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Liquidation Triggers and the Valuation of Equity and Debt*

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Abstract

Net-worth covenants, as introduced by Black and Cox (1976), provide the firm's bondholders with the right to force reorganization or liquidation if the value of the firm falls below a certain threshold. In the event of default, however, many bankruptcy codes stipulate an *automatic stay* of assets that prevent bondholders from triggering liquidation and thus impact many positive net-worth covenants.

To consider this impact on a corporation's capital structure we develop a general model of liquidation driven by a liquidation trigger. This trigger accumulates with time and severity of distress. In addition, current distress periods may have greater weight than old ones. The tractability of the approach stems from its ability to allow parameters appropriate for different legal rules and types of bondholder safety covenants.

The proposed model includes several well-known models, like Merton, Black-Cox and others. We show how to valuate various types of corporate securities by using this model. Numerical results and sensitivity analysis are presented for selected basic cases.

1. Introduction

The modeling of default is crucial in determining the value of corporate securities. The classic structural approach, based on Black and Scholes (1973) and pioneered by Merton (1974), assumes that default can only occur in the event that the total value of the firm at maturity is less than the contractual payment due on the debt. To cope with the possibility of early default prior to bond maturity, Black-Cox (1976, hereafter BC) developed a “first passage” model, which assumes that the time of default is the first instance that the market value of the assets of the issuer has fallen below a specified distress threshold that precipitates immediate liquidation of the firm’s assets. This distress threshold marks the trigger for the liquidation of firm’s assets in subsequent structural models as well [see Brennan and Schwartz (1978), Longstaff and Schwartz (1995), Leland (1994), Ericsson and Reneby (1998) and others].

As noted by Leland (1994) and Ericsson and Reneby (1998), there are several ways to determine and justify a distress threshold. A legal interpretation is based on the practice in many countries to determine the financial distress threshold in corporate law. The minimum value of the firm’s assets signaling legal financial distress is usually related to the total nominal value of outstanding debt. An economic approach views the distress threshold as the level of asset value necessary for the firm to retain sufficient credibility to continue operations. A contractual interpretation for the existence of the distress threshold is based on positive net worth covenants that enable bondholderss to force reorganization or liquidation in the event the value of the firm falls below a pre-determined threshold.

In practice the onset of financial distress does not necessarily lead to immediate liquidation of the firm's assets by its debtholders. Often, for political and social considerations, bankruptcy laws favor firm continuation and creditors cannot immediately liquidate the firm.[1] Legal bankruptcy protection, like Chapter 11 of the U.S. Bankruptcy Code, enables a firm experiencing financial distress to renegotiate its outstanding debt. As noted by Bebchuk and Chang (1992) and Bebchuk (1997, 2002), once an insolvent company files for reorganization under Chapter 11, an "automatic stay" prevents debtholders from seizing assets until a reorganization plan is adopted. If the parties fail to agree on a reorganization plan, the supervising court would eventually convert the bankruptcy proceedings to a Chapter 7 liquidation of the firm's assets. While the supervising court has not decided on liquidation procedure the debtholders cannot obtain any value from the company before they agree with the equityholders on the division of the firm's assets.

Empirical studies have found that the average time period between the indication of financial distress and its resolution ranges between two to three years for firms that renegotiate their claim under Chapter 11 of the U.S. Bankruptcy Code.[2] Moreover, empirical studies show that most firms emerge from Chapter 11 and only few are actually liquidated.[3] Empirical studies focusing on the distinction between the bankruptcy event and the division of the firm's assets between the different stakeholders point out that liquidation models based on "first passage", as introduced first by BC (1976), are invalid, since reaching the threshold does not automatically trigger either liquidation or reorganization of the firm's assets.

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[1] See White (1994).
[2] See Frank and Torous (1989), Betker (1995), Gilson (1997) and Helwege (1999).
[3] According to Weiss (1990) and Gilson, John and Lang (1990), only 5% of the firms were liquidated under Chapter 7 after filing for Chapter 11 protection, while according to Morse and Show (1988) 15% - 25% of these firms are liquidated.

To address the discrepancy between bankruptcy and the liquidation of the firm's assets recent work on capital structure and securities valuation suggests that liquidation occurs only if the value of the firm's assets has reached the distress threshold and remains below this threshold for a prolonged period of time. Fan and Sundaresan (2000) suggest that when the firm is in default, borrowers stop making the contractual coupon and start servicing the debt strategically until the firm's asset value returns to a level above the distress threshold.

In this spirit, a liquidation model developed by François and Morellec (2002, hereafter FM), assumes that the firm issues perpetual debt with contractual coupon payments, and liquidation occurs when the value of its assets dips below the distress threshold and remains below that level for an interval exceeding a pre-determined 'grace' period. If the firm's asset value rebounds and rises above the distress threshold before the pre-determined grace period has elapsed, the procedure is discontinued and the invisible "distress clock" is reset to zero. According to this approach, while debt is strategically serviced automatically after the value of the firm crosses the distress threshold, liquidation is declared only after the predetermined grace period has run its course.

Moraux (2002), points out that according to this liquidation model, each time firm value falls below the threshold level an additional grace period is granted without reference to previous instances of insolvency. Asset value could remain below the threshold level for the majority of the duration of debt without the firm being liquidated. To overcome this deficiency, Moraux (2002) assumes that the firm issues zero-coupon debt, and proposes that liquidation is triggered when the *total* time that the firm's asset value spends under the distress threshold ("excursion time") exceeds a pre-determined grace period. Consequently, the previous "distress clock" is not reset to zero when the value of the firm's assets rebounds above the threshold. In this manner, the liquidation model becomes highly path-dependent, since it accumulates the entire history of a firm's financial distress. However, by simply accumulating excursion time this model gives a company's history of financial distress an equal weight in triggering liquidation.

This model may distort the empirical nature of the liquidation process, which while affected by the 'big picture', is not necessary triggered by it. This path-dependent liquidation process might have "too strong a memory". Under this model, a firm may be liquidated even if the value of the firm's assets has recently spent only a very short period of time under the distress threshold (since years ago it had spent an extensive period below the threshold and as a result the total time has reached the predetermined graced period).

A second drawback of the above two liquidation models (FM and Moraux) results from not considering the impact of distress severity on the decision to liquidate firm's assets. These two models do not differentiate between cases in which firm value is below but close to the distress threshold and cases firm value falls far below the threshold level. The decision to liquidate is often a function of degree as well as time.

To overcome the drawbacks of these two methods and to describe liquidation in a more realistic and flexible manner, we develop a model in which the impact of prior transgressions on the liquidation trigger is a function of both the severity of the current distress period and the distance of past events from it. According to our model, liquidation is triggered when the *total weighted* time that firm value has spent under the distress threshold exceeds the pre-determined grace period. By applying this process one can increase the weight of recent and/or severe distress periods over early and/or mild distress periods on the decision to liquidate the firm's assets. This method allows the invisible “distress clock to move in syncopated rhythm, and debtholders can claim value when the pre-determined grace period is violated.

Our general model for liquidation can be adapted to a wide array of legal regimes and contractual arrangements. Each legal and economic environment can be efficiently translated into an appropriate liquidation procedure. At one extreme, we can exclude or significantly reduce the impact of distress episodes in the distant past on the liquidation trigger. At the other extreme, we can weigh each distress observation equally. Our model, therefore, is a general model incorporating both FM (2002) and Moraux (2002).

The nature of the different liquidation procedures has important ex-ante consequences, since participants in the market agree to finance the company on terms that reflect the possible ex-post outcomes, which may include reorganization or liquidation of a firm’s assets. By employing our approach one can directly value different types of corporate securities and analyze complex capital structure scenarios for various liquidation procedures. We provide numerical examples to investigate how the length of the grace period, liquidation decay factors, distress severity, leverage ratios and the firm’s asset volatility affect both asset prices and credit spreads.

The model presented in this paper assumes a simple capital structure with one type of zero-coupon debt. However, it can be easily extended to a case in which the firm has issued both senior and junior debt, convertible bonds or warrants.

The remainder of the paper is organized as follows: Section 2 specifies the assumptions, and the valuation models for equity and debt are derived. Section 3 specifically outlines the pricing of zero-coupon bonds under our model. Section 4 is devoted to explaining existing liquidation models and to highlighting the advantage of our model over these models. In section 5, we numerically analyze the main implications of the models for asset pricing. Conclusions are presented in section 6.

The appendices contain numerical examples that are omitted in the main body of the text.

2. A Liquidation Model with Adjustable Memory

In this section we construct a general and adaptive liquidation model with adjustable memory to estimate the value of various corporate securities. According to our model, liquidation is triggered when the exponentially weighted cumulative time that firm value has spends under the distress threshold exceeds a fixed exogenous amount of time. Using our model, it is possible to increase the impact of late and/or severe distress periods over early and/or mild distress periods on the decision to liquidate. We rely on standard structural approach assumptions: assets are continuously traded in an arbitrage-free and complete market with riskless borrowing or lending at a constant rate r. The instantaneous standard deviation of the rate of return of the firm, σ, is constant; the value of the firm's assets, V_t, is independent of the capital structure of the firm, and is well described under the risk neutral measure Q, by the following stochastic differential equation:

$$dV_t = (r - \delta)V_t dt + \sigma V_t dW_t \qquad (1)$$

where W is a standard Brownian motion and δ is the firm's payout ratio.

We suppose that the firm has outstanding only equity and a single bond issue with a promised final payment of P. The firm goes bankrupt in one of two ways:

either if the value of the firm's assets falls below a time dependent threshold level, denoted by K_t, at any time prior to debt maturity, or if the value of the assets is less than some constant F at debt maturity.[4]

According to the BC (1976) model, the default event allows the creditor to force immediate liquidation through its safety covenants. In our model, as in FM (2002) and Moraux (2002) models, default and liquidation are distinct events. We assume that liquidation is declared when the total weighted excursion time (hereafter "cumulative distress time", or CDT), exceeds a pre-determined grace period, denoted by d. In order to determine the CDT we define the time dependent threshold level K_t according to BC (1976):

$$K_t = \lambda F e^{-r(T-t)} \quad \text{where}: 0 \le \lambda \le 1 \tag{2}$$

In this exponential form the threshold level is a constant fraction of the promised final payment. Let us define the following random variable:

$$g_t^K = \sup\left\{ s \le t \middle| \, V_s = K_s \right\} \tag{3}$$

when g_t^K is the last time before t that firm value crossed threshold K_s. The state variable for the liquidation trigger (the CDT) is defined by I_t^K, which is equal to the exponentially weighted excursion time at date t, and defined:

[4] Usually this parameter is set to equal the principal P of debt as in Merton (1974) and in subsequent models. However, if liquidation costs are incurred at maturity, as in Anderson and Sundaresan (1996), this may not accurately reflect the value of debt.

$$I_t^K = \int_0^{g_t^K} e^{-\beta(t-s)} f(V_s)ds + \int_{g_t^K}^{t} e^{-\gamma(t-s)} f(V_s)ds \qquad (4)$$

where β is the decay factor for past distress periods and γ is the decay factor of the last distress period. As β and γ increase, the impact of past distress periods become decreasingly meaningful to the decision to liquidate the firm. The function $f(V_t)$ defines the impact of the severity of the distress event on the liquidation trigger. We model $f(V_t)$ as follows:

$$f(V_t) = \begin{cases} e^{\alpha\left[\frac{K_t - V_t}{K_t}\right]} & V_t \le K_t \\ 0 & V_t > K_t \end{cases} \qquad (5)$$

where $\alpha \ge 0$.

Accordingly, the decision to liquidate a firm's assets does not depend solely on the duration of the distress periods or its continuity, as described in Moraux (2002) and FM (2002) respectively, but also on the distance of past distress periods from the present and on the severity of distress i.e. the degree to which firm value falls below the threshold. Liquidation occurs the first time that the CDT extends beyond d. The liquidation time is denoted by θ^K, and it is defined mathematically by:

$$\theta^K = \inf\left\{ t > 0 \,\middle|\, I_t^K \ge d, V_t \le K_t \right\} \qquad (6)$$

In the special case where α=0, the severity of the distress period has no impact on the liquidation procedure and the CDT can be calculated by the expression:

$$I_t^K = \int_0^{g_t^K} e^{-\beta(t-s)} 1_{\{V_s \le K_s\}} ds + \int_{g_t^K}^{t} e^{-\gamma(t-s)} 1_{\{V_s \le K_s\}} ds \qquad (7)$$

where $1_{\{V_S \le K_B\}}$ is the characteristic function that receives the value of one if firm value is below the distress threshold level, and zero otherwise.

In our setup, shareholders hold a corporate security with payoffs equivalent to a complex Parisian call option on the value of the firm's assets. The shareholders have a residual claim on the cash flows generated by the firm's assets unless the weighted excursion time under the distress threshold has reached the pre-determined grace period, d . The bondholders hold a corporate security with payoffs equivalent to a complex Parisian option with rebate. The following examples are of special interest since they pertain to previous contributions of the literature. In all of these examples α=0.

Example 1. *When $\beta \to +\infty$ and $\gamma = 0$, liquidation procedure occurs at the first point in time that the firm value process has spent consecutively more than the pre-specified grace period below the threshold K_t. Thus, when $\beta \to +\infty$ and $\gamma = 0$, we get the François and Morellec (2002) liquidation procedure.*

In this example the CDT is accumulated only during the current distress period, where past distress periods do not influence the liquidation trigger. At the one extreme, when $d = 0$, the FM model gets, as a special case, the standard modeling of default and liquidation [see Leland (1994)]. At the other extreme, when $d > (T - t)$, i.e. the grace period is longer than the maturity of debt, default never leads to liquidation before debt maturity and the FM (2002) model takes on, as a special case, the standard model for default and reorganization [see Anderson and Sundaresan (1996) or Fan and Sundaresan (2000)].

Example 2. *When $\beta = 0$ and $\gamma = 0$, liquidation occurs the first time the firm value spends a total time greater than the pre specified grace period below K_t. Thus, when $\beta = 0$ and $\gamma = 0$, we have the Moraux (2002) liquidation procedure.*

Under this parameterization, each distress period is weighted equally and each period has the same influence on I_t^K. At the one extreme, when $d = 0$, no extra survival time below the distress threshold is allowed, default leads to immediate liquidation of the firm's assets and we get, as a special case, the BC (1976) liquidation model. At the other extreme, when $d \geq (T - t)$, liquidation can occur only at debt maturity, and the model collapses to the basic structural approach introduced by Merton (1974).

In the next section a valuation model for equity and debt is derived under various liquidation procedures. The procedure by which the firm's value is divided among the different claimants as a result of a financial distress is a crucial issue in the pricing of the various contingent claims. FM (2002), following Sundaresan and Fan (2000), assume that during default periods a sharing of cash flow rule results from a bargaining game among the claimants. In this form, reorganization of the firm's assets becomes a necessary precondition for subsequent liquidation, which is a costly procedure. This framework is somehow inconsistent with the actual nature of the bankruptcy procedure, as described by Bebchuk (1997), which prevents debtholders from extracting any value before they agree with the equityholders on the division of the firm's assets. Therefore, we assume in our framework that debtholders can extract value only after the value of the firm assets has exhausted the predetermined grace period under the distress threshold. This approach is consistent with BC (1976) that refers to the threshold level as the point at which debtholders can takeover the firm's assets

3. The Valuation of Defaultable Bonds

In this section we evaluate the various corporate securities by considering the simple case of a firm with risky assets V_t, which is financed by equity, S_t, and one debt obligation, maturing at time T, with par value P, and market value B_t. The bond contract conveys to debtholders, under a *protective covenant*, the right to force liquidation at any time $t \in [0, \mathrm{T}]$, if asset value equals or is lower than an exogenous threshold level K_t. However, the debtholders succeed to force liquidation only when the liquidation trigger, I_t^K, exceeds the predetermined grace period, d. At liquidation, debtholders would receive V_{θ^K} at time θ^K; and equityholders would receive nothing. At debt maturity, T, assuming no early liquidation has been declared, equityholders would receive the maximum between zero and the difference between the firm assets value, V_T, and the promised face value, P. Equityholder rights are isomorphic to the payoff function of a European complex Parisian option, using the indicator function $1_{\{\theta^K > T\}}$ the equityholders payoff is given by the following function:

$$\mathrm{S}(V_T, T, I_T^K, T) = (V_T - P)^+ \, 1_{\{\theta^K > T\}} = \begin{cases} V_T - P & \text{if } V_T > P \text{ and } \theta^K > T \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

The value of the equityholders claim at any time prior to debt maturity $t \in [0,T]$, provided that default has not occurred by time t, is expressed by:

$$S(V_t, t, I_t^K, T) = e^{-rT} E_t^Q[(V_T - P)^+ 1_{\{\theta^K > T\}}] \tag{9}$$

where $E_t[\cdot]$ denotes the conditional expectation under a risk neutral measure Q, given available information at time t.[5]

The value of the zero-coupon bond is decomposed to two sources of value: firstly, its value at maturity, assuming the firm is not prematurely liquidated, and secondly, its value if the firm is liquidated before debt maturity, T, since the pre-determined grace period d was violated by the weighted excursion distress time. As noted by BC (1976), although those two components are mutually exclusive, they are both possible outcomes. Accordingly each contributes to the present value of both equity and debt.[6] The price of a zero coupon bond, B, with maturity $T > t$, under the risk neutral probability measure, is given by:

$$B(V_t, t, I_t^K, T) = \mathrm{E}_t^Q[\min(V_T, P) e^{-rT} 1_{\{\theta^K > T\}}] + \mathrm{E}_t^Q[V_{\theta^K} e^{-r\theta^K} 1_{\{\theta^K \le T\}}] \tag{10}$$

Roughly speaking, the payoff at time $\theta_d \wedge T$ is given according to the no liquidation scenario (the left expression at the right hand side of the equation), in which debtholders receive the minimum between the value of the firm's assets and the par value of debt, or alternatively, should early liquidation take place (the right expression

[5] We use the risk neutral valuation assuming the market is perfect.

[6] Black and Cox (1976) decompose firm value into two additional components: the upper boundary of the security value if the firm is reorganized and the value of the payouts it will potentially receive.

at the right hand side of the equation), debtholders receive the value of the firm's assets at that time.

The next step in evaluating the firm's capital structure is to calculate the zero-coupon yield spread. Practitioners typically quote corporate bond prices in terms of the spread of their yield-to-maturity over the riskless interest rate. The firm credit spread at time t, denoted by sp_t is calculated as:

$$sp_t = \frac{\ln\left(\frac{B_t}{P}\right)}{-(T-t)} - r \tag{11}$$

Given the above assumptions, we can derive the governing partial differential equations and boundary conditions that should be solved to value the firm's stocks and bonds as a function of the three state variables V, t, I.

The relevant form of the valuation equation for the stock, S, will be:

$$\frac{\partial S}{\partial t} + \frac{\sigma^2 V^2}{2}\frac{\partial^2 S}{\partial V^2} + (r-\delta)V\frac{\partial S}{\partial V} - rS + \frac{\partial S}{\partial I} = 0 \tag{12}$$

The boundary conditions are as follows:

$$S(V_T, T, I_T^K, T) = \max\,(V_T - P, 0) \qquad \textit{for} \;\; 0 \le I_T^K < d \tag{13}$$

$$S(V_t, t, d, T) = 0 \tag{14}$$

The relevant form of the valuation equation for the bond, B, will be:

$$\frac{\partial B}{\partial t}+\frac{\sigma^2 V^2}{2}\frac{\partial^2 B}{\partial V^2}+(r-\delta)V\frac{\partial B}{\partial V}-rS+\frac{\partial B}{\partial I}=0 \tag{15}$$

The boundary conditions are as follows:

$$B(V_T, T, I_T^K, T)=\min(V_T, P) \qquad for:\ 0\le I_T^K < d \tag{16}$$

$$B(V_t, t, d, T)=V_t \tag{17}$$

Bergman (1985) has developed a general procedure for pricing path-dependent contingent claims and applied the procedure to the case of the averaging claims. A new term that is proportional to the rate of change of the average is introduced to the Black-Scholes equation. Haber, Schönbucher and Wilmott (1999) have used this extension for the pricing of Parisian options, where a new state variable I_t^K gives rise to a modified form of the Black-Scholes equation. In a standard Parisian option, the clock variable $I_t^{K_B}$ is reset to zero once the price falls below K_t, where in the Parisian contract the knocked-out feature is activated only if the cumulative time spent below K_t exceeds some prescribed value. In our formulation, these two contracts constitute private cases of the more general specification of the virtual distress clock. We let the speed at which I_t^K accumulates be dependent on the distance between the distress threshold and firm value, thus large deviations below the threshold are heavily weight. Countdown speed is also dependent on the distance of previous distress periods from current time, giving greater weight to recent events.

4. The Limitation of Past Liquidation Models Based on the Excursion Time

In this section we describe the two existing liquidation models that are based on excursion time as developed by FM (2002) and Moraux (2002). By using two numerical examples we illustrate the anomalous behavior stemming from each of them, and demonstrate how, through rational determination of the parameters, our general liquidation model can prevent such anomalies.

In both examples, we consider a leveraged firm that issues only one stock and one zero-coupon bond maturing in 10 years. The debtholders are protected by a safety covenant that allows them to declare bankruptcy when the value of the firm's assets is less than the distress threshold K_t. For each of the liquidation models, the distress period before liquidation lasts at most one year, so $d = 1$. However, the state variable that triggers liquidation, I_t^K, is treated differently in each model. According, to Moraux's (2002) *total cumulative excursion method*, liquidation occurs when the value of the firm's assets accumulates more than one year under the threshold level, and thus in our setting equation (7) is parameterized as follow: $\alpha = \beta = \gamma = 0$. According to FM's (2002) *consecutive excursion method*, liquidation occurs when the value of the firm's exceeds a consecutive one-year period under the distress threshold, and thus: $\alpha = \gamma = 0$ and $\beta \to \infty$. To illustrate our adjustable liquidation model we have chosen a third set of parameters: $\alpha = \gamma = 0,$ and $\beta = 0.25$. This parameterization constitutes a special case of our *weighted cumulative excursion method.*

In the first example, depicted in figure 1, the value of the firm's assets between the middle of the second year and the beginning of the seventh year has accumulated three years under the distress threshold. Figure 2 shows the value of the state variable that triggers liquidation according to each method. After 3.5 years, the cumulative distress period is greater than a year and liquidation is triggered according to the *total cumulative excursion model.* The *weighted cumulative excursion method* reduces the impact of previous distress periods and liquidation is postponed by four months, since the excursion periods are not consecutive. In the *consecutive excursion method* the bankruptcy occurs no less than seven times, however, liquidation is avoided since none of these comprise a consecutive twelve-month period. The safety covenant is not respected despite the fact that the firm was in dire financial straits for a prolonged period of time.

In the second example, as described in figures 3 and 4, the value of the firm's assets crosses the distress threshold at the end of the third year for the first time and stays there for a consecutive time of ten months until firm value rebounds above the threshold. Liquidation procedures are not triggered under any of the three models since the liquidation state variable I_t^K is less than one (10/12). In the middle of the ninth year the value of the firm once again falls below the threshold level and stays there three consecutive months. According to the *consecutive excursion method,* the distress clock is reset and liquidation procedures are not initiated after three month. We receive similar results for the *weighted cumulative excursion method,* since the liquidation state variable has fallen to the value of 0.21 from 0.83 given the fact that firm value remained above the threshold for more than five years. In contrast, according to the *cumulative excursion method,* liquidation is warranted. The distress clock is not reset or "moved back" and liquidation is declared immediately after two months. The state variable that triggers default has not forgotten or reduced the impact of the distress period that occurred in the distant past. Since the sensitivity of the trigger in this exaggerates the risk engendered in the current distress episode, one can conclude that the consecutive excursion model has a too strong memory.

5. Numerical Implementation and Sensitivity Analysis

We now turn to the implementation of the model for calculating bond prices, equity prices and the credit spread of a levered firm. We describe the procedure for model implementation, then, comparative statics are presented.

A. Numerical implementation

A first step toward numerical solution is discretizing the partial differential equations for V, and I . Since in most cases, an analytical solution is not available, we need to employ a numerical solution. We follow the Monte-Carlo simulation approach since it is easy to implement and applicable for a wide range of problems presented in this paper. We briefly describe the method, which is discussed extensively in Boyle (1977), Broadie and Glasserman (1996) and Boyle et al. (1997).

Our procedure is as follows:

A. We generate independent replications of the firm's asset value, V , for n short intervals of length Δt and approximate equation (1) as:

$$V_{t+\Delta t}^{(i)} = V_t \exp\left[\left(r - \delta - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t} Z^{(i)}\right] \tag{18}$$

for $i = 1,...,n$, where $Z^{(i)}$ is a standard normal random variable.

B. At each time step, we calculate the value of the discrete threshold level as: $K_i^D = Pe^{-r(T-i\Delta t)}$. To replace the discrete monitored threshold level with a continuous threshold, we use the Broadie, Glasserman and Kou (1995) approximation:

$$K_i^C = K_i^D e^{\rho\sigma\sqrt{\Delta t}} \tag{19}$$

where $\rho = -\zeta\left(\frac{1}{2}\right)\Big/\sqrt{2\pi} \approx 0.5826$ and ζ is the Riemann Zeta function.

C. At each time step along the price path, i, we calculate the weighted excursion period $I_t^{V_B}$ by approximating equation (3) and (4), and then check if the CDT exceeds the pre- determined grace period, θ^K.

D. Firm value is divided among the various claimants according to equations (9) and (10), based on the earliest between the two events T (bond maturity) and θ^K (end of the grace period). Each claim payoff is discounted by the risk free rate.

E. Repeat steps (A)-(C) to collect samples of the bond in a risk- neutral world.

F. Calculate the mean of the samples to estimate the value of the various claims.

B. Application and Analysis

To demonstrate our approach, we consider some realistic examples and perform a sensitivity analysis of the bond price, the equity price and the credit spread with respect to a number of parameters. In order to emphasize the impact of our method on the value of the various corporate claims, we also compare our results to existing structural methods for modeling credit spreads.

As a base case we assume a firm with capital structure which comprising of one stock and one zero-coupon bond with $P = 109.926$ and $T = 5$. Firm value comes to 100 and, as a result the leverage ratio, (which is defined as $LR = Pe^{-rT} / V_t$), is equal to $LR = 0.9$. The risk free interest rate is $r = 4\%$, the pre-determined grace period is $d = 0.25$, the firm asset volatility is 30%, and no payout is expected $(\delta = 0)$. The parameter α is set to zero, and as a result, the distress severity has no influence on the CDT. The parameter γ equals zero as well, which means that each observation on the last distress period has an equal impact on the decision to liquidate the firm's assets. Bondholders hold a contract which enables them takeover the firm at the time the value of the firm's assets is smaller than the discounted balance of debt, and as a result, the distress threshold, as in the BC (1976) model, parallels the secured discounted balance and equals: $K_t = Pe^{-r(T-s)}$. To isolate the impact of the deviations from the provisions of the bondholder's contract on claim value, we assume an absence of costs pertaining to liquidation and financial distress.

We now analyze the determinants of the level of credit spreads and corporate securities values. Table 1 lists the numerical estimates of corporate securities within various structural frameworks of default and liquidation. The credit spread according to the Merton model comes to 5.1%. This high spread stems from the model's underlying assumption that neither liquidation nor default can occur before the contractual maturity of debt, and thus, in instances of financial distress, debtholders cannot extract value from the firm prior to maturity. At the other extreme, BC (1976) assumes that the firm's assets are immediately liquidated upon heating the distress threshold. If this threshold is equal to the secured discounted debt balance, there is no effective credit risk and the credit spread is equal to zero. Figure 7 and table 1 show that as the decay factor of previous distress periods increases, the value of debt decreases. At the extreme, as in FM (2002), when $\beta \to \infty$, the distress clock is reset whenever the firm's asset value crosses the distress threshold from below. When the grace period is short, and equal to one month, the gap between the credit-spread according to the *total cumulative excursion method* $(\beta = 0, \gamma = 0)$ and the *consecutive excursion method* $(\beta \to \infty, \gamma = 0)$ is relatively small and equal to 46 basis points. However, when the grace period increases to three months, and the violation of the terms of the safety covenant are more severe, the gap becomes crucial, reaching 70 basis points. In this case, modeling the true nature of the liquidation procedure becomes more valuable; intermediate values of β may capture the true nature of liquidation procedure more appropriately. When the grace period rises to one year, the gap between the two extreme cases declines to 67 basis points, since the probability of early liquidation by any method is extremely decreased.

Figure 5 and table 2 show that credit spreads increase with asset volatility. However, the impact of an increase of beta has a larger effect when volatility is high. As reported, when σ=40%,. the increase of beta from zero to infinity increases the credit spread by 100 basis points, while a similar shift in beta when σ equals 30% changes the spread by only 50 basis points. The explanation for this observation is fairly straightforward. As volatility increases, the probability that the firm value will fall below the threshold level more frequently increases as well. Although the total cumulative excursion time may approximate that of a less volatile firm, the length of each distress period is shortened, and as a result, the impact of past distress periods on the liquidation trigger is reduced and the violation of the safety covenant becomes more severe.

Figure 6 (and table 6) provides estimated credit spreads for a combination of financial leverage ratios (LR) and decay factors for previous distress periods (β). As beta increases, the gap between the credit-spreads of the two leveraged firms increases. When β=0, the credit spreads of the two leveraged firms are equal to 1.84% and 1.97%. However, when $\beta \to \infty$, the credit spreads amount to 2.36% and 3.76%. The gap between the credit spreads of the two leveraged firms according to Merton model comes to 2.87%. This important outcome emphasizes the fact that as the liquidation trigger is less sensitive to the impact of past distress periods, an increase in financial leverage decreases debtholder protection.

Table 3 summarizes the impact of distress severity (α) on the securities values and credit spreads. Not surprisingly, when the parameter α increases, bondholder protection becomes more efficient. When α increases from zero to thirty, we observed a sharp decline in the credit spread from 184 and 219 basis points to 111 and 118 basis points respectively for β values of 0 and 1.5. As α converges to infinity, any decrease of the firm's asset value below the threshold will spark immediate liquidation and the result will be a convergence to BC (1976) model.

6. Conclusion

We present a simple and general structural model for the valuation of corporate securities where the bondholders' right to force reorganization or liquidation of a distressed firm is immediate and may consume time. To evaluate the impact on corporation capital structure, we develop a general liquidation model driven by a liquidation trigger. Unlike other models, our trigger accommodates a greater number of scenarios and enables a more accurate assessment of financial distress. The liquidation trigger accumulates over time, but is also dependent on the degree to which the threshold is violated. In addition, recent distress episodes can carry a higher influence than older episodes.

We show that by applying the appropriate liquidation parameters, our model converges to François and Morellec (2002) liquidation model, in which liquidation is triggered if the value of the firm's assets has exceeds a consecutive excursion time. Moreover, our general model also accommodates Moraux's (2002) liquidation model, which assumes that liquidation occurs if total excursion time exceeds a pre-determined grace period. While these two models may describe the liquidation procedure accurately for a specific bond indenture or for a specific legal regime, our model, as illustrated in this paper, covers a wide array of legal precepts and contractual arrangements. All of the liquidation models presented above, may be viewed as a middle ground approach between the Merton's framework, where liquidation occurs only upon debt maturity and the Black-Cox model, where reorganization of the firm's assets is invoked when a minimum threshold is violated during the lifetime of the debt.

We illustrate the applicability of our model for the valuation of firms with simple capital structures and we present both comparative statics and sensitivity analysis of the various corporate claims for various indenture provisions, legal regimes and corporate capital structures.

A natural direction for future research is to apply the model to environments characterized by empirically supported dynamics of risk- free short rates and observed credit spreads. Additional features such as interim payments, taxes, liquidation costs, debt subordination and alternative bond indentures can be incorporated as well. Although not a trivial task, exploring these directions may be rewarding in providing new guidance for risk measurement and pricing, as well as for supporting empirical findings and observed behavior patterns in the fixed income and equity markets.

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Table 1

Corporate credit spread and the value of the firm's capital structure for various grace periods and past period decay factors

Scenario	β	*Equity value*	*Debt Value*	*Credit spread*
Base case	$\beta = 0$	17.93	82.07	1.84%
	$\beta = 1.5$	19.31	80.69	2.19%
	$\beta = 3$	20.03	79.97	2.36%
	$\beta \to \infty$	20.74	79.26	2.54%
$d = 0.083$ (One month)	$\beta = 0$	14.46	85.54	1.02%
	$\beta = 1.5$	15.01	84.99	1.14%
	$\beta = 3$	15.27	84.73	1.21%
	$\beta \to \infty$	16.41	83.59	1.48%
$d = 1$ (One year)	$\beta = 0$	24.17	75.83	3.43%
	$\beta = 1.5$	26.53	73.47	4.06%
	$\beta = 3$	26.66	73.34	4.09%
	$\beta \to \infty$	26.67	73.33	4.10%
$d = T$ (Merton 1974)		30.25	69.75	5.10%
$d = 0$ (BC 1976)		10.0	90.0	0.0%

Parameters for the base case are the risk free interest rate $r = 4\%$, the volatility of the firm's assets volatility $\sigma = 30\%$, $P = 109.926$ and $T = 5$. The firm asset value equals 100, and as a result, the leverage ratio, which is defined as $LR = Fe^{-rT}/V_t$, equals $LR = 0.9$. The pre- determined grace period: $d = 0.25$, no payout is delivered $(\delta = 0)$. The liquidation model parameters α and γ are set at zero.

Table 2

Corporate credit spread and the value of the firm's capital structure for various asset volatilities, leverage value and past period decay factor.

Scenario	β	*Equity value*	*Debt Value*	*Credit spread*
Base case	$\beta = 0$	17.93	82.07	1.84%
	$\beta = 1.5$	19.31	80.69	2.19%
	$\beta = 3$	20.03	79.97	2.36%
	$\beta \to \infty$	20.74	79.26	2.54%
$d = T$ (Merton 1974)		30.25	69.75	5.10%
$d = 0$ (BC 1976)		10.0	90.0	0.0%
$\sigma = 40\%$	$\beta = 0$	21.02	78.98	2.61%
	$\beta = 1.5$	22.90	77.10	3.09%
	$\beta = 3$	23.89	76.11	3.35%
	$\beta \to \infty$	24.82	75.18	3.60%
$d = T$ (Merton 1974)		36.21	63.78	7.97%
$LR = 0.95$ ($P = 116.03$)	$\beta = 0$	13.93	86.07	1.97%
	$\beta = 1.5$	15.46	84.54	2.33%
	$\beta = 3$	16.27	83.73	2.53%
	$\beta \to \infty$	21.28	78.72	3.76%
$d = T$ (Merton1974)		28.18	71.82	5.60%
$d = 0$ (BC 1976)		5.00	95.00	0.00%

Parameters for the base case are similar to those of table 1.

Table 3

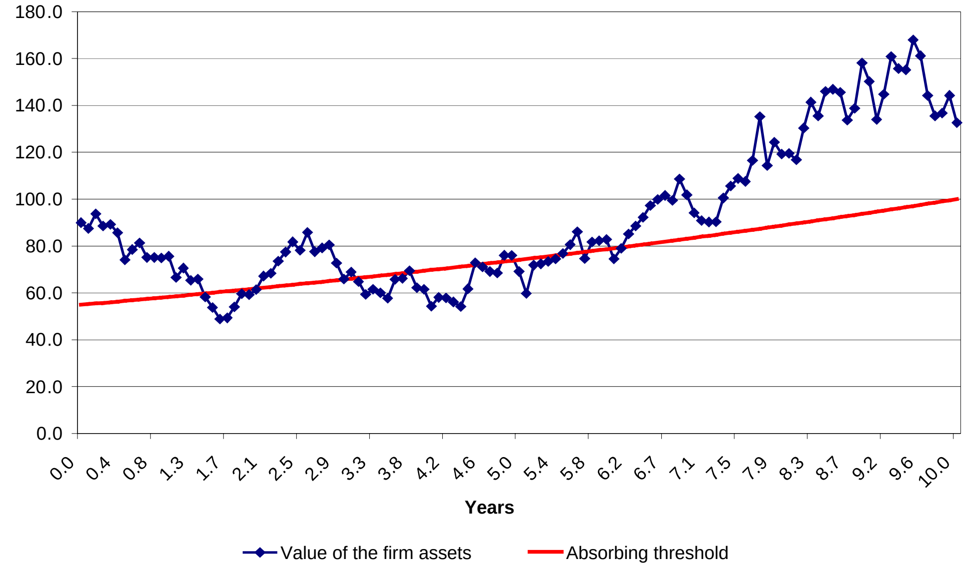
Corporate credit spread and the value of the firm's capital structure for various liquidation model parameters

Scenario	β	*Equity value*	*Debt Value*	*Credit spread*
Base case	$\beta = 0$	17.93	82.07	1.84%
	$\beta = 1.5$	19.31	80.69	2.19%
	$\beta = 3$	20.03	79.97	2.36%
	$\beta \to \infty$	20.74	79.26	2.54%
$\alpha = 3$	$\beta = 0$	17.42	82.58	1.72%
	$\beta = 1.5$	18.54	81.46	1.99%
	$\beta = 3$	19.09	80.91	2.13%
	$\beta \to \infty$	19.77	80.23	2.30%
$\alpha = 30$	$\beta = 0$	14.85	85.15	1.11%
	$\beta = 1.5$	15.17	84.83	1.18%
	$\beta = 3$	15.30	84.70	1.21%
	$\beta \to \infty$	15.69	84.31	1.31%

Parameters for the base case are similar to those of table 1.

Figure 1

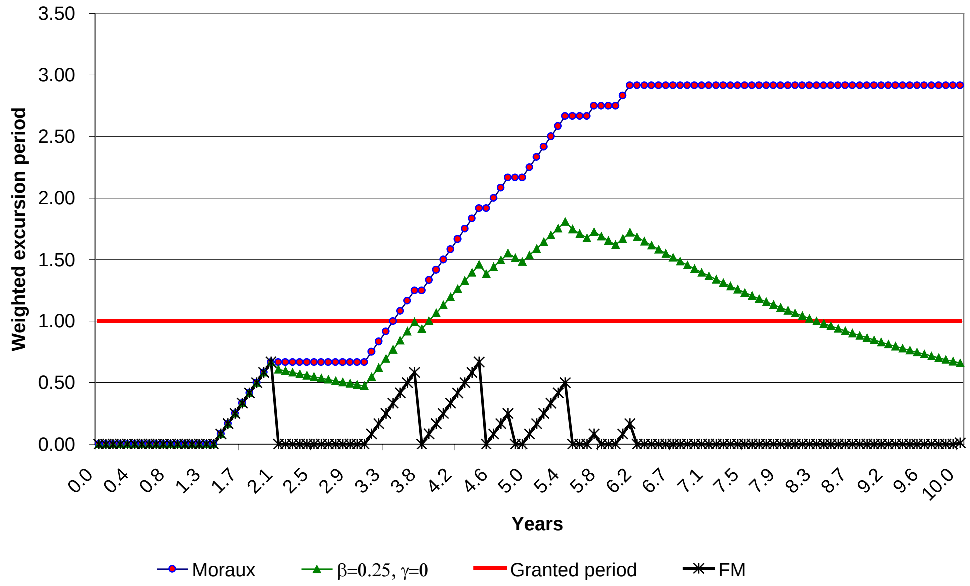
Example 1: Simulation of the firm's asset value and the distress threshold.



In Figure 1 we simulate one path of the distress threshold and firm value over a ten- year period, as discussed in example 1 in chapter 4. The distress threshold is worth $K_t = Pe^{-rt}$, where $r = 0.04$ and $P = 100$.

Figure 2

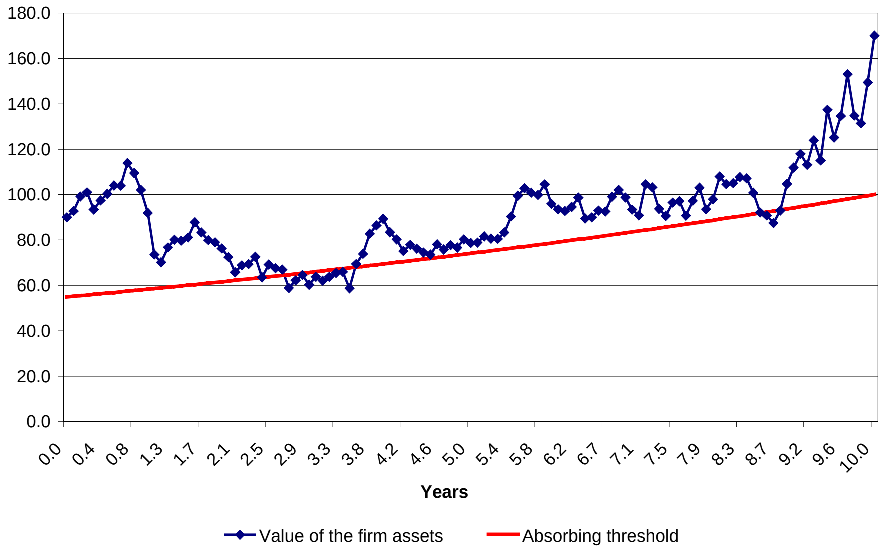
Example 1: Simulation of excursion time according to alternative trigger models.



In figure 2 the weighted excursion period is calculated for the firm's path value presented in figure 1. The grace (change terminology on chart as well, if accepted) period is set at $d = 1$.

Figure 3

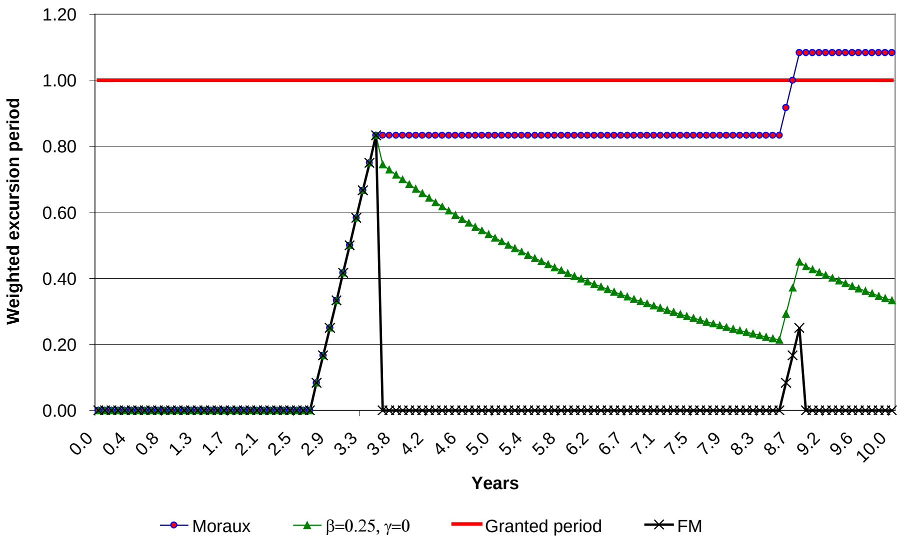
Example 2: Simulation of firm value and the distress threshold.



In Figure 3 we simulate one path of the distress threshold and the firm's asset value over a ten- year period, as discussed in example 2 in chapter 4. The distress threshold is $K_t = Pe^{-rt}$, where $r = 0.04$ and $P = 100$.

Figure 4

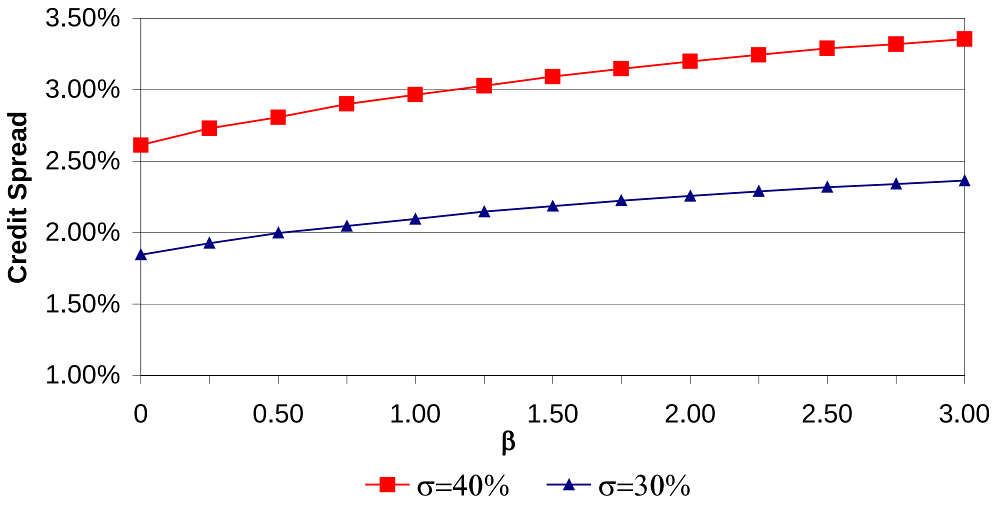
Example 2: Simulation of excursion time according to alternative trigger models.



In figure 4, the weighted excursion period is calculated for the firm's path value presented in figure 1. The grace (change on chart as well if terminology is accepted) period is set at $d = 1$

Figure 5

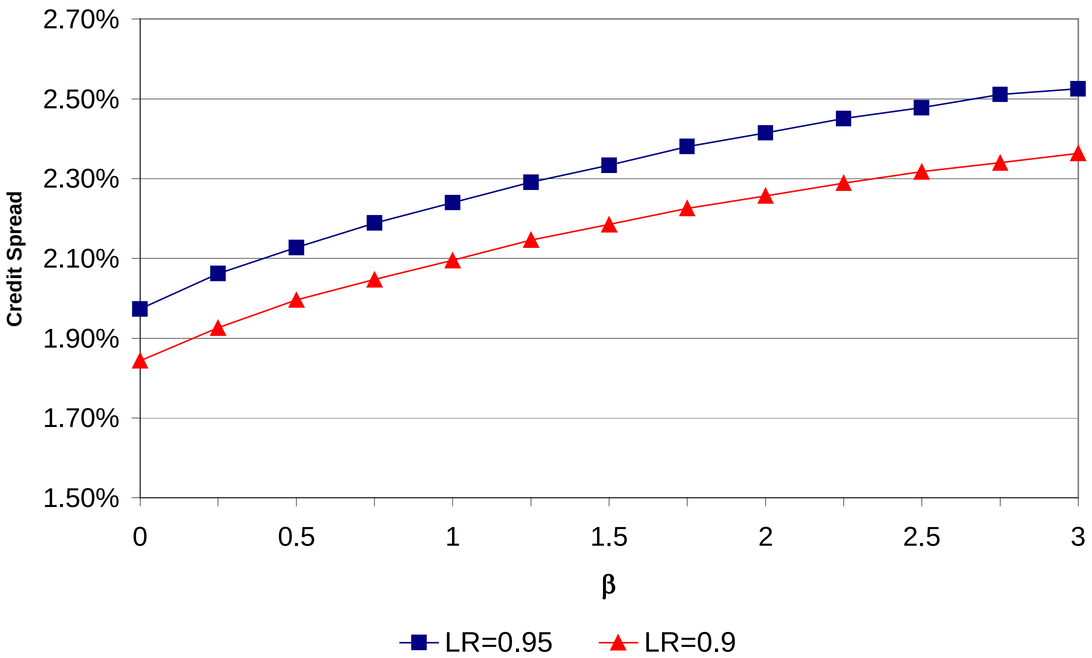
Corporate credit spread as a function of past distress decay factor (β) and asset volatility (σ).



Parameters: See table 1.

Figure 6

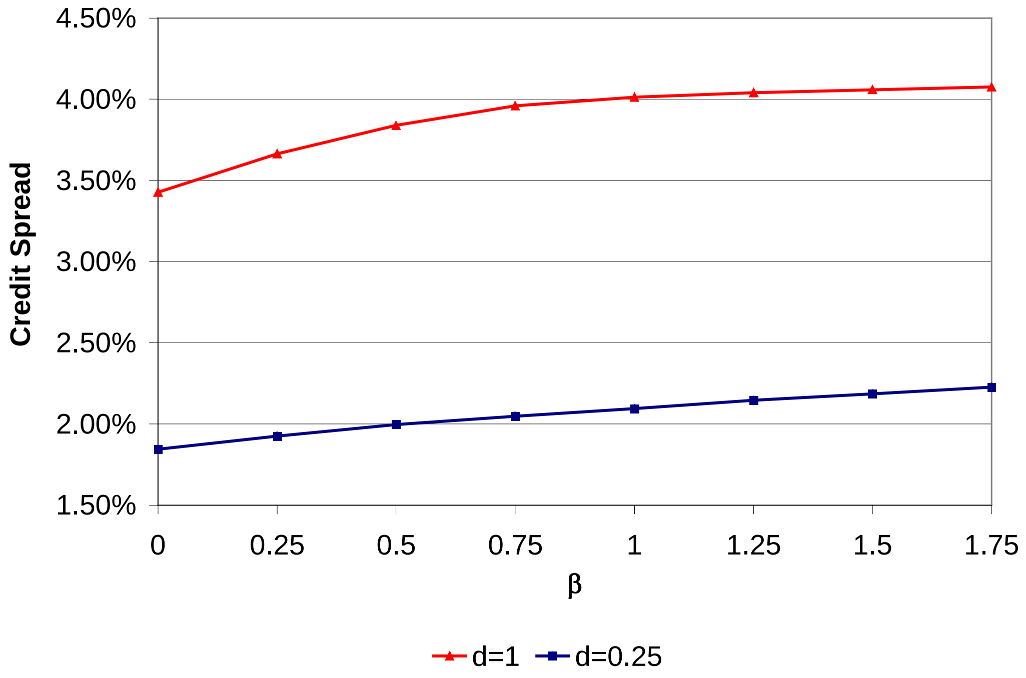
Corporate credit spread as a function of past distress decay factor (β) and leverage ratio (LR).



Parameters: See table 1.

Figure 7

Corporate credit spread as a function of past distress decay factor (β) and grace period (d).



Parameters: See table 1.