Happiness Maintenance and Asset Prices

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Abstract

This paper explores the implications of investors' everyday mild feelings for aggregate asset returns. To this end, it introduces a novel class of state dependent preferences - happiness maintenance preferences - into the standard Mehra and Prescott (1985) economy by allowing investors' coefficient of relative risk aversion to depend partly on their current feelings, which, in turn, are a function of the current state of the economy. Consistent with recent evidence from experimental psychology (see for example Isen (1999)), good times bring about a positive mood for investors and a heightened pain from any potential loss. In an attempt to maintain their good mood, investors become less willing to bear any portfolio risk, i.e. they become more risk averse.

Extremely mild procyclical changes (a standard deviation of about one percentage point) in investors' risk aversion are sufficient to bring the implications of a simple dynamic model of asset pricing in line with the historically observed stylized features of asset returns, without relying on unreasonable values of the behavioral parameters. With a realistic consumption process, the model is capable of accounting for a sizable equity premium in line with the one observed in the US data. It also performs well with respect to other financial statistics, such as the average risk-free rate, the volatility and predictability of stock returns and the Sharpe ratio. Being able to match the equity premium, it implies that aggregate fluctuations have important welfare costs.

Keywords: state dependent utility, affect and decision making, equity premium puzzle.

JEL Codes: D81, D91, E44, G12.

"Every man is a suffering-machine and a happiness-machine combined. The two functions work together harmoniously, with a fine and delicate precision, on the give-and-take principle. For every happiness turned out in the one department, the other stands ready to modify it with a sorrow or pain" (Mark Twain, The Mysterious Stranger)

1 Introduction

The relationship between emotions and individual behavior is a central theme of modern psychology. A large body of experimental evidence as well as simple introspection support the view that emotions color the way we go about our everyday decisions in many important respects. For instance, it is widely recognized that "gut feelings" experienced at the moment of making a decision, such as worry, fear, dread, anxiety, and hope, to name a few, can play a critical role in the choice one eventually makes. Economists have been aware of these simple observations at least since Adam Smith[1]. Nevertheless, the vast majority of asset pricing models maintain the assumption that consumers' valuation of relevant economic variables can be fairly well represented as a given unemotional (utility) function of the enduring, intrinsic properties of the physical outcomes. The recent stream of research in behavioral finance (see Thaler (1992), Barberis and Thaler (2001) and Shleifer (2000) for extensive surveys) is no exception[2].

There are, of course, good reasons to maintain this assumption. Samuelson (1937) acknowledges that emotions are among the psychological factors that affect the values of the behavioral parameters of a model, but, nevertheless, considers a careful study of such dependence as outside the realm of economics[3]. The numerous documented empirical failures of asset pricing models constitute grounds to question, on a purely pragmatic basis, the ability of the assumption of unemotional consumers to shed light on the properties of aggregate asset returns. In fact, over the last fifteen years, empirical studies have extensively documented the failure of traditional consumption-

[1]"When we are about to act, the eagerness of passion will seldom allow us to consider what we are doing with the candour of an indifferent person. The violent emotions which at that time agitate us, discolour our view of things, even when we are endeavouring to place ourselves in the situation of another, and to regard the objects that interest us in the light in which they will naturally appear to him...This self-deceit, this fatal weakness of mankind, is the source of half the disorders of human life." (Adam Smith, 1759).

[2]In fact, Hirshleifer (2001), in his detailed survey of the approach to asset pricing based on the psychology of investors, acknowledges the possibility that feelings may affect people's perceptions of and choices with respect to risk, but notices that at present we lack a careful "analysis of the effects of currently experienced emotions on current prices."

[3]Friedman (1962) perhaps best epitomizes this view: "The relativity, i.e. nonconstancy, of wants has a number of important implications...Despite these qualifications, economic theory proceeds largely to take wants as fixed. This is primarily a case of division of labor. The economist has little to say about the formation of wants; this is the province of the psychologist. The economist's task is to trace the consequences of any given sets of wants. The legitimacy of and justification for this abstraction must rely ultimately, as with any other abstraction, on the light that is shed and the power to predict that is yielded by the abstraction".

based asset pricing models to account for the historically observed level, variation and cyclical behavior of asset returns. Two of the most prominent failures that have been identified are the so-called equity premium puzzle and the risk-free rate puzzle. Regardless of the calibration and for behaviorally realistic individual attitudes toward risk, this class of models is not able simultaneously to generate risk premia that correspond to a six per cent annual equity premium discussed in Mehra and Prescott (1985), match the level of returns, and replicate aggregate fluctuations. But there are also sound and more substantive reasons to explore the stock market implications of emotions. Recent advances in experimental psychology have shown that emotions are not inherently more unpredictable or erratic than cognitive deliberations (Damasio, 1994; Zajonc, 1998) and that they can be experimentally measured in many useful ways (Loewenstein and Lerner, 2001; Isen, 1999). It is commonplace among financial industry professionals and in the popular press to account for movements in asset returns in terms of fear or hope or anxiety. Finally, financial advisors customarily use terms such as subjective "psychological comfort" as a criterion to determine the composition of individual portfolios.

Motivated by these pragmatic and substantive reasons, I study the effects of investors' currently experienced emotions on asset prices. I parsimoniously and selectively incorporate insights from experimental psychology about the determinants and the effects of this class of emotions into a tractable consumption/saving problem in order to develop an affect-based theory of asset pricing[4]. This is a challenging task. The generality needed to build an asset pricing model is clearly not achieved by any psychological theory or experiment on emotions. Psychologists are far from understanding all the details of the complex constellation of individual emotional states. Nevertheless, the contention of the present research is that some important pieces of the puzzle are in place and theory and experiments have accumulated a basis of knowledge which is solid enough to build interesting generalizations. The model is necessarily simplistic given that its purpose is to illustrate which type of insights into asset pricing can be gained by taking seriously the state of the art knowledge of emotions. The hope is that as further knowledge is developed we will be able to build more articulated and possibly more realistic asset pricing models.

Based on the hypothesis that feelings form the neural and psychological substrate of investors' preferences[5], I focus on a special class of affect-dependent preferences, whose motivation is provided by a well documented stylized feature of emotions experienced at the moment of decision making, happiness maintenance: investors' risk aversion depends partly on their current affective state, which, in turn, is a function

[4]See Mehra and Sah (2002) for a complementary analysis of the effects of incidental influences on the volatility of asset returns.

[5]This approach is shared by some recent studies, none of which focuses on asset pricing. Loewenstein (1996) reviews this related literatures on, and models how, visceral factors such as hunger, fatigue, sexual desire, moods, emotions, pain and drug cravings affect preferences between different goods in models with no uncertainty.

of the current state of the economy. In particular, good times bring about a positive mood for investors and, consistent with the experimental evidence (Isen (1999) and others), a heightened pain from any potential loss. In an attempt to maintain their mood, investors become less willing to bear any portfolio risk, i.e. they become more risk averse. A simple calibration exercise shows that a dynamic model of asset pricing based on investors' preferences that display such a happiness maintenance feature is capable of accounting for a sizable equity premium in line with the one observed in the US post-war data, and performs well with respect to other financial statistics, such as the Sharpe ratio and the volatility and predictability of equity returns. Being able to match the equity premium, it implies that aggregate fluctuations have important welfare costs. Mild procyclical changes (a standard deviation of about one percentage point) in investors' risk aversion over wealth are sufficient to bring the implications of the model in line with the historically observed first two moments of asset returns. Moreover, the empirical performance of the model obtains with a realistic consumption process and does not rely on unreasonable values of the behavioral parameters, such as individual risk aversion and rate of time preference. According to the hypothesis I pursue, one can usefully conceptualize the historically observed high equity premium as a "happiness maintenance" premium, inasmuch as it has been a reward for both the potential excess riskiness of the actual consumption stream associated with investing in the stock market and the higher perceived threat to investors' psychological well-being when holding equities in their portfolios. In this sense, immediate emotions, by increasing the perceived risk associated with equity, can contribute to resolving some of the more prominent asset pricing puzzles.

Related literature This work is related not only to various strands of the equilibrium asset pricing literature but also to recent studies of emotions within the literature on psychology and economics.

The consumption-based asset pricing literature on the equity premium puzzle is vast and is beyond the scope of this work to survey (see, rather, Campbell (1999) or Kocherlakota (1996)). My exercise is close in spirit to the generalizations of either the time or the state-separability assumption of Von Neumann-Morgenstern preferences pursued in the literature, respectively, by introducing habit formation preferences (Sundaresan (1989), Constantinides (1990), Campbell and Cochrane (1999)) or risk-sensitivity and precautionary motives (Tallarini (2000), Hansen, Sargent and Wang (2002)). In particular, Campbell and Cochrane (1999) show that a specification of state-dependent preferences motivated by habit-persistence enjoys some success in matching the main asset pricing facts, yet does not appear to provide a fully satisfactory solution to the equity premium puzzle since it still requires unrealistically high and strongly countercyclical effective risk aversion on the part of individuals to account for the size of the historically observed equity premia. Melino and Yang (2003) use state-dependent recursive preferences to show that modest variation in the elasticity of intertemporal substitution improves the asset pricing performance of a habit-like model with strongly countercyclical effective risk aversion. Danthine et al. (2003) and Gordon and St-Amour (2000) also study the asset pricing implications of state-dependent preferences over consumption only. Danthine et al. (2003) stress the non-stationarity of the implied pricing kernel as a limiting aspect of this class of preferences. To bypass this problem, Gordon and St-Amour (2000) introduce an *ad hoc* normalization parameter they estimate from the data. Once so normalized, their pricing-kernel still needs a strongly countercyclical risk aversion to fit asset markets data. Finally, in Barberis, Huang and Santos (2001) a strongly countercyclical loss aversion provides an at least partial resolution of the equity premium puzzle. Happiness-maintenance preferences share with these models the focus on state-dependent risk aversion. Yet, in sharp contrast to other specifications of state-dependent preferences, they imply a stationary pricing kernel and rely on a low and *mildly pro*cyclical risk aversion as the key force behind their ability to account for asset market facts.

Zou (1994) and Bakshi and Chen (1996) study preferences based on an interpretation of Max Weber's spirit of capitalism as the pursuit of wealth for its own sake. As in happiness maintenance, wealth enters the utility function directly. Bakshi and Chen (1996) find that if investors derive a direct enjoyment from wealth, the observed volatility of wealth is a risk factor for stock returns. However, this provides only a partially satisfactory resolution of the equity premium puzzle (Campbell (1993, 1999)): if consumption is smooth and wealth is volatile, this itself is a puzzle that must be explained, not an exogenous fact that can be used to resolve other puzzles. The present work complements these previous (partial equilibrium) wealth-based models by providing a mechanism to account for the observed volatility of wealth endogenously within a general equilibrium context that takes explicitly into account the restrictions imposed on wealth by the budget constraint of investors. Moreover, a novel feature of happiness maintenance preferences is that wealth levels (relative to consumption) are an extra source of risk associated with stock holding.

Saunders (1993), Hirshleifer and Shumway (2001), Kamstra, Kramer and Levi (2001), Kamstra, Kramer and Levi (2000) document an empirical relationship between events such as weather or length of the day and asset returns and interpret it as suggestive evidence of an impact of a special class of emotions, moods, on asset prices. Mehra and Sah (2001) formally explore such a relationship within an equilibrium asset pricing model. They consider the potential effects of moods, i.e. small fluctuations in investors' subjective preferences (discount factors and attitudes towards risk) on the volatility of equity prices. They find that such fluctuations may have significant implications for understanding the volatility of the prices of financial assets. More direct evidence of an emotional reaction of investors to risk is provided in Lo and Repin (2001) who study the psychophysiology of risk processing of a sample of stock market traders and find significant correlation between changes in the traders' cardiovascular variables and market volatility.

Caplin and Leahy (2001), Laibson (2001), Koszegi (2002), and Bernheim and

Rangel (2002) develop decision theoretic frameworks which explicitly incorporate emotions. Both the class of emotions studied here and the focus on asset pricing distinguish the present work from these studies.

Outline of the paper The first section motivates the affect-maintenance hypothesis through a selective survey of the findings of experimental psychologists. It then develops a formal representation of individual preferences that is broadly consistent with the experimental evidence. It further shows that happiness maintenance preferences are general enough to be derived from an intuitive set of axioms yet parsimonious enough to be embedded into a standard equilibrium asset pricing model along the lines of Mehra and Prescott (1985). The second section characterizes equilibrium returns in an economy populated by investors with happiness-maintenance preferences and builds intuition on the relationship between the implied excess return on equities over bonds and the features of investors' hedonic risk aversion. A proof of equilibrium existence and a simple calibration exercise are the core of the third section, which investigates the quantitative asset pricing implications of affect-maintenance. The fourth section concludes. Algebraic derivations and proofs are confined to the Appendix A.

2 An affect-based capital asset pricing model

Immediate emotions[6] are experienced at the time of decision making and arise from factors related to the decision at hand, including the individual's environment. The term *affect* is used to define this broad class of emotional states, of which background moods, happiness or sadness[7] are perhaps the most prominent exemplars. Recent decision making research has seen an increased interest in the role of affect: a growing body of studies demonstrates that affect is a strong conditioner of individual preferences and indicates that *mild* affective states can markedly influence everyday thought processes (Loewenstein et al. (2001), Slovic et al. (2002)). It is worth emphasizing that such mild everyday affective states are more frequent and relatively less intense than visceral states, i.e. strong and infrequent "negative emotions (e.g., anger, fear), drive states (e.g.,hunger, thirst, sexual desire), and feeling states (e.g., pain) that grab people's attention and motivate them to engage in specific behav-

[6]Anticipated (or expected) emotions are, instead, typically not experienced in the immediate present, but expected to be experienced in the future. They consist of predictions about the emotional consequences of decision outcomes. A thorough analysis of emotions and decison making is well beyond the scope of the present work (Loewenstein and Lerner (2001) is an up to date review; Loewenstein (2000) surveys the field with special reference to economics). Eisenberg et al (1996) find that anxiety, a typical anticipatory emotion, correlates with risk aversion.

[7]It may also be viewed as a quality (e.g. goodness or badness) associated with a stimulus. Finucane et al. (2002) stress the tendence of these two conceptions to be related, so that I will use the term intercheangeably.

iors" (Loewenstein, 2000). Experiments have uncovered many regularities of how different affective states shape individual risk preferences and impact choices under uncertainty. Broadly speaking, affect has been recognized to do so by changing probability assessments (e.g., Johnson and Tversky (1983)) and valuation of outcomes (e.g. Isen and Patrick (1983)). A series of studies (see Isen (1999) for a thorough review) documents that individuals in different emotional states display different willingness to gamble. In particular, a stylized fact is that people who feel good are *more* risk averse than people who feel neutral, in particular when the stakes are high. The idea is that people have a motive for positive affect maintenance: people who feel good risk losing that state, as well as any tangible stake, if they lose a gamble, since losing might undermine their good mood. Therefore, with more to lose than controls, they are more risk averse than controls. Interestingly enough, this choice pattern has been observed notwithstanding a general tendency of individuals in a positive affective state to make optimistic probability assessments.

For example, Isen and Patrick (1983) conducted an experiment to study the influence of positive affect on choices under uncertainty. Participants, a large sample of college students, were randomly assigned to two groups: positive affect was induced only in participants in one group by receipt of a small gift, a McDonald's gift certificate worth \$.50. Subjects were given ten poker chips and told that these chips represented their credit for participating in the study. Risk preferences were measured in terms of the amount of chips actually bet by the two groups of participants in a game of roulette. They found that individuals in a positive mood bet significantly less than controls on gambles with a meaningful probability of losing (about 20% chance of winning). In particular, individuals in a neutral state bet on average about six times as many chips as individuals in a positive mood[8].

The finding that individuals in good mood are more risk averse has been replicated with different measures of risk preferences. For example, Isen and Geva (1987) used the level of the probability of winning before accepting a bet of fixed amount and found again that, when a meaningful amount was at stake, namely their whole endowment of chips, individuals in a positive mood, in contrast to those in a control group, set a level for the probability of winning as a cutoff point for accepting a given gamble on average about 30% higher than controls. Isen et al. (1984) documented that individuals in whom a positive mood had been induced by receipt of a small gift expressed greater preference in a lottery choice for a \$1 ticket rather than a \$10 ticket relative to a control group. Nygren et al. (1996) provided stronger support for an influence of affect on risk taking: they asked participants in whom positive affect had been induced, as well as no manipulation controls, to make actual betting decisions in twelve different three-outcome gambles. The mean bet value of affect condition participants was found to be consistently lower of about 30% than controls, regardless of the riskiness of the gambles, i.e. the ratio of the probability of winning and loosing or of the amounts.

[8]The latter bet on average only about half chip!

While these early results indicated a tendency toward conservatism in risk preferences, Isen et al. (1988) focused more directly on the notion of risk aversion typically employed in economics and finance. They examined the slope of the utility associated with various outcomes, as a function of positive affect induced by means of a small bag of candy. Both control and affect condition participants were asked to make choices between pairs of simple 50-50 gambles in such a way that a set of indifference points could be found and individual utility functions constructed. The average utility curves were computed for the two groups and people in whom positive affect had been induced displayed a steeper utility function than controls. Fong and McCabe (1999) replicate the essence of these findings within a very careful experimental setup that through the adoption of auction theoretic techniques (see Kagel (1995)) enables them to avoid potential difficulties with the studies mentioned so far, especially associated with the possible role of uncontrolled variables, the lack of monetary incentives and the lack of mechanism to ensure that truthful revelation of private values of the lottery was a dominant strategy. They endowed their subjects with lottery tickets and let them bid for the tickets in both a sealed-bid and an English auction. Subjects could earn up to $10 in each lottery or as little as zero in each. They found that average exit price is significantly lower for subjects whose mood had been improved by a minor manipulation, indicated a higher risk aversion in affect subjects.

The perspective suggested by these findings is well described by the idiom: don't push your luck. It is worth contrasting it with the findings of illusion of control or "gambling with the house money" of Thaler and Johnson (1990), which motivate the work of Barberis et al. (2001). As suggested in Arkes et al. (1988), the presence of a meaningful loss might be the crucial determinant of the discrepancy between the findings of the two classes of experiments. In one experiment, where a meaningful loss was nonexistent, affect participants exhibited relatively more risk-prone behavior compared to controls. In a second experiment dealing with insurance buying behavior where participants were forced to focus on potential loss, positive affect participants displayed a greater risk aversion than did controls. Nygren et al. (1996) further illustrates this point: positive affect participants significantly overestimated the probability of winning while participants in the control group did not, in accord with the findings of studies such as Johnson and Tversky (1983). Nevertheless, in actual gambling situations, affect condition participants were much less likely to gamble than were controls.

2.1 Setup

Affect maintenance preferences are introduced in an otherwise standard "endowment economy" (Lucas (1978), Mehra and Prescott (1985)), populated by a large number of infinitely-lived investors, who are identical with respect to their preferences, endowments and expectations. Given these assumptions, it is customary to aggregate the investors into a single "stand-in", representative agent who each period is faced with a consumption/saving problem. There is only one consumption good. The only source of income in the economy is a large number of identical and infinitely-lived fruit trees, each in fixed supply. Without loss of generality, the supply of trees is normalized to unity and it is assumed that there exists one tree per individual, so that the amount of fruit produced by a tree in period t, denoted y_t, represents the output or dividend per capita. Fruits are non-storable, cannot be used to increase the number of trees and can only be used for consumption. They are uncertain and evolve according to $y_{t+1} = x_{t+1}y_t$, where $x_{t+1} \in \{\lambda_1, \ldots, \lambda_n\}$ is the growth rate of output which follows a given stationary stochastic process to be detailed on later. Each tree has a single perfectly divisible equity claim outstanding on it. In each period there is a spot market for the consumption good and a financial market in which equity shares are exchanged at a price p_t. Consequently, the gross rate of return on equity holdings from period t to period $t+1$ is defined as $R_{t+1} = \frac{p_{t+1}+d_{t+1}}{p_t}$. A one-period risk-free asset in zero net supply at a price p_t^f completes the description of the "technology" side of the economy. It pays a gross interest rate $R_t^f = \frac{1}{p_t^f}$.

Investors' utility has a hedonic component which depends on the performance of their portfolios. More precisely, investors derive utility from a composite good, g_t, which includes both current (per-capita) consumption, c_t, and current (per-capita) financial wealth, w_t. They rank random sequences of the composite good according to

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(g_t)$$

where

$$u(g_t) = \frac{g_t^{1-\alpha}}{1-\alpha}$$

and $\beta \in (0,1)$ is the subjective discount factor, $E_0[\cdot]$ is the expectations operator conditional on the information available at time zero, and $\alpha > 0$ has the conventional interpretation of the parameter of relative risk aversion. Notice that $u(\cdot)$ is chosen to be iso-elastic to facilitate comparison with previous studies and, perhaps more importantly, to insure stationarity of the price/dividend ratio and returns.

The composite good, g_t, represents the main departure from standard assumptions and much of the remaining part of this section details its key features and its connection with happiness maintenance. In contrast to standard consumption-based asset pricing models, investors' financial wealth, w_t, enters their preferences directly over and above the indirect utility of the consumption services it provides[9], that is

$$g_t = g(c_t, w_t; \theta_t) = c_t^{1-\theta_t} w_t^{\theta_t}$$

[9] I am not the first to study investors' preferences that depend directly on their financial wealth. Pigou (1947) elaborated on the notion of amenity utility provided by wealth, in the form of power, sense of security, and control from having resources. Kurz (1968) develops an optimal growth model in which the utility function is also sensitive to the per capita capital stock of the economy. Carroll

The parameter $\theta_t \in [0,1]$ controls the (relative) demand for financial happiness: values of θ_t close to the lower (upper) bound of the $[0,1]$ interval correspond to a low (high) demand for happiness relatively to consumption. Financial wealth represents a straightforward measure of financial performance. It is introduced directly into the utility function to capture in a straightforward yet parsimonious way the wide range of non-consumption related pleasures associated with ownership of financial assets, such as, to name a few, power and social status, but also sense of security and control from having resources. Throughout the paper, it is assumed that investors value total financial wealth as defined by the value of the beginning-of-period asset holding, s_t, and dividends, y_t, at the current prices, p_t. Formally,

$$w_t = (p_t + y_t)\, s_t$$

Affect-maintenance preferences are modelled as an instance of state-dependent preferences[10] by postulating a state-dependent demand for financial happiness. Appendix A gives a standard set of axioms and a representation theorem for these preferences. To develop intuition on the connection between θ_t and happiness maintenance, it is useful to rewrite the utility function[11] as

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\alpha}}{1-\alpha} \left(\frac{w_t}{c_t}\right)^{(1-\alpha)\theta_t} \tag{1}$$

Investors' expected utility has a decision component, which depends on current consumption, and a hedonic component, which depends on the performance of their portfolios. Financial income *relative* to consumption is assumed to provide a good first approximation indicator of this performance and, hence, a direct source of, at least financial, happiness. This appears to be broadly consistent with an empirically well established stylized fact of the relationship between individual emotional well-being and aggregate economic conditions[12] (Easterlin (2001), (2000), (1974), (1995); Blanchflower and Oswald (2000); Diener and Oishi (2000)): there is no clear cut trend,

(2000) shows a model with direct utility from wealth might help to explain the high saving rates of the rich. Carroll (2002) further shows that direct utility from wealth can help understanding why portfolios of the rich are heavily skewed toward investments in their own privatelyheld businesses. Zou (1994) and Bakshi and Chen (1996) study preferences based on an interpretation of Max Weber's spirit of capitalism as the pursuit of wealth for its own sake. The loss aversion motive studied in Barberis, Huang and Santos (2001) is yet another instance of preferences that depend on changes in financial wealth.

[10]Laibson (2001) and Loewenstein (2000) take an analogous approach to study the effect of emotions in different contexts and with a focus on different questions from the ones studies here.

[11]Notice that the utility function is non-separable in consumption and wealth.

[12]Methodologies used to assess subjective wellbeing include large-scale national surveys, daily experience sampling, longitudinal studies, and controlled experiments. These surveys typically ask questions such as: "Taking all things together, how would you say things are these dayswould you say youre very happy, fairly happy, or not too happy these days?" While life satisfaction and happiness are somewhat different concepts, responses are highly correlated and hence these concepts

positive or negative, in self-reported subjective well-being over periods of 20 to 30 years in rich countries. In particular, in the United States between 1946 and 1991, per capita real income has risen by a factor of 2.5, but happiness, on average, remained constant. Empirical research on happiness and the business cycle (Di Tella et al (2001)) confirms that movements in reported individual well-being display significant correlation with macroeconomic variables such as Gross Domestic Product. This holds true after controlling for the personal characteristics of respondents, country fixed-effects, year dummies, and country-specific time trends. Finally, psychometric studies[13] lends further support to the view that income affects subjective well-being in a systematic fashion. In fact, a series of studies (perhaps most notably Ehrhardt et al. (2000) and Larsen and Diener (1985)) finds that life-satisfaction is stable over time, with autocorrelations that range in the $(.4, .6)$ interval: even if emotions are found to fluctuate a great deal over time, when averaged over several months these shifts reveal that individuals' mean levels of emotion exhibit a significant degree of coherence and stability over time and across situations which is not attributable to artifact of self-report measurements. In the context of the present model, where all income is financial, the term $\left(\frac{w_t}{c_t}\right)^{(1-\alpha)\theta_t}$ then summarizes the possibly many reasons why income affects investors' happiness, other than by providing them future consumption services, and, by entering directly the utility function, formalizes the intuition that investors' utility from consumption depends on their experienced happiness.

One important implication of (1) is that a mean preserving spread of financial wealth (relative to consumption) directly reduces investors' utility. If financial wealth is a source of happiness, the desire to maintain such happiness should determine the size of the reduction of investors' utility. In fact, while investors' relative risk aversion over g_t gambles, α, is constant, investors' hedonic risk aversion, a_t, that is their risk aversion over portfolio risk, is a simple function of their demand for financial happiness. In fact, by definition[14]

$$a_t = (\alpha - 1)\,\theta_t + 1 \in (1, \alpha)$$

As far as $\alpha > 1$ - a restriction maintained throughout the paper - a_t is increasing in θ_t. In this sense, for any given α, θ_t determines by how much a mean preserving spread of financial wealth (relative to consumption) reduces investors' utility. Of

are typically joined under the broader rubric of subjective wellbeing. A host of validation studies have suggested that these questions reveal something meaningful and can indeed be interpreted as reflecting levels of well-being. For instance, those who report being happier are typically rated by others as happier, and tend to smile and laugh more. Self-reported happiness also correlates with both heart rate and electrical activity in the brain. Measures of subjective well-being have relatively high test-retest correlations, similar microeconometric structure across time and space, and are highly correlated with related questions. See Frey and Stutzer (2002) for an up to date survey of the happiness research literature in economics.

[13]See Diener and Lucas (2000) for a survey of the literature on subjective emotional well-being.

[14]See Appendix A for details.

course, given $\theta_{t,}$ a higher α translates into a higher hedonic risk aversion. This offers a particularly parsimonious device to introduce happiness maintenance and preserve, at the same time, the constancy of relative risk aversion over consumption. Consistent with the previously described findings of experimental psychologists (Isen (1999) and others), the following particularly simple specification is chosen:

$$\theta_t = \theta \overline{x}_t (n), \qquad \theta > 0 \tag{2}$$

where

$$\overline{x}_t (n) = \frac{1}{n+1} \sum_{\tau=0}^{n} x_{t-\tau} = \frac{1}{n+1} \sum_{\tau=0}^{n} \frac{y_{t-\tau}}{y_{t-\tau-1}}$$

is the average of the recent n states of the economy and θ_t is an increasing function of the state of the economy so that, for any given α, a_t is higher in good times. This is broadly consistent with the view that risk aversion is *pro*cyclical, that is in good times investors become *more* risk averse toward financial wealth (relative to consumption) in an attempt to maintain their happiness. The parameter θ controls average hedonic risk aversion. Moreover, together with n, it determines the intensity of the happiness maintenance motive, that is how sensitive investors' hedonic risk aversion is to changes in the state of the economy. In fact, with higher values of θ differences in hedonic risk aversion across states are more pronounced for any realization of the underlying states of the economy. The parameter n controls how far back in the past investors look to determine whether times are good or bad. If $n = 0$ (2) simplifies to

$$\theta_t = \theta x_t = \theta \frac{y_t}{y_{t-1}}$$

In this case good times are measured simply by the current state of the economy. If $n \geq 1$ a mean of the recent past states of the economy measures investors' hedonic risk aversion. This is broadly consistent with the psychological evidence on incidental emotions (see Loewenstein et al. (1999) and Gilbert and Wilson (2002) for a careful review of the experiments) which documents the existence of a durability or projection effect: individuals exaggerate the degree to which their future moods will resemble their current or recent past moods. This specification can be interpreted either as a formalization of the case when investors' current moods are affected by the recent economic trend or when their views about the current times are formed by extrapolating from the recent past, a feature often encountered in the accounts of the popular press.

Under some particular values of θ_t the model collapses into basic consumption - based or wealth - based models that have been previously studied in the literature and hence provide a useful benchmark to gauge the contribution of happiness maintenance. In particular, if $\theta_t = \theta \neq 0$, the model reduced to a standard wealth-based setup of the type studied, even though in a different context, by Bakshi and Chen (1996) and Epstein and Zin (1989). When $\theta_t = \theta = 0$, the model reduces to the standard consumption-based asset pricing framework.

In summary, affect-maintenance preferences are defined by:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(g_t)$$

where

$$u(g_t) = \frac{\left(c_t^{1-\theta_t} w_t^{\theta_t}\right)^{1-\alpha}}{1-\alpha} \tag{3}$$

and θ_t is defined in (2). Notice that $U(\cdot)$ is increasing in both consumption and wealth and investors are assumed to have rational expectations on the fundamentals of the economy. The choices of an investor with affect-dependent preferences are fully characterized by the triple $(\beta, \alpha, \theta_t)$, i.e., respectively, by his subjective rate of time preference, β, his relative risk aversion, α, and his relative demand for financial happiness, θ_t. Variables (y_t, x_t, θ_t) are sufficient relative to the entire history of shocks up to, and including, time t for predicting the subsequent evolution of the economy. They thus constitute legitimate state variables for the model.

2.2 The consumption-saving problem

The problem of the "stand-in" investor, viewed in period 0, is, given the asset price function, $p_t = p(y_t, x_t, \theta_t)$, the initial state of the economy, y_0, his initial asset holdings, s_0, and his initial affective state θ_0 to choose a sequence of plans for consumption, c_t, and end-of-period asset holdings, s_{t+1}, that maximizes his present discounted expected utility subject to his budget constraint. Formally, the investor chooses consumption and asset holdings that solve the following problem

$$\max_{\{c_t, s_{t+1}\}} E \sum_{t=0}^{\infty} \beta^t u(g(c_t, w_t; \theta_t)) \tag{4}$$

$$\begin{aligned} c_t + p_t s_{t+1} &= (p_t + y_t) s_t = w_t \\ c_t &> 0, s_{t+1} \in (0, 1] \\ & s_0, y_0 \ \theta_0 \text{ given} \end{aligned}$$

where $u(g_t)$ is defined in (3).

The functional equation associated with the investor's maximization problem is:

$$\begin{aligned} V(s_t, y_t, \theta_t) &= \max_{\{c_t, s_{t+1}\}} \{u(g(c_t, w_t; \theta_t)) + \beta E_t V(s_{t+1}, y_{t+1}, \theta_{t+1})\} \\ c_t + p_t s_{t+1} &= (p_t + y_t) s_t = w_t \\ c_t &> 0, s_{t+1} \in (0, 1] \\ & s_0, y_0, \theta_0 \text{ given} \end{aligned} \tag{5}$$

where $E_t(x_t) = \int x_t dF(y_{t+1}, x_{t+1}; y_t, x_t)$ is the expectation operator. The first order and envelope conditions are respectively:

$$u_1\left(g\left((p_t + y_t)s_t - p_t s_{t+1}, (p_t + y_t)s_t; \theta_t\right)\right)p_t = \beta E_t V_1\left(s_{t+1}, y_{t+1}, \theta_{t+1}\right)$$

and

$$\begin{aligned} V_1\left(s_t, y_t, \theta_t\right) &= u_1\left(g\left((p_t + y_t)s_t - p_t s_{t+1}, (p_t + y_t)s_t; \theta_t\right)\right)(p_t + y_t) \\ &\quad + u_2\left(g\left((p_t + y_t)s_t - p_t s_{t+1}, (p_t + y_t)s_t; \theta_t\right)\right)(p_t + y_t) \end{aligned}$$

where u_1 and u_2 denote the partial derivative of the utility function with respect to consumption and wealth respectively. Consequently, simple algebraic manipulations deliver[15] the following Euler equation:

$$\begin{aligned} & u_1\left(g\left(c_t, w_t; \theta_t\right)\right)p_t \\ = \; & \beta E_t\left[\left(u_1\left(g\left(c_{t+1}, w_{t+1}; \theta_{t+1}\right)\right) + u_2\left(g\left(c_{t+1}, w_{t+1}; \theta_{t+1}\right)\right)\right)\left(p_{t+1} + y_{t+1}\right)\right] \end{aligned} \tag{6}$$

This is the fundamental equation for the pricing of capital assets. It equates the loss in utility associated with respectively postponing consumption from today to tomorrow and carrying one additional unit of capital to the discounted expected utility of the resulting additional consumption next period. The intuition for it is common to a broad class of wealth-based asset pricing models: by reducing consumption by p_t units in the first period, the agent can purchase one unit of the asset, thereby raising consumption by s_{t+1} units in the second period. This consumption/saving decision, however, entails a portfolio adjustment which has a direct effect on investor's utility as indicated by the second term on the right-hand side of equation (6). The distinctive feature of the affect maintenance model is that the extent to which a given portfolio adjustment is expected to change investors' utility depends on how the (expected) future state of the economy affects investors' hedonic risk aversion through its dependence on θ_{t+1}. Moreover, investors fully anticipate changes in their hedonic risk aversion and, consequently, can fully hedge this extra source of uncertainty.

The Euler equation is derived using only the preferences and the budget constraint of the investor. Before exploiting the restrictions imposed by equilibrium on asset returns, it is useful to consider the consequences of the individual budget constraint and preferences for asset returns. Given the preferences specified in (3) , the risk-free interest rate is given by

$$R_f = \frac{1}{\beta E\left[\left(\frac{c_{t+1}}{c_t}\right)^{(1-\alpha)(1-\theta_t)-1}\left(\frac{w_{t+1}}{w_t}\right)^{(1-\alpha)\theta_t} k_{t+1}\left(\frac{c_{t+1}}{w_{t+1}}\right)^{(1-\alpha)(\theta_t - \theta_{t+1})}\right]} \tag{7}$$

[15]For the details on the derivation of the Euler see Appendix A.

where $k_{t+1} \equiv \left(\frac{1-\theta_{t+1}}{1-\theta_t}\right)\left(1+\frac{\theta_{t+1}}{1-\theta_{t+1}}\frac{c_{t+1}}{w_{t+1}}\right)$, θ_t and θ_{t+1} are defined in (2). This can be equivalently written as

$$R_f = \frac{1}{\beta E\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\left(\frac{w_{t+1}/c_{t+1}}{w_t/c_t}\right)^{1-a_t} k_{t+1}\left(\frac{c_{t+1}}{w_{t+1}}\right)^{a_{t+1}-a_t}\right]}$$

where $a_t = (\alpha-1)\,\theta_t + 1$ is the hedonic risk aversion. Two parametric choices provide a useful benchmark to gauge the contribution of happiness maintenance to the determination of expected returns. In particular, when $\theta_t = \theta_{t+1} = \theta = 0$, the risk-free rate reduces to the one predicted by the standard consumption-based asset pricing framework of Mehra and Prescott (1985), that is

$$R_f = \frac{1}{\beta E\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\right]} \tag{8}$$

In analogy with (8), under affect-maintenance preferences the risk-free rate is ceteris paribus high when the investor is impatient, i.e. when he has a low rate of time preference, β, when he has a high consumption smoothing preference, that is when his coefficient of relative risk aversion, α, is high, and when consumption growth is high. If $\theta_t = \theta_{t+1} = \theta \neq 0$, we have

$$R_f = \frac{1}{\beta E\left[\left(\frac{c_{t+1}}{c_t}\right)^{(1-\alpha)(1-\theta)-1}\left(\frac{w_{t+1}}{w_t}\right)^{(1-\alpha)\theta}\tilde{k}_{t+1}\right]}$$

where $\tilde{k}_{t+1} \equiv \left(1+\frac{\theta}{1-\theta}\frac{c_{t+1}}{w_{t+1}}\right)$. Affect maintenance, in analogy with standard wealth-based asset-pricing models, adds wealth growth as a determinant of returns. One can then think of the way wealth fluctuations contribute to the determination of the risk-free rate in analogy with the way consumption changes do in a traditional consumption-based model. However, given consumption and wealth growth, there is a first sense in which affect maintenance contributes to lower the risk-free rate: if investors derive direct utility from their portfolio wealth, then, relative to the case of Mehra and Prescott (1985), they will attempt to reduce current consumption and increase current saving. The attempt to increase current saving puts downward pressure on the real interest rate, as indicated by the third term in (7).

Relative to these polar cases, the risk-free rate under affect-maintenance preferences has one distinctive feature: there is an added source of uncertainty or volatility due to the fact that now risk-aversion itself changes. This affects the risk-free rate in two ways: since investors are uncertain about how much marginal utility they will derive tomorrow from any extra unit of portfolio wealth, there is an increase in perceived risk from the third term in (7) which leads them to seek safety in the risk-free asset, driving its price up or, equivalently, its return down. Moreover, not just wealth changes, but the level of their future wealth (relative to consumption) itself becomes a crucial determinant of the risk-free rate. In fact, the last term in (7) indicates that for a given expected level of wealth (relative to their consumption) tomorrow, affect-maintenance adds an important component of uncertainty as $a_{t+1} - a_t$ changes stochastically with changes in the state of the economy. This creates a further motive for investors to smooth their consumption. If they are frustrated in their attempts to do so, this increase in perceived risk leads them to seek safety in the risk-free asset, hence bidding its price up and lowering its return. Importantly, this higher uncertainty depresses the price of risky assets. In fact, the expected return on equity is given by

$$1 = \beta E\left[\left(\frac{c_{t+1}}{c_t}\right)^{(1-\alpha)(1-\theta_t)-1}\left(\frac{w_{t+1}}{w_t}\right)^{(1-\alpha)\theta_t} k_{t+1}\left(\frac{c_{t+1}}{w_{t+1}}\right)^{(1-\alpha)(\theta_t-\theta_{t+1})} R_{t+1}\right] \tag{9}$$

or equivalently as

$$1 = \beta E\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\left(\frac{w_{t+1}/c_{t+1}}{w_t/c_t}\right)^{1-a_t} k_{t+1}\left(\frac{c_{t+1}}{w_{t+1}}\right)^{a_{t+1}-a_t} R_{t+1}\right]$$

where $a_t = (\alpha - 1)\,\theta_t + 1$ is the hedonic risk aversion. Given the specification of the investor's preferences, it is straightforward[16] to observe that the (conditional) expected premium demanded by the investor to hold his wealth in equities, $E\Pi = ER_{t+1} - R_f$ will be equal to

$$R_f cov\left(-\left(\frac{c_{t+1}}{c_t}\right)^{(1-\alpha)(1-\theta_t)-1}\left(\frac{w_{t+1}}{w_t}\right)^{(1-\alpha)\theta_t} k_{t+1}\left(\frac{c_{t+1}}{w_{t+1}}\right)^{(1-\alpha)(\theta_t-\theta_{t+1})}, R_{t+1}\right) \tag{10}$$

or equivalently as

$$R_f cov\left(-\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\left(\frac{w_{t+1}/c_{t+1}}{w_t/c_t}\right)^{1-a_t} k_{t+1}\left(\frac{c_{t+1}}{w_{t+1}}\right)^{a_{t+1}-a_t}, R_{t+1}\right)$$

The expected premium, as stated in (10), depends upon the covariation of the intertemporal marginal rate of substitution and the return on equities, as customarily implied by consumption-based asset pricing models. This covariability is what results in risk. Contrary to standard consumption-based asset pricing models, if investors were risk neutral, i.e. if $\alpha = 0$, the covariance term would not be zero. In other words, an investor who is indifferent to consumption variations per se but has affect-maintenance preferences would still require a premium to hold equities. In this sense, affect maintenance adds two crucial determinants of the expected rate of return on

[16] See Appendix A for the algebraic derivations.

equities and, consequently, of the expected premium: since investors are uncertain about how much marginal utility they will derive tomorrow from any extra unit of portfolio wealth, there is an increase in perceived risk from the third term in (10). Moreover, under affect maintenance wealth risk which characterizes standard wealth-based asset pricing model gains an extra term: not just wealth changes, but the *level* of their future wealth (relative to consumption) itself becomes a crucial determinant of the premium. In fact, the last term in (10) indicates that for a given expected level of wealth (relative to their consumption) tomorrow, affect maintenance adds an important component of uncertainty as $a_{t+1} - a_t$ changes stochastically with changes in the state of the economy. This creates a further motive for investors to smooth their consumption. If they are frustrated in their attempts to do so, this increase in perceived risk leads them to require a higher return on risky assets for any given level of consumption and wealth risk. It is worth stressing that the (expected) future level of the ratio of consumption to wealth is bounded below one even in a growing economy, since the budget constraint of the investor holds. In contrast to previous models with state-dependent preferences such as Danthine et al. (2003) and Gordon and St-Amour (2000), this implication of happiness maintenance preferences enables the model to rely on a "level" effect as an extra source of risk while retaining the stationarity of returns.

3 Aggregate asset pricing implications of happiness maintenance

To what extent can the most prominent asset pricing puzzles, namely the "risk-free rate" and "equity premium" puzzles, be resolved by the introduction of affect-maintenance preferences? More generally, can affect maintenance contribute to building a satisfactory analytical account of the main stylized facts of financial markets? To address this questions it is necessary to characterize equilibrium asset prices and returns in an economy populated by investors with preferences defined as in (3). Given the fixed supplies of goods and assets, the determination of the quantity choices in a competitive equilibrium in this economy is trivial: all fruits are consumed during the period in which they are produced, i.e., $c_t = y_t$, where c_t is the per-capita consumption in period t, and the representative investor willingly holds all his wealth in the risky asset, $s_t = s_{t+1} = 1$. Since consumer-investors are assumed to have identical preferences, per-capita consumption of the representative investor equals aggregate consumption, which then equals aggregate output. Equilibrium, then, is characterized by the asset price function that supports this allocation, that is by the function $p_t = p\left(y_t, x_t, h_t\right)$ that solves

$$\begin{aligned} & u_1\left(g\left(y_t, p_t + y_t; \theta_t\right)\right) p_t \\ = \; & \beta E_t\left[\left(u_1\left(g\left(y_{t+1}, p_{t+1} + y_{t+1}; \theta_{t+1}\right)\right) + u_2\left(g\left(y_{t+1}, p_{t+1} + y_{t+1}; \theta_{t+1}\right)\right)\right)\left(p_{t+1} + y_{t+1}\right)\right] \end{aligned}$$

Loosely speaking, the optimality conditions that correspond to the solution of the investor's problem defined in (4) and the requirement of market clearing in the aggregate provide the fundamental equations for pricing the risk-free and risky asset. Given (3), this equation can be equivalently written[17] as

$$\begin{aligned} & (1-\theta_t) f_t (f_t+1)^{(1-\alpha)\theta_t} \\ = \;& \beta E\left[(1-\theta_{t+1}) x_{t+1}^{1-\alpha}\left(1+\frac{\theta_{t+1}}{1-\theta_{t+1}}(f_{t+1}+1)^{-1}\right)(f_{t+1}+1)^{1+(1-\alpha)\theta_{t+1}}\right] \end{aligned} \quad (11)$$

where $f_t = \frac{p_t}{y_t}$ is the price-dividend ratio function. For the probability structure specified in the next section, Appendix A contains a proof of the existence of equilibrium. The following proposition offers a characterization of equilibrium asset returns when investors have affect-maintenance preferences.

Proposition 1 *Given the preferences in* (3) *the equilibrium risk-free interest rate and equilibrium expected return on equity satisfy, respectively,*

$$R_f = \frac{1}{\beta E\left[x_{t+1}^{-\alpha}\left(\frac{f_{t+1}+1}{f_t+1}\right)^{(1-\alpha)\theta_t} k(f_{t+1})(f_{t+1}+1)^{(1-\alpha)(\theta_{t+1}-\theta_t)}\right]} \quad (12)$$

and

$$1 = \beta E\left[x_{t+1}^{-\alpha}\left(\frac{f_{t+1}+1}{f_t+1}\right)^{(1-\alpha)\theta_t} k(f_{t+1})(f_{t+1}+1)^{(1-\alpha)(\theta_{t+1}-\theta_t)} R_{t+1}\right] \quad (13)$$

where $k(f_{t+1}) = \left(\frac{1-\theta_{t+1}}{1-\theta_t}\right)\left(1+\frac{\theta_{t+1}}{1-\theta_{t+1}}(f_{t+1}+1)^{-1}\right)$, θ_t *and* θ_{t+1} *are defined in* (2) *and* $f_t = \frac{p_t}{y_t}$.

Proof. *see Appendix A.* ■

Calibrations of equation (12) in the traditional asset-pricing literature typically use historical data on $\frac{c_{t+1}}{c_t} = x_{t+1}$ to estimate expected consumption growth. When these calibrations use conventional values of the preference parameters β and α, the resulting risk-free rate is much higher than its historical average of one or two percent per year in the United States. This finding has been dubbed the "risk-free rate puzzle" by Weil (1989). Moreover, calibrations of the average equity premium implied by the analog of equation (13) using conventional values of the preference parameters β and α typically yield an equity premium that is much smaller than the historical average equity premium of about 6% per year in the United States. This is in essence the so called "equity premium puzzles" (Mehra and Prescott, 1985).

[17]See Appendix A for the details of the derivation.

3.1 A quantitative assessment

In order to gain first insight into the quantitative effect of affect maintenance on aggregate asset returns, this section presents numerically computed solutions to the problem of the investor defined in (4) for various parameter choices and uses this solution to compute the associated time averaged risk-free rate, market rate, and risk premium as implied by equations (12) and (13). The choice of specific values of the behavioral parameters and of the aggregate growth rate of output, x_t, is crucial for the empirical evaluation of the model.

The "technology" side of the model is entirely standard. As in Mehra and Prescott (1985), the aggregate growth of output x_t follows a Markov chain and the number of states n is limited to two (λ_1, λ_2) ,with transition probabilities given by Π and defined as follows

$$\begin{aligned}
\lambda_1 &= 1+\mu+\delta \qquad (14)\\
\lambda_2 &= 1+\mu-\delta \\
\Pi &= \begin{bmatrix} \pi & 1-\pi \\ 1-\pi & \pi \end{bmatrix}
\end{aligned}$$

The parameters μ, δ and π are chosen to match respectively the mean, standard deviation and first order autocorrelation of aggregate consumption growth. More precisely, as customary in the consumption-based asset pricing literature, the values of the parameters are chosen to match the historically observed values in the time series for aggregate consumption in the US economy between 1889 and 1985. The values required are $\mu = 0.018$, $\delta = 0.036$, $\pi = 0.43$.

The remaining parameters $(\beta, \alpha, \theta_t)$ pertain to investors' preferences and have all clear behavioral interpretations and, consequently, can be chosen on the ground of realism in the light of introspection and evidence from field studies. Before presenting the main findings, it is then important to discuss the criteria that enlighten these choices since, to a large extent, the puzzling nature of the implications of traditional-asset pricing models is a matter of tastes, in the sense that the stand one takes on it depends crucially on ones views on the reasonableness of given values of preference parameters.

It is customary to choose a time preference coefficient, β, close to and lower than one. A negative rate of time preference has been shown to be effective in "solving" the "risk-free rate puzzle", but introspection provides a strong argument in support of a positive rate of time preference. Consequently, β is chosen to be equal to .99.

Ever since Mehra and Prescott (1985) most discussions of the asset pricing puzzles deem as reasonable risk aversion coefficients within the $(0, 10)$ interval. Casual introspection, field studies and experiments provide compelling support to a risk aversion coefficient lower than 10, given that an α beyond 10 would imply rejections of consumption (and wealth) bets that most investors and in fact most subjects in experiments do not turn down[18]. A risk aversion coefficient close to the lower bound of the interval $(0, 10)$ is particularly important from the perspective of the present work, since the many documented asset pricing puzzles suggest the need of exploring analytical avenues that can account for the observed stylized facts of asset markets without resorting to high values of risk aversion. Given that $\alpha > 1$ is needed to insure consistency between the chosen specification of θ_t and happiness maintenance, α is taken to be equal to 3, a relatively low value which does not violate this restriction. Moreover, a comparative dynamics exercise is undertaken for values of risk aversion within the $(2, 10)$ interval.

Given these parametric choices, the calibration exercise consists in asking whether there are behaviorally reasonable parametrizations of happiness demand, θ_t, under which the model delivers a satisfactory asset pricing performance. Given the simple specification chosen for θ_t in (2), addressing this questions requires determining which ranges of the two happiness demand parameters, θ and n, are reasonable given the assumptions on the aggregate growth of output x_t. Recall that for the composite good g_t to be well defined θ_t has to lie at every point of time within the $[0, 1]$ interval. This suggests a first straightforward restriction for $\theta \in \left[0, \frac{1}{\overline{x}_t(n)}\right]$, for every t. Psychological experiments do not provide firm ground to further restrict the choice of θ within this interval. Moreover, introspection seems to provide limited guidance as well. The focus of the present study on asset pricing, rather than on the demand for happiness *per se,* suggests to take a pragmatic stand on this question. Consequently, the following interval is considered: $[0, 0.5]$. This interval is conservative since it does not allow the demand for consumption to be relatively higher than that for happiness. Moreover, it has the advantage of constraining hedonic risk aversion to be always lower than α and less volatile than the underlying state of the economy. In fact, recall that investors' hedonic risk aversion, given the chosen $\alpha = 3$, is simply

$$a_t = 2\theta_t + 1 = 2\theta\overline{x}_t(n) + 1$$

and its standard deviation is

$$\sigma(a_t) = \sigma(\theta_t) = 2\theta\sigma(\overline{x}_t(n)) \leq 2\theta\delta$$

Given the mean and standard deviation of aggregate consumption growth, $\theta_t \in [0, 0.5]$ implies $a_t \in (1, 2)$ and $\sigma(a_t) \in (0, 0.036)$. This parametric choice appears to be broadly consistent with the emphasis of happiness maintenance experiments on *mild* everyday emotions, which, in the current context, are measured by a low and mildly volatile hedonic risk aversion. Finally, given that the model is calibrated on yearly frequency, it is intuitive to constrain n within the $[0, 5]$ interval so as to roughly

[18] A noptable exception is Kandel and Stambaugh (1991), who have argued that there is no compelling reason to restrict the coefficient of relative risk aversion to be less than ten, as the results of such experiments are very sensitive to details of their specification, including particularly the size of the gamble one considers.

cover the average length of a business cycle. In this way, investors span at most an entire cycle in their assessment of the recent trend of economic conditions. Table 1 contains a summary of the parameter values .

An often heard source of criticism of preference-based accounts of stock market facts (see Zin (2001) for a recent forceful restatement) blames them for deviating from the discipline of structural modelling and possibly obtaining spurious results due to the extra degrees of freedom the modeler enjoys by adding free behavioral parameters. Zin (2001) suggests using non-sample-based criteria to determine whether a given preference structure is reasonable. The clear behavioral interpretation of θ_t and the tight mapping between its stochastic properties and those of hedonic risk aversion can be considered as non-sample-based criteria to discriminate between alternative parametric choices of happiness demand. In this respect, the spirit of the present calibration exercise can be usefully contrasted with two of the most prominent existing preference-based accounts of stock market facts, namely habit-persistence and prospect theory inasmuch as neither the θ_t process is forced to match any of the stochastic properties of returns as Campbell and Cochrane (1999)'s habit process is, nor its specification allows for a relatively large number of parameters whose reasonableness is problematic to gauge outside the data matching exercise as in Barberis et al. (2001).

Computing returns Expected returns cannot be solved for in closed form and need to be computed using numerical methods. As in Mehra and Prescott (1985), the definition of returns can be used to rewrite equation (13) in terms of the price-dividend ratio. Generally speaking, if the pricing kernel does not depend upon the level of consumption (as here), we would not expect asset prices to depend on the levels either. Thus it is natural to assume that the price of equities is $p^e(c_t, \lambda_i, \theta_i)$ $= f_i c_t$, where $f_i = \frac{p^e(c_t,\lambda_i,\theta_i)}{c_t}$ is a price-dividend ratio function related to λ_i, the growth rate of output, both directly and through the dependence of θ_t on λ_i. Under the assumed probability structure the Euler equation (11) in general defines a system of (nonlinear) first-order difference equations in unknown price-dividend ratios which can be solved using numerical methods. With these price relationships it is relatively straightforward to compute the conditional and unconditional expectation of asset returns using their definition[19].

The number of first-order difference equations and price-dividend ratios defined by (11) and the methods adopted to actually solve these equations depend on the value of n. In particular, when $n = 0$, $\theta_i = \theta\lambda_i$ and hence the price-dividend ratio function depends only on the growth rate of output, that is $f_i = \frac{p^e(c_t,\lambda_i)}{c_t}$. The Euler equation (11) in this case defines a system of two (nonlinear) first-order difference equations in two unknown price-dividend ratios which can be solved in complete analogy with the traditional Mehra and Prescott (1985) methodology. It is clear

[19]Details on the remaining part of the computation are given in Appendix A.

then that when $n = 0$ the model precludes time-variation in expected returns. This feature is not unintentional, since it highlights an important distinctive feature of happiness maintenance with respect to other models of state-dependent preferences such as, for example, habit-persistence: the ability of the model to generate a high and highly volatile premium does not depend on time-varying expected returns. For $n \in [1, 5]$ the dependence of θ_i on the past realizations of aggregate dividend growth induces an element of time-variation in expected returns. This feature has a limited impact on the predicted premium but will be shown to improve the fit of the model with respect to a broader set of stylized facts of financial markets. It also makes the number of first-order difference equations and price-dividend ratios defined by (11) different from the case of $n = 0$ since now $f_i = \frac{p^e(c_t, \lambda_i, \theta_i)}{c_t}$ depends effectively on two states, λ_i and θ_i. This dependence renders the traditional solution methodology of less straightforward application. To circumvent this difficulty, standard simulation methods are employed to solve (11) . In particular, a simple parametrized expectation algorithm is adopted (see Marcet and Marshall (2002) for a detailed description of the algorithm) and the solved system is then simulated to generate a long times series of 50,000 data points to compute summary statistics[20].

3.1.1 The risk-free rate, the equity premium and the volatility puzzles

The first column of Table 2 displays summary statistics for value-weighted S&P 500 stock market data from 1889 to 1985. This is an updated version of the long sample used in the seminal contribution of Mehra and Prescott (1985) and since then used as a classical benchmark to evaluate the empirical performance of asset pricing models. As already mentioned, the mean excess return of equities over relatively riskless securities such as bonds, usually referred to as the equity premium, is slightly higher than six percent, with riskless securities paying an average return of about one percent. Moreover, riskless securities have displayed a significantly lower volatility of returns than equities with a difference in the order of ten percentage points. Consequently, the so called Sharpe ratio, which is defined as the mean of the equity premium divided by its standard deviation, has been in the neighborhood of .3.

To facilitate comparison, the second column of Table 2 reports the predictions of the traditional consumption-based asset pricing model - to which the present model reduces when $\theta_t = \theta_{t+1} = 0$ - under the parameters chosen. This offers a quantitative counterpart to the previous discussion of asset pricing puzzles: in essence, the predicted premium is one order of magnitude smaller than the actual one, the risk free rate is too high and both returns display excessively low volatilities. As a result, the implied Sharpe ratio is too low.

The third column of Table 2 shows that, contrary to partial equilibrium wisdom (see Bakshi and Chen (1996) for an example), the predictions of a basic wealth-based model are virtually indistinguishable from those of the consumption-based model.

[20]The fortran codes to solve and simulate the model are available upon request from the author.

In fact, when the latter is generalized to allow for a constant demand for happiness - a case to which the present model reduces when $\theta_t = \theta_{t+1} = \theta \neq 0$ - and the restrictions that the budget constraint poses on the relationship between wealth and consumption are taken into consideration, the introduction of a demand for wealth only increases prices hence lowering *both* returns, but it has virtually no effect on the premium and on the volatilities of returns. This parallels the arguments articulated in Campbell (1993, 1999) and lends further support to a classical criticism of wealth-based models: if consumption is smooth and wealth is volatile, this itself is a puzzle that must be explained, not an exogenous fact that can be used to resolve other asset pricing puzzles.

The fourth column of Table 2 contrasts these disappointing results with the predictions of the affect-maintenance model. Happiness demand is on average equal to 0.25 and, hence, hedonic risk aversion is on average 1.5 and has a standard deviation of about 0.01. The implied premium is more than ten times bigger than either the wealth-based or the consumption-based benchmarks. Moreover, stock market volatility ceases to be a puzzle. A low level of risk aversion and a reasonable rate of time preference can be reconciled with the basic facts of asset markets, i.e. a fairly stable and low average return on riskless securities and a sizable and fairly volatile premium of equities over bonds, granted that investors have a reasonably low and *mildly* procyclical hedonic risk aversion.

Table 3 is presented to give insight into the way happiness maintenance affects expected returns. It contrasts the scenario of the long sample of Table 2, that is to say a combination of relatively low riskless return, high premium and high volatility, with the post-war scenario of relatively higher riskless return, lower premium and lower volatilities. A slightly lower and less volatile demand for happiness provides a possible rationale for this second case since lowering hedonic risk aversion reduces the wedge between the returns on riskless and risky securities mostly by increasing the return on riskless securities.

Volatility bounds and the market price of risk To complete the illustration of the quantitative effects of happiness maintenance on asset returns, it is useful to consider Hansen and Jagannathan (1991)'s statement of the equity premium puzzle. The Sharpe ratio for asset i equals the assets risk premium divided by its standard deviation puts a lower bound on the volatility of the stochastic discount factor. The logic of Hansen and Jagannathan (1991) implies that the largest possible Sharpe ratio is given by the conditional standard deviation of the log stochastic discount factor. More formally,

$$\left| \frac{E\left(R^i\right) - R^f}{\sigma\left(R^i\right)} \right| \leq \frac{\sigma_t\left(m_{t+1}\right)}{E_t m_{t+1}}$$

where $\frac{E\left(R^i\right)-R^f}{\sigma\left(R^i\right)}$ is the Sharpe ratio, $\frac{\sigma_t\left(m_{t+1}\right)}{E_t m_{t+1}}$ is the market price of risk and m_{t+1} is the stochastic discount factor or pricing kernel. The Sharpe ratio is limited by the volatility of the stochastic discount factor. In this formulation, the "equity premium puzzle" lies in the fact that data on asset market returns and prices imply values of the market price of risk that are too high to be reconciled with many particular models of the stochastic discount factor. This is because these theories make the conditional standard deviation of the stochastic discount factor too small. To see this, Figure 1 plots values of the market prices of risk implied by a traditional consumption-based asset pricing model with power utility - to which the affect maintenance model reduces when $\theta_t = \theta_{t+1} = 0$ - under different values of the coefficient of relative risk aversion. Even with a risk aversion at the upper limit of the plausible interval $(1, 10)$, the market price of risk implied by the model falls short from satisfying the Hansen-Jagannathan bounds.

It is straightforward to derive the stochastic discount factor for an economy with happiness maintenance preferences:

$$m_{t,t+1} = \left(\frac{c_{t+1}}{c_t}\right)^{(1-\alpha)(1-\theta_t)-1} \left(\frac{w_{t+1}}{w_t}\right)^{(1-\alpha)\theta_t} k_{t+1} \left(\frac{c_{t+1}}{w_{t+1}}\right)^{(1-\alpha)(\theta_t-\theta_{t+1})}$$

where $k_{t+1} \equiv \left(\frac{1-\theta_{t+1}}{1-\theta_t}\right)\left(1 + \frac{\theta_{t+1}}{1-\theta_{t+1}}\frac{c_{t+1}}{w_{t+1}}\right)$, θ_t and θ_{t+1} are defined in (2) or equivalently

$$m_{t,t+1} = \left(\frac{c_{t+1}}{c_t}\right)^{-\alpha} \left(\frac{w_{t+1}/c_{t+1}}{w_t/c_t}\right)^{1-a_t} k_{t+1} \left(\frac{c_{t+1}}{w_{t+1}}\right)^{a_{t+1}-a_t}$$

where $a_t = (\alpha - 1)\,\theta_t + 1$ is the hedonic risk aversion.

By simply inspecting this expression, one could conjecture that happiness maintenance contributes to increase the conditional volatility of the stochastic discount factor, since, broadly speaking, they add two sources of volatility: the consumption/wealth ratio, w/c, and hedonic risk aversion, a_t. Figures 2 to 4 show that, thanks to these components, for low and moderately procyclical hedonic risk aversion, the equilibrium stochastic discount factor implied by happiness-maintenance preferences lies well within Hansen-Jagannathan bounds.

Inspecting the mechanism What drives the asset pricing implications of happiness maintenance? At the outset one might suspect that the somewhat striking asset pricing implications of the model are the result of hidden unreasonable behavioral assumptions, such as, for example, an equilibrium risk aversion which is much higher than α or an elasticity of intertemporal substitution which is much lower than $\frac{1}{\alpha}$. Nevertheless, in contrast to other types of state-dependent preferences such as habit-persistence, the specification chosen for affect maintenance highlights the fact that in equilibrium risk aversion is overall constant and equal to α. Moreover, while the elasticity of intertemporal substitution is indeed different from $\frac{1}{\alpha}$ and is a complicated function of the other parameters of the model[21], under the benchmark

[21] Appendix A contains a precise derivation of the elasticity of intertemporal substitution.

parametrization of the model it is straightforward to compute and is equal 0.4, hence well within the range of what is commonly assumed in the consumption-based asset pricing literature.

Tables 4.1 to 4.3 provide some insight into what might lie beneath the asset pricing performance of the model. In particular, Table 4.1 shows that affect maintenance shares the intuition of traditional models as to the effects of increasing the coefficient of relative risk aversion, α, on the premium. As would be expected, the aggregate risk premium rises as agents become more risk averse. While qualitatively this is in complete analogy with what predicted by a traditional model with power utility, the quantitative effect of increasing risk aversion under happiness-maintenance preferences is remarkably strong.

Further, Table 4.1 shows that increasing hedonic risk aversion decreases the risk-free rate. This latter fact is easily explained by noting that, greater affective risk aversion, by causing agents to value a certain wealth unit next period even more highly, will induce them to be willing to pay even more today for the claim to such certain wealth. Hence the risk-free rate of return is observed to fall. Noticeably, these results are robust across a number of happiness modes. Finally, as shown in Table 4.2, the volatility of asset returns increases in a directly proportional fashion with the volatility of risk aversion.

Table 4.3 shows that hedonic risk aversion is key to the satisfactory empirical performance of the model. Indeed, as far as the model has a low and mildly procyclical hedonic risk aversion, with an average in the order of 1.5 and a standard deviation in the order of 0.01, it can easily generate sizable premia with realistic volatilities. The message is that there is a wide range of combinations of values of θ and α under which the model can match the Sharpe ratio, that is under which the model can generate a sizable and volatile premium of equities over bonds. With a risk aversion as high as 10, an average happiness demand as low as 0.04 is sufficient to bring the predicted equity premium in line with the data. Vice versa, even with a risk aversion as low as 2, there is an acceptable value of average happiness demand for which the model can deliver a premium which is of an order of magnitude comparable to the one observed in the data.

3.1.2 Cyclicality of returns and the correlation puzzle

The first column of Table 5 reports some salient facts concerning the variations of asset returns and prices at the business cycle frequencies, namely the fact that the expected equity premium, conditional volatility and the Sharpe ratio are countercyclical, while the (*ex post*) price-dividend ratio and the equity premium are procyclical. Moreover, on average the equity premium is low and negative in bad times and high and positive in good times. It should come as no surprise that the procyclical risk aversion characterizing happiness maintenance preferences generates premia and expected premia (and a Sharpe ratio) that naturally fit the cyclical patterns found in the data. Further, it significantly attenuates the counterfactual countercyclical price-dividend ratio implied by the standard consumption-based model.

Perhaps more importantly, the unconditional correlation between stock returns and consumption growth predicted by the model is well below the almost 1 predicted by the standard model while still being higher than in the data. In this sense, happiness maintenance alleviates, even though it does not fully resolve, the so called "correlation puzzle" of Cochrane and Hansen (1992).

3.1.3 Auto-correlations and cross-correlations of returns

Table 6 reports autocorrelations and cross-correlations of returns, excess returns and of the price-dividend ratio. With respect to the consumption-based benchmark - whose implications are reported in the first column - the most notable change is that happiness-maintenance preferences, as far as the recent past performance of the economy contributes to shape investors' current hedonic risk aversion, bring the pattern of autocorrelations of the price-dividend ratio in line with the data. Moreover, returns and the premium become negatively correlated at longer horizons.

Shorter horizon autocorrelations and cross-correlations are roughly comparable to those of the standard model and constitute perhaps the biggest source of embarrassment for the happiness maintenance model. The simple to the extreme specification of the happiness demand process is certainly fundamental in driving this results and breaking the perfect positive correlation between θ_t and the state of the economy x_t postulated when $n = 0$ would likely improve the fit of the model. Nevertheless, it would violate the spirit of the calibration exercise which is not meant to force the model to fit asset market data but rather is an exploration of the potential of behaviorally realistic parametrizations of happiness maintenance to account for the stylized facts of financial markets.

3.1.4 Long-horizon predictability

Can the mild changes of hedonic risk aversion induced by happiness maintenance help the model to reproduce the observed patters of predictability of asset returns? To answer this question, Table 7 reports the results of regressions of cumulative log returns over a k-year horizon on lagged price-dividend ratio for $k = 1, 2, 3, 5$ and 10. More precisely, the table reports slope coefficients, β_k, and R_k^2 obtained from running the following regression from the simulated data

$$r_{t+1} + r_{t+2} + ... + r_{t+k} = \alpha_k + \beta_k (p_t - d_t) + \epsilon_{k,t}$$

where r_t is log return. For ease of comparison the corresponding values in the data and in the standard consumption-based model are reported in the first and second columns of Table 7 respectively. As far as the recent past performance of the economy contributes to shape investors' current hedonic risk aversion, the stylized pattern documented by Campbell and Shiller (1988) are well replicated by the model: the coefficients are negative; they start low and then increase. Finally, the R^2 increases with the return horizon.

Table 8 offers an alternative perspective on predictability through the excess volatility of stock prices. It reports the percentage of the variance of the price-dividend ratio accounted for by the covariance of the price-dividend ratio with future returns or by the covariance of the price-dividend ratio with future dividend growth. The rationale for this exercise is provided by the following approximate decomposition of the variance of log price-dividend ratio derived in Campbell (1991)

$$var\left(p_t - d_t\right) \approx \sum_{j=1}^{k} \rho^j cov\left(p_t - d_t, \Delta d_{t+j}\right) - \sum_{j=1}^{k} \rho^j cov\left(p_t - d_t, r_{t+j}\right)$$

where $\rho = \frac{\overline{p/d}}{1+\overline{p/d}}$ is defined at the steady state. In the data, both covariances are negative and a large fraction of total volatility is accounted for by the covariance of the price-dividend ratio with future returns. To gauge the magnitude of this phenomenon, consider that, for example, for the 1889-1985 period at a 15 year lag 101 percent of the total variance of the price-dividend ratio is accounted for by the covariance of the price-dividend ratio with future returns and only -10 percent is accounted by the covariance of the price-dividend ratio with future dividend growth. Moreover, it is at about a 15 years horizon that the forecasts of future returns account for the entire overall volatility. While the basic consumption-based model is grossly at odds with these facts, as far as the recent past performance of the economy contributes to shape investors' current hedonic risk aversion, the implications of the affect maintenance model are easy to square with the data.

3.1.5 Long-horizon volatility

Why does the stock market fluctuate? Table 9 investigates whether happiness maintenance has a say about stock market volatility at long horizons and presents the results of regressions of long-horizon log stock price changes on long-horizon log consumption changes using simulated data. Barsky and DeLong (1993) study a similar question and using aggregate stock market data find that these regressions yield coefficient invariably greater than one and as high as 1.61 at a 20-year horizon. Once again, as the first column of Table 9 shows, this findings are a source of embarrassment for the traditional-consumption based asset pricing model. Not so for the happiness maintenance model which consistently delivers coefficient significantly greater than one and can easily generate coefficients as high as about 6. Finally, at the 20-year horizon the model delivers estimates which are in line with the 1.61 estimate of Barsky and DeLong (1993). In this sense, mild procyclical changes in hedonic risk aversion constitute a natural and quantitatively realistic mechanism behind the volatility of financial markets at long horizons.

3.1.6 The welfare costs of aggregate fluctuations

Under happiness maintenance aggregate fluctuations are likely to entail sizable welfare costs. A simple computation in the spirit of Lucas (1987) illustrates this point. For purely illustrative purposes, consider perhaps the simplest version of the model when $\theta_t = 1$, or equivalently $a_t = a = \alpha$ and wealth growth is iid lognormal with mean g_w and standard deviation σ_w. Denote the log mean wealth growth rate by

$$\alpha_w = \ln E\left(\frac{w_{t+1}}{w_t}\right) = g_w + \frac{\sigma_w^2}{2}$$

Define α_w^* as the "certainty equivalent" growth rate of wealth, that is the growth rate in a non-stochastic economy that gives investors the same level of utility. Then it is straightforward to derive[22] the following measure for the welfare cost of aggregate fluctuations

$$\Delta\alpha_w = \alpha\frac{\sigma_w^2}{2} \tag{15}$$

We can contrast this measure with the one computed in Lucas (1987), that is

$$\Delta\alpha_c = \alpha\frac{\sigma_c^2}{2}$$

With an $\alpha = 2$, σ_c and g_c of the order of one percent annually imply that consumers would only trade *one hundredth* of a percentage point of growth for the complete elimination of fluctuations. By contrast, taking as customary (Cochrane and Hansen (1992), Epstein and Zin (1991)) the return on the NYSE value-weighted index as a proxy for $\frac{w_{t+1}}{w_t}$, σ_w is of the order of twenty percent annually and g_w is about seven percent. Hence, with $\alpha = 2$ consumer-investors would trade about *one half* of a percentage point of growth for the complete elimination of fluctuations, an estimate that is more than an order of magnitude higher than that indicated by the calculation of Lucas (1987). This is likely to be a lower bound for estimates one might obtain from a model with happiness-maintenance preferences, where aggregate fluctuations change investors' demand for happiness. Unfortunately, it is not possible to derive closed forms solutions for the welfare costs of aggregate fluctuations in the full fledged model with happiness maintenance preferences and one has to resort to numerical methods to derive empirical estimates. A careful pursuit of this question is somewhat peripheral with respect to the main focus of the present paper and is left for future work. For the sake of the present analysis (15) is a simple yet powerful illustration of the order of magnitude of the discrepancy one is likely to find between the estimates of the costs of aggregate fluctuations in standard consumption-based models and in models with a demand for happiness.

[22] Appendix A contains details of the derivation.

3.1.7 Robustness checks

The results collected in Table 10 show that the asset pricing performance of the happiness maintenance model is robust to alternative specifications of the happiness demand process which are consistent with the finding of psychologists and with the view that risk aversion is *pro*cyclical but not to alternative theories about the influence of happiness on risk aversion.

In fact, the second column of Table 10 contrasts the implication of the happiness maintenance model, which are reported for ease of comparison in the first column, with those of a model where happiness demand and hedonic risk aversion are *counter*cyclical. More precisely, it reports summary statistics of asset returns when investors' preferences are defined as in (3) but the happiness demand process θ_t is perfectly negatively correlated with consumption growth, rather than positively correlated as under happiness maintenance. The effects of this departure from the baseline model are significant: the premium shrinks to about a quarter of the value with procyclical hedonic risk aversion under the same parametrization mostly due to the fact that the risk-free rate is four times as high.

Nevertheless, the empirical predictions of the model are robust to alternative specifications that retain the procyclicality of happiness demand and hedonic risk aversion. For example, as the third column of Table 10 shows, the implied returns the model would predict if one specified a separate process for θ_t are virtually indistinguishable from those of the benchmark calibration.

4 Conclusion

Drawing on ingredients from outside the usual domain of economic theories of decision making can help to make some otherwise puzzling features of financial markets more comprehensible. In particular, an equilibrium model simple to the extreme has been used to show that *mild* everyday feelings have rich implications for aggregate asset returns. Happiness maintenance, a well documented feature of the immediate emotional perception of risk, by increasing the risk associated with equity contributes to resolve some of the most prominent documented asset pricing puzzles, such as the risk-free rate, equity premium and volatility puzzles and offers an intuitive rationale for why business cycles entail nonnegligible welfare costs. Finally, it provides a perspective over a broad set of stylized features of financial markets, such as, for example, the predictability of asset returns and the volatility of asset prices at long horizons.

Perhaps most notably, the model does not depart from conventional asset pricing wisdom along any dimensions other than investors' preferences and its satisfactory empirical performance is accomplished in a relatively parsimonious way by adopting a one-parameter formulation of happiness-maintenance preferences. This formulation has the additional advantage of lending itself to a straightforward behavioral interpretation, hence offering firm ground on which to judge the reasonableness of the parametric values chosen. In this sense, the results presented are encouraging as they represent one instance of a viable preference-based account of stock market facts and show that a deeper understanding of emotions may extend our knowledge of financial markets in many important respects.

References

[1] Arkes, H., L. Herren, and Alice M. Isen, 1988, "The Role of Potential Loss in the Influence of Affect on Risk-Taking Behavior," Organizational Behavior and Human Decision Processes, 41, pp. 181-193.

[2] Bakshi, Gurdip S., and Zhiwu Chen, 1996, "The Spirit of Capitalism and Stock-Market Prices," American Economic Review, 86(1), pp. 133-157.

[3] Barberis, Nicholas, Ming Huang, and Tano Santos, 2001, "Prospect Theory and Asset Prices," Quarterly Journal of Economics, 116 (1), pp. 1-53.

[4] Barberis, Nicholas, and Richard Thaler, 2001, "A Survey of Behavioral Finance," mimeo, University of Chicago.

[5] Barsky, Robert, and J. Brad De Long, 1993, "Why does the stock market fluctuate?," Quarterly Journal of Economics, 107, pp. 291-311.

[6] Bernheim, Douglas, and Antonio Rangel, 2002, "Addiction, Conditioning, and the Visceral Brain," mimeo, Stanford University.

[7] Blanchflower, David G., and Andrew J. Oswald, 2000, "Well-Being Over Time in Britain and the USA," NBER Working Paper No. 7487, Cambridge, MA.

[8] Campbell, John Y., 1991, "A Variance Decomposition for Stock Returns," Economic Journal, 101, pp. 157-179.

[9] Campbell, John Y., 1993, "Intertemporal Asset Pricing Without Consumption Data," American Economic Review, 83, pp. 487-512.

[10] Campbell, John Y., 1999, "Asset Prices, Consumption, and the Business Cycle," in John Taylor and Michael Woodford, eds.: Handbook of Macroeconomics, Vol. 1, North-Holland, Amsterdam.

[11] Campbell, John Y., and John H. Cochrane, 1999, "By force of habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," Journal of Political Economy, 107, pp. 205-251.

[12] Campbell, John Y., and Robert J. Shiller, 1988, "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," Review of Financial Studies, 1, pp. 195-228.

[13] Caplin, Andrew, and John Leahy, 1999, "Psychological Expected Utility Theory ad Anticipatory Feelings," Quarterly Journal of Economics, 116 (1), pp. 1-53.

[14] Carroll, Christopher D., 2000, "Why Do the Rich Save So Much?," in Does Atlas Shrug? The Economic Consequences of Taxing the Rich ,ed. by J. B. Slemrod, Forthcoming, Harvard University Press.

[15] Carroll, Christopher D., 2002, "Portfolios of the rich," Mimeo, Johns Hopkins University.

[16] Cochrane, John H., and Lars Peter Hansen, 1992, "Asset Pricing Lessons for Macroeconomics," in O. J. Blanchard and S. Fischer, eds.: NBER Macroeconomics Annual 1992, MIT Press, Cambridge, Mass.

[17] Cole, Harold L., George J. Mailath, and Andrew Postlewaite, 1995, "Incorporating Concern for Relative Wealth into Economic Models," Minneapolis Fed Quarterly Review, 19(3), pp.12-21.

[18] Constantinides, George, 1990, "Habit Formation: A Resolution of the Equity Premium Puzzle," Journal of Political Economy, 98, pp. 519-543.

[19] Damasio Antonio R., 1994, Descartes' Error: Emotion, Reason, and the Human Brain, New York, Avon.

[20] Di Tella, Raphael, Andrew J MacCulloch, and Andrew J Oswald, 2001, "The Macroeconomics of Happiness," Mimeo, Harvard University.

[21] Diener, Ed, and Richard E. Lucas, 2000, "Subjective Emotional Well-Being," in M. Lewis and J. M. Haviland Eds. Handbook of Emotions (2nd Ed.), New York, Guilford, pp. 325-337.

[22] Diener, Ed, and Shigehiro Oishi, 2000, "Money and Happiness: Income and Subjective Well-Being Across Nations," in Ed Diener and Eunkook M. Suh Eds., Culture and Subjective Well-Being, Cambridge, MA: MIT Press, pp. 185-218.

[23] Dreze, Jacques H., and Aldo Rustichini, 2001, "State-Dependent Utility and Decision Theory," in Barbera S., Peter Hammond and C. Seidl Eds., Handbook of Utility Theory, Volume II, Kluwer.

[24] Easterlin, Richard A., 1974, "Does Economic Growth Improve the Human Lot? Some Empirical Evidence," in Paul A. David and Melvin W. Reder Eds., Nations and Households in Economic Growth, New York: Academic Press, pp. 89-125.

[25] Easterlin, Richard A., 1995, "Will Raising the Incomes of All Increase the Happiness of All?," Journal of Economic Behaviour and Organization, 27(1), pp. 35-48.

[26] Easterlin, Richard A., 2000, "The Worldwide Standard of Living Since 1800," Journal of Economic Perspectives, 14(1), pp. 7-26.

[27] Easterlin, Richard A., 2001, "Income and Happiness: Towards a Unified Theory," Economic Journal, 111(473), pp. 465-484.

[28] Ehrhardt, Joop J., Willem E. Saris, and Ruut Veenhoven, 2000, "Stability of Life-satisfaction over Time," Journal of Happiness Studies, 1(2), pp. 177-205.

[29] Eisenberg, A. E., Jonathan Baron, and M. E. P. Seligman, 1996, "Individual Differences in Risk Aversion and Anxiety," Mimeo, University of Pennsylvania.

[30] Epstein, Lawrence, and Stanley Zin, 1989, "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," Econometrica, 57, pp. 937-969.

[31] Epstein, Lawrence, and Stanley Zin, 1991, "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Investigation," Journal of Political Economy, 99, pp. 263-286.

[32] Frey, Bruno S., and Alois Stutzer, 2002, "What Can Economists Learn from Happiness Research?," Journal of Economic Literature, 40, pp. 402-435.

[33] Fong, Christina, and Kevin McCabe, 1999, "Are Decisions under Risk Malleable?," Proceedings of the National Academy of Sciences USA, 96, pp. 10927-10932.

[34] Friedman, Milton, 1962, Price Theory, Chicago, Aldine.

[35] Danthine, Jean-Pierre, John B., Donaldson, Chrisos, Giannikos, and Hany, Guirguis, 2003, "On the Consequences of State Dependent Preferences for the Pricing of Financial Assets," CEPR Discussion Paper #3697.

[36] Gilbert, Daniel T., and Timothy D. Wilson, 2002, "Affective Forecasting," Mimeo, University of Virginia.

[37] Gordon, Stephen, and Pascal St-Amour, 2000, "A preference regime model of bull and bear markets," American Economic Review, 90(4), pp. 1019-33.

[38] Hansen, Lars P., and Ravi Jagannathan, 1991, "Restrictions on Intertemporal Marginal Rates of Substitution Implied by Asset Returns," Journal of Political Economy, 99, pp. 225-262.

[39] Hansen, Lars Peter, Thomas J. Sargent, and Neng Wang, 2002, "Robust Permanent Income and Pricing with Filtering," Macroeconomic Dynamics, 6, pp. 40-84.

[40] Hirshleifer, David, 2001, "Investor Psychology and Asset Pricing", Journal of Finance, 56(4), pp. 1533-1597.

[41] Hirshleifer, David, and Tyler Shumway, 2001, "Good Day Sunshine; Stock Returns and the Weather," Mimeo, Ohio State University.

[42] Inglehart, Ronald, and Hans-Dieter Klingemann, 2000, Genes, Culture and Happiness, Cambridge, MIT Press.

[43] Isen, Alice M., 1999, "Positive Affect and Decision Making," in Lewis M. and J. M. Haviland Eds., Handbook of Emotions, New York, The Guilford Press.

[44] Isen, Alice M., and Nehemia Geva, 1987, "The Influence of Positive Affect on Acceptable Level of Risk: The Person with a Large Canoe has a Large Worry," Organizational Behavior and Human Decision Processes, 39, pp. 145-154.

[45] Isen, Alice M., Thomas E. Nygren, and Gregory F. Ashby, 1988, "Influence of Positive Affect on the Subjective Utility of Gains and Losses: It is Just not Worth the Risk," Journal of Personality and Social Psychology, 55, pp. 710-717.

[46] Isen, Alice M., and Robert Patrick, 1983, "The Effect of Positive Feelings on Risk-Taking: When the Chips are Down," Organizational Behavior and Human Performance, 31, pp. 194-202.

[47] Isen Alice, A. Pratkanis, Paul Slovic, and L. Slovic, 1984, "The Influence of Positive Affect on Risk Preference," Mimeo, 92nd Annual Meeting of the American Psychological Association, Toronto, Canada.

[48] Johnson, E. J., and Amos Tversky, 1983, "Affect, Generalization, and the Perception of Risk," Journal of Personality and Social Psychology, 45, pp. 20-31.

[49] Kagel, John, 1995, "Auctions: A Survey of Experimental Research," in J. H. Kagel and A. E. Roth Eds., The Handbook of Experimental Economics, Princeton, N.J., Princeton University Press.

[50] Kamstra, Mark J., Lisa A. Kramer, and Maurice D. Levi, 2000, "Losing Sleep at the Market: The Daylight-Savings Anomaly," American Economic Review, 90, pp. 1005-1011.

[51] Kamstra, Mark J., Lisa A. Kramer, and Maurice D. Levi, 2001, "Winter Blues: Seasonal Affective Disorder (SAD), the January Effect, and Stock Market Returns," Mimeo, University of British Columbia.

[52] Kandel, Shmuel, and Robert F. Stambaugh, 1991, "Asset Returns and Intertemporal Preferences," Journal of Monetary Economics, 27, pp. 37-91.

[53] Karni, Edi, 1985, Decision Making Under Uncertainty, Harvard University Press, Cambridge, MA.

[54] Kocherlakota, Narayana, 1996, "The Equity Premium: It's Still a Puzzle," Journal of Economic Literature, 34, pp. 42-71.

[55] Koszegi, Botond, 2002, "Anticipation in Observable Behavior," Mimeo, University of California Berkeley.

[56] Kurz, Mordecai, 1968, "Optimal Economic Growth and Wealth Effects," International Economic Review, 9, pp. 348-357.

[57] Laibson, David, 2001, "A Cue-Theory of Consumption," Quarterly Journal of Economics, 116 (1), pp. 54-75.

[58] Larsen, R.J., and E. Diener, 1985, "A Multitrait-Multimethod Examination of Affect Structure: Hedonic Level and Emotional Intensity," Personality and Individual Differences, 6, pp. 631-636.

[59] Lo, Andrew W., and Dmitry V. Repin, 2001, "The Psychophysiology of Real-Time Financial Risk Processing," NBER Working Paper No. w8508.

[60] Loewenstein, George F., 2000, "Emotions in Economic Theory and Economic Behavior," American Economic Review, 90, pp. 426-432.

[61] Loewenstein, George F., Elke U. Weber, Christopher K. Hsee, and Edward S. Welch, 2001, "Risk as Feelings," Psychological Bulletin, 127(2), pp.

[62] Loewenstein, George F., and Jennifer S. Lerner, 2001, "The Role of Affect in Decision Making," in Davidson R., Klaus Scherer and H. Goldsmith Eds., Handbook of Affective Science, New York, Oxford University Press.

[63] Lucas, Robert E. Jr., 1978, "Asset Pricing in an Exchange Economy," Econometrica, 46, pp. 1429-1445.

[64] Lucas, Robert E. Jr., 1987, Models of Business Cycles, Basil Blackwell, Cambridge, Mass.

[65] Marcet, Albert and David Marshall, 2002, "Convergence of Approximate Model Solutions to Rational Expectations Equilibria Using the Method of Parameterized Expectations," forthcoming, Journal of Economic Dynamics and Control.

[66] Mehra, Rajnish and Prescott Edward, 1985, "The Equity Premium: a Puzzle," Journal of Monetary Economics, 15, pp. 145 - 161.

[67] Mehra, Rajnish, and Raaj Sah, 2002, "Mood Fluctuations, Projection Bias, and Volatility of Equity Prices," forthcoming, Journal of Economic Dynamics and Control.

[68] Melino, Angelo, and Alan X., Yang, 2003, "State Dependent Preferences can explain the Equity Premium Puzzle," forthcoming, Review of Economic Dynamics.

[69] Milnor J., 1997, Topology from the Differentiable Viewpoint, Princeton University Press, Princeton, 2nd edition.

[70] Myerson, Roger B., 1991, Game Theory: Analysis of Conflict, Harvard University Press, Cambridge, MA.

[71] Nygren, Thomas E., Alice M. Isen, Pamela J. Taylor, and Jessica Dulin, 1996, "The Influence of Positive Affect on the Decision Rule in Risk Situations: Focus on Outcome rather than Probability," Organizational Behavior and Human Decision Processes, 66, pp. 59-72.

[72] Pigou, Arthur Cecil, 1947, "Economic Progress in a Stable Environment," Economica, 14, pp. 180-188.

[73] Samuelson, Paul A., 1937, "A Note on the Measurement of Utility," Review of Economic Studies, 4, pp. 155-161.

[74] Saunders, E. M. J., 1993, "Stock Prices and Wall Street Weather," American Economic Review, 83, pp. 1337-1345.

[75] Shleifer, Andrei, 2000, Inefficient Markets: An Introduction to Behavioral Finance, Oxford University Press.

[76] Slovic, Paul, Melissa L. Finucane, Ellen Peters, and Donald G. MacGregor, 2002, "The Affect Heuristic," in Gilovich, T., D. Griffin, and D. Kahneman Eds., Intuitive Judgment: Heuristics and Biases, Cambridge University Press.

[77] Smith, Adam, 1759, Theory of Moral Sentiments, London, George Bell & Sons.

[78] Sundaresan, Suresh M., 1989, "Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth," Review of Financial Studies, 2, pp. 73-88.

[79] Tallarini, Thomas, 2000, "Risk-Sensitive Real Business Cycles," Journal of Monetary Economics, 45(3), pp. 507-532.

[80] Thaler, Richard, Ed., 1992, Advances in Behavioral Finance, Russell Sage Foundation.

[81] Weil, Philippe, 1989, "The Equity Premium Puzzle and the Riskfree Rate Puzzle," Journal of Monetary Economics, 24, pp. 401-421.

[82] Zajonc, Robert B., 1998, "Emotions," in Gilbert, Daniel, Susan T. Fiske, and Gardner Lindzey Eds., The Handbook of Social Psychology, 4th Edition, Oxford University Press, New York, pp. 591-632.

[83] Zhou, Heng-fu, 1994, "The Spirit of Capitalism and Long-Run Growth," European Journal of Political Economy, 10, pp. 279-293.

[84] Zin, S. E., 2001, "Are Behavioral Asset-Pricing Models Structural?," Journal of Monetary Economics, 49, pp. 215-228.

5 Appendix A: derivations and proofs

5.1 Axioms and representation theorems for state-dependent utility

Technically, the specification chosen for the affect-dependent utility belongs to the wider class of state-dependent utility functions. The structure of the preferences underlying state-dependent utility functions is relatively well understood. Karni (1985) and more recently Dreze and Rustichini (2001) present a thorough analysis of alternative axiomatizations. I follow Myerson (1991) and give a list of axioms and a representation theorem for state-dependent preferences of the type informally illustrated in the text.

5.1.1 Notation

For any finite set Z, let $\Delta(Z)$ denote the set of probability distributions over Z. That is, define:

$$\Delta(Z) = \left\{ q : Z \to R \mid \sum_{y \in Z} q(y) = 1 \text{ and } q(z) \geq 0, \forall z \in Z \right\}$$

Let X denote the set of possible prizes that the decision maker could ultimately get, Ω denote the set of possible states of the world, and assume both X and Ω are finite. Define a lottery to be any function f that specifies a nonnegative real number $f(x \mid t)$, for every prize x in X and every state t in Ω, such that $\sum_{x \in X} f(x \mid t) = 1$ for every t in Ω.

Let L denote the set of all such lotteries. That is,

$$L = \{ f : \Omega \to \Delta(X) \}$$

For any state t in Ω and any lottery f in L, $f(\cdot \mid t)$ denotes the probability distribution over X designated by f in state t. That is,

$$f(\cdot \mid t) = \{ f(x \mid t) \}_{x \in X} \in \Delta(X)$$

Let Ξ denote the set of all events, S, so that

$$\Xi = \{ S \mid S \subseteq \Omega \text{ and } S \neq \emptyset \}$$

For any two lotteries f and g in L and any event S in Ξ, we write $f \succsim_S g$ if and only if (iff) the lottery f would be at least as desirable as g, in the opinion of the decision-maker, if he knew that the true state of the world was in the set S. In other words, $f \succsim_S g$ iff the decision-maker would be willing to choose the lottery f when he has to choose between f and g and he knows only that the event S has occurred. Given the relation $\succsim_S$,we can define

$$\begin{aligned} f &\succ _S g \text{ iff } f \succsim_S g \text{ and } g \succsim_S f \\ f &\sim _S g \text{ iff } f \succsim_S g \text{ and } g \not\sim_S f \end{aligned}$$

where $f \succ_S g$ and $f \sim_S g$ have the customary meanings of (conditional) indifference and (conditional) strict preference. Naturally, $\succsim_\Omega$, $\succ_\Omega$ and $\sim_\Omega$correspond to the familiar $\succsim, \succ$ and $\sim$, that is when no conditioning event is considered, we refer to prior preferences.

For any number α such that $0 \leq \alpha \leq 1$, and for any two lotteries f and g in L, $\alpha f + (1 - \alpha) g$ denotes the lottery in L such that

$$(\alpha f + (1 - \alpha) g)(x \mid t) = \alpha f (x \mid t) + (1 - \alpha) g (x \mid t)$$

for all $x \in X$ and $t \in \Omega$.

Finally, a conditional-probability function on Ω is any function $p : \Xi \to \Delta(\Omega)$ that specifies nonnegative conditional probabilities $p(t \mid S)$ for every state t in Ω and every event S, such that $p(t \mid S) = 0$ if $t \notin S$ and $\sum_{r \in S} p(r \mid S) = 1$.

5.1.2 Axioms

The axioms are to hold for all lotteries e, f, g and h in L, for all events S and T in Ξ, and for all numbers α and β between 0 and 1:

Axiom 2 (Completeness) *$f \succsim_S g$ or $g \succsim_S f$.*

Axiom 3 (Transitivity) *If $f \succsim_S g$ and $g \succsim_S h$, then $f \succsim_S h$.*

Axiom 4 (Relevance) *If $f(\cdot \mid t) = g(\cdot \mid t)$, $\forall t \in S$, then $f \sim_S g$.*

Axiom 5 (Monotonicity) *If $f \succ_S h$ and $0 \leq \beta < \alpha \leq 1$, then $\alpha f + (1 - \alpha) h \succ_S \beta f + (1 - \beta) h$.*

Axiom 6 (Continuity) *If $f \succsim_S g$ and $g \succsim_S h$, then there exists some number γ such that $0 \leq \gamma \leq 1$ and $g \sim_S \gamma f + (1 - \gamma) h$.*

Axiom 7 ((Strict) objective substitution) *If $e (\succ_S) \succsim_S f$ and $g \succsim_S h$ and $0 (<) \leq \alpha \leq 1$, then $\alpha e + (1 - \alpha) g (\succ_S) \succsim_S \alpha f + (1 - \alpha) h$.*

Axiom 8 ((Strict) subjective substitution) *If $f (\succ_S) \succsim_S g$ and $f \succsim_T g$ and $S \cap T = \varnothing$, then $f (\succ_{S \cup T}) \succsim_{S \cup T} g$.*

Axiom 9 (Interest) *For every state t in Ω, there exist prizes y and z in X such that $[y] \succ_{\{t\}} [x]$, where $[\cdot]$ denotes the lottery that always gives the prize for sure.*

5.1.3 A representation theorem

A utility function can be any function from $X \times \Omega$ into the real numbers, $\Re$. A utility function is state-independent iff there exists some function $U : X \rightarrow \Re$, such that $u(x,t) = U(x)$, for all x and t.

Theorem 10 *The eight axioms are jointly satisfied if and only if there exists a utility function $u : X \times \Omega \rightarrow \Re$ and a conditional-probability function $p : \Xi \rightarrow \Delta(\Omega)$ such that:*

$$\begin{aligned}
\max_{x \in X} u(x,t) &= 1 \text{ and } \min_{x \in X} u(x,t) = 0, \forall t \in \Omega; \\
p(R \mid T) &= p(R \mid S)\, p(S \mid T), \\
\forall R, \forall S, \forall T &: R \subseteq S \subseteq T \subseteq \Omega, S \neq \emptyset; \\
f &\succsim {}_S g \text{ iff } E_p[u(f) \mid S] \geq E_p[u(g) \mid S], \\
\forall f, g &\in L, \forall S \in \Xi,
\end{aligned}$$

where $E_p[u(f) \mid S] = \sum_{t \in S} p(t \mid S) \sum_{x \in X} u(x,t) f(x \mid t)$ is the expected utility value of the prize determined by f, when $p(\cdot \mid S)$ is the probability distribution for the true state of the world.

Proof. *see Myerson (1991).* ■

5.1.4 Discussion and caveats

Axiom 11 (State neutrality) *For any two states r and t in Ω, if $f(\cdot \mid t) = f(\cdot \mid t)$ and $g(\cdot \mid t) = g(\cdot \mid t)$ and $f \succsim_r g$, then $f \succsim_t g$.*

Theorem 12 *Given the axioms above, state neutrality is also satisfied if and only if the conditions of the representation theorem can be satisfied with a state-independent utility function.*

Proof. *see Myerson (1991).* ■

5.2 Definition, existence and uniqueness of the equilibrium solution

This Appendix provides a more formal definition of equilibrium for an exchange economy populated by investors with preferences defined as in (3). It also contains a proof that such equilibrium exists.

5.2.1 Definition of equilibrium

Equilibrium is defined by a pair of functions, $p : \Re^+ \rightarrow \Re^+$, the asset pricing function, and $v(s, y, \theta; p)$, a value function, such that:

1. $v : \Re^{+} \times \Re^{+} \rightarrow \Re^{+}; v(s, y, \theta; p(\circ)) = \max E_0 \left[\sum_{t=0}^{\infty} \beta^t u(g_t)\right]$, subject to $c_t + p_t s_{t+1} \leq s_t(p_t + y_t)$ given $F(\cdot)$, $s_0 = \widehat{s}_0 < 1, y_0, \theta_0$.

2. $s_{t+1} = s_t = 1$, $c_t = y_t$.

5.2.2 Existence of equilibrium

This section proves the existence of a (bounded and strictly positive) equilibrium price-dividend function for probability structure (14). The main complication in establishing existence derives from the endogeneity of the pricing kernel induced by the dependence of the utility function on wealth and, in equilibrium, on the price-dividend function. Such endogeneity prevents us from characterizing the Euler equation as a non-linear counterpart of the linear Fredholm equations much studied in the consumption-based asset pricing tradition.

Under the assumed probability structure, the Euler equation (11) defines the following system of two non-linear equations in two unknown price-dividend ratio functions:

$$\begin{aligned} f_1 (f_1+1)^{-a_1} - \beta \left[\begin{array}{c} \pi_{11}\lambda_1^{1-\alpha}\left(1 - \frac{a_1}{1-\alpha}\frac{1}{f_1+1}\right)(f_1+1)^{1-a_1} \\ +\pi_{12}\lambda_2^{1-\alpha}\left(1 - \frac{a_2}{1-\alpha}\frac{1}{f_2+1}\right)(f_2+1)^{1-a_2} \end{array} \right] &= 0 \\ f_2 (f_2+1)^{-a_2} - \beta \left[\begin{array}{c} \pi_{21}\lambda_1^{1-\alpha}\left(1 - \frac{a_1}{1-\alpha}\frac{1}{f_1+1}\right)(f_1+1)^{1-a_1} \\ +\pi_{22}\lambda_2^{1-\alpha}\left(1 - \frac{a_2}{1-\alpha}\frac{1}{f_2+1}\right)(f_2+1)^{1-a_2} \end{array} \right] &= 0 \end{aligned} \quad (16)$$

where

$$\left[\begin{array}{cc} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{array} \right] \equiv \left[\begin{array}{cc} \pi & 1-\pi \\ 1-\pi & \pi \end{array} \right]$$

To simplify notation, we notice that (16) can be rewritten as

$$\begin{aligned} f_1 (f_1+1)^{-a_1} - \beta \left[\begin{array}{c} \pi_{11}\lambda_1^{1-\alpha}(f_1+1+\gamma_1)(f_1+1)^{-a_1} \\ +\pi_{12}\lambda_2^{1-\alpha}(f_2+1+\gamma_2)(f_2+1)^{-a_2} \end{array} \right] &= 0 \\ f_2 (f_2+1)^{-a_2} - \beta \left[\begin{array}{c} \pi_{21}\lambda_1^{1-\alpha}(f_1+1+\gamma_1)(f_1+1)^{-a_1} \\ +\pi_{22}\lambda_2^{1-\alpha}(f_2+1+\gamma_2)(f_2+1)^{-a_2} \end{array} \right] &= 0 \end{aligned}$$

$$\begin{aligned} f_1 - \beta \left[\pi_{11}\lambda_1^{1-\alpha}(f_1+1+\gamma_1) + \pi_{12}\lambda_2^{1-\alpha}(f_2+1+\gamma_2)\frac{(f_2+1)^{-a_2}}{(f_1+1)^{-a_1}}\right] &= 0 \\ f_2 - \beta \left[\pi_{21}\lambda_1^{1-\alpha}(f_1+1+\gamma_1)\frac{(f_1+1)^{-a_1}}{(f_2+1)^{-a_2}} + \pi_{22}\lambda_2^{1-\alpha}(f_2+1+\gamma_2)\right] &= 0 \end{aligned}$$

$$\begin{aligned} x_1 - \beta\left[\pi_{11}\lambda_1^{1-\alpha}(x_1+\gamma_1) + \pi_{12}\lambda_2^{1-\alpha}(x_2+\gamma_2)\frac{x_2^{-a_2}}{x_1^{-a_1}}\right] - 1 &= 0 \\ x_2 - \beta\left[\pi_{21}\lambda_1^{1-\alpha}(x_1+\gamma_1)\frac{x_1^{-a_1}}{x_2^{-a_2}} + \pi_{22}\lambda_2^{1-\alpha}(x_2+\gamma_2)\right] - 1 &= 0 \end{aligned}$$

$$\begin{aligned} x_1 - \beta\left[\pi_{11}\lambda_1^{1-\alpha}(x_1+\gamma_1) + (1-\pi_{11})\lambda_2^{1-\alpha}(x_2+\gamma_2)\frac{x_1^{a_1}}{x_2^{a_2}}\right] - 1 &= 0 \\ x_2 - \beta\left[\pi_{21}\lambda_1^{1-\alpha}(x_1+\gamma_1)\frac{x_2^{a_2}}{x_1^{a_1}} + (1-\pi_{21})\lambda_2^{1-\alpha}(x_2+\gamma_2)\right] - 1 &= 0 \end{aligned}$$

where $x_1 \equiv f_1 + 1$ and $x_2 \equiv f_2 + 1$. We can than denote (16) as

$$G(x,\phi) = \left[\begin{array}{c} G_1(x,\pi_1) \\ G_2(x,\pi_2) \end{array}\right] = 0$$

where $x \equiv (x_1, x_2)$, $\phi \equiv (\pi_1, \pi_2)$, $\pi_1 \equiv (\pi_{11}, \pi_{12})$, $\pi_2 \equiv (\pi_{21}, \pi_{22})$.

We resort to a fixed point argument (see Milnor (1997) for a detailed treatment) to show that a solution to G exists.

It is understood that all parameters other than the probabilities are taken as given. Let

$$\zeta = \{(x,\phi) \mid G(x,\phi) = 0\} \subset \mathbb{R}^2 \times (\Delta)^2$$

We start by proving the following

Lemma 13 *ζ is a smooth manifold.*

Proof. *By perturbing G with respect to π_1, we need to show that, for an arbitrarily fixed open and full Lebesgue set of parameter values $(\beta, \lambda_1, \lambda_2, \alpha, a)$, the Jacobian of the map G with respect to π and x, $D_{\pi,x}G$, has full rank. To this end we study the Jacobian of the map G with respect to π and x. By definition, we have*

$$D_{\pi,x}G = \left[\begin{array}{cc} \alpha(\lambda_1) - \alpha(\lambda_2) & 0 \\ 0 & \hat{\alpha}(\lambda_1) - \hat{\alpha}(\lambda_2) \end{array}\right]$$

where we define $\alpha(\lambda_1) \equiv \lambda_1^{1-\alpha}(x_1+\gamma_1)$, $\alpha(\lambda_2) \equiv \lambda_2^{1-\alpha}(x_2+\gamma_2)\frac{x_1^{a_1}}{x_2^{a_2}}$, $\hat{\alpha}(\lambda_1) \equiv \lambda_1^{1-\alpha}(x_1+\gamma_1)\frac{x_2^{a_2}}{x_1^{a_1}}$, $\hat{\alpha}(\lambda_2) \equiv \lambda_2^{1-\alpha}(x_2+\gamma_2)$.

Evidently, $D_{\pi,x}G$ is onto if $\alpha(\lambda_1) - \alpha(\lambda_2) \neq 0$ (or, equivalently, $\alpha(\lambda_1) \neq \alpha(\lambda_2)$) and $\hat{\alpha}(\lambda_1) - \hat{\alpha}(\lambda_2) \neq 0$ (or, equivalently, $\hat{\alpha}(\lambda_1) \neq \hat{\alpha}(\lambda_2)$). However,

$$\alpha(\lambda_1) = \alpha(\lambda_2) \Leftrightarrow \hat{\alpha}(\lambda_1) = \hat{\alpha}(\lambda_2)$$

Suppose, then, that these equalities hold. (16) *simplifies to*

$$\begin{aligned} x_1 &= \beta\lambda_1^{1-\alpha}\left(x_1+\gamma_1\right)+1 \\ x_2 &= \beta\lambda_1^{1-\alpha}\left(x_1+\gamma_1\right)\frac{x_2^{a_2}}{x_1^{a_1}}+1 \end{aligned}$$

By taking the ratio, we obtain

$$\frac{x_1-1}{x_2-1}=\frac{x_1^{a_1}}{x_2^{a_2}} \tag{17}$$

We need to verify the existence of an open and full Lebesgue measure set of parameter values such that

$$\frac{x_1-1}{x_2-1}\neq\frac{x_1^{a_1}}{x_2^{a_2}}$$

When $\pi_{11}=1$ *and* $\pi_{21}=0$ (16) *simplifies to*

$$\begin{aligned} x_1-1 &= \beta\lambda_1^{1-\alpha}x_1+\beta\lambda_1^{1-\alpha}\gamma_1 \\ x_2-1 &= \beta\lambda_2^{1-\alpha}x_2+\beta\lambda_2^{1-\alpha}\gamma_2 \end{aligned}$$

$$\begin{aligned} x_1-\beta\lambda_1^{1-\alpha}x_1 &= \beta\lambda_1^{1-\alpha}\gamma_1+1 \\ x_2-\beta\lambda_2^{1-\alpha}x_2 &= \beta\lambda_2^{1-\alpha}\gamma_2+1 \end{aligned}$$

$$\begin{aligned} x_i\left(1-\beta\lambda_i^{1-\alpha}\right) &= \beta\lambda_i^{1-\alpha}\gamma_i+1 \\ x_i &= \frac{\beta\lambda_i^{1-\alpha}\gamma_i+1}{1-\beta\lambda_i^{1-\alpha}} \end{aligned}$$

$$\begin{aligned} x_1 &= \frac{\beta\lambda_1^{1-\alpha}\gamma_1+1}{1-\beta\lambda_1^{1-\alpha}}\equiv x_1^* \\ x_2 &= \frac{\beta\lambda_2^{1-\alpha}\gamma_2+1}{1-\beta\lambda_2^{1-\alpha}}\equiv x_2^* \end{aligned}$$

and

$$\begin{aligned} x_1^*-1 &= \frac{\beta\lambda_1^{1-\alpha}\left(1+\gamma_1\right)}{1-\beta\lambda_1^{1-\alpha}} \\ x_2^*-1 &= \frac{\beta\lambda_2^{1-\alpha}\left(1+\gamma_2\right)}{1-\beta\lambda_2^{1-\alpha}} \end{aligned}$$

Thus

$$\frac{x_1^*-1}{x_2^*-1}=\frac{\beta\lambda_1^{1-\alpha}\left(1+\gamma_1\right)\left(1-\beta\lambda_2^{1-\alpha}\right)}{\beta\lambda_2^{1-\alpha}\left(1+\gamma_2\right)\left(1-\beta\lambda_1^{1-\alpha}\right)}$$

and

$$\frac{\left(x_1^*\right)^{a_1}}{\left(x_2^*\right)^{a_2}}=\left(\frac{\beta\lambda_1^{1-\alpha}\left(1+\gamma_1\right)}{1-\beta\lambda_1^{1-\alpha}}\right)^{a_1}\left(\frac{1-\beta\lambda_2^{1-\alpha}}{\beta\lambda_2^{1-\alpha}\left(1+\gamma_2\right)}\right)^{a_2}$$

Consider now the function

$$H\left(x,a\right)=\frac{x_1^*-1}{x_2^*-1}-\frac{\left(x_1^*\right)^{a_1}}{\left(x_2^*\right)^{a_2}}$$

It is straightforward to show that $\frac{\partial H}{\partial a}\neq 0$. *In fact, we have*

$$\begin{aligned}\frac{\partial H}{\partial a}&=-\frac{\left(x_1^*\right)^{a_1}}{\left(x_2^*\right)^{a_2}}\log\frac{\left(x_1^*\right)^{\lambda_1}}{\left(x_2^*\right)^{\lambda_2}}\\&=-\frac{\left(x_1^*\right)^{a_1}}{\left(x_2^*\right)^{a_2}}\left(\lambda_1\log x_1^*-\lambda_2\log x_2^*\right)\end{aligned}$$

Clearly, there exists an open and full Lebesgue measure set of parameter values $(\beta,\lambda_1,\lambda_2,\alpha,a)$ *such that* $\frac{\partial H}{\partial a}\neq 0$ *or, equivalently,* ζ *is a smooth manifold.* ■

Lemma 14 *There exists a regular value of the map* $proj\left(\zeta\right)\rightarrow\Delta$, π^* *such that* $\#\left[proj^{-1}\left(\pi^*\right)\right]=odd$.

Proof. *Fix* $\pi_{11}=1$ *and* $\pi_{21}=0$. *Then* (16) *simplifies to a system of two linear equations*

$$\begin{aligned}x_1-1&=\beta\lambda_1^{1-\alpha}x_1+\beta\lambda_1^{1-\alpha}\gamma_1\\x_2-1&=\beta\lambda_2^{1-\alpha}x_2+\beta\lambda_2^{1-\alpha}\gamma_2\end{aligned}$$

$$\begin{aligned}x_1&=\frac{\beta\lambda_1^{1-\alpha}\left(1+\gamma_1\right)}{1-\beta\lambda_1^{1-\alpha}}+1\\x_2&=\frac{\beta\lambda_2^{1-\alpha}\left(1+\gamma_2\right)}{1-\beta\lambda_2^{1-\alpha}}+1\end{aligned}$$

Clearly, the solution is unique. ■

Lemma 15 *The map* $proj\left(\zeta\right)\rightarrow\Delta$ *is proper, that is* $proj^{-1}\left(\pi\right)$ *is compact for each compact subset of probability.*

Proof. *It suffices to show that* $1<proj^{-1}\left(\pi\right)<\infty$.

Suppose that (without loss of generality) $x_1 = 1$. Then

$$\beta\left[\pi_{11}\lambda_1^{1-\alpha}\left(1+\gamma_1\right)+\left(1-\pi_{11}\right)\lambda_2^{1-\alpha}\left(x_2+\gamma_2\right)\frac{1}{x_2^{a_2}}\right]=0$$

which is obviously impossible.

Suppose, by contradiction, that $\exists \pi^n \to \pi^h$ such that $\left\|x^h\right\| \equiv \left\|x\left(\pi^h\right)\right\| \to \infty$. We distinguish two cases (∞ is symmetric to zero and therefore ignored):

1. $\frac{\left(x_1^h\right)^{a_1}}{\left(x_2^h\right)^{a_2}} \to K > 0$

   The second equation becomes

$$1=\beta\left[\pi_{21}\lambda_1^{1-\alpha}\left(\frac{x_1^h}{x_2^h}+\frac{\gamma_1}{x_2^h}\right)\frac{\left(x_2^h\right)^{a_2}}{\left(x_1^h\right)^{a_1}}+\left(1-\pi_{21}\right)\lambda_2^{1-\alpha}\left(1+\frac{\gamma_2}{x_2^h}\right)\right]+\frac{1}{x_2^h}$$

   thus in the limit

$$1=\beta\left[\pi_{21}\lambda_1^{1-\alpha}\lim_{h\to\infty}\left(\frac{x_1^h}{x_2^h}\right)K^{-1}+\left(1-\pi_{21}\right)\lambda_2^{1-\alpha}\right]$$

   The first equation (again dividing by x_1 and taking the limit) becomes

$$1=\beta\left[\pi_{11}\lambda_1^{1-\alpha}\left(1+\frac{\gamma_1}{x_1^h}\right)+\left(1-\pi_{11}\right)\lambda_2^{1-\alpha}\left(\frac{x_2^h}{x_1^h}+\frac{\gamma_2}{x_1^h}\right)\frac{\left(x_1^h\right)^{a_1}}{\left(x_2^h\right)^{a_2}}\right]+\frac{1}{x_1^h}$$
$$1=\beta\left[\pi_{11}\lambda_1^{1-\alpha}+\left(1-\pi_{11}\right)\lambda_2^{1-\alpha}\lim_{h\to\infty}\left(\frac{x_2^h}{x_1^h}\right)K\right]$$

   Since $\left\|x^h\right\| \to \infty$ and $\frac{\left(x_1^h\right)^{a_1}}{\left(x_2^h\right)^{a_2}} \to K > 0$, then $x_1^h \to \infty$ and $x_2^h \to \infty$. Since $a_1 \neq a_2$, either $\lim_{h\to\infty}\left(\frac{x_2^h}{x_1^h}\right) = \infty$ or $\lim_{h\to\infty}\left(\frac{x_1^h}{x_2^h}\right) = \infty$. Hence, $\frac{\left(x_1^h\right)^{a_1}}{\left(x_2^h\right)^{a_2}} \to K > 0$ is impossible.

2. $\frac{\left(x_1^h\right)^{a_1}}{\left(x_2^h\right)^{a_2}} \to 0$

   By repeating the same procedure, we have

$$1=\beta\left[\pi_{11}\lambda_1^{1-\alpha}+\left(1-\pi_{11}\right)\lambda_2^{1-\alpha}\lim_{h\to\infty}\frac{\left(x_1^h\right)^{a_1-1}}{\left(x_2^h\right)^{a_2-1}}\right]$$
$$1=\beta\left[\pi_{21}\lambda_1^{1-\alpha}\lim_{h\to\infty}\frac{\left(x_2^h\right)^{a_2-1}}{\left(x_1^h\right)^{a_1-1}}+\left(1-\pi_{21}\right)\lambda_2^{1-\alpha}\right]$$

If $\frac{\left(x_1^h\right)^{a_1}}{\left(x_2^h\right)^{a_2}} \to 0$, *then either* $\lim_{h\to\infty} \frac{\left(x_1^h\right)^{a_1-1}}{\left(x_2^h\right)^{a_2-1}} = 0$ *or* $\lim_{h\to\infty} \frac{\left(x_1^h\right)^{a_1-1}}{\left(x_2^h\right)^{a_2-1}} = \infty$. *Hence* $\frac{\left(x_1^h\right)^{a_1}}{\left(x_2^h\right)^{a_2}} \to 0$ *is impossible.*

■

We are now in a position to state the following

Theorem 16 *There exists a bounded and strictly positive equilibrium price-dividend function for probability structure* (14)

Proof. *The statement follows directly from Lemmas 13-15.* ■

5.3 Details of the derivations in the text

5.3.1 Hedonic Relative Risk Aversion

Consider the a-temporal case where the outcome $l \in L$ is independent of the preference state $s \in S$, with probabilities given by P_l and P_s respectively. It is straightforward to derive the hedonic risk aversion of the investor by using the definition of relative risk aversion. In fact,

$$\begin{aligned} u(g) &= EU(C, W/C, S) \\ &= \sum_{l\in L}\sum_{s\in S} P_l P_s C_l^{1-\alpha} \frac{W/C_l^{(1-\alpha)\theta_s}}{1-\alpha} \\ &= \sum_{l\in L} P_l \mathcal{V}(W/C_l) \end{aligned}$$

where $\mathcal{V}(W/C_l) = \sum_{s\in S} P_s C_l^{1-\alpha} \frac{W/C_l^{(1-\alpha)\theta_s}}{1-\alpha}$ is the state independent utility function, that is a linear combination with positive weights of conditionally isoelastic concave functions, and thus concave. Moreover, given that S and L are orthogonal, the curvature of $\mathcal{V}(W/C_l)$ captures the investors' attitude toward atemporal risk. Hence, the Arrow-Pratt coefficient of relative risk aversion with respect to (wealth relative to consumption) lotteries on L is

$$RRA_{W/C} = -W/C \frac{\mathcal{V}_{WW}}{\mathcal{V}_w}$$

$$
\begin{aligned}
\mathcal{V}_W &= \sum_{s\in S} P_s C_l^{1-\alpha}\theta_s W/C_l^{(1-\alpha)\theta_s-1} \\
\mathcal{V}_{WW} &= \sum_{s\in S} P_s C_l^{1-\alpha}\theta_s\left(\left(1-\alpha\right)\theta_s-1\right)W/C_l^{(1-\alpha)\theta_s-2} \\
RRA_W &= -W/C\frac{\sum_{s\in S} P_s C_l^{1-\alpha}\theta_s\left(\left(1-\alpha\right)\theta_s-1\right)W/C_l^{(1-\alpha)\theta_s-2}}{\sum_{s\in S} P_s C_l^{1-\alpha}\theta_s W/C_l^{(1-\alpha)\theta_s-1}} \\
&= -\sum_{s\in S}\left(\left(1-\alpha\right)\theta_s-1\right) \\
&= \sum_{s\in S}\left(\left(\alpha-1\right)\theta_s+1\right)
\end{aligned}
$$

If preferences are state-independent, i.e. $\theta_s = \theta\ \forall s$, then the coefficient of relative risk aversion is constant and equal to $(\alpha-1)\theta+1$. The coefficient of relative risk aversion for lotteries that are conditional on the realization of a given state s is $(\alpha-1)\theta_s+1$. Since each period is associated with a single preference state, $(\alpha-1)\theta_t+1$ can be interpreted as the coefficient of relative risk aversion for static lotteries.

5.3.2 Intertemporal elasticity of substitution

Recall the Euler

$$
1 = \beta E\left[g_c^{(1-\alpha)(1-\theta_t)-1} g_w^{(1-\alpha)\theta_t} k_{t+1}\left(\frac{c_{t+1}}{w_{t+1}}\right)^{(1-\alpha)(\theta_t-\theta_{t+1})} R_{t+1}\right]
$$

where $g_c = \frac{c_{t+1}}{c_t}$ and $g_w = \frac{w_{t+1}}{w_t}$. and $k_{t+1} \equiv \left(\frac{1-\theta_{t+1}}{1-\theta_t}\right)\left(1+\frac{\theta_{t+1}}{1-\theta_{t+1}}\frac{c_{t+1}}{w_{t+1}}\right)$ Along a balanced growth path with constant interest rates we have $g_c = g_w = g$ and the Euler becomes

$$
\begin{aligned}
1 &= \beta\left(g^{-\alpha}+\frac{\theta}{1-\theta}g^{-\alpha}\frac{c}{w}\right)(1+r) \\
1 &= \beta g^{-\alpha}\left(1+\frac{\theta}{1-\theta}\frac{c}{w}\right)(1+r)
\end{aligned}
$$

It is straightforward to observe that if we take the term $\frac{c}{w}$ as exogenous and ignore the dependence of wealth on returns, then

$$
\frac{dg}{dr} = \frac{1}{\alpha}
$$

Nevertheless, using the investors' budget constraint

$$
\frac{w_{t+1}}{w_t} = R_{t+1}\left(1-\frac{c_t}{w_t}\right)
$$

and the balanced growth path assumption we can write

$$g = (1+r)\left(1 - \frac{c}{w}\right)$$

which provides $\frac{c}{w}$ as the following function of r

$$\frac{c}{w} = 1 - \frac{g}{1+r}$$

substituting for $\frac{c}{w}$ into the Euler we have

$$1 = \beta g^{-\alpha}\left(1 + \frac{\theta}{1-\theta}\left(1 - \frac{g}{1+r}\right)\right)(1+r)$$

Taking logs we have

$$\begin{aligned} 0 &= \log\beta - \alpha\log g + \log\left(1 + \frac{\theta}{1-\theta}\left(1 - \frac{g}{1+r}\right)\right) + \log(1+r) \\ 0 &\approx (\beta - 1) - \alpha(g-1) + \frac{\theta}{1-\theta}\left(1 - \frac{g}{1+r}\right) + r \\ g &= \frac{\beta - 1 + \alpha + \frac{\theta}{1-\theta} + r}{\alpha + \frac{\theta}{1-\theta}\frac{1}{1+r}} \end{aligned}$$

Hence, the elasticity of intertemporal substitution is

$$\begin{aligned} \frac{dg}{dr} &= \frac{\alpha + \frac{\theta}{1-\theta}\frac{1}{1+r} + \left(\beta - 1 + \alpha + \frac{\theta}{1-\theta} + r\right)\frac{\theta}{1-\theta}\frac{1}{(1+r)^2}}{\left(\alpha + \frac{\theta}{1-\theta}\frac{1}{1+r}\right)^2} \\ &= \frac{1}{\alpha + \frac{\theta}{1-\theta}\frac{1}{1+r}}\left[1 + \frac{\theta}{1-\theta}\frac{\beta - 1 + \alpha + \frac{\theta}{1-\theta} + r}{\left(\alpha + \frac{\theta}{1-\theta}\frac{1}{1+r}\right)(1+r)^2}\right] \end{aligned}$$

5.3.3 Euler equations and returns

Starting from (5), using (3) and substituting for c_t into the objective function from the constraint, we can rewrite the problem as:

$$\begin{aligned} V(s_t, y_t, \theta_t) &= \max_{s_{t+1}}\left\{\begin{array}{c} \frac{\left([(p_t+y_t)s_t - p_t s_{t+1}]^{(1-\theta_t)}[(p_t+y_t)s_t]^{\theta_t}\right)^{1-\alpha}}{1-\alpha} \\ +\beta E_t V(s_{t+1}, y_{t+1}, \theta_{t+1}) \end{array}\right\} \\ c_t &> 0, s_{t+1} \in (0,1] \\ & \quad s_0, y_0, \theta_0 \text{ given} \end{aligned}$$

The first order condition with respect to s_{t+1} is

$$\begin{aligned} & \beta E_t V_1(s_{t+1}, y_{t+1}, \theta_{t+1}) \\ = \; & (1-\theta_t)\left[(p_t + y_t)s_t - p_t s_{t+1}\right]^{(1-\alpha)(1-\theta_t)-1}\left[(p_t+y_t)s_t\right]^{(1-\alpha)\theta_t} p_t \end{aligned}$$

The envelope with respect to s_t is

$$V_1(s_t, y_t, \theta_t) = \begin{array}{c} (1-\theta_t)\left[(p_t+y_t)s_t - p_t s_{t+1}\right]^{(1-\alpha)(1-\theta_t)-1}\left[(p_t+y_t)s_t\right]^{(1-\alpha)\theta_t}(p_t+y_t) \\ +\theta_t\left[(p_t+y_t)s_t - p_t s_{t+1}\right]^{(1-\alpha)(1-\theta_t)}\left[(p_t+y_t)s_t\right]^{(1-\alpha)\theta_t-1}(p_t+y_t) \end{array}$$

Hence, substituting back for consumption from the budget constraint and using the definition of w_t, the Euler equation can be written as

$$\begin{aligned}
& (1-\theta_t)\, c_t^{(1-\alpha)(1-\theta_t)-1} w_t^{(1-\alpha)\theta_t} p_t \\
= \; & \beta E_t\left\{\left[\begin{array}{c} (1-\theta_{t+1})\, c_{t+1}^{(1-\alpha)(1-\theta_{t+1})-1} w_{t+1}^{(1-\alpha)\theta_{t+1}} \\ +\theta_{t+1} c_{t+1}^{(1-\alpha)(1-\theta_{t+1})} w_{t+1}^{(1-\alpha)\theta_{t+1}-1} \end{array}\right](p_{t+1}+y_{t+1})\right\} \\
& (1-\theta_t)\, c_t^{(1-\alpha)(1-\theta_t)-1} w_t^{(1-\alpha)\theta_t} p_t \\
= \; & \beta E_t\left\{(1-\theta_{t+1})\, c_{t+1}^{(1-\alpha)(1-\theta_{t+1})-1} w_{t+1}^{(1-\alpha)\theta_{t+1}}\left(1+\frac{\theta_{t+1}}{1-\theta_{t+1}}\frac{c_{t+1}}{w_{t+1}}\right)(p_{t+1}+y_{t+1})\right\}
\end{aligned}$$

Simple algebraic manipulations deliver the Euler equation (9) that appears in the text.

Premium For the expected premium, using the definition of the risk-free rate we have

$$\begin{aligned}
1 \;=\; & \beta E\left[\left(\frac{c_{t+1}}{c_t}\right)^{(1-\alpha)(1-\theta_t)-1}\left(\frac{w_{t+1}}{w_t}\right)^{(1-\alpha)\theta_t} k_{t+1}\left(\frac{c_{t+1}}{w_{t+1}}\right)^{(1-\alpha)(\theta_t-\theta_{t+1})}\right] ER_{t+1} \\
& +cov\left(\left(\frac{c_{t+1}}{c_t}\right)^{(1-\alpha)(1-\theta_t)-1}\left(\frac{w_{t+1}}{w_t}\right)^{(1-\alpha)\theta_t} k_{t+1}\left(\frac{c_{t+1}}{w_{t+1}}\right)^{(1-\alpha)(\theta_t-\theta_{t+1})}, R_{t+1}\right) \\
1 \;=\; & \frac{ER_{t+1}}{R_f} \\
& +cov\left(\left(\frac{c_{t+1}}{c_t}\right)^{(1-\alpha)(1-\theta_t)-1}\left(\frac{w_{t+1}}{w_t}\right)^{(1-\alpha)\theta_t} k_{t+1}\left(\frac{c_{t+1}}{w_{t+1}}\right)^{(1-\alpha)(\theta_t-\theta_{t+1})}, R_{t+1}\right)
\end{aligned}$$

$$\begin{aligned}
R_f &= ER_{t+1} + R_f cov\left(m_{t+1}, R_{t+1}\right) \\
ER_{t+1} - R_f &= R_f cov\left(-m_{t+1}, R_{t+1}\right)
\end{aligned}$$

where $m_{t+1} = \left(\frac{c_{t+1}}{c_t}\right)^{(1-\alpha)(1-\theta_t)-1}\left(\frac{w_{t+1}}{w_t}\right)^{(1-\alpha)\theta_t} k_{t+1}\left(\frac{c_{t+1}}{w_{t+1}}\right)^{(1-\alpha)(\theta_t-\theta_{t+1})}$.

5.3.4 Computation of returns

To write the Euler as a (nonlinear) first order difference equation in the price-dividend ratio, recall

$$1 = \beta E\left[\frac{1-\theta_{t+1}}{1-\theta_t}\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\frac{(w_{t+1}/c_{t+1})^{-a_{t+1}}}{(w_t/c_t)^{-a_t}}\left(1+\frac{\theta_{t+1}}{1-\theta_{t+1}}\frac{c_{t+1}}{w_{t+1}}\right)R_{t+1}\right]$$

Define $f_t = \frac{p_t}{y_t}$ to be the price-dividend ratio and observe that in equilibrium we can write $w_t = (f_t+1)\, y_t$. The Euler, then, becomes

$$1 = \beta E\left[\frac{1-\theta_{t+1}}{1-\theta_t}\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\frac{(f_{t+1}+1)^{-a_{t+1}}}{(f_t+1)^{-a_t}}\left(1+\frac{\theta_{t+1}}{1-\theta_{t+1}}\frac{c_{t+1}}{(f_{t+1}+1)\, y_{t+1}}\right)R_{t+1}\right]$$
$$1 = \beta E\left[\frac{1-\theta_{t+1}}{1-\theta_t}\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\frac{(f_{t+1}+1)^{-a_{t+1}}}{(f_t+1)^{-a_t}}\left(1+\frac{1-\theta_{t+1}}{1-\theta_t}\frac{1}{f_{t+1}+1}\right)R_{t+1}\right]$$

since by definition returns are $R_{t+1} = \frac{p_{t+1}+y_{t+1}}{p_t} = \frac{\left(\frac{p_{t+1}}{y_{t+1}}+1\right)y_{t+1}}{\frac{p_t}{y_t}y_t} = \frac{f_{t+1}+1}{f_t}\frac{y_{t+1}}{y_t}$, we have

$$1 = \beta E\left[\frac{1-\theta_{t+1}}{1-\theta_t}\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\frac{(f_{t+1}+1)^{-a_{t+1}}}{(f_t+1)^{-a_t}}\left(1+\frac{1-\theta_{t+1}}{1-\theta_t}\frac{1}{f_{t+1}+1}\right)\frac{f_{t+1}+1}{f_t}\frac{y_{t+1}}{y_t}\right]$$
$$1 = \beta E\left[\frac{1-\theta_{t+1}}{1-\theta_t}\left(\frac{c_{t+1}}{c_t}\right)^{1-\alpha}\frac{(f_{t+1}+1)^{-a_{t+1}}}{(f_t+1)^{-a_t}}\left(1+\frac{1-\theta_{t+1}}{1-\theta_t}\frac{1}{f_{t+1}+1}\right)\frac{f_{t+1}+1}{f_t}\right]$$

Hence, the Euler can be rewritten as

$$f_t\,(f_t+1)^{-a_t} = \beta E\left[\frac{1-\theta_{t+1}}{1-\theta_t}\left(\frac{c_{t+1}}{c_t}\right)^{1-\alpha}\left(1+\frac{1-\theta_{t+1}}{1-\theta_t}\frac{1}{f_{t+1}+1}\right)(f_{t+1}+1)^{1-a_{t+1}}\right]$$

5.3.5 Welfare costs of aggregate fluctuations

When $\theta_t = 1$, $a_t = \alpha$ and wealth growth is iid lognormal, the instantaneous utility becomes

$$\begin{aligned} u_t &= \frac{\left(c_t^{1-\theta_t} w_t^{\theta_t}\right)^{1-\alpha}}{1-\alpha} \\ &= \frac{w_t^{1-\alpha}}{1-\alpha} \end{aligned}$$

Up to a constant, expected utility is

$$
\begin{aligned}
& \frac{1}{1-\alpha} E \sum_j \beta^j w_{t+j}^{1-\alpha} \\
= \; & \frac{w_t^{1-\alpha}}{1-\alpha} E \sum_j \beta^j \left(\frac{w_{t+j}}{w_t}\right)^{1-\alpha} \\
= \; & \frac{w_t^{1-\alpha}}{1-\alpha} \sum_j \beta^j e^{(1-\alpha)j\left[g_w+(1-\alpha)\frac{\sigma_w^2}{2}\right]} \\
= \; & \frac{w_t^{1-\alpha}}{(1-\alpha)\left[1-\beta e^{(1-\alpha)\left[g_w+(1-\alpha)\frac{\sigma_w^2}{2}\right]}\right]}
\end{aligned}
$$

The log mean wealth growth rate is

$$
\alpha_w = \ln E\left(\frac{w_{t+1}}{w_t}\right) = g_w + \frac{\sigma_w^2}{2}
$$

In terms of α_w, expected utility is

$$
\begin{aligned}
& \frac{w_t^{1-\alpha}}{(1-\alpha)\left[1-\beta e^{(1-\alpha)\left(\alpha_w-\frac{\sigma_w^2}{2}\right)+(1-\alpha)^2\frac{\sigma_w^2}{2}}\right]} \\
= \; & \frac{w_t^{1-\alpha}}{(1-\alpha)\left[1-\beta e^{(1-\alpha)\alpha_w-\alpha(1-\alpha)\frac{\sigma_w^2}{2}}\right]}
\end{aligned}
$$

Define α_w^* as the "certainty equivalent" growth rate of wealth, that is the growth rate in a non-stochastic economy that gives investors the same level of utility. Then

$$
\frac{w_t^{1-\alpha}}{(1-\alpha)\left[1-\beta e^{(1-\alpha)\alpha_w^*}\right]} = \frac{w_t^{1-\alpha}}{(1-\alpha)\left[1-\beta e^{(1-\alpha)\left(\alpha_w-\alpha\frac{\sigma_w^2}{2}\right)}\right]}
$$

which implies

$$
\begin{aligned}
\alpha_w^* &= \left(\alpha_w - \alpha\frac{\sigma_w^2}{2}\right) \\
\Delta\alpha_w &= \alpha\frac{\sigma_w^2}{2}
\end{aligned}
$$

6 Appendix B: Tables and figures

Table 1 - Summary of parameter values

Parameter	Value
β	0.99
α	3
θ_t	$(0, 0.5)$
$\mu = E\left(c_{t+1}/c_t\right)$	0.018
$\delta = \sigma\left(c_{t+1}/c_t\right)$	0.036
π	0.43

Table 2 - Summary of unconditional first and second moments of returns in the benchmark calibration

	US data (MP sample)	No Happiness ($\theta_t = \theta_{t+1} = 0$)	No HM ($\theta_t = \theta_{t+1} = 0.24$)	HM ($\theta = 0.24, n = 2$)
$E\left(R^f\right)$	0.80	5.74	4.64	0.84
$E\left(R^e\right)$	6.98	6.22	5.24	6.95
$E\left(R^{ep}\right)$	6.18	0.48	0.60	6.11
$\sigma\left(R^f\right)$	5.44	1.57	2.20	5.55
$\sigma\left(R^e\right)$	19.02	4.87	5.65	23.17
$\sigma\left(R^{ep}\right)$	18.53	4.60	5.20	22.57
$\frac{E(R^{ep})}{\sigma(R^{ep})}$	0.33	0.10	0.11	0.27

All statistics are annualized and in percent terms. See Table 1 for a summary of the parameter values.

Notation: $E(\cdot)$ denotes unconditional expected value, $\sigma(\cdot)$ denotes unconditional standard deviation; HM abbreviates Happiness Maintenance.

Data sources: Mehra and Prescott (1985), Campbell (1999). Period: 1889-1985.

Implied hedonic risk aversion: $E(a_{t+1}) = 1.5$; $\sigma(a_{t+1}) = 0.009$; $a_{t\max} = 1.52$; $a_{t\min} = 1.48$.

Table 3 - Summary of unconditional first and second moments of returns in the Post-War calibration

	US data (Post-War)	No Happiness ($\theta_t = \theta_{t+1} = 0$)	No HM ($\theta_t = \theta_{t+1} = 0.23$)	HM ($\theta = 0.23, n = 2$)
$E\left(R^f\right)$	1.68	5.74	4.69	1.56
$E\left(R^e\right)$	6.89	6.22	5.28	6.47
$E\left(R^{ep}\right)$	5.36	0.48	0.59	4.91
$\sigma\left(R^f\right)$	2.23	1.57	2.19	5.06
$\sigma\left(R^e\right)$	16.7	4.87	5.63	20.59
$\sigma\left(R^{ep}\right)$	16.8	4.60	5.18	19.98
$\frac{E(R^{ep})}{\sigma(R^{ep})}$	0.32	0.10	0.11	0.25

All statistics are annualized and in percent terms. See Table 1 for a summary of the parameter values.

Notation: $E(\cdot)$ denotes unconditional expected value, $\sigma(\cdot)$ denotes unconditional standard deviation; HM abbreviates Happiness Maintenance.

Data sources: Mehra and Prescott (1985), Campbell (1999). Period: 1947-1985.

Implied hedonic risk aversion: $E(a_{t+1}) = 1.48$; $\sigma(a_{t+1}) = 0.009$; $a_{t\max} = 1.5$; $a_{t\min} = 1.46$.

Table 4.1 - Comparative dynamics: α, θ

		$E\left(R^f\right)$	$E\left(R^e\right)$	$E\left(R^{ep}\right)$	$\sigma\left(R^f\right)$	$\sigma\left(R^e\right)$	$\sigma\left(R^{ep}\right)$	$\frac{E(R^{ep})}{\sigma(R^{ep})}$
	$\alpha = 1.1$	2.36	2.56	0.19	0.71	4.04	3.98	0.05
$\theta = 0.24$	$\alpha = 2$	2.85	3.75	0.90	1.87	8.12	7.90	0.11
	$\alpha = 3$	0.84	6.95	6.11	5.55	23.17	22.57	0.27
	$\alpha = 1.1$	2.48	2.66	0.17	0.68	3.94	3.88	0.04
$\theta = 0.18$	$\alpha = 2$	3.27	3.89	0.63	1.64	6.66	6.45	0.10
	$\alpha = 3$	3.46	5.60	2.15	3.69	12.69	12.14	0.18
	$\alpha = 1.1$	2.60	2.76	0.16	0.63	3.85	3.80	0.04
$\theta = 0.12$	$\alpha = 2$	3.62	4.09	0.46	1.39	5.59	5.41	0.09
	$\alpha = 3$	4.49	5.59	1.09	2.62	8.38	7.95	0.14
	$\alpha = 1.1$	2.72	2.87	0.15	0.60	3.77	3.72	0.04
$\theta = 0.06$	$\alpha = 2$	3.94	4.30	0.36	1.20	4.83	4.67	0.08
	$\alpha = 3$	5.16	5.85	0.69	2.00	6.18	5.85	0.12
	$\alpha = 1.1$	2.85	2.99	0.14	0.56	3.69	3.65	0.04
$\theta = 0.00$	$\alpha = 2$	4.27	4.55	0.29	1.04	4.23	4.10	0.07
	$\alpha = 3$	5.74	6.22	0.48	1.57	4.87	4.60	0.10

All statistics are annualized and in percent terms.

Parameter values: $\beta = 0.99$, $n = 2$.

Notation: $E(\cdot)$ denotes unconditional expected value, $\sigma(\cdot)$ denotes unconditional standard deviation.

Data sources: Mehra and Prescott (1985), Campbell (1999). Period: 1889-1985.

Table 4.2 - Comparative dynamics: θ, n

		$E(R^f)$	$E(R^e)$	$E(R^{ep})$	$\sigma(R^f)$	$\sigma(R^e)$	$\sigma(R^{ep})$	$\frac{E(R^{ep})}{\sigma(R^{ep})}$
$\theta = 0.24$	$n = 0$	3.66	8.73	5.07	19.03	28.60	20.45	0.25
	$n = 2$	0.84	6.95	6.11	5.55	23.17	22.57	0.27
	$n = 4$	3.58	5.33	1.75	3.65	11.31	10.70	0.16
	$n = \infty$	4.58	5.19	0.61	2.54	5.98	5.41	0.11
$\theta = 0.18$	$n = 0$	4.23	6.88	2.63	12.07	18.54	13.78	0.19
	$n = 2$	3.46	5.60	2.15	3.69	12.69	12.14	0.18
	$n = 4$	4.33	5.39	1.05	2.74	8.22	7.74	0.14
	$n = \infty$	4.87	5.43	0.56	2.16	5.53	5.09	0.11
$\theta = 0.12$	$n = 0$	4.78	6.25	1.47	7.43	12.23	9.62	0.15
	$n = 2$	4.49	5.59	1.09	2.62	8.38	7.95	0.14
	$n = 4$	4.86	5.61	0.75	2.20	6.56	6.17	0.12
	$n = \infty$	5.16	5.68	0.52	1.91	5.24	4.87	0.11

All statistics are annualized and in percent terms.

Parameter values: $\beta = 0.99$, $\alpha = 3$.

Notation: $E(\cdot)$ denotes unconditional expected value, $\sigma(\cdot)$ denotes unconditional standard deviation.

Data sources: Mehra and Prescott (1985), Campbell (1999). Period: 1889-1985.

Table 4.3 - Inspecting the mechanism: a_t

		$E\left(R^f\right)$	$E\left(R^{ep}\right)$	$\frac{E(R^{ep})}{\sigma(R^{ep})}$
US data		0.80	6.18	0.32
$\alpha = 2,$	$\theta = 0.42$	0.21	4.27	0.23
$\alpha = 3,$	$\theta = 0.24$	0.84	6.11	0.27
$\alpha = 4,$	$\theta = 0.16$	2.36	6.12	0.28
$\alpha = 5,$	$\theta = 0.12$	3.71	6.19	0.30
$\alpha = 6,$	$\theta = 0.10$	5.05	6.16	0.32
$\alpha = 7,$	$\theta = 0.08$	6.22	6.23	0.33
$\alpha = 8,$	$\theta = 0.06$	7.41	6.18	0.35
$\alpha = 9,$	$\theta = 0.05$	8.44	6.24	0.37
$\alpha = 10,$	$\theta = 0.04$	9.54	6.11	0.39

All statistics are annualized and in percent terms.

Parameter values: $\beta = 0.99$, $n = 2$.

Notation: $E(\cdot)$ denotes unconditional expected value, $\sigma(\cdot)$ denotes unconditional standard deviation.

Data sources: Mehra and Prescott (1985), Campbell (1999). Period: 1889-1985.

Table 5 - Cyclicality

	US data	No Happiness $\theta_t = \theta_{t+1} = 0$	HM $n = 0$	HM $n = 2$	HM $n = 5$
$E\left(R_L^{ep}\right)$	$(-14.63, -11.24)$	-4.1	-14.2	-12.1	-7.1
$E\left(R_H^{ep}\right)$	$(10.54, 13.66)$	5.0	20.3	25.0	10.0
$E_t\left(R^{ep}\right)$	c-cyclical	none	none	c-cyclical	c-cyclical
$\sigma_t\left(R^{ep}\right)$	c-cyclical	none	none	p-cyclical	p-cyclical
R^f	none	0.14	0.15	0.14	0.14
R_{t+1}^e	0.48	0.98	0.80	0.82	0.91
p_t/d_t	p-cyclical	-0.15	-0.15	-0.08	-0.09
$\frac{E_t(R^{ep})}{\sigma_t(R^{ep})}$	c-cyclical	none	none	c-cyclical	c-cyclical

All statistics are annualized and in percent terms.

Notation: $E(\cdot)$ denotes unconditional expected value, $\sigma(\cdot)$ denotes unconditional standard deviation; $E_t(\cdot)$ denotes conditional expected value, $\sigma_t(\cdot)$ denotes conditional standard deviation; HM abbreviates Happiness Maintenance.

Data sources: Cecchetti et al. (2000), Campbell (1999). Period: 1889-1985

Table 6 - Autocorrelations and cross-correlations

	US data	No Happiness $\theta_t = \theta_{t+1} = 0$	HM $n = 0$	HM $n = 2$	HM $n = 5$
$\rho\left(R_t^f, R_{t-1}^f\right)$	0.87	-0.15	-0.15	-0.14	-0.15
$\rho\left(R_t^f, R_{t-2}^f\right)$	0.73	0.01	0.01	0.04	0.01
$\rho\left(R_t^f, R_{t-3}^f\right)$	0.69	0.00	0.00	-0.02	0.00
$\rho\left(R_t^f, R_{t-5}^f\right)$	0.60	0.00	0.00	-0.01	0.00
$\rho\left(R_t^e, R_{t-1}^e\right)$	-0.03	-0.32	-0.55	-0.25	-0.33
$\rho\left(R_t^e, R_{t-2}^e\right)$	-0.17	0.03	0.06	0.17	0.03
$\rho\left(R_t^e, R_{t-3}^e\right)$	-0.06	0.00	0.01	-0.46	-0.01
$\rho\left(R_t^e, R_{t-5}^e\right)$	-0.04	-0.01	-0.01	-0.03	0.09
$\rho\left(R_t^{ep}, R_{t-1}^{ep}\right)$	0.03	-0.01	-0.05	-0.04	-0.03
$\rho\left(R_t^{ep}, R_{t-2}^{ep}\right)$	-0.22	-0.02	-0.01	0.02	-0.02
$\rho\left(R_t^{ep}, R_{t-3}^{ep}\right)$	-0.06	0.00	0.01	-0.46	-0.01
$\rho\left(R_t^{ep}, R_{t-5}^{ep}\right)$	-0.14	0.00	0.00	-0.03	-0.01
$\rho\left(p_t/d_t, p_{t-1}/d_{t-1}\right)$	0.78	-0.15	-0.15	0.55	0.63
$\rho\left(p_t/d_t, p_{t-2}/d_{t-2}\right)$	0.59	0.01	0.01	0.33	0.54
$\rho\left(p_t/d_t, p_{t-3}/d_{t-3}\right)$	0.54	0.00	0.00	-0.04	0.43
$\rho\left(p_t/d_t, p_{t-5}/d_{t-5}\right)$	0.36	0.00	0.00	0.00	0.21
$\rho\left(R_t^f, R_t^e\right)$	-0.09	0.33	0.70	0.25	0.35
$\rho\left(R_{t+1}^e, p_t/d_t\right)$	-0.12	-0.33	-0.70	-0.42	-0.33

All statistics are annualized and in percent terms.
Notation: $\rho(\cdot,\cdot)$ denotes correlation; HM abbreviates Happiness Maintenance.
Data sources: Campbell (1999). Period: 1889-1985

Table 7 - Long-horizon predictability

	US data	No Happiness $\theta_t = \theta_{t+1} = 0$	HM $n = 0$	HM $n = 2$	HM $n = 5$
β_1	-1.5	-15.6	-10.2	-6.9	-5.2
β_2	-3.0	-15.6	-10.2	-10.8	-6.5
β_3	-3.7	-15.7	-10.2	-10.2	-7.7
β_5	-6.6	-15.1	-10.1	-10.4	-11.1
β_{10}	-12.1	-14.7	-10.0	-10.2	-10.5
R_1^2	4%	7%	43%	28%	13%
R_2^2	8%	5%	43%	44%	15%
R_3^2	10%	4%	43%	41%	18%
R_5^2	19%	3%	41%	42%	25%
R_{10}^2	39%	2%	38%	40%	21%

All statistics are annualized.

Estimated coefficients and R^2 in regressions of k-year horizon of log returns on the lagged log pride-dividend ratio, $r_{t+1} + r_{t+2} + ... + r_{t+k} = \alpha_k + \beta_k (p_t - d_t) + \epsilon_{k,t}$.

US data are based on annual NYSE data. Period: 1889-1985. Source: Campbell (1999), Cecchetti et al. (2000).

Notation: β_k denotes 10×coefficient; HM abbreviates Happiness Maintenance.

Parameter values: $\beta = 0.99$, $\alpha = 2$, $\theta = 0.24$.

Table 8 - Volatility tests

	No Happiness	HM	HM	HM
	$\theta_t = \theta_{t+1} = 0$	$n = 0$	$n = 2$	$n = 5$
r^e_{t+1}	166	114	46	41
Δd_{t+1}	-57	-3	–1	-4
r^e_{t+5}	153	100	99	80
Δd_{t+5}	-59	-3	-1	-3
r^e_{t+10}	150	101	98	96
Δd_{t+10}	-56	-3	-1	-2
r^e_{t+15}	140	99	96	94
Δd_{t+15}	-48	-2	0	-1
r^e_{t+20}	144	101	97	94
Δd_{t+20}	-49	-2	0	0

All statistics are annualized.

Percentage of variance of the (log) price/dividend ratio explained by future (log) equity returns or (log) dividend growth, that is $100 \times \sum_{j=1}^{k} \rho^j cov\left(p_t - d_t, x_{t+j}\right) / var\left(p_t - d_t\right),$ where x_{t+j} is either $-r^e_{t+k}$ or Δd_{t+k}.

Notation: HM abbreviates Happiness Maintenance.

Table 9 - Long-horizon volatility

	No Happiness $\theta_t = \theta_{t+1} = 0$	HM $n = 0$	HM $n = 2$	HM $n = 5$
β_1	1.1	6.3	5.3	2.4
β_2	1.1	3.7	5.0	2.1
β_5	1.0	2.9	4.9	2.0
β_{10}	1.0	2.5	4.5	2.0
β_{20}	1.0	2.2	3.9	1.7
β_{40}	1.0	1.8	2.8	1.4
R_1^2	98%	65%	69%	83%
R_2^2	98%	43%	65%	80%
R_5^2	98%	35%	61%	80%
R_{10}^2	98%	31%	52%	80%
R_{20}^2	98%	31%	40%	75%
R_{40}^2	98%	31%	36%	75%

All statistics are annualized.

Coefficients and R^2 in regressions of k-year horizon of the difference in (log) prices on the difference in (log) consumption, $p_t - p_{t-k} = \alpha + \beta_k (c_t - c_{t-k}) + \epsilon_{k,t}$.

Notation: HM abbreviates Happiness Maintenance.

Table 10 - Summary of unconditional first and second moments of returns under alternative specifications of the happiness demand process

	HM ($\theta = 0.24, n = 2$)	Countercyclical ($\theta = 0.24, n = 2$)	HM ($\theta_h = 0.25 > \theta_l = 0.23$)
$E\left(R^f\right)$	0.84	3.85	0.84
$E\left(R^e\right)$	6.95	5.41	6.95
$E\left(R^{ep}\right)$	6.11	1.55	6.11
$\sigma\left(R^f\right)$	5.55	2.89	5.55
$\sigma\left(R^e\right)$	23.17	13.84	23.17
$\sigma\left(R^{ep}\right)$	22.57	13.50	22.57
$\frac{E(R^{ep})}{\sigma(R^{ep})}$	0.27	0.11	0.27

All statistics are annualized and in percent terms. For parameter values see Table 1.

Notation: $E(\cdot)$ denotes unconditional expected value, $\sigma(\cdot)$ denotes unconditional standard deviation; HM abbreviates Happiness Maintenance.

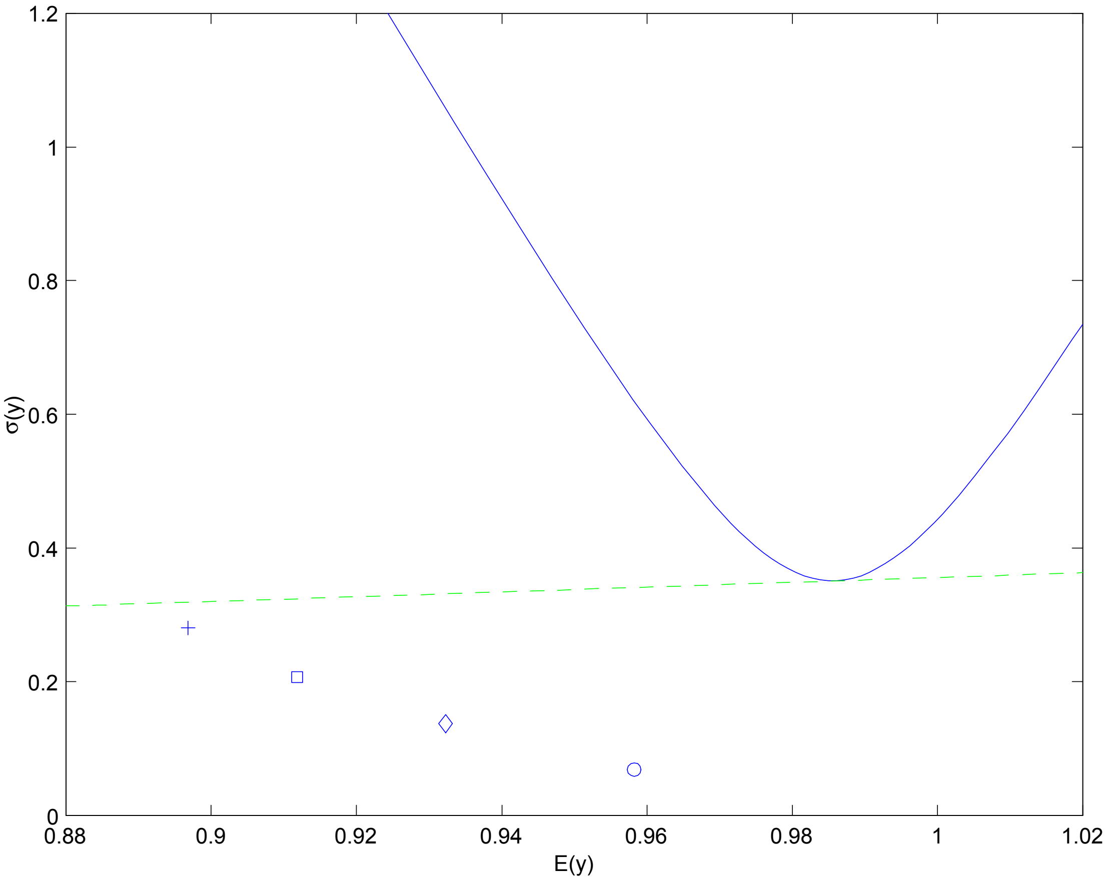

Figure 1: The equity premium puzzle, $\theta = 0$

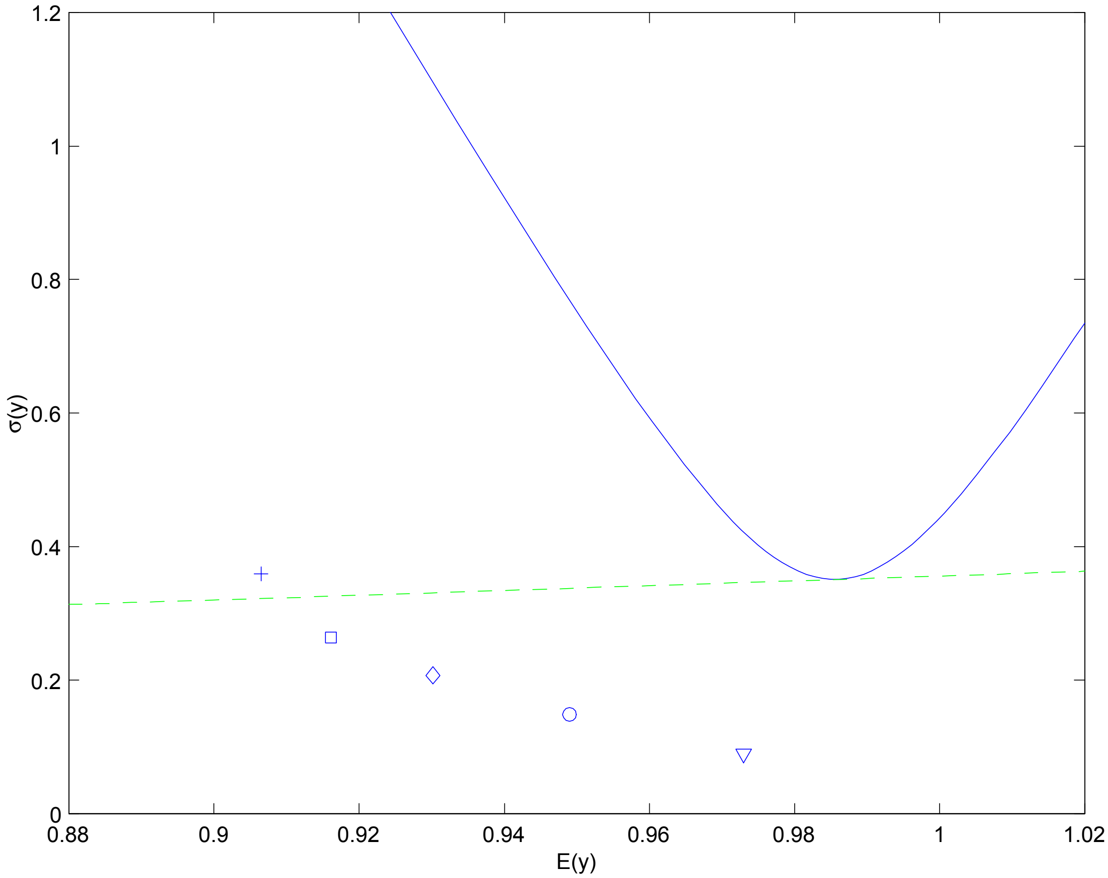

Figure 2: Happiness maintenance, $\theta = .05$

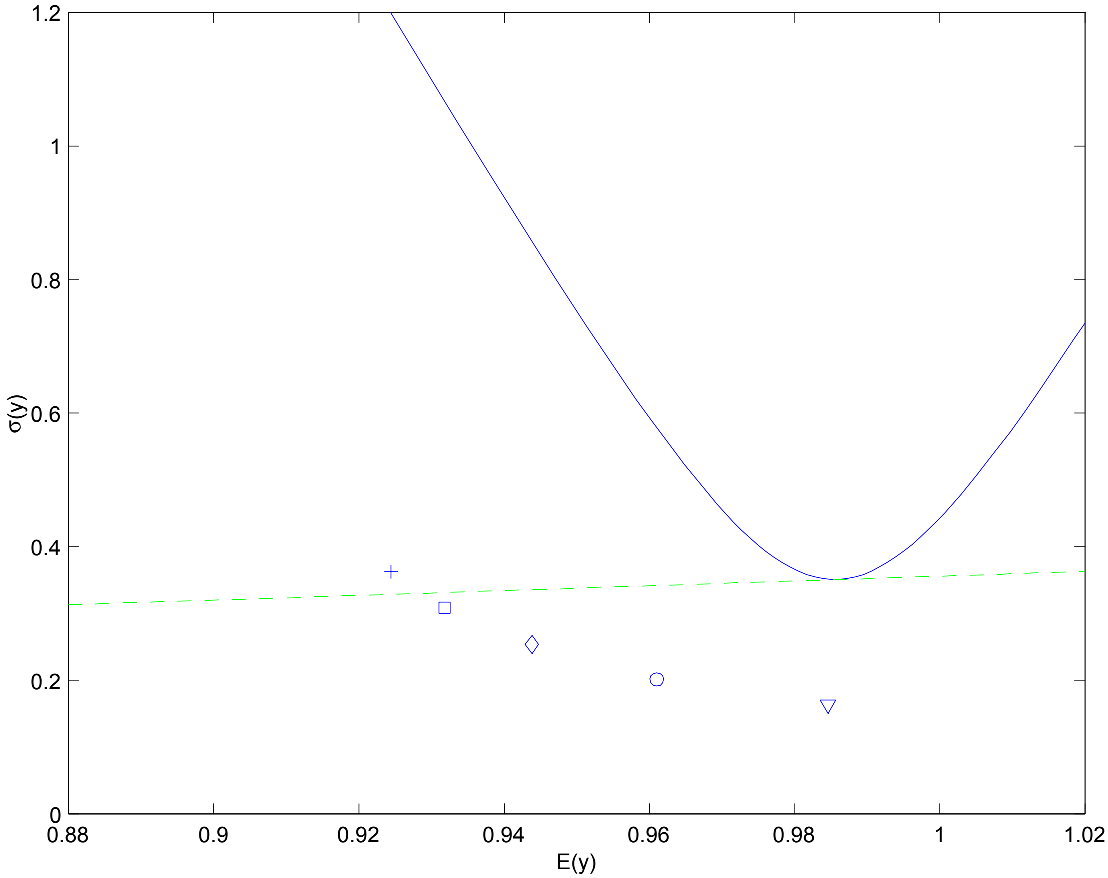

Figure 3: Happiness maintenance, $\theta = .15$

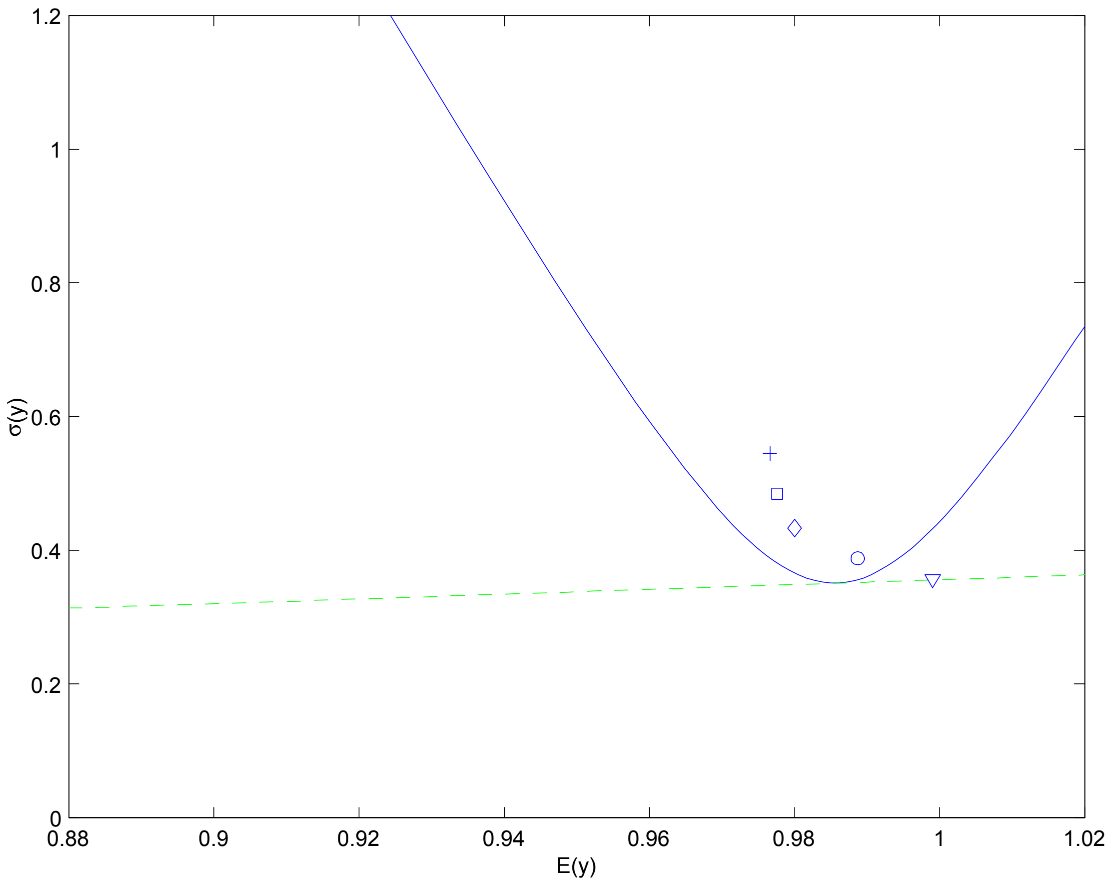

Figure 4: Happiness maintenance, $\theta = .24$