

Information Asymmetry,
Corporate Debt Financing and Optimal
Investment Decisions: A Reduced Form Approach

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Abstract

Under the assumption of information asymmetry between market investors and firm managers, a reduced form model of a firm is developed in order to derive optimal investment strategies and capital structures while taking into account the effects of dividend policies and taxes. The motivation of the reduced form approach lies in its empirical implementation tractability. Closed-form solutions for debt issuance prices and debt values from firm managers' perspective are derived. Considering the inconsistency between the two prices incurred from the asymmetric information, a firm's problem of optimal investment risk determination is presented and solved by trading off two opposing effects: asset substitution and default cost. Furthermore, the optimal dividend policy and tax benefits from debt interest payment are also considered, and the application of the model in portfolio management is discussed. Finally, two simple examples are provided. Under these two specific settings, the optimal investment policies are derived explicitly to illustrate the implementation of the model proposed in this paper and demonstrate the general consistency of the results implied by our methodology and the traditional structural framework.

1 Introduction

Firm managers are continually faced with choices among various options for their firms' future investments. Their investment decisions depend not only on the initial capital requirements and risk exposure of these different investment options, but also on their firm's current budget and costs of debt issuance. Furthermore, their decisions will often influence the firm's future asset returns and the potential default or bankruptcy risk. Due to the limited capital available, in order to undertake investment projects, a firm typically needs to raise funds from external sources of finance. Two general solutions are usually considered: raising additional equity capital, and debt financing. In this paper, we focus on the impact of corporate debt financing on investment decisions, and thus we assume initially that the issuance of additional equity is too costly to pursue in view of the agency costs and market imperfections. This assumption can be relaxed, as we will discuss in the sequel, by changing the initial constraint on capital budgets.

In order to derive optimal investment strategies and corporate financing policies, we develop here a continuous-time reduced form model of the firm. We model a firm's asset return as a general geometric Brownian motion with stochastic volatility, whose growth rate and volatility are determined by firm managers through their investment decisions, as well as market shocks exposed to the firm. In view of this general setting of asset-value dynamics, our approach is distinct from those of the traditional structural credit risk models (e.g., Merton (1974), Black and Cox (1976)), in which the asset return is specified as an exogenous process, usually a log-normal process with constant volatility. Moreover, our modeling strategy is different from models with completely endogenous asset-value dynamics, such as that in Basak and Shapiro (2003), in that the market impacts on the asset return of the firm are also emphasized here.

When using structural models for credit risk, a difficulty arises in calculating the hitting (default) probability when the asset dynamics are other than log-normal. In order to overcome this problem and to provide a tractable dynamic framework for implementation, we adopt a reduced form credit risk model for pricing the debt (bonds) issued by a firm. In particular, a simple point process with stochastic jump intensity is applied to modeling defaults as originally proposed in Lando (1998). The intensity of this default indicator process is again mutually determined both by the firm's investment risk exposure and by macroeconomic factors.

Another factor that we consider is the existence of information asymmetry between market investors and firm managers. More specifically, it is assumed that market investors have information on macroeconomic factors (e.g., risk-free interest rates) and a part of the firm's information that is shared by the market.

However, they have no access to the information about the firm's investment decisions. Due to the lack of information, they would demand an extra yield on the bonds issued by the firm. Since the investment decisions affect the future asset return and default risk of the firm, the manager's inside information brings an inconsistency between the two parties on the values of the debt issued. It is reasonable to assume that the debt is issued according to the investor's price. For the firm's managers, their investment decisions may create an advantage on the price of the debt if they increase the investment risk, a phenomenon called the "asset substitution effect" in Jensen and Meckling (1976). Meanwhile, however, an increase in investment risk exposure will lead to a potential asset loss upon default. Here default, or a credit event, typically includes bankruptcy, failure to meet its cash obligations or restructuring. Therefore there exists a trade-off when choosing investment strategies to balance these two opposing effects. This paper focuses principally on the problem of determining an optimal investment policy in this context with the goal of maximizing the firm's equity value.

The partial information about the firm shared by market investors, as mentioned above, is characterized in a reduced form; i.e., we import a credit index process for each firm to characterize its credit quality information that market investors can access. This idea was first proposed by Hull and White (2002) to measure a firm's default risk. Chen and Filipović (2003a, 2003b) proposed a specific affine model of credit indices for both single-party and multi-entities, in which they show how to derive a firm's credit index value from the corresponding bond spreads and demonstrate that the credit index value of a firm is generally consistent with its investment grade given by financial agencies (e.g., Moody's and Standard & Poor's). Similar work can also be found in Albanese et al. (2003) and Rouady and Jeanblanc (2002). Here we use the same setting as in Chen and Filipović (2003a); that is, we assume that the higher the credit index value, the worse a firm's financial situation, and a zero value of the corresponding credit index implies perfect financial health of a firm. Note that the credit index in this paper is treated as the synthesis of all the firm specific information accessible to the market; however, in order to capture macroeconomic factors, we also consider the short rate as another state variable. Together with the credit index of a firm, these two variables are modeled by a positive two-dimensional affine process. A general extension to modeling macroeconomic factors as a multi-variable model is straightforward to derive. However, the one-factor short rate model considered here is sufficient for our purposes.

A related line of work on capital structure theory can be traced back as early as to Modigliani and Miller (M-M)(1958, 1963) who derive the optimal amount of debt by balancing the tax deductions from interest payments against the default costs. Harris and Raviv (1991) provide a comprehensive survey on

different variants of the original M-M model. More recently, Leland (1994), Leland and Toft (1996), and Leland (1998) examine corporate debt values and capital structure in a unified analytical structural framework and derive the debt valuation formula and optimal leverage for a firm. Fan and Sundaresan (2000) investigate the interactions between debt holders and equity holders in a game-theoretic setting and study the effects of dividend policies on debt valuation. A remarkable and insightful work on modeling a firm is that of Jarrow and Purnanandam (2003), in which the authors differentiate the finance distress and insolvency, and demonstrate the existence of the finance distress state provides the firm the hedging incentives. However, their formulation is too general for implementation, and the authors have not considered the issue of asymmetric information, which is one of the main focuses of this paper.

The structure of this article is as follows. In Section 2, we introduce the model and discuss debt valuation issues in detail. In Section 3, we formulate the firm's optimization problem and derive the optimal investment strategy and its capital structure. In Section 4, we generalize the original model to allow consideration of optimal dividend policies and tax effects on investment decisions. Two specific examples are discussed in Section 5 to illustrate the implementation of our model and the general consistency between our reduced form methodology and the structural framework. A brief conclusion is given in Section 6.

2 The Model

Consider a continuous trading economy with finite time horizon $[0, T^*]$ and a filtered probability space $(\Omega, (\mathcal{F}_t)_{0 \leq t \leq T^*}, \mathcal{F}, \mathbb{P})$ representing the randomness in the economy during this time horizon. Suppose there exists an equivalent martingale measure \mathbb{Q} and a risk-free short rate process r_t , which, under measure \mathbb{Q} , follows the same dynamics as in Cox, Ingersoll and Ross (1985):

$$dr_t = (b_r + \beta_r r_t)dt + \sigma_r \sqrt{r_t} dW_t^r, \quad (1)$$

where b_r , β_r and σ are constants, and W^r is an adapted \mathbb{Q} -Brownian motion. Here $|\beta_r|$ represents the mean reversion rate of the short rate to the long-run mean $|\frac{b_r}{\beta_r}|$. Therefore at time $t \geq 0$, the price of a risk-free zero-coupon bond with time to maturity $T - t$ ($T > t$) can be written as follows:

$$\pi(t, T) = \exp\{\phi^T(T - t) + \psi_r^T(T - t)r_t\}, \quad (2)$$

where

$$\begin{aligned}\psi_r^T(T-t) &= \frac{2(1 - e^{\gamma_r(T-t)})}{(\gamma_r - \beta_r)(e^{\gamma_r(T-t)} - 1) + 2\gamma_r}, \\ \phi^T(T-t) &= \frac{2b_r}{\sigma_r^2} \log \left(\frac{2\gamma_r e^{(\gamma_r - \beta_r)(T-t)/2}}{(\gamma_r - \beta_r)(e^{\gamma_r(T-t)} - 1) + 2\gamma_r} \right) \\ \text{with } \gamma_r &= \sqrt{\beta_r^2 + 2\sigma_r^2}.\end{aligned}$$

As mentioned above, in order to model the firm-specific information shared by market investors, we apply and define the credit index Y_t of a firm following the process

$$dY_t = (b_y + \beta_y Y_t + \beta_{ry} r_t)dt + \sigma_y \sqrt{Y_t} dW_t^y, \quad (3)$$

b_y , β_y , β_{ry} and σ_y are constants, and W^y is again an adapted \mathbb{Q} -Brownian motion independent of W^r . The constant $\beta_{ry} \geq 0$ measures the correlation between the short rate and the credit index process¹.

2.1 Modeling Investment Options

It is assumed that, at time 0, a firm of interest is faced with different investment options. In order to derive an optimal decision about how to choose among these investments, each optional investment project is characterized by its unique risk exposure factor α , and the associated initial capital requirement is denoted by $I(\alpha)$. Let \mathcal{B} denote the set of investment decisions open to the firm managers; thus $\alpha \in \mathcal{B}$. The impact of an investment opportunity on the dynamics of the firm's asset return is modeled as follows.

Assumption 1. *It is assumed that \mathcal{B} is a compact subset of \mathbb{R}_+ . Moreover, given the decision $\alpha \in \mathcal{B}$ at time 0, the asset return obeys the following dynamics under the equivalent martingale measure \mathbb{Q} :*

$$\frac{dA_t}{A_t} = (r_t + G(\alpha))dt + \sqrt{\alpha + \sigma_A^2 Y_t} dW_t^A, \quad A_0 = I(\alpha), \quad (4)$$

where A_t denotes the asset value at time t of a firm invested in investment project α , $G : \mathcal{B} \mapsto \mathbb{R}_+$ is an increasing function characterizing the excessive growth rate, σ_A is a constant, and W^A denotes a \mathbb{Q} -Brownian motion which is mutually independent of W^r and W^y . Moreover, it is assumed that the function $I : \mathcal{B} \mapsto \mathbb{R}_+$ is also increasing.

This formulation of investment return dynamics is in the spirit of several important and well-accepted economic facts. First, the entire dynamics of the return not only depends on the firm's investment strategy and its financial health (measured by the credit index Y_t), but also is related to macroeconomic

situations (measured by the short-term interest rate r_t). In view of the intrinsic interdependence between a firm's return and its financial credit quality, we model the asset return process by setting its volatility to be directly related to the credit index. Meanwhile, the volatility of the return also depends on the risk exposure factor α of a specific investment strategy, which we use to characterize each investment decision uniquely. By requiring $G(\alpha)$ to be an increasing function, the model is consistent with the economic intuition that the investors will require a higher return when taking a higher risk. This intuition has been demonstrated by Vassalou and Xing (2003) for stock investors. Furthermore, since $G(\alpha)$ is positive, we admit that a firm may have arbitrage opportunities usually generated by its special expertise in the area (e.g., patents). As argued in Jarrow and Purnanandam (2003), under some circumstances, given arbitrage opportunities, the firm can become infinitely large with zero capital necessary. Therefore, to exclude this situation, we stipulate that there exists an initial capital requirement $I(\alpha)$ for each investment option α . Requiring that the function I be increasing implies that a higher expected return needs a higher initial input. Besides this formulation, we also limit the arbitrage opportunities by proposing the following assumption on default.

Assumption 2. *Let $T_{\mathcal{D}}$ denote the default time of a firm. It is assumed that the default indicator $1_{\{T_{\mathcal{D}} \leq t\}}$ follows a doubly stochastic point process with stochastic intensity $\Lambda_t = \ell + \lambda_r r_t + \lambda_y Y_t + J(\alpha)$, where the constants ℓ, λ_r , and λ_y are non-negative, and the function $J : \mathcal{B} \mapsto \mathbb{R}_+$ is increasing.*

Thus, Assumption 2 indicates that the riskier the investment strategy chosen, the higher the firm's default risk as characterized by the extra jump intensity $J(\alpha)$. However, this assumption also includes the macroeconomic influence on a firm's default intensity, as well as the firm's financial health. This modeling strategy is different from those proposed in Leland (1994) and Leland and Toft (1996), in which the default (boundary) is determined endogenously by the firm.

Remark 1. *According to Jarrow and Purnanandam (2003), the intensity component $J(\alpha) + \ell$ can be partially interpreted as the operational or disaster risk caused by malfeasance of management or by an unexpected structural shift in the economy.*

2.2 Debt Valuation for Firm Managers

In order to apply valuation formulas proposed by Lando (1998), we make the following technical assumption.

Assumption 3. *It is assumed that the information set for firm managers at time t is represented by \mathcal{F}_t , where $\mathcal{F}_t = \mathcal{H}_t \vee \mathcal{N}_t$ with $\mathcal{H}_t := \sigma\{(r_s, Y_s, A_s) : 0 \leq s \leq t\}$ and $\mathcal{N}_t := \sigma\{1_{\{T_{\mathcal{D}} \leq s\}} : 0 \leq s \leq t\}$, for each $t \in [0, T^*]$.*

Under this assumption, we have the following lemma, due to Lando (1998).

Lemma 1. *For any $0 \leq t < T$, and any T -contingent claim with random payoff $X1_{\{T_D > T\}}$, where $X \in \mathcal{H}_T$, the price at time t of the claim is given by*

$$P(t, T) = 1_{\{T_D > t\}} \mathbb{E} \left[e^{-\int_t^T (r_s + \Lambda_s) ds} X \middle| \mathcal{H}_t \right]. \quad (5)$$

From Lemma 1, we can derive the following proposition concerning debt valuation by firm managers.

Proposition 1. *From the firm's perspective, the price of a zero-coupon bond issued with time to maturity $T - t$ and default recovery at maturity is given by*

$$\begin{aligned} B_f(\alpha, t, T) &= \mathbb{E} \left[e^{-\int_t^T r_s ds} (\delta_f 1_{\{T_D \leq T\}} + 1_{\{T_D > T\}}) \middle| \mathcal{F}_t \right] \\ &= (1 - \delta_f) \pi_f(\alpha, t, T) 1_{\{T_D > t\}} + \delta_f \pi(t, T), \end{aligned} \quad (6)$$

where δ_f denotes the recovery rate and

$$\pi_f(\alpha, t, T) = \exp\{\phi^f(T - t) + \psi_r^f(T - t)r_t + \psi_y^f(T - t)Y_t\} \quad (7)$$

with ϕ^f , ψ_r^f and ψ_y^f satisfying the following Riccati equations:

$$\begin{aligned} \frac{d\phi^f(t)}{dt} &= b_r \psi_r^f(t) + b_y \psi_y^f(t) - \ell - J(\alpha), \quad \phi^f(0) = 0, \\ \frac{d\psi_r^f(t)}{dt} &= \frac{1}{2} \sigma_r^2 (\psi_r^f(t))^2 + \beta_r \psi_r^f(t) + \beta_{ry} \psi_y^f(t) - 1 - \lambda_r, \quad \psi_r^f(0) = 0, \\ \frac{d\psi_y^f(t)}{dt} &= \frac{1}{2} \sigma_y^2 (\psi_y^f(t))^2 + \beta_y \psi_y^f(t) - \lambda_y, \quad \psi_y^f(0) = 0. \end{aligned} \quad (8)$$

Remark 2. *It is, of course, quite convenient for implementation if explicit expressions are known for the exponent functions ϕ^f , ψ_r^f and ψ_y^f . If $\beta_{ry} = 0$ in (8), we can obtain such expressions, namely,*

$$\begin{aligned} \psi_r^f(t) &= \frac{2(\lambda_r + 1)(1 - e^{\tilde{\gamma}_r t})}{\tilde{\gamma}_r(e^{\tilde{\gamma}_r t} + 1) - \beta_r(e^{\tilde{\gamma}_r t} - 1)}, \quad \tilde{\gamma}_r = \sqrt{\beta_r^2 + 2\sigma_r^2(1 + \lambda_r)}, \\ \psi_y^f(t) &= \frac{2\lambda_y(1 - e^{\gamma_y t})}{\gamma_y(e^{\gamma_y t} + 1) - \beta_y(e^{\gamma_y t} - 1)}, \quad \gamma_y = \sqrt{\beta_y^2 + 2\sigma_y^2\lambda_y}, \\ \phi^f(t) &= \frac{2b_r(\lambda_r + 1)}{\sigma_r^2} \log \left(\frac{2\tilde{\gamma}_r e^{(\tilde{\gamma}_r - \beta_r)t/2}}{(\tilde{\gamma}_r - \beta_r)(e^{\tilde{\gamma}_r t} - 1) + 2\tilde{\gamma}_r} \right) \\ &\quad + \frac{2b_y\lambda_y}{\sigma_y^2} \log \left(\frac{2\gamma_y e^{(\gamma_y - \beta_y)t/2}}{(\gamma_y - \beta_y)(e^{\gamma_y t} - 1) + 2\gamma_y} \right) - (\ell + J(\alpha))t. \end{aligned} \quad (9)$$

2.3 Debt Valuation from the Market's Perspective

As mentioned above, since market investors do not have access to a firm's asset information and its managers' investment strategy, the "rational" debt price for market investors is different from that of the firm managers. The following assumption specifies this difference.

Assumption 4. *It is assumed that the information known to the market investors at time t is given by $\mathcal{G}_t := \sigma\{(r_s, Y_s) : 0 \leq s \leq t\}$, for $t \in [0, T]$. Moreover due to the lack of information on investment decision α , investors use the default intensity*

$$\Lambda_t^* = \ell^* + \lambda_r^* r_t + \lambda_y^* Y_t,$$

where, for simplicity, it is assumed that

$$\ell^* = \ell, \quad \lambda_r^* = \lambda_r, \quad \lambda_y^* = \lambda_y.$$

Assumption 4 postulates that market investors will value the debt under his own pricing measure with the dynamics of the default different from that used by the firm's managers. Since this measure is different from the martingale measure of the firm, it follows that the market investors' valuation is biased. However, it is further assumed that the investor demands an extra yield $\ell_e \geq 0$ on the bonds.

Following Lemma 1 and Proposition 1, from the market's perspective, we obtain the price of a zero-coupon bond as follows:

$$B_m(t, T) = e^{-\ell_e(T-t)} [(1 - \delta_m) \pi_m(t, T) 1_{\{T_D > t\}} + \delta_m \pi(t, T)], \quad (10)$$

where δ_m denotes the recovery rate, and it is assumed that $\delta := \delta_m = \delta_f$, and

$$\pi_m(t, T) = \exp\{\phi^m(T-t) + \psi_r^m(T-t)r_t + \psi_y^m(T-t)Y_t\} \quad (11)$$

with ϕ^m , ψ_r^m and ψ_y^m satisfying the following Riccati equations.

$$\begin{aligned} \frac{d\phi^m(t)}{dt} &= b_r \psi_r^m(t) + b_y \psi_y^m(t) - \ell, \quad \phi^m(0) = 0, \\ \frac{d\psi_r^m(t)}{dt} &= \frac{1}{2} \sigma_r^2 (\psi_r^m(t))^2 + \beta_r \psi_r^m(t) + \beta_{ry} \psi_y^m(t) - 1 - \lambda_r, \quad \psi_r^m(0) = 0, \\ \frac{d\psi_y^m(t)}{dt} &= \frac{1}{2} \sigma_y^2 (\psi_y^m(t))^2 + \beta_y \psi_y^m(t) - \lambda_y, \quad \psi_y^m(0) = 0. \end{aligned} \quad (12)$$

Comparing (12) with (8), it is straightforward to show that

$$\psi_r^m(t) = \psi_r^f(t), \quad \psi_y^m(t) = \psi_y^f(t), \quad \phi^m(t) = \phi^f(t) + J(\alpha)t.$$

Therefore we have the following proposition.

Proposition 2. *Under Assumptions 2, 3 and 4, if a firm does not default at time t , the difference on the debt (with time to maturity $T - t$) value between market investors and the firm is given by*

$$\begin{aligned}\kappa(\alpha, t, T) &= \frac{B_f(\alpha, t, T)}{B_m(t, T)} \\ &= e^{\ell_e(T-t)} + (1 - \delta) \left(e^{-J(\alpha)(T-t)} - 1 \right) \frac{\pi_m(t, T)}{B_m(t, T)}.\end{aligned}\quad (13)$$

In particular, if we assume zero-recovery upon default (i.e., $\delta = 0$) we further have

$$\kappa(\alpha, t, T) = e^{(\ell_e - J(\alpha))(T-t)}.\quad (14)$$

Since it is reasonable to assume that the debt will be issued according to the price given by market investors, from Proposition 2 we can see that firm managers may increase the risk factor α to increase $J(\alpha)$ so that they can obtain a price advantage (i.e., $J(\alpha) > \ell_e$) when issuing the debt due to the limited information held by market investors.

2.4 Default Costs

As is well known (e.g., Fischer, Heinkel and Zechner (1989), Goldstein, Ju and Leland (2001)), default is costly due to deadweight losses including third party costs, liquidation costs, agency costs, etc. In this paper, these costs will be taken as exogenously given. In particular, we make the following assumption.

Assumption 5. *It is assumed that when default occurs before the terminal date of an investment with exposure factor α , the default costs consist of two parts: a fixed component $\rho_0(\alpha)$, and another component related to the investment asset $\rho_1(\alpha)A_T$, where T is the terminal date of the investment, and $\rho_0 : \mathcal{B} \mapsto \mathbb{R}_+$ and $\rho_1 : \mathcal{B} \mapsto [0, 1]$ are increasing functions.*

The fixed (first) part of the costs can be related to legal and other fees associated with default, and the flexible (second) part can be related to the depreciation of the asset due to the liquidation costs. It is assumed that the deadweight losses on the asset occur at the terminal date of the investment if a firm defaults before. Therefore the expected default loss is derived in the following proposition.

Proposition 3. *Let T denote the terminal date of the investment. Under As-*

sumption 5, we have the expected default cost at time $t(t < T)$ given by

$$C_{\mathcal{D}}(t) = \rho_0(\alpha)(\pi(t, T) - \pi_f(\alpha, t, T)1_{\{T_{\mathcal{D}} > t\}}) + \rho_1(\alpha)A_t \left(e^{G(\alpha)(T-t)} - \pi_a(\alpha, t, T)1_{\{T_{\mathcal{D}} > t\}} \right), \quad (15)$$

where $\pi(t, T)$ and $\pi_f(\alpha, t, T)$ are defined in (2) and (7), and

$$\pi_a(\alpha, t, T) = \exp\{\phi^a(T-t) + \psi_r^a(T-t)r_t + \psi_y^a(T-t)Y_t\} \quad (16)$$

with ϕ^a , ψ_r^a and ψ_y^a satisfying the following Riccati equations.

$$\begin{aligned} \frac{d\phi^a(t)}{dt} &= b_r\psi_r^a(t) + b_y\psi_y^a(t) + G(\alpha) - \ell - J(\alpha), \quad \phi^a(0) = 0, \\ \frac{d\psi_r^a(t)}{dt} &= \frac{1}{2}\sigma_r^2(\psi_r^a(t))^2 + \beta_r\psi_r^a(t) + \beta_{ry}\psi_y^a(t) - \lambda_r, \quad \psi_r^a(0) = 0, \\ \frac{d\psi_y^a(t)}{dt} &= \frac{1}{2}\sigma_y^2(\psi_y^a(t))^2 + \beta_y\psi_y^a(t) - \lambda_y, \quad \psi_y^a(0) = 0. \end{aligned} \quad (17)$$

With these results in hand, we are now ready to set up and solve the investment optimization problem for the firm.

3 Optimal Investment Decision

In this section, we fix the current time at 0 and assume that the firm has not defaulted yet. At time 0, the firm is faced with various investment options represented by the decision set \mathcal{B} , and all these investment projects share the same terminal date denoted by T_0 . Suppose that at time 0, the firm holds initial capital C_0 in hand; however, for simplicity, it is assumed that this initial capital can not exceed the initial investment requirement for any investment project; i.e.,

$$C_0 \leq I(\alpha), \quad \forall \alpha \in \mathcal{B}.$$

Therefore, in order to carry out the investment project, the firm needs to raise funds externally. As mentioned in the previous section, it is assumed first that equity capital is inelastic, which implies that the investment can only be financed by issuing debt. As mentioned in Jarrow and Purnanandam (2003), this restriction can be easily relaxed by setting the initial capital C_0 to be a flexible function dependent on state variables α and Y_0 , which will be discussed below. It is further assumed that the firm will issue a zero-coupon bond for debt financing with maturity T_0 and face value L .

Using the results in Section 2 and the initial budget constraint, we can determine the debt face value L needed to be issued. Without considering the

dividend and capital reserve, the initial budget constraint becomes

$$I(\alpha) = L_0^m + C_0,$$

where L_0^m is the debt value viewed by the market, which is assumed to be the sale price of the debt. From (10), we have that

$$L_0^m = L \cdot B_m(0, T_0), \quad (18)$$

where $B_m(0, T)$ is defined in (10). It follows that the debt face value L necessary to issue is equal to

$$\frac{I(\alpha) - C_0}{B_m(0, T_0)},$$

which is due at time T_0 .

3.1 Firm's Objective Function

The firm managers will act to serve the interests of shareholders, and therefore we assume that the objective of the firm is to maximize the present value of equity at time T_0 . Since the equity is equal to the asset value less liabilities, based on the previous discussion we can write the objective function F as follows.

$$\begin{aligned} F(\alpha) = & \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} [A_{T_0}(1 - \rho_1(\alpha)1_{\{T_D \leq T_0\}}) - \rho_0(\alpha)1_{\{T_D \leq T_0\}}] \middle| \mathcal{F}_0 \right] \\ & - \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} L(\delta 1_{\{T_D \leq T_0\}} + 1_{\{T_D > T_0\}}) \middle| \mathcal{F}_0 \right], \end{aligned} \quad (19)$$

with the budget constraint

$$L = \frac{I(\alpha) - C_0}{B_m(0, T_0)}, \quad A_0 = I(\alpha). \quad (20)$$

The first term on the right hand side (RHS) of (19) is the expected value of the asset considering the deadweight losses upon default. The second term is the expected value of the liability. Note that due to the information asymmetry, the value of the debt issued is not equal to L_0^m from the firm managers' perspective. The optimal investment strategy can be derived by solving the following optimization problem.

$$\begin{aligned} \alpha^* = & \arg \max_{\alpha \in \mathcal{B}} \left\{ \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} [A_{T_0}(1 - \rho_1(\alpha)1_{\{T_D \leq T_0\}}) - \rho_0(\alpha)1_{\{T_D \leq T_0\}}] \middle| \mathcal{F}_0 \right] \right. \\ & \left. - \frac{I(\alpha) - C_0}{B_m(0, T_0)} \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} (\delta 1_{\{T_D \leq T_0\}} + 1_{\{T_D > T_0\}}) \middle| \mathcal{F}_0 \right] \right\}. \end{aligned} \quad (21)$$

Remark 3. *It is obvious to see that there exists an optimal solution for (21) if*

the decision set \mathcal{B} has finitely many elements.

3.2 The Optimal Choice of Investment Strategy

In view of Remark 3, we assume that the following regularity conditions hold throughout the remainder of the paper. This assumption allows us to solve the optimization problem (21) analytically.

Assumption 6. *It is assumed that the decision set $\mathcal{B} = [0, B_0]$ for some $B_0 > 0$, and that the functions G, I, J, ρ_0 , and ρ_1 are in $C^2(\mathcal{B})$.*

Now we first obtain the analytical expression of the objective function F in terms of α . This derivation is straightforward from Propositions 2 and 3.

Lemma 2. *The objective function F of a firm defined in (19) admits the following explicit expression.*

$$\begin{aligned} F(\alpha) = & (1 - \rho_1(\alpha))I(\alpha)e^{G(\alpha)T_0} + \rho_0(\alpha)\pi_f(\alpha, 0, T_0) + \rho_1(\alpha)I(\alpha)\pi_a(\alpha, 0, T_0) \\ & - \kappa(\alpha, 0, T_0)(I(\alpha) - C_0) - \rho_0(\alpha)\pi(0, T_0), \end{aligned} \quad (22)$$

where $\pi(0, T_0)$, $\pi_f(\alpha, 0, T_0)$ and $\pi_a(\alpha, 0, T_0)$ are defined in (2), (7) and (16), respectively, and $\kappa(\alpha, 0, T_0)$ can be found in (13). Moreover, since the functions G, I, J, ρ_0 , and ρ_1 are continuous with respect to α , so is F , and therefore F has a maximum on the compact set $[0, B_0]$.

The first term on the RHS of (22) is increasing with respect to α , the second term is decreasing in α , while the third and fourth terms are not monotonic in α . A straightforward approach to the optimization problem is to obtain the first order condition. By differentiating F with respect to α , we have the first order optimality condition as

$$\begin{aligned} 0 = & I'(\alpha) \left[(1 - \rho_1(\alpha))e^{G(\alpha)T_0} + \rho_1(\alpha)\pi_a(\alpha, 0, T_0) - \kappa(\alpha, 0, T_0) \right] \\ & + I(\alpha) \left[(1 - \rho_1(\alpha))G'(\alpha)T_0e^{G(\alpha)T_0} + \rho_1(\alpha)(G'(\alpha) - J'(\alpha))T_0\pi_a(\alpha, 0, T_0) \right] \\ & + I(\alpha) \left[(\rho_1'(\alpha))e^{G(\alpha)T_0} + \rho_1'(\alpha)\pi_a(\alpha, 0, T_0) \right] + \rho_0'(\alpha)\pi_f(\alpha, 0, T_0) - \rho_0'(\alpha)\pi(0, T_0) \\ & - \rho_0(\alpha)J'(\alpha)T_0\pi_f(\alpha, 0, T_0) - (I(\alpha) - C_0)\kappa'(\alpha, 0, T_0), \end{aligned} \quad (23)$$

where we have used the relations

$$\begin{aligned} \pi_f'(\alpha, 0, T_0) &= -J'(\alpha)T_0\pi_f(\alpha, 0, T_0), \\ \pi_a'(\alpha, 0, T_0) &= (G'(\alpha) - J'(\alpha))T_0\pi_a(\alpha, 0, T_0), \end{aligned}$$

which follow from (7), (8), (16) and (46). Although it is easy to solve (23) numerically given the functions G, I, J, ρ_0 , and ρ_1 , it is different to tell how

those other factors (e.g., the deadweight loss ρ_0 , ρ_1 or initial capital C_0) affect the determination of the optimal decision α^* . In order to see this, we assume zero recovery rate upon default²; i.e., $\delta = 0$. Then the objective function can be further simplified as

$$F(\alpha) = I(\alpha)e^{G(\alpha)T_0} - \rho_1(\alpha)I(\alpha)e^{G(\alpha)T_0} \left(1 - \Gamma_2(x, T_0)e^{-J(\alpha)T_0}\right) + [\rho_0(\alpha)\Gamma_1(x, T_0) + (C_0 - I(\alpha))e^{\ell_\epsilon T_0}] e^{-J(\alpha)T_0} - \rho_0(\alpha)\pi(0, T_0), \quad (24)$$

where $x := (r_0, Y_0)$ and

$$\begin{aligned} \Gamma_1(x, T_0) &= \exp\{\tilde{\phi}^f(T_0) + \psi_r^f(T_0)r_0 + \psi_y^f(T_0)Y_0\}, \\ \Gamma_2(x, T_0) &= \exp\{\tilde{\phi}^a(T_0) + \psi_r^a(T_0)r_0 + \psi_y^a(T_0)Y_0\} \end{aligned}$$

with

$$\tilde{\phi}^f(T_0) = \phi^f(T_0) + J(\alpha)T_0, \quad \tilde{\phi}^a(T_0) = \phi^a(T_0) - (G(\alpha) - J(\alpha))T_0.$$

In view of (8) and (46), the functions $\Gamma_1(x, T_0)$ and $\Gamma_2(x, T_0)$ are independent of α . Therefore (24) enables us to clarify the relationship between different factors and the optimal decision α^* . The first term on the RHS of (24) represents the expected return of the initial investment, which increases when the risk-exposure rises. And thus this term may encourage firm managers to opt for a more aggressive investment strategy. The second term comes from the dead weight loss due to the liquidation incurred from default. From the general theory of Riccati equations (see e.g., Birkhoff and Rota (1989)), we know that $\Gamma_2(x, T_0) \in (0, 1]$, for each $x \in \mathbb{R}_+^2$ and $T_0 \in \mathbb{R}_+$. Therefore the second term is decreasing with respect to α , which implies that the default cost factor $\rho_1(\alpha)$ will force firm managers to reach a more conservative investment policy. Compared with the first two terms, the effect of α on the third term is rather complicated. Generally speaking, in order to avoid a huge default loss on the initial capital and decrease the likelihood of a fixed default cost expense, the firm will tend to choose the investment project with low risk. However, if for all $\alpha \in \mathcal{B}$, $(I(\alpha) - C_0)e^{\ell_\epsilon T_0} - \rho_0(\alpha)\Gamma_1(x, T_0) \geq 0$, then driven by price advantages gained from liability holders due to the market investors' biased price for purchasing the debt, the firm prefers to choose higher-risk strategy. Since we already assume that

$$I(\alpha) \geq C_0, \quad \forall \alpha \in \mathcal{B}, \quad (25)$$

from the above discussion, we can see that the fixed default cost $\rho_0(\alpha)$ serves as a *wedge* to make firm managers hesitate to increase the risk exposure of their investment. This result is consistent with the one in Basak and Shapiro (2003). The optimal investment strategy is, of course, derived by balancing all of the

above factors.

4 General Extensions of the Model

In this section, we discuss several possible extensions of the original model to incorporate some other elements when determining the investment risk exposure α .

4.1 Equity Capital, Dividend Policy and Capital Reserve

As mentioned above, the external source of fund raising may not be restricted to debt financing; issuing additional equity can be an alternative means of fund raising despite the extra costs due to market imperfections. Moreover, dividend policies and capital reserve have also been ignored in the above analysis. Now we will generalize the model to take these factors into account. Since the dividends are paid to shareholders, the objective function of the firm will become the expected value of equity plus the dividend paid currently. In fact, if we assume that the reserved capital is kept in a risk-free money market savings account (with the growth rate equal to the short rate), and it is not subject to a deadweight loss at default, then this reserved capital is equivalent to the dividend in terms of their having identical present expected values³. Therefore we need to consider only the optimal dividend policy, and the firm's optimization problem becomes

$$\begin{aligned} \max_{\alpha \in \mathcal{B}, d \in \mathbb{R}_+} \left\{ \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} [A_{T_0}(1 - \rho_1(\alpha)1_{\{T_D \leq T_0\}}) - \rho_0(\alpha)1_{\{T_D \leq T_0\}}] \middle| \mathcal{F}_0 \right] \right. \\ \left. - \tilde{L} \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} (\delta 1_{\{T_D \leq T_0\}} + 1_{\{T_D > T_0\}}) \middle| \mathcal{F}_0 \right] + d \right\}, \end{aligned} \quad (26)$$

with the budget constraint

$$\tilde{L}_0^m + C_0 + C_e(\alpha, d) = I(\alpha) + d, \quad A_0 = I(\alpha),$$

where d denotes the dividends paid at time 0, \tilde{L}_0^m denotes the expected value of the debt \tilde{L} from the market investors' perspective, and $C_e(\alpha, d)$ represents the additional equity capital raised by the firm. On assuming that C_e is a positive and increasing function of the dividend d , it follows that equity capital is costly, but is neither perfectly elastic nor inelastic, as mentioned in Jarrow and Purnanandam (2003). Again from (10), we have that

$$\tilde{L} = \frac{I(\alpha) - C_0 - C_e(\alpha, d) + d}{B_m(0, T_0)}.$$

Therefore, on further assuming $C_e \in C^2(\mathcal{B} \times \mathbb{R}_+)$, it is straightforward to solve (26) using Lagrangian optimization. Finally it is necessary to restrict

$$\frac{\partial C_e(\alpha, d)}{\partial \alpha} < \frac{dI(\alpha)}{d\alpha}, \quad \forall (\alpha, d) \in \mathcal{B} \times \mathbb{R}_+$$

in order to exclude the case in which the initial equity capital is sufficient to cover any level of investment.

4.2 Coupon Payments and Tax Benefits

In Section 3, we presumed that the firm issues debt in the form of zero-coupon bonds. However, in order to benefit from the tax deductibility of coupon payments, a firm usually chooses to issue debts with coupons, and liability holders also prefer to invest in bonds with interest payments. Let ξ denote the coupon payment rate and Δ represent the interval between two adjacent coupon payment dates. It is assumed that $T_0 = N \cdot \Delta$ and there is zero-recovery at default. Without loss of generality, we also assume that $\ell_e = 0$, otherwise we can specify $\tilde{J}(\alpha) = J(\alpha) - \ell_e$ instead of the original $J(\alpha)$. It is easy to derive the coupon bond pricing formulas for market investors:

$$B_{mc}(0, T_0, \xi, \Delta) = \sum_{i=1}^N \xi \cdot \Delta \cdot \pi_m(0, i\Delta) + \pi_m(0, T_0), \quad (27)$$

where π_m is defined in (11). However, a firm can benefit fully⁴ from the tax deductions provided by interest payments. Therefore the debt value from the firm's perspective is given by

$$B_{fc}(\alpha, 0, T_0, \xi, \Delta, \tau) = \sum_{i=1}^N (1 - \tau) \xi \cdot \Delta \cdot \pi_f(\alpha, 0, i\Delta) + \pi_f(\alpha, 0, T_0), \quad (28)$$

where $\tau\xi$ is the tax benefit and π_f is defined by (7). Thus the ratio between the two prices is given by

$$\begin{aligned} \kappa_c(\alpha, 0, T_0) &= \frac{B_{fc}(\alpha, 0, T_0, \xi, \Delta, \tau)}{B_{mc}(0, T_0, \xi, \Delta, \tau)} \\ &= \frac{\sum_{i=1}^N (1 - \tau) \xi \cdot \Delta \cdot \pi_f(\alpha, 0, i\Delta) + \pi_f(\alpha, 0, T_0)}{\sum_{i=1}^N \xi \cdot \Delta \cdot \pi_m(0, i\Delta) + \pi_m(0, T_0)}. \end{aligned} \quad (29)$$

Since $\pi_f(\alpha, 0, T_0) = \pi_m(0, T_0) \cdot e^{-J(\alpha)T_0}$, $\kappa_c(\alpha, 0, T_0)$ decreases as T_0 increases, and it follows that short term debt does not exploit the tax benefits as completely as long term debt. Moreover it is easy to see that the optimization problem (22) does not change except for replacing $\kappa(\alpha, 0, T_0)$ with $\kappa_c(\alpha, 0, T_0)$. Therefore a

firm will prefer to issue debt with coupon payments if and only if $\kappa_c(\alpha, 0, T_0) < \kappa(\alpha, 0, T_0)$.

Since $\kappa(\alpha, 0, T_0) = e^{-J(\alpha)T_0}$, we have

$$\begin{aligned} \frac{\kappa_c(\alpha, 0, T_0)}{\kappa(\alpha, 0, T_0)} &= e^{J(\alpha)T_0} \frac{\sum_{i=1}^N (1-\tau)\xi \cdot \Delta \cdot \pi_f(\alpha, 0, i\Delta) + \pi_f(\alpha, 0, T_0)}{\sum_{i=1}^N \xi \cdot \Delta \cdot \pi_m(0, i\Delta) + \pi_m(0, T_0)} \\ &= \frac{\sum_{i=1}^N (1-\tau)\xi \cdot \Delta \cdot \pi_m(0, i\Delta)e^{J(\alpha)(N-i)\Delta} + \pi_m(0, T_0)}{\sum_{i=1}^N \xi \cdot \Delta \cdot \pi_m(0, i\Delta) + \pi_m(0, T_0)}. \end{aligned} \quad (30)$$

Now it is easy to see that $\kappa_c(\alpha, 0, T_0) < \kappa(\alpha, 0, T_0)$ if and only if

$$\tau > 1 - \frac{\sum_{i=1}^N \pi_m(0, i\Delta)}{\sum_{i=1}^N \pi_m(0, i\Delta)e^{J(\alpha)(N-i)\Delta}}, \quad (31)$$

which implies that firm managers will not take advantage of tax benefits by issuing coupon bonds unless the deduction amount is large enough compared with the threshold given in (31). On the other hand, from (31), we can see that the riskier the investment strategy chosen, the larger is this threshold, and thus the more likely that firm managers will issue zero-coupon bonds. This result is consistent with the result provided by the M-M model that there exists a tradeoff between tax benefits and external costs of potential default.

4.3 Applications in Portfolio Management

In reality, a firm is faced with only finitely many investment opportunities. However, this does not mean that the decision set \mathcal{B} defined previously is finite. By diversifying the investment and setting up an appropriate portfolio, firm managers can choose any risk exposure α in a convex region. The following discussion shows how to relate the risk exposure factor α , the initial investment requirement $I(\alpha)$ and the excessive growth rate $G(\alpha)$ to the portfolio position held by the firm. For simplicity, it is assumed that $\sigma_A = 0$ in (4).

Suppose there are M investment options with return dynamics and initial input requirements given by

$$\frac{dV_i(t)}{V_i(t)} = (r_t + \mu_i)dt + \sigma_i dW_t^i, \quad V_i(0) \geq v_i, \quad (32)$$

where $i \in \{1, 2, \dots, M\}$, $\mu := (\mu_1, \dots, \mu_M)^T \in \mathbb{R}^M$, $\sigma := \text{diag}(\sigma_1, \dots, \sigma_M) \in \mathbb{R}_+^{M \times M}$, and $W := (W^1, W^2, \dots, W^M)$ denotes an M -dimensional standard \mathbb{Q} -Brownian motion. It is assumed that $\mu_1 = 0$ and $\sigma_1 = 0$, which represents the risk-free money market account. Let $\theta := (\theta_1, \dots, \theta_M)^T \in \mathbb{R}_+^M$ denote an investment policy, where $\sum_{i=1}^M \theta_i = 1$. Then by Itô's Lemma, it is easy to

deduce that the dynamics of the investment asset value A are described by

$$\frac{dA_t}{A_t} = (r_t + \langle \theta, \mu \rangle) dt + \langle \theta, \sigma dW_t \rangle. \quad (33)$$

By comparing (33) with (4), we have that

$$G(\alpha) = \langle \theta, \mu \rangle, \quad \text{with} \quad \alpha = \sum_i^M \theta_i^2 \sigma_i^2, \quad (34)$$

and the decision set $\mathcal{B} = [0, \sigma^*]$, where $\sigma^* = \max\{\sigma_1, \sigma_2, \dots, \sigma_M\}$. Moreover we can also derive the initial investment requirement as follows:

$$A_0 = I(\alpha) = \max_{i \in \{1, 2, \dots, M\}} \left\{ \frac{v_i}{\theta_i} \right\}. \quad (35)$$

Therefore we transform the original optimization problem of determining the optimal risk exposure α into the optimization problem of choosing the portfolio θ . Using the method proposed previously, the optimal portfolio position of a firm can be derived straightforwardly.

5 Simple Examples

In order to illustrate the determination of the optimal investment risk exposure α using the model proposed in this paper, we will take several simple examples and solve the corresponding optimization problem.

5.1 Proportional Deadweight Losses

The first example is given by the following additional assumptions to simplify the problem.

Assumption 7. *It is assumed that the decision set $\mathcal{B} = [0, 10]$, $\ell_e = 0$, $\sigma_A = 0$ in (4), and*

$$\begin{aligned} G(\alpha) &= G_0 \log(1 + \alpha), & I(\alpha) &= C_0(1 + \alpha), & J(\alpha) &= J_0 \cdot \alpha, \\ \rho_0(\alpha) &= \rho_0 * \alpha * C_0, & \rho_1(\alpha) &= 1 - e^{-\rho_1 \alpha}. \end{aligned}$$

Moreover the firm will issue the debt in the form of zero-coupon bonds with zero default recovery, and dividends are not paid.

The maximal risk exposure factor α is set at 10, which means that the maximal volatility of asset return is around 300% given by the investment risk. We can see that all those functions set in Assumption 7 are positive and increasing,

as already presumed in the previous discussion, and that the function I satisfies $I(\alpha) \geq C_0$, for all $\alpha \in \mathcal{B}$, which is consistent with (25). Meanwhile, from Assumption 7, we can see that $G(0) = \rho_0(0) = \rho_1(0) = 0$, which implies that when a firm chooses the risk-free investment, the excessive growth rate is equal to zero, as are deadweight losses. Now from (24), it follows that the objective function is given by

$$F(\alpha) = C_0 (e^{-\rho_1 \alpha} + (1 - e^{-\rho_1 \alpha}) \Gamma_2(x, T_0) e^{-J_0 \alpha T_0}) (1 + \alpha)^{G_0 T_0 + 1} + C_0 (\rho_0 \Gamma_1(x, T_0) - 1) \alpha e^{-J_0 \alpha T_0} - \rho_0 C_0 \alpha \pi(0, T_0). \quad (36)$$

Without loss of generality, we set $C_0 = 1$. In order to solve the optimization problem numerically, we need to preset several parameters. For the structural parameters $(b_r, b_y, \beta_r, \beta_y, \beta_{ry}, \sigma_r, \sigma_y, \ell, \lambda_r, \lambda_y)$, their values are taken from the empirical results in Chen and Filipović (2003a); i.e.,

$$\begin{aligned} \sigma_r &= 5.89 \times 10^{-3}, & \sigma_y &= 2.555, \\ b_r &= 0.01167, & b_y &= 1.6328 \times 10^{-5}, \\ \beta_r &= -0.15492, & \beta_{ry} &= 0.23006, & \beta_y &= -1.472, \\ \ell &= 0, & \lambda_r &= 0.26365, & \lambda_y &= 0.10613, \\ r_0 &= 0.05, & Y_0 &= 0.07 \quad (\text{corresponding to Aaa credit rating}). \end{aligned}$$

These estimates are provided by empirical testing across more than 650 corporate bonds and 150 treasury bonds. The parameter G_0 is set at 0.06, and thus we have

$$e^{G(10)} = (1 + 10)^{G_0} = 1.15,$$

which means that, under the martingale measure \mathbb{Q} , the maximal yearly expected return of the investment asset is around 15% over that of the risk-free money market account. Moreover the parameter J_0 is set to be equal to 0.2. Since $e^{J(10)} = e^{10 \cdot J_0} = 7.4$, this means the most risky investment decision ($\alpha = 10$) can increase default risk at most seven times.

First we examine the relationship between the risk exposure factor α and the objective function F . We fix $T_0 = 1$, and plot the function $F(\alpha)$ when choosing three different sets of deadweight loss factors: ρ_0 and ρ_1 . The result is shown in Figure 1. We can see that the objective function is a smooth function with a non-trivial⁵ maximum in the region $[0, 10]$.

[Figure 1 about here.]

Moreover it is clear to see the optimal investment risk exposure α^* and the maximal expected equity value $F(\alpha^*)$ both decrease when the deadweight loss factors ρ_0 and ρ_1 increase. This is consistent with the intuition that a firm will decrease its risk exposure if default losses rise.

Now we start to investigate the relationship between α^* and ρ_0, ρ_1 , respectively. Figures 2 and 3 show explicitly that the optimal investment risk factor α and the maximum of the objective function are monotonically decreasing with respect to both loss factors ρ_0 and ρ_1 . Actually we can also see that the optimal choice of the risk exposure of investment is more sensitive to the default losses proportional to the asset than to the fixed component of default costs.

[Figure 2 about here.]

Finally, we examine the impact of the investment span T_0 on the optimal strategy. Figure 4 shows the change of the optimal risk exposure α^* and the objective function $F(\alpha^*)$ with respect to increases in T_0 . Unexpectedly, we find that α^* does not have a monotonic relationship with the span T_0 . As T_0 increases from one year to four years, firm managers should reduce the risk-exposure α^* first due to the dramatic increase of default risk at the order of $J(\alpha)e^{-J(\alpha)T_0}$. However, as T_0 exceeds four years, firm managers should increase the investment risk exposure in view of the higher expected asset return, although the default risk also rises. Therefore as the investment span increases, the optimal risk exposure α^* is determined by balancing these two opposing effects. The monotonic relationship between the maximal equity value $F(\alpha^*)$ and the investment span T_0 shown in Figure 4 is easy to understand since the longer the investment span, the more return the investors demand.

[Figure 3 about here.]

5.2 The Alternative Cap-Type of Deadweight Losses

The following example is examined in order to be compared with the case that is discussed in detail in Purnanandam (2003) using the traditional structural framework. Instead of the previous assumption of proportional deadweight losses, the default cost is alternatively assumed as

$$C_{\mathcal{D}}(T) = (A_T - M)^+ 1_{\{T_{\mathcal{D}} \leq T\}},$$

and the initial capital requirement $I(\alpha)$ is fixed at $3C_0$ regardless of the investment risk exposure α , while the excess growth rate $G(\alpha)$, the extra jump intensity $J(\alpha)$ and other variables follow the same settings as in the previous example. Therefore in this case, the final gain of the investment is capped by a positive bound M in the event of default, which implies that the smaller this bound, the larger the potential deadweight loss. In this case, we can also derive the objective function of the firm as follows, and the detailed formulas are

provided in the appendix.

$$\begin{aligned}
F(\alpha) &= \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} [A_{T_0} - (A_{T_0} - M)^+ 1_{\{T_D \leq T_0\}}] \right] - \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} L 1_{\{T_D > T_0\}} \right] \\
&= I(\alpha) e^{G(\alpha)T_0} - \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} [(A_{T_0} - M)^+ - (A_{T_0} - M)^+ 1_{\{T_D > T_0\}}] \right] \\
&\quad - (I(\alpha) - C_0) e^{-J(\alpha)T_0} \\
&= 3C_0 e^{G(\alpha)T_0} - (Q_1(\alpha, x, T_0) - MQ_2(\alpha, x, T_0)) \\
&\quad + (Q_3(\alpha, x, T_0) - MQ_4(\alpha, x, T_0)) - 2C_0 e^{-J(\alpha)T},
\end{aligned} \tag{37}$$

where

$$\begin{aligned}
Q_1(\alpha, x, T_0) &= \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} A_{T_0} 1_{\{A_{T_0} \geq M\}} \right], \\
Q_2(\alpha, x, T_0) &= \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} 1_{\{A_{T_0} \geq M\}} \right], \\
Q_3(\alpha, x, T_0) &= \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} A_{T_0} 1_{\{A_{T_0} \geq M\}} 1_{\{T_D > T_0\}} \right], \\
Q_4(\alpha, x, T_0) &= \mathbb{E} \left[e^{-\int_0^{T_0} r_s ds} 1_{\{A_{T_0} \geq M\}} 1_{\{T_D > T_0\}} \right].
\end{aligned} \tag{38}$$

The explicit formulas for Q_1 , Q_2 , Q_3 and Q_4 are given in the appendix using the Fourier transform analysis as shown by (47)-(50).

By maximizing the objective function (37) with respect to different default cost factors M , the relationship between the optimal investment risk exposure α^* and the bound M is displayed in Figure 5. In contrast to the monotonic relationship between α^* and the deadweight loss factors ρ_0 and ρ_1 in the proportional deadweight loss case, we find a non-monotonic U -shaped relationship between the optimal risk exposure and the default cost factor M . Purnanandam (2003) also obtain the similar result in the structural framework. While it is quite intuitive to see that when the bound M is larger, the firm will choose a larger risk exposure, since potential default cost is smaller, it is necessary to elaborate the reason why the firm will still choose the high risk exposure when the default loss becomes significant large. When M is close to the liability, the equity value of a firm becomes a barrier option whose value is increasing in the volatility of the asset value. Therefore in the high deadweight loss circumstance, the firm may choose the high risk exposure. However, this conjecture needs to seek the further empirical evidence to support. Moreover, this example shows that our methodology generally produces consistent results as in the structural framework, but with a lot more tractability in terms of the implementations⁶

[Figure 4 about here.]

6 Conclusion

In the setting of information asymmetry between firm managers and market investors, this paper has developed a reduced model to analyze the optimal investment policy and capital structure while considering dividends and tax effects. The reduced form methodology facilitates the implementation as well as the theoretical analysis.

Under the model we have established, debt values (from the firm managers' point of view) and issuing prices (from market investors' perspective) are both derived in closed form. Due to the inside information held by firm managers, they may issue the debt at higher price by increasing their investment risk. We demonstrate that firm managers determine their optimal investment risk exposure by balancing two opposing factors: the pricing advantages gained from the liability holders, and deadweight losses upon the default. Because of the tractability of the model, the solution to this optimization problem has easily been characterized and is straightforward to solve numerically. Moreover we have generalized our model to consider tax effects and optimal dividend policies. We have found that unless the tax benefit is large enough, the firm would still choose issuing zero-coupon bonds, and that threshold has also been derived. Moreover we have discussed the applications of our model in portfolio management and have provided two simple examples to show the tractability of the model in practice and their general consistency with the structural framework. However, the latter one is rather hard for implementations. An interesting direction for future research is the empirical implementation of the model through estimation of a firm's model parameters.

A Appendix

In this Appendix, we provide the proofs of the propositions given in this paper.

A.1 Proof of Proposition 1

Proof. By Lemma 1, we have that for any $0 \leq t < T$,

$$\begin{aligned} B_f(\alpha, t, T) &= \mathbb{E} \left[e^{-\int_t^T r_s ds} [(1 - \delta_f)1_{\{T_D > T\}} + \delta_f] \middle| \mathcal{F}_t \right] \\ &= 1_{\{T_D > t\}}(1 - \delta_f)\mathbb{E} \left[e^{-\int_t^T (r_s + \Lambda_s) ds} \middle| \mathcal{H}_t \right] + \delta_f \pi(t, T), \quad (39) \end{aligned}$$

where $\pi(t, T)$ is defined in (2). Let $\pi_f(\alpha, t, T) := \mathbb{E} \left[e^{-\int_t^T (r_s + \Lambda_s) ds} \middle| \mathcal{H}_t \right]$. Since $M_t := \pi_f(\alpha, t, T)e^{-\int_0^t (r_s + \Lambda_s) ds}$ is an \mathcal{H} -martingale up to time T , by Itô's lemma

we arrive at the following partial differential equation (PDE) for π_f :

$$0 = \frac{1}{2}\sigma_r^2 r \frac{\partial^2 \pi_f}{\partial r^2} + \frac{1}{2}\sigma_y^2 Y \frac{\partial^2 \pi_f}{\partial Y^2} + (b_r + \beta_r r) \frac{\partial \pi_f}{\partial r} + (b_y + \beta_y Y + \beta_{ry} r) \frac{\partial \pi_f}{\partial Y} + \frac{\partial \pi_f}{\partial t} - (r + \Lambda) \pi_f, \quad \pi_f(\alpha, T, T) = 1. \quad (40)$$

Now suppose

$$\pi_f(\alpha, t, T) = \exp\{\phi^f(T-t) + \psi_r^f(T-t)r_t + \psi_y^f(T-t)Y_t\}.$$

On substituting this form into (40) and separating variables, we see in order to satisfy (40), it is necessary and sufficient that the functions $\phi^f(t)$, $\psi_r^f(t)$, and $\psi_y^f(t)$ solve the Riccati equations defined in (8). Therefore we complete the proof. \square

A.2 Proof of Proposition 3

Proof. Under Assumption 5, it is easy to see that

$$\begin{aligned} C_{\mathcal{D}}(t) &= \mathbb{E} \left[e^{-\int_t^T r_s ds} \mathbf{1}_{\{T_{\mathcal{D}} \leq T\}} (\rho_0 + \rho_1 A_T) \middle| \mathcal{F}_t \right] \\ &= \rho_0 \mathbb{E} \left[e^{-\int_t^T r_s ds} \mathbf{1}_{\{T_{\mathcal{D}} \leq T\}} \middle| \mathcal{F}_t \right] + \rho_1 \mathbb{E} \left[e^{-\int_t^T r_s ds} \mathbf{1}_{\{T_{\mathcal{D}} \leq T\}} A_T \middle| \mathcal{F}_t \right]. \end{aligned}$$

From Lemma 1 and Proposition 1, it follows that

$$\begin{aligned} \mathbb{E} \left[e^{-\int_t^T r_s ds} \mathbf{1}_{\{T_{\mathcal{D}} \leq T\}} \middle| \mathcal{F}_t \right] &= \mathbb{E} \left[e^{-\int_t^T r_s ds} (1 - \mathbf{1}_{\{T_{\mathcal{D}} > T\}}) \middle| \mathcal{F}_t \right] \\ &= \pi(t, T) - \pi_f(\alpha, t, T) \mathbf{1}_{\{T_{\mathcal{D}} > t\}}. \end{aligned}$$

Again by Lemma 1, we have

$$\begin{aligned} \mathbb{E} \left[e^{-\int_t^T r_s ds} \mathbf{1}_{\{T_{\mathcal{D}} \leq T\}} A_T \middle| \mathcal{F}_t \right] &= \mathbb{E} \left[e^{-\int_t^T r_s ds} (1 - \mathbf{1}_{\{T_{\mathcal{D}} > T\}}) A_T \middle| \mathcal{F}_t \right] \\ &= \mathbb{E} \left[e^{-\int_t^T r_s ds} A_T \middle| \mathcal{F}_t \right] \\ &\quad - \mathbf{1}_{\{T_{\mathcal{D}} > t\}} \mathbb{E} \left[e^{-\int_t^T (r_s + \Lambda_s) ds} A_T \middle| \mathcal{H}_t \right]. \end{aligned}$$

From (4), it is easy to see that

$$\mathbb{E} \left[e^{-\int_t^T r_s ds} A_T \middle| \mathcal{F}_t \right] = A_t e^{G(\alpha)(T-t)}.$$

Let us denote

$$S_t := \mathbb{E} \left[e^{-\int_t^T (r_s + \Lambda_s) ds} A_T \middle| \mathcal{H}_t \right].$$

Then, with similar arguments as in the proof of Proposition 1, we have the following PDE for S :

$$\begin{aligned}
0 = & \frac{1}{2}(\sigma_A^2 Y + \alpha)A^2 \frac{\partial^2 S}{\partial A^2} + \frac{1}{2}\sigma_r^2 r \frac{\partial^2 S}{\partial r^2} + (r + G(\alpha))A \frac{\partial S}{\partial A} + (b_r + \beta_r r) \frac{\partial S}{\partial r} \\
& + \frac{1}{2}\sigma_y^2 Y \frac{\partial^2 S}{\partial Y^2} + (b_y + \beta_y Y + \beta_{ry} r) \frac{\partial S}{\partial Y} + \frac{\partial S}{\partial t} - (r + \Lambda)S, \quad S_T = A_T.
\end{aligned} \tag{41}$$

Now suppose that

$$S_t = A_t \exp\{\phi^a(T-t) + \psi_r^a(T-t)r_t + \psi_y^a(T-t)Y_t\}.$$

Then, a necessary and sufficient condition for S to satisfy (41) is that the functions $\phi^a(t)$, $\psi_r^a(t)$, and $\psi_y^a(t)$ solve the Riccati equations defined in (46). \square

A.3 Derivations of Equations (33), (34) and (35)

Since $A = \sum_{i=1}^M A_i$, where $A_i = \theta_i A$ for each $i \in \{1, 2, \dots, M\}$, and

$$\frac{dA_i(t)}{A_i(t)} = (r_t + \mu_i)dt + \sigma_i dW_t^i,$$

we have from by Itô's Lemma that

$$\begin{aligned}
dA_t &= \sum_{i=1}^M A_i(t) [(r_t + \mu_i)dt + \sigma_i dW_t^i] \\
&= \sum_{i=1}^M \theta_i A_t [(r_t + \mu_i)dt + \sigma_i dW_t^i] \\
&= A_t [(r_t + \langle \theta, \mu \rangle)dt + \langle \theta, \sigma dW_t \rangle],
\end{aligned} \tag{42}$$

which is (33). In (34), the function $G(\alpha)$ is easy to see, while α can be obtained in view of the fact that

$$\langle \theta, \sigma dW_t \rangle = \left(\sum_{i=1}^M \theta_i^2 \sigma_i^2 \right) \sum_{i=1}^M \frac{\theta_i \sigma_i}{\sum_{i=1}^M \theta_i^2 \sigma_i^2} dW_t^i,$$

and $\sum_{i=1}^M \frac{\theta_i \sigma_i}{\sum_{i=1}^M \theta_i^2 \sigma_i^2} W_t^i$ is a \mathbb{Q} -Brownian motion given by Lévy's Characterization theorem. Therefore (34) is proved. Finally since we have the initial constraint on $A_i(0) \geq v_i$, for each $i \in \{1, 2, \dots, M\}$, (35) is straightforward to derive. \square

A.4 Derivations of the Q Functions in (37)

Here we will apply the Fourier transform analysis used in Heston (1993), and Duffie, Pan and Singleton (2000). First we need to define some notation. Let us denote

$$\begin{aligned}\varphi_{a,b}(\nu, x, A_0, T_0, \lambda_0, \lambda_1, \lambda_2) &:= \mathbb{E} \left[e^{-\int_0^{T_0} (\lambda_0 + \lambda_1 r_s + \lambda_2 Y_s) ds} e^{(a+j\nu b) \ln(A_{T_0})} \right] \\ \Phi_{a,b}(z, x, A_0, T_0, \lambda_0, \lambda_1, \lambda_2) &:= \mathbb{E} \left[e^{-\int_0^{T_0} (\lambda_0 + \lambda_1 r_s + \lambda_2 Y_s) ds} e^{a \ln(A_{T_0})} \mathbf{1}_{\{b \ln(A_{T_0}) \leq z\}} \right].\end{aligned}\quad (43)$$

The following relationship is derived by Duffie, Pan and Singleton (2000, Proposition 2).

$$\begin{aligned}\varphi_{a,b}(\nu, x, A_0, T_0, \lambda_0, \lambda_1, \lambda_2) &= \int_{\mathbb{R}} e^{j\nu z} d\Phi_{a,b}(z, x, A_0, T_0, \lambda_0, \lambda_1, \lambda_2), \\ \Phi_{a,b}(z, x, A_0, T_0, \lambda_0, \lambda_1, \lambda_2) &= \frac{\varphi_{a,0}(\nu, x, A_0, T_0, \lambda_0, \lambda_1, \lambda_2)}{2} \\ &\quad - \frac{1}{\pi} \int_0^\infty \frac{\text{Im}[\varphi_{a,b}(\nu, x, A_0, T_0, \lambda_0, \lambda_1, \lambda_2) e^{-j\nu z}]}{\nu} d\nu,\end{aligned}\quad (44)$$

where $\text{Im}(c)$ denotes the imaginary part of $c \in \mathbb{C}$.

Now following the same approach in Appendix A.3, we have

$$\varphi_{a,b}(\nu, x, A_0, T_0, \lambda_0, \lambda_1, \lambda_2) = e^{\phi^c(T_0) + \psi_r^c(T_0)r_0 + \psi_y^c(T_0)Y_0 + (a+j\nu b) \ln(A_0)}, \quad (45)$$

where ϕ^c , ψ_r^c and ψ_y^c satisfying the following Riccati equations.

$$\begin{aligned}\frac{d\phi^c(t)}{dt} &= \frac{\alpha}{2}(a + j\nu b)^2 + b_r \psi_r^c(t) + b_y \psi_y^c(t) + (G(\alpha) - \alpha/2)(a + j\nu b) - \lambda_0, \quad \phi^c(0) = 0, \\ \frac{d\psi_r^c(t)}{dt} &= \frac{1}{2}\sigma_r^2 (\psi_r^c(t))^2 + \beta_r \psi_r^c(t) + \beta_{ry} \psi_y^c(t) + (a + j\nu b) - \lambda_1, \quad \psi_r^c(0) = 0, \\ \frac{d\psi_y^c(t)}{dt} &= \frac{1}{2}\sigma_y^2 (\psi_y^c(t))^2 + \beta_y \psi_y^c(t) - \lambda_2, \quad \psi_y^c(0) = 0.\end{aligned}\quad (46)$$

Therefore $\Phi_{a,b}$ can explicitly calculated from $\phi_{a,b}$ through the Fourier inversion (44). Finally by (38) and Lemma 1, we have

$$Q_1(\alpha, x, T_0) = \Phi_{1,-1}(-\ln(M), x, 2C_0, T_0, 0, 1, 0), \quad (47)$$

$$Q_2(\alpha, x, T_0) = \Phi_{0,-1}(-\ln(M), x, 2C_0, T_0, 0, 1, 0), \quad (48)$$

$$Q_3(\alpha, x, T_0) = \Phi_{1,-1}(-\ln(M), x, 2C_0, T_0, \ell + J(\alpha), 1 + \lambda_r, \lambda_y), \quad (49)$$

$$Q_4(\alpha, x, T_0) = \Phi_{0,-1}(-\ln(M), x, 2C_0, T_0, \ell + J(\alpha), 1 + \lambda_r, \lambda_y), \quad (50)$$

so that the objective function $F(\alpha)$ in (37) can be explicitly calculated. \square

Notes

¹In order to retain the positivity of Y_t , we restrict β_{ry} to be positive.

²Similar results are obtained under a non-zero recovery assumption but the analysis is more involved.

³Strictly speaking, capital reserve may decrease the deadweight loss cost due to liquidation; however this issue is beyond the scope of this paper.

⁴According to U.S. tax codes, tax deductibility on coupon payments must be subject to the condition that a firm produces earnings before interest and taxes (EBIT) at least as large as the coupon payment ξ for a fiscal year, which is not considered in this paper. For detail, see Goldstein, Ju and Leland (2001).

⁵The maximum is not obtained at either end of the region.

⁶In the structural framework, even the case of proportional deadweight losses is hard to be solved without Monte Carlo simulations.

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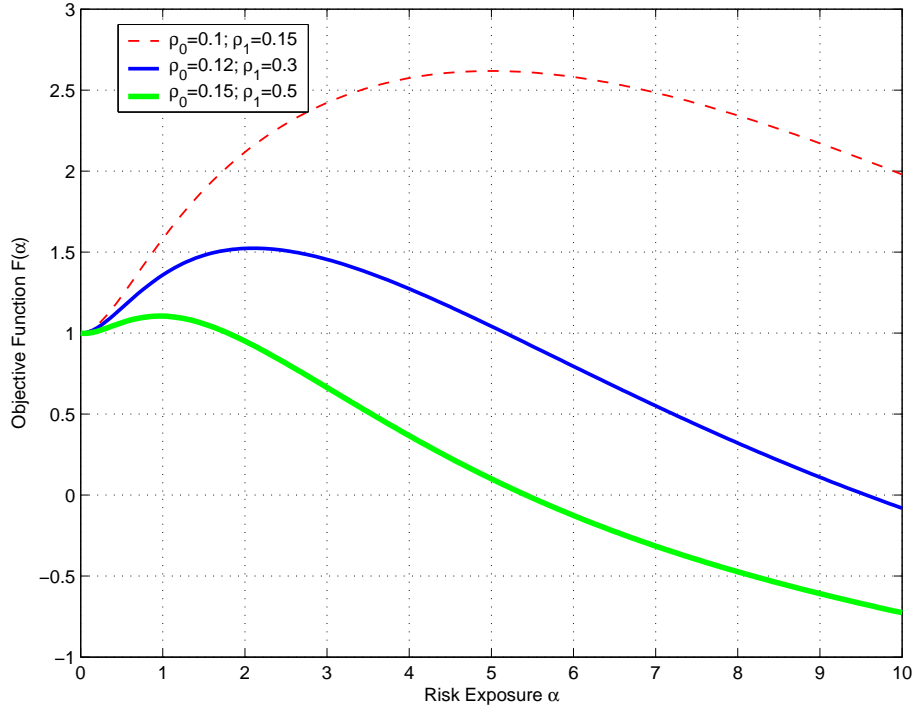


Figure 1: **The objective function F as a function of risk exposure α .** The objective functions F are plotted for three different sets of deadweight loss factors: $\rho_0 = 0.1; \rho_1 = 0.15$, $\rho_0 = 0.12; \rho_1 = 0.3$ and $\rho_0 = 0.15; \rho_1 = 0.5$. Here T_0 is fixed at 1, and the objective functions are calculated according to (36).

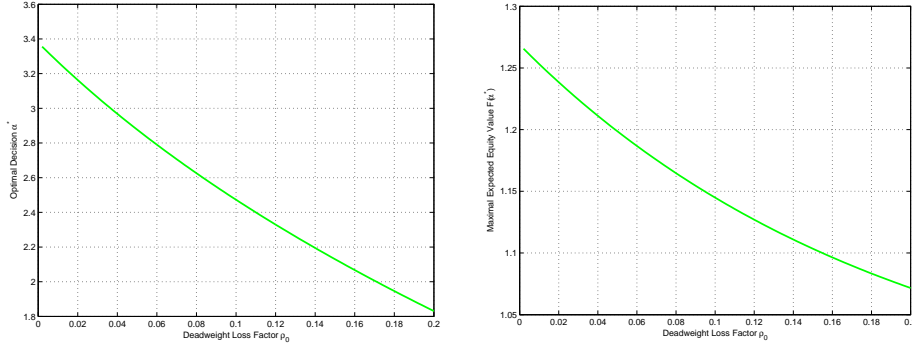


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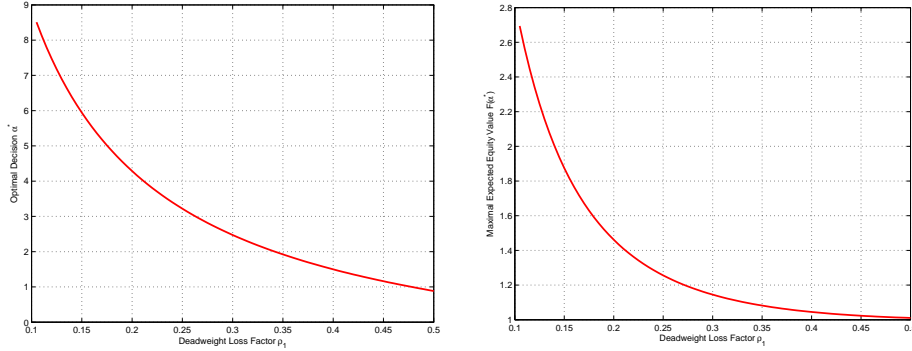


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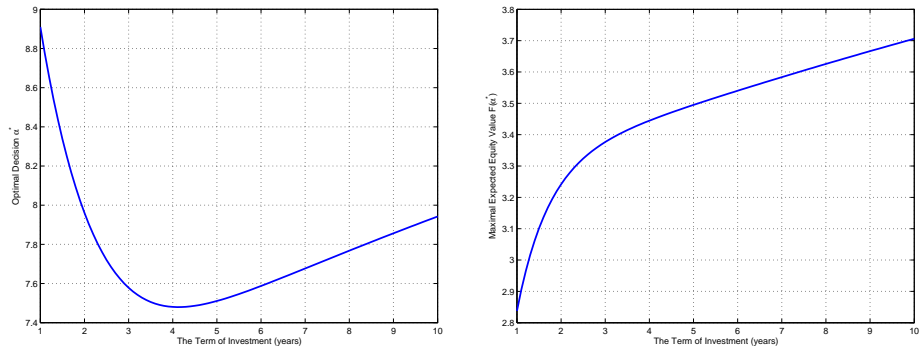


Figure 4: **Sensitivity of optimal risk exposure α^* and the maximal equity value $F(\alpha^*)$ to the investment span T_0 .** We fix $\rho_0 = 0.1$ and $\rho_1 = 0.1$ and plot the optimal investment risk exposure α^* as a function of the span T_0 . For each T_0 , the optimal risk exposure is found and the corresponding maximal equity value is calculated according to (36).

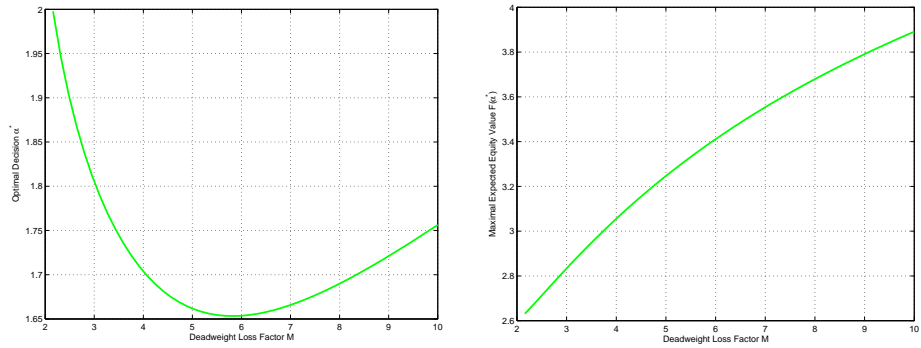


Figure 5: **Sensitivity of optimal risk exposure α^* and the maximal equity value $F(\alpha^*)$ to the default cost factor M .** Without loss of generality, we fix $C_0 = 1$ and plot the optimal investment risk exposure α^* as a function of the bound M . For each M , the optimal risk exposure is found and the corresponding maximal equity value is calculated according to (37).