The quality of the signal matters – A note on imperfect observability and the timing of moves*

Wieland Müller
Humboldt–University†

February 3, 1999

Abstract

In a recent study Huck and Müller (1998) report that–in contrast to Bagwell's (1995) prediction–first movers in a simple experimental market do not lose their commitment power in the presence of noise. The present note shows that it is the quality of the signal and not the knowledge about the physical timing of moves that is responsible for these experimental results. Additionally, the findings reported here provide further evidence that the positional order protocol cannot induce non–equilibrium play.

1. Introduction

Bagwell (1995) shows in the context of a two–stage two–player game that any noise associated with the observation of the first mover's choice eliminates the first mover advantage. In a recent study Huck and Müller (1998) experimentally assess the behavioral relevance of this claim. More precisely, they implemented several versions of a simple two–person sequential–move game similar to an example given by Bagwell (1995). These versions varied in the quality of the signal informing the second mover. In treatments with noise-levels up to 10 % they observe play settling down close to the Stackelberg outcome favoring the first mover (contrary to Bagwell's prediction). In the treatment with 20 % noise play coincides with

*I wish to thank Werner Güth, Steffen Huck and Jörg Oechssler for helpful comments and Sabine Kröger and Christiane von Trotha for helping to conduct the experiments. Financial support from the TMR project "Savings and Pensions" of the European Commission and the Deutsche Forschungsgemeinschaft, Sonderforschungsbereich 373 "Quantifikation und Simulation ökonomischer Prozesse", Humboldt-Universität zu Berlin is gratefully acknowledged.

†Institute for Economic Theory III, Spandauer Strasse 1, 10178 Berlin, Germany, Fax +49 30 20935704, email *wmueller@wiwi.hu–berlin.de.*

the Stackelberg respectively Cournot outcome roughly half of the time. The explanation for the behavior in the noisy–leader games is simple: No matter what the level of noise is, second movers tend to identify the signal with the action taken by first movers and play a best response against the assumed action. Whereas this is learned and exploited by first movers in the games with lower levels of noise, this is not the case in the game with high noise. Thus, for experienced players no support for Bagwell's claim is found. However, there is some support for the so–called noisy Stackelberg equilibrium; an equilibrium in mixed strategies that converges to the Stackelberg outcome as the noise goes to zero. This equilibrium in mixed strategies (one of two such equilibria) has been emphasized by van Damme and Hurkens (1997).

The motivation of the present note is twofold: First, I will explore the boundaries of the results reported in the above-mentioned study. If the noise level is strictly between 25 and 75 % the equilibria in mixed strategies do no longer exist so that the Cournot equilibrium in pure strategies is the unique game theoretic prediction. What happens in that case? Will second movers still adapt to the signal? Secondly, I explore to which extent the findings of the earlier study can be explained by the physical timing of decisions. Of course, second movers in this study knew–in addition to the imperfect signal–that the first mover has already taken his action. So, is it the case that the (quality of the) signal doesn't matter that much and that all that matters to the results is the knowledge about the physical timing of moves?

Therefore, I implement two further versions of the game mentioned above: In one version the level of noise is so high that the there is no equilibrium in mixed strategies and in another version second movers do not receive a signal at all; they simply know that the first mover has already taken his action.[1] If subjects continue to play near the Stackelberg equilibrium in both cases this would question the support for the noisy Stackelberg equilibrium found in the previous study.

Of course, the latter treatment is a further investigation of the positional order protocol that has been proven in a number of studies to affect behavior in games in which first movers' actions are not observable (see e.g. Cooper et al (1993), Camerer et al (1996), Rapoport (1997), and Güth et al. (1998)). It is, however, questionable whether behavior in the noisy–leader games can be explained by the positional order effect or by 'virtual observability'. The latter principle has been named by Camerer et al. (1996) and was stated in the following way: "Fix a game of imperfect information in which players do not observe earlier moves. Erase the information sets and compute the subgame–perfect equilibria. Then

[1] Note that this treatment corresponds with a situation in which the noise level equals 50%.

restore the information sets and check if the subgame–perfect equilibrium the first mover prefers is a Nash equilibrium in the restored game. If so, play that equilibrium. If not, ignore timing and play a Nash equilibrium" (p. 5). As already noted by Camerer et al (1996) virtual observability does not select the Stackelberg equilibrium in the game investigated (for details see below) since it is not a Nash equilibrium in the restored game. Thus, the positional order protocol cannot induce non-equilibrium play, a claim, supported by the findings of Güth et al (1998).

The remainder of the paper is organized as follows: Section 2 presents an analysis of the implemented game and introduces the experimental design. In Section 3 the experimental methods and procedures are described. The results of the experiments are presented in Section 4. Finally, the findings will be discussed in Section 5.

2. Analysis and experimental design

I study a 2–player game which is similar to the example provided by Bagwell (1995, p. 272). The first mover (or Stackelberg leader) can choose between S and C. Afterwards the second mover (or follower) receives a signal about the leader's decision. The signal is either s or c. After each signal the follower has two choices called S_s and C_s in case the signal was s and S_c and C_c in case the signal was c. Figure 2.1 shows the extensive form game for the case of a perfect signal.

Let $\varepsilon = \text{prob}(c \mid S) = \text{prob}(s \mid C)$ be the probability of receiving the wrong signal. If $\varepsilon = 0$ the strategy vector $(S, (S_s, C_c))$ is the unique subgame–perfect equilibrium. As Bagwell has shown, as soon as $\varepsilon > 0$ (and $\varepsilon < 1$), i.e. as soon as there is even the slightest amount of noise the unique equilibrium in pure strategies is the Cournot equilibrium $(C, (C_s, C_c))$. Here the leader chooses C and–expecting this–the follower ignores his signal and always chooses C. For $0 < \varepsilon < 1/4$, the pure Cournot equilibrium is accompanied by two equilibria in mixed strategies:

$$\text{prob}(S) = 1 - \varepsilon, \quad \text{prob}(S_s) = 1 \quad \text{and} \quad \text{prob}(S_c) = \frac{1 - 4\varepsilon}{2 - 4\varepsilon} \tag{2.1}$$

and

$$\text{prob}(S) = \varepsilon, \quad \text{prob}(S_s) = \frac{1}{2 - 4\varepsilon} \quad \text{and} \quad \text{prob}(S_c) = 0. \tag{2.2}$$

Note that the mixed strategy equilibrium (2.1) converges to the Stackelberg outcome and the mixed strategy equilibrium (2.2) converges to the Cournot out-

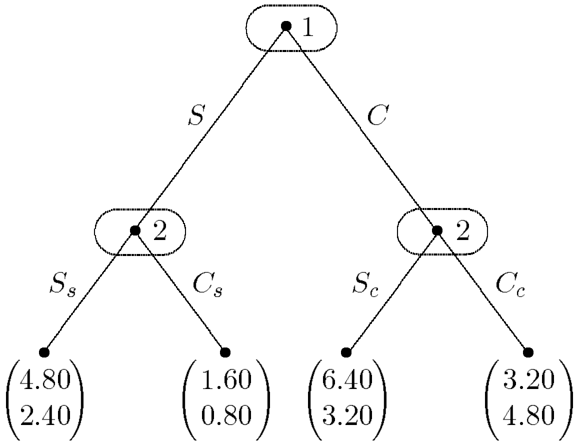

Figure 2.1: The game in case of a perfect signal.

come as the noise, ε, goes to zero. If, however, $\varepsilon \in (.25, .75)$ the Cournot equilibrium in pure strategies is the unique game theoretic solution of the game. This can be seen most easily by inspecting the strategic form of the above game.

I implement two versions of this game. In treatment NOISE the parameter $\varepsilon = .4$, i.e. the probability of receiving the wrong signal is equal to 40%.[2] In treatment POP (Positional Order Protocol), after the first mover has made his choice, the second mover receives no signal at all. From a game theoretic perspective the mere knowledge of physical timing is irrelevant. Thus, the game played in treatment POP is equivalent to the game in which both players decide simultaneously. The unique Nash equilibrium in that case is both players choosing action C. As already mentioned in the introduction the principle "virtual observability" also predicts this outcome.

In both treatments the game is played for ten successive rounds with full anonymity between subjects employing a random matching procedure ensuring that nobody would meet the same opponent twice.

Summarizing the theoretical predictions one should expect

Prediction A: In both treatments most outcomes will coincide with the Cournot outcome.

[2] In the four treatments reported in Huck and Müller (1998) ε was respectively set equal to 0, .01, .1 and .2.

If–in contrast to the theoretical predictions–the timing affects the outcomes in both games in such a way that the first mover is favored one should expect

Prediction B: A considerable fraction of the outcomes will coincide with the Stackelberg outcome.

3. Method and procedure

The experiments reported in this study were conducted at Humboldt University in November 1998. Forty six subjects participated in the two sessions. Twenty four subjects were allocated to treatment NOISE and another twenty two subjects to treatment POP. The participants were undergraduate students of economics or business administration.

The experiments were run with pen & paper. Subjects were seated in large lecture rooms with enough space between them to rule out communication. The randomly assigned roles were kept fix through the entire experiment. The instructions informed participants that there would be ten rounds of the experiment with individual feedback between the rounds and that the matching would be random, but that nobody would meet the same opponent twice. Sessions lasted about one hour. Subjects' average total earnings were DM 43.17.[3]

The frame of both treatments was identical and as neutral as possible. The game was illustrated by a graph, players were labelled A (first mover) and B (second mover), and choices were simply labelled l(eft) and r(ight) for the first and L and R for the second mover. In treatment NOISE first movers received a small white sticker and an envelope. They had to write down their decision on the sticker. Then they stuck it inside the envelope and wrote their codenumber on the envelope. After that subjects carried out the chance move by grabbing numbered chips out of an urn containing 100 chips. Depending on the chosen chip the experimenter wrote the signals on the envelopes which were sealed and collected afterwards.[4] The sealed envelopes were then handed out to the followers who had to write code numbers and decisions on them. When all subjects acting as a follower had made their decisions they were allowed to open the envelopes to learn about the actual decisions of their partners. After that the envelopes were passed back to the leaders in order to inform them about the reaction of the followers. This completed a round.

[3]Note that subjects in treatment NOISE received an additional flat payment of DM 10 to compensate them for the longer time they had to spent.

[4]The second mover got the wrong signal if the chip showed the numbers from 61 to 100 otherwise she got the right signal.

Treatment POP was conducted similarly with the exception that there was no chance move and that second movers did not get any information about the first mover's choice.

4. Results

Table 4.1 summarizes the results of the two treatments. For each round the table shows the total absolute frequencies of first and second movers' decisions at their respective information sets. The two bottom lines show aggregated choices across rounds.

Consider first treatment NOISE: In the first round of this treatment two thirds of the first movers commit themselves to the Cournot action C while one third chooses action S. Only three of the twelve second movers do not decide according to the Nash equilibrium prediction in round one by choosing S_s after receiving signal s. In the course of the session first movers continue to choose rather action C than action S. And in the tenth round all first movers have learned to choose action C. With regard to second movers one observes that after receiving signal c most subjects react by choosing action C_c whereas after receiving signal s actions S_s and C_s are chosen about half of the time (see also the bottom line of Table 4.1). However, in the tenth round only 3 second movers violate the unique Nash equilibrium prediction by choosing action S_s after receiving signal s. Thus, subjects in this treatment appear to play the Cournot equilibrium.

Next, consider treatment POP: With regard to first movers one observes that behavior is rather stable over the rounds. In all rounds at most 2 out of 11 first movers choose action S. The S-choices in rounds 5 to 8 do not stem from the same subjects. In the last two rounds all first movers choose action C. Regarding the behavior of second movers one observes that during the first five rounds there are only few subjects choosing S rather than C. However, from round 6 on–with only one exception–all second movers choose action C. It is interesting to note that first movers seem to know that "anteriority" isn't advantageous in that game since they do not even seriously attempt to try to select the Stackelberg outcome (see Table 4.1).

Summarizing these results one has

Observation 1. *In both treatments subjects learn to play the unique Nash equilibrium; play clearly converges to the Cournot outcome.*

Thus, the experimental results strongly support Prediction A and falsify Prediction B.

	Noise					Pop	
	S		C		Wrong	S	C
Round	S_s	C_s	S_c	C_c	Signals	S	C
1st	4		8		$2S \to c$	1	10
	3	3	-	6	$4C \to s$	4	7
2nd	6		6		$3S \to c$	2	9
	3	3	3	3	$3C \to s$	2	9
3rd	3		9		$3C \to s$	1	10
	4	2	1	5		4	7
4th	2		10		$2S \to c$	–	11
	1	2	1	8	$3C \to s$	4	7
5th	3		9		$1S \to c$	2	9
	1	3	4	4	$2C \to s$	2	9
6th	3		9		$1S \to c$	2	9
	4	3	–	5	$5C \to s$	–	11
7th	4		8		$3S \to c$	2	9
	2	1	2	7	$2C \to s$	–	11
8th	1		11		$7C \to s$	2	9
	4	4	1	3		–	11
9th	2		10		$3C \to s$	–	11
	4	1	2	5		–	11
10th	–		12		$7C \to s$	–	11
	3	4	–	5		1	10
Aggr.	28		92			12	98
Choices	29	26	14	51		17	93

Table 4.1: Summary of experimental results: Treatment Noise (left) and treatment Pop (right).

5. Discussion

In Huck and Müller (1998) it is found that–in contrast to Bagwell's prediction–first movers (in a simple experimental market) do not lose their commitment power in the presence of noise. Instead they found some support for the so–called noisy Stackelberg equilibrium favoring the first mover that is selected by an approach suggested by van Damme and Hurkens (1997). The motivation of the present study is to assess this finding and to explore whether the results can at least partially be explained by the fact that second movers–although not knowing what exactly happened–do know that first movers have already decided. This question arises naturally since in a number of studies, it was shown that the physical timing of decisions may serve as a selection device favoring the player who moves first. However, the results of this study point to the following facts. First, the quality of the signal and not (the knowledge of) the timing of moves matters: If the quality of the signal is very low such that the noisy Stackelberg equilibrium no longer exists, play clearly converges to the Cournot outcome. Second, the results of the previous study cannot be explained by a timing effect favoring the first mover: If second movers do not receive any information about the first movers' action, play clearly converges to the Cournot outcome. Taken together, these results suggests that the evidence for the noisy Stackelberg equilibrium found in Huck and Müller (1998) should be taken seriously. So further experimental work dealing with that issue is called for. Furthermore, the results of treatment POP confirm the conjecture of Camerer et al. (1996) that "virtual observability" doesn't select the Stackelberg outcome in the game at hand. On a more general level this result provides further evidence that the positional order protocol cannot induce non–equilibrium play (see Güth et al 1998).

References

[1] Bagwell, K. (1995): Commitment and Observability in Games, *Games and Economic Behavior,* **8**, 271-280.

[2] Camerer, C., Knez, M. and Weber, R. A. (1996): Timing and virtual observability in ultimatum bargaining and 'weak link' coordination games, *Social Science Working Paper No. 970*, California Institute of Technology

[3] Cooper, R., DeJong, D. V., Forsythe, R. and Ross, T. W. (1993): Forward induction in the Battle–of–the–Sexes games, *American Economic Review*, **83**, 1303-1316.

[4] Güth, W., Huck, S., and Rapoport, A (1998): The limitations of the positional order effect, *Journal of Economic Behavior and Organization*, **34**, 313-325.

[5] Huck, S, and Müller, W. (1998): Perfect versus imperfect observability–An experimental test of Bagwell's result, *Discussion paper No. 120*, Humboldt–University Berlin.

[6] Rapoport, A. (1997): Order of play in strategically equivalent games in extensive form, *International Journal of Game Theory* 26, 113 - 136.

[7] van Damme, E., and Hurkens, S. (1997): Games with Imperfectly Observable Commitment, *Games and Economic Behavior*, **21**, 282-308.

Appendix

Translated Instructions of treatment Noise

[As explained in the text treatment Pop was conducted in the same way as treatment Noise except for the fact that second movers did not get a signal at all. The following instruction were adapted accordingly.]

Read this sheet carefully. In case of questions, give notice! We will then come to you and answer them privately.

Welcome to our experiment, in which you can earn some money—depending on your decisions and the ones of randomly chosen other participants. The rules are quite simple. Look at the following decision tree:

[Figure of decision tree, similar to Figure 2.1 with the exceptions that players are labeled "A" and "B", resp., and that player 1's actions are labeled "l" and "r" and player 2's "L" and "R".]

First, A decides between "l" and "r". Then, before making his own choice, B is informed about the decision of A. This information is only partially reliable. This is because it is determined by chance whether B receives the correct information or a wrong one. This works as follows: After A has made his decision, we take a random draw out of 100 chips. These chips are numbered from 1 to 100. If A draws one of the first 60 chips, B will receive the correct information about A's decision. This means that if A has chosen "l" (respectively "r"), we will tell B that A has chosen "l" (respectively "r"). If A draws a chip with a number from 61 to 100, B receives an incorrect information. This means that if A has chosen "l" (respectively "r"), we will tell B that A has chosen "r" (respectively "l"). After B has received the information about A's choice, he has to make his decision. This means that B has to decide between "L" and "R". The real decision of A (which, as explained above, need not necessarily be identical with the one transmitted to B) and the decision of B determine the payoffs.

There are four possible cases:

A chooses "l", B chooses "L": In this case A receives 4,80 DM and B receives 2,40 DM.

A chooses "l", B chooses "R": In this case A receives 1,60 DM and B receives 0,80 DM.

A chooses "r", B chooses "L": In this case A receives 6,40 DM and B receives 3,20 DM.

A chooses "r", B chooses "R": In this case A receives 3,20 DM and B receives 4,80 DM.

The procedure is as follows: A writes his decision whether to choose "l" or "r" on a little sticker. Then he sticks the sticker on the inside of an envelope, without closing it. After the random draw has decided about the information to be transmitted to B, we write the according information on the envelope and close it. Then each envelope is given to a randomly chosen B. B receives the envelope and writes his decision on it, without opening it. Finally, all envelopes are collected by the experimenters.

This is the end of the first round of the experiment. After this there will be nine further rounds, in each of which you will be randomly matched with a different participant. We will ensure that you will be matched with ten different participants during the ten rounds.

After each of the ten rounds all participants are informed about the outcome of their round.

To guarantee anonymity you receive a code number. Please keep your code card carefully, because you will only obtain your payoff, when showing this card. In addition to this, the code number ensures your anonymity towards us and the participant you are matched with.

Your total payoff is the sum of the single payoffs of the ten rounds. In addition to this you will receive DM 10 independent of the outcome of the rounds [only in treatment NOISE].

You have the role A [B].