GARCH DIAGNOSIS WITH PORTMANTEAU BICORRELATION TEST AN APPLICATION ON THE MALAYSIA'S STOCK MARKET

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ABSTRACT

This study employed the Hinich portmanteau bicorrelation test (Hinich and Patterson, 1995; Hinich, 1996) as a diagnostic tool to determine the adequacy of the GARCH model in describing the returns generating process of Malaysia's stock market, specifically the Kuala Lumpur Stock Exchange Composite Index (KLSE CI). The bicorrelation results demonstrated that, while GARCH model is commonly applied to financial time series, this model cannot provide an adequate characterization for the underlying process of KLSE CI. Further investigation using the windowed test procedure revealed that this was due to the presence of episodic non-stationarity in the data, which could not be captured by any kind of ARCH or GARCH model, even after modifications to the specifications of the GARCH model. Thus, this study points to the need to continue the search for a parsimonious and congruent model capable of capturing the episodic features presence in the returns series of KLSE CI.

Keywords: GARCH; Non-linearity; Non-stationarity; Data generating process; Bicorrelation; Malaysian stock market.

INTRODUCTION

After the stock market crash of October 19, 1987, interest in non-linear studies has experienced a tremendous rate of development. This has come about because the frequency of large moves in stock markets has been greater than would be expected under a normal distribution (Hsieh, 1991: 1839). This observed trend was echoed by Campbell *et al.*, who wrote "A natural frontier for financial econometrics is the modelling of non-linear phenomenon" (1997: 467).

The main driving forces behind this phenomenal growth are the developments in the mathematical and statistical analysis of dynamic systems. The richness of these new non-linear tests lies in their ability to uncover a more complex form of dependencies in a time series that otherwise appear to be random. This is further supported by growing views that the observable world is nonlinearly dynamic (Pesaran and Potter, 1993; Campbell *et al.*, 1997; Barnett and Serletis, 2000).

However, there are literally unlimited numbers of possible non-linear models that could potentially describe the returns generating process for financial time series. It is widely accepted that the non-linear dependencies in financial time series are very well described by the Autoregressive Conditional Heteroscedasticity (ARCH) model introduced by Engle (1982) or its extension Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model by Bollerslev (1986). The GARCH model has become enormously popular for modelling financial time series over the past 15 years. This popularity is evidenced by the incorporation of GARCH estimation into major software packages (see example, Brooks, 1997; McCullough and Renfro, 1999; Brooks *et al.*, 2001)

The simple GARCH (1,1) model for {y(t)} can be written as:

$$y(t) = \varepsilon_t h_t^{1/2}$$
$$\varepsilon_t | \psi_{t-1} \sim N(0, h_t)$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \qquad (1)$$

In the literature, various procedures have been utilized or proposed to test whether the GARCH formulation represents an adequate characterization of the data. It has been suggested that the BDS test can be used as a general test of model mis-specification (Brock *et al.*, 1991). Since the BDS test has reasonable power against the GARCH model, it has been used as a diagnostic tool to determine the adequacy of the GARCH model for the detected non-linear structure. In this case the standardized residuals from the fitted GARCH model were subjected to the BDS test. The null hypothesis then became one that the specified GARCH model was sufficient to model the non-linear structure in the data against an unspecified alternative that it was not. If the BDS test cannot reject the null using appropriate critical values derived from simulation, then the GARCH model was assumed to be an adequate characterization of the data. This procedure has been followed by various authors (see, for example, Hsieh, 1989, 1991; Krager and Kugler, 1993; Abhyankar *et al.*, 1995; Opong *et al.*, 1999; McMillan and Speight, 2001).

However, the problem with using the BDS test in this manner arises because the asymptotic distribution could not approximate very well the BDS statistic applied to standardized residuals of ARCH, GARCH and EGARCH (Brock *et al.*, 1991; Hsieh, 1991). The Monte Carlo results from Brooks and Heravi (1999) provided further evidence against the sole use of the BDS test as a general mis-specification test for GARCH model. Specifically, their results showed that the BDS test cannot reliably discriminate between different types of non-linear structure, such as a self-exciting threshold autoregressive (SETAR) model and a GARCH model, in which both are considered as competing data descriptions.

Some authors have suggested that the application of existing non-linearity tests in a sequential way can provide deeper insight into the nature of non-linear generating mechanism of a time series (Barnett *et al.*, 1995, 1997; Barnett and Serletis, 2000; Ashley and Patterson, 2001). Lim (2001) utilized the differing power of the BDS and Hinich bispectrum test in detecting GARCH as an alternative framework for checking the adequacy of the GARCH model. In this case, the low power of the Hinich bispectrum test relative to the BDS test for the GARCH model suggested that the bispectrum test is useful as a marker for the GARCH model. However, this approach has received hardly any attention from other researchers.

The Hinich portmanteau bicorrelation test (Hinich and Patterson, 1995; Hinich, 1996), which was designed to detect episodes of transient serial dependencies within a data series, is an alternative technique to check the adequacy of a GARCH model as valid data descriptions. From the Monte Carlo simulations given in Hinich and Patterson (1995), it was demonstrated that the bicorrelation test has better small sample properties and did not have the stiff data requirements of many of its competitors, such as the BDS test. Several studies (see Hinich and Patterson, 1995; Brooks and Hinich, 1998; Brooks *et al.*, 2000) have utilized the bicorrelation test to examine the validity of specifying a GARCH error structure.

This study employed the Hinich portmanteau bicorrelation test as a diagnostic tool to determine the adequacy of the GARCH model in describing the returns generating process of Malaysia's stock market. The focus of this paper is on the assumption of strict stationarity of GARCH model. The application to the Malaysian stock market, specifically the Kuala Lumpur Stock Exchange Composite Index (KLSE CI) is certainly worth investigation since very few studies have been conducted on financial markets of developing countries, especially the utilization of advances in non-linear techniques.

This paper reviews in the following section some features of the Malaysia's stock market, the Kuala Lumpur Stock Exchange (KLSE). Section III gives a brief description of the data and the methodology used in this study. Section IV presents the empirical results as well as the analysis of the findings. Finally, concluding remarks are given at the end of the paper.

THE KUALA LUMPUR STOCK EXCHANGE (KLSE)

In Malaysia, the Kuala Lumpur Stock Exchange (KLSE) is the only body approved by the Minister of Finance, under the provisions of the Securities Industry Act, 1983, as the stock exchange in the country. The KLSE is a self-regulatory organization with its own memorandum and articles of association, as well as rules which govern the conduct of its members in securities dealings. The KLSE is also responsible for the surveillance of the market place, and for the enforcement of its listing requirements which spell out the criteria for listing, disclosure requirements and standards to be maintained by listed companies.

Although the history of KLSE can be traced to the 1930s, the public trading of shares in Malaysia only really began in 1960 when the Malayan Stock Exchange (MSE) was formed. When the Federation of Malaysia was formed in 1963, with Singapore as a component state, the MSE was renamed the Stock Exchange of Malaysia (SEM). With the secession of Singapore from the Federation of Malaysia in 1965, the common stock exchange continued to function but as the Stock Exchange of Malaysia and Singapore (SEMS).

The year 1973 was a major turning point in the development of the local securities industry, for it saw the split of SEMS into The Kuala Lumpur Stock Exchange Berhad (KLSEB) and the Stock Exchange of Singapore (SES). The split was opportune in view of the termination of the currency interchangeability arrangements between Malaysia and Singapore. Although the KLSEB and SES were deemed to be separate exchanges, all the companies previously listed on the SEMS continued to be listed on both exchanges.

When the Securities Industry Act 1973 was brought into force in 1976, a new company called the Kuala Lumpur Stock Exchange (KLSE) took over the operations of KLSEB as the stock exchange in Malaysia. Its function was to provide a central market place for buyers and sellers to transact business in shares, bonds and various other securities of Malaysian listed companies. On 1 January 1990, following the decision on the “final split” of the KLSE and SES, all Singaporean incorporated companies were de-listed from the KLSE and vice-versa for Malaysian companies listed on the SES.

The companies listed on the KLSE are listed either on the Main Board or the Second Board. The Second Board, which complements the Main Board, was established on 11 November 1988 to enable smaller companies with strong growth potential to seek a listing on the Exchange. Each board is further classified by sectors which reflect the core business of these companies.

The KLSE computes an index for each of the main sectors traded on the bourse - industrial, finance, property, tin and plantation sectors - and the second board. However, the most widely followed, by far, is the KLSE Composite Index (KLSE CI). The KLSE CI was introduced in 1986 after it was found that there was a need for a stock market index which would serve as a more accurate indicator of the performance of the Malaysian stock market and the economy. At that time, there was effectively no index which represented the entire market.

The KLSE CI satisfies stringent guidelines and was arrived at only after rigorous screening of the component companies that were eventually selected to compose the index. In 1995, the number of component companies was increased to 100, and will be limited to this number, although the actual component companies may change from time to time. The KLSE CI is constructed by using the weighted average method, where the weight used is the number of ordinary shares outstanding.

METHODOLOGY

The Data

In this study, we utilized the daily closing values of Kuala Lumpur Stock Exchange Composite Index (KLSE CI) for the sample period of 2 January 1990 to 28 June 2002. The price series obtained were used to compute a set of continuously compounded percentage returns for the KLSE CI, using the relationship:

$$r_t = 100^* \ln(P_t/P_{t-1}) \tag{2}$$

where P_t is the closing price of the stock on day t, and P_{t-1} the rate on the previous trading day.

Hinich Portmanteau Bicorrelation Test

In the windowed test procedure of Hinich and Patterson (1995), a correlation portmanteau test similar to the Box-Pierce Q-statistic is developed for the detection of linear serial dependencies within a window. For detecting non-linear serial dependencies, the procedure uses a bicorrelation portmanteau test, which can be considered as a time-domain analog of the bispectrum test statistic. In applying these tests, the full sample is broken down into smaller windows of data. If the full data sample does exhibit significant linear or non-linear ser ial dependencies, but there are only a few windows that are significant, then this suggests that the data may instead be characterized by episodes of transient dependencies. In another words, it is the activity of these few windows that is actually driving the results of the overall sample.

In this section, we provide a brief description of the test statistics used in this windowed test procedure. A full theoretical derivation of the test statistics and a number of Monte Carlo simulations to assess their size and power are given in Hinich and Patterson (1995) and Hinich (1996).

Let the sequence {x(t)}denote the sampled data process, where the time unit t is an integer. The test procedure employs non-overlapped data window, thus if n is the window length, then the k-th window is $\{x(t_k), x(t_k+1),\ldots x(t_k+n-1)\}$. The next non-overlapped window is $\{x(t_{k+1}), x(t_{k+1}+1),\ldots x(t_{k+1}+n-1)\}$, where $t_{k+1} = t_k+n$. The null hypothesis for each window is that x{t} are realizations of a stationary pure noise process[1] that has zero bicovariance. The alternative hypothesis is that the process in the window is random with some non-zero correlations $C_{xx}(r) = E[x(t)x(t+r)]$ or non-zero bicorrelations $C_{xxx}(r,s) = E[x(t)x(t+r)x(t+s)]$ in the set $0 < r < s < L$, where L is the number of lags.

Define Z(t) as the standardized observations obtained as follows:

$$Z(t) = \frac{x(t) - m_x}{s_x} \tag{3}$$

for each t = 1, 2,………,n where m_x and s_x are the sample mean and sample standard deviation of the window.

The sample correlation is:

$$C_{ZZ}(r) = (n-r)^{-\frac{1}{2}} \sum_{t=1}^{n-r} Z(t)Z(t+r) \tag{4}$$

The C statistic, which is developed for the detection of linear serial dependencies within a window, is defined as:

$$C = \sum_{r=1}^{L} [C_{ZZ}(r)]^2 \quad \sim \chi^2_{(L)} \tag{5}$$

[1] A stationary time series is called pure-noise or pure white-noise if $x(n_1),\ldots n_N$. A white noise time series, by contrast is one for which the autocovariance function is zero for all lags. Whiteness does not imply that x(n) and x(m) are independent for $m \neq n$ unless the series is Gaussian.

The (r,s) sample bicorrelation is:

$$C_{ZZZ}(r,s) = (n-s)^{-1}\sum_{t=1}^{n-s} Z(t)Z(t+r)Z(t+s) \quad \text{for } 0 \leq r \leq s \tag{6}$$

The H statistic, which is developed for the detection of non-linear serial dependencies within a window, is defined as:

$$H = \sum_{s=2}^{L}\sum_{r=1}^{s-1} G^2(r,s) \qquad \sim \chi^2_{(L-1)(L/2)} \tag{7}$$

where $G(r,s) = (n-s)^{\frac{1}{2}} C_{ZZZ}(r,s)$

In both the C and H statistics, the number of lags L is specified as $L = n^b$ with $0 < b < 0.5$, where b is a parameter under the choice of the user. Based on the results of Monte Carlo simulations, Hinich and Patterson (1995) recommended the use of b=0.4 in order to maximize the power of the test while ensuring a valid approximation to the asymptotic theory.

A window is significant if either the C or H statistic rejects the null of pure noise at the specified threshold level. This study uses a threshold of 0.01. In this case, the chance of obtaining a false rejection of the null is approximately one out of every 100 windows. With such a low-level threshold, it would minimize the chance of obtaining false rejections of the null hypothesis indicating the presence of dependencies where these actually do not exist. It is possible to use the bicorrelation test to examine whether a GARCH formulation represents an adequate characterization of the data. This is achieved by transforming the returns into a set of binary data, where

$$\{y(t)\}: \qquad \begin{aligned} y(t) &= 1 \quad \text{if } Z(t) > 0 \\ y(t) &= -1 \quad \text{if } Z(t) < 0 \end{aligned} \tag{8}$$

If Z(t) are generated by a GARCH process whose innovations $\{\varepsilon_t\}$ are symmetrically distributed around a zero mean, then the binary set {y(t)} will be a stationary pure noise series. The justification for the binary transformation is that it turns a GARCH into a pure noise[2]. To put it differently, a GARCH process that has symmetric innovations produces independently distributed binary output. The binary transformed data has moments which are well-behaved with respect to the asymptotic theory. Therefore, if the null of pure noise is rejected by the C or H statistics, then there are statistical structures present in the data that cannot be captured by a GARCH model. The rejection may be due to serial dependence in the innovations but this violates a critical assumption for GARCH model.

EMPIRICAL RESULTS

Table 1 provides summary statistics for the returns series in order to get a better view of some of the important statistical features. The means are quite small. The KLSE CI returns series exhibit some degree of positive or right-skewness. On the other hand, the distributions are highly leptokurtic, in which the tails of its distribution taper down to zero more gradually than do the tails of a normal distribution. Not surprisingly, given the non-zero skewness levels and excess kurtosis demonstrated within these series of returns, the Jarque-Bera (JB) test strongly rejects the null of normality.

[2] Though ARCH/GARCH is a martingale difference process, and thus white noise, it is not pure noise.

TABLE 1
Summary Statistics for KLSE CI Returns Series

	KLSE CI Returns Series
Sample Period	2/1/1990-28/6/2002
No. of observations	3259
Mean	0.007818
Median	0.000000
Maximum	20.81737
Minimum	-24.15339
Std deviation	1.679596
Skewness	0.457460
Kurtosis	38.02681
JB normality test statistic	166713.4
p-value	(0.000000)*

*Denotes very small value

Table 2 presents the correlation (C) and bicorrelations (H) test statistics for the binary transformed data set {y(t)} covering the full sample period. The results show that the null of pure noise is strongly rejected by the C and H statistic for KLSE CI returns series. This implies that these return series cannot be generated by a strongly stationary pure noise process as required by the GARCH model. In other words, there are statistical structures present in the returns series of KLSE CI that cannot be captured by a GARCH model.

TABLE 2
C and H Statistics for Whole Sample of Binary Trasformed Data

	KLSE CI Returns Series
Sample Period	2/1/1990-28/6/2002
No. of observations	3259
No. of lags	25
No. of bicorrelations	300
p-value	
- C Statistic	0.0000*
- H Statistic	0.0000*

*Denotes extremely small *p*-value.

The next challenge was to investigate the persistency of the underlying non-stationarity for the KLSE CI returns series. One approach is to break the whole sample into relatively narrow windows. The window length should be sufficiently long to provide adequate statistical power and yet short enough for the data generating process to have remained roughly constant. In this study, the data are split into a set of non-overlapping windows of 35 observations in length, approximately seven trading weeks.

Table 3 presents the results for the windowed testing. The results show that the null of pure noise is rejected in 22 windows by the C statistic, which is equivalent to 23.66%. On the other hand, there are 33 significant H windows. In those significant windows, the rejections of the null were due to either a significant C or H, or to both. The final row of Table 3 provides the number of windows where the rejection of the null occurs. The results reveal that the number of significant windows is larger than the 1% one would expect purely by chance, given the nominal threshold level of 0.01. The findings from this windowed test procedure demonstrated that the underlying KLSE CI returns series are characterized by transient epochs of dependencies surrounded by long periods of pure noise, and hence violates the critical assumption of strict stationarity as required by the GARCH process.

TABLE 3
Windowed-Test Results for Binary Transformed Data

	KLSE CI Returns Series
Total number of windows	93
No. of lags	4
No. of bicorrelations	6
Significant C windows	22 (23.66%)
Significant H windows	33 (35.48%)
Total number of significant C or H windows	34 (36.56%)

CONCLUSIONS

The results from our econometric investigation using the Hinich portmanteau bicorrelation test reveal that the null of pure noise is strongly rejected by the C and H statistics. This implies that there are statistical structures present in the returns generating process of KLSE CI that cannot be captured by a GARCH model. Further investigation using the windowed test procedure found that the returns series are characterized by transient epochs of dependencies surrounded by long periods of pure noise, which violates the critical assumption of strict stationarity as required by the GARCH process. Thus, this episodic non-stationarity present in the data could not be captured by any kind of ARCH or GARCH model, even after modifications to the specifications of the GARCH model. As pointed out by Brooks and Hinich (1998), the search for a parsimonious and congruent model capable of capturing the episodic features of the exchange rates returns series must continue.

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