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Integrating Success Factors and Group Attitudes into the Valuation of a Company

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Abstract: In the process of valuation of a company usually a number of experts work together in a team. The valuation formulas are usually based on several financial figures like the expected discounted cash flows and the expected cost of capital. But it is also based on an evaluation of the success factors, which are the causes of economic prosperity of the company. Especially in the new market or in the e-business these soft criteria determine the value of the companies. In this paper a Comparative Company Approach (CCA) based on the Analytic Hierarchy Process (AHP) (Saaty, 1990) is suggested. A company has to be compared to its competitors according to the predetermined criteria – especially the success factors. The results of this kind of benchmarking process are the relative values of the companies included in the investigation. The general method and the aggregation of individual benchmarking results will be described mathematically. The new idea is to use AHP as a basis for a Comparative Company Approach: Knowing the results of the AHP-benchmarking and the monetary value of the competitors (e.g. stock capitalization) it is possible to calculate a range of estimates for the monetary value of the company considered.

Keywords: Analytic Hierarchy Process, AHP, Multi-objective Decision Making, Group Decision Making, Valuation

1 Hierarchical analysis of the problem

Using the technique of the Analytic Hierarchy Process (AHP) (Saaty, 1990) we first have to define a hierarchy of soft and hard criteria (success factors and financial numbers) which are important for the valuation of a company. Within this Comparative Company Approach (CCA) (e.g. Damodaran, 1994, pp. 15-16) we assume that the criterion weights are the same for all companies within a certain industry. A conceptual example of such a hierarchy is shown in figure 1 (see next page).

For a formal description we introduce the following symbols:

- I number of elements (criteria, alternatives = companies) of the decision problem [-]
- i, j elements of the decision problem (criteria, alternatives = companies); $i, j \in \{0, ..., I\}$
- Γ_i Set of elements (criteria, alternatives = companies) which have impact to element (criteria) i

For simplification it is assumed that the nodes of the hierarchy are numbered topologically, i. e. $j<i$ holds if $i \in \Gamma_j$. The top node is numbered as 0 and gives a description of the problem (in AHP-terminology: the goal).

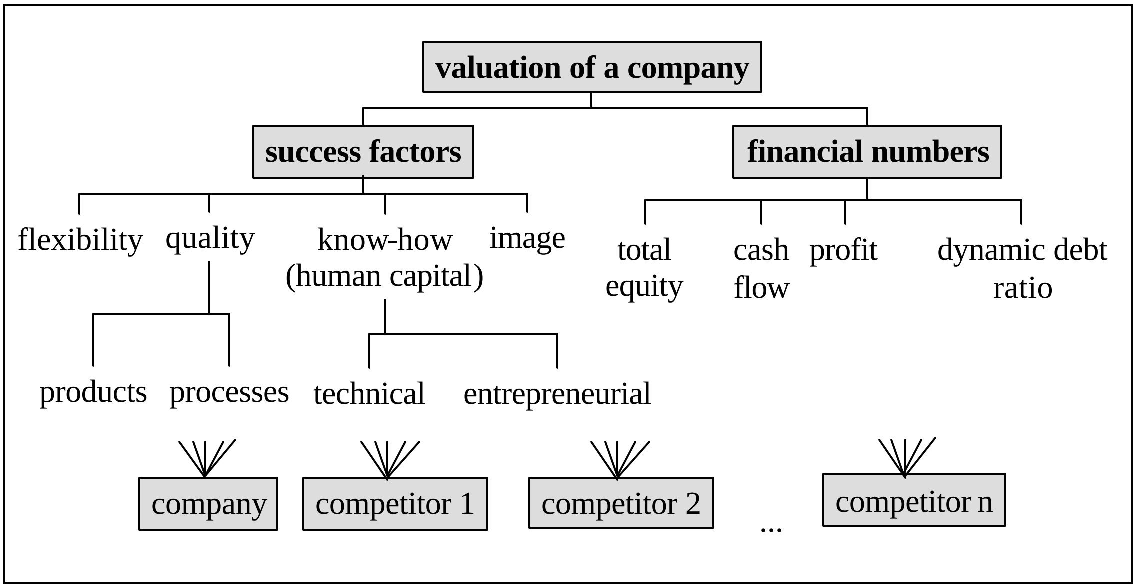

Figure 1: Example of a hierarchy for the valuation of a company

2 Determination of local priorities

After the determination of the hierarchy the importance of each criterion has to be evaluated. Further the different companies are to be compared with respect to the different criteria. (Hafner (1988) uses AHP only for the determination of the criterion weights.) The relative importance/ value of an element with respect to a higher-level element of the hierarchy is defined as (local) priority:

p_{ij} priority of element i with respect to element j [-], $i \in \Gamma_j, j \in \{0, ..., I\}$

The priorities are normalized to the sum $\sum_{i \in \Gamma_j} p_{ij} = 1$. The concrete determination depends on the nature of criteria/comparisons.

<u>Case 1: Hard criteria, which are proportional to the priority</u>

For hard criteria, like total equity or a pre-calculated sum of discounted cash flows, which are proportional to the priority, the local priorities can easily be calculated:

$$p_{ij} := \frac{w_{ij}}{\sum_{k \in \Gamma_j} w_{kj}}$$

w_{ij} "value" of company i for the hard criterion j [-], $i \in \Gamma_j, j \in \{0, ..., I\}$

Case 2: Hard criteria, which are reciprocal to the priority

For hard criteria, like the dynamic debt ratio (i.e. total debt / cash flow), which are reciprocal to the priority, the calculation of the local priorities is slightly more complicated:

$$p_{ij} := \frac{\frac{1}{\overline{p}_{ij}}}{\sum\limits_{k \in \Gamma_j} \frac{1}{\overline{p}_{kj}}} = \frac{\left(\sum\limits_{m \in \Gamma_j} w_{mj} \right) \Big/ w_{ij}}{\sum\limits_{k \in \Gamma_j} \left(\left(\sum\limits_{m \in \Gamma_j} w_{mj} \right) \Big/ w_{kj} \right)}$$

$\overline{p}_{ij}$ priority of element i with respect to element j *assuming that "proportional importance" exists* [-], $i \in \Gamma_j$, $j \in \{0, ..., I\}$

For practical purpose the calculation can be done in two steps: First the local priorities $\overline{p}_{ij}$ are calculated assuming that there is a proportional importance. Second the local priorities p_{ij} are calculated on the basis of the reciprocals of the $\overline{p}_{ij}$.

Case 3: Soft criteria and criterion weights

For soft criteria, like all different kinds of success factors, and for the determination of criterion weights the AHP technique derives the local priorities from comparison matrices. The elements of a matrix are defined by pairwise comparisons of companies with respect to a criterion or by pairwise comparisons of criteria with respect to a higher-level criterion. The comparisons are made by the use of the scale suggested by Saaty (1990) or any other reasonable scale (table 1).

Saaty-scale	1 equally good (large, important) 3 moderately better (larger, more important) 5 strongly better (larger, more important) 7 very strongly better (larger, more important) 9 extremely better (larger, more important)
	Intermediate values 2, 4, 6, 8 and reciprocals (e.g. 1/2; 1/7;) are also usable.

Table 1: Saaty-scale

For further discussion the following symbols were introduced:

$A_j = (a_{ikj})$ comparison matrix for criterion j, $j \in \{0, ..., I\}$

a_{ikj} result of comparisons of criterion/company i versus criterion/company k with respect to criterion j [-]

$p_j = (p_{ij})$ priority vector for criterion j, $j \in \{0, ..., I\}$

The entries in the comparison matrix are substitutes for the relations of the corresponding local priorities, $a_{ikj} = p_{ij} / p_{kj}$. The quadratic matrix A_j is reciprocal as $a_{kij} = p_{kj} / p_{ij} = 1 / a_{ikj}$. For k = i, $a_{ikj} = 1$ holds. Therefore the number of pairwise comparisons reduces to n(n–1)/2, where n is the number of columns/rows of matrix A_j. The matrix A_j is consistent if $a_{ikj} \cdot a_{kmj} = a_{imj}$ for any i and m (transistency). For a consistent matrix the calculation of the local priority vector is easy. The local priorities are given by normalizing any column k of the matrix: $p_{ij} = a_{ikj} \Big/ \sum_{m \in \Gamma_j} a_{imj}$.

As human individuals are not necessary consistent in their votings the transistency can be missed, especially for larger n. In the case of inconsistency we have to make the following considerations: Multiplying a consistent matrix A_j with the vector p_j we get $A_j \cdot p_j = n \cdot p_j$. In this formulation p_j is known to be the eigenvector of A_j with the eigenvalue n. For an inconsistent matrix A_j we have to find a nontrivial solution for the eigenvalue problem $A_j \cdot p_j = \lambda_{max} \cdot p_j$ where λ_{max} is the greatest of n possible eigenvalues and the corresponding eigenvector p_j represents the priority vector. Nevertheless it is important to achieve a consistent or an almost consistent comparison matrix. Therefore the degree of consistency is measured in the following way: For a consistent matrix λ_{max}=n and for a inconsistent matrix $\lambda_{max} \geq$n holds. Therefore the consistency index CI:=(λ_{max}-n)/(n-1) is a possible measure for the degree of inconsistency. Defining the figure RI as the consistency index for a completely randomly generated matrix of size n, we can measure the quality of consistency with the consistency ratio CR:= CI/RI. The figure CR describes the inconsistency, which occurred in the actual voting as a percentage of the inconsistency, which will occur by completely randomly voting. As a rule of thumb a value of CR ≤ 0.1 will be accepted. Otherwise a re-voting of the comparison-matrix has to be performed.

Extension to Case 3: Aggregation of single votings to a group voting

Usually a number of experts is engaged in the process of benchmarking and determining the criterion weights. Therefore we have to aggregate the individual votings to a group comparison matrix. With the arithmetic mean as well as with the median it is possible to get an inconsistent group matrix from consistent individual votings. Furthermore with the arithmetic mean it is also possible to get a non-reciprocal group matrix. Therefore we use the geometric mean:

$$a_{ikj} := \sqrt[Z]{\prod_{z=1}^{Z} a_{ikjz}}$$

Z	number of individuals
z	index of individual, $z \in \{1, ..., Z\}$
a_{ikjz}	result of comparisons of criterion/company i versus criterion/company k with respect to criterion j made by individual z [-], $z \in \{1, ..., Z\}$

To measure the diversity within the group we can calculate a “geometric variance” which is also used in software package “Team Expert Choice™ Version 9.5”. It is defined as follows:

$$\mathrm{var}_{ikj} := \log \sqrt[Z]{\prod_{z=1}^{Z} \max\left\{\frac{a_{ikjz}}{a_{ikj}}, \frac{a_{ikj}}{a_{ikjz}}\right\}} \Big/ \log U$$

U Upper bound of the scale used for pairwise comparisons [-].
For the Saaty-scale U=9 holds.

This measure of diversity is defined on [0, 1]. During the voting process it is important to show the diversity and to discuss the meaning of the criterion if the geometric variance exceeds a pre-determined value.

3 Relative values and estimation of the monetary value of a company

The relative values of the companies can be calculated by synthesizing the (local) priorities of the total hierarchy. Defining

p_i overall priority of element i [-], $i \in \{0, ..., I\}$

we can use the recursive calculation formula for the overall priority of element i:

$$p_0 = 1$$
$$p_i := \sum_{j \mid i \in \Gamma_j} p_{ij} \cdot p_j \quad \text{for } i \in \{1, \ldots, I\}$$

The overall priorities for the companies, i.e. the nodes i for which $\Gamma_i = \emptyset$ holds, represent the total relative values of the companies included in the comparison. Defining

m_i monetary value of company i [monetary units], $i \in \{0, ..., I\}$,

e.g. capitalized stock, we can calculate different estimates (optimistic, pessimistic, average) for the value of a certain company $\hat{i}$ if we know the monetary values of its competitors j:

$$m_{\hat{i}}^{opt} := \max_j \{m_j / p_j\} \cdot p_{\hat{i}}$$
$$m_{\hat{i}}^{pes} := \min_j \{m_j / p_j\} \cdot p_{\hat{i}}$$
$$m_{\hat{i}}^{ave} := \left(\sum_j m_j \Big/ \sum_j p_j\right) \cdot p_{\hat{i}}$$

The calculation for company A is demonstrated in table 2 (MU monetary units; for clarification the index i is substituted by the company name A, B, C or D):

company a	**total relative value** [-] b	**known value of the competitor** [MU] c	**multiplier** [MU] d=c/b	**estimation for the value of company A** [MU] e=d*0.275	
A	0.275	**?**			
B	0.242	35,000	144,628	**39,773**	
C	0.247	40,000	161,943	**44,534**	optimistic
D	0.236	30,000	127,119	**34,958**	pessimistic
Total B, C, D	0.725	105,000	144,828	**39,828**	average

Table 2: Example for calculating estimates for the monetary value of a company

Of course, it is recommended to perform a sensitivity analysis to see how the range of estimates will change if the criterion weights would be chosen differently. With the sensitivity analysis it is also possible to identify the critical success factors. Companies are encouraged to invest in these critical success factors to improve their competitive position.

4 Conclusion

In this paper a method has been suggested which enables to combine success factors and financial data within the process of the valuation of a company. The method is based on the Analytic Hierarchy Process (AHP). The results of the AHP approach can be interpreted as relative values of the companies included in the investigation. The new idea is to take the results of the AHP as input data for a Comparative Company Approach (CCA). With this further calculation it is possible to obtain a range of estimates for the value of a company.

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