

# **Enlargement and the Balance of Power: an Experimental Study**

by

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## **Abstract**

Many important decisions are taken according to weighted majority rule. Power indices predict that enlargement of the voting body may affect the balance of power between the original members even if their number of votes and the decision rule remain constant. Some of the existing voters may actually gain, a phenomenon known as the *paradox of new members*. We test for this effect using laboratory experiments. Participants propose and vote on how to divide a budget according to weighted majority voting rules, and we measure the voting power of a player by his average payoff in the experiment. By comparing voting power across voting bodies of varying size, we find empirical support for the paradox of new members. Our results also allow an assessment of the predictive performance of standard power indices.

Keywords: voting, power indices, experiments, paradox of new members

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## **INTRODUCTION**

Many important collective bodies make decisions by weighted majority voting: different members control different numbers of votes. Examples are the Electoral College in the United States, the International Monetary Fund, and the Council of Ministers in the European Union.

Different voting weights typically reflect differences in sizes of populations or resources of the states. However, it is well known that the voting power of a state may be very imperfectly correlated with its voting weight (see e.g. Banzhaf, 1965). For example, a state controlling 51% of the votes has all the power under simple majority voting. The imperfect correlation between weights and power makes institutional design more complex. In order to analyze voting power a number of power indices have been proposed, the most important of which are the Shapley (1953) value or Shapley-Shubik (1964) index, and the Banzhaf (1965) index. These indices often predict unintended consequences of institutional change. For example, in the context of changes in the IMF voting rules, Dreyer and Schotter (1980) claim that there is “a noticeable discrepancy between what one would think the consequences of the voting changes would be and what they actually are”.

A particular instance of counterintuitive effects of institutional change is the paradox of new members (Brams and Affuso, 1976). Brams and Affuso show that when a new member is added to a voting body the power indices of some original members may increase, even if the original members’ weights and the decision rule remain unchanged. The reason is that the addition of the new member may present some of the existing members but not others with greater opportunities to form winning coalitions.

Brams and Affuso’s examples are not merely theoretical curiosities. For EU enlargements it has been viewed as desirable that “the existing balance of power between the Member States ... be broadly preserved” (Enlargement of the Community: Transitional Period and Institutional implications, 1978; cited in Brams and Affuso, 1985); between 1973 and 1986 the voting weights of existing members and the percentage of votes needed for a majority have been kept constant in enlargements of the EU Council of Ministers (see e.g. Felsenthal and Machover, 2001, table I). However, formal power analysis suggests the existing balance of power

was *not* preserved in any enlargement: every enlargement increased either the Shapley value or Banzhaf index for some existing member state (Brams and Affuso, 1985).<sup>2</sup>

Of course, whether the enlargements have actually increased an original member state's voting power is more difficult to ascertain. A natural empirical measure of voting power is the proportion of the benefits from legislation that accrue to a voter, but in order to use this one would need information on the material consequences of enlargement of the voting body. Two difficulties immediately present themselves. First, changes in the membership of the voting body are usually accompanied by other changes in the legislative process, so that it may be difficult to disentangle the effect of enlargement from changes in these other factors. Second, the material consequences of membership may be difficult to measure, and also be influenced by numerous other factors. For example, even for the case of the EU, where weights and decision rules have remained stable across enlargements, it is difficult to measure the benefits of membership for a particular member, let alone how these benefits have changed due, *ceteris paribus*, to enlargement.<sup>3</sup>

These difficulties with uncovering empirical relationships between voting structure and voting power have led to a heavy reliance on assumptions about the relevant power index for formal analysis. The objective of our paper is to use laboratory experiments to empirically examine the relation between the distribution of voting weights and voting power. In our experiment participants propose and vote on how to distribute a fixed budget among themselves.<sup>4</sup> This approach allows us to use a natural empirical measure of voting power -- the average share of the budget realized by a voter -- and relate it to voting weight, decision rule, and the composition of the voting body. In particular we study three treatments, corresponding to the examples in Brams and Affuso (1976), and examine how the balance of power between the original parties is affected by the addition of a new member. We find significant discrepancies between empirical measures of voting power and standard power indices. However, in a comparative static sense the theoretical predictions of Brams and Affuso are borne out. In particular, the empirical voting power of an existing

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<sup>2</sup> The paradox has also theoretically occurred in the US Electoral College (see Brams and Affuso, 1976) and a weak version of it has been claimed to be common in the Dutch parliament (see van Deemen and Rusinowska, 2003).

<sup>3</sup> There have been attempts to correlate voting weights and net financial transfers in the EU, but these are qualified by the substantial difficulties with obtaining accurate net transfer data: see Baldwin (1994).

<sup>4</sup> Thus we focus on purely distributive policies.

member increases with the addition of a new member, thus the paradox of new members is observed.

### **THREE VOTING GAMES**

Our three treatments correspond to the examples used in Brams and Affuso (1976). In all treatments there is a ‘strong’ player with three votes and two ‘weak’ players with two votes each. In our first treatment these three players comprise the voting body, and five votes are needed to pass a proposal. Here, the strong player has veto power, since no proposal can be passed without her votes, and so we refer to this as our VETO treatment. Our second treatment is identical except that only four votes are needed to pass a proposal. As a consequence, no player has a strategic advantage over the others, since any two members have enough votes to pass a proposal. This is reflected by all sophisticated measures of voting power, which assign equal power to each player, and thus we refer to this as our SYMMETRIC treatment. In our third treatment five votes are needed to pass a proposal, and there is an additional member with a single vote: we refer to this as our ENLARGED treatment.

In all treatments proposals specify how to divide a budget of 120 points. Table 1 contains predicted voting powers for each treatment. The predicted voting power of a player is the number of points they can expect to gain; the predictions are based on the core (which, when empty, is replaced by Schmeidler’s (1969) extension, the Nucleolus), the two best-known power indices (Shapley and Banzhaf) and a naïve power index that assigns payoffs proportionally to the voting weights.<sup>5</sup>

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<sup>5</sup> Conceptually the Shapley value seems the most appropriate power index for distributive decisions (Roth, 1977a, 1977b; Laruelle and Valenciano, 2005). However, there are some arguments in favor of the Banzhaf index (see Felsenthal and Machover, 1998, p.174 for a discussion), and the Nucleolus (see Montero, 2005).

**Table 1. Summary Predictions**

VETO Treatment

		Voting Power			
	Votes*	Core/ Nucleolus	Shapley	Banzhaf	Proportional
Player 1	3	120	80	72	51
Player 2	2	0	20	24	34
Player 3	2	0	20	24	34

\* 5 votes required to pass a proposal

SYMMETRIC Treatment

		Voting Power			
	Votes*	Nucleolus	Shapley	Banzhaf	Proportional
Player 1	3	40	40	40	51
Player 2	2	40	40	40	34
Player 3	2	40	40	40	34

\* 4 votes required to pass a proposal

ENLARGED Treatment

		Voting Power			
	Votes*	Nucleolus	Shapley	Banzhaf	Proportional
Player 1	3	40	50	50	45
Player 2	2	40	30	30	30
Player 3	2	40	30	30	30
Player 4	1	0	10	10	15

\* 5 votes required to pass a proposal

Relative to VETO, ENLARGED allows us to test whether the paradox of new members occurs when the total number of votes required to pass a proposal is held constant. As with any enlargement, the naïve, proportional, power index predicts all the original members lose voting power and so the paradox will not occur. However, the other measures predict that the weak players' voting power will increase. This is because the addition of the new member eliminates the strong player's veto power, and so the other original members no longer depend on the strong player.<sup>6</sup>

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<sup>6</sup> For this type of enlargement, where a player loses veto power, it is intuitive that other original members will be empowered. However, this is not a general result. Suppose four voters have 4, 2, 1, and 1 votes respectively, and 7 votes are required to pass a proposal (so that the two larger players have veto power). If a fifth voter with 4 votes is added the large players lose veto power, but the other original members are worse-off according to the Shapley and Banzhaf measures and unaffected according to the Nucleolus.

Relative to SYMMETRIC, ENLARGED allows us to test the paradox of new members when a simple majority of votes is required to pass a proposal. Here, the effect on the original members is less intuitive. Intuitively, the weak original members are worse-off after enlargement: they no longer have enough votes to enforce a proposal on their own, and now need the cooperation of the new member. The large player becomes the most powerful in relative terms, but it is not clear whether his power should increase in absolute terms; this is indeed the case according to the Banzhaf and Shapley measures. The Nucleolus makes the extreme prediction that the existing members are unaffected.

## **THE EXPERIMENT**

### **Design and Procedures**

The experiment comprised twelve sessions conducted at the University of Nottingham using subjects recruited by e-mail from a university-wide pool of undergraduate students. Four sessions were conducted with each treatment, and involved either 12 subjects (VETO and SYMMETRIC treatments) or 16 subjects (ENLARGED treatment) per session. Thus, 160 subjects participated in total.

All sessions used an identical protocol. Upon arrival, subjects were given a written set of instructions that the experimenter read aloud.<sup>7</sup> Subjects then reviewed the instructions on their computer screens and were allowed to ask questions by raising their hands and speaking to the experimenter in private. Subjects were not allowed to communicate with one another throughout the session, except via the decisions they entered on their terminal.

The decision-making phase of the session then consisted of 10 rounds. At the beginning of each round subjects were assigned to groups of either three or four (depending on treatment). Subjects were not told who of the other people in the room were in their group, and group compositions changed from round to round. In particular, the same set of subjects was never matched together twice. At the beginning of each round subjects were also assigned roles, determining how many votes they controlled, and roles also varied across rounds. In every round subjects entered decisions anonymously, so that it was not possible to build up a reputation across rounds. For statistical reasons, prior to the first round we formed two equally-

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<sup>7</sup> A copy of the instructions for the VETO treatment can be found in Appendix A.

sized subsets of subjects and then formed groups from within these subsets; no information passed across the two subsets, and so this procedure ensured that each session resulted in two independent observations.

Each round had a random time limit between five and ten minutes for groups to bargain over the division of 120 points. If no agreement were reached before the random deadline, each group member would earn zero points. Bargaining proceeded as follows. Any subject could put a proposal on the table by completing a proposal form on the left side of their screen. Once a proposal was on the table all members of the group would see it on the right side of their screens. Any subject was also able to replace their proposal with another at any time until the round ended. Thus at any time there may be up to three proposals (VETO and SYMMETRIC treatments) or up to four proposals (ENLARGED treatment) on the table. Subjects could indicate which proposals were acceptable or unacceptable, and by indicating a proposal was acceptable they placed their votes in favor of that proposal. The first proposal to receive the required number of votes was enforced, and subjects received the points specified in that proposal.

At the end of the experiment subjects were privately paid according to their accumulated point earnings from all 10 rounds, using an exchange rate of 3p per point (VETO and SYMMETRIC treatments) or 4p per point (ENLARGED treatment). Earnings averaged £12 per subject in all sessions, and ranged from a minimum of £2.12 to a maximum of £20.40.<sup>8</sup> Sessions lasted, on average, 35 minutes, with no session taking longer than 55 minutes.

### **Overview of results**

In our design there is a possibility of negotiations breaking down, since each round had a randomly determined time limit between five and ten minutes. In fact, this time restriction did not bind in our experiment. In VETO, where there is a unique core allocation giving the entire budget to the strong player, agreements are quickly reached: only in two of 160 games did bargaining extend beyond five minutes, and even then agreements were reached before the random deadline. The games used in SYMMETRIC and ENLARGED have an empty core, and our prior expectations were

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<sup>8</sup> At the time of the experiment the exchange rate was approximately £1 = \$1.85.

that subjects would find it more difficult to reach an agreement.<sup>9</sup> As it turned out, an agreement was reached in every game well before there was any danger of the random deadline coming into effect, and the maximum duration of negotiations in these treatments was 2¼ minutes. Thus, there are no disagreement outcomes in any of the 480 games of our experiment.<sup>10</sup>

Figure 1 shows how voting power -- measured as average earnings for a given player-role -- develops across rounds for each treatment. As shown in the left panel, the strong player's voting power is greatest in VETO and is smallest in SYMMETRIC. Correspondingly, the weak players have most voting power in SYMMETRIC and least in VETO.<sup>11</sup>

--- Figure 1 about here ---

It is also evident in Figure 1 that earnings change across rounds, particularly in VETO and ENLARGED. To analyze these dynamics we computed Spearman rank correlation coefficients between the strong player's voting power and round for each matching group. In SYMMETRIC four coefficients are positive and four are negative, and so we cannot reject the null hypothesis that power is equally likely to increase or decrease with experience using a two-sided sign-test (p-value = 1.000). On the other hand, for VETO and ENLARGED all coefficients are positive, and so we can reject the null hypothesis that power is equally likely to increase or decrease with experience (p-value = 0.008). Further analysis of these treatments shows that the significant increase in the strong player's voting power occurs in earlier rounds: in the last four rounds we find no evidence of a relationship between voting power and round (VETO p-value = 0.727; ENLARGED p-value = 1.000). Thus, the strong player's voting power can be described as initially increasing, before stabilizing around 91 in the last four rounds of VETO and around 69 in the last four rounds of

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<sup>9</sup> As Kahan and Rapoport (1984, p.67) have argued, "... in a coreless game, some constraints on coalitional activity must be operative in the society, or else it will become ensnared in the endless rounds of successive secessions from proposed [payoff configurations]."

<sup>10</sup> The early agreements in our experiment contrast sharply with results from two-person (Roth *et al.*, 1988, Gächter and Riedl, 2005) and three-person bargaining (Bolton *et al.*, 2003), where there are pronounced deadline effects.

<sup>11</sup> We also noted considerable asymmetry between the earnings of the two weak players in some matching groups. In particular, earnings were significantly higher for player 2 than player 3. However, all the results reported below, with one exception noted later, apply whether the earnings of players 2 and 3 are regarded separately or pooled.

ENLARGED. By comparison, in SYMMETRIC the strong player's voting power in the last four rounds is 47.

Several patterns in the data suggest the changes in early rounds reflect a learning process as subjects become more familiar with strategic aspects of the game. For example, in the first round agreements to divide the 120 points equally among all members were quite frequent, occurring in 17 of 48 ( $\approx 35\%$ ) groups. For many subjects this must have seemed a natural and acceptable outcome. However, as rounds progressed equal divisions were observed less frequently. In the last round only 2 of 48 groups ( $\approx 4\%$ ) agreed upon an equal division. Figure 2 shows that the frequency of equal divisions decreases over rounds in all three treatments.

--- Figure 2 about here ---

Broadly speaking, proposals in which the pie was equally divided tended to be replaced by proposals allocating zero to at least one player. Since in all three treatments a proposal can be implemented without unanimous support, a winning coalition can form that excludes some players, and the members of this coalition maximize their point earnings by allocating zero points to outsiders. In all treatments we observed an increase in the frequency of *minimal winning coalitions* across rounds, as shown in Figure 3. Across all three treatments, minimal winning coalitions formed in 22 of 48 groups ( $\approx 46\%$ ) in the first round, compared with 41 of 48 ( $\approx 85\%$ ) in the last round.

--- Figure 3 about here ---

In VETO, minimal winning coalitions, which must include the strong player, formed in 123 of 160 ( $\approx 77\%$ ) games overall, and in 53 of 64 ( $\approx 83\%$ ) games in the last four rounds. In SYMMETRIC, minimal winning coalitions, which can be comprised of any two players, formed in 126 of 160 ( $\approx 79\%$ ) games overall, and in 59 of 64 ( $\approx 92\%$ ) games in the last four rounds. In ENLARGED there can be two different types of minimal winning coalition: the strong player with one of the weak players, or the two weak players with the new member. In this treatment, the first type of coalition was much more common. Minimal winning coalitions excluding the

strong player formed in only 8 of 160 (5%) games, and in only 1 of 64 ( $\approx 2\%$ ) games in the last four rounds; minimal winning coalitions including the strong player occurred in 116 of 160 ( $\approx 73\%$ ) games, and in 58 of 64 ( $\approx 91\%$ ) games in the last four rounds.<sup>12</sup>

When minimal winning coalitions formed, the budget was often split equally among its members. However, Figure 4 shows that the extent to which this occurred varied substantially across treatments. In SYMMETRIC the proportion of 60-60 divisions was rather stable and averaged 80% over all rounds, but in the other two treatments there are fewer cases of equal division within a minimal winning coalition. In ENLARGED, 36% of minimal winning coalitions split the budget evenly between its members. In VETO, where any minimal winning coalition must include the strong player, minimal winning coalitions agreed a 60-60 division 37% of the time in the first round, but this proportion decreased to zero by the last round.

---Figure 4 about here ---

In order to further study departures from equal division within minimal winning coalitions we restrict attention to minimal winning coalitions involving the strong player, and study how the strong player's share varies across rounds and treatments (see Figure 5). The set of minimal winning coalitions featuring the strong player is the same in all treatments: she can form a winning coalition with either weak player. However, very different patterns are evident across treatments. In VETO the strong player earned 77 points on average in the first round, compared with 98 points in the last round. Thus, as rounds progressed the strong player was demanding and getting larger shares of the pie. In SYMMETRIC we see a stable pattern: when the strong player is included in a minimal winning coalition she gets about half the pie. In ENLARGED the strong player's earnings within minimal winning coalitions averaged more than 60 points in every matching group, but were nevertheless substantially lower than in VETO, being stable around 69 points. Thus, because the weak players can form a winning coalition with the new member, this threat appears

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<sup>12</sup> The finding that some subjects are willing to propose distributions that give zero to another subject, and others are willing to vote for such a proposal, is consistent with other experiments on multi-person bargaining (see Güth and van Damme, 1998, Fréchet *et al.*, 2003, Kagel *et al.*, 2005, and Okada and Riedl, 2005).

to limit the strong player's ability to extract larger shares of the pie from one of the weak players.

---Figure 5 about here ---

We find the comparison between the strong player's power in a minimal winning coalition in the SYMMETRIC treatment and her power in the ENLARGED treatment particularly interesting. The strong player's earnings within a minimal winning coalition differ significantly across these two treatments, whether we focus on all rounds (Wilcoxon two-sided p-value = 0.003) or just the last four rounds (Wilcoxon two-sided p-value = 0.004), and this suggests that the amount the strong player gets when she forms a coalition with one of the weak players depends on the alternative coalition opportunities available to the weak players. This contrasts with the predictions of theories of ex post payoff division. Gamson's (1961) theory predicts ex post payoff division proportional to the voting weights, so that the strong player should get 72 points and the weak player should get 48 points when they form a coalition in both treatments.<sup>13</sup> Other theories of ex post payoff division also fail to capture the significant difference. The Bargaining Set (Davis and Maschler, 1967), Kernel (Davis and Maschler, 1965) and the Aspiration solution concepts (see Bennett, 1983) all predict that the coalition of the strong and one of the weak players divide the 120 points equally in both treatments.

### **Voting power: tests of hypotheses**

For formal comparisons of voting power we use non-parametric tests applied to sets of independent observations. We first examine how voting power varies with voting weights within treatments. Table 2 presents the strong and weak player's voting power for each treatment.<sup>14</sup>

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<sup>13</sup> Fréchette *et al.* (2005) also found that Gamson's law performs badly in a game similar to our SYMMETRIC treatment.

<sup>14</sup> Recall that there are no disagreements. Thus, in VETO and SYMMETRIC the strong player's earnings plus twice the weak player's earnings equals 120 points. In ENLARGED the strong player's earnings plus twice the weak player's earnings is less than 120 points. The residual is the earnings of the new member. Averaging over all rounds this player earns 5.6 points per round, while averaging over the last four rounds she earns 0.9 points per round.

**Table 2. Empirical measures of voting power**

	VETO		SYMMETRIC		ENLARGED	
	all rounds	last 4 rounds	all rounds	last 4 rounds	all rounds	last 4 rounds
strong player	80.6	91.1	46.2	47.2	59.7	68.9
weak player	19.7	14.4	36.9	36.4	27.3	25.1

In VETO the strong player earns more than a weak player in every single matching group, whether we focus on all rounds or just the last four rounds. Thus the voting power of the strong player is significantly greater than that of the weak player (one-sided sign-test p-value = 0.004), as predicted by all the power indices given in Table 1. The comparison of strong and weak player earnings in the SYMMETRIC treatment provides the one case in which asymmetries between the two weak players affect our results. We find that the earnings of the strong player are not significantly different from the earnings of player 2 (all rounds: two-sided sign-test p-value = 0.727; last four rounds: two-sided sign-test p-value = 1.000) but are significantly greater than the earnings of player 3 (all rounds: one-sided sign-test p-value = 0.035; last four rounds: one-sided sign-test p-value = 0.035). Averaging over the two weak players, we find that the strong player earns significantly more if we consider all rounds (one-sided sign-test p-value = 0.035), but not if we focus on the last four rounds (one-sided sign test, p-value = 0.363).<sup>15</sup> Finally, for ENLARGED, we find that the strong player earns significantly more than a weak player in every single matching group whether we look at all rounds or just the last four rounds (one-sided sign-test p-value = 0.004). This last finding is not predicted by the Nucleolus, but is consistent with the other power indices given in Table 1.

For each index and treatment we used two-sided sign tests to test the hypothesis that the strong player's voting power is equally likely to be above or below the prediction. None of the indices performed well in this sense. In the case of the Nucleolus it is not surprising that we can reject the hypothesis in VETO<sup>16</sup>, since any deviation from the point prediction must be below, but we can also reject the

<sup>15</sup> In this treatment 101/160 ( $\approx 63\%$ ) games result in two-player coalitions who divide the pie 60-60 between themselves. However player 2 was included in such coalitions more than twice as often as player 3.

<sup>16</sup> Kagel *et al.* (2005) obtain a similar result with a structured protocol for negotiations: the share of the veto player is well below the equilibrium prediction.

hypothesis in ENLARGED.<sup>17</sup> Likewise, the naïve index is rejected in VETO (p-value = 0.008) and ENLARGED (p-value 0.008). For the case of the Shapley and Banzhaf indices we also reject in ENLARGED (for both indices p-value = 0.008), but their performances in VETO depends on whether we focus on all or just the last four rounds. Using all rounds we cannot reject the null hypothesis for either index (Shapley: p-value = 1.000; Banzhaf: p-value = 0.289).<sup>18</sup> If, however, we focus on the last four rounds we reject the null hypothesis at a 10% significance level for both indices (p-value = 0.070 in both cases).

Brams and Affuso (1976) use the Shapley and Banzhaf power indices to predict that an original member's voting power increases when a new member is admitted to a voting group. We now compare treatments in order to test this prediction. First we consider the comparison between VETO and ENLARGED. The Shapley and Banzhaf indices, and indeed indices based on the core (or its extension), predict that this will increase the voting power of the weak player. Our experiment delivers strong empirical support for this prediction, since the weak player's earnings are significantly higher in ENLARGED (all rounds: one-sided Wilcoxon test p-value = 0.006; last four rounds: one-sided Wilcoxon test p-value = 0.006).

Next we consider the comparison between treatments SYMMETRIC and ENLARGED. Brams and Affuso predict, on the basis of the Shapley and Banzhaf indices, that the addition of the new player with one vote will increase the strong player's voting power. Interestingly, this prediction does not follow from other indices, such as the Nucleolus. In our experiment the strong player earns significantly more in ENLARGED than SYMMETRIC (all rounds: one-sided Wilcoxon test p-value = 0.003; last four rounds: one-sided Wilcoxon test p-value = 0.001). Thus the paradox of enlargement, as predicted by Brams and Affuso, is observed in our experiment.

## CONCLUSIONS

Our experiment provides empirical measures of how voting weights and voting rules influence voting power in weighted voting games. Our focus is on the three games

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<sup>17</sup> In every single matching group, regardless of whether we consider all or last four rounds, the strong player earns more than predicted in ENLARGED (p-value = 0.008).

<sup>18</sup> In fact the average earnings of the strong player is remarkably close to the Shapley value. Murnighan and Roth (1977) obtain a similar result.

discussed in the seminal work of Brams and Affuso (1976). These games illustrate how the enlargement of a voting body may benefit an original member, even if the decision rule and her relative voting weight *vis-à-vis* the other original members are held constant.

In one of the cases studied, a strong player loses her veto power when a new member is added, and this strengthens the power of weaker members. This prediction is shared by sophisticated power indices, and also receives strong empirical support from our experiment. On the other hand, individual power indices on which this comparative static prediction could be based do not deliver good point predictions. When the strong player is a veto player she gets significantly less than the entire pie (i.e. her core allocation), but in the later rounds of the session she gets significantly more than predicted by the Shapley or Banzhaf indices. In the larger voting body the strong player attains significantly more than predicted by any of the power indices considered.

In the second case studied by Brams and Affuso the prediction that enlargement will benefit an original member is more controversial, since it relies on the particular power indices they use. The Shapley and Banzhaf indices predict that the strong player benefits from the addition of a new member, while the Nucleolus concept predicts no change in voting power. Here our experiment supports Brams and Affuso's comparative static prediction: the paradox of new members is observed in our data.

Our results underscore the important point made by voting theorists: that formal analysis is required in order to accurately predict the effects of changes in voting bodies. While an important part of such analysis should be based on theoretical analysis of the properties of different voting weights and rules, we also argue that empirical methods have an important complementary role. Empirical evidence is particularly valuable when, as in one of the cases we study, alternative solution concepts make different predictions about the effects of institutional changes.

## APPENDIX A

### Instructions

#### Introduction

This is an experiment about group decision-making. There are other people in this room who are also participating in this experiment. You must not talk to them or communicate with them in any way during the experiment. The experiment will take about one hour, and at the end you will be paid in private and in cash. The amount of money you earn will depend on the decisions that you and the other participants make.

In this experiment you will participate in ten rounds. In each round you will be in a group with two other people, but you will not know which of the other people in this room are in your group. The people in your group will change from round to round, and in particular you will never be matched with the same set of two other people twice. The decisions made by you and the other people in your group will determine how many points you earn in that round. At the end of the experiment you will be paid according to your total point earnings from all ten rounds. You will be paid 3p per point.

#### Description of a round

At the beginning of each round, you will be randomly allocated a subject identification number, either 1, 2, or 3. (Thus, your identification number may change from round to round.) Each person controls a number of votes depending on his or her identification number as follows:

Subject Identification Number	1	2	3
Number of Votes	3	2	2

In each round you and the other people in your group have 120 points to divide. You and the other people in your group can make proposals about how these points are to be divided among the group members. You and the other people in your group can also cast votes in favour of proposals. The first proposal to receive *five* votes will be enforced. When a proposal is enforced the round ends and each person earns the number of points specified in that proposal.

There will be a time limit for each round. This time limit will be some whole number of seconds between 300 and 600, but you will not be informed of the exact time limit. This means that the round could end suddenly at any time between 300 seconds (five minutes) and 600 seconds (ten minutes). If no proposal has been enforced when the round ends, each person in your group will earn zero points in the round.

All rounds will be identical except that your subject number may change from round to round, the other people in your group may change from round to round, and the time limit may change from round to round.

### **How you make proposals**

At the beginning of a round your computer screen will look like the one shown in Figure 1. On the left side of the screen there is a form for making proposals. To make a proposal you must specify the number of points that each person in your group will receive. For each person you can type in any whole number between 0 and 120, but the total number of points received by the group members must add up to 120. When you have completed a proposal you click on the "submit" button to submit it.

Your proposal will then appear on the right side of the screen, in the list of "proposals on the table." A sample screen is shown in Figure 2, except that the entries marked XXX will be the numbers you entered in your proposal. As long as the round has not already ended, you can amend your proposal by simply completing a new proposal and submitting it. The new proposal will replace the old one.

### **How proposals are enforced**

Once a proposal is on the table, all the people in your group will see it on the right side of their screens. At any time there may be up to three proposals on the table, one submitted by each person in your group. For each proposal there is an "accumulated votes" counter that informs all people in the group of how many votes are currently in favour of the proposal. When a proposal is submitted it automatically receives the votes of the person who submitted it. Thus, during the round your screen might look like the one shown in Figure 3 (except that the entries marked XXX will correspond to the decisions made by participants).

For each proposal on the table you can indicate if it is acceptable by clicking on the "acceptable" button next to that proposal. If you do this, the proposal will receive your votes. If you change your mind after indicating that a proposal is acceptable you will be able to withdraw your support by clicking on the "unacceptable" button. The "accumulated votes" counter will change to keep track of how many votes are currently in favour of the proposal.

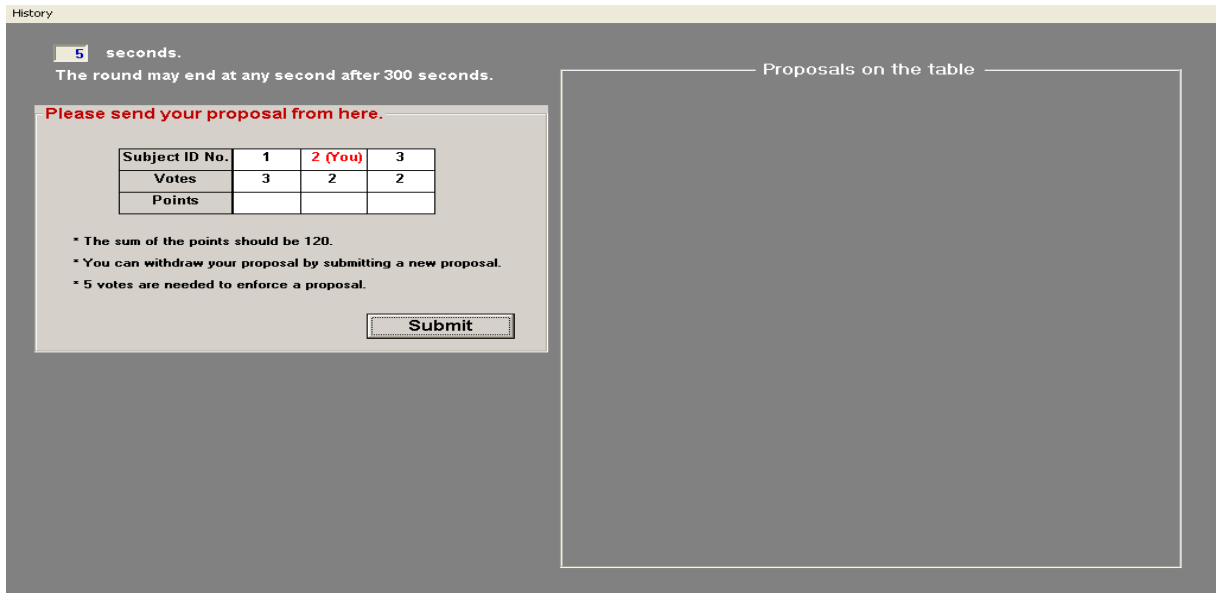
You can use your votes to support more than one proposal. However, the first proposal to have five votes in its favour will be the one that is enforced, and this proposal will determine how many points you receive.

### **Ending the session**

At the end of round ten your total points from all rounds will be converted to cash at a rate of 3p per point and you will be paid this amount in private and in cash.

Now, please click on the "start" button and begin reviewing the instructions on your screen. If you have any questions raise your hand and a monitor will come to your desk to answer them. When you have finished reading through the screens reviewing the instructions, click on the ready button to indicate that you are ready to begin the decision-making part of the experiment. When everyone in the room is ready, decision-making will begin.

# Figure 1



# Figure 2

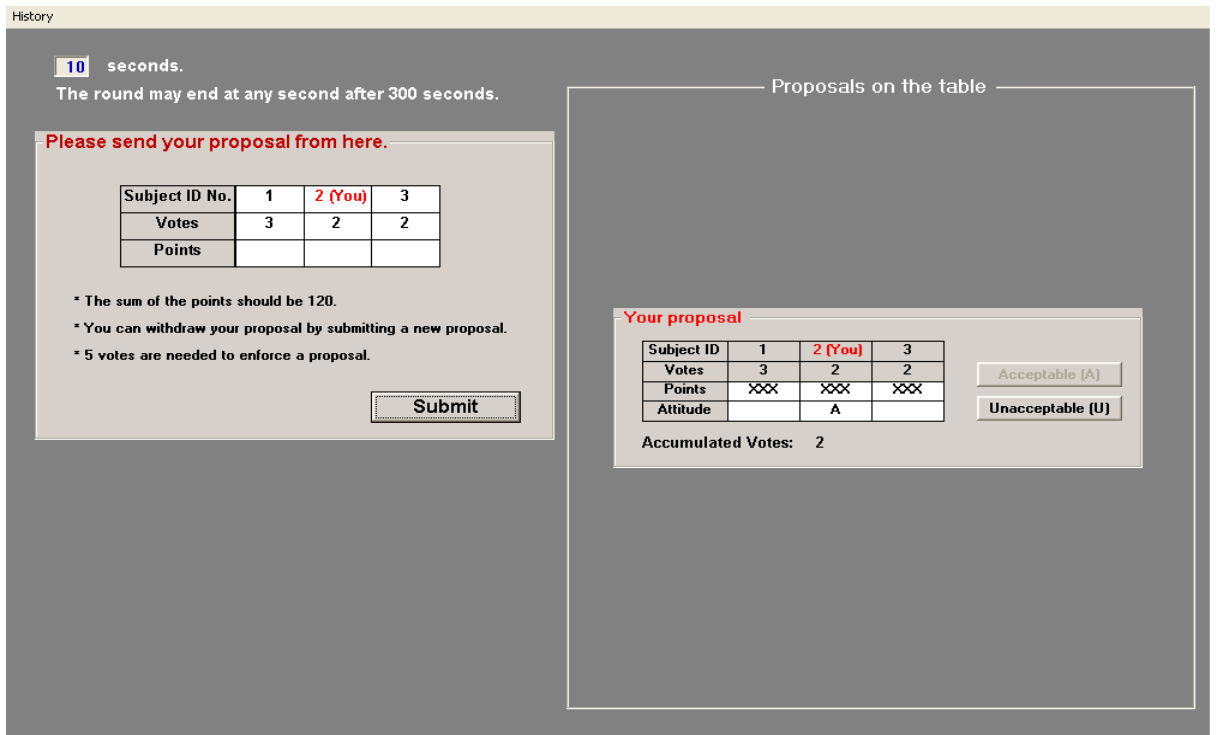


Figure 3

History

**10** seconds.  
The round may end at any second after 300 seconds.

Please send your proposal from here.

Subject ID No.	1	2 (You)	3
Votes	3	2	2
Points			

- \* The sum of the points should be 120.
- \* You can withdraw your proposal by submitting a new proposal.
- \* 5 votes are needed to enforce a proposal.

---

Proposals on the table

**1's proposal**

Subject ID	1	2 (You)	3
Votes	3	2	2
Points	xxx	xxx	xxx
Attitude	A		

Accumulated Votes: 3

**Your proposal**

Subject ID	1	2 (You)	3
Votes	3	2	2
Points	xxx	xxx	xxx
Attitude		A	

Accumulated Votes: 2

**3's proposal**

Subject ID	1	2 (You)	3
Votes	3	2	2
Points	xxx	xxx	xxx
Attitude			A

Accumulated Votes: 2

## REFERENCES

- Baldwin, Richard. 1994. *Towards an Integrated Europe*. CEPR, London.
- Banzhaf, John F. 1965. "Weighted Voting Doesn't Work: a Mathematical Analysis." *Rutgers Law Review* 19: 317-45.
- Bennett, Elaine. 1983. "The Aspiration Approach to Predicting Coalition Formation and Payoff Distribution in Sidepayment Games." *International Journal of Game Theory* 12, 1-28.
- Brams, Steven J., and Paul J. Affuso. 1976. "Power and Size: a New Paradox." *Theory and Decision* 7: 29-56.
- Brams, Steven J., and Paul J. Affuso. 1985. "New Paradoxes of Voting Power on the EC Council of Ministers." *Electoral Studies* 4: 135-39.
- Bolton, Gary E., Chatterjee, Kalyan and Kathleen L. McGinn. 2003. "How Communication Links Influence Coalition Bargaining: A Laboratory Investigation." *Management Science* 49: 583-598.
- Davis, Morton and Michael Maschler. 1965. "The Kernel of a Cooperative Game." *Naval Research Logistics Quarterly* 12, 223-259.
- Davis, Morton and Michael Maschler. 1967. "Existence of Stable Payoff Configurations for Cooperative Games," in: *Essays in Mathematical Economics in Honor of Oskar Morgenstern* (ed. by Martin Shubik). Princeton University Press.
- Dreyer, Jacob S., and Andrew Schotter. 1980. "Power Relationships in the International Monetary Fund: the Consequences of Quota Changes." *Review of Economics and Statistics* 62: 97-106.
- Felsenthal, Dan S., and Moshé Machover. 1998. *The Measurement of Voting Power*. Edward Elgar Publishing.
- Felsenthal, Dan S., and Moshé Machover. 2001. "The Treaty of Nice and Qualified Majority Voting." *Social Choice and Welfare* 18: 431-464.
- Fréchette, Guillaume R., Kagel, John H., and Steven F. Lehrer. 2003. "Bargaining in Legislatures: An Experimental Investigation of Open versus Closed Amendment Rules." *American Political Science Review* 97: 221-232.
- Fréchette, Guillaume R., Kagel, John H., and Massimo Morelli. 2005. "Gamson's Law versus Non-Cooperative Bargaining Theory." *Games and Economic Behavior* 51, 365-390.

- Gächter, Simon and Arno Riedl. 2005. "Moral Property Rights in Bargaining with Infeasible Claims." *Management Science* 51: 249-263.
- Gamson, William A. 1961. "A Theory of Coalition Formation." *American Sociological Review* 26, 373-382.
- Güth, Werner and Eric van Damme. 1998. "Information, Strategic Behavior, and Fairness in Ultimatum Bargaining: An Experimental Study." *Journal of Mathematical Psychology* 42: 227-247.
- Kagel, John H., Sung, Hankyoung, and Eyal Winter. 2005. "Multilateral Bargaining in the Presence of a Veto Player: An Experimental Analysis." Typescript.
- Kahan, James P. and Amnon Rapoport. 1984. *Theories of Coalition Formation*. Lawrence Earlbaum Associates.
- Laruelle, Annick and Federico Valenciano. 2005. "Bargaining in Committees as an Extension of Nash's Bargaining Theory." *Journal of Economic Theory*, forthcoming.
- Montero, Maria. 2005. "Noncooperative Foundations of the Nucleolus in Majority Games." *Games and Economic Behavior*, forthcoming.
- Murnighan, Keith J., and Alvin E. Roth. 1977. "The Effects of Communication and Information Availability in an Experimental Study of a Three-Person Game." *Management Science* 23: 1336-1348.
- Okada, Akira and Arno Riedl. 2005. "Inefficiency and Social Exclusion in a Coalition Formation Game: Experimental Evidence." *Games and Economic Behavior* 50: 278-311.
- Roth, Alvin E. 1977a. "The Shapley Value as a von Neumann-Morgenstern Utility." *Econometrica* 48: 657-664.
- Roth, Alvin E. 1977b. "Utility Functions for Simple Games." *Journal of Economic Theory* 16: 481-489.
- Roth, Alvin E., Murnighan, J. Keith and Francoise Schoumaker. 1988. "The Deadline Effect in Bargaining: Some Experimental Evidence." *American Economic Review* 78: 155-162.
- Schmeidler, David. 1969. "The Nucleolus of a Characteristic Function Game." *SIAM Journal on Applied Mathematics* 17: 1163-1170.
- Shapley, Lloyd S. 1953. "A Value for n-Person Games," in: Kuhn, H. W. and A.W. Tucker (Eds.), *Contributions to the Theory of Games II*, Princeton University Press, 307-317.

- Shapley, Lloyd S., and Martin Shubik. 1954. "A Method for Evaluating the Distribution of Power in a Committee System." *American Political Science Review* 48: 787-792.
- van Deemen, Adrian and Rusinowska, Agnieszka. 2003. "Paradoxes of Voting Power in Dutch Politics." *Public Choice* 115: 109-137.

Figure 1. Voting power of strong and weak players

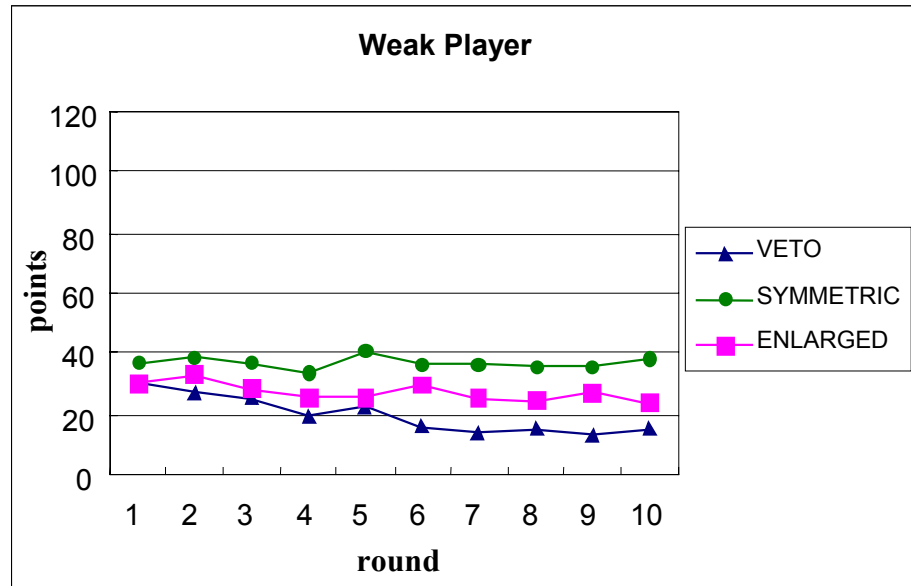
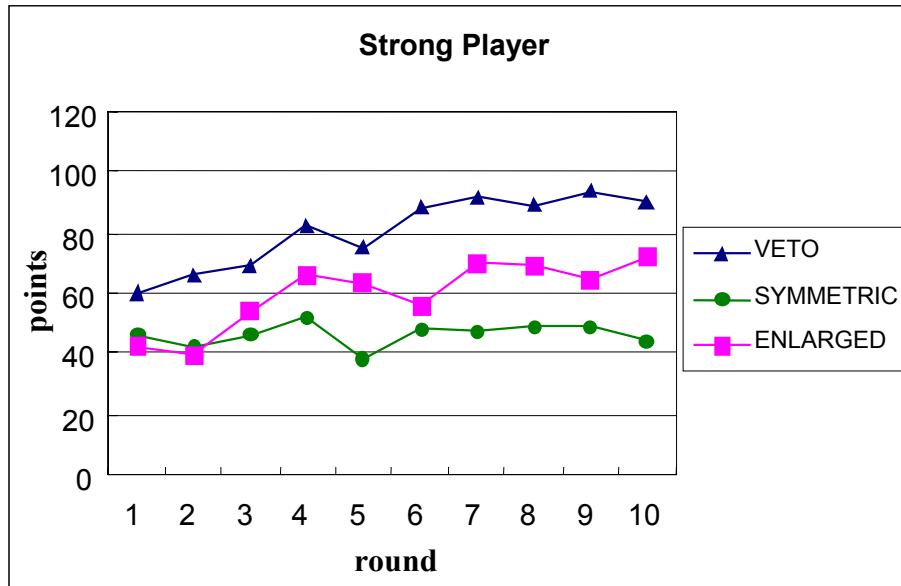


Figure 2. Proportion of equal divisions between all players

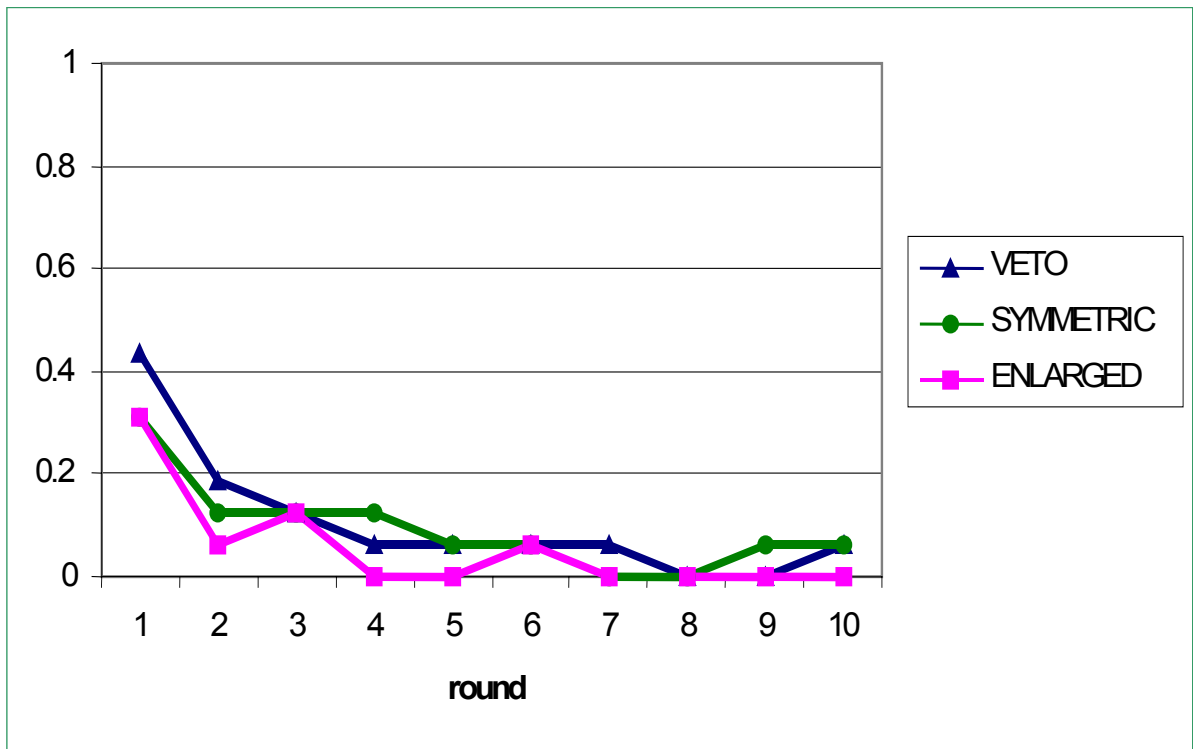
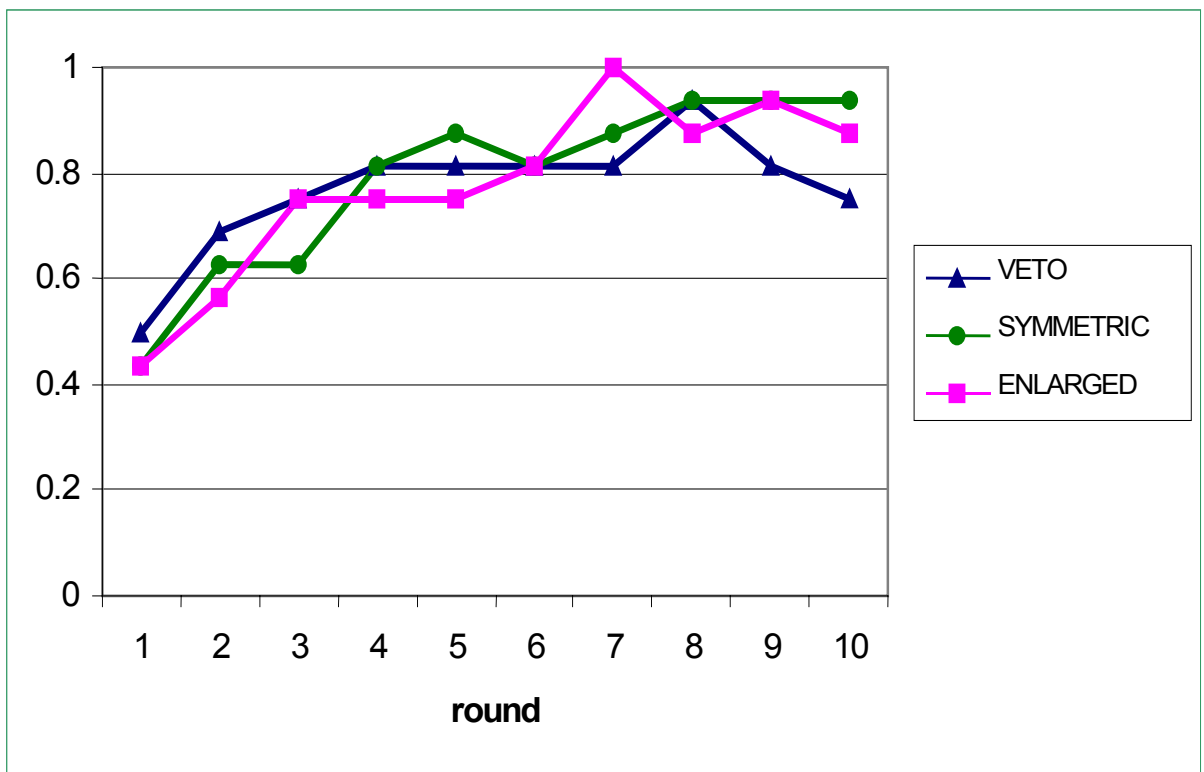
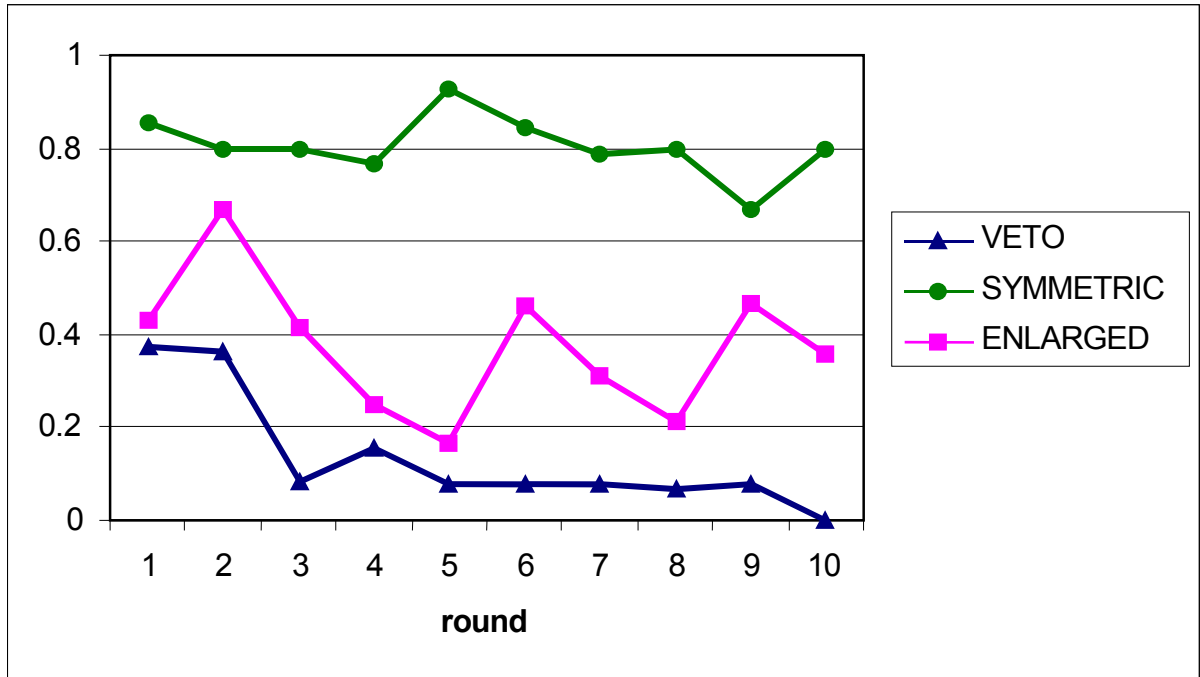


Figure 3. Proportion of minimal winning coalitions



**Figure 4. Proportion of minimal winning coalitions that divide equally**



**Figure 5. Strong player's voting power within minimal winning coalitions**

