (from market value GM to intrinsic value M_i)

Equation 2 basically says that the return of the risk-free asset is equal to the harmonic mean of the market's returns (as previously stated all returns are gross returns).

These returns are not returns in the classical sense because they do not describe price changes over a period of time. Consider an envelope containing either \$140 with probability 0.5 or \$70 with probability 0.5 that has a market price of \$100. Buying the envelope at \$100 to open it and cash its content would result in either a return of 1.4 or a return of 0.7. These are intrinsic returns and do not describe a change in the market price of the envelope. Intrinsic returns can be viewed as describing the price change determined by a given scenario i becoming a certainty (because the price of asset A would go from P_A to A_i).

Another interesting result, considering the striking similarity to the CAPM, is the following (see Appendix A for a proof):

$$\frac{E(R_A)-R_f}{E(R_M)-R_f}=\frac{Cov(R_A,\frac{1}{R_M})}{Cov(R_M,\frac{1}{R_M})} \tag{3}$$

where

R_A, R_M, R_f are all intrinsic returns

$E(R_A)$ and $E(R_M)$ are probability-weighted arithmetic expectations ($\sum pR$).

Using α as a notation for the ratio of covariances on the right-hand side, equation 3 becomes:

$$E(R_A)=R_f+\alpha\left(E(R_M)-R_f\right) \tag{4}$$

where

$$\alpha = \frac{Cov(R_A, \frac{1}{R_M})}{Cov(R_M, \frac{1}{R_M})}$$

The α coefficient defined above is generally a good approximation of $\frac{Cov(R_A, R_M)}{Cov(R_M, R_M)}$ (the intrinsic return β). Moreover, if R_M has a continuous normal distribution, α can be shown[2] to equal the intrinsic return β. In section II (equation 6) we'll see that intrinsic return βs are approximately equal to market price return βs.

The bottom line is that under RVT's assumptions (unique time value of money rate, and risk-indifferent, geometric mean maximizing investors with homogeneous expectations) equilibrium prices determine intrinsic expected returns that satisfy the CAPM equation.

Again, these returns are not returns in the classical sense. They do not describe market price movements over a period of time but rather returns from market price to underlying intrinsic values. Consistently successful investor Warren Buffett appears to focus on intrinsic returns, while paying little attention to returns in the classical sense.

II. Explaining β under the RVT framework

The RVT predicts that, as long as expectations about the future do not change, asset prices will all increase over time at the same rate T (the unique time value of money rate[3]). To explain asset specific βs under the RVT framework it is sufficient to assume that the set of scenario probabilities fluctuates in time around a central set of values $\{p_i\}$. Such assumption does not take much away from RVT's plausibility, since it is only natural for expectations to fluctuate in time due to a variety of factors.

A change in probabilities will determine a change in asset prices, thus generating market price returns (these are returns in the classical sense). It can be shown that these market price returns will exhibit asset specific β coefficients. Moreover, these β coefficients will closely approximate the β coefficients of the assets' intrinsic returns.

For simplicity and in light of equation 4, I will actually calculate α, which as previously stated closely approximates β. If the fluctuation around {pi} is random, has a small amplitude, and a uniform probability distribution spanning the n-fold interval $\{p_i\text{-}dp , p_i\text{+}dp\}$, it can be shown that α is closely approximated by (see Appendix B for details):

$$\hat{\alpha} \approx 1 + \frac{1}{\sum_{i=1}^{n} p_i \frac{A_i}{M_i}} \cdot \frac{Cov'(\frac{A}{M}, \ln M)}{Cov'(\ln M, \ln M)} \tag{5}$$

where:

$\hat{\alpha} = \dfrac{Cov(\hat{R}_A, \dfrac{1}{\hat{R}_M})}{Cov(\hat{R}_M, \dfrac{1}{\hat{R}_M})}$ is the α coefficient (“hat” indicates α refers to market price returns)

$\hat{R}_A$ and $\hat{R}_M$ are market price returns generated by the fluctuation of probabilities

A and M refer to the intrinsic values of asset A and the market respectively

Equation 5 basically says that different assets will exhibit different βs when probabilities fluctuate in time. The covariances on the right-hand side of equation 5 are not weighted with probabilities[4], they are computed as if all scenarios have the same probability p equal to 1/n. To indicate this a prime sign ′ has been added to the notation.

The right-hand side of equation 5 appears to closely approximate $\dfrac{1}{\sum p \, {}^{A}\!/\!{}_{M}} \cdot \dfrac{Cov(A, M)}{Cov(M, M)}$ (which is equal to $\dfrac{Cov(R_A, R_M)}{Cov(R_M, R_M)}$). Since $\hat{\alpha}$ closely approximates β as previously discussed, we have:

$$\frac{Cov(\hat{R}_A, \hat{R}_M)}{Cov(\hat{R}_M, \hat{R}_M)} \approx \frac{Cov(R_A, R_M)}{Cov(R_M, R_M)} \qquad (6)$$

where:

$\hat{R}_A$ and $\hat{R}_M$ are market price returns (returns in the classical sense)

R_A and R_M are intrinsic returns

Equation 6 basically says that market price return βs are equal to intrinsic return βs. In other words, business operations risk determines market risk. Assuming probabilities fluctuate in time is therefore sufficient to explain the connection between market risk and business operations risk.

III. Does the CAPM equation hold under the RVT?

Although intrinsic returns satisfy equation 3, which closely resembles the CAPM equation, market price returns (generated by the fluctuation of probabilities) do not satisfy the CAPM equation.

The fluctuation of probabilities around {pi} will not change the fact that, over the long term, all market prices grow at the same rate T. However, asset prices will have different arithmetic expected returns. It can be shown (see Appendix C for details) that, for small probability fluctuations, the following approximation holds reasonably well in most cases:

$$E(\hat{R}_A) \approx E(\hat{R}_f) + \hat{\gamma}\left(E(\hat{R}_M) - E(\hat{R}_f)\right) \tag{7}$$

where:

$$\hat{\gamma} = \frac{Cov(\hat{R}_A, \frac{1}{\hat{R}_A})}{Cov(\hat{R}_M, \frac{1}{\hat{R}_M})}$$

$\hat{R}_A$ and $\hat{R}_M$ indicate market price returns for asset A and the market

$\hat{R}_f$ indicates market price returns for the risk-free asset

Using an approach similar to the one described in Appendix B (for $\hat{\alpha}$), it can be shown that, for small fluctuations, $\hat{\gamma}$ is closely approximated by:

$$\hat{\gamma} \approx 1 + \frac{2}{\sum_{i=1}^{n} p_i \frac{A_i}{M_i}} \cdot \frac{Cov'(\frac{A}{M}, \ln M)}{Cov'(\ln M, \ln M)} + \frac{1}{\sum_{i=1}^{n} p_i \frac{A_i}{M_i}} \cdot \frac{Cov'(\frac{A}{M}, \frac{A}{M})}{Cov'(\ln M, \ln M)} \qquad (8)$$

Equations 7 and 8 indicate that RVT market price returns generated by probabilities fluctuation do not satisfy the CAPM equation, primarily because the $\hat{\gamma}$ coefficient is significantly different than β. However, $E(\hat{R}_A)$ would appear to be a linear function of the market premium $(E(\hat{R}_M) - E(\hat{R}_f))$, with $E(\hat{R}_f)$ as the intercept.

Time value of money and the fluctuation of probabilities are only two of the factors that can affect returns. Intrinsic returns also affect market price returns over the long term since, as time passes, the "true" intrinsic value would become apparent and the market price would adjust accordingly. The fluctuation of T itself (which affects growth and non-growth stocks differently), the fluctuation of payoffs, and random new information are also relevant. All these factors need to be considered in order to assess CAPM's degree of compatibility with the RVT framework. Considering the above results, RVT's predictions about market price returns cannot be in sharp contrast with the CAPM (especially if we further assume risk aversion).

Anyway, an empirical test to reject the RVT cannot be conclusive unless investors are aware of the cumulative return maximization strategy, which doesn't seem to be the case at present.

IV. Additional comments about RVT's implications for finance theory

If we were to assume that investors use expectations about future prices instead of expectations about intrinsic values (plausible since investors nowadays appear to focus on price movements), than the RVT would pretty much explain the CAPM equation. This would be a fairly spectacular result considering the assumptions made (risk indifference, unique time value of money, cumulative return maximization). However, it would entail making the same mistake the CAPM does, which is to look at market prices' behavior in order to determine what these prices should be. The RVT's attractiveness comes mainly from its ability to determine market prices based solely on expectations about business operations. Moreover, predicting short-term price movements seems a futile exercise, in spite of its popularity. Market prices do not obey immutable laws of physics, they are determined by investors' behavior, which can change as they acquire new knowledge. Stretching RVT's logic in order to explain the CAPM equation, and losing RVT's main appeal in the process, seems to make little, if any, sense.

Finance theory is maybe too concerned with explaining market data, and not concerned enough with investigating optimal investing strategies. To find such strategies it should be useful to look at those employed by consistently successful investors. It would be interesting to estimate the probability of an investor matching Warren Buffett's record by simply being lucky.

The RVT represents a dramatic departure from mainstream finance theory because it adopts geometric mean maximization as a new rationality criterion. I believe recasting all major finance theory achievements in RVT's framework is possible, and potentially beneficial in furthering understanding of finance phenomena. For instance, it is easy to show that Modigliani & Miller proposition I holds under the RVT. Indeed, debt/equity financing decisions affect asset intrinsic values within any given scenario but not the market's intrinsic value or the scenario's probability. Consequently, the total capitalization of the market (given by $GM = \prod M^p$) does not depend on debt/equity financing decisions either. It can be similarly shown that mergers and spin-offs do not add value to the market (unless cost savings are involved). The RVT is obviously helpful in explaining why employed investors invest more in stocks than retired investors, since investors maximize the geometric mean of their entire wealth, which includes the present value of future income. Other results, in various areas of finance, can be similarly investigated.

Due to the very nature of the geometric mean, widespread adoption of cumulative return maximization would make markets deeper and consequently less volatile. Trading volumes would likely decrease.

Alb [February 2004] illustrates, using numerical examples, that diversification is not necessarily indicative of risk aversion. Investors that only care about cumulative returns would still diversify (even into less attractive investments), not to reduce portfolio variance, but to increase cumulative returns. The CAPM relies heavily on the assumption investors are risk-averse, although investors being risk-indifferent or risk-loving is, at least theoretically, possible. The RVT does not depend on (and consequently does not exclude) any risk preference assumption. The RVT could accommodate a situation in which some investors are risk-averse, some risk-loving, and some risk-indifferent, and still predict an equilibrium that would not differ sharply from the real world.

Conclusions

Latane [1967] shows that maximizing the geometric expected return will certainly outperform any other strategy in terms of cumulative return. Alb [2001] assumes time value of money is unique and rational investors only care about maximizing the cumulative return of their wealth, and shows that equilibrium can only be reached when all investors own the market and prices are given by equation 1.

This paper shows that intrinsic returns predicted by the RVT do satisfy the CAPM equation. The paper also shows that the intrinsic return of the risk-free asset equals the probability-weighted harmonic mean of market intrinsic returns. Intrinsic returns refer to arithmetic returns from equilibrium price to underlying intrinsic values.

Asset specific βs can be explained under the RVT by simply assuming that scenario probabilities fluctuate in time around a central value. Moreover these market price return βs will closely approximate the βs calculated using intrinsic returns. This result clarifies the connection between business operations risk and market price risk.

Market price expected returns (determined by probability fluctuations) do not satisfy the CAPM. However, for small fluctuations, expected returns would appear to be linearly related to the market premium, with the risk-free return as the intercept. The long-term effects of intrinsic returns on market price returns, and other types of fluctuations, need to be considered but RVT's predictions cannot be in sharp contrast with the CAPM.

Finance theory goes to great lengths in order to explain why the CAPM equation holds in the real world. It postulates risk aversion, assumes investors "require" a particular return out of a particular asset, and heavily relies on utility functions. The RVT shows that a lot can be explained with fewer assumptions. Why should there be more than one time value of money rate? What exactly prevents investors from being risk-indifferent (or risk-loving)? And do they really pay attention to utility functions? The RVT only relies on the assumption investors want to maximize cumulative returns. This assumption appears reasonable because cumulative returns are popular, and because they can be maximized (in the sense that, over the long term, they do tend to a limit).

The above mentioned results significantly strengthen RVT's ability to explain observable finance phenomena. Recasting finance theory's major results into an RVT framework appears possible and potentially beneficial.

Appendix A

The proof of equation 3 follows.

Considering the definition of expectations and equation 2, and factoring out R_f we can write:

$$\frac{E(R_A)-R_f}{E(R_M)-R_f}=\frac{\sum_i p_i R_{A_i}\cdot\sum_i \frac{p_i}{R_{M_i}}-1}{\sum_i p_i R_{M_i}\cdot\sum_i \frac{p_i}{R_{M_i}}-1} \tag{A1}$$

From equation 1 the following equality easily follows:

$$1=\sum p_i \frac{R_{A_i}}{R_{M_i}} \tag{A2}$$

Replacing 1 in the numerator and denominator on the right-hand side of equation A1 we get:

$$\frac{E(R_A)-R_f}{E(R_M)-R_f}=\frac{\sum_i p_i R_{A_i}\cdot\sum_i \frac{p_i}{R_{M_i}}-\sum p_i \frac{R_{A_i}}{R_{M_i}}}{\sum_i p_i R_{M_i}\cdot\sum_i \frac{p_i}{R_{M_i}}-\sum p_i \frac{R_{M_i}}{R_{M_i}}} \tag{A3}$$

It is now obvious that we can use Cov(x,y)=E(xy)-E(x)E(y) to rewrite equation A3 as:

$$\frac{E(R_A)-R_f}{E(R_M)-R_f}=\frac{Cov(R_A,\frac{1}{R_M})}{Cov(R_M,\frac{1}{R_M})} \tag{A4}$$

Appendix B

The proof of equation 5 follows.

We previously showed the following approximation holds very well for gross returns close 1:

$$\frac{Cov(\hat{R}_A, \frac{1}{\hat{R}_M})}{Cov(\hat{R}_M, \frac{1}{\hat{R}_M})} \approx \frac{Cov(\hat{R}_A, \hat{R}_M)}{Cov(\hat{R}_M, \hat{R}_M)} \tag{B1}$$

Similarly we can show the following approximation holds very well for gross returns close to 1:

$$\frac{Cov(\hat{R}_A, \hat{R}_M)}{Cov(\hat{R}_M, \hat{R}_M)} \approx \frac{Cov(\ln \hat{R}_A, \ln \hat{R}_M)}{Cov(\ln \hat{R}_M, \ln \hat{R}_M)} \tag{B2}$$

Considering equation 1, returns generated by a probability change from $\{p_i\}$ to $\{p_i+dp_i\}$ can be written as:

$$\hat{R}_M = \prod_{i=1}^{n} M_i^{\;dp_i} \quad \text{and} \quad \hat{R}_A = \left(1 + \frac{\sum_i dp_i \frac{A_i}{M_i}}{\sum_i p_i \frac{A_i}{M_i}} \right) \cdot \prod_{i=1}^{n} M_i^{\;dp_i} \tag{B3}$$

Taking the logarithm:

$$\ln \hat{R}_M = \sum_{i=1}^{n} dp_i \ln M_i \quad \text{and} \quad \ln \hat{R}_A = \ln \left(1 + \frac{\sum_i dp_i \frac{A_i}{M_i}}{\sum_i p_i \frac{A_i}{M_i}} \right) + \sum_{i=1}^{n} dp_i \ln M_i \tag{B4}$$

Since $\frac{\sum dp \frac{A}{M}}{\sum p \frac{A}{M}}$ goes to zero when $\{dp_i\}$ goes to zero (small fluctuations):

$$\ln\left(1+\frac{\sum dp\frac{A}{M}}{\sum p\frac{A}{M}}\right)\approx\frac{\sum dp\frac{A}{M}}{\sum p\frac{A}{M}} \tag{B5}$$

Considering B4, B5, and developing the covariances, then taking $\sum p\frac{A}{M}$ outside[5] we get:

$$\frac{Cov(\ln\hat{R}_A,\ln\hat{R}_M)}{Cov(\ln\hat{R}_M,\ln\hat{R}_M)}\approx 1+\frac{1}{\sum p\frac{A}{M}}\cdot\frac{\sum_{i,j}\frac{A_i}{M_i}\ln M_j Cov(dp_i,dp_j)}{\sum_{i,j}\ln M_i\ln M_j Cov(dp_i,dp_j)} \tag{B6}$$

Since the probabilities must add up to 1 we have that $\sum dp_i = 0$ and consequently:

$$\sum_{i,j}Cov(dp_i,dp_j)=0 \tag{B7}$$

Due to the symmetry of the fluctuation we have that:

$Cov(p_i,p_i)=V$ for all i, and also $Cov(p_i,p_j)=C$ for all i # j (B8)

Replacing B8 in B7 we get that:

$$V=(1-n)C \tag{B9}$$

Using B9 in B6, eliminating C, factoring out n^2, rearranging a bit, and using Cov(x,y)=E(xy)-E(x)E(y) we get[6] equation 5.

Appendix C

The "proof" of equation 7 follows.

Due to the symmetry of $\hat{R}_A$ around 1, $\hat{R}_A$ and $\frac{1}{\hat{R}_A}$ have identical distributions, consequently:

$$Cov(\hat{R}_A, \frac{1}{\hat{R}_A}) = 1 - E^2\left(\hat{R}_A\right) \tag{C1}$$

The same is true for $\hat{R}_M$ and, since for small fluctuations $1 + \hat{R}_A \approx 1 + \hat{R}_M \approx 2$, we can write:

$$\frac{E\left(\hat{R}_A\right) - 1}{E\left(\hat{R}_M\right) - 1} \approx \frac{Cov(\hat{R}_A, \frac{1}{\hat{R}_A})}{Cov(\hat{R}_M, \frac{1}{\hat{R}_M})} \tag{C2}$$

The ratio of covariances on the right-hand side of equation C2 is the very definition of $\hat{\gamma}$, and we know that for small fluctuations the following approximation holds very well:

$$\hat{\gamma} \approx \frac{Cov(\hat{R}_A, \hat{R}_A)}{Cov(\hat{R}_M, \hat{R}_M)} = \frac{Var(\hat{R}_A)}{Var(\hat{R}_M)} \tag{C3}$$

Applying equation 8 to the risk-free asset and then $Cov(a(x),b(y))=E_x(a_x)E_y(b_y)Cov(x,y)$, we get that $\hat{\gamma}$ for the risk-free asset is in most situations close to 0 (again $'$ indicates expectations are not probability-weighted):

$$\hat{\gamma}_f \approx \left(1 - \frac{E'(\frac{1}{M^2})}{E(\frac{1}{M}) \cdot E'(\frac{1}{M})} \right)^2 \approx 0 \qquad \text{(C4)}$$

Equations C3 and C4 imply that, in general, $E(\hat{R}_f)$ will be much closer to 1 than $E(\hat{R}_M)$ and $E(\hat{R}_A)$. Consequently, replacing 1 with $E(\hat{R}_f)$ on the left-hand side of C2 will not “ruin” the approximation. Hence equation 7 holds reasonably well in most cases.

REFERENCES

Alb, Silviu I. “The Relative Value Theory.” Economics Working Paper Archive at WUSTL, June 2001, http://econpapers.hhs.se/paper/wpawuwpfi/0106003.htm

Alb, Silviu I. “Maximizing Long-Term Cumulative Returns.” Journal of Wealth Management, Vol. 6, No. 4, (February 2004), pp. 29-32

Faugere, Christophe, and Julian Van Erlach. “A General Theory of Stock Market Valuation and Return.” Economics Working Paper Archive at WUSTL, November 2003, http://econpapers.hhs.se/paper/wpawuwpfi/0311005.htm

Latane, Henry A. “Criteria for Choice among Risky Ventures.” Journal of Political Economy, Vol. 67, No. 2 (April 1967), pp. 144-155

Samuelson, Paul. “Lifetime Portfolio Selection by Dynamic Stochastic Programming.” Review of Economics and Statistics, Vol. 51, No. 3 (August 1969), pp. 239-246

Von Neumann, John L., and Oscar Morgenstern. The Theory of Games and Economic Behavior. Princeton, NJ: Princeton University Press, 1944

FOOTNOTES

[1] Samuelson [1969] rejects the geometric mean saying it does not maximize utility, but Von Neumann and Morgenstern [1944] introduce utility by postulating the arithmetic mean, and consequently, as Alb [February 2004] points out, using utility to reject the geometric mean appears to be a logical fallacy.

[2] Equality follows from applying $Cov(a(x),b(y))=E_x(a_x)E_y(b_y)Cov(x,y)$, which holds if variables x and y are normally distributed and functions a() and b() are differentiable (see Rubinstein [1976] for a proof). However, even if R_A and R_M are not normally distributed, α is still a decent approximation of β, especially if R stays close to 1, in which case we could approximate $1/R \approx 2-R$, and replace in α to get $\alpha \approx \beta$. We'll later use $R \approx 1+\ln(R)$.

[3] This is in sharp contrast to mainstream financial theory, which predicts asset prices will increase at different rates depending on the β coefficient. Empirically testing which prediction appears more accurate is not a trivial task. Faugere and Erlach [2003] find evidence long-term return closely approximates the return on risk-free debt.

[4] This follows from assuming that all scenario probabilities fluctuate within intervals (p_i-dp , p_i-dp) that have all the same width. Assuming the intervals' width $2dp_i$ is proportional to p_i would lead to a similar result with covariances being "almost" probability-weighted.

[5] We approximate the $\sum p\frac{A}{M}$ term by its value at the central probability set $\{p_i\}$. The term becomes a constant and, consequently, can be factored out. A more precise calculation should be possible using Taylor series (and might be worthwhile).

[6] It can be similarly shown that in a world where market prices are given by the arithmetic mean of intrinsic values, $\hat{\alpha}$ would be approximately equal to $\frac{\sum p_i M_i}{\sum p_i A_i} \cdot \frac{Cov'(A,M)}{Cov'(M,M)}$.