What is the Link Between Margin Loans and Stock Market Bubbles?*

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Abstract

As a reaction to the general suspicion that margin loans had been a key element of the stock market boom and crash in the late 1920s, the Federal Reserve Bank was empowered to regulate margin lending with the Securities and Exchange Act. The efficacy of the Federal Reserve's margin policy has extensively been studied empirically. However, there still exists no formal rationale for the regulation of margin lending. In this paper, we demonstrate in a principal-agent model that the availability of margin loans can cause the development of a stock market bubble through inducing investors to pay more for a stock than its fundamental value. We show that the emergence of a margin loan induced bubble can be ruled out by an initial margin requirement and thus provide a formal rationale for margin regulation.

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Key Words: asset pricing, asset price bubbles, margin loans, margin regulation.

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1 Introduction

> *"The stock market is in its most dramatic boom ever. [...] Margin debt is soaring; it has increased 87% in the past year. In the midst of this record-breaking boom, the Federal Reserve Board remains silent about the speculative level of the market, neither commenting that the market is too high nor using its powers over margin requirements to dampen the markets."*[1]

As this statement by Robert J. Shiller, shortly before the bursting of the so-called "internet bubble", illustrates, there is a widespread belief of a connection between margin loans and stock market booms.

The supposition that margin lending may have an important impact on stock prices first came up after the stock market crash of October 1929. The fact that the volume of margin loans outstanding had been extraordinary high in the late 1920s raised the general suspicion that margin loans had fueled the stock market boom that preceded the 1929 crash and that margin calls had aggravated the crash. This suspicion led to an intense discussion about margin lending and its impact on stock prices.[2]

As a consequence of the discussion about the role of margin lending in the 1929 stock market crash, in 1934 the Federal Reserve Board was given the authority to regulate margin loans with the Securities Exchange Act.[3] In the same year, the Board of Governors of the Federal Reserve System established an "initial margin requirement" for margin loans granted by brokers and dealers.[4] This requirement sets a minimum equity position on the date of a securities purchase that is financed by a margin loan.

Between 1934 and 1974 the Federal Reserve changed the initial margin requirement 22 times. However, after a number of empirical studies had raised serious doubts whether

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[1] Shiller (2000), page A46.

[2] See, for example, Roelse (1930), Eiteman (1932, 1933), Livermore (1932), or Palyi (1932).

[3] From an examination of the congressional hearing on the bill to empower the Federal Reserve to regulate margin lending, Moore (1966) identified three major objectives behind the introduction of margin regulation: the reduction of excessive credit used in stock speculation to leave more for productive uses, the protection of investors from going too deep into debt to invest in stocks, and the reduction of stock market price volatility.

[4] The Federal Reserve's margin regulation was later extended to margin loans granted by other lending institutions. For an overview of the regulation of margin loans see, e.g., Fortune (2000).

the variations of the margin requirement had the intended effects,[5] the Federal Reserve gave up its active margin policy in 1974.

The stock market crash of October 1987 and the studies that followed[6] rekindled interest in margin policy and spawned a political and academic debate on the effects of margin lending and margin requirements. In particular, an empirical study by Hardouvelis (1988) who claimed that historical evidence supported the proposition that margin requirements could be used to control stock market volatility revitalized academic interest in margin policy. The results of Hardouvelis were disputed by several subsequent studies[7] and the empirical literature on the effects of margin regulation expanded significantly.

Recently, the combination of a booming stock market and soaring margin debt in the late 1990s led to congressional hearings on the economic effects of margin lending[8] and to an intense debate on the possible contribution of margin regulation in the prevention of stock market bubbles.[9] Furthermore, an empirical study by Fortune (2001) found a significant positive relationship between the volume of margin loans outstanding and the level and the volatility of stock prices in the subsequent month.

Therefore, there is an enduring discussion about the impact of margin loans and margin regulation on stock prices. The effects of margin lending and the Federal Reserve's margin policy on stock prices have extensively been studied empirically. In contrast, the analytical research on the effects of margin loans and margin regulation is scarce.[10]

The aim of this paper is to provide a formal rationale for the regulation of margin lending by demonstrating in an analytical model that the availability of margin loans can cause

[5] See, for example, Cohen (1966), Moore (1966), Largay and West (1973), and Officer (1973).

[6] See, for example, Katzenbach (1987), Brady (1988), or U.S. Securities and Exchange Commission (1988).

[7] See, for example, Ferris and Chance (1988), Kupiec (1989), Salinger (1989), Schwert (1989a,b), and Hsieh and Miller (1990).

[8] See U.S. Congress (2000).

[9] See, for example, Bartlett (2000), Shiller (2000), and the remarks by Alan Greenspan, chairman of the Federal Reserve Board, in Federal Open Market Committee (1996) and Federal Reserve Bank of Kansas City (2002).

[10] As far as we know, there is no previous analytical study of the effects of margin lending on stock prices and there exist only four analytical studies of the effects of (changes in) margin requirements, i.e. Heal (1984), Goldberg (1985), Kupiec and Sharpe (1991), and Chowdhry and Nanda (1998).

the development of a stock market bubble through inducing investors to pay more for a stock than its fundamental value.[11]

Thus, the three major questions this paper deals with are:

1. What is the link between margin loans and stock market bubbles?
2. Under which conditions do margin loans induce investors to pay more for a stock than its fundamental value?
3. Can the emergence of margin loan induced stock market bubbles be ruled out by margin regulation and, if so, how should the Federal Reserve's margin policy look like?

The commonly supposed link between margin lending and the development of stock market bubbles is the pyramiding hypothesis which is quite intuitive, but has not been analyzed formally yet.[12] In this paper we do not analyze the pyramiding hypothesis neither. Instead, we provide another possible link between margin lending and the emergence of stock market bubbles, i.e. we formally demonstrate that an agency problem between investors who are financed by a margin loan and the institution granting the margin loans can induce the investors to pay more than the fundamental value for a risky asset.

In their seminal paper on agency theory Jensen and Meckling (1976) looked at, among other things, the incentive effects associated with debt. They demonstrated that an owner-manager of a (at least partly) debt-financed firm faces strong incentives to increase the risk of the firm's investments. This phenomenon is commonly referred to as the "risk shifting problem" or "asset substitution problem" in the literature.[13]

Allen and Gale (2000) applied the phenomenon of risk shifting/asset substitution to an asset pricing context. They constructed a principal-agent model to illustrate that investors who can borrow money to invest in a risky asset are willing to pay a higher price for this

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[11] Note that the term asset price bubble is commonly used to designate a situation in which the price of an asset exceeds its fundamental value; see, e.g., Tirole (1985), Flood and Hodrick (1990), or Allen et al. (1993). Following this definition, a stock market bubble is a situation in which stock prices exceed the stocks' fundamental values.

[12] For the pyramiding hypothesis see, for example, Garbade (1982) and Kupiec (1998).

[13] See, for example, Myers (1977), Haugen and Senbet (1981), Gavish and Kalay (1983), John and John (1993), or Leland (1998).

asset than they are if their investment is completely equity financed. This is due to the fact that credit-financed investors do not bear the full cost of borrowing when the investment turns out badly. If the return from the investment is lower than the repayment the lending institution requires, the investors declare bankruptcy and thus avoid full repayment of the loan. When the return is high, however, the investors keep the remainder of the investment's return after repayment of the loan.[14]

In Allen and Gale (2000) the lending institution writes simple debt contracts with the investors.[15] In this paper we apply the basic idea of Allen and Gale (2000) to the case that the investors borrow money via margin loans. An investor who purchases securities on margin has to deposit a certain percentage of the purchase price, often (partly) in the form of other securities, with the lending institution as collateral. Furthermore, the margin loan financed securities stay with the lender and also serve as collateral. Therefore, margin loans are secured by the investor's equity in the margin account and the securities which have been purchased from the margin loan. However, as (at least a part of) the collateral consists of risky securities, margin loans nevertheless bear a default risk. Hence, the basic idea of Allen and Gale (2000) is applicable to margin loans.

The remainder of this paper is organized as follows: In Section 2 we develop our model for the analysis of how the availability of margin loans can contribute to the emergence of asset price bubbles. After defining a basic framework in which investments have to be completely equity financed as there is no lending institution and determining the fundamental value of a risky asset within this setting (Section 2.1), we extend the framework to a principal-agent model by introducing a margin loan (Section 2.2). In Section 3 we look at the maximum price the investors are willing to pay for a risky asset in this principal-agent setting and compare it to the asset's fundamental value derived in Section 2.1. After determining the additional net return the investors expect from a margin loan financed investment in the risky asset (Section 3.1), we derive a necessary and a sufficient condition under which the investors are willing to pay more for the risky asset than its fundamental value (Section 3.2). Finally, we provide an economic interpretation of these conditions (Section 3.3).

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[14] In Allen and Gale (2003a) this basic idea is used to consider the relationship between monetary policy and asset price bubbles. And in Allen and Gale (2003b) the model of Allen and Gale (2000) is applied to the analysis of the effect of stock market interlinkages on asset price bubbles.

[15] Simple debt contracts in the sense of Gale and Hellwig (1985).

2 Development of the model

The term bubble is commonly used to designate a situation in which the price of an asset exceeds its fundamental value. Therefore, as a first step, we develop a simple framework to determine the fundamental value of an asset.

2.1 A simple framework to determine the fundamental value of a risky asset

The assumptions of the basic model are:

Assumption 1: There is only one period, beginning at t_0 and ending at t_1.

Assumption 2: There are two assets, a safe one and a risky one. For each dollar invested in the safe asset in t_0, the return is $r_s > 1$ in t_1. The return of the risky asset in t_1 is uncertain and binomially distributed.

Assumption 3: All investors are risk neutral, fully rational and seek to maximize their expected return in t_1.

Assumption 4: The investors expect the return per unit of the risky asset to be $r_u > 0$ with a probability of $w_u \in (0, 1)$ and $r_d \in (0, r_u)$ with a probability of $w_d = 1 - w_u$ in t_1.

In this easy framework, the maximum price the investors are willing to pay per unit of the risky asset in t_0 can easily be calculated. From Assumption 4 it can be seen that the investors expect the return per unit of the risky asset to be

$$E_r = w_u \cdot r_u + w_d \cdot r_d \tag{1}$$

in t_1.

Since all investors are assumed to be risk neutral, the maximum price they are willing to pay for the risky asset, P^f, will equate the expected marginal returns from investing 1 \$ in the two assets[16] in the way that

$$\frac{1}{P^f} \cdot E_r = r_s. \tag{2}$$

As a result, the investors' reservation price in t_0 for one unit of the risky asset in the basic model is

$$P^f = \frac{E_r}{r_s}. \tag{3}$$

As can be seen from Equation (3), in the simple framework of our basic model the investors' reservation price in t_0 for one unit of the risky asset, P^f, is the present value of the return in t_1 the investors expect per unit of the asset, where the discount rate is the investors' capital cost. Thus, P^f corresponds to the standard definition of an asset's fundamental value.[17] Therefore, in the following, we will refer to P^f as the fundamental value of the risky asset and define a bubble as follows:

Definition 1: If the price of the risky asset in t_0 exceeds the asset's fundamental value, P^f, it contains a non-fundamental component and hence there is a bubble.

2.2 The introduction of a margin loan into the model

After having determined the fundamental value of the risky asset in Section 2.1, we now introduce an institution into our model that offers a margin loan and thereby extend the basic framework of Section 2.1 to a principal-agent model.

[16] As $r_s > 1$, the best alternative to an investment in the risky asset is to invest the money in the safe asset.

[17] See, for example, Flood and Hodrick (1990) or Campbell and Kyle (1993).

Assumption 5: Each investor has an initial wealth of 1 \$ which he/she invests in the risky asset at a price of P^f in t_0.[18]

Presuming that the investors do not keep their initial wealth riskless in their pocket is consistent to the assumptions of $r_s > 1$ and of fully rational investors seeking to maximize their expected return in t_1.[19]

If an investor purchases a security "on margin", he/she has to deposit a certain percentage of the purchase price, often in the form of other securities, with the lending institution as collateral. In the following, we will look at the case that the collateral is the risky asset, hence the investors will borrow against their initial portfolio of $1/P^f$ units of the risky asset.

Now, still in t_0, we introduce an institution into the model that offers a margin loan:[20]

Assumption 6: Each investor is offered to borrow up to $q^{max} > 0$ \$ for each 1 \$ of value of his/her initial portfolio via a margin loan. If an investor borrows $q \in (0,\ q^{max}]$ in t_0, the lending institution requires a repayment of $q \cdot i$ in t_1 with $i \geq r_s$. If the investor is unable to repay $q \cdot i$ in t_1 from the return on the investment financed by the margin loan, the lender can use the return on the investor's initial portfolio to fulfill its claim. Therefore, the investor's initial portfolio serves as collateral.

q^{max} can be thought of as the "loan-to-value ratio" of the margin loan. Obviously, it is just another form of an initial margin requirement.[21] As we assume a one-period setting, we do not incorporate a maintenance margin requirement in our model. Fortune (2003) has shown that maintenance margin requirements in reality only rarely come into play, namely in the case of extreme price declines, and are therefore of minor relevance.

[18] With this assumption we presume the risky asset to be perfectly divisible which is certainly another simplification of our model, yet a standard assumption in asset pricing models.

[19] See Assumptions 2 and 3.

[20] Therefore, there are two successive events in t_0. First, each investor purchases $1/P^f$ units of the risky asset. Then, the margin loan is introduced into the model.

[21] We will return to this fact in Section 4.

3 Analysis of the model

In Section 2.2 we have developed the principal-agent model for our analysis of the effect of the availability of margin loans on asset prices. In this section, we first determine the highest price P^m the investors are willing to pay for the risky asset when the purchase is financed by a margin loan.[22] Then, we compare this price to the asset's fundamental value we have derived in Section 2.1 to deduce the conditions under which the availability of margin loans can lead to the emergence of a bubble by inducing the investors to pay a price for the risky asset that exceeds P^f.

As P^f in Section 2.1, P^m will equate the additional net returns the investors expect from an investment in the risky asset and in the best alternative in our extended framework. Therefore, as a first step of our determination of P^m, we determine the investors' best alternative to making use of the margin loan and investing the borrowed money in the risky asset.

As we have presumed that $i \geq r_s > 1$, no investor will make use of the margin loan to keep the borrowed amount in his/her pocket. The additional net return in t_1 from borrowing $q > 0$ and investing the borrowed money in the safe asset is $ANR_s = q \cdot (r_s - i) \leq 0$ and cannot be positive as $i \geq r_s$. Therefore, there is no reason for the investors to make use of the margin loan to invest in the safe asset. Hence, the best alternative to borrowing q and investing the money in the risky asset is not to make use of the margin loan offer at all,[23] and thus the opportunity cost of investing the margin loan in the risky asset is zero.

As a result, P^m can be calculated from the following equation:

$$ANR_r(P^m) = 0 \tag{4}$$

with $ANR_r(P)$ being the function for the additional net return an investor expects from making use of the margin loan if he/she invests the borrowed money in the risky asset at a price of P.

[22] Therefore, P^m is the investors' reservation price for the risky asset when the investment is financed by a margin loan. Remember that P^f is the investors' reservation price for the risky asset when the purchase is equity financed.

[23] For $i = r_s$, ANR_s is zero. Therefore, if $i = r_s$, the investors are indifferent between not making use of the margin loan and borrowing q to invest the borrowed money in the safe asset.

The determination of $ANR_r(P)$ is quite complicated. In Section 3.1 we first show that concerning $ANR_r(P)$ two price intervals have to be distinguished and then derive $ANR_r(P)$. Then, in Section 3.2 we calculate P^m from Equation (4) for both price intervals and deduce a necessary and a sufficient condition under which P^m is higher than P^f.

3.1 The expected additional net return from a margin loan financed investment in the risky asset

The additional net return an investor expects from an investment financed by a margin loan can be calculated by comparing the investor's expected total return in t_1 with and without the investment.

As a first step to determine $ANR_r(P)$, we look at the *actual* additional net return in t_1 of an investor who has made use of the margin loan offer and has invested the money in q/P units of the risky asset in t_0, ANR_r^A, in dependence of the risky asset's actual return in t_1. Presuming that $r_u^A > P \cdot i$ and $r_d^A \in (0, r_u^A)$,[24] we distinguish five different possible cases for $ANR_r^A(r_u^A, r_d^A)$ (see Figure 1):

In case A1 the actual return per unit of the risky asset is r_u^A in t_1. As $r_u^A > P \cdot i$, the investor is able to completely repay $q \cdot i$ from the return on the borrowed q alone.[25]

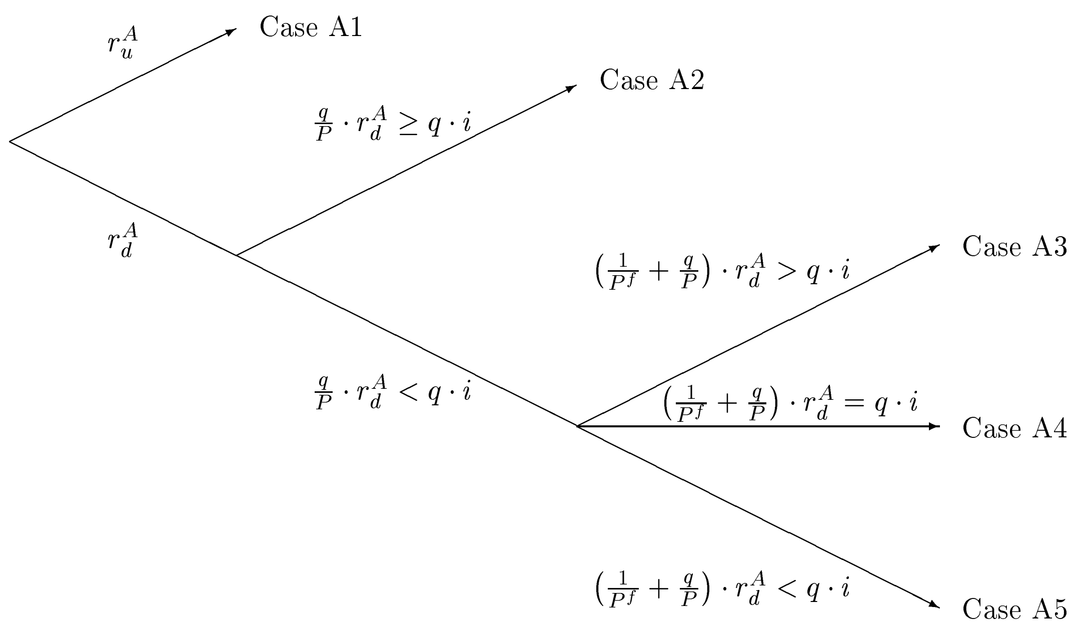
In cases A2 to A5 the risky asset's actual return per unit is r_d^A in t_1. Although r_d^A is lower than r_u^A, in case A2 it is nevertheless still high enough such that, as in case A1, the return on the investor's initial portfolio does not have to be touched for the repayment of $q \cdot i$. In contrast, in cases A3 to A5 the investor is not able to completely repay $q \cdot i$ from the return on the q/P units of the risky asset he has bought from the margin loan alone and hence the lending institution gets (a part of) the return on his/her initial portfolio.

In case A3 the lender gets only a part of the return on the investor's initial $1/P^f$ units of the risky asset as the return on his/her total portfolio is higher than the repayment the

[24] Assuming $r_u^A/P > i$ excludes some possible cases from our analysis. We will show in footnote 29 that these cases are not relevant for the determination of $ANR_r(P)$.

[25] As $r_u^A > P \cdot i \Leftrightarrow q/P \cdot r_u^A > q \cdot i$, the actual return on the q/P units of the risky asset the investor has purchased from the margin loan is higher than the repayment the lending institution requires.

Figure 1: The five different possible cases for $ANR_r^A(r_u^A, r_d^A)$.



lending institution requires.[26] r_d^A/P^f is the maximum amount the investor is able to use to fulfill the lender's remaining claim for the case that the return from the margin loan financed investment in the risky asset is not sufficient to completely repay $q \cdot i$. Case A4 is such that this is just enough whereas in case A5 the investor is not able to completely repay the margin loan in t_1 even from the return on his/her total portfolio.

Case A1:

As $r_u^A/P > i$, in case A1 the investor gains an additional net return from the margin loan. Thus, his/her actual total return in t_1, TR^A, in case A1 is

$$TR^A = \underbrace{\frac{r_u^A}{P^f}}_{from\ initial\ portfolio} + \underbrace{q \cdot \left(\frac{r_u^A}{P} - i\right)}_{from\ margin\ loan} > \frac{r_u^A}{P^f} > 0.$$

[26] The investor's total portfolio consists of his/her initial portfolio of $1/P^f$ units of the risky asset and the margin loan financed portfolio of q/P units of the risky asset.

The actual additional net return from the margin loan financed investment in the risky asset in case A1 is

$$ANR_r^A = q \cdot \left(\frac{r_u^A}{P} - i \right) > 0.$$

<u>Case A2:</u>
As $r_d^A/P \geq i$, in case A2 the margin loan again generates an additional net return that is not negative. The investor's actual total return in case A2 is

$$TR^A = \underbrace{\frac{r_d^A}{P^f}}_{from\ initial\ portfolio} + \underbrace{q \cdot \left(\frac{r_d^A}{P} - i \right)}_{from\ margin\ loan} \geq \frac{r_d^A}{P^f} > 0$$

with an actual additional net return from the margin loan of

$$ANR_r^A = q \cdot \left(\frac{r_d^A}{P} - i \right) \geq 0.$$

<u>Case A3:</u>
Due to $(1/P^f + q/P) \cdot r_d^A > q \cdot i$, in case A3 the investor's actual total return is still positive with

$$TR^A = \underbrace{\frac{r_d^A}{P^f}}_{from\ initial\ portfolio} + \underbrace{q \cdot \left(\frac{r_d^A}{P} - i \right)}_{from\ margin\ loan} > 0.$$

Like in case A2, the actual additional net return from the margin loan in case A3 is

$$ANR_r^A = q \cdot \left(\frac{r_d^A}{P} - i \right) < 0,$$

but now this term is negative as $r_d^A/P < i$.

Case A4:

As $\left(1/P^f + q/P\right) \cdot r_d^A = q \cdot i$, the actual return in t_1 on the investor's total portfolio is just sufficient to completely repay $q \cdot i$ and TR^A in case A4 is

$$\begin{aligned} TR^A &= \frac{r_d^A}{P^f} + q \cdot \left(\frac{r_d^A}{P} - i\right) \\ &= \underbrace{\frac{r_d^A}{P^f}}_{from\ initial\ portfolio} - \underbrace{\frac{r_d^A}{P^f}}_{from\ margin\ loan} = 0. \end{aligned}$$

The investor's actual additional net return from the margin loan in case A4 is negative with

$$\begin{aligned} ANR_r^A &= q \cdot \left(\frac{r_d^A}{P} - i\right) \\ &= -\frac{r_d^A}{P^f} < 0. \end{aligned}$$

Case A5:

Due to $\left(1/P^f + q/P\right) \cdot r_d^A < q \cdot i$, the investor, as in case A4, has to give the whole return on his total portfolio to the lending institution and thus his/her total return in t_1 in case A5 is zero:

$$TR^A = \underbrace{\frac{r_d^A}{P^f}}_{from\ initial\ portfolio} - \underbrace{\frac{r_d^A}{P^f}}_{from\ margin\ loan} = 0.$$

The investor's actual additional net return from the margin loan in case A5, as in case A4, is

$$ANR_r^A = -\frac{r_d^A}{P^f} < 0.$$

The investor's liability is limited to the return on his/her initial portfolio. Therefore, if the return on the investor's total portfolio of $1/P^f + q/P$ units of the risky asset in t_1 is not sufficient to completely repay $q \cdot i$, i.e. if $\left(1/P^f + q/P\right) \cdot r_d^A < q \cdot i$,[27] the investor

[27] See case A5 in Figure 1.

defaults on the margin loan.[28] Thus, although the margin loan is secured by the investor's initial portfolio of $1/P^f$ units of the risky asset, it nevertheless bears a default risk.

Up to this point, we have analyzed the possible cases for the actual additional net return of borrowing q and investing the money in the risky asset in dependence of the risky asset's actual return in t_1, $ANR_r^A(r_u^A, r_d^A)$. Now, we look at the expected additional net return of such an investment as a function of the price P the investor pays for the asset in t_0, $ANR_r(P)$. Therefore, in the following we will focus on $ANR_r(P)$ for given r_s, w_u, r_u, r_d, i, q^{max}, and q.

As a consequence of the investor's limited liability, concerning $ANR_r(P)$ we distinguish two intervals of prices $P < r_u/i$:[29]

Definition 2: Price interval I1 contains all prices $P < r_u/i$ for which the investor expects to be able to completely repay $q \cdot i$ in t_1 irrespective of the risky asset's payoff state, i.e. all $P < r_u/i$ for which $(1/P^f + q/P) \cdot r_d$ is at least as high as $q \cdot i$.

Definition 3: Price interval I2 contains all prices $P < r_u/i$ for which the investor expects to default in the low payoff state of the risky asset, i.e. all $P < r_u/i$ for which $(1/P^f + q/P) \cdot r_d$ is lower than $q \cdot i$.[30]

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[28] The margin loan's default risk is increasing the more the investor pays for the risky asset. This is due to the fact that the investor defaults in the risky asset's low payoff state if $(1/P^f + q/P) \cdot r_d^A < q \cdot i$ and $\partial((1/P^f + q/P) \cdot r_d^A)/\partial P = -q \cdot r_d^A/P^2 < 0$ whereas $\partial(q \cdot i)/\partial P \equiv 0$.

[29] If the price P an investor who makes use of the margin loan to invest in the risky asset pays for the asset is higher than or equal to r_u/i, then even the return the investor expects from the investment in the high payoff state, $q/P \cdot r_u$, is not higher than the repayment of $q \cdot i$ the lending institution requires. As there is no reason for a rational investor to pay such a price, we will therefore exclude all prices $P \geq r_u/i$ from our analysis in the following.

[30] From $\partial((1/P^f + q/P) \cdot r_d)/\partial P = -q \cdot r_d/P^2 < 0$ it can be seen that prices from the interval I2 are higher than P from the interval I1. $(1/P^f + q/P) \cdot r_d = q \cdot i \Leftrightarrow P = q \cdot r_d/(q \cdot i - r_d/P^f)$ is the price that separates price interval I1 from interval I2. If $q \cdot r_d/(q \cdot i - r_d/P^f) \geq r_u/i$, there is only price interval I1.
For example, for $r_s = 1.1$, $w_u = w_d = 0.5$, $r_u = 2$, $r_d = 0.5$, $i = 1.15$, and $q = 0.6$, P^f is $E_r/r_s = (0.5 \cdot 2 + 0.5 \cdot 0.5)/1.1 \approx 1.136364$. As $(1/P^f + q/P) \cdot r_d = q \cdot i \Leftrightarrow P = q \cdot r_d/(q \cdot i - r_d/P^f) = 0.6 \cdot 0.5/(0.6 \cdot 1.15 - 0.5/1.136364) = 1.2$ and $r_u/i = 2/1.15 \approx 1.739130$, price interval I1 contains all prices $P > 0$ up to $P = 1.2$ and price interval I2 contains all prices $1.2 < P < 1.739130$.

From our results concerning ANR_r^A the function for the additional net return an investor expects from a margin loan financed investment in the risky asset in dependence of the price P he/she pays for the asset, $ANR_r(P)$, can be derived for the two intervals:

<u>Interval I1:</u>
For prices from the interval I1,

$$ANR_r(P) = \underbrace{w_u \cdot q \cdot \left(\frac{r_u}{P} - i\right)}_{>0} + \underbrace{w_d \cdot q \cdot \left(\frac{r_d}{P} - i\right)}_{\geq -w_d \cdot \frac{r_d}{P^f}} \tag{5}$$

$$= q \cdot \left(\frac{E_r}{P} - i\right). \tag{6}$$

The first part of the sum in Equation (5) stems from the investor's expectations concerning the high payoff state of the risky asset and is, due to $P < r_u/i \Leftrightarrow r_u/P > i$, always positive.
The second term of Equation (5) represents the additional net return the investor expects from borrowing q and investing the money in the risky asset for the case that the risky asset's return is r_d in t_1. It is nonnegative for prices up to r_d/i and negative for higher P. This is due to the fact that for $P \leq r_d/i$, the investor expects to be able to completely repay $q \cdot i$ from the return on the margin loan financed investment alone even in the low payoff state of the risky asset. For $P > r_d/i$ he/she expects to have to use (a part of) the return on his/her initial portfolio, r_d/P^f, for the repayment of $q \cdot i$ in the low payoff state so that his/her expected total return is lowered by the margin loan. For prices P for which $(1/P^f + q/P) \cdot r_d > q \cdot i$ holds, the second term of Equation (5) is higher than $-w_d \cdot r_d/P^f$ as the investor expects that the lending institution will at most get a part of the return on his/her initial portfolio in t_1. For $(1/P^f + q/P) \cdot r_d = q \cdot i$, the second term is equal to $-w_d \cdot r_d/P^f$ as the investor expects that the lender will get all payments from his/her initial $1/P^f$ units of the risky asset. The expected return on his/her initial portfolio in the low payoff state of the risky asset, r_d/P^f, is the maximum amount the investor can lose.

Interval I2:

For prices from the interval I2, $ANR_r(P)$ is

$$ANR_r(P) = \underbrace{w_u \cdot q \cdot \left(\frac{r_u}{P} - i\right)}_{>0} + \underbrace{w_d \cdot \left(-\frac{r_d}{P^f}\right)}_{<0}. \tag{7}$$

As in Equation (5), the first term of Equation (7) represents the additional net return the investor expects from a margin loan financed investment in the risky asset for the risky asset's high payoff state and is always positive due to $P < r_u/i \Leftrightarrow r_u/P > i$.

The second part of the sum in Equation (7), again standing for the additional net return the investor expects for the risky asset's low payoff state, is always negative. This is due to the fact that, as for the case that $(1/P^f + q/P) \cdot r_d = q \cdot i$ of interval I1, for all prices from the interval I2 the investor expects to lose the whole return on his/her initial portfolio to the lending institution in the risky asset's low payoff state.

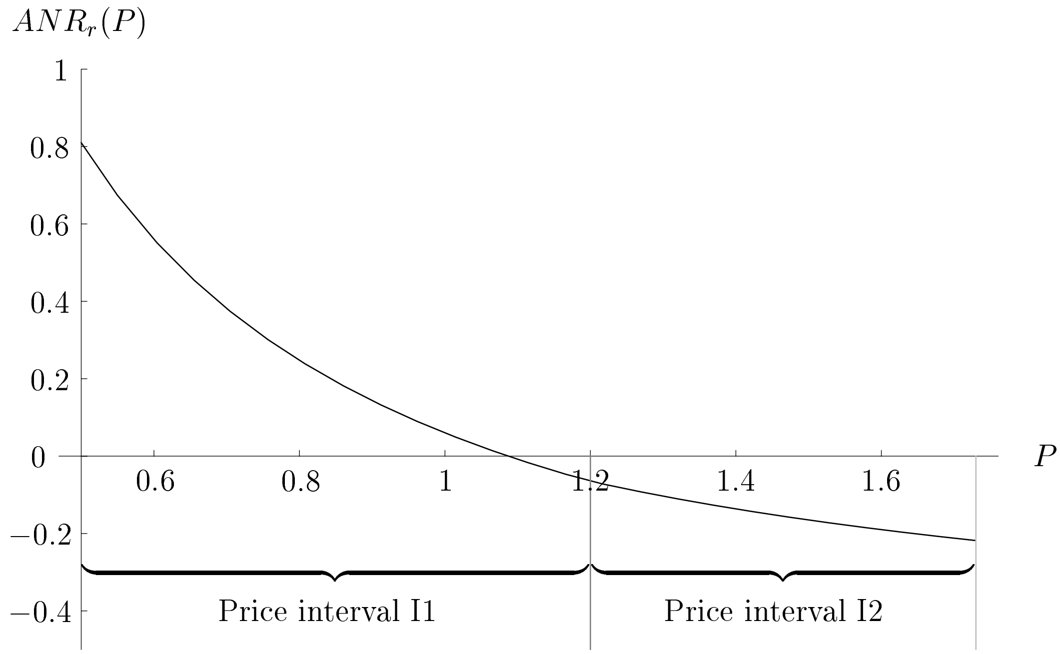
Therefore, for prices $P < r_u/i$, the function for the additional net return an investor expects from a margin loan financed investment in the risky asset in dependence of P, $ANR_r(P)$, is[31]

$$ANR_r(P) = \begin{cases} w_u \cdot q \cdot \left(\frac{r_u}{P} - i\right) + w_d \cdot q \cdot \left(\frac{r_d}{P} - i\right) & \text{for } \left(\frac{1}{P^f} + \frac{q}{P}\right) \cdot r_d \geq q \cdot i \\ w_u \cdot q \cdot \left(\frac{r_u}{P} - i\right) - w_d \cdot \frac{r_d}{P^f} & \text{for } \left(\frac{1}{P^f} + \frac{q}{P}\right) \cdot r_d < q \cdot i. \end{cases} \tag{8}$$

An example for $ANR_r(P)$ which is based on the parameter constellation of the example in footnote 30 is presented in Figure 2:

[31] Remember that we have shown that prices P for which $P \geq r_u/i \Leftrightarrow q/P \cdot r_u \leq q \cdot i$ are not relevant for our analysis of P^m. Hence, all prices for which the investor expects to default in both payoff states, i.e. all P for which $(1/P^f + q/P) \cdot r_u < q \cdot i$, and thus $ANR_r(P)$ is $-w_u \cdot r_u/P^f - w_d \cdot r_d/P^f$ are excluded from our analysis.

Figure 2: Example for $ANR_r(P)$.



Note: In this figure an example for $ANR_r(P)$ when $r_s = 1.1$, $w_u = w_d = 0.5$, $r_u = 2$, $r_d = 0.5$, $i = 1.15$, and $q = 0.6$ is presented for $P \in [0.5, 1.739130)$. As we have shown in footnote 30, for this parameter constellation price interval I1 contains all prices $P > 0$ up to $P = 1.2$ and price interval I2 consists of all prices $P \in (1.2, 1.739130)$. $ANR_r(P)$ is $0.5 \cdot 0.6 \cdot (2/P - 1.15) + 0.5 \cdot 0.6 \cdot (0.5/P - 1.15)$ in price interval I1 and $0.5 \cdot 0.6 \cdot (2/P - 1.15) - 0.5 \cdot 0.5/1.136364$ in price interval I2.

Figure 2 reveals that $ANR_r(P)$ is strictly monotonically decreasing in both price intervals. This can as well be seen from the negative first order derivatives of $ANR_r(P)$ with respect to P for both price intervals:[32]

$$\frac{\partial ANR_r(P)}{\partial P} = -\frac{w_u \cdot q \cdot r_u}{P^2} - \frac{w_d \cdot q \cdot r_d}{P^2} \tag{9}$$

$$= -\frac{q \cdot E_r}{P^2} < 0 \tag{10}$$

for interval I1 and

[32] Strictly speaking, $ANR_r(P)$ is not differentiable at $(1/P^f + q/P) \cdot r_d = q \cdot i \Leftrightarrow P = q \cdot r_d/(q \cdot i - r_d/P^f)$. Rather, $ANR_r(P)$ has a kink, where the left-hand first order derivative of $\partial ANR_r(P)/\partial P = -q \cdot E_r/P^2$ and the right-hand first order derivative of $\partial ANR_r(P)/\partial P = -w_u \cdot q \cdot r_u/P^2$ have to be distinguished. We will nevertheless not handle this price as an extra case, but point to possible problems of this simplification whenever they arise in the following.

$$\frac{\partial ANR_r(P)}{\partial P} = -\frac{w_u \cdot q \cdot r_u}{P^2} < 0 \tag{11}$$

for interval I2.[33] This result is not surprising as the return the investor expects from investing the margin loan in the risky asset decreases the more he/she pays for the asset whereas the margin loan's cost is independent of P.[34]

3.2 The conditions under which $\mathbf{P^m > P^f}$

After having determined $ANR_r(P)$ in Section 3.1, we are now able to calculate the maximum price the investors are willing to pay for the risky asset when the purchase is financed by a margin loan, P^m, from Equation (4).

With our result for $ANR_r(P)$,[35] Equation (4) can be written as:[36]

$$0 = \begin{cases} w_u \cdot q \cdot \left(\frac{r_u}{P^m} - i\right) + w_d \cdot q \cdot \left(\frac{r_d}{P^m} - i\right) & \text{for } \left(\frac{1}{P^f} + \frac{q}{P^m}\right) \cdot r_d \geq q \cdot i \\ w_u \cdot q \cdot \left(\frac{r_u}{P^m} - i\right) - w_d \cdot \frac{r_d}{P^f} & \text{for } \left(\frac{1}{P^f} + \frac{q}{P^m}\right) \cdot r_d < q \cdot i \end{cases} \tag{12}$$

Prices from the interval I2 are higher than P from the interval I1.[37] Therefore, to investigate whether it is possible that P^m is higher than P^f we calculate P^m for the case that this price is in interval I2, i.e. if $(1/P^f + q/P^m) \cdot r_d$ is lower than $q \cdot i$, and compare our result for P^m to the risky asset's fundamental value derived in Section 2.1.

---

[33] The convexity of $ANR_r(P)$ in both price intervals which is visible in Figure 2 is confirmed by the second order derivatives of $ANR_r(P)$ with respect to P which are $\partial^2 ANR_r(P)/\partial P^2 = 2 \cdot q \cdot E_r \cdot P^{-3} > 0$ for price interval I1 and $\partial^2 ANR_r(P)/\partial P^2 = 2 \cdot w_u \cdot q \cdot r_u \cdot P^{-3} > 0$ for price interval I2. Note that, as we have mentioned in footnote 32, $ANR_r(P)$ is not differentiable at $P = q \cdot r_d \cdot P^f/(q \cdot i \cdot P^f - r_d)$.

[34] The return the investor expects from investing the margin loan in the risky asset is $q \cdot E_r/P$ in both price intervals. The margin loan's cost is $q \cdot i$ in price interval I1 and r_d/P^f in price interval I2.

[35] See Equation (8).

[36] A closer look at Equation (12) reveals that P^m is always lower than r_u/i when it is calculated from Equation (12). For $P^m \geq r_u/i \Leftrightarrow r_u/P^m \leq i$, neither $w_u \cdot q \cdot (r_u/P^m - i) + w_d \cdot q \cdot (r_d/P^m - i) = 0$ nor $w_u \cdot q \cdot (r_u/P^m - i) - w_d \cdot r_d/P^f$ can hold.

[37] See footnote 30.

As can be seen from Equation (12), if P^m is in price interval I2, the investors' reservation price for the risky asset when the purchase is financed by a margin loan can be calculated from

$$0 = w_u \cdot q \cdot \left(\frac{r_u}{P^m} - i\right) - w_d \cdot \frac{r_d}{P^f}.$$

It can be shown that if P^m is in interval I2, it is possible that $P^m > P^f$ and thus that the availability of a margin loan leads to the emergence of a bubble:

$$\begin{aligned} 0 &= w_u \cdot q \cdot \left(\frac{r_u}{P^m} - i\right) - w_d \cdot \frac{r_d}{P^f} \\ \Leftrightarrow \quad P^m &= P^f \cdot \frac{w_u \cdot q \cdot r_u}{P^f \cdot w_u \cdot q \cdot i + w_d \cdot r_d}. \end{aligned} \tag{13}$$

P^m is higher than the risky asset's fundamental value P^f if

$$\begin{aligned} w_u \cdot q \cdot r_u &> P^f \cdot w_u \cdot q \cdot i + w_d \cdot r_d \\ \Leftrightarrow \quad w_u \cdot q \cdot \left(\frac{r_u}{P^f} - i\right) - \frac{w_d \cdot r_d}{P^f} &> 0. \end{aligned} \tag{14}$$

Therefore, from Equation (13) it can be seen that if Condition (14) holds, P^m is higher than P^f, and thus the fact that the investors' investment is financed by a margin loan induces them to pay more than the asset's fundamental value for a risky asset.

Condition (14) can be interpreted as follows:
$w_u \cdot q \cdot \left(r_u/P^f - i\right) - w_d \cdot r_d/P^f$ is equal to $ANR_r(P)$ if $P = P^f$ and P^f is in price interval I2. In Section 3.1 we have shown that $\partial ANR_r(P)/\partial P$ is negative in both price intervals.[38] Therefore, the investors are only willing to pay a price for the risky asset that exceeds P^f if at least $ANR_r(P^f)$ is positive.

[38] See the derivatives in (10) and (11).

Inserting P^m from Equation (13) into the condition for P^m being in price interval I2, this latter condition can be transformed to a form that is independent of P^m:

$$
\begin{aligned}
& \left(\frac{1}{P^f} + \frac{q}{P^m}\right) \cdot r_d < q \cdot i \\
\Leftrightarrow \quad & q \cdot i > \frac{r_d}{P^f} + \frac{q \cdot r_d \cdot (P^f \cdot w_u \cdot q \cdot i + w_d \cdot r_d)}{w_u \cdot q \cdot r_u \cdot P^f} \\
\Leftrightarrow \quad & q \cdot \frac{i}{r_s} > \frac{r_d}{w_u \cdot (r_u - r_d)}. \qquad (15)
\end{aligned}
$$

Therefore, we have shown that if P^m is in price interval I2, it is possible that it is higher than the asset's fundamental value P^f. As a next step, we will look whether P^m can be higher than P^f when P^m is in price interval I1.

From Equation (12) it can be seen that for the case that it is in price interval I1, P^m can be calculated from

$$0 = w_u \cdot q \cdot \left(\frac{r_u}{P^m} - i\right) + w_d \cdot q \cdot \left(\frac{r_d}{P^m} - i\right).$$

It can be shown that if P^m is in interval I1, it is not higher than P^f and hence the availability of a margin loan does not cause a bubble:

$$
\begin{aligned}
& 0 = w_u \cdot q \cdot \left(\frac{r_u}{P^m} - i\right) + w_d \cdot q \cdot \left(\frac{r_d}{P^m} - i\right) \\
\Leftrightarrow \quad & 0 = \frac{w_u \cdot r_u + w_d \cdot r_d}{P^m} - (w_u + w_d) \cdot i \\
\Leftrightarrow \quad & P^m = \frac{E_r}{i}. \qquad (16)
\end{aligned}
$$

As the risky asset's fundamental value is E_r/r_s,[39] Equation (16) reveals that if P^m is in price interval I1, it cannot be higher than P^f as $i \geq r_s$.

[39] See Equation (3).

A closer look at Equations (3) and (16) moreover reveals that if P^m is in price interval I1, the investors' reservation price for the risky asset in t_0 again, as in Section 2.1, is the present value of the return in t_1 they expect per unit of the asset with the investors' capital cost being the discount rate. However, in contrast to Section 2.1, this time the investors' capital cost is i instead of r_s.[40]

After having determined P^m for the case that this price is in interval I1, we are able to transform the condition for P^m being in price interval I1 to a form that is independent of P^m by inserting our result of Equation (16) into this condition:

$$\begin{aligned} & \left(\frac{1}{P^f} + \frac{q}{P^m}\right) \cdot r_d \geq q \cdot i \\ \Leftrightarrow \quad & \frac{r_d \cdot r_s}{E_r} + \frac{q \cdot r_d \cdot i}{E_r} \geq q \cdot i \\ \Leftrightarrow \quad & q \cdot \frac{i}{r_s} \leq \frac{r_d}{w_u \cdot (r_u - r_d)}. \end{aligned} \tag{17}$$

Therefore, we have shown that

- for $q \cdot i/r_s \leq r_d/(w_u \cdot (r_u - r_d))$, the maximum price the investors are willing to pay for the risky asset when the purchase is financed by a margin loan is
    1. in price interval I1, hence the investors expect to be able to completely repay $q \cdot i$ in t_1 irrespective of the risky asset's payoff state for P^m,
    2. $P^m = E_r/i \leq P^f$ and thus the availability of a margin loan does not induce the investors to pay more than the fundamental value for the risky asset,
- for $q \cdot i/r_s > r_d/(w_u \cdot (r_u - r_d))$, P^m is
    1. in price interval I2, hence the investors expect to default in the low payoff state of the risky asset for P^m,
    2. is $P^f \cdot w_u \cdot q \cdot r_u/(P^f \cdot w_u \cdot q \cdot i + w_d \cdot r_d)$ and higher than P^f if $w_u \cdot q \cdot (r_u/P^f - i) - w_d \cdot r_d/P^f$ is positive.

[40] In the basic model in which we have determined P^f, the investors had to invest their own money. As we have shown that the best alternative to an equity financed investment in the risky asset is to invest the money in the safe asset and receive a return of r_s per dollar invested, the investors' capital cost in the basic model is r_s.

Therefore, Conditions (14) and (15) are necessary conditions for $P^m > P^f$. Moreover, it can be shown that Condition (15) always holds when Condition (14) holds, and thus that Condition (14) is not only a necessary but also a sufficient condition for P^m being higher than P^f:

The condition for P^m being higher than P^f when P^m is in price interval I2 (Condition (14)) can be transformed to

$$
\begin{aligned}
& w_u \cdot q \cdot r_u - w_u \cdot q \cdot i \cdot P^f - w_d \cdot r_d > 0 \\
\Leftrightarrow \quad & w_u \cdot q \cdot i \cdot \left(r_u \cdot \frac{r_s}{i} - E_r \right) > w_d \cdot r_d \cdot r_s \\
\Rightarrow \quad & q \cdot \frac{i}{r_s} > \frac{w_d \cdot r_d}{w_u \cdot \left(r_u \cdot \frac{r_s}{i} - E_r \right)}.
\end{aligned} \tag{18}
$$

The last transformation holds as

$$
\begin{aligned}
& w_u \cdot q \cdot \left(\frac{r_u}{P^f} - i \right) - \frac{w_d \cdot r_d}{P^f} > 0 \\
\Leftrightarrow \quad & w_u \cdot q \cdot r_u \cdot r_s - w_d \cdot r_d \cdot r_s > w_u \cdot q \cdot i \cdot E_r \\
\Leftrightarrow \quad & r_u \cdot \frac{r_s}{i} - \underbrace{\frac{w_d \cdot r_d \cdot r_s}{w_u \cdot q \cdot i}}_{>0} > E_r.
\end{aligned}
$$

Therefore, if Condition (14) holds, $q \cdot i / r_s$ is higher than $w_d \cdot r_d / (w_u \cdot (r_u \cdot r_s / i - E_r))$. With this result it can be shown that Condition (15) always holds when Condition (14) holds:[41]

$$
\begin{aligned}
& \frac{w_d \cdot r_d}{w_u \cdot \left(r_u \cdot \frac{r_s}{i} - E_r \right)} \geq \frac{r_d}{w_u \cdot (r_u - r_d)} \\
\Leftrightarrow \quad & w_d \cdot r_d \cdot w_u \cdot r_u - w_d \cdot {r_d}^2 \cdot w_u \geq r_d \cdot w_u \cdot \left(r_u \cdot \frac{r_s}{i} - E_r \right) \\
\Leftrightarrow \quad & w_d \cdot (r_u - r_d) \geq r_u \cdot \frac{r_s}{i} - w_u \cdot r_u - w_d \cdot r_d \\
\Leftrightarrow \quad & w_d \cdot r_u \geq r_u \cdot \frac{r_s}{i} - w_u \cdot r_u \\
\Leftrightarrow \quad & i \geq r_s.
\end{aligned}
$$

[41] The first transformation holds when Condition (14) holds as in this case $r_u \cdot r_s / i$ is higher than E_r; see the derivation of Condition (18).

Therefore, we have shown that if Condition (14) holds, P^m is in price interval I2. Thus, Condition (14) is not only a necessary but also a sufficient condition for P^m being higher than the risky asset's fundamental value.

The results derived in Section 3.2 are summed up in Tables 1 and 2:

Table 1: Results concerning P^m.

Condition	P^m is in price interval	$P^m =$
$q \cdot \frac{i}{r_s} \leq \frac{r_d}{w_u \cdot (r_u - r_d)}$ (17)	I1	$\frac{E_r}{i} \quad [\leq P^f]$ (16)
$q \cdot \frac{i}{r_s} > \frac{r_d}{w_u \cdot (r_u - r_d)}$ (15)	I2	$\frac{P^f \cdot w_u \cdot q \cdot r_u}{P^f \cdot w_u \cdot q \cdot i + w_d \cdot r_d}$ (13)

Table 2: The two conditions for $P^m > P^f$.

Necessary condition	Sufficient condition
$q \cdot \frac{i}{r_s} > \frac{r_d}{w_u \cdot (r_u - r_d)}$ (15)	$w_u \cdot q \cdot \left(\frac{r_u}{P^f} - i\right) - \frac{w_d \cdot r_d}{P^f} > 0$ (14)

3.3 An economic interpretation of Conditions (14) and (15)

In Section 3.2 we have shown that when the investment is financed by a margin loan, it is possible that the maximum price the investors are willing to pay for the risky asset is higher than the asset's fundamental value. In this section we first examine why the fact that the investment is financed by a margin loan under certain conditions induces the investors to pay more than P^f for the risky asset. Then, we take a closer look at the two conditions for $P^m > P^f$, i.e. Conditions (14) and (15), to investigate in which situations these conditions are most likely to hold.

Our finding that, under certain conditions, investors are willing to pay more for the risky asset when their investment is margin loan financed than they are if their investment is equity financed contradicts the result of neo-classical finance theory that investment decisions are independent of the investors' capital structure.[42]

The fact that there is a potential relationship between capital structure and investment decisions has first been observed by Jensen and Meckling (1976) who identified the so-called *asset substitution/risk shifting* problem as a major component of the agency costs of debt. Allen and Gale (2000) have applied this phenomenon to an asset pricing context. They have demonstrated in a principal-agent framework that when investors can borrow money via a simple debt contract[43] in order to invest in a risky asset, they are willing to pay more for the asset than they are if their investment is equity financed.

Allen and Gale have traced their finding back to a risk shifting problem between the lender and the investors. Credit-financed investors do not bear the full cost of borrowing when the investment turns out badly. In payoff states in which the return from the investment is lower than the repayment the lending institution requires, the investors declare bankruptcy and thus avoid full repayment of the loan. When the return is high, however, the investors keep the remainder of the investment's return after repayment of the loan. In contrast to the model of Allen and Gale (2000), in our model the lending institution does not write simple debt contracts with the investors but grants margin loans. Hence, the loan is secured by the borrowing investor's initial portfolio of $1/P^f$ units of the risky asset which serves as collateral. However, we have shown that the margin loan nevertheless bears a default risk, namely the risk that the return on the borrowing investor's total portfolio in t_1 is not sufficient to completely repay $q \cdot i$.[44] Therefore, our finding that it is possible that the investors are willing to pay more than the asset's fundamental value for the risky asset when the purchase is financed by a margin loan, can be explained by an interpretation similar to that given in Allen and Gale (2000).

We have shown that for prices from the interval I2, the repayment the lending institution requires in t_1, $q \cdot i$, is higher than the return the investors expect from their total portfolio in the risky asset's low payoff state, $(1/P^f + q/P) \cdot r_d$. Hence, for these prices the investors expect to default on the margin loan if the risky asset's return per unit is r_d in t_1. Therefore, for prices from the interval I2 only the additional net return the investors

42 See, for example, Modigliani and Miller (1958).

43 See, for example, Townsend (1979) or Gale and Hellwig (1985) for simple debt contracts.

44 See case A5 in Figure 1.

expect from the margin loan for the high payoff state, $q \cdot (r_u/P - i)$, (negatively) depends on P.[45] The additional net return the investors expect from the margin loan for the low payoff state of the risky asset, however, is $-r_d/P^f$ regardless of the price P they pay for the asset.[46] Hence, if Condition (15) holds, and thus P^m is in price interval I2, there is the same risk shifting problem as that documented in Allen and Gale (2000). In contrast to that of equity financed investors, the additional net return investors who are financed by a margin loan expect from an investment in the risky asset only partly (negatively) depends on P^m. Therefore, if P^m is in price interval I2, it is possible[47] that margin loan financed investors are willing to pay more for the risky asset than equity financed investors are.

If Condition (15) does not hold and thus P^m is in price interval I1, however, even for P^m the investors do not expect to default on the loan but believe to be able to completely repay $q \cdot i$ in t_1 irrespective of the risky asset's payoff state. Therefore, there is no risk shifting problem as the additional net return the investors expect from the margin loan financed investment (negatively) depends on P for both payoff states[48] and thus P^m is not higher than P^f.[49]

We have shown in Section 2.1 that $P^f = E_r/r_s$ (see Equation (3)) and in Section 3.2 that $P^m = E_r/i$ (see Equation (16)) if $q \cdot i/r_s$ is not higher than $r_d/(w_u \cdot (r_u - r_d))$ (see Condition (17)) and thus if there is no risk shifting problem. As $i \geq r_s$, it can be concluded from these results that if there is no risk shifting problem, P^m cannot be higher than P^f.

[45] See the first term of Equation (7).

[46] See the second part of the sum in Equation (7).

[47] Remember that Condition (15) is just a necessary condition for $P^m > P^f$. The investors are only willing to pay more than the asset's fundamental value for the risky asset if Condition (14) holds; see Table 2.

[48] See Equation (5).

[49] The asymmetry of ANR_r for the two price intervals concerning its sensitivity to P can also be seen from the first order derivatives of ANR_r with respect to P; see the derivatives in (10) and (11).

With our result for P^m for the case that Condition (15) holds[50] it can be shown that if there is a risk shifting problem, P^m is higher than E_r/i:[51]

$$
\begin{aligned}
& P^f \cdot \frac{w_u \cdot q \cdot r_u}{P^f \cdot w_u \cdot q \cdot i + w_d \cdot r_d} > \frac{E_r}{i} \\
\Leftrightarrow \quad & w_u \cdot q \cdot i \cdot w_d \cdot (r_u - r_d) > w_d \cdot r_d \cdot r_s \\
\Leftrightarrow \quad & q \cdot \frac{i}{r_s} > \frac{r_d}{w_u \cdot (r_u - r_d)}. \qquad (15)
\end{aligned}
$$

We have shown that if Condition (15) holds, P^m is higher than E_r/i. Therefore, for $i = r_s$, the existence of a risk shifting problem is sufficient for $P^m > P^f$. As $P^f = E_r/r_s$ is higher than E_r/i for $i > r_s$, however, the existence of a risk shifting problem is only necessary but not sufficient for $P^m > P^f$ when the margin loan's cost are higher than the cost of the investors' equity. Rather, if $i > r_s$, for $P^m > P^f$ moreover[52] Condition (14) must hold.

Therefore, in the following we will refer to the necessary condition for $P^m > P^f$, Condition (15), as the risk shifting condition and to the sufficient condition for $P^m > P^f$, Condition (14), as the bubble condition (see Table 3).

Table 3: The risk shifting condition and the bubble condition.

Risk shifting condition	Bubble condition
$q \cdot \frac{i}{r_s} > \frac{r_d}{w_u \cdot (r_u - r_d)}$ (15)	$w_u \cdot q \cdot \left(\frac{r_u}{P^f} - i\right) - \frac{w_d \cdot r_d}{P^f} > 0$ (14)

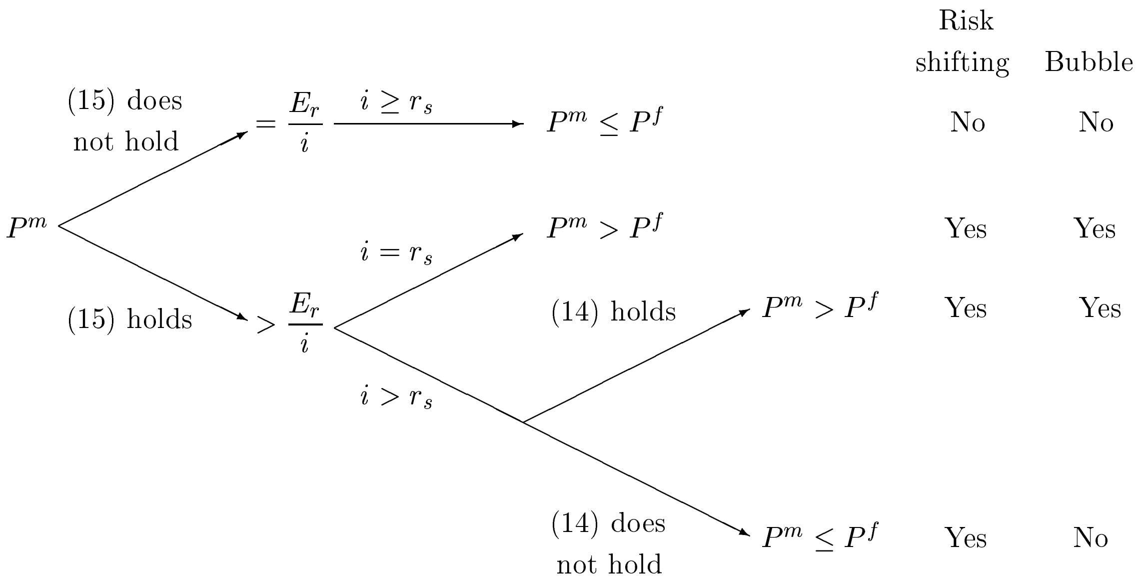
[50] See Equation (13).

[51] Remember that we have shown that if Condition (15) does not hold, P^m is E_r/i.

[52] Remember that we have shown that Condition (15) always holds when Condition (14) holds.

In Figure 3 the results of our analysis of Conditions (14) and (15) are collected:

Figure 3: Results concerning Conditions (14) and (15).



The risk shifting condition can be written as

$$RSC > 0$$

with

$$RSC = q \cdot \frac{i}{r_s} - \frac{r_d}{w_u \cdot (r_u - r_d)}. \tag{19}$$

From the sign of the first order partial derivatives of RSC with respect to the input parameters of Equation (19) the impact of each of these parameters on the probability that the risk shifting condition holds can be seen. The first order partial derivatives of RSC with respect to r_s, i, q, r_u, r_d, and w_u are given in Table 4 in Appendix 6.1. From Table 4 it can be seen that the probability of a risk shifting problem between the lending institution and the investors is the higher

1. the lower the return on the safe investment, r_s,

2. the higher the repayment the lending institution requires per dollar of margin loan borrowed, i,

3. the more the investors borrow in t_0, q,

4. the more risky, in terms of $r_u - r_d$, the investors assume an investment in the risky asset to be, and

5. the more optimistic about the prospects of an investment in the risky asset, in terms of a high r_u and w_u, the investors are.

The bubble condition can be written as

$$BC > 0$$

with

$$\begin{aligned} BC &= w_u \cdot q \cdot \left(\frac{r_u}{P^f} - i \right) - \frac{w_d \cdot r_d}{P^f} \\ \Leftrightarrow \quad BC &= \frac{r_s \cdot (w_u \cdot q \cdot r_u - w_d \cdot r_d)}{w_u \cdot r_u + w_d \cdot r_d} - w_u \cdot q \cdot i.^{53} \end{aligned} \tag{20}$$

From the sign of the first order partial derivatives of BC with respect to the input parameters of Equation (20) the impact of each of these parameters on the probability that the bubble condition holds can be seen. The first order partial derivatives of BC with respect to r_s, i, q, r_u, r_d, w_u, and w_d are given in Table 5 in Appendix 6.2. It can be seen that the probability that the availability of margin loans leads to the emergence of a bubble is the higher

1. the lower the repayment the lending institution requires per dollar of margin loan borrowed and

2. the more risky, in terms of $r_u - r_d$, the investors assume an investment in the risky asset to be.

[53] Remember that $P^f = E_r/r_s = (w_u \cdot r_u + w_d \cdot r_d)/r_s$; see Equations (1) and (3).

4 Margin regulation

In Section 3 we have shown that under Assumptions 1 to 6, the availability of margin loans can lead to the emergence of a bubble by inducing investors to pay more than the asset's fundamental value for a risky asset. In this section, we investigate whether, and if so how, the development of a bubble in our model can be prevented by margin regulation.

As a first step, we introduce an institution that regulates margin loans into our model:

Assumption 7: There is a regulator that can set an initial margin requirement $m \in (0, 1]$ in t_0. Under an initial margin requirement of m, the maximum amount of margin loan each investor is allowed to borrow in t_0 is $1/m - 1$.[54] The regulator is able to observe the return on the safe asset r_s, the margin loan's conditions (i, q^{max}), and the investors' expectations concerning the return of the risky asset in t_1 (w_u, w_d, r_u, r_d).

We have presumed that the lending institution offers each investor to borrow up to $q^{max} > 0$. Therefore, only if

$$\frac{1}{m} - 1 < q^{max} \tag{21}$$

$$\Leftrightarrow \qquad m > \frac{1}{1 + q^{max}}, \tag{22}$$

the initial margin requirement actually limits the maximum amount of margin loan each investor is able to borrow in t_0.[55]

[54] Remember that we have assumed that each investor has an initial wealth of 1 \$; see Assumption 5. For example, under an initial margin requirement of 0.5, the maximum amount each investor is allowed to borrow in t_0 is $1/0.5 - 1 = 1$. Note that if the regulator sets m to 1, the maximum amount of margin loan each investor is able to borrow is $1/1 - 1 = 0$ and thus margin lending is completely suspended.

[55] In Section 2.2 we have already pointed to the fact that the loan-to-value ratio q^{max} is just another manifestation of an initial margin requirement. Actually, q^{max} can easily be transformed into an initial margin requirement by $1/(1 + q^{max})$. For example, if $q^{max} = 0.25$, $1/(1 + q^{max})$ is $1/1.25 = 0.8$ which is exactly the equity financed part of the investor's new portfolio when the investor completely makes use of the margin loan.

And as

$$\frac{\partial \left(\frac{1}{m} - 1\right)}{\partial m} = -\frac{1}{m^2} < 0, \tag{23}$$

the regulator limits margin lending the more the higher it sets m when Condition (22) holds.

Therefore, the situation at which we look at in this section is that r_s, the margin loan's conditions ($i \geq r_s$ and $q^{max} > 0$), and the investors' expectations are given exogenously and can be observed by the regulator. Before the investors make their choice whether, and if so to which extent, they make use of the offered margin loan, the regulator can set an initial margin requirement m.

4.1 The suitability of an initial margin requirement to rule out the emergence of a bubble

Before we look at the question whether the development of a bubble in our model can be prevented by an initial margin requirement, we exclude some possible parameter constellations as they are not relevant for our analysis in this section.

In Section 3.2 we have shown that the bubble condition (14) is a necessary condition for P^m being higher than P^f. Therefore, the availability of margin loans can only lead to the emergence of a bubble if

$$w_u \cdot q \cdot \left(\frac{r_u}{P^f} - i\right) - \frac{w_d \cdot r_d}{P^f} > 0 \tag{14}$$

$$\Leftrightarrow \qquad r_u > \underbrace{\frac{w_d \cdot r_d}{w_u \cdot q}}_{>0} + i \cdot P^f. \tag{24}$$

From Condition (24) it can be seen that, irrespective of q, the bubble condition can only hold if r_u is higher than $i \cdot P^f$. Hence, if r_u is not higher than $i \cdot P^f$, no margin regulation is required to rule out the emergence of a margin loan induced bubble. Therefore, parameter constellations for which $r_u \leq i \cdot P^f$ can be excluded from our analysis in this section.

Assumption 8: Parameter constellations for which r_u is not higher than $i \cdot P^f$ are not relevant in this section and are therefore excluded from our analysis.

It can be shown that by setting the initial margin requirement to an appropriate level, the regulator can prevent the development of a margin loan induced bubble. In Section 3.2 we have derived two necessary conditions for $P^m > P^f$, i.e. Conditions (14) and (15).[56] The risk shifting condition (15) is

$$q \cdot \frac{i}{r_s} > \frac{r_d}{w_u \cdot (r_u - r_d)} \tag{15}$$

$$\Leftrightarrow \qquad q > \frac{r_s \cdot r_d}{i \cdot w_u \cdot (r_u - r_d)}. \tag{25}$$

Condition (25) reveals that for the emergence of a bubble it is necessary that the maximum amount of margin loan each investor is able to borrow in t_0 is higher than $r_s \cdot r_d/(i \cdot w_u \cdot (r_u - r_d))$. And we have shown that if $1/m - 1 \leq q^{max}$, then $1/m - 1$ is the maximum amount each investor is able to borrow in t_0.[57]

Hence, by setting m to such a level that

$$\frac{1}{m} - 1 \leq \frac{r_s \cdot r_d}{i \cdot w_u \cdot (r_u - r_d)}$$

$$\Leftrightarrow \qquad m \geq \frac{1}{\frac{r_s \cdot r_d}{i \cdot w_u \cdot (r_u - r_d)} + 1}, \tag{26}$$

the regulator can prevent that Condition (15) respectively Condition (25) can hold and thus that P^m can be higher than P^f.[58] Therefore, by setting the initial margin requirement

[56] See Table 3. Moreover, we have shown that Condition (14) is not only a necessary but a sufficient condition for $P^m > P^f$.

[57] See Condition (22).

[58] A closer look at Condition (26) reveals that, due to $r_s \cdot r_d/(i \cdot w_u \cdot (r_u - r_d)) > 0$, it is never necessary to completely suspend margin lending by setting m to 1 to prevent the emergence of a margin loan induced bubble.

to an appropriate level the regulator can rule out that the availability of margin loans can lead to the emergence of a bubble in our model.

The bubble condition (14) is

$$w_u \cdot q \cdot \left(\frac{r_u}{P^f} - i\right) - \frac{w_d \cdot r_d}{P^f} > 0 \tag{14}$$

$$\Leftrightarrow \qquad q \cdot w_u \cdot (r_u - i \cdot P^f) > w_d \cdot r_d \tag{27}$$

$$\Rightarrow \qquad q > \frac{w_d \cdot r_d}{w_u \cdot (r_u - i \cdot P^f)}.^{59} \tag{28}$$

Condition (28) reveals that if the bubble condition holds, q must be higher than $w_d \cdot r_d/(w_u \cdot (r_u - i \cdot P^f))$. From this result it can be concluded that P^m can only be higher than P^f if the maximum amount each investor is able to borrow in t_0 is higher than $w_d \cdot r_d/(w_u \cdot (r_u - i \cdot P^f))$. Therefore, by setting m to such a level that[60]

$$\frac{1}{m} - 1 \leq \frac{w_d \cdot r_d}{w_u \cdot (r_u - i \cdot P^f)}$$

$$\Rightarrow \qquad m \geq \frac{1}{\frac{w_d \cdot r_d}{w_u \cdot (r_u - i \cdot P^f)} + 1}, \tag{29}$$

the regulator can prevent that Condition (14) respectively Condition (28) can hold and thus that P^m can be higher than P^f.

4.2 Implications for the Federal Reserve's margin policy

In Section 4.1 we have shown that by setting the initial margin requirement to such a level that Condition (26) or (29) holds, the regulator can rule out the emergence of a margin loan induced bubble in our model. By taking a closer look at Conditions (26) and

[59] Remember that we have excluded all parameter constellations for which r_u is not higher than $i \cdot P^f$ from our analysis in this section; see Assumption 8. Therefore, the last transformation is correct.

[60] As we have excluded all parameter constellations for which $r_u \leq i \cdot P^f$ (see Assumption 8), the transformation is correct.

(29), we investigate which conclusions concerning the question how the Federal Reserve's margin policy should look like can be drawn from our model in this section.

The answer to the question how margin policy should look like crucially depends on the objective(s) of margin regulation. In the remainder of this section we will investigate the implications of our results for the Federal Reserve's margin policy under the following assumption:

Assumption 9: The Federal Reserve aims to prevent that the availability of margin loans leads to the emergence of a stock market bubble.

The minimum m for which Condition (26) holds is

$$m_1^* = \frac{1}{\frac{r_s \cdot r_d}{i \cdot w_u \cdot (r_u - r_d)} + 1}. \tag{30}$$

Equation (30) can be written as

$$m_1^* = \frac{1}{x + 1} \tag{31}$$

with

$$x = \frac{r_s \cdot r_d}{i \cdot w_u \cdot (r_u - r_d)}. \tag{32}$$

The minimum m for which Condition (29) holds is[61]

$$m_2^* = \frac{1}{\frac{w_d \cdot r_d}{w_u \cdot (r_u - i \cdot P^f)} + 1} \tag{33}$$

$$\Leftrightarrow \quad m_2^* = \frac{1}{\frac{w_d \cdot r_d \cdot r_s}{w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d)} + 1}. \tag{34}$$

[61] Remember that $P^f = E_r/r_s = (w_u \cdot r_u + w_d \cdot r_d)/r_s$; see (1) and Equations (3).

Equation (34) can be written as

$$m_2^* = \frac{1}{y+1} \tag{35}$$

with

$$y = \frac{w_d \cdot r_d \cdot r_s}{w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d)}. \tag{36}$$

From Equations (31) and (32) it can be seen that the minimum m for which Condition (26) holds, m_1^*, is is the higher (lower) the lower (higher) $x = r_s \cdot r_d / (i \cdot w_u \cdot (r_u - r_d))$.[62] And a closer look at Equations (35) and (36) reveals that the minimum m for which Condition (29) holds, m_2^*, is the higher (lower) the lower (higher) $y = w_d \cdot r_d \cdot r_s / (w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d))$.[63]

From the sign of the first order partial derivatives of x and y with respect to the input parameters of Equations (32) and (36) some conclusions about how the Federal Reserve's margin policy should look like under Assumption 9 can be drawn. In Tables 6 and 7 in Appendices 6.3 and 6.4 the first order partial derivatives of x and y with respect to r_s, i, r_u, r_d, w_u, and w_d are presented.[64]

We have shown that the risk shifting condition cannot hold if m is at least $m_1^* = 1/(r_s \cdot r_d/(i \cdot w_u \cdot (r_u - r_d)) + 1)$.[65] Therefore, Table 6 reveals that if the Federal Reserve not only aims to rule out the emergence of a margin loan induced stock market bubble,

[62] x is always positive. Therefore, the first order derivative of m_1^* with respect to x which is $\partial m_1^* / \partial x = -(x+1)^{-2}$ is always negative.

[63] We have excluded all parameter constellations for which r_u is not higher than $i \cdot P^f$ from our analysis in this section; see Assumption 8. And for $r_u > i \cdot P^f$, y is positive; see Equation (33). Therefore, the first order derivative of m_2^* with respect to y which is $\partial m_2^* / \partial y = -(y+1)^{-2}$ is always negative.

[64] The results presented in Table 7 necessarily hold only for parameter constellations for which r_u is higher than $i \cdot P^f$. Yet, we have shown that for $r_u \leq i \cdot P^f$, no margin regulation is required to rule out $P^m > P^f$.

[65] See Equation (30) and the derivation of Conditions (25) and (26).

but moreover seeks to prevent that there is a risk shifting problem between the lending institution and the investors, it should limit margin lending the more[66]

1. the lower the riskless rate of return,

2. the higher the interest rate lending institutions charge on margin loans,

3. the more risky an investment in stocks is commonly assumed to be,[67] and

4. the more optimistic about the prospects of an investment in stocks the investors are.[68]

We have shown that the Federal Reserve limits margin lending the more the higher it sets the initial margin requirement. It can be shown that for $r_u > i \cdot P^f$,[69] m_2^* is lower than or equal to m_1^*:[70]

$$\frac{1}{\frac{w_d \cdot r_d}{w_u \cdot (r_u - i \cdot P^f)} + 1} \leq \frac{1}{\frac{r_s \cdot r_d}{i \cdot w_u \cdot (r_u - r_d)} + 1}$$

$$\Leftrightarrow \quad w_d \cdot i \cdot (r_u - r_d) \geq r_s \cdot (r_u - i \cdot P^f)$$

$$\Leftrightarrow \quad w_d \cdot i \geq r_s - w_u \cdot i$$

$$\Leftrightarrow \quad i \geq r_s.$$ [71]

Therefore, under Assumption 8, m_2^* is lower than (for $i > r_s$) or equal to (for $i = r_s$) m_1^*. Hence, if the Federal Reserve's seeks to limit margin lending only as much as necessary

66 Remember that the Federal Reserve limits margin lending the more the higher it sets the initial margin requirement; see the derivative in (23).

67 A closer look at Table 6 reveals that m_1^* is the higher (lower) the higher (lower) r_u and the lower (higher) r_d.

68 From Table 6 it can be seen that m_1^* is the higher (lower) the higher (lower) w_u and r_u.

69 Remember that we have excluded all parameter constellations for which $r_u \leq i \cdot P^f$ from our analysis in this section; see Assumption 8.

70 See Equation (30) for m_1^* and Equation (33) for m_2^*.

71 Remember that $P^f = E_r/r_s = (w_u \cdot r_u + w_d \cdot r_d)/r_s$; see Equations (3) and (1).

to rule out the emergence of a margin loan induced stock market bubble, it should set the initial margin requirement to m_2^*.[72] Table 7 reveals that if the Federal Reserve aims to limit margin lending only as much as necessary to prevent $P^m > P^f$, it should limit margin lending the more

1. the higher the riskless rate of return,
2. the lower the interest rate lending institutions charge on margin loans, and
3. the more risky an investment in stocks is commonly assumed to be.[73]

5 Summary and conclusions

This paper aimed to answer three major questions:

1. *What is the link between margin loans and stock market bubbles?*

   In this paper, we have provided a new rationale for a link between margin loans and stock market bubbles. We have demonstrated in a principal-agent model that a risk shifting problem between investors who are financed by a margin loan and the institution granting the margin loans can induce the investors to pay more than the fundamental value for a risky asset. This is due to the fact that although margin loans are secured by collateral, they nevertheless bear a default risk as (at least a part of) the collateral consists of risky securities. Hence, margin loan financed investors do not have to bear the full cost of borrowing in payoff states in which they default on the loan. Therefore, they are willing to bid up the prices of stocks

[72] Certainly, as the Federal Reserve has to use proxies for the input parameters of Equation (34), it is not able to (deliberately) set the initial margin requirement in a way that it is exactly equal to $m_2^* = 1/(w_d \cdot r_d/(w_u \cdot (r_u - i \cdot P^f)) + 1)$. And if $m < 1/(w_d \cdot r_d/(w_u \cdot (r_u - i \cdot P^f)) + 1)$, the Federal Reserve does not totally prevent that the availability of margin loans can lead to the emergence of a bubble. However, the higher the Federal Reserve sets the initial margin requirement the more it limits the extent of a possible bubble. This is due to the fact that the Federal Reserve limits margin lending the more the higher it sets m. And the first order partial derivative of P^m for price interval I2 (see Equation (13)) with respect to q is positive; remember that we have shown in Section 3.2 that the availability of margin loans can only lead to a bubble if P^m is in price interval I2.

[73] A closer look at Table 7 reveals that m_2^* is the higher (lower) the higher (lower) r_u and the lower (higher) r_d.

above the stocks' fundamental values as they expect to not to have to bear the full cost of borrowing in low payoff states.

2. *Under which conditions do margin loans induce investors to pay more for a stock than its fundamental value?*

   From our theoretical model we have derived a necessary and a sufficient condition under which the availability of margin loans induces investors to pay more than the asset's fundamental value for a risky asset. We have referred to the necessary condition as the risk shifting condition and to the sufficient condition as the bubble condition.

   - <u>The risk shifting condition:</u>

     If the necessary condition holds, there is a risk shifting problem between the investors and the lending institution. The investors expect to default on the margin loan in the risky asset's low payoff state[74] and the maximum price they are willing to pay for the asset is higher than the present value of the return they expect from the asset, where the discount rate is the interest rate the lender charges on the margin loan. We have shown that the risk shifting condition is the more likely to hold

     - the lower the riskless rate of return,
     - the higher the interest rate on margin loans,
     - the more the investors borrow,
     - the more risky the investors assume an investment in the risky asset to be, and
     - the more optimistic the investors are about the prospects of an investment in the risky asset.

   - <u>The bubble condition:</u>

     We have defined the fundamental value of an asset as the maximum price equity-financed investors are willing to pay for the asset. In our theoretical model, this maximum price is the present value of the return the investors expect from the asset, where the discount rate is the riskless rate of return. If the risk shifting condition holds, the investors are willing to pay more for the asset than the present value of the expected return from the asset, where the discount rate is the interest rate on the margin loan. Therefore, if and

74 Remember that we have presumed that the return of the risky asset is binomially distributed; see Assumption 2.

only if the interest rate on the margin loan is equal to the riskless rate of return, the risk shifting condition is a sufficient condition for the emergence of a margin loan induced bubble.[75] Otherwise, the availability of margin loans only induces investors to bid up the price of an asset above its fundamental value if the bubble condition holds. We have shown that the bubble condition is the more likely to hold

– the lower the interest rate on margin loans and
– the more risky the investors assume an investment in the risky asset to be.

These theoretical results suggest that the availability of margin loans is the more likely to lead to the development of a stock market bubble the lower the interest rate on margin loans. Furthermore, if we take stock market price volatility as a proxy for the risk margin loan financed investors associate with an investment in the stock market, our model indicates that the probability that margin loan financed investors bid up stock prices above their fundamental values is the higher the higher stock market volatility.

3. *Can the emergence of margin loan induced stock market bubbles be ruled out by margin regulation and, if so, how should the Federal Reserve's margin policy look like?*

We have demonstrated that in our theoretical model a regulator can rule out the emergence of a margin loan induced bubble by the imposition of an appropriate initial margin requirement. From our results it can be seen that the minimum initial margin requirement the regulator must set to prevent the development of a bubble is the higher the higher the risk the investors associate with an investment in the risky asset. If we take stock market price volatility as a proxy for the risk investors who are financed by a margin loan associate with an investment, this finding suggests that if the Federal Reserve aims to prevent stock market bubbles, it should set its initial margin requirement the higher the higher stock market volatility.

Previous empirical studies indeed found that stock market price volatility had a significant impact on the Federal Reserve's margin policy. However, the results concerning the direction of the impact are ambiguous. While the finding of a positive relationship between changes in the Federal Reserve's margin requirement and lags of changes in stock market volatility by Hsieh and Miller (1990) is in line with our

[75] We have excluded the possibility that the interest rate on the margin loan is lower than the riskless rate of return; see Assumption 6.

theoretical results, Schwert (1989a,b) found a negative impact of changes in stock market volatility on the margin requirement.

We have used a stark set of assumptions in our model. Among other things, we have presumed a one-period setting, risk-neutral investors, and binomially distributed asset returns. These and other strong simplifications have been necessary to make the basic effect we wanted to show as clear as possible. Having served their purpose some of the assumptions can now be relaxed in the direction of greater realism and relevance. For example, the pyramiding hypothesis could be analyzed in a multi-period model in which a rise in the price of the risky asset allows the investors to borrow additional funds on the risen value of their portfolio. Furthermore, by looking at the return a lending institution expects from granting a margin loan it could be analyzed under which conditions lending institutions may set the conditions of margin loans in a way that margin loan financed investors are willing to pay more than the fundamental value for stocks.[76]

Finally, we have employed our model to analyze the impact of margin loans on stock prices. However, the risky asset in our model can be interpreted in a number of ways. And the essential feature of the loan in our model, i.e. that it is fully secured by collateral but nevertheless contains a default risk as a part of the collateral consists of the risky asset which is bought from the loan, is not unique to margin loans. Therefore, our model can also be applied to markets other than the stock market. For example, the purchase of real estate usually is (at least partly) financed by credit, where the object purchased from the credit serves as collateral. Hence, our model is as well applicable to the real estate market.

[76] Remember that in our model the conditions of the margin loan are given exogenously.

6 Appendix

6.1 The first order partial derivatives of RSC

Table 4: The first order partial derivatives of RSC.

Parameter	First order partial derivative	Sign
r_s	$-q \cdot \frac{i}{{r_s}^2}$	$-$
i	$\frac{q}{r_s}$	$+$
q	$\frac{i}{r_s}$	$+$
r_u	$-\frac{-r_d \cdot w_u}{(w_u \cdot (r_u - r_d))^2}$	$+$
r_d	$-\frac{w_u \cdot r_u}{(w_u \cdot (r_u - r_d))^2}$	$-$
w_u	$-\frac{-r_d \cdot (r_u - r_d)}{(w_u \cdot (r_u - r_d))^2}$	$+$

6.2 The first order partial derivatives of BC

Table 5: The first order partial derivatives of BC.

Parameter	First order partial derivative	Distinction of cases	Sign
r_s	$\frac{w_u \cdot q \cdot r_u - w_d \cdot r_d}{w_u \cdot r_u + w_d \cdot r_d}$	$q \cdot w_u \cdot r_u > w_d \cdot r_d$ $q \cdot w_u \cdot r_u = w_d \cdot r_d$ $q \cdot w_u \cdot r_u < w_d \cdot r_d$	$+$ 0 $-$
i	$-w_u \cdot q$		$-$
q	$\frac{r_s \cdot w_u \cdot r_u}{w_u \cdot r_u + w_d \cdot r_d} - w_u \cdot i$	$r_s \cdot r_u > i \cdot E_r$ $r_s \cdot r_u = i \cdot E_r$ $r_s \cdot r_u < i \cdot E_r$	$+$ 0 $-$
r_u	$\frac{r_s \cdot w_u \cdot q \cdot (w_u \cdot r_u + w_d \cdot r_d)}{(w_u \cdot r_u + w_d \cdot r_d)^2} - \frac{r_s \cdot (w_u \cdot q \cdot r_u - w_d \cdot r_d) \cdot w_u}{(w_u \cdot r_u + w_d \cdot r_d)^2}$		$+$
r_d	$\frac{-r_s \cdot w_d \cdot (w_u \cdot r_u + w_d \cdot r_d)}{(w_u \cdot r_u + w_d \cdot r_d)^2} - \frac{r_s \cdot (w_u \cdot q \cdot r_u - w_d \cdot r_d) \cdot w_d}{(w_u \cdot r_u + w_d \cdot r_d)^2}$		$-$
w_u	$\frac{r_s \cdot q \cdot r_u \cdot (w_u \cdot r_u + w_d \cdot r_d)}{(w_u \cdot r_u + w_d \cdot r_d)^2} - \frac{r_s \cdot (w_u \cdot q \cdot r_u - w_d \cdot r_d) \cdot r_u}{(w_u \cdot r_u + w_d \cdot r_d)^2} - q \cdot i$	$\frac{(1+q) \cdot r_s \cdot r_u \cdot w_d \cdot r_d}{{E_r}^2} > q \cdot i$ $\frac{(1+q) \cdot r_s \cdot r_u \cdot w_d \cdot r_d}{{E_r}^2} = q \cdot i$ $\frac{(1+q) \cdot r_s \cdot r_u \cdot w_d \cdot r_d}{{E_r}^2} < q \cdot i$	$+$ 0 $-$
w_d	$\frac{-r_s \cdot r_d \cdot (w_u \cdot r_u + w_d \cdot r_d)}{(w_u \cdot r_u + w_d \cdot r_d)^2} - \frac{r_s \cdot (w_u \cdot q \cdot r_u - w_d \cdot r_d) \cdot r_d}{(w_u \cdot r_u + w_d \cdot r_d)^2}$		$-$

$\underline{\partial BC/\partial r_u}$ is positive as

$$r_s \cdot w_u \cdot q \cdot (w_u \cdot r_u + w_d \cdot r_d) > r_s \cdot (w_u \cdot q \cdot r_u - w_d \cdot r_d) \cdot w_u.$$

$$\Leftrightarrow \quad q \cdot w_u \cdot r_u + q \cdot w_d \cdot r_d > q \cdot w_u \cdot r_u - w_d \cdot r_d$$

$$\Leftrightarrow \quad \underbrace{(1+q) \cdot w_d \cdot r_d}_{>0} > 0.$$

$\underline{\partial BC/\partial r_d}$ is negative as

$$-r_s \cdot w_d \cdot (w_u \cdot r_u + w_d \cdot r_d) < r_s \cdot (w_u \cdot q \cdot r_u - w_d \cdot r_d) \cdot w_d.$$

$$\Leftrightarrow \quad -w_u \cdot r_u - w_d \cdot r_d < w_u \cdot q \cdot r_u - w_d \cdot r_d$$

$$\Leftrightarrow \quad \underbrace{(1+q) \cdot w_u \cdot r_u}_{>0} > 0.$$

And $\underline{\partial BC/\partial w_d}$ is negative as

$$-r_s \cdot r_d \cdot (w_u \cdot r_u + w_d \cdot r_d) < r_s \cdot (w_u \cdot q \cdot r_u - w_d \cdot r_d) \cdot r_d.$$

$$\Leftrightarrow \quad -w_u \cdot r_u - w_d \cdot r_d < w_u \cdot q \cdot r_u - w_d \cdot r_d$$

$$\Leftrightarrow \quad \underbrace{(1+q) \cdot w_u \cdot r_u}_{>0} > 0.$$

6.3 The first order partial derivatives of x

Table 6: The first order partial derivatives of x.

Parameter	First order partial derivative	Sign	Impact[77]on m_1^*
r_s	$\frac{r_d}{i \cdot w_u \cdot (r_u - r_d)}$	$+$	Negative
i	$\frac{-r_s \cdot r_d}{i^2 \cdot w_u \cdot (r_u - r_d)}$	$-$	Positive
r_u	$\frac{-r_s \cdot r_d \cdot i \cdot w_u}{(i \cdot w_u \cdot (r_u - r_d))^2}$	$-$	Positive
r_d	$\frac{r_s \cdot i \cdot w_u \cdot (r_u - r_d) - r_s \cdot r_d \cdot (-i \cdot w_u)}{(i \cdot w_u \cdot (r_u - r_d))^2}$	$+$	Negative
w_u	$\frac{-r_s \cdot r_d \cdot i \cdot (r_u - r_d)}{(i \cdot w_u \cdot (r_u - r_d))^2}$	$-$	Positive

[77] Remember that we have shown that $\partial m_1^* / \partial x < 0$; see footnote 62.

6.4 The first order partial derivatives of y

Table 7: The first order partial derivatives of y.

Para-meter	First order partial derivative	Distinction of cases	Sign	Impact[78] on m_2^*
r_s	$\frac{w_d \cdot r_d \cdot w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d)}{(w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d))^2} - \frac{w_d \cdot r_d \cdot r_s \cdot w_u \cdot r_u}{(w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d))^2}$		$-$	Positive
i	$\frac{-w_d \cdot r_d \cdot r_s \cdot w_u \cdot (-w_u \cdot r_u - w_d \cdot r_d)}{(w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d))^2}$		$+$	Negative
r_u	$\frac{-w_d \cdot r_d \cdot r_s \cdot w_u \cdot (r_s - i \cdot w_u)}{(w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d))^2}$		$-$	Positive
r_d	$\frac{w_d \cdot r_s \cdot w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d)}{(w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d))^2} - \frac{w_d \cdot r_d \cdot r_s \cdot w_u \cdot (-i \cdot w_d)}{(w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d))^2}$		$+$	Negative
w_u	$\frac{-w_d \cdot r_d \cdot r_s \cdot (r_u \cdot r_s - 2 \cdot i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d)}{(w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d))^2}$	$w_u < \frac{r_u \cdot r_s - i \cdot E_r}{i \cdot r_u}$	$-$	Positive
		$w_u = \frac{r_u \cdot r_s - i \cdot E_r}{i \cdot r_u}$	0	No impact
		$w_u > \frac{r_u \cdot r_s - i \cdot E_r}{i \cdot r_u}$	$+$	Negative
w_d	$\frac{r_d \cdot r_s \cdot w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d)}{(w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d))^2} - \frac{w_d \cdot r_d \cdot r_s \cdot w_u \cdot (-i \cdot r_d)}{(w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d))^2}$		$+$	Negative

[78] Remember that we have shown that $\partial m_2^* / \partial y < 0$; see footnote 63.

$\underline{\partial y/\partial r_s}$ is negative as

$$
\begin{aligned}
& w_d \cdot r_d \cdot w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d) < w_d \cdot r_d \cdot r_s \cdot w_u \cdot r_u \\
\Leftrightarrow \quad & r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d < r_s \cdot r_u \\
\Leftrightarrow \quad & \underbrace{-i \cdot E_r}_{<0} < 0.
\end{aligned}
$$

$\underline{\partial y/\partial i}$ is positive as

$$
\begin{aligned}
& -w_d \cdot r_d \cdot r_s \cdot w_u \cdot (-w_u \cdot r_u - w_d \cdot r_d) > 0 \\
\Leftrightarrow \quad & E_r > 0.
\end{aligned}
$$

$\underline{\partial y/\partial r_u}$ is negative if $r_s > i \cdot w_u$. With Assumption 8 we have excluded all parameter constellations for which r_u is not higher than $i \cdot P^f$ as they are not relevant for our analysis. And $r_u > i \cdot P^f$ implies $r_s > i \cdot w_u$:[79]

$$
\begin{aligned}
& r_u > i \cdot P^f \\
\Leftrightarrow \quad & r_u \cdot r_s > i \cdot (w_u \cdot r_u + w_d \cdot r_d) \\
\Leftrightarrow \quad & r_s > i \cdot w_u + \underbrace{\frac{i \cdot w_d \cdot r_d}{r_u}}_{>0}.
\end{aligned}
\tag{37}
$$

Therefore, for the parameter constellations that are relevant for our analysis, $\partial y/\partial r_u$ is negative.

[79] Remember that $P^f = E_r/r_s = (w_u \cdot r_u + w_d \cdot r_d)/r_s$; see Equations (1) and (3).

$\underline{\partial y/\partial r_d}$ is positive if

$$\begin{aligned} & w_d \cdot r_s \cdot w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d) > w_d \cdot r_d \cdot r_s \cdot w_u \cdot (-i \cdot w_d) \\ \Leftrightarrow \quad & r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d > -i \cdot w_d \cdot r_d \\ \Leftrightarrow \quad & r_s > i \cdot w_u. \end{aligned}$$

We have shown that $r_u > i \cdot P^f$ implies $r_s > i \cdot w_u$.[80] Therefore, for the parameter constellations we look at, $\partial y/\partial r_d$ is positive.

Finally, it can be shown that for $r_s > i \cdot w_u$, $\underline{\partial y/\partial w_d}$ is positive:

$$\begin{aligned} & r_d \cdot r_s \cdot w_u \cdot (r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d) > w_d \cdot r_d \cdot r_s \cdot w_u \cdot (-i \cdot r_d) \\ \Leftrightarrow \quad & r_u \cdot r_s - i \cdot w_u \cdot r_u - i \cdot w_d \cdot r_d > -i \cdot w_d \cdot r_d \\ \Leftrightarrow \quad & r_s > i \cdot w_u. \end{aligned}$$

[80] See the derivation of Condition (37).

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