Return-Volume Dependence and Extremes in International Equity Markets

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Abstract

This paper reconsiders return-volume dependence for the U.S. and six international equity markets. We contribute to previous work by proposing surprise volume as a new proxy for private information flow and apply extreme value theory in studying dependence for large volume and return, i.e. under situations of market stress. Results from a GARCH-M model indicate that surprise volume is superior in explaining conditional variance and reveals a positive market risk premium. Under conditions of market stress, the return-volume dependence is weaker, albeit mostly significant. The results for the U.S. market are most pronounced in that surprise volume explains ARCH- as well as leverage-effects and, under market stress, the return-volume dependence remains significant and symmetric. For the European and Asian markets, however, the dependence is weaker with asymmetry under market stress, i.e. small minimal returns show lower volume dependence than large maximal returns. We argue that our results are more consistent with a Gennotte and Leland (1990) misinterpretation hypothesis for market crashes than with cascade or behavioral explanations which associate high volume with steep price declines.

Key Words: trading volume, return-volume dependence, mixture of distributions hypothesis, extreme returns, bivariate extremal dependence, market crashes
 JEL Classification: C13, G10, G15

1. Introduction

The uncertainty of equity returns is commonly measured by the equity returns' variance. However, empirical evidence indicates that a stationary normal distribution is only a very approximate unconditional model for equity returns. In particular, unconditional return distributions tend to be fat-tailed.[1] Providing a possible explanation for the latter phenomenon, it can be proposed that conditional return variance may vary over time.

While models of conditional autoregressive heteroskedasticity (ARCH) have become the most commonly used econometric models of time-varying variance, they do not offer an economic explanation of the phenomenon. Given the stylized fact that there are fat-tails in the distribution of returns, the objective in this paper is to examine to what degree fat-tailedness is associated with trading volume. We are interested in two related features of the return-volume relationship. First we re-examine the relationship under "normal" market conditions. We then enter into a new area of empirical research by studying conditions of market stress. The relationship between returns, trading volume and conditional volatility under "normal" market conditions is potentially important in understanding how information is transmitted to the market as well as embedded in stock prices. The return volume relationship under market stress is central to an understanding of crashes and rapid run-ups in stock prices.

We contribute in several ways to previous work on return-volume dependence, which was mostly focusing on the U.S. equity market. First, we study dependence for the U.S. as well as for six additional large international equity markets: Holland, France, Germany, Hong Kong, the U.K., and Japan. Second, we propose a new proxy measure of private information flow, called "surprise volume", which is defined as unpredictable above-average trading volume. This measure of heterogeneity offers an economically intuitive indicator of private information arrival as commonly used in the trading volume literature based on noisy rational expectations. Third, we conduct a study on the joint distribution of extreme trading

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[1]The empirical study of fat-tailed return distributions has a long tradition. Mandelbrot (1963) proposed the sum-stable class of distribution functions as a model of returns. More recently, results from statistical extreme value theory have been applied in finance, providing results on the tail behavior of a broad class of stationary fat-tailed return distributions. Empirical studies include Jansen and de Vries (1991), Longin (1996) and Jondeau and Rockinger (1999) on tail behavior as well as Longin and Solnik (2001), Poon et al. (2001), and Silvapulle and Granger (2001) who also consider extremal dependence.

volumes and returns by applying a model from bivariate extreme value theory. Lastly, we study separately the asymptotic joint distribution of large drops and run-ups in stock prices. Our objective in focusing on extreme returns and volumes is to infer how volume, information flow and returns depend on each other in the tails of their joint distribution. We already suspect that this dependence is not "usual" in the sense that it seems difficult to associate large market moves with observable information events; see e.g. Haugen et al. (1991). Also, the results in Gallant et. al. (1992) and Balduzzi et. al. (1996) suggest that the tail-behavior of the price-volume relation may differ from that of the overall joint distribution. However, the findings are not based on an asymptotic model of extremes and do not allow for a characterization of situations of market stress. The extreme value model allows us to examine the price-volume relationship under market stress, which includes the examination of potential asymmetry between booms and crashes, as well as to test the implications of models of market stress.

Using daily data over the period from January 1988 to November 1997, we apply a new flexible filtering technique to capture stochastic trends, seasonality, and linear dependence in trading volume. We then retest many of the features as documented for the U.S. market. In particular, when fitting a GARCH-M model, we find that price variability and volume are significantly related for all international markets excluding the French. Surprise volume –i.e. unpredictable above-average trading activity– is significantly associated with the variation in conditional return variance. Volume surprises also reveal a positive risk-return relation. The latter finding is consistent with volume as a proxy of private information flow which plays an important role in the determination of equity return premiums. In fact, while these features are quite robust over our cross-section of markets, we find the U.S. serving as a benchmark market yielding the most pronounced results. Only for the U.S. market, surprise volume completely dominates GARCH as well as leverage effects in explaining volatility persistence.

Excess kurtosis in the returns of our sample of international markets can generally be captured quite well by a GARCH-M *cum* volume model. However, the model cannot fully explain fat-tailedness; our examination of the asymptotic behavior of extreme returns and volume can shed light on this characteristic. We find that although the relationship between return variability and trading volume is on average weaker for extreme observations, independence between extremal absolute return and volume can be rejected for five out of seven markets. The finding of a weaker return variability-volume dependence in the tails can be attributed

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to the very largest return and especially volume observations. The dependence relation is quite symmetric for the majority of the markets in our sample, i.e. the decrease in dependence for large extremal returns and small extremal volume is of the same order of magnitude as the one for small extremal returns and large extremal volume. Interestingly, however, in Holland and Japan, the dependence relation is weaker for large extremal volume, indicating large volume being a bad proxy for information flow. In France and Germany, the extremal dependence relation is the weakest in our sample. The occurrence of sharp price changes under moderate trading volume, as particularly in France and Germany, is consistent with the explanation of market crashes in terms of the misinterpreted-trade hypothesis by Gennotte and Leland (1990).

Furthermore, we use the extreme value model to estimate the price-volume dependence for negative and positive extremal returns separately. We again find the U.S. market to serve as a benchmark where the return variability-volume dependence remains symmetric in the upper and lower tail. That is, the estimated elasticity of price decreases to volume is equal to the elasticity for price increases. This result can not be attested to the European and Asian markets of our sample. The estimation results for Holland, the U.K., Hong Kong, and Japan demonstrate that prices and volume are less dependent for large price decreases than for large price increases. This finding cannot be captured in existing models of the return-volume relation which imply symmetry. Also, since it seems hard to argue that short-sale restrictions are more substantial in the U.S. than in the other countries, the documented cross-sectional elasticities do not seem to be consistent with the Hong and Stein (2003) hypothesis that investors are rational but crashes occur because of more intrusive restrictions on selling than buying stocks. At the same time, in none of the countries do we see the dependence between returns and volume strengthening for large extremal volume, i.e. we do not observe high volume that tends to be accompanied by sharp price changes as would be predicted by cascading or behavioral explanations of market crashes.

The remainder of the paper is organized as follows. A brief review on earlier findings on the return-volume dependence in the U.S. equity market is given in Section 2. Section 3 outlines the methodological framework including the GARCH and the extreme value model. Section 4 introduces the data set and describes the approach to filtering market volume. Model estimation and the empirical results are given in Section 5. Section 6 contains the conclusion.

2. The Mixture Model and Earlier Empirical Findings

Fat-tailed return distributions can be explained by non-stationary models with time-varying variance. Two common approaches to time-varying variance are models of the autoregressive conditional heteroskedasticity class and the subordination model. The first approach is based on discrete time series modeling documented in a vast collection of papers; a classical survey is given by Bollerslev et al. (1992). The second approach provides a theoretical explanation of fat-tailed return distributions in which the distribution of prices and volume is jointly subordinate to an unobservable directing process. The model goes back to Clark (1973), among others. An extension within the Grossman and Stiglitz (1980) noisy rational expectations framework is given by Tauchen and Pitts (1983). In the literature, the model is also referred to as the mixture model or the mixture of distributions hypothesis.

The subordination model is especially appealing from a market microstructure perspective, since it makes assumptions about the underlying process which jointly generates price changes and trading volume. Returns over a fixed period such as one trading day are given as a random sum of subperiod returns with common distribution function. The arrival of traders is driven by an unobservable directing process. An implication of the model is a positive dependence between return variability and the unobservable directing process, since a higher trading intensity increases the variability over a fixed return measurement period. The noisy rational expectations framework offers an economic interpretation of the unobservable directing process. Assuming that traders share a common information set and additionally receive noisy private information signals, trading will occur due to differences in private information sets. Therefore, proxy variables for the directing process are for example the number of transactions or trading volume.[2]

Karpoff (1987) reviews early empirical studies on trading volume and return variability. The results show a positive relation between price variability and volume consistent with the mixture of distributions hypothesis. Lamoureux and Lastrapes (1990) examine Clark's subordination hypothesis in the context of a GARCH model with a volume variable. The authors find that volume can explain

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[2]Since in general, the latter is more easily observed, volume is frequently used in empirical investigations, although the number of transactions might be a preferable variable; see e.g. Jones et al. (1994). Also order flow, i.e. the net of buyer and seller initiated transactions, may provide a useful proxy variable as shown by Marsh and Rock (1986) for equity markets and more recently by Evans and Lyons (2002) in the area of exchange rate modeling.

GARCH effects in U.S. single stocks. We will use an extended Lamoureux and Lastrapes model in the following.

Gallant et al. (1992) estimate the joint conditional return-volume density for the U.S. market over a 57 year period. The authors detrend their price and volume series and apply seasonal adjustments in order to provide an approximately stationary return and volume series. In line with the earlier studies, a positive return volatility-volume dependence is found. The estimation results indicate that the positive dependence relation has non-linear features. Whereas the relation is nearly flat for negative standardized volume realizations, it is increasing for positive standardized volume. For volume observations above a level of about two, the relation weakens (see their figure 6). The authors also find that a negative conditional risk-return relation turns into a positive one when volume is incorporated into the analysis. Additionally, the results suggest that a leverage effect is jointly explainable by returns and volume.

Lamoureux and Lastrapes (1994) extend the Tauchen and Pitts bivariate mixture model by assuming a dependent directing process and find no evidence that volatility persistence in stock returns is explainable by volume data. Richardson and Smith (1994) test a joint set of moment restrictions of the mixture of distributions model. Under a lognormally distributed directing process[3], the fit seems superior as compared to other distributional assumptions, however overall evidence is not overwhelming. Andersen (1996) adopts a noisy rational expectations framework and explicitly allows for non-informational trading and common information arrivals. He shows that this improves the empirical fit of the implemented moment restrictions. In a dynamic setting, the model allows for a reduction in the estimated volatility persistence of U.S. single stocks.[4]

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[3]This is a common assumption in the literature. Note however, that it is not strictly model-consistent as a continuous distribution can only be regarded as an approximation to the underlying integer valued directing process.

[4]Evidence on the model remains diverse: Liesenfeld (1998) tests Andersen's model by assuming a dependent directing process. Like Lamoureux and Lastrapes, he finds no evidence that volatility persistence is explainable by volume data. Bollerslev and Jubinski (1999) suggest that long-memory characteristics in the directing process may help to explain earlier model rejections. Fleming et al. (2001) propose a modification of the mixture model under the independence assumption. In the present paper we argue that it theoretically seems difficult to justify the assumption of dependence in a directing process which represents unobservable information flow, since information is commonly regarded to be an independent random phenomenon.

3. The Methodological Framework

This section contains the methodological framework for our empirical investigation. First, a brief outline of the mixture of distributions hypothesis is given, focusing on its empirically relevant implications. The corresponding GARCH model specification follows. After a discussion of the return variability-volume relation for large observations, the bivariate extreme value model is specified.

3.1. Return and Volume in the Mixture Model

The evolution of prices and volume in the mixture of distributions model can either be measured with respect to standard calendar time or with respect to some operational time measure. A natural measure of operational time is given by the random number of transactions $N(\tau)$ in an intra-day time interval $[0; \tau]$. Intraday equilibrium asset prices P_{τ_i} and corresponding trade volumes V_{τ_i} are observed at random time points. The counting sequence $\{N(\tau)\}_{0 \leq \tau \leq 1}$ is called directing process, where $N_t = N_t(1)$ denotes the number of transactions for the tth trading day. Observed intra-day prices $\{P_{\tau_i}\}_{1 \leq \tau_i \leq N_t}$ and trade volumes $\{V_{\tau_i}\}_{1 \leq \tau_i \leq N_t}$ are subordinated to a common directing process. Assuming continuously compounded returns, $R_{\tau_i} = \ln(P_{\tau_i}/P_{\tau_{i-1}})$, it follows that the cumulative day t return R_t and the cumulative volume V_t is given by the random sums $R_t = \sum_{i=1}^{N_t} R_{\tau_i}$ and $V_t = \sum_{i=1}^{N_t} V_{\tau_i}$, respectively. This setup forms the basis of the mixture of distributions hypothesis.

For a comprehensive summary of the model's moment conditions refer to Harris (1987). Assuming finite second moments, an independent price and directing process, and *iid* intra-day returns for simplicity, it follows: $Var(R_t) = Var(N_t)E(R_{\tau_i})^2 + E(N_t)Var(R_{\tau_i})$. The latter result shows that the unconditional daily return variance is a function of the unobservable directing process. When the daily return expectation is small, one can write: $Var(R_t) \approx E(N_t)Var(R_{\tau_i})$. For the conditional daily return variance it follows: $Var(R_t | N_t = n_t) = n_t Var(R_{\tau_i})$. Another important model implication is the prediction of a positive covariance between squared returns and volume, $Cov(R_t^2; V_t) > 0$ and hence also between absolute returns and volume, $Cov(|R_t|; V_t) > 0$. The mixture of distributions hypothesis provides a formal basis for the assertion that trading volume relates to daily return volatility.

In an empirical setting the unobservable directing process must be specified.

In Clark's classical model, prices and volume are assumed to be driven by an *iid* directing process that represents the number of transactions. Under the information flow interpretation of the directing process, more structure is imposed. Using an economic model such as the noisy rational expectations framework, trading volume is explained by two components, namely by individual non-informational trading and by differences in the traders' private information sets. In this setting, the directing process can be characterized as an unobservable flow of noisy private information. Assuming that non-informational trading volume is given by some stationary random V'_t, one may write: $V_t = V'_t + \sum_{i=1}^{N'_t} V'_{\tau_i}$. Trading volume is obviously a relevant observable variable for the unobservable information flow, but it serves as a proxy only. When common information events on day t cause price variability h_t^2 without trading, conditional daily variance is

$$Var(R_t | N'_t = n'_t) = h_t^2 + n'_t Var(R_{\tau_i}). \qquad (3.1)$$

In order to perform a univariate test of the mixture of distributions hypothesis, the exogenous unobservable directing process in (3.1) is proxied by realizations of an observable endogenous volume variable. Particularly, the information flow interpretation of the directing process has two implications. First, above-average volume which proxies the realizations $\sum_{i=1}^{n'_t} v'_{\tau_i}$ should explain return variance better than volume itself. Second, the same holds for unpredictable volume being a more suitable proxy for the number of day t information arrivals n'_t than volume itself.

3.2. The GARCH Model Specification

A GARCH with mean specification (GARCH-M, Engle et al. (1987)) is chosen to fit the overall conditional return variability-volume dependence. Apart from the standard GARCH(1,1) terms, the variance equation contains an asymmetric ARCH-term. In the first model, no volume variable enters the variance equation. A volume variable, as motivated by the preceding section, is added in models 2 and 3.

3.2.1. Conditional Return

For all model specifications, the conditional return equation contains a constant, a first order autoregressive and an in-mean term:

$$R_t = \mu + \rho R_{t-1} + \lambda \sigma^2_{t,\mathcal{F}_t} + \epsilon_t, \quad \epsilon_t \sim (0, \sigma^2_{t,\mathcal{F}_t}), \quad t = 1,...,T. \qquad (3.2)$$

The noise terms ϵ_t are independent draws from a possibly fat-tailed distribution function with conditional variance $\sigma^2_{t,\mathcal{F}_t}$. The AR(1) term ρR_{t-1} allows for possible linear dependence in index returns due to asynchronous trading of the member stocks, a phenomenon that is well-documented in the literature. Conditional variance is related to conditional return by the in-mean term $\lambda \sigma^2_{t,\mathcal{F}_t}$. A justification for the above in-mean specification can be provided e.g. by the evidence of Genotte and Marsh (1993) who find an approximately linear variance return relation in monthly U.S. equity returns. Note that, as shown in the model of the latter authors, a theoretical statement about the sign of the relation is ambiguous in a consumption based asset pricing framework and depends on the risk aversion of the representative investor. As pointed out by Gallant et al. (1992) and Golsten et al. (1993), this makes empirical calibration an important issue.

3.2.2. Conditional Variance

The conditional return variance $\sigma^2_{t,\mathcal{F}_t} = Var(R_t \mid \mathcal{F}_t)$ in equation (3.2) is assumed to follow a discrete Markov process conditioned on the set $\mathcal{F}_t = \{\epsilon^2_{t-1}, \sigma^2_{t-1}\}$ for given initial $(\epsilon^2_0, \sigma^2_0)$. Referring back to equation (3.1), three different model specifications are chosen. The base model (model specification 1) defines the variance component h^2_t by

$$\sigma^2_{t,\mathcal{F}_t} = h^2_t \equiv \omega_0 + \omega_1 \epsilon^2_{t-1} + \omega_2 \epsilon^2_{t-1} I_{\{\epsilon_{t-1}<0\}} + \omega_3 \sigma^2_{t-1,\mathcal{F}_{t-1}}, \qquad (3.3)$$

where $I_{\{A\}}$ denotes an indicator variable for the event A. The term $\epsilon^2_{t-1} I_{\{\epsilon_{t-1}<0\}}$ is introduced in order to allow a possible asymmetric response of conditional variance to positive and negative return innovations. In case of a positive coefficient ω_2, a negative return shock in t will cause an increased return variance in $t+1$. This observation is then frequently denoted as a "leverage effect".

Models 2 and 3 include a stationary random variable U_t which measures abnormal trading volume. The latter is given by $U_t = V_t - \overline{V}_t$, where $\overline{V}_t$ denotes average volume for period t. Under $\mathcal{F}_t = \{\epsilon^2_{t-1}, \sigma^2_{t-1}, U_t\}$, the conditional variance equation in model specification 2 is given by

$$\sigma^2_{t,\mathcal{F}_t} = h^2_t + \omega_4 U_t. \qquad (3.4)$$

The parameter ω_4 is predicted to be positive under the prediction of the mixture model. An implication of the mixture of distributions hypothesis based on a noisy rational expectations framework is that only trading due to private information

arrivals, not overall trading, should relate to return variability. Interpreting the directing process as an unobservable flow of private information as in equation (3.1), it is above-average volume that should serve as a potential proxy for information flow.[5] Using $U_t I_{\{U_t>0\}}$ as volume variable, a corresponding non-linear specification is chosen in model 3

$$\sigma^2_{t,\mathcal{F}_t} = h^2_t + \omega_4 U_t I_{\{U_t>0\}}. \tag{3.5}$$

To summarize, specification 1 is a standard GARCH-M model with a leverage term (equations (3.2) and (3.3)). Model specification 2 uses all realizations of the volume variable in order to explain conditional return variance (equations (3.2) and (3.4)). In contrast, model 3 only uses above-average volume proxying private information flow (equations (3.2) and (3.5)).

3.3. Extremal Returns and Volume

An especially interesting domain for testing the predictions of the mixture of distributions hypothesis is certainly in the tails of the joint return-volume distribution. Of course, if the return variability-volume relation was linear, a separate examination of the tails would be redundant. However, there is reason to believe that non-linearities in the relation make it worthwhile to address the question of whether the tails of the return distribution can be explained by trading volume. A positive answer can be interpreted as evidence for the dependence prediction of the mixture of distributions model. Otherwise, the question arises as to why the relation changes and how this phenomenon can be explained.

There are at least two potential theoretical explanations of why a breakdown of the return variability-volume dependence may occur in the tails. First, extreme volume observations might be a bad proxy for the unobservable directing process. As volume is not only given by the number of transactions but also by the number of shares traded, errors in the measurement of the directing process are unavoidable and might be particularly biased for large volume observations. This problem, denoted as the bad proxy hypothesis, is hard to deal with in detail when no transactions data are available. However, under the assumption that extreme volumes overestimate the magnitude of the directing process, a testable implication of the hypothesis is that the correlation of extremal volume with extremal

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[5]Lagged volume was found to be much less predicitve in this and previous studies of return-volume dependence, also providing evidence of an information flow interpretation.

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return variability is weaker than in the overall distribution. Particularly, extreme volume should on average be observable jointly with moderate return variability, i.e. moderate absolute returns. Second, extreme return observations might not be due to information events, but due to temporary market illiquidity. A corresponding hypothesis put forward by Gennotte and Leland (1990) assumes that liquidity constraints may cause non-informational traders to have an impact on return variability. This is because traders may occasionally misinterpret non-informational liquidity trading as being informational.[6] As opposed to the standard rational expectations model, high return variability may thus occur in the absence of common information releases and/or high trading activity. A typical application of the model is to market crashes, i.e. large price declines under moderate volume where a change in the common information set of the traders is improbable; see Haugen et al. (1991) for related empirical evidence. A testable implication is that the return variability-volume correlation weakens in the tails and particularly that extreme returns are jointly observable together with moderate trading volume.

3.4. The Extreme Value Model

Testing for return variability-volume dependence in the tails involves inferring whether there is statistical evidence that the asymptotic joint distribution of absolute returns and volume converges to two independent marginal distributions or not. This is done by examining extreme return and volume observations. Since the data in the tails are sparse by definition, we rely on a statistical model in order to robustify our results. This model is given in the context of extreme value theory as outlined for example in Embrechts et al. (1997) and Coles (2001).

3.4.1. Univariate Model

Classical univariate extreme value theory examines the asymptotic distribution of properly normalized extrema of a series of *iid* random variables. Although the returns R_t from the GARCH model of Section 3.2. are not *iid,* a stationary marginal return distribution function $F_R(r)$ can be assumed to exist under quite

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[6]A note on private information versus liquidity: Under liquidity constraints, large transactions have a price-impact which rational discretionary liquidity traders will avoid as outlined in the Admati and Pfleiderer (1988) model. If liquidity traders do not time their trades, their trading may be misinterpreted as informational following the Gennotte and Leland (1990) framework.

general conditions on the model parameters.[7] It then follows that the return maximum $M_{R,T} = \max(R_1, ..., R_T)$[8] obeys the same types of limiting distributions as in the *iid*-case, with the only difference that the extreme return observations will tend to occur in clusters.

Therefore, classical extreme value theory provides a valid approach to make inferences about the extremal behavior of the stationary return series $(R_t)_{1 \leq t \leq T}$. With the norming constants $a_T > 0$ and $b_T \in \mathbb{R}$, the asymptotic distribution of the return maximum is derived as the limit of $\Pr\{a_T^{-1}(M_{R,T} - b_T) \leq r\} = F_R^T(a_T r + b_T)$ as $T \to \infty$. If a limiting distribution exists, it has to be a member of the class of distribution functions which satisfy the max-stability condition

$$\lim_{T \to \infty} F_R^T(a_T r + b_T) = H_R(r),$$

where H_R is a non-degenerate distribution function. A general representation of the distribution functions satisfying the above stability condition is given by the generalized extreme value distribution (GEV):

$$H_{R;\xi}(r) = \exp\left[-\{1 + \xi r\}^{-1/\xi}\right], \quad 1 + \xi r > 0. \tag{3.6}$$

The shape parameter, $\xi \in \mathbb{R}$, also denoted as tail index, characterizes the extremal behavior of the returns. Particularly, the GEV corresponds to the Fréchet distribution for $\xi > 0$,

$$\Phi_\xi(r) = \begin{cases} 0, & r \leq 0 \\ \exp\left[\{-r\}^{-1/\xi}\right], & r > 0 \end{cases},$$

to the Weibull extreme value distribution for $\xi < 0$,

$$\Psi_\xi(r) = \begin{cases} \exp\left[-\{-r\}^{1/\xi}\right], & r < 0 \\ 1, & r \geq 0 \end{cases},$$

and to the Gumbel distribution in the limit when $\xi \to 0$:

$$\Lambda(r) = \exp\left[-\exp\{-r\}\right].$$

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[7]Related results can be found for example in Mikosch and Stărică(2000) for the GARCH(1,1) model and in Borkovec and Klüppelberg (1998) for an AR(1) process with heteroskedastic noise.

[8]Clearly, for F_R defined on the entire real axis, the results hold equivalently for minima since $\min(R_1, ..., R_T) = -\max(-R_1, ..., -R_T)$.

The location-scale family of the GEV distribution additionally includes a location parameter $\mu \in \mathbb{R}$ and a scale parameter $\psi > 0$. Its distribution function, which is particularly useful for estimation purposes, is given by:

$$H_{R;\xi,\mu,\psi}(r) = \exp\left[-\left\{1 + \xi\left(\frac{r-\mu}{\psi}\right)\right\}^{-1/\xi}\right], \quad 1 + \xi\left(\frac{r-\mu}{\psi}\right) > 0.$$
(3.7)

3.4.2. Bivariate Model

Bivariate extreme value theory describes the asymptotic joint behavior of normalized componentwise maxima of a vector of random variables. Given the stationary sequence of volume data U_t, the random return-volume vectors (R_t, U_t) have a common joint distribution function $F(r, u)$. With $M_{U,T} = \max(U_1, ..., U_T)$, the focus is now on the limit of the joint probability distribution of the maxima

$$\lim_{T\to\infty} \Pr\{a_{R,T}^{-1}(M_{R,T} - b_{R,T}) \le r, \; a_{U,T}^{-1}(M_{U,T} - b_{U,T}) \le u\} =$$

$$= \lim_{T\to\infty} F^T(a_{R,T}r + b_{R,T}, \; a_{U,T}u + b_{U,T}) = H(r, u),$$
(3.8)

where $a_{R,T} > 0$, $b_{R,T} \in \mathbb{R}$ and $a_{U,T} > 0$, $b_{U,T} \in \mathbb{R}$ again denote suitable norming constants. It follows from the univariate case that the marginal distributions $H(r, \infty)$ and $H(\infty, u)$ are each given by the GEV distributions $H_R(r)$ and $H_U(u)$.

Since a random variable from one type of the three limiting extreme value distributions can mathematically be easily transformed into one of the two others, it is a matter of convenience which unique marginal distribution is used in the bivariate model. Hence, without loss of generality, standard Fréchet margins can be assumed. For the return variable it follows $H(r, \infty) = \Phi_1(r) = \exp(-r^{-1})$ under the transformation $\widetilde{R} = \{1 + \xi\left((R - \mu)/\psi\right)\}^{1/\xi}$; the volume variable follows accordingly. By the representation theorem of Pickands (1981) it can then be shown that H satisfying the stability condition (3.8) has to be of the form:

$$H(r, u) = \exp\left\{-\left(\frac{1}{r} + \frac{1}{u}\right)A\left(\frac{r}{r+u}\right)\right\}, \quad r, \; u > 0.$$
(3.9)

The function $A(w)$ with $w = r/(r+u)$ characterizes bivariate extremal dependence. If $A(w) = 1$, $0 < w < 1$, the tails of the joint distribution are independent, whereas $A(w) = \max(w, 1 - w)$, $0 < w < 1$, indicates perfect dependence.

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Several parametric models can be chosen for the dependence function $A(w)$. A standard model is the bivariate logistic model with a single parameter $\gamma \geq 1$; see also Smith et al. (1990), Klüppelberg and May (1998), and Coles (2001). The choice of the symmetric function $A_\gamma(w) = \{(1-w)^\gamma + w^\gamma\}^{1/\gamma}$ in (3.9) yields a joint extreme value distribution function

$$H_\gamma(r, u) = \exp\left\{-\left(\frac{1}{r^\gamma} + \frac{1}{u^\gamma}\right)^{1/\gamma}\right\}. \tag{3.10}$$

The above model is chosen for the investigation of extremal returns and volume as there is no a priori implication from the mixture model regarding asymmetric dependence. On the other hand, both, the bad proxy and the misinterpreted-trade hypothesis as outlined in Section 3.3, predict asymmetric features for the tail dependence. Therefore, in the empirical part of the paper, the results for the above model are reviewed with respect to possible dependence asymmetries.

4. The Dataset and Volume Adjustments

This section briefly introduces the data set used in the empirical study and describes the adjustments made to the raw volume series.

4.1. The Dataset

The dataset consists of daily closing price data p_t and turnover data v_t for seven major international stock market indices in the time period January 4, 1988 to November 14, 1997, with $t = 1, ..., 2575$. All data were obtained from Datastream. The countries together with the symbol for the corresponding index, are Holland (Amsterdam EOE Index, 'AEX'), France (CAC 40, 'CAC'), Germany (DAX 30, 'DAX'), Hong Kong (Hang Seng Index, 'HSI'), the U.K. (FTSE 100, 'FTS'), the U.S. (Standard and Poor's 500, 'SPX') and Japan (Topix, 'TPX').

The study of international capitalization-weighted market index data enables us to analyze the return-volume behavior of a broad class of stocks which typically show above average liquidity. The data set represents a local currency perspective. Apart from the case of the DAX-index, dividends are not included in the calculation of the indices. Since focus on return-volume dependence and the tail-properties of the joint distribution, we do not regard these as important issues in our following examination.

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4.2. Volume Adjustments

The dependence implication of the mixture of distributions model can be tested by examining the relation between the proxy variable for the directing process and a measure of return variability. If trading volume is related to the underlying directing process and the relation is linear in the given range of observations, then there should be a significant correlation between the volume variable and return variability. As outlined in Section 3.1, an information flow interpretation of the directing process implies characteristics that differ from those of a pure transaction based interpretation of the directing process. Hence, the utilization of volume data in empirical specifications is not straightforward.

Non-stationarity and time-series dependence of volume data is a major issue in empirical investigations of the return-volume relationship. In the present application the task is to filter a stationary, uncorrelated series of trading volume. For other investigations on returns and volume which apply adjustments to the raw volume series see Gallant et al. (1992), Campbell et al. (1993), and Andersen (1996). As also proposed for example in Lo and Wang (2000), we follow an approach which imposes rather little structure on the raw volume series of the different markets in our sample. However, in contrast to these previous studies, we do not apply moving averages but a more flexible data-efficient filtering technique to capture stochastic trends in volume. Also, we additionally apply the linear Box/Jenkins time-series methodology to capture seasonality and dependence.

4.2.1. Trends in Volume: Filtering Abnormal Volume

In a first step, we transform the volume data by the application of the natural logarithm. As volume data are restricted to non-negative values, the distribution of the resulting series $\ln v_t$ is better approximated by the normal. The transformation also improves the stationarity properties of the volume series under a long run constant volume growth rate and a volume variance that relates to the volume level.
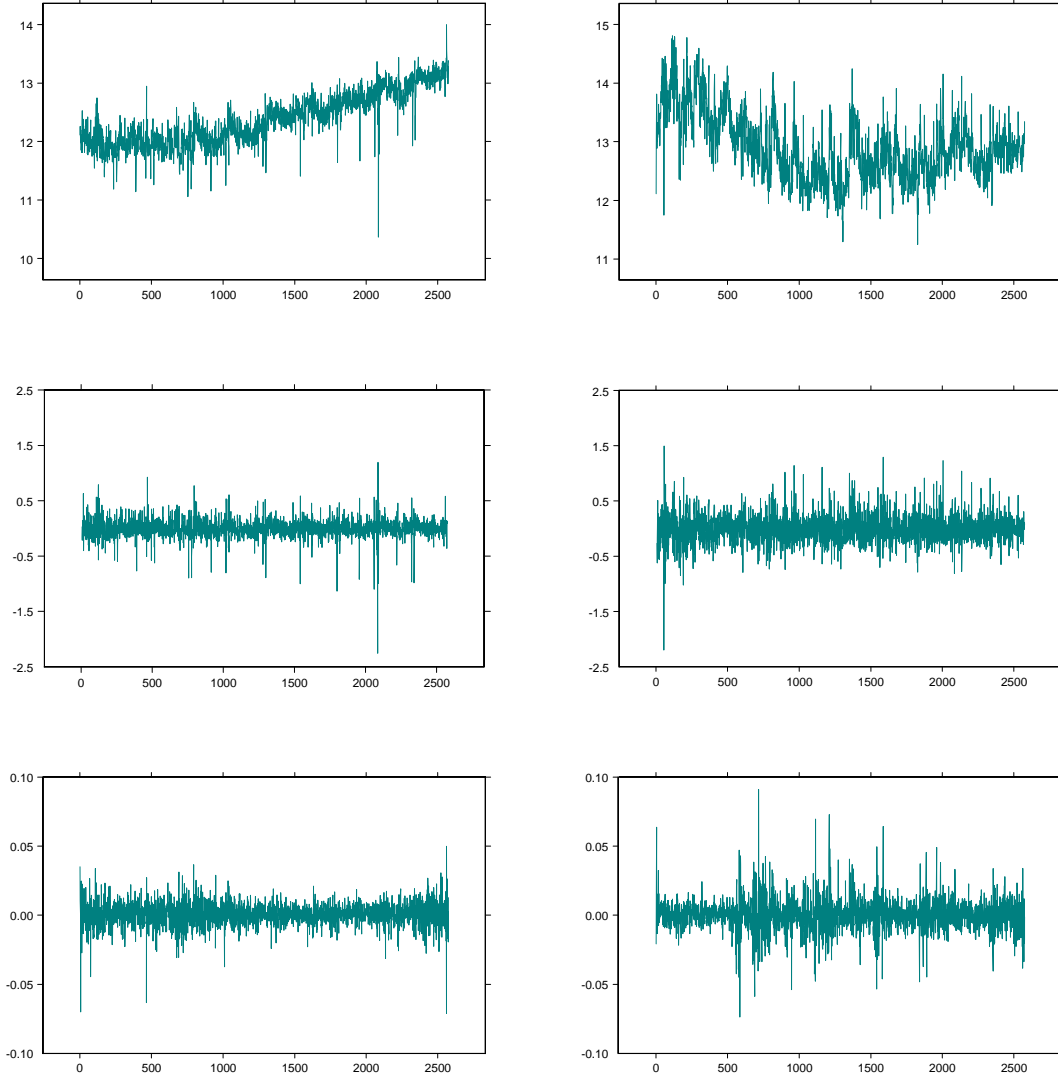
Figure 1: Logarithmic volume $\ln v_t$, filtered abnormal volume $u_{t,2}$ and continuously compounded returns r_t for the S&P 500 (left column) and the Topix (right column).

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In order to remove a stochastic time trend in overall volume, the Hodrick and Prescott (1997) filtering method is applied. The method minimizes the sum of squared deviations between the original and the smoothed series given a smoothness constraint for the fitted series.[9] The output of the method provides a series of normal logarithmic trading volume. Taking the difference between actual and normal volume yields a stationary series of abnormal logarithmic volume. This series, denoted by $u_{t,1}$, is used as a volume series in our empirical investigation.

4.2.2. Volume Seasonality and Dependence: Filtering Uncorrelated Abnormal Volume

In a second step, the time-series dependence structure of the abnormal volume series $u_{t,1}$ is examined within a linear filter framework. Applying the classical Box/Jenkins-methodology shows that a suitable overall representation of the data is given by a first order autoregressive moving average with a five-weekday seasonality component. In order not to overfit, the chosen ARIMA$((1,0,1) \times (1,0,1)_5)$-model is estimated for all markets' abnormal volume series $u_{t,1}$. Model estimation is carried out by least squares. This can be considered a useful approach even in the presence of fat-tailedness in the volume series (compare Embrechts et al. (1997)).

The estimation results show that the volume series exhibit a high degree of linear predictability. The adjusted R^2-statistics for the fitted models are 0.205, 0.179, 0.326, 0.321, 0.567, 0.283 and 0.526 for the Dutch, French, German, U.K., Hong Kong, U.S. and Japanese markets, respectively. The finding of a statistically significant weekly seasonality implies volume level effects for each weekday.[10] Filtering the $u_{t,1}$-series by the fitted model yields a residual series of uncorrelated abnormal logarithmic volume $u_{t,2}$. This is the second volume variable used in the empirical investigation. A graphical illustration of the data for the S&P 500 and the Topix is given in Figure 1.

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[9]A value of $5 \cdot 10^6$ is chosen for the smoothing parameter in the Hodrick and Prescott filter. The results are not sensitive with respect to the choice of this parameter. Common alternative filtering techniques are rolling centered moving averages.

[10]To keep matters simple, we did not model potential monthly seasonality. For previous empirical results on weekday effects see also Foster and Viswanathan (1993).

5. Empirical Results

The introduction raised a number of issues concerning the relation between return and volume. Based on the adjusted volume series, the relation between fat-tailed return distributions and trading volume is examined, where the information flow interpretation of the directing process and its empirical validation is of special interest. We also analyze the role of volume with respect to conditional expected return and a possible asymmetric response to positive versus negative return innovations. Finally, the extremal dependence between return variability and volume is addressed.

5.1. Estimates of the GEV Distribution

A statistical method of making inferences about the tails of the return distributions is by estimating the parameters of the GEV distribution. In the finance literature, this approach has been used for example by Longin (1996).

Two different approaches are commonly applied for inferences about the GEV given in equation (3.7): maximum likelihood estimation (MLE) and the method of probability weighted moments (PWM). Considering MLE, Prescott and Walden (1980) point out that the usual regularity conditions hold under the assumption $\xi > -1/2$. PWM-estimation is an alternative approach, which is especially promising in small samples. Hosking et al. (1985) derive the method under the assumption $\xi < 1$. Valid asymptotic standard errors follow with $\xi < 1/2$. Fortunately, these parameter restrictions do not imply severe limitations in financial applications where typically $0 \leq \xi < 1/2$.

Due to its better theoretical foundation and a moderate given sample size, ML-estimation is chosen for fitting the marginal GEV distributions. The PWM-estimator is used additionally in order to check for a proper convergence of the iterative MLE. In case of non-convergence of the MLE, PWM estimators are given, which is indicated in the presentation of our results (Tables I and III).

Table I: Estimates of the GEV for Absolute Returns and Abnormal Volume

The Table gives parameter estimates of marginal extreme value distributions for the maxima of the absolute returns $|r_t|$ and the uncorrelated abnormal volume data $u_{t,2}$. The estimates are calculated by maximum likelihood if not otherwise stated. $\ln L$ is the log-likelihood at the estimated parameter vector.

| | max($/r_{tl}/$) | | | | max(u_{t2}) | | | |
	m	y	x	$\ln L$	m	y	x	$\ln L$
AEX	0.0153 (0.000703)	0.00619 (0.000475)	0.251 (0.0808)	396.4	0.431 (0.0196)	0.192 (0.0167)	0.160‡ (0.120)	–2.45
CAC	0.0194 (0.000656)	0.00627 (0.000539)	0.194 (0.0777)	399.2	0.395 (0.0161)	0.160 (0.0170)	0.306 (0.106)	9.94
DAX	0.0175 (0.000839)	0.00764 (0.000693)	0.298 (0.0811)	368.6	0.414 (0.0247)	0.216 (0.0209)	0.294 (0.118)	–25.8
FTS	0.0143 (0.000449)	0.00432 (0.000384)	0.213 (0.0991)	441.6	0.322 (0.0138)	0.128 (0.0109)	0.309 (0.0753)	35.5
HSI	0.0223 (0.000951)	0.00876 (0.000807)	0.438 (0.0848)	342.8	0.465 (0.0192)	0.187 (0.0167)	–0.0630‡ (0.0599)	15.8
SPX	0.0143 (0.000545)	0.00526 (0.000480)	0.230 (0.0741)	417.6	0.252 (0.0119)	0.112 (0.00944)	0.0910‡ (0.0702)	66.4
TPX	0.0186 (0.00110)	0.00724 (0.000963)	0.268 (0.0707)	354.4†	0.419 (0.0187)	0.183 (0.0160)	0.0764‡ (0.102)	8.93

All estimates apart form those marked with ‡ are significantly different from zero at the 95% confidence level. † PWM parameter estimate since MLE does not converge after a given maximum of 100 iterations, $\ln L$ is calculated at the estimated PWM parameters.

The parameters of the extreme value distribution are estimated based on monthly extrema which are chosen as a compromise between convergence towards the extreme value model and the available amount of data. Monthly subsample maxima $r_{m,T}$ are extracted from the absolute daily return observations $|r_t|$. For the volume data, we pick monthly maxima $u_{m,T}$ from the filtered abnormal volume data $u_{t,2}$, where $m = 1, ..., 118$. For a detailed description of the estimation technique see also Embrechts et al. (1997).

Parameter estimates are summarized in Table I. The results show that the maximal absolute returns of all markets are in the domain of attraction of the Fréchet extreme value distribution, i.e. they exhibit Pareto-like tail behavior. All the estimates of the tail coefficient ξ are significantly larger than zero. The estimate of the tail coefficient is largest for the Hang Seng index. The estimates for the other markets range from 0.298 for the DAX index down to 0.194 for the CAC index, indicating decreasingly fat-tails in these markets. Interestingly, the maxima from the filtered volume series provide evidence partly for the Fréchet, partly for the Gumbel extreme value distribution where $\xi = 0$. Note that the Gumbel law indicates exponential tail behavior, which is given for example under the normal distribution.

5.2. Overall Return-Volume Dependence

The estimation of the GARCH-M models in equations (3.2) to (3.5) is based on the quasi maximum likelihood (QML) methodology. Deviations from normality in the noise terms are accounted for by using the Bollerslev and Wooldridge (1992) approach. This yields the parameter estimates $(\widehat{\mu}, \widehat{\rho}, \widehat{\lambda}, \widehat{\omega}_0, \widehat{\omega}_1, \widehat{\omega}_2, \widehat{\omega}_3, \widehat{\omega}_4)$ and their standard errors.

Apart from fitting the standard GARCH model of Section 3.2 (model 1), the models with volume variable in the conditional variance equation (model 2 and model 3) are fitted each alternatively with the realizations $u_t = u_{t,1}$ or $u_t = u_{t,2}$. A judgment of the empirical fit of the five different model specifications is based on the Akaike information criterion (AIC). The latter is defined as the fitted log-likelihood penalized for the number of parameters in the model, given by: AIC $\equiv -2 \cdot \widehat{\ln L} + 2 \cdot \#(parameters)$.

Table II: Empirical Fit of Alternative GARCH Models

AIC is the Akaike information criterion derived as the log-likelihood at the estimated parameter vector penalized for the number of parameters. The s-statistic denotes sample skewness, the k-statistic sample excess kurtosis of the returns r_t or the fitted model residuals. $u_{t,1}$ denotes abnormal logarithmic volume, $u_{t,2}$ denotes uncorrelated abnormal logarithmic volume.

	r_t: s / k	Model 1: AIC	s / k	Model 2 $u_{t,1}$: AIC	s / k	$u_{t,2}$: AIC	s / k	Model 3 $u_{t,1}>0$: AIC	s / k	$u_{t,2}>0$: AIC	s / k
AEX	−0.267 / 5.51	−6.65	−0.610 / 5.86	−6.52	−0.287 / 5.00	−6.55	−0.494 / 3.73	−6.67	−0.394 / 4.10	−6.69~	−0.313 / 3.80
CAC	−0.156 / 3.11	−6.25	−0.441 / 3.67	−6.25	−0.397 / 3.12	−6.26~	−0.441 / 3.51	−6.25	−0.437 / 3.57	−6.25	−0.444 / 3.83
DAX	−0.961 / 14.57	−6.28	−1.27 / 15.83	−6.20	−1.483 / 17.31	−6.31	−1.17 / 10.15	−6.32	−0.601 / 5.92	−6.42~	−0.542 / 3.44
FTS	0.0617 / 2.19	−6.90	−0.00631 / 1.24	−6.80	−0.0360 / 1.19	−6.82	−0.0565 / 1.11	−6.94	−0.223 / 0.917	−6.95~	−0.179 / 0.630
HSI	−1.88 / 37.31	−5.92	−0.730 / 6.41	−5.75	−1.41 / 14.07	−5.82	−1.34 / 12.67	−5.95	−0.532 / 5.01	−6.03~	−0.691 / 3.92
SPX	−0.671 / 7.96	−6.90	−1.00 / 8.45	−6.71	−0.750 / 7.47	−6.79	−0.528 / 4.54	−6.95	−0.602 / 4.05	−6.98~	−0.892 / 6.67
TPX	0.337 / 6.93	−6.47	0.249 / 2.84	−6.47	0.249 / 2.84	−6.29	−0.0455 / 2.93	−6.47	0.251 / 2.86	−6.51~	−0.0329 / 2.67

~ denotes minimal AIC among all five GARCH-M specifications.

Table III: Parameters of the GEV for Absolute GARCH-M Model Residuals

The Table gives parameter estimates of marginal extreme value distributions for the maxima of the absolute fitted residuals from model 2 and model 3 each with uncorrelated abnormal volume in the conditional variance equation. The estimates are calculated by maximum likelihood if not otherwise stated. $\ln L$ is the log-likelihood at the estimated parameter vector.

	Model 2				Model 3			
	m	y	x	$\ln L$	m	y	x	$\ln L$
AEX	0.0153 (0.000702)	0.00615 (0.000468)	0.240 (0.0835)	397.8	0.0153 (0.000705)	0.00616 (0.000469)	0.238 (0.0841)	397.8
CAC	0.0194 (0.000671)	0.00515 (0.000636)	0.178 (0.0723)	398.7	0.0194 (0.000668)	0.00634 (0.000522)	0.181 (0.0736)	398.8
DAX	0.0172 (0.000815)	0.00750 (0.000651)	0.309 (0.0821)	370.1	0.0171 (0.000782)	0.00721 (0.000673)	0.345 (0.0860)	372.3
FTS	0.0143 (0.000467)	0.00449 (0.000395)	0.183‡ (0.0982)	439.1	0.0142 (0.000472)	0.00456 (0.000429)	0.188‡ (0.111)	480.4
HSI	0.0219 (0.000965)	0.00871 (0.000841)	0.464 (0.0918)	327.1	0.0216 (0.000928)	0.00848 (0.000825)	0.474 (0.0883)	329.6
SPX	0.0142 (0.000535)	0.00524 (0.000482)	0.197 (0.0744)	420.4	0.0138 (0.000520)	0.00522 (0.000573)	0.238 (0.0912)	418.1
TPX	0.0167 (0.000686)	0.00646 (0.000565)	0.253 (0.0545)	363.9†	0.0166 (0.000674)	0.00602 (0.000581)	0.246 (0.0542)	364.7†

All estimates apart form those marked with ‡ are significantly different from zero at the 95% confidence level. † PWM parameter estimate since MLE does not converge after a given maximum of 100 iterations, $\ln L$ calculated at the estimated PWM parameters.

5.2.1. Empirical Fit to the Specified Log-Likelihood

In Table II the AIC statistic is given as a diagnostic for overall model fit of the five different GARCH specifications.[11]

The results show that model 3 with the uncorrelated abnormal volume variable provides the most suitable model specification. When using the AIC to select the model with the best fit for each market, this model is chosen for six out of seven markets. The exception is the French market, where all estimated specifications show results that are equivalent to the model without volume variable. The AIC statistics for model 2 also demonstrate that volume *per se* does not necessarily improve model fit when taking the increased number of model parameters into account.

These findings on overall model fit indicate that positive innovations to the volume variable have superior power in explaining conditional return variance in the given GARCH model setting. This is somewhat surprising considering the fact that roughly half of the volume information is neglected. However, based on the discussion in Section 3.1, the finding provides evidence for the information flow interpretation of the directing process. This interpretation is also consistent with uncorrelated abnormal volume providing a better fit than the raw abnormal volume variable. As uncorrelated above-average volume –surprise volume– is a superior proxy for unobservable information flow, the results suggest that private information flow rather than volume per se is correlated with return volatility.[12]

5.2.2. Distribution of the Model Residuals

In the following, the model residuals are given by the standardized residuals $\widehat{\epsilon}_t/\widehat{\sigma}_{t,\mathcal{F}_t}$ of the fitted GARCH models. The distributional characteristics of the residuals provide evidence on the extent to which fat-tails in the return series can

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[11]While private information flow is approximately controlled for by the volume variable in models 2 and 3, the effect of a possible accumulation of common information arrivals on Mondays is not captured by the GARCH models. Note that empirically a model extension including a Monday-dummy variable does not seem too promising. A Monday-dummy shows slight AIC improvements for the Dutch and the Hong Kong market, while the AIC increases or stays constant in all other cases. The estimated parameters of our models remain basically unchanged.

[12]This result does not imply however, that total volume is generally non-informative. Espejel (1997) provides empirical evidence in a microstructure context showing that below average volume can also be informative although to a weaker extend than above average volume.

be explained by the fitted model specifications.[13]

First, we consider kurtosis estimates. Results for the raw return series are reported in the first column of Table II which demonstrate significant excess kurtosis relative to the normal for all markets. The estimates range from a moderate value of 2.19 for the U.K. to up to 37.31 for the Hong Kong market. Table II further provides estimates of excess kurtosis of the residual series from the GARCH models. The GARCH model without volume variable (model 1) yields a decrease in the estimated excess kurtosis of model residuals versus raw returns only for three out of the seven markets (U.K., Hong Kong and Japan). Obviously, the model without volume variable does not unambiguously explain fat-tailedness of the raw return series. Adding the volume variable reduces excess kurtosis in the fitted model residuals to relatively moderate levels for all markets but the French. The largest reduction in excess kurtosis is achieved under the model with the best fit according to the AIC criterion. Excess kurtosis estimates below values of four result for the AEX, the CAC, the DAX , the Topix and even for the Hang Seng index. The reduction in sample kurtosis is relatively low for the S&P 500. Interestingly, sample kurtosis nearly vanishes for the fitted GARCH residuals of the U.K. market.

Table II also reports sample skewness. Most market return distributions are negatively skewed, where the U.K. and Japanese markets are exceptions. Return sample skewness is larger than sample skewness in the residuals from model 3 (with uncorrelated volume variable) for the DAX, the Hang Seng and the Topix, while it is smaller for all the other markets. This indicates that the GARCH model with volume variable explains some of the skewness in the returns of the DAX, the Hang Seng and the Topix. For the other markets, smaller sample skewness together with reduced sample kurtosis in the model residuals as compared to raw returns suggests that fat-tailedness for positive returns is better explained than fat-tailedness for negative returns. We conclude that in contrast to kurtosis, the results do not reveal a common pattern on the relation between skewness and the GARCH cum volume effects.

Considering the results on the asymptotic extremal distributional properties

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[13]Correlation analysis reveals that there is some positive dependence between the volume variable and the standardized model residuals. Fitting a generalized model where the volume variable also enters the conditional return equation (3.2), shows that the correlation vanishes while the in-mean term and especially volume jointly explain conditional return. All our following basic results on the distribution of the model residuals and the significance of the other model parameters remain the same.

of the residuals shows that there is still strong evidence of fat-tailedness. Table III reports the estimated GEV parameters for the absolute residuals of GARCH model specifications 2 and 3. The hypothesis of exponential tail behavior (i.e. a Gumbel type extreme value distribution) has to be rejected for all indices with the exception of the FTSE 100. This asymptotic result obviously confirms the finding of low sample excess kurtosis for the U.K. market (see also Table II). Comparing the results of Table III with those of Table I shows that the residuals for the AEX, CAC, FTS, SPX and TPX yield roughly the same or lower estimates of the tail coefficient while the estimates are even larger for the DAX and the HSI data. Hence, although sample kurtosis decreases substantially for most markets when GARCH effects and return variability-volume dependence is taken into account, the fitted GARCH models do not offer a complete explanation for fat-tailed equity returns.
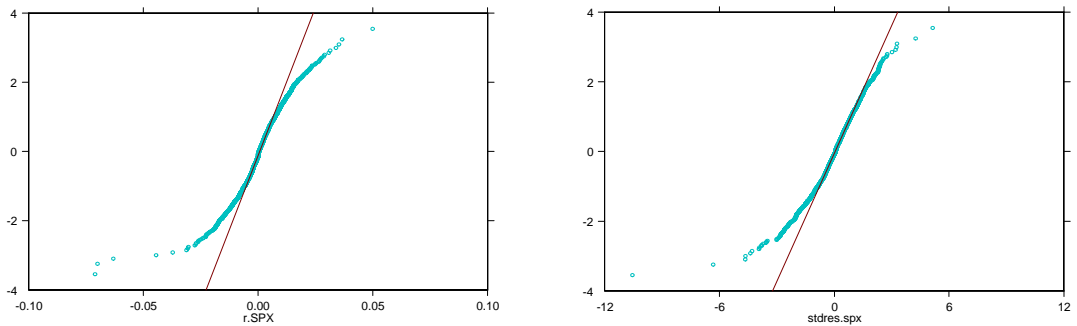

Figure 2: S&P 500 QQ-plots of returns (left) and standardized residuals (right) on the x-axis each against quantiles of the normal distribution.

An illustration of these observations on fat-tailedness is given in Figures 2 to 4, where return or residual quantiles are plotted against the quantiles of a theoretical distribution function yielding so-called QQ-plots; see e.g. Embrechts et al. (1997). All residual series are taken from GARCH model specification 3 with the uncorrelated volume variable. Figure 2 shows S&P 500 return and residual quantiles in comparison to normal quantiles. The deviations from the plotted straight line demonstrate fat-tailed behavior compared to the normal distribution. As can be seen in the plot of Figure 3, the student-t distribution with four (eight) degrees of freedom shows a good fit to negative (positive) S&P 500 residuals. QQ-plots for the CAC and the Topix model residuals are given in Figure 4. It

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can generally be noted that the class of student distributions provides a useful distributional approximation. However, the plots also show single outlying tail observations which are not captured by the student distribution.

Figure 3: S&P 500 QQ-plots of standardized residuals on the x-axis against quantiles of the student-t distribution with four degrees of freedom (left) and with eight degrees of freedom (right).
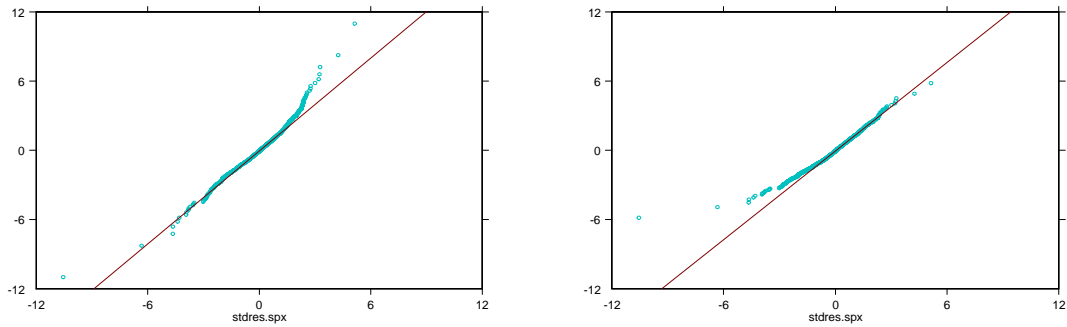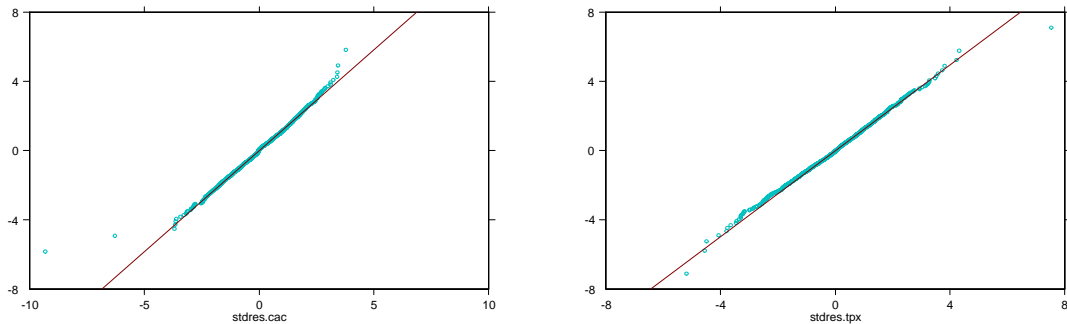

Figure 4: QQ-plot of standardized CAC residuals on the x-axis against quantiles of the student-t distribution with eight degrees of freedom (left). QQ-plot of standardized Topix residuals on the x-axis against quantiles of the student-t distribution with six degrees of freedom (right).

Table IV: Time Series Properties of the GARCH Model Residuals

Sample autocorrelations, partial autocorrelations and Ljung-Box Q-statistics for the standardized residuals and squared standardized residuals of model 3 with uncorrelated positive abnormal volume in the conditional variance equation.

	lag l	e/s:	SACF	SPACF	$Q(l)$	Prob.	e^2/s^2:	SACF	SPACF	$Q(l)$	Prob.
AEX	1		0.024	0.024	1.5333	0.216		-0.045**	-0.045**	5.3350**	0.021
	2		-0.010	-0.011	1.8151	0.404		-0.011	-0.013	5.6400	0.060
	3		0.032	0.032	4.4472	0.217		0.019	0.018	6.6170	0.085
	4		0.007	0.005	4.5688	0.334		0.049**	0.050**	12.751**	0.013
CAC	1		0.004	0.004	0.0393	0.843		-0.017	-0.017	0.7358	0.391
	2		0.009	0.009	0.2414	0.886		0.001	0.001	0.7411	0.690
	3		-0.002	-0.002	0.2485	0.969		0.004	0.005	0.7932	0.851
	4		0.041**	0.041**	4.5709	0.334		0.009	0.009	0.9939	0.911
DAX	1		0.068**	0.068**	12.061**	0.001		-0.024	-0.024	1.5002	0.221
	2		0.006	0.001	12.149**	0.002		0.020	0.020	2.5788	0.275
	3		0.036	0.035	15.450**	0.001		0.065**	0.066**	13.633**	0.003
	4		0.010	0.006	15.728**	0.003		0.069**	0.072**	25.912**	0.000
FTS	1		0.025	0.025	1.6359	0.201		-0.014	-0.014	0.5258	0.468
	2		0.019	0.018	2.5490	0.280		0.092**	0.092**	22.253**	0.000
	3		-0.009	-0.010	2.7728	0.428		0.076**	0.080**	37.291**	0.000
	4		-0.004	-0.004	2.8248	0.588		0.029	0.023	39.449**	0.000
HSI	1		0.051**	0.051**	6.3062**	0.012		-0.013	-0.013	0.4148	0.520
	2		0.043**	0.040**	10.840**	0.004		-0.017	-0.017	1.1223	0.571
	3		0.078**	0.074**	25.886**	0.000		0.024	0.023	2.4945	0.476
	4		0.041**	0.032	29.971**	0.000		-0.007	-0.007	2.6229	0.623
SPX	1		-0.006	-0.006	0.0974	0.755		-0.005	-0.005	0.0720	0.788
	2		-0.019	-0.019	1.0718	0.585		0.034	0.034	2.9721	0.226
	3		-0.042**	-0.042**	5.5432	0.136		0.047**	0.047**	8.5735**	0.036
	4		-0.026	-0.027	7.2670	0.122		0.034	0.034	11.584**	0.021
TPX	1		0.039**	0.039**	3.8554**	0.050		-0.021	-0.021	1.0949	0.295
	2		0.038	0.037	7.6628**	0.022		-0.021	-0.021	2.1792	0.336
	3		0.032	0.029	10.342**	0.016		0.010	0.009	2.4277	0.489
	4		0.048**	0.044**	16.212**	0.003		0.010	0.010	2.6694	0.615

** denotes significance at the 95% confidence level. Note that the assumed $1/\sqrt{T}$ standard errors for the sample (partial) autocorrelations are conservative bounds when testing for dependence in the squared residuals (see the discussion in the text part).

5.2.3. Time Series Properties of the Model Residuals

The time series properties of the residuals from model 3 which contains the uncorrelated volume variable $u_{t,2}$ are examined in Table IV. The given statistics test for dependence in the residuals and their squared values. They include sample autocorrelations (SACF), sample partial autocorrelations (SPACF) and Ljung/Box statistics $Q(l)$ up to a lag of $l = 4$ trading days. It is well-known that for *iid* random variables with finite variance, the sample autocorrelations are asymptotically normal with a standard error of $1/\sqrt{T}$. The Ljung/Box statistic $Q(l)$ tests for uncorrelatedness up to lag l, where, again, the variance of the variables has to exist.

The results for the GARCH residuals indicate that the AR(1) term in the conditional return equation (3.2) cannot account for the dependence in all of the index returns. Based on the $Q(4)$-statistic, uncorrelatedness has to be rejected for three markets. While the DAX and the Topix shows evidence of an AR(2) dependence structure, the Hang Seng yields highly significant PACF estimates up to lag three. Market microstructure might explain part of these results.[14] The implication of this model violation is weak, however. Autocorrelation in the residuals can be reduced by introducing additional higher order autoregressive terms in the conditional return equation (3.2). Additional estimation results not reported here, show that changes in parameter estimates are small and do not impact our following conclusions.

As the results of Table III indicate, the statistics for examining the dependence structure of the *squared* residuals in Table IV must be interpreted more cautiously. The hypothesis $\xi > 0.25$ cannot be rejected for the model residuals based on common significance levels. Therefore, the existence of the fourth moment of the residual series is questionable. Mikosch and Stărică (2000), for example, point out that, when the moment is nonexistent, convergence of the sample autocorrelation function is much slower than the usual $1/\sqrt{T}$–rate, since the limiting distribution will be given by a stable law. The empirical results in Table IV show that the autocorrelation statistics for the CAC, the Hang Seng and the Topix stay within their usual $1/\sqrt{T}$–bands, indicating that the squared residuals are uncorrelated. For the Dutch and the U.S. market, autocorrelation statistics lie somewhat outside

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[14]In the Japanese market, limits are imposed on daily stock price changes. Comparable trading rules are given in Hong Kong, where admissible price ranges are defined depending on current bid and ask prices, the previous closing price and the lowest transaction price of the day.

the $1/\sqrt{T}$–bands. Evidence against uncorrelatedness is strongest for the DAX and especially the FTSE 100 index. Here, the model might not capture volatility persistence completely. Still, the results indicate that the GARCH model with volume variable can adequately describe a great amount of dependence in squared returns, which is a highly significant feature for all markets.

5.2.4. Results: Normal Market Conditions

In the following, the discussion of the estimation results is based on the GARCH model specifications 1 and 3, the latter containing the uncorrelated volume variable $u_{t,2}$. Table V gives the results for the model with no volume variable, Table VI gives the results for the model with volume variable.

Conditional Variance Considering the return variability-volume dependence, the uncorrelated abnormal volume variable provides strong evidence for the mixture of distributions hypothesis. Six out of seven parameter estimates of ω_4 in Table VI are significant at the 99 percent confidence level.

The CAC-index is the only data set showing an insignificant positive return variability-volume dependence. Note that although the volume variable is insignificant, the constant ω_0 in the variance equation has the lowest significance level in the sample, which indicates that model 3 describes time variation in variance quite well.[15] Another exception in the conditional return variability-volume dependence is found for the U.S. market. The S&P 500 is the only index where the volume variable is highly significant while the GARCH coefficients are statistically insignificant. Comparing the results of Table VI to those reported in Table V, volume obviously captures large parts of the volatility persistence in returns which is highly significant in a model without the volume variable. By setting the variables equal to their unconditional means in the conditional variance equation (3.5), our results indicate that roughly 60 percent of the U.S. market volatility can be attributed to common information arrivals together with statistically insignificant volatility persistence, whereas roughly 40 percent can be attributed to above-average trading activity.

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[15]These particular results for the French market may relate to the definition of the CAC volume series; see `www.euronext.fr` for details. The authors thank Bertrand Maillet for pointing out this issue.

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Table V: GARCH Model without Volume Variable

Estimation results for the GARCH-M model without volume variable (model 1). The *t*-statistics are based on variance estimates according to Bollerslev and Wooldridge (1992).

	m	r	l	w_0	w_1	w_2	w_3	w_4
AEX	0.000426 (1.17)	0.0192 (0.89)	3.08 (0.67)	$3.48\ 10^{-6}$*** (2.77)	0.0624*** (3.38)	0.0414 (1.27)	0.875*** (38.48)	--
CAC	−0.000475 (−0.69)	0.0500** (2.40)	7.33 (1.26)	$7.79\ 10^{-6}$*** (2.63)	0.0300** (2.18)	0.0821*** (3.08)	0.863*** (30.16)	--
DAX	0.000228 (0.45)	0.0292 (0.99)	2.63 (0.63)	$1.03\ 10^{-5}$** (2.27)	0.0525** (2.37)	0.130 (1.36)	0.803*** (14.68)	--
FTS	−0.000607 (−1.31)	0.0657*** (3.12)	17.27** (2.20)	$1.76\ 10^{-6}$*** (3.63)	0.0317*** (2.90)	0.0337*** (2.64)	0.923*** (65.51)	--
HSI	0.000519 (1.21)	0.142*** (5.62)	1.32 (0.53)	$1.44\ 10^{-5}$*** (3.26)	0.0549* (1.92)	0.192*** (3.17)	0.780*** (21.96)	--
SPX	0.0000471 (0.15)	0.0354* (1.67)	7.15 (1.32)	$1.28\ 10^{-6}$*** (3.22)	0.0164 (1.42)	0.0428** (2.46)	0.941*** (62.41)	--
TPX	−0.0000199 (−0.079)	0.124*** (5.77)	−0.00162 (−0.0006)	$1.88\ 10^{-6}$*** (3.64)	0.0277** (2.27)	0.142*** (4.83)	0.892*** (61.76)	--

*, ** and *** denotes significance for a double-sided test at the 90%, 95% and 99% confidence level, respectively.

Table VI: GARCH Model with Volume Variable

Estimation results for the GARCH-M model with uncorrelated positive abnormal volume in the conditional variance equation (model 3 with $u_{t,2}$). The t-statistics are based on variance estimates according to Bollerslev and Wooldridge (1992).

	m	r	l	w_0	w_1	w_2	w_3	w_4
AEX	0.000228 (0.86)	0.0222 (0.96)	5.67 (1.55)	$2.27\ 10^{-6}$** (2.02)	0.0781*** (2.89)	0.139** (2.55)	0.736*** (20.56)	$7.51\ 10^{-5}$*** (4.85)
CAC	−0.000481 (−0.66)	0.0503** (2.42)	7.36 (1.20)	$7.87\ 10^{-6}$* (1.68)	0.0230* (1.86)	0.0856*** (3.06)	0.854*** (24.10)	$8.76\ 10^{-6}$ (0.52)
DAX	−0.00171*** (−6.27)	−0.0366 (−0.97)	17.81*** (5.34)	$2.03\ 10^{-5}$*** (7.12)	0.0838** (2.07)	0.224*** (2.77)	0.360*** (7.50)	$3.77\ 10^{-4}$*** (5.92)
FTS	−0.00202*** (−4.38)	0.0319 (1.37)	39.79*** (4.59)	$2.81\ 10^{-5}$*** (9.16)	0.00158 (0.087)	0.116*** (3.41)	0.209*** (3.67)	$1.91\ 10^{-4}$*** (8.44)
HSI	−0.000785*** (−3.07)	0.0927*** (3.94)	6.48*** (5.59)	$1.04\ 10^{-5}$* (1.73)	0.0144 (0.46)	0.326*** (5.37)	0.600*** (9.65)	$2.83\ 10^{-4}$*** (7.43)
SPX	−0.00172*** (−3.69)	0.0127 (0.40)	35.68*** (4.22)	$3.31\ 10^{-5}$*** (8.37)	0.134 (1.31)	−0.0623 (−0.57)	0.0107 (0.230)	$4.31\ 10^{-4}$*** (6.94)
TPX	−0.000227*** (−7.12)	0.116*** (5.14)	14.03*** (5.36)	$4.98\ 10^{-6}$** (2.52)	0.0405 (1.49)	0.312*** (5.17)	0.647*** (18.15)	$1.34\ 10^{-4}$*** (5.63)

*, ** and *** denotes significance for a double-sided test at the 90%, 95% and 99% confidence level, respectively.

The result of a statistically insignificant volatility persistence component for the U.S. market is particularly interesting as it is achieved under the uncorrelated volume variable. A possible explanation for this puzzling finding is that random common and private information arrivals are positively cross-correlated. When common information events cause private information flow and hence above-average volume as indicated by the empirical findings in Hiemstra and Jones (1994), volatility persistence is found in model 1 while abnormal volume captures volatility persistence in model 3. Our results for the U.S. market are therefore consistent with the hypothesis that volatility persistence is explainable by return-volume causality.

For all other markets, the ω_2 and the ω_3 coefficients in Table VI are significant at the 99 percent level. Comparing the results to those in Table V shows that volume can at least partly explain volatility persistence in these markets as well, since the magnitude of the ω_3 coefficients decreases remarkably. The volume variable accounts for roughly 25 percent of market volatility for the AEX, the DAX and the FTSE 100 and for about 10 percent in the case of the Hang Seng and the Topix.

Evidence of the so-called leverage effect in Table V shows that the effect is significant for five out of seven markets in model 1. When the volume variable is added, the ω_2 coefficient becomes significant for the Dutch and German markets, but insignificant for the U.S. market (see Table VI). The latter result for the U.S. market is in line with the finding of volume dominating GARCH effects. The results for the non-U.S. markets suggest that the leverage effect is stronger than the standard ARCH effect, i.e. there is evidence of an asymmetric response of volatility with respect to positive and negative return innovations.

Conditional Return As the mixture of distributions model does not make predictions about the risk-return relation and since the implications of economic theory are ambiguous, it is of particular interest to empirically study the role of volume. The results presented here show that volume can increase the significance of the GARCH in-mean component. Our finding of a positive return-variance relation is in accordance with the results for the U.S. market in Gallant et al. (1992).[16]

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[16]Using monthly data, Gennotte and Marsh (1993) find a positive, though not highly significant relation. A negative relation for monthly data is found in Golsten et al. (1993).

Table V illustrates that in the GARCH-M setting without volume, the evidence of a significant conditional risk-return relation is weak. Only the U.K. data show a significant positive relation. When the volume variable is introduced into the model in Table VI, the results change to positive coefficients for all markets, where significance at the 99 percent level is given for five out of seven markets. The results indicate that the risk premium is varying through time, positively depending on conditional variance. As the conditional variance can be explained by volume, it seems that this evidence helps to explain, in an economically plausible way, the puzzling positive volume return premium as documented for example in Karpoff (1987) and more recently in Gervais et al. (2001). However, since the volume variable is essential for the explanation of the conditional risk-return relation, risk considerations can only partly explain a time-varying risk premium in international market returns (see also the footnote in Section 5.2.2). In other words, there is also evidence of a positive high volume premium, i.e. conditional expected returns are closely related to the arrival of private information.[17]

Comparing the results of Table V with those of Table VI furthermore gives some results which show that autocorrelation in index returns is related to abnormal trading volume. Particularly, above-average trading volume can explain positive autocorrelation in U.K. and U.S. index returns (compare also Table IV). Evidence for the other markets of our sample is weaker. The strong autocorrelation in the Asian equity market returns cannot be explained, indicating microstructure issues as outlined in Section 5.2.3.

5.3. Return Variability-Volume Dependence in the Tails

The examination of the return variability-volume dependence in the tails of the joint return-volume distribution is based on estimates of the bivariate extreme value model. A test of the null hypothesis of independence is performed. Additionally, asymmetries in the dependence structure are analyzed considering the predictions of the bad proxy and the misinterpreted-trade hypothesis as outlined in Section 3.3.

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[17]In order to explain the positive sign of the relation, on average, the positive price premium that informed buyers pay to uninformed sellers tends to be larger than the negative price premium informed sellers give to uninformed buyers. This corresponds for example to a microstructure finding by Kalay et al. (2002) who document that buys are more informative and have a larger price impact than sells.

5.3.1. Estimation

Following Tawn (1988), the parameters of the marginal GEV distributions (3.7) are first estimated for the absolute returns and the abnormal volume observations individually. This was done in Section 5.1, where the results were summarized in Table I. In a second step, the dependence parameter of the joint limiting extreme value distribution function $H_\gamma(r, u)$ given by (3.10) is estimated. With the corresponding density function $h_\gamma(r, u)$ and $M = 118$ standardized subsample extremes $\widetilde{r}_{m,T}$ and $\widetilde{u}_{m,T}$ from our dataset, the ML-estimate of the dependence parameter is given by:

$$\widehat{\gamma} = \arg\max_{\gamma \in [1, \infty)} \sum_{m=1}^{M} \ln h_\gamma(\widetilde{r}_{m,T}, \widetilde{u}_{m,T}). \tag{5.1}$$

For $\gamma > 1$ the usual regularity conditions of the MLE are satisfied. The estimate of γ yields the parametric estimate of the dependence function $\widehat{A}_\gamma(w)$.

5.3.2. Testing for Independence

Under independent upper tails of the absolute returns and the abnormal volume variable, the dependence parameter γ equals one. At the boundary $\gamma = 1$, the score function of the mth observation,

$$s_m = \left. \frac{d \ln h_\gamma(\widetilde{r}_{m,T}, \widetilde{u}_{m,T})}{d\gamma} \right|_{\gamma=1}, \tag{5.2}$$

has expectation $E(s_m) = 0$ and variance $E(s_m^2) = \infty$. It can be shown that the normalized score statistic of the log likelihood function then converges to a standard normal distribution:

$$\left[\tfrac{1}{2} M \ln M \right]^{-\frac{1}{2}} \sum_{m=1}^{M} s_m \xrightarrow{d} N(0; 1). \tag{5.3}$$

As the rate of convergence is rather slow, Tawn (1988) recommends the use of simulated critical values for inference in small samples. When the tails of absolute returns and abnormal volume are dependent, i.e. for $\gamma > 1$, it follows that $E(s_m) > 0$.

5.3.3. Asymmetric Dependence

The bivariate extremal model estimated above is based on the assumption of symmetric dependence. The latter implies that dependence is strongest under equally large standardized absolute return and volume extremes and that an equal reduction in the magnitude of either standardized extreme yields the same reduction in dependence as measured by $A(w)$.

Possible asymmetric dependence is for example captured by an alternative non-parametric estimator of the dependence function derived in Pickands (1981). Under unit Fréchet margins, and $\widetilde{w} = [(1/\widetilde{u})/(1/\widetilde{u} + 1/\widetilde{r})] = \widetilde{r}/(\widetilde{r} + \widetilde{u})$, the estimator is given by:

$$\widehat{A}(\widetilde{w}) = M \left(\sum_{m=1}^{M} \min \left\{ [(1 - \widetilde{w})\widetilde{r}_{m,T}]^{-1}; [\widetilde{w}\widetilde{u}_{m,T}]^{-1} \right\} \right)^{-1}, \quad 0 < \widetilde{w} < 1. \tag{5.4}$$

Furthermore, the score statistic (5.2) can provide information about the dependence between extremal return and volume observations versus the magnitude of each extremal observation. In order to test for a linear relation between the score statistic and the magnitude of the extreme observations, the following regression model is used:

$$s_m = \beta_0 + \beta_1 \left| r_{m,T} \right| + \beta_2 u_{m,T} + \eta_m. \tag{5.5}$$

If there is no linear relationship, the constant model,

$$s_m = \alpha + v_m, \tag{5.6}$$

should perform equally well with model (5.5). The coefficient α in the latter model is predicted to be positive in the dependence case.

The estimation of the above models by standard OLS would make statistical inference non-robust with respect to the independence case which, as noted above, implies $E(s_m) = 0$ and $E(s_m^2) = \infty$. In order to robustify the inference, a Huber M-estimator is used. Estimation is carried out by iteratively re-weighted least squares, where a cutoff point of 1.345 is chosen. For details refer for example to Venables and Ripley (1999).

Table VII: Tail Dependence between Volume and Absolute Returns

The correlation coefficient *corr* between absolute returns $|r_t|$ and uncorrelated abnormal volume $u_{t,2}$ for the overall joint distribution is estimated based on the Newey and West heteroskedasticity and autocorrelation consistent procedure. ML-estimates of g are given with standard errors and the normalized score statistic values in parenthesis. $[x \pm 1se]$ denotes estimates of g under the return tail coefficient estimate plus or minus one standard deviation. The extremal correlation coefficient is $corr_g$. Weighted least squares estimates of the coefficients of the regression models are given with asymptotic *t*-statistics in parenthesis.

	corr ($|r_t|$, $u_{t,2}$)	g [$g|x_R \pm 1se$]	$corr_g$ (max($|r_t|$), max($u_{t,2}$))	a	MAD	b_1	b_2	MAD
AEX	0.13	1.18†† (0.0723 / 2.86) [1.20 / 1.15]	0.16	0.29''' (2.66)	0.834	2.73 (0.25)	−1.28*** (−3.12)	0.802
CAC	0.095	1.02 (0.0648 / 0.32) [1.03 / 1.01]	0.020	0.11 (1.06)	1.01	−48.89*** (−5.58)	−5.51*** (−5.55)	0.874
DAX	0.12	1.01 (0.0617 / 0.18) [1.02 / 1.00]	0.013	0.13' (1.29)	0.908	−13.85*** (−2.57)	−1.79*** (−7.89)	0.722
FTS	0.24	1.29†† (0.0864 / 5.51) [1.31 / 1.28]	0.24	0.42''' (3.34)	0.819	58.86*** (2.48)	−0.027 (−0.050)	0.956
HSI	0.21	1.13† (0.0711 / 2.07) [1.15 / 1.11]	0.12	0.16' (1.46)	1.04	0.52 (0.042)	−1.46*** (−2.67)	0.751
SPX	0.26	1.21†† (0.0781 / 7.26) [1.22 / 1.19]	0.18	0.24' (1.50)	0.699	4.85 (0.44)	0.11 (0.089)	0.754
TPX	0.23	1.05† (0.0520 / 1.66) [1.06 / 1.04]	0.050	0.078 (0.57)	1.22	−2.82 (−0.57)	−1.14* (−1.85)	1.05

Assuming the standardized extremes being true values, *, **, ***, ', '', ''' denotes significance at the 90, 95 and 99% confidence level for a double-sided or a single-sided test, respectively. †, †† denote significance at the 90 and 95% confidence level for a single-sided test, respectively, with critical values of 1.61 and 2.39 for a sample size of 118 as given by the results in Tawn (1988, Table 1).

5.3.4. Results: Market Stress

Return Variability-Volume Dependence in the Tails The results for return variability-volume dependence are given in Table VII. In order to compare the estimation results with evidence for the overall unconditional return variability-volume dependence, Newey/West (1987) heteroskedasticity and autocorrelation consistent estimates of the correlation between $|r_t|$ and u_t are given in the first column. The results for the dependence parameter $\widehat{\gamma}$, its standard error and Tawn's normalized score statistic (5.3) are reported in column two. The extremal correlation coefficient, $\widehat{corr}_\gamma(M_{R,T}; M_{U,T}) = \widehat{\gamma}^{-1}\Gamma(2/\widehat{\gamma})^{-1}\Gamma(1/\widehat{\gamma})^2 - 1$, is given in column three.[18] As the tail coefficient estimates $\widehat{\xi}$ are particularly subject to sampling error, a sensitivity analysis is performed. Column two additionally gives the estimates of the dependence parameters which result for a one standard deviation variation in the estimated tail coefficient of the return distribution. The results turn out to be quite stable, showing that a potential errors-in-variables problem due to the estimation of the marginal GEV parameters in a first step is not too severe.

The parametric estimation results in Table VII show that the tail independence hypothesis for the joint distribution of absolute returns and abnormal volume can be rejected for five out of the seven markets in the sample. The two-step estimation procedure gives a confidence level of 90 percent based on Tawn's score statistic under the assumption that the parameters of the GEV distribution are known. The dependence relation is most significant for the U.S. and the U.K. market. For all the markets with exception of the Dutch and the U.K., the estimated extremal correlation between absolute returns and volume is smaller than the overall correlation estimate. For the CAC and the DAX market data, the hypothesis of tail independence for absolute returns and volume cannot be rejected. This particularly indicates that return variability-volume dependence for the DAX, which was found to be highly significant in the GARCH model setting, breaks down in the tails. For the French market the finding is not surprising as the volume variable has no significant explanatory power in the GARCH model. Generally however, it can be concluded that there is evidence of return variability-volume dependence also in the tails of the joint return-volume distribution.

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[18]Note that dependence in the extreme value model is based on the range of the monthly data as measured by the observed monthly maximum. Our Newey/West daily estimator of overall return variability-volume dependence is also a consistent estimator of dependence on a monthly basis.

Evidence on an Asymmetric Dependence Function Referring to Section 5.3.3, non-parametric estimates (5.4) of the dependence function for all indices except the CAC are given in Figure 5. The plots of a smoothed version of the non-parametric estimator suggest some asymmetry particularly for the AEX, DAX and TPX data. While dependence for the DAX is stronger than under the symmetric model for small $w < 1/2$, the same holds for $w > 1/2$ in the case of the AEX and the Topix. Recalling the definition $w = r/(r + u)$, these results indicate that dependence weakens for large (absolute standardized) return observations for the German market and for large (abnormal standardized) volume observations for the Dutch and Japanese markets. As discussed in Section 3.3, this is evidence for the misinterpreted-trade hypothesis, especially for the German market, and evidence of the bad proxy hypothesis for the other two markets.

The regression models (5.5) and (5.6) offer a different perspective in addressing the question of asymmetry. Estimation results are given in columns four to eight in Table VII. The regression results for model (5.6) yield a significantly positive estimate of the intercept α for the AEX, DAX, FTS, HSI and SPX. These results confirm the conclusions drawn from the Tawn-Statistic (5.3). Under the regression model, evidence of dependence turns out to be weaker for the Topix, but stronger for the DAX. The rescaled median absolute deviation (MAD) provides a robust estimate of the standard deviation of the residuals. It shows that for the S&P 500 and the FTSE 100, model (5.5) does not explain variation in the score statistic better than the simple constant model (5.6). We therefore conclude that empirically, there is no linear relationship as suggested by the model. In other words, evidence of asymmetry is weak.

For the other markets, the MAD-statistics indicate that model (5.5) has better explanatory power than model (5.6). Evidence of asymmetry in the price variability-volume dependence is given for the Dutch, Hong Kong and Japanese markets. An examination of the estimated regression equations shows that large extremal volume is significantly negatively correlated with the score-statistic. As in the previous section, this can be interpreted as evidence consistent with the bad proxy hypothesis, i.e. extremal volume is a bad proxy for information flow. The results for the CAC and the DAX show that the score statistic is also significantly negatively correlated with the extreme absolute return observations. This again indicates evidence consistent with the misinterpreted-trade hypothesis. That is, the weakness of the extremal return variability-volume dependence in the French and German markets may be regarded as related to temporary liquidity issues.
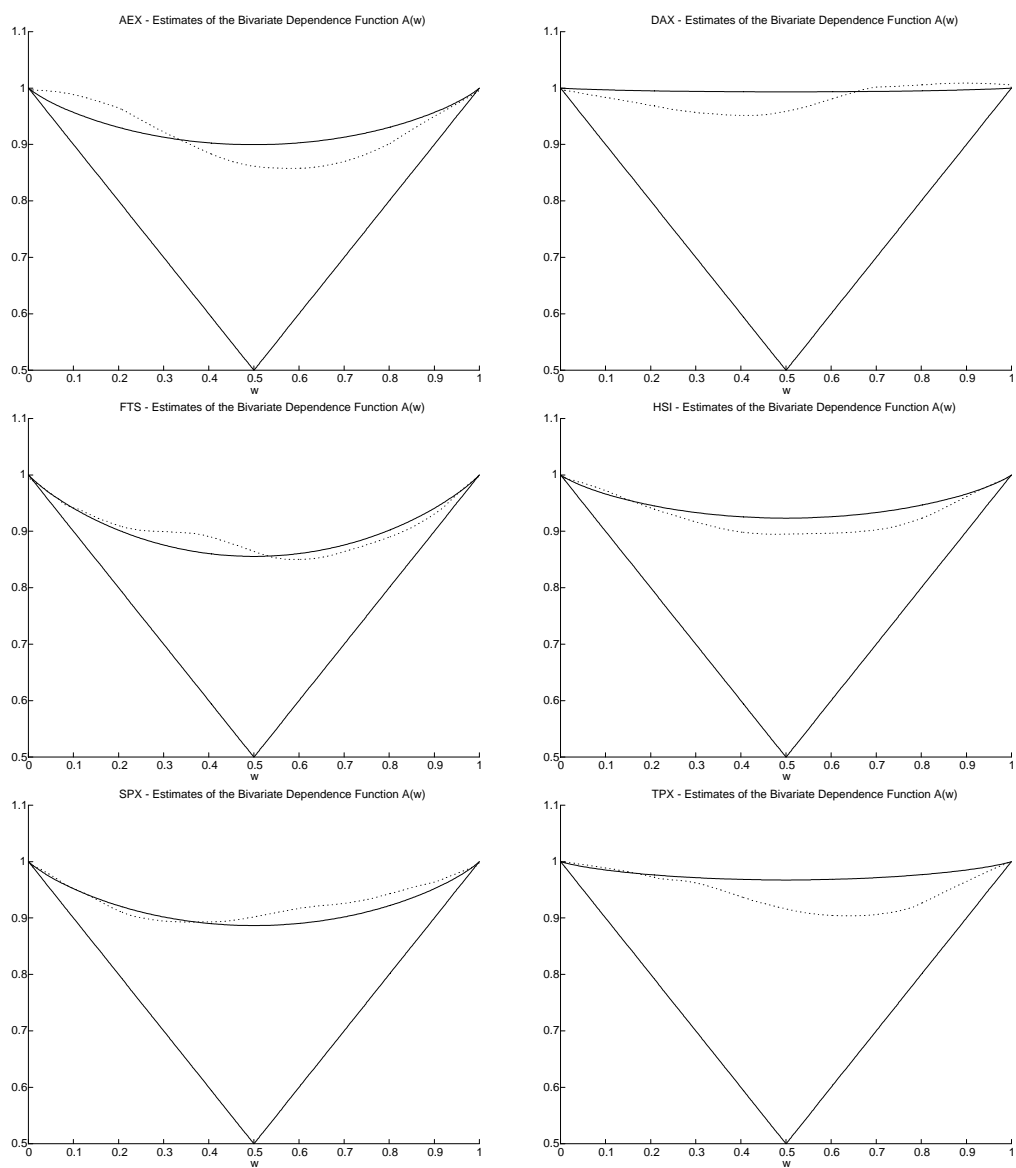
Figure 5: Parametric (solid) and smoothed non-parametric (dotted) estimates of the bivariate dependence function $A(w)$, $w = r/(r + u)$, for the AEX, the DAX, the FTSE 100, the Hang Seng, the S&P 500 and the Topix.

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Table VIII: Tail Dependence between Volume and Absolute, Positive, and Negative Returns

ML-estimates of g are given with the corresponding standard errors in parenthesis. The extremal correlation coefficient is given by $corr_g$.

	g $M{=}118$	$corr_g$ (max($|r_t|$), max(u_{t2}))	g $M{=}118$	$corr_g$ (max(r_t), max(u_{t2}))	g $M{=}118$	$corr_g$ (max($-r_t$), max(u_{t2}))	g $M{=}117$	$corr_g$ (max$_m(-r_t)$, max$_{m-1}(u_{t2})$)
AEX	1.18 (0.0723)	0.16	1.16 (0.0691)	0.14	1.08 (0.0621)	0.076	1' (-)	0
CAC	1.02 (0.0648)	0.020	1' (-)	0	1.02 (0.0625)	0.020	1.15 (0.0710)	0.14
DAX	1.01 (0.0617)	0.013	1' (-)	0	1' (-)	0	1.03 (0.0528)	0.029
FTS	1.29 (0.0864)	0.24	1.24 (0.0775)	0.21	1.10 (0.0630)	0.094	1.07 (0.0752)	0.067
HSI	1.13 (0.0711)	0.12	1.14 (0.0681)	0.13	1.03 (0.0638)	0.029	1' (-)	0
SPX	1.21 (0.0781)	0.18	1.16 (0.0720)	0.14	1.15 (0.0765)	0.14	1' (-)	0
TPX	1.05 (0.0520)	0.050	1.11 (0.0637)	0.10	1' (-)	0	1.01 (0.0577)	0.010

' the MLE of the dependence parameter hits the boundary of the parameter space.

Altogether the results suggest that a weakening of the return variability-volume dependence in the tails can be attributed to the very largest return and especially volume observations. Also, in none of the markets do we see the dependence between returns and volume significantly strengthening for large extremal volume.

Asymmetric Return-Volume Dependence in the Tails   So far the results considered return variability-volume dependence in the tails by examining dependence between monthly maxima of $|r_t|$ and u_t. Table VIII gives a comparison of the results on tail dependence when the dependence parameter γ is estimated separately for three different cases: dependence between maximum r_t and u_t, i.e. the upper tail return-volume dependence, dependence between maximum $-r_t$ and u_t, i.e. the lower tail return-volume dependence, and finally dependence between maximum $-r_t$ and preceding months u_t's, i.e. a one-month lagged lower tail return-volume dependence.

The results in columns two to seven of Table VIII indicate that dependence is weaker on average when not only the size but also the sign of the return observations is considered. For the German, French and the U.S. market, there is little difference in the return-volume dependence in the upper versus lower tail. This is not surprising for the German and the French market where the dependence relation is found to be insignificant. Hence, the U.S. market is the only market in the sample for which return variability-volume dependence in the tails is significant, symmetric, and, as the last two columns of Table VIII indicate, contemporaneous.

For all the other markets with significant extremal return variability-volume dependence, the estimation results indicate that return-volume dependence is stronger in the upper tail of the return distribution.[19]   This provides evidence that the relation between minimal returns and volume is weaker than the relation between maximal returns and volume. Under the plausible assumption that investors do not face more substantial short-sale restrictions in the U.S. as compared to the other markets in our sample, cross-sectional evidence does not seem to be consistent with a hypothesis that crashes occur under releases of hidden information which is in turn accumulated due to short-sale restrictions. Under such hypothesis the return-volume dependence in the lower tail of the return distribution should be stronger for the non-U.S. markets.

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[19]These findings of tail asymmetry are reconfirmed for less liquid emerging markets in a study by Qi (2001); this supports a liquidity story in explaining asymmetry.

When we consider the lagged lower tail relationship in the last two columns of Table VIII, cross-sectional evidence of crash-predictability in the sense of a volume-return lead-lag relation turns out to be weak. Interestingly, lagged dependence is strongest for France, where contemporaneous dependence is weak.

In summary, in none of the markets extreme stock price drops tend to have a higher correlation with volume than do extreme stock price upswings. Overall evidence of crash-predictability in the sense of extremal volume leading extremal return is weak.

6. Conclusion

This paper provides empirical evidence on the return-volume relationship for a cross-section of seven large international equity markets. The results show that trading volume is important to the understanding of expected equity returns, return variability and large price movements. Consistent with the mixture of distributions hypothesis, we find an overall significant return variability-volume dependence. It turns out that the U.S. market, where the S&P 500 proxies the largest and most liquid national stock market portfolio, has characteristics that appear more straightforward than those for the other markets in the study. For the U.S. benchmark, surprise volume explains ARCH- as well as leverage-effects and, under market stress, the return variability-volume dependence remains significant and symmetric.

Referring to the cross-section of equity markets, our empirical results provide several implications which shed a new light on the return variability-volume relation in international stock markets:

- As opposed to standard applications of volume in financial modeling, we find surprise volume to be a superior proxy for unobservable information flow. Apart from being more informative, surprise volume, within a non-linear specification of the conditional variance equation, significantly drives conditional volatility and expected return dynamics.

- Surprise volume, which is time-series uncorrelated, is found to explain fat-tails and particularly ARCH-effects. This is contrary to a hypothesis in the mixture of distributions literature where ARCH-type volatility persistence is assumed to be due to time series correlation in the proxy variable for unobservable information flow.

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- Surprise volume explaining ARCH-effects is consistent with return-volume causality as documented in Hiemstra and Jones (1994). When large return innovations cause lagged volume innovations, these may exhibit low time series correlation and still explain lagged increased conditional return volatility.

- Asymmetric volatility, or the so-called leverage effect, appears to be a feature which vanishes for the U.S. market but partly strengthens for the other markets once surprise volume is introduced. It seems an open question as to whether negative returns cause larger lagged heterogeneity among investors than positive returns.

Our empirical findings are partly consistent, partly inconsistent with models for understanding the mechanics of financial crises and market stress. The return-volume relation in the tails of the joint return-volume distribution exhibits the following characteristics:

- The price variability-volume dependence is significant, albeit weaker in the tails. Further analysis suggests that weaker dependence is mostly due to the very largest absolute return and especially large volume observations. This may indicate large volume being a bad proxy for private information flow.

- Estimating dependence in the upper return-volume and lower return-volume tail separately, indicates that, on average, the return-volume dependence is weaker for minimal returns and volume than for maximal return and volume. This finding is inconsistent with the mixture model which does not provide implications regarding such asymmetric dependence.

- Empirical evidence of asymmetric dependence appears to be consistent with the non-linearity in price elasticity as put forward with the Gennotte and Leland (1990) mis-interpreted trade hypothesis. Large run-ups in prices tend to be accompanied by corresponding trading activity, while large price drops may occur without substantial trading volume. Evidence by Haugen et al. (1991) shows that such drops also tend not to be explainable by public information releases. This supports a liquidity explanation of drops consistent with the Gennotte and Leland model.

- In none of the markets do we see the dependence between return variability and volume strengthen for large extremal volume. Also, evidence of asymmetric dependence predicts price drops under moderate volume. These findings appear to be inconsistent with models predicting negative skewness and hence the probability of crashes to be larger conditional on high levels of trading volume such as the Hong and Stein (2003) hidden information hypothesis for market crashes.

- While negative skewness in stock returns was found to be predictable by preceding abnormal trading volume in Chen et al. (2001), the authors indicate that forecasting crashes remains a difficult task (p. 348). We test for extremal dependence in extreme abnormal volumes during one month and subsequent month's extreme returns. Our evidence of crash-predictability in the sense of extremal volume leading extremal return is weak.

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