

Bidding Strategies of Sequential First Price Auctions Programmed by Experienced Bidders*

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ABSTRACT

This paper considers bidding automata programmed by experienced subjects in sequential first price sealed bid auction experiments. These automata play against each other in computer tournaments. The risk neutral subgame perfect Nash equilibrium strategy of the independent private value model serves as a benchmark. The equilibrium strategy does not describe any of the heterogeneous automata programs submitted by subjects and does not always perform better than average in the tournament.

Keywords: Experimental Economics, First-Price Sealed-Bid Auctions, Sequential Auctions, Independent Private Value Model, Finite Automata

JEL Classifications: C12, C13, C72, C92, D44

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1. INTRODUCTION

Sequential auctions are used to auction off identical lots of fish, flowers, wine, real-estate and other goods, lot by lot in a sequence of auction stages. The lots are usually well proportioned such that each buyer demands not more than one of them. Although sequential auctions are old and empirically important (see Cassady (1967)) most of the literature is quite recent, motivated by Ashenfelter (1989) and his observation of a «declining price anomaly»¹. Ashenfelter reports that in auction markets for wine and arts the price for two subsequent, identical lots is twice as likely to decrease than to increase, when it does not stay the same. This behavioral pattern which is also called the «afternoon effect» violates what Ashenfelter coined «the law of one price». The idea that equal lots should be priced equally is intuitive and had been established rigorously by Weber (1983) for risk neutral bidders with unit demands in the independent private value model.

Since Ashenfelter's observation, price declines have been reported to occur in many sequential auction markets². Indeed, there have been several attempts to explain the «declining price anomaly». For instance, McAfee and Vincent (1993) analyze sequential auctions with risk-averse bidders but can explain the decline theoretically only under the unlikely assumption of non-decreasing absolute risk aversion. Black and de Meza (1993) show the decline under application of a buyer's option that assigns the buyer of the first lot the right to buy the remaining lots at the same price³. Ashenfelter (1989) reports however that this option has been adopted by various auction houses precisely to attenuate this declining price phenomenon and that observed prices appear to decrease more severely in auctions where such an option is absent.

¹ In fact, the observation that prices decline in sequential auction was made earlier in the literature. Buccola (1982) reported it for livestock auctions and Burns (1985) in an experimental study with professional wool buyers. Ashenfelter (1989) brought it to the attention of a wider audience of economists.

² See Ashenfelter and Genesove (1990), McAfee and Vincent (1993), Chanel et al. (1996), Pesando and Shum (1996), Beggs and Graddy (1997), Ginsburgh (1998), Laffont et al. (1998), Pezanis-Christou (2000) and Romeu (2001).

³ Other theoretical approaches that address price declines in sequential auctions as a consequence of supply or demand uncertainty are Engelbrecht-Wiggans (1994), Bernhardt and Scoones (1994), Gale and Hausch (1994), Pezanis-Christou (1996), Burguet and Sakovicz (1997), Jeitschko (1999), Katzman (1999) and Neugebauer and Pezanis-Christou (2002).

Price declines have been also reported under controlled laboratory conditions, though the evidence is mixed. Burns (1985) reports price declines in an experiment with professional wool traders⁴. She also ran experiments with a control group of student subjects and reports that they managed to learn in the laboratory market and that price declines vanished after the early periods had past. Keser and Olson (1996) report price declines in sequential first price sealed bid auctions. Their experiment is very short as it consists only of 20 sequential auctions. In Neugebauer and Pezanis-Christou (2002) we use a very similar design in our control treatment as the one reported in Keser and Olson (1996), but we run longer sessions involving 100 sequential auctions. Likewise Burn's (1985) observation, we find that price declines vanish after subjects have gained some experience in the environment⁵. We come to this result although subjects' behavior deviates significantly from the risk neutral Nash equilibrium prediction. In the theoretical model by Weber (1983), bids are independent of prices as they depend linearly on values. The data reported in Neugebauer and Pezanis-Christou (2002) do neither support linearity nor price independence of bids.

In this paper, we take a closer look at the bidding strategies in sequential auction experiments. The exercise is quite simple. Experienced experimental subjects are asked to formulate a profit maximizing strategy. The revealed strategies permit a comparison of predicted strategies and observed ones. The experimental approach is called the strategy method and goes back to Selten (1967). It has been applied to many experimental settings: Selten (1967), Fader and Hauser (1988) and Selten et al. (1997) applied it to oligopolies, Axelrod (1984) to the prisoner's dilemma, Mitzkewitz and Nagel (1993) and Rapoport and Fuller (1995) to bargaining, Selten and Buchta (1999) to the first price sealed bid auction, Seale and Rapoport (2000) to a tacit coordination game, and Fischbacher et al. (2001) to public goods. Likewise Selten et al. (1997), the present paper reports on finite automata programmed by experienced experimental subjects to run a computer tournament⁶. The bidding automaton that

⁴ A professional trader stated afterwards that they had behaved in the experiment just as in the real world market. Burns ascribes the price decline in the wool market to the fact that bidders often act as agents for others and are used to place higher bids in order to ensure that they fill their orders.

⁵ Other experimental work on sequential auctions, not related to the discussion, can be found in Frahm and Schrader (1970) and Pitchik and Schotter (1988).

⁶ Axelrod (1984) and Fader and Hauser (1988) run tournaments with automata, too, but automata were not programmed by subjects who gathered experience in an interactive setting prior to the programming task. A related branch of research is «agent-based modeling». The literature surveyed and linked to human subject experiments by Duffy (2004) follows up on Gode and Sunder's (1993) seminal work in which random automata («zero-intelligent agents») are applied to double-auction markets.

generates the greatest average earning in a series of sequential auctions is awarded a prize.

The main focus of the paper is thus not on the price anomaly, although we consider also the price sequences that result from the application of the bidding automata. The approach goes a step beyond and elicits the components that experimental subjects take into account when they decide to bid in sequential auctions. The knowledge of the determinants of individual behavior may prove to be important in constructing descriptive models of behavior to explain anomalies. In fact, it is a good question whether the strategy method is an ideal approach to uncover behavioral strategies. Selten et al. (1997, p. 552) point to some limitations: «it would be wrong to assert that there is no difference between a programmed strategy and spontaneous behavior. The strategy method cannot completely reveal the structure of spontaneous behavior.» Hence, the present study should be understood as complementary to Neugebauer and Pezani-Christou (2002).

The remainder of the paper is organized as follows. Section 2 introduces to Weber's model and section 3 describes the experimental design. In the fourth section, the results are reported: Weber's model is challenged as a descriptive theory by the observation that subjects' bidding functions depend on observed prices. Moreover, the risk neutral Nash equilibrium strategy can not be recommended as a profit maximizing strategy for the computer tournament, because it is outperformed in a market of bidding automata submitted by subjects of an interactive experiment. Finally, section 5 concludes.

2. THEORETICAL CONSIDERATIONS

Consider a market in which a seller offers $k > 0$ identical lots of some commodity to $n > k$ potential buyers. He offers the lots in a sequence of k auction stages. Buyers demand exactly one lot each, such that they submit sealed bids $b_k \in [0, 1]$ at every auction stage $k = \{1, \dots, k\}$ as long as their demand is not met. Hence, the number of potential buyers and the number of lots decrease from one stage to the next. Buyers are assumed risk neutral and have private values $x \sim U[0, 1]$ iid —independently drawn from the uniform distribution over the unit interval—. A buyer's surplus is given by the difference between her value and the commodity's price. The price is determined according to the first price rule: the buyer who submits the highest bid is awarded the lot of the auction stage at a price equal to her bid. William Vickrey (1961) showed the existence and the uniqueness of a symmetric risk neutral Nash equilibrium (hereafter RNNE) for the single lot case. Let $x_{(i)}$ denote the order statistic of the i^{th} highest value, the RNNE (in case $k=1$) is formally represented by equation (1).

$$b(x) = E[x_{(2)} | x = x_{(1)}] = x \frac{n-1}{n} \quad (1)$$

A buyer applies the RNNE as in eq. (1) if her bid equals the expectation about the second highest value given she has the highest value. Using backward induction, Weber (1983) showed the existence and the uniqueness of a subgame perfect RNNE in the multiple lot case. Equation (2) represents the generalized RNNE (to $k > 1$) for the uniform distribution over the unit interval.

$$b_{\kappa}(x) = E[x_{(k+1)} | x = x_{(\kappa)}] = x \frac{n-k}{n-\kappa+1}, \forall \kappa \leq k \quad (2)$$

where the index $\kappa \leq k$ indicates the auction stage. The RNNE bid can be interpreted as the buyer's expectation about the $(k+1)^{\text{th}}$ highest value given that she has the highest value among the (remaining) potential buyers⁷. An important result of Weber (1983) is that the expected price sequence in sequential auctions is constant; Ashenfelter (1989) termed this «the law of one price». The result is formally represented in equation (3).

$$\begin{aligned} E[p_{\kappa}] &= E[b_{\kappa}(x_{(\kappa)})] = E\left[x_{(\kappa)} \frac{n-\kappa}{n-\kappa+1}\right] \\ &= E[x_{(\kappa)}] \frac{n-k}{n-\kappa+1} = \frac{n-\kappa+1}{n+1} \frac{n-k}{n-\kappa+1} \\ &= \frac{n-k}{n+1} \end{aligned} \quad (3)$$

The expected price at stage κ is the expected RNNE bid of the κ^{th} highest value, the one expected to win the auction stage. Equation (3) reveals that this price does not depend on stage κ , thus it must be constant. A bidder's expected payoff from participating in the auction is represented in equation (4), computed by adding up the expected payoffs from all auction stages.

⁷ For a detailed derivation of the result see Neugebauer and Pezanis-Christou (2002).

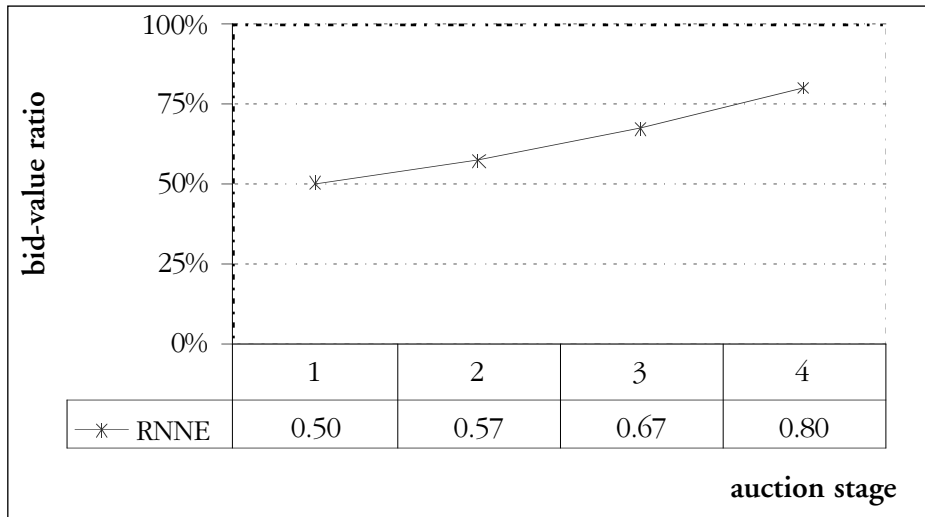
$$E[\pi] = \frac{\sum_{\kappa=1}^k E[x_{(\kappa)} - p_{\kappa}]}{n} = \frac{\sum_{\kappa=1}^k n - \kappa + 1}{n(n+1)} - \frac{k(n-k)}{n(n+1)} = \frac{k(k+1)}{2n(n+1)} \quad (4)$$

Before the experiment is discussed, let us briefly reflect on two theoretical implications of the RNNE. First, it seems remarkable that the unique subgame perfect RNNE is only a best response against itself. In other words, the RNNE is a best response at each stage only if applied by all other bidders in the market. Generally it is not optimal to stick to the RNNE, if another bidder deviates from it. Therefore, it is questionable whether the RNNE can be a good descriptive theory that can explain behavior in the market. Second, in theory the lot is awarded always to the bidder with the highest value. The consequence is that the prices reflect values of bidders who have left the market. The price contains no valid information about the values of the ones who remain in the market, and thus, it is useless with respect to the prediction of future behavior. Hence, theory suggests the same behavior whether prices are revealed or not. In fact, it appears odd that people at sequential auctions should ignore the prices paid in the same market.

3. DESIGN & THEORETICAL PREDICTIONS

The auction design uses the parameter values $n=8$ and $k=4$. Eight potential buyers participate in a first price sealed bid auction market where four identical lots are auctioned off. At the beginning of every auction each buyer receives a randomly drawn independent private value from the interval 0 to 100. Unless a buyer has won a lot in the auction, she submits a sealed bid between 0 and 100 at every auction stage. If she is awarded a lot, she has to wait until the end of the auction. The winner's surplus in an auction stage is the difference between her value and her bid. In the RNNE, the expected price at every stage of the auction is 44.44 and the expected profit of a buyer is 13.88. The RNNE bid-value ratios are displayed in figure 1. Potential buyers are predicted to bid half their value at the first auction-stage, and $4/7$, $2/3$ and $4/5$ of their value at the second, third and fourth stage, respectively.

Figure 1. The RNNE bidding strategy



Previous to the computer tournament, we set up an experiment with the same parameter values involving salient rewards. The experiment was a spontaneous decision task, in which subjects submitted up to 400 bids in 100 sequential auctions and earned their accumulated surpluses⁸. Thereafter, subjects were offered to participate in a computer tournament by submitting their best paying strategy on a sheet of paper⁹. Upon submission, the programmability of each strategy was checked. A strategy requires bidding functions of all possible values for each of the four auction stages. Where necessary the experimentalist asked for revision. Subjects were informed that their strategies would be applied as automata to markets of sequential auctions as they had experienced and that price information feedback would be given at each auction stage. The winning strategy in the tournament, which was going to win a prize of \$75, would be the one that produced the greatest average earnings. The winner would be paid the prize a month after she submitted the strategy for the tournament.

A total of 48 subjects submitted strategies for the tournament. Although the tournament conditions were identical for all participants, the experienced conditions in the

⁸ The experiment took about two hours. See Neugebauer and Pezani-Christou (2002) for details.

⁹ The instructions are appended to the paper.

previous auction experiments were not all the same. The auction experiments were spontaneous decision tasks in which subjects faced one of four experience conditions: in the first treatment, hereafter referred to as INTERACTION, 8 subjects from a room of 16 interacted with each other in a market. The remaining 8 subjects did not interact with another but each subject competed in a market with 7 computerized bidders. A computerized bidder was represented by a bid applying the RNNE strategy to an independent value. The highest bids including the one of the experimental subject won the lots in the auction stages¹⁰. Since subjects believed to interact with other students¹¹, we refer to the second treatment as BELIEVED INTERACTION¹². Both treatments, INTERACTION and BELIEVED INTERACTION provided price information feedback at the end of each auction stage. Experience conditions in the third treatment were identical to the ones in the second one with the difference that subjects were aware of facing computerized competitors. They were informed that the computerized competitors would bid a constant fraction of their independent values at every stage, but they were not told the fraction that applied¹³. The third treatment is referred to as COMPUTERIZED and the fourth one is called NO PRICE. The latter treatment did not provide price information feedback, but applied otherwise identical conditions to the third treatment. Subjects in NO PRICE received only qualitative information feedback about whether they won a lot or not. In fact, in case they won a lot they knew the price equaled their bid. The third and the fourth treatment involved 16 participants each. All 48 participants were economics undergraduates. The experiment was run at the «Laboratorio de Investigaciones en Economía Experimental» —LINEEX— which is joined by the University Valencia and by the University Jaime I Castellon and at the Centre for Experimental Economics of the University York —EXEC.

¹⁰ In case of a tie, the winner of an auction stage was selected at random.

¹¹ The treatments involved instructions in which students were told that they would compete with 7 others in an auction market. Participants in BELIEVED INTERACTION were not explicitly notified that the competitors they faced were all computerized.

¹² Participants in INTERACTION represent one market in the study of Neugebauer and Pezanis-Christou (2002). In order to set up the exact conditions of Neugebauer and Pezanis' experiment, 16 subjects were necessary in the room. However, since the data of only one interactive market was needed, we used the remaining 8 subjects for a pilot experiment on computerized Nash markets. The automata in BELIEVED INTERACTION can be included into the present study, since the instructions to the computer tournament were flawless.

¹³ Cox, Smith and Walker (1987), Harrison (1989) and Neugebauer and Selten (2003) ran sessions on single unit first price sealed bid auctions with computerized bidders. Harrison revealed the bid-value ratio of the RNNE automaton to experimental subjects.

In the weeks after the experiment, the strategies were introduced into a PASCAL program to run the tournament¹⁴. The program resembles the experimental market conditions: 8 bidding automata representing the subjects' strategies are matched to a market where they compete at four auction stages for fictitious lots. At the end of each auction stage price information feedback is provided. The market closes after 100 sequential auctions. The average payoff and the mean bid-value ratio are computed for each automaton after 1000 runs, i.e., after 100,000 sequential auctions. In INTERACTION and BELIEVED INTERACTION, the same 8 automata compete in the 100,000 auctions, whereas in COMPUTERIZED and NO PRICE, each 16 automata are randomly assigned to two markets of 8 before each run. The four most profitable automata of each treatment (according to the average payoff in 100,000 sequential auctions) were selected to run a finale tournament. The 16 automata participating in the finale (4 most profitable ones x 4 treatments) were matched in the same way and the most profitable bidding automaton was determined to be the winner of the tournament.

4. RESULTS

In this section, we consider the responses of the experienced experimental subjects to the question: 'What is the best performing strategy in the sequential auction when price information feedback is supplied?' The submitted bidding strategies are compared to the RNNE in two ways. First, we consider the submitted bidding functions. Secondly, we check the profitability of the RNNE strategy in each treatment. For this purpose, we run additional tournaments in which the RNNE substitutes the poorest performing strategy of a treatment. The resulting average bids, prices and payoffs are reported below.

The automata programs that represent the subjects' strategies submitted for the tournament are detailed on the left hand side of the tables A1 – A4 in the appendix. We need to introduce some notation. Each strategy contains a bidding function for each of the four auction stages, denoted by b_1 , b_2 , b_3 and b_4 , respectively. Prices in the sequential auction are denoted correspondingly by p_1 , p_2 , p_3 , p_4 , and the values are represented by x . A chance move is denoted by number-pairs: the second number and the first number indicate the random number to be drawn and its likelihood of

¹⁴ The PASCAL code is available from the author upon request.

occurrence, respectively. In case a program refers to earlier rounds, the letter t denotes the current round, $t-1$ the previous one, etc. On the right hand side of the tables A1 – A2 the average bid-value ratios are depicted and contrasted with the RNNE graph. Automata programs are arranged by treatments: table A1, A2, A3 and A4 exhibit INTERACTION, BELIEVED INTERACTION, COMPUTERIZED, and NO PRICE, respectively. Within table A1 – A4, automata are ordered according to their profitability in the computer tournament, work spaces where subjects were seated are indicated by the PC number. In table A1 ‘Interaction 1 – PC 11’ indicates that the subject who submitted the most profitable strategy in INTERACTION was seated at work space 11 during the spontaneous decision task. The average payoff per sequential auction is indicated for each automaton at the bottom, in case of ‘Interaction 1’ the average payoff is 10.94. As pointed out above, the four most profitable automata of each market made it to the finale. The RANK numbers 1, 2, ..., 16 indicate the performance of the automata in the finale.

A brief examination of the bidding strategies reveals that they are heterogeneous and different from the RNNE. Some of the strategies are considerably more complicated than the RNNE bidding function, which increases linearly in values and consecutively from one stage to the next. Only two automata programs in COMPUTERIZED (‘Computerized 12’ and Computerized 15’) and two in NO PRICE (‘No Price 2’ and ‘No Price 6’) are linear increasing in values, and less than half of the automata induce consecutively increasing bid-value ratios as can be verified from the plots in the tables A1 – A4. The automata programs that violate these properties are recorded in the first and second row of table 1. Some automata programs include lotteries rather than deterministic bids as recorded in the third row of table 1. In the tournament, lotteries can be approximated by their expected values. Hence, the occurrence of the lotteries in the bidding functions is not too bothersome for the theory. A troublesome feature of the submitted strategies is the dependence on observed prices. Only under NO PRICE conditions strategies are formulated independently of prices. However, more than half (17 out of 30) of the strategies from the treatments in which subjects experienced price information feedback prior to submitting strategies for the tournament exhibit price dependence. In INTERACTION all automata programs but ‘Interaction 2’ include price information feedback. Table 1 identifies the automata programs that violate price independence in the fourth row. In Neugebauer and Pezanis-Christou (2002), price dependence of bids was inferred from the data. The bidding strategies submitted for the sequential auction tournament provide broad support for this observation. Its implication is that the RNNE is a poor descriptor of behavior in these markets, since it describes bidding in terms of value only.

Table 1. Bidding automata disagreeing with theory arranged by treatment*

	INTERACTION	BELIEVED INTERACTION	COMPUTERIZED	NO PRICE
1) non-linear	7/7	8/8	13/15 {1,2,...,11,13,14}	12/14 {1,3,4,5,7,8,...,16}
2) non-increasing in stages	3/7 {1,3,5}	3/8 {2,7,8}	7/15 {1,2,3,5,7,10,11}	10/14 {1,3,4,6,7,11,12,13,14,16}
3) chance dependent	4/7 {3,4,5,6}	2/8 {1,7}	1/15 {5}	4/14 {4,7,14,16}
4) price dependent	6/7 {1,3,4,5,6,7}	5/8 {1,5,6,7,8}	6/15 {2,7,8,10,11,14}	-/14

* The numerator represents the number of observations, the denominator the sample size. Incomplete programs are excluded from this summary.

Furthermore, Neugebauer and Pezanis-Christou's data suggest overbidding above the RNNE in the first three stages but no overbidding in the last stage. In the tournament, the automata induce average bid-value ratios above the RNNE predictions in stages 1-3 in the treatments with price information feedback and no significant differences in stage 4. The results are hence in line with the ones of Neugebauer and Pezanis. The average deviations between the bid-value ratios and the RNNE and the test results are summarized in Table 2. In NO PRICE, the automata produce no significant differences from the RNNE in stages 1-3 and significant underbidding in stage 4. The interpretation of the result is difficult without the knowledge of the data from the spontaneous decision task, which we have not yet analyzed. However, apparently the experience of price information feedback conditions induced subjects to submit different bidding strategies than without this experience, which is even more suggested by the resulting price sequences plotted in Figure 2. Prices are generally above the expected ones of the RNNE. While prices in NO PRICE decline from the first to the fourth stage, they increase for the first stages in all other treatments, in which subjects received price information feedback. The significant price changes between stages do not only contradict to the law of one price of Weber (1983) they also stand in marked contrast to the observations of Ashenfelter (1989) and Neugebauer and Pezanis-Christou (2002). Yet, they are compatible with the data of Neugebauer and Pezanis-Christou where the average price sequence over the last 50 sequential auctions followed the same qualitative pattern as in INTERACTION (but without significant changes)¹⁵: increases for the first three stages and a decline

¹⁵ Remember, participants of INTERACTION represent one of six markets in Neugebauer and

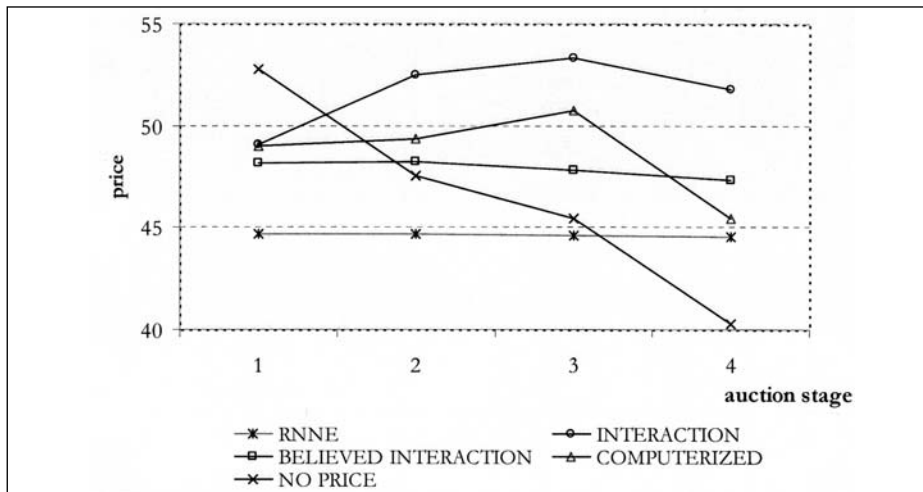
for the last stage, the price at the fourth stage being greater than the one at the first stage and smaller than the one at the second stage. In summary, all behavioral patterns that result in the tournament seem to be consistent with those reported in Neugebauer and Pezani-Christou. This is a remarkable result which suggests that the bidding programs capture crucial aspects of the spontaneous decision task.

Table 2. Deviation of average bid-value ratios from the RNNE by treatment*

Auction-Stage	INTERACTION	BELIEVED INTERACTION	COMPUTERIZED	NO PRICE
1	.146 (.027)	.120 (.012)	.139 (.005)	.051 (.198)
2	.137 (.063)	.084 (.036)	.165 (.001)	.029 (.331)
3	.106 (.062)	.030 (.401)	.120 (.003)	-.047 (.198)
4	.017 (.496)	-.056 (.183)	.003 (.363)	-.146 (.008)

* P-values of two-tailed Wilcoxon-Signed-Ranks-Test in parenthesis. The null hypothesis assumes that average bid-value ratios equal the RNNE. Incomplete programs are excluded.

Figure 2. The price sequences



Pezanis (2002). In the experiment, which preceded the submission of a strategy, subjects bid spontaneously up to 400 times in 100 sequential auctions.

As has been pointed out, the RNNE does not describe behavior well. However, if the RNNE automaton is more profitable than other strategies in sequential auction markets, it justifies itself and its usage could be recommended without caring too much about its aptness as descriptive theory.

In order to check whether the RNNE performs better than the other automata we substitute the least profitable automaton in every treatment by the RNNE automaton and run the tournament again. Since the automata programs in the tables A1 – A4 are ordered according to their performance in the tournament the excluded programs are the last ones in the tables. The tournament results are recorded in table 3. The first row indicates the average payoff of the strategies without inclusion of the RNNE. For instance, in INTERACTION automata ‘Interaction 1’ – ‘Interaction 8’ earn on average 9.66 per sequential auction. The average payoff in the other treatments is greater; and is greatest in NO PRICE. The second row reveals the relative performance of the RNNE, when it is substituted for the least profitable automaton in the treatment, and its excess over the average payoff in absolute terms. In INTERACTION, for instance, the RNNE automaton earns 0.077 less than the average (when it substitutes ‘Interaction 8’) and thus it yields the 5th highest payoff within the market of 8 automata. The RNNE automaton performs well in the other three treatments, in which the participants only experienced RNNE automata before they submitted their strategy. In COMPUTERIZED it is the best performing one, in the BELIEVED INTERACTION it is second best and in NO PRICE it is third best. Unfortunately, INTERACTION, in which the RNNE is an underperformer, is the most realistic treatment. We conclude that the RNNE does neither explain the strategies of the experiment nor can we recommend its application to yield maximum profit in an interactive market.

Finally, the third row in table 3 reports the average payoff of the four most profitable strategies in the finale. The automata that participated in the finale are identified with strategies 1-4 of each treatment. For instance, ‘Interaction 1’-‘Interaction 4’ earned on average 12.14 and were placed at RANKS 1-3 and 11 in the finale tournament. It is remarkable that the automata submitted in INTERACTION performs better than the other strategies including the RNNE. The individual RANKS of the finale tournament are reported with the automata in the appendix. For instance, ‘RANK 1’ identifies the most profitable automaton in the finale tournament. – And the winner is ‘Interaction 2’.

Table 3. Average payoff by treatment

	INTERACTION	BELIEVED INTERACTION	COMPUTERIZED	NO PRICE
1) average payoff	9.66	11.20	10.25	12.24
2) RNNE payoff- average payoff	-0.077	0.609	1.373	1.169
rank RNNE /sample size	5 /8	2 /8	1 /16	3 /16
3) average payoff finale	12.14	11.87	11.39	11.65
RANKS in the finale*	1, 2, 3, 11	4, 5, 6, 10	8, 9, 13, 15	7, 12, 14, 16

* The finale corresponds to the tournament of automata 1-4 from all treatments.

5. CONCLUSION

The present paper reports on bidding automata programmed by experienced experimental subjects in sequential auction markets. Salient rewards were introduced by a prize awarded to the most profitable automaton in a computer tournament. The tournament applied Weber's (1983) model of sequential first price sealed bid auctions in the independent private value framework. Although all automata were programmed for the same tournament, subjects' experience prior to the programming task differed. Before submitting an automata program, subjects ran one of four experimental treatments, all representing possible implementations of Weber's model. In three treatments subjects received price information feedback at each auction stage; in the fourth one, information was limited to qualitative feedback about winning or not winning a lot. The provision of price information induced an obvious treatment effect: subjects who experienced price information feedback developed bidding automata that included price information, whereas no participant of the qualitative feedback treatment included price information in their bidding automata programs. Furthermore, three treatments involved computerized competitors applying the equilibrium bidding strategy, and one treatment involved interaction between participants. Although the automata programmed by the participants of the interactive sequential auctions produced a relatively low average payoff when matched with each other. They performed comparatively well when matched with the equilibrium automaton or when matched with the automata programmed by participants of the other treatments.

The discovery that bids are conditioned on prices of earlier auction stages is at odds with the theoretical solution that predicts bids to depend on values only. As

Neugebauer and Pezani-Christou (2002) also inferred price dependence of bids from the data of sequential auction markets, in which bids were spontaneously submitted in 100 sequential auctions, we conclude that price information feedback matters for behavior in the experiment. Hence, bidding theories that neglect price information feedback must be poor behavioral descriptors.¹⁶ The finding that bids might depend on observed prices can be crucially relevant for the understanding of behavior in empirical auction markets when identical lots are supplied. Ashenfelter (1989) reported from real world auction markets of wine and arts that identical, subsequent lots are frequently knocked down at the same price. In fact, Ashenfelter was rather concerned with the frequency of price declines compared to increases in sequential auctions. Nevertheless, his modal observation involved equal prices for subsequent lots.

The risk neutral Nash equilibrium bidding strategy is only a best reply against itself. When applied unilaterally, its relative performance turned out to be poor in the tournament with heterogeneous automata programmed by subjects who experienced the interactive sequential auction setting. From this point of view, the equilibrium strategy seems to be neither a good predictor nor an above average earner. To put it bluntly, in order to program a winning automaton for the tournament we must recommend practicing bidding in the interactive auction experiment rather than studying the symmetric equilibrium strategy.

As pointed out in the introduction of the paper, there might be limitations of the strategy method which recommend its use only as a complementary research tool. Nevertheless, the comparative advantage of the strategy method, as applied in this work, over the standard spontaneous decision inquiry approach is that the determinants of the bidding functions do not have to be inferred from data, but are revealed by the individuals themselves. The analysis in this paper does not include a fit of the spontaneous data with the submitted strategies for the tournament. Therefore, we cannot tell how well the formulated strategies approximate individual behavior. Nevertheless, the reported behavioral patterns are in line with the results of earlier research (cf. Neugebauer and Pezani-Christou (2002)). In the future, we might focus more attention on the relation between the two approaches

¹⁶ The dependence of bids on observations at earlier stages can be accommodated by different learning models. See Camerer (2003) for a survey.

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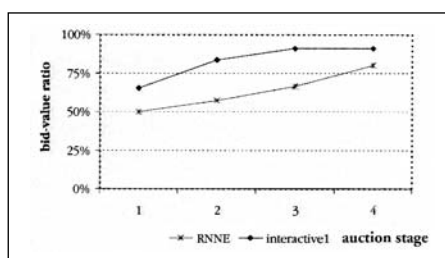
APPENDIX

Table A1. Automata INTERACTION

Interaction 1 - PC 11 - RANK 11

- 1st bid: $b_1 = .5x$ if $x > 50$, $b_1 = 30$ if $x \in [34, 50]$, $b_1 = 29$
if $x = 33$, $b_1 = 25$ if $x \in [26, 32]$, $b_1 = x - 1$ if $x < 26$
2nd bid: $b_2 = p_1 + 5$ if $x > p_1 + 5$, $b_2 = x - 1$ otherwise
3rd bid: $b_3 = p_2 + 5$ if $x > p_2 + 5$, $b_3 = x - 1$ otherwise
4th bid: $b_4 = x - 1$

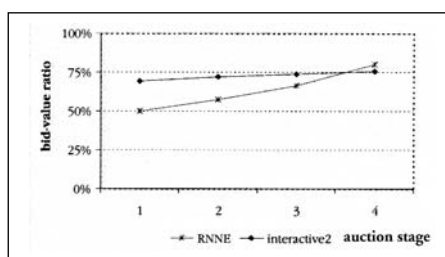
Average auction payoff 10.94



Interaction 2 - PC 2 - RANK 1

- 1st bid: $b_1 = 49$ if $x > 59$, $b_1 = 48$ if $x \in [50, 59]$, $b_1 = 39$ if
 $x \in [45, 49]$, $b_1 = x - 5$ if $x \in [6, 44]$, $b_1 = 1$ otherwise
2nd bid: $b_2 = 54$ if $x > 59$, $b_2 = 49$ if $x \in [50, 59]$,
 $b_2 = 39$ if $x \in [45, 49]$, $b_2 = b_1$ otherwise
3rd bid: $b_3 = 59$ if $x > 59$, $b_3 = 54$ if $x \in [56, 59]$, $b_3 = 49$ if
 $x \in [50, 55]$, $b_3 = 39$ if $x \in [45, 49]$, $b_3 = b_2$ otherwise
4th bid: $b_4 = 69$ if $x > 70$, $b_4 = 64$ if $x \in [65, 70]$, $b_4 = 59$ if
 $x \in [60, 64]$, $b_4 = x - 1$ if $x \in [50, 54]$, $b_4 = 39$ if $x \in [45,$
 $49]$, $b_4 = x - 5$ if $x \in [6, 44]$, $b_4 = x - 1$ otherwise

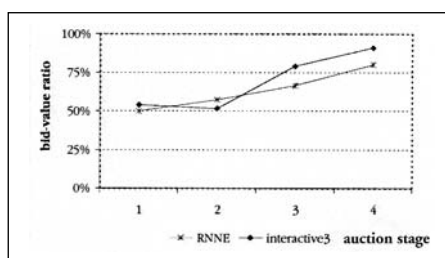
Average auction payoff 10.79



Interaction 3 - PC 15 - RANK 2

- 1st bid: $b_1 = (1/3, 48; 1/3, 49; 1/3, 50)$ if $x > 55$,
 $b_1 = (1/(x-3), 1; 1/(x-3), 2; \dots; 1/(x-3), x-3)$
if $x > 4$, $b_1 = 1$ if $x < 5$
2nd bid: $b_2 = p_1$ if $(x > p_1$ and $b_1 > p_1 - 3)$, $b_2 = b_1$ (if $x > 55$
and **not** $(x > p_1$ and $b_1 > p_1 - 3)$), $b_2 = (1/(x-2), 1; 1/(x-2), 2; \dots; 1/(x-2), x-2)$ if $x > 4$, $b_2 = 1$ if $x < 5$
3rd bid: $b_3 = p_2$ if $(x > p_2$ and $b_2 > p_2 - 3)$, $b_3 = (1/7, 48;$
 $1/7, 49; \dots; 1/7, 54)$ if $x > 55$ and **not** $(x > p_2$ and
 $b_2 > p_2 - 3)$, $b_3 = x - 2$ if $x < 56$
4th bid: $b_4 = x - 6$ if $x > 55$, $b_4 = x - 1$ if $x < 6$

Average auction payoff 10.22



Interaction 4 - PC 13 - RANK 3

- 1st bid: $b_1 = .5x$
2nd bid: $b_2 = p_1 + 3$ if $x > \max(p_1 + 3, 60)$, $b_2 = b_1$ otherwise
3rd bid: $b_3 = p_2 + (.5, 2)(.5, 3)$ if $b_3 > p_2 + 3$, $b_3 = b_2$ otherwise
4th bid: $b_4 = p_3 - 4$ if $x > p_3 - 4$, $b_4 = x - 5$ otherwise

Average auction payoff 10.11

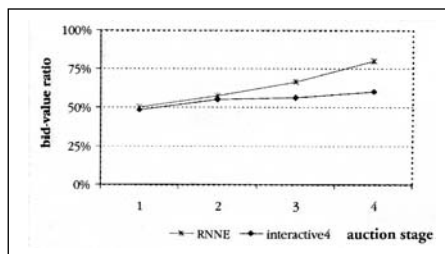
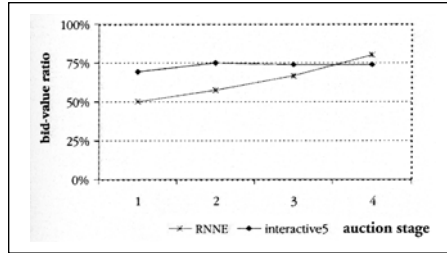


Table A1 INTERACTION continued

Interaction 5 - PC 4

- 1st bid: $b1=.6x$ if $x>60$, $b1=x-10$ if $x \in [41,60]$,
 $b1=x-(.5,2;5,3)$ if $x<41$
- 2nd bid: $b2=p1+2$ if $x>\max(p1+2,60)$,
 $b2=x-3$ otherwise
- 3rd bid: $b3=b2$
- 4th bid: $b4=.8x$ if $x>60$, $b4=x-5$ if $x \in [41,60]$,
 $b4=b3$ otherwise

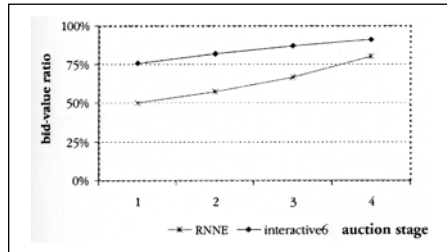
Average auction payoff 10.10



Interaction 6 - PC 8

- 1st bid: $b1=.5x$ if $x>74$, $b1=45$ if $x \in [46,74]$,
 $b1=x-1$ if $x<46$
- 2nd bid: $b2=p1+3$ if $x>p1+3$, $b3=b2$ otherwise
- 3rd bid: $b3=b2+(1/6,5;1/6,6;1/6,7;1/6,8;1/6,9;1/6,10)$
if $x>b2+10$, $b3=x-1$ otherwise
- 4th bid: $b4=b3+(1/6,5;1/6,6;1/6,7;1/6,8;1/6,9;1/6,10)$
if $x>b3+10$, $b4=x-1$ otherwise

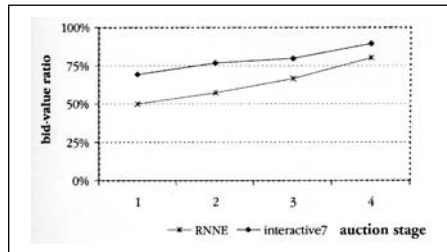
Average auction payoff 9.46



Interaction 7 - PC 6

- 1st bid: $b1=39$ if $x>44$, $b1=31$ if $x \in [32,44]$,
 $b1=x-1$ otherwise
- 2nd bid: $b2=p1$ if $x>p1$, $b2=b1$ otherwise
- 3rd bid: $b3=p2$ if $x>p2$, $b3=b2$ otherwise
- 4th bid: $b4=p3+10$ if $x>p3+10$, $b4=x-1$ otherwise

Average auction payoff 8.03



Interaction 8 - PC 10 **incomplete**

- 1st bid: $b1=(1/11,40;1/11,41;...;1/11,50)$ if $x>69$,
 $b1=?$ Otherwise
 - 2nd bid: $b2=p1+(.5,2;5,3)$ if $x>p1+3$, $b2=?$ Otherwise
 - 3rd bid: $b3=p2-(.5,2;5,3)$ if $x>p2$, $b3=?$ Otherwise
 - 4th bid: $b4=x-(?)$
- (the program applies a zero bid where not indicated)

Average auction payoff 7.66

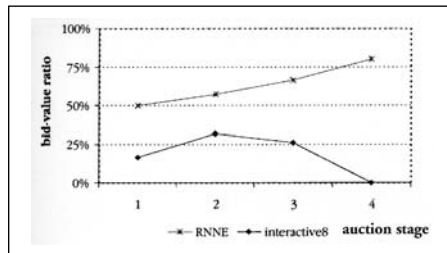
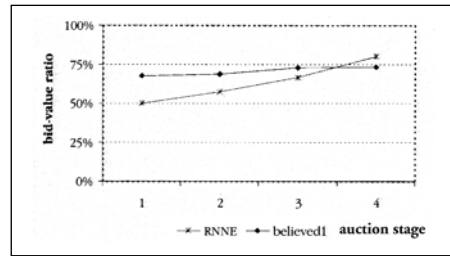


Table A2 Automata BELIEVED INTERACTION

Believed-Interaction 1 - PC 9 – RANK 6

1st bid: $b_1 = (1/9, 41; 1/9, 42; \dots; 1/9, 49)$ if $x > 50$, $b_1 = 41$
if $x \in [47, 50]$, $b_1 = x - 5$ if $x \in [6, 46]$, $b_1 = x - 1$ if $x < 6$
2nd bid: $b_2 = b_1$
3rd bid: $b_3 = p_2 + 3$ if $(p_2 > p_1 \text{ and } x > p_2 + 3)$, $b_3 = \lfloor .5(p_1 + p_2) \rfloor$ if $(p_2 < p_1 \text{ and } x > p_1)$, $b_3 = b_2$ otherwise
4th bid: $b_4 = 70$ if $x > 70$, $b_4 = p_3 + (.5, 2; .5, 3)$
if $x \in [p_3 + 3, 70]$, $b_4 = b_3$ otherwise

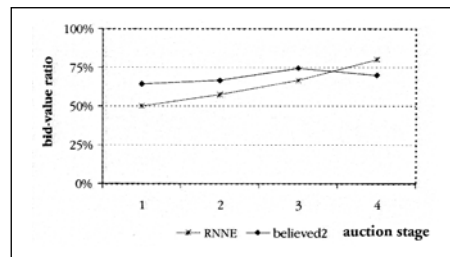
Average auction payoff 12.62



Believed-Interaction 2 - PC 1 – RANK 4

1st bid: $b_1 = 43$ if $x > 60$, $b_1 = 37$ if $x \in [40, 60]$, $b_1 = x - 5$ if $x \in [6, 39]$, $b_1 = x - 1$ if $x < 6$
2nd bid: $b_2 = 53$ if $x > 70$, $b_2 = 37$ if $x \in [40, 70]$, $b_2 = 33$ if $x \in [36, 39]$, $b_2 = x - 3$ if $x < 36$
3rd bid: $b_3 = 56$ if $x > 70$, $b_3 = 4$ if $x \in [50, 70]$, $b_3 = x - 5$ if $x \in [6, 49]$, $b_3 = x - 1$ if $x < 6$
4th bid: $b_4 = 66$ if $x > 70$, $b_4 = x - 10$ if $x \in [50, 70]$, $b_4 = x - 7$ if $x \in [8, 49]$, $b_4 = x - 1$ if $x < 8$

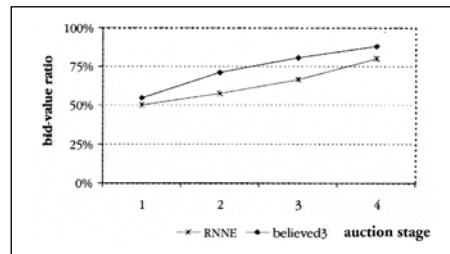
Average auction payoff 11.95



Believed-Interaction 3 - PC 16 – RANK 10

1st bid: $b_1 = 44$ if $x > 50$, $b_1 = x - 12$ if $x \in [20, 50]$, $b_1 = x - 4$ otherwise
2nd bid: $b_2 = b_1 + 3$
3rd bid: $b_3 = b_2 + 5$ if $x > b_2 + 5$, $b_3 = x - 1$ otherwise
4th bid: $b_4 = b_3 + 7$ if $x > b_3 + 7$, $b_4 = x - 1$ otherwise

Average auction payoff 11.94



Believed-Interaction 4 - PC 7 – RANK 5

1st bid: $b_1 = .5x$ if $x > 60$, $b_1 = x - 7$ if $x \in [21, 60]$,
 $b_1 = x - 1$ if $x < 21$
2nd bid: $b_2 = b_1 + 7$ if $x > 60$, $b_2 = x - 4$ if $x \in [21, 60]$,
 $b_2 = x - 1$ if $x < 21$
3rd bid: $b_3 = b_2 + 5$ if $x > 60$, $b_3 = x - 2$ if $x \in [21, 60]$,
 $b_3 = x - 1$ if $x < 21$
4th bid: $b_4 = b_3 + 5$ if $x > 60$, $b_4 = b_3$ otherwise

Average auction payoff 11.87

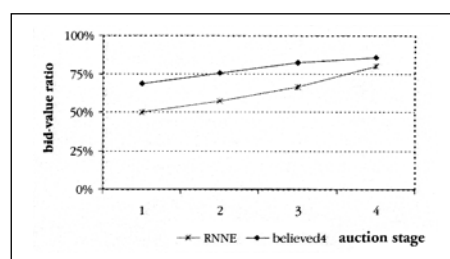
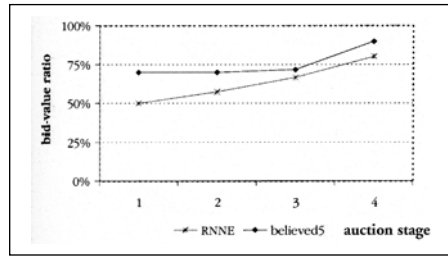


Table A2 BELIEVED INTERACTION continued

Believed-Interaction 5 - PC 5¹⁷

- 1st bid: $\{b_1=50 \text{ if } t=1, b_1=p_1(t=1) \text{ if } t=2, b_1=(p_1(t=1)+p_1(t=2))/2 \text{ if } t=3, b_1=p_1(t-1)+p_1(t-2)+p_1(t-3))/3 \text{ if } t \geq 3\}$ if $x > \max(60, \max(p_1(t-1, \dots, t-3)))$
 $b_1=x-15$ if $x \in [31, \max(60, \max(p_1(t-1, \dots, t-3)))]$,
 $b_1=x-5$ if $x \in [6, 30]$, $b_1=x-1$ if $x < 6$
- 2nd bid: $b_2=p_1(t)$ if $x > p_1$, $b_2=b_1$ otherwise
- 3rd bid: $b_3=p_2+2$ if $x > p_2+2$, $b_3=b_2$ otherwise
- 4th bid: $b_4=p_3+4$ if $x > p_3+4$, $b_4=x-1$ otherwise

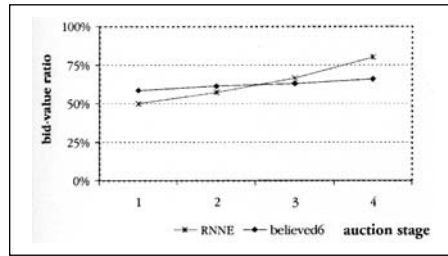
Average auction payoff 11.60



Believed-Interaction 6 - PC 3

- 1st bid: $b_1=40$ if $x \in [66, 100]$, $b_1=45$ if $x \in [51, 65]$,
 $b_1=34$ if $x \in [41, 50]$, $b_1=25$ if $x \in [31, 40]$, $b_1=15$
if $x \in [21, 30]$, $b_1=5$ if $x \in [11, 20]$, $b_1=x-1$ if $x < 10$
- 2nd bid: $b_2=p_1-2$ if $x > p_1$, $b_2=b_1$ otherwise
- 3rd bid: $b_3=p_2+2$ if $(x > p_2+2 \text{ and } p_1 < p_2)$,
 $b_3=b_2$ if $(p_1 > p_2 \text{ or } x < p_2+3)$
- 4th bid: $b_4=b_3$, $b_4=p_3+10$ if $x > p_3+10$,
 $b_4=p_3$ if $x \in [p_3, p_3+10]$,

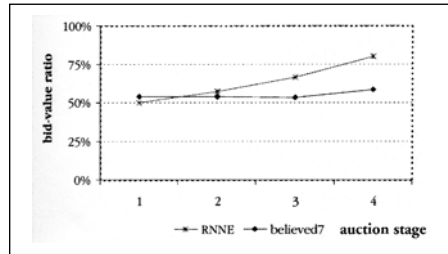
Average auction payoff 11.28



Believed-Interaction 7 - PC 12

- 1st bid: $b_1=\min\{(1/11, 40; 1/11, 41; \dots; 1/11, 50),$
 $x-(1/11, 30; 1/11, 31; \dots; 1/11, 40)\}$ if $x > 64$,
 $b_1=x-10$ if $x < 65$
- 2nd bid: $b_2=b_1$ if $p_1 < 51$, $b_2=b_1+2$ if $p_1 > 50$
- 3rd bid: $b_3=b_2$
- 4th bid: $b_4=65$ if $(p_3 > p_2 \text{ and } x > 80)$, $b_4=x-10$
if $(p_3 > p_2 \text{ and } x \in [60, 80])$, $b_4=b_3$ otherwise

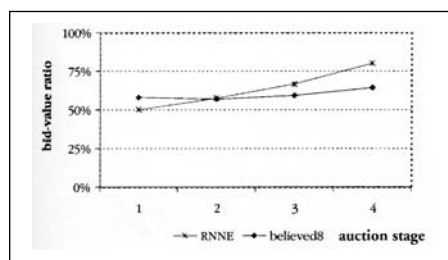
Average auction payoff 10.72



Believed-Interaction 8 - PC 14

- 1st bid: $b_1=35$ if $x > 60$, $b_1=x-10$ if $x \in [16, 60]$, $b_1=x-3$
if $x \in [4, 15]$, $b_1=x-1$ if $x < 4$
- 2nd bid: $b_2=b_1$
- 3rd bid: $b_3=45$ if $(p_2 > p_1 \text{ and } x > 60)$, $b_3=b_2$ otherwise
- 4th bid: $b_4=b_3+(p_3-p_2)$ if $(p_3 > p_2 \text{ and } x > \max(b_3+p_3-p_2, 61))$,
 $b_4=x-7$ if $x \in [16, 60]$, $b_4=b_3$ otherwise

Average auction payoff 7.63



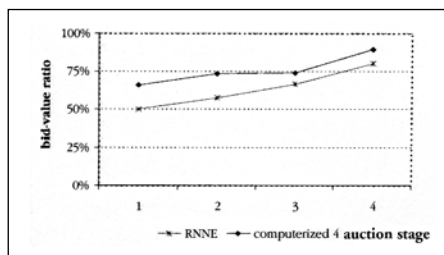
¹⁷ The letter t refers to the auction sequence in the simulation, $t=1$ is the first auction of $T=100$ repetitions. Thus, $t-1$ refers to the previous sequential auction and not to the previous auction stage.

Table A3 Automata COMPUTERIZED

Computerized-Price 1 - PC 11 - RANK 13

- 1st bid: $b_1=46$ if $x>69$, $b_1=41$ if $x \in [50,69]$,
 $b_1=x-9$ if $x \in [15,49]$, $b_1=1$ otherwise
2nd bid: $b_2=48$ if $x>79$, $b_2=46$ if $x \in [50,79]$,
 $b_2=x-9$ if $x \in [15,49]$, $b_2=1$ otherwise
3rd bid: $b_3=51$ if $x>79$, $b_3=46$ if $x \in [50,79]$,
 $b_3=x-9$ if $x \in [15,49]$, $b_3=1$ otherwise
4th bid: $b_4=56$ if $x>74$, $b_4=47$ if $x \in [60,74]$,
 $b_4=x-9$ if $x \in [15,49]$, $b_4=1$ otherwise

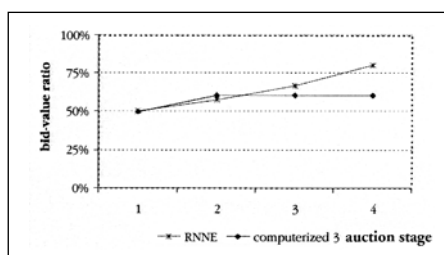
Average auction payoff 12.44



Computerized-Price 2 - PC 12 - RANK 9

- 1st bid: $b_1=45$ if $x>55$, $b_1=42$ if $x \in [45,55]$,
 $b_1=x/2$ if $x \in [25,44]$, $b_1=x-1$ if $x<25$
2nd bid: $b_2=p_1$ if $x>p_1$, $b_2=x-1$ otherwise
3rd bid: $b_3=\max(p_1, p_2)+5$ if $x>\max(p_1, p_2)+5$,
 $b_3=x-1$ otherwise
4th bid: $b_4=x-20$ if $x>70$, $b_4=x-10$ if $x \in [60,70]$,
 $b_4=x-5$ if $x \in [25,59]$, $b_4=x-1$ if $x<25$

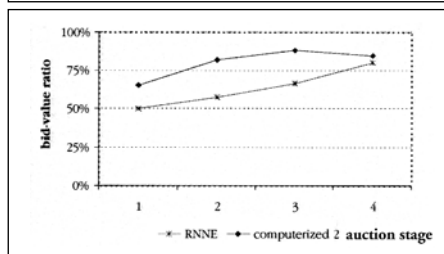
Average auction payoff 11.87



Computerized-Price 3 - PC 13 - RANK 15

- 1st bid: $b_1=x/2$
2nd bid: $b_2=.6x$
3rd bid: $b_3=.6x$
4th bid: $b_4=.6x$

Average auction payoff 11.81



Computerized-Price 4 - PC 9 - RANK 12

- 1st bid: $b_1=.6x$ if $x>60$, $b_1=.7$ if $x<61$
2nd bid: $b_2=.65x$ if $x>60$, $b_2=.75$ if $x<61$
3rd bid: $b_3=b_2$
4th bid: $b_4=.8x$ if $x>65$, $b_4=.9x$ if $x \in [51,65]$,
 $b_4=.95x$ if $x<51$

Average auction payoff 11.79

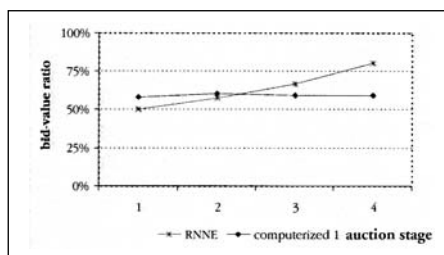
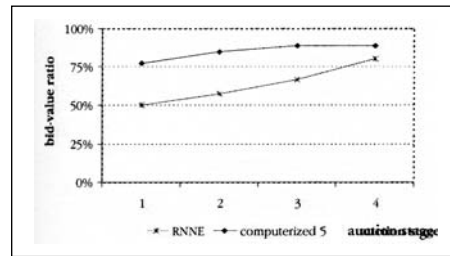


Table A3 COMPUTERIZED continued

Computerized-Price 5 - PC 5

- 1st bid: $b1=(.5,47)(.5,48)$ if $x>54$,
 $b1=x-5$ if $x \in [35,54]$, $b1=x-1$ if $x<35$
 2nd bid: $b2=p1+5$ if $x>\max(p1+5,54)$, $b2=x-5$
 if $x \in [55, p1+5]$, $p1>50$, $b2=b1$ otherwise
 3rd bid: $b3=p2+3$ if $x>\max(55, p2+3)$, $b3=x-3$ if $x \in [35, \max(55, p2+3)]$, $p2>52$, $b3=x-1$ otherwise
 4th bid: $b4=p3+3$ if $x>\max(55, p3+3)$, $b4=b3$ otherwise

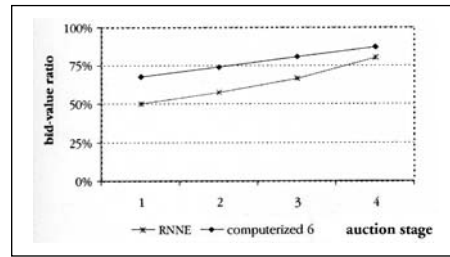
Average auction payoff 11.71



Computerized-Price 6 - PC 15

- 1st bid: $b1=35$ if $x>38$, $b1=x-1$ otherwise
 2nd bid: $b2=43$ if $x>56$, $b2=40$ if $x \in [41,56]$,
 $b2=x-1$ if $x<41$
 3rd bid: $b3=50$ if $x>56$, $b3=45$ if $x \in [46,56]$,
 $b3=x-1$ if $x<46$
 4th bid: $b4=60$ if $x>62$, $b4=x-2$ if $x \in [38,62]$,
 $b4=x-1$ if $x<38$

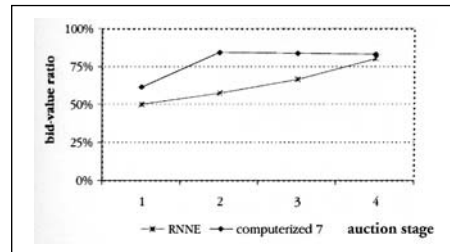
Average auction payoff 11.55



Computerized-Price 7 - PC 8

- 1st bid: $b1=\min\{x-20,50\}$ if $x>39$,
 $b1=\max\{x-5,0\}$ if $x<40$
 2nd bid: $b2=p1+5$ if $x>p1+6$, $b2=x-2$ if $x<p1+7$
 3rd bid: $b3=p2+5$ if $x>p2+6$, $b3=x-2$ if $x<p2+7$
 4th bid: $b4=p3+5$ if $x>p3+6$, $b4=x-2$ if $x<p3+7$

Average auction payoff 11.35



Computerized-Price 8 - PC 14

- 1st bid: $b1=x-35$ if $x>76$, $b1=x-20$ if $x \in [66,76]$,
 $b1=x-15$ if $x \in [56,65]$, $b1=x-10$ if $x \in [35,55]$,
 $b1=x/2$ if $x<35$
 2nd bid: $b2=p1+1$ if $x>p1+1$, $b2=x-1$ if $x<p1+2$
 3rd bid: $b3=p2+1$ if $x>p2+1$, $b3=x-1$ if $x<p2+2$
 4th bid: $b4=p3+1$ if $x>p3+1$, $b4=x-1$ if $x<p3+2$

Average auction payoff 11.13

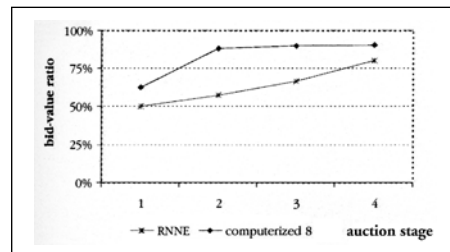


Table A3 COMPUTERIZED continued

Computerized-Price 9 - PC 3

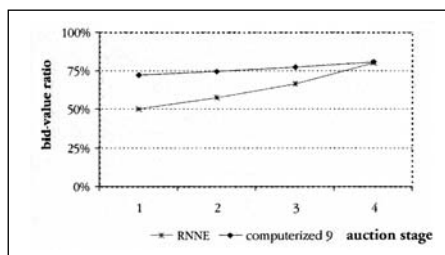
1st bid: $b_1=40$ if $x>41$, $b_1=x-1$ otherwise

2nd bid: $b_2=42$ if $x>43$, $b_2=x-1$ otherwise

3rd bid: $b_3=45$ if $x>46$, $b_3=x-1$ otherwise

4th bid: $b_4=48$ if $x>49$, $b_4=x-1$ otherwise

Average auction payoff 10.78



Computerized-Price 10 - PC 6

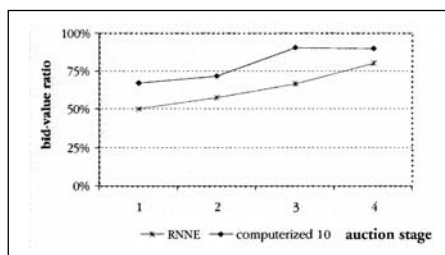
1st bid: $b_1=2/3x$

2nd bid: $b_2=p_1+3$, $x>p_1+3$, $b_2=2/3x$ otherwise

3rd bid: $b_3=p_2+2$, $x>p_2+2$, $b_3=2/3x$ otherwise

4th bid: $b_4=b_3$

Average auction payoff 10.41



Computerized-Price 11 - PC 7

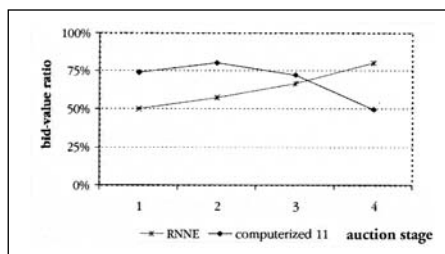
1st bid: $b_1=42$ if $x>43$, $b_1=x-1$ otherwise

2nd bid: $b_2=p_1-1$ if $x>p_1$, $b_2=x-1$ otherwise

3rd bid: $b_3=b_2-2$

4th bid: $b_4=x/2$

Average auction payoff 10.18



Computerized-Price 12 - PC 1

1st bid: $b_1 = 1/3 x$

2nd bid: $b_2 = ? x$

3rd bid: $b_3 = 2/3 x$

4th bid: $b_4 = x-5$, $x>5$, $b_4=1$ otherwise

Average auction payoff 10.16

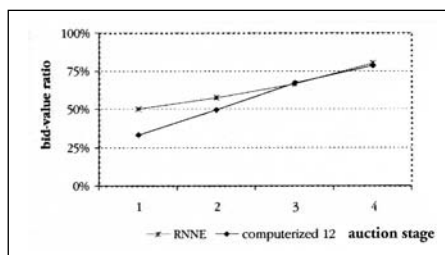
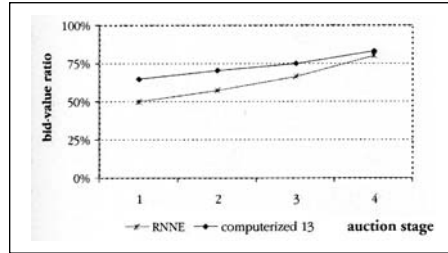


Table A3 COMPUTERIZED continued

Computerized-Price 13 - PC 2

1st bid: $b_1=36$ if $x>1.25*36$, $b_1=.8x$ otherwise2nd bid: $b_2=40$ if $x>40/.85$, $b_2=.85x$ otherwise3rd bid: $b_3=43$ if $x>43/.9$, $b_3=.9x$ otherwise4th bid: $b_4=49$ if $x>49/.95$, $b_4=.95x$

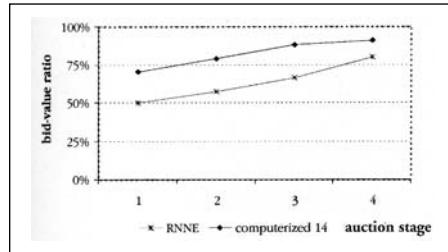
Average auction payoff 10.04



Computerized-Price 14 - PC 4

1st bid: $b_1=38$, if $x>39$, $b_1=x-1$ otherwise2nd bid: $b_2=p_1$, if $x>p_1$, $b_2=x-1$ otherwise3rd bid: $b_3=(x-p_1)/2+p_1$ if $x>p_1$, $b_3=x-1$ otherwise4th bid: $b_4=65$ if $x>66$, $b_4=x-1$ otherwise

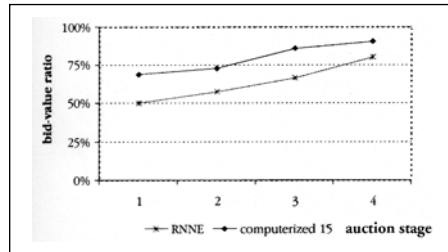
Average auction payoff 9.83



Computerized-Price 15 - PC 10

1st bid: $b_1=40$ if $x>42$, $b_1=x-2$ otherwise2nd bid: $b_2=40$ if $x>41$, $b_2=x-1$ otherwise3rd bid: $b_3=60$ if $x>61$, $b_3=x-1$ otherwise4th bid: $b_4=75$ if $x>76$, $b_4=x-1$ otherwise

Average auction payoff 9.02



Computerized-Price 16 - PC 16

Subject did not submit any strategy

(the program applies a zero bid where not indicated)

Average auction payoff 0

Table A4 Automata NO PRICE

Computerized-No-Price 1 - PC 5 – RANK 12

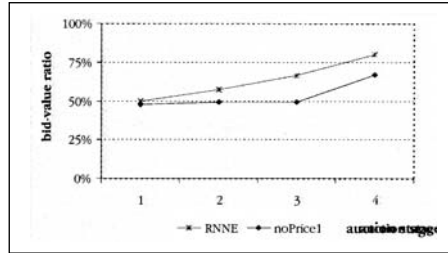
1st bid: $b_1 = .5x$ if $x > 45$, $b_1 = x - 10$ if $x < 46$

2nd bid: $b_2 = .5x$

3rd bid: $b_3 = .5x$

4th bid: $b_4 = 2/3 x$

Average auction payoff 14.32



Computerized-No-Price 2 - PC 10 – RANK 7

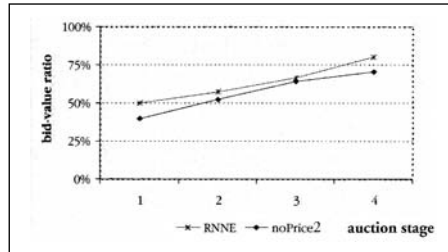
1st bid: $b_1 = .4x$

2nd bid: $b_2 = b_1 + .2(x - b_1)$

3rd bid: $b_3 = b_1 + .4(x - b_1)$

4th bid: $b_4 = b_1 + .5(x - b_1)$

Average auction payoff 14.21



Computerized-No-Price 3 - PC 7 – RANK 14

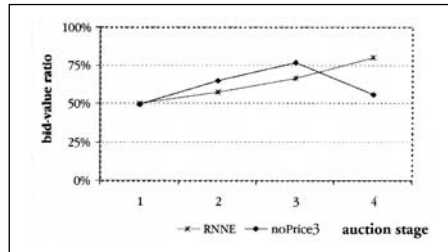
1st bid: $b_1 = x/2$

2nd bid: $b_2 = b_1 + 5$ if $x > 10$, $b_2 = b_1 + (x - b_1)/2$ otherwise

3rd bid: $b_3 = b_2 + 5$ if $x > 15$, $b_3 = b_2 + (x - b_2)/2$ otherwise

4th bid: $b_4 = x - 10$

Average auction payoff 13.82



Computerized-No-Price 4 - PC 12 – RANK 16

1st bid: $b_1 = .5x$ if $x > 30$, $b_1 = x - (.2, 1; .2, 2; \dots; 2, 5)$ if $x < 31$

2nd bid: $b_2 = .55x$ if $x > 30$, $b_2 = b_1$ otherwise

3rd bid: $b_3 = .6x$ if $x > 30$, $b_3 = b_2$ otherwise

4th bid: $b_4 = .55x$ if $x > 30$, $b_4 = b_3$ otherwise

Average auction payoff 13.76

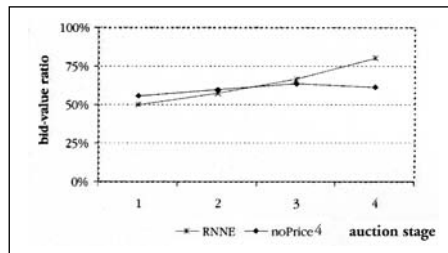
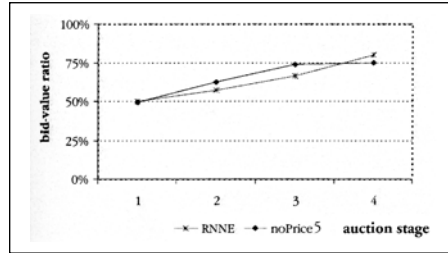


Table A4 NO PRICE continued

Computerized-No-Price 5 - PC 1

1st bid: $b_1 = .5x$ 2nd bid: $b_2 = 5/8 x$ 3rd bid: $b_3 = 45$ if $x > 70$, $b_3 = .75 x$ if $x < 71$ 4th bid: $b_4 = 50$ if $x > 70$, $b_4 = .75 x$ if $x < 71$

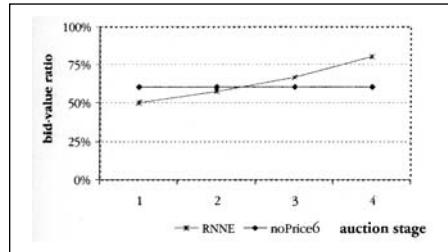
Average auction payoff 13.66



Computerized-No-Price 6 - PC 13

1st bid: $b_1 = .6x$ 2nd bid: $b_2 = .6x$ 3rd bid: $b_3 = .6x$ 4th bid: $b_4 = .6x$

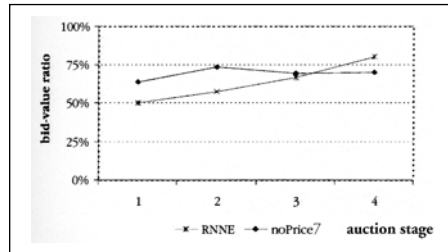
Average auction payoff 13.43



Computerized-No-Price 7 - PC 9

1st bid: $b_1 = (1/11, 40; 1/11, 41; \dots; 1/11, 50)$ if $x > 70$,
 $b_1 = .6x$ if $x \in [20, 70]$, $b_1 = .8x$ if $x < 20$ 2nd bid: $b_2 = (1/11, 50; 1/11, 51; \dots; 1/11, 60)$ if $x > 70$,
 $b_2 = .7x$ if $x \in [20, 70]$, $b_2 = .9x$ if $x < 20$ 3rd bid: $b_3 = 50$ if $x > 70$, $b_3 = .8x$ if $x \in [20, 70]$,
 $b_3 = .9x$ if $x < 20$ 4th bid: $b_4 = b_3$

Average auction payoff 13.39



Computerized-No-Price 8 - PC 3

1st bid: $b_1 = \max\{.5x, 30\}$ if $x > 49$, $b_1 = 30$ if $x \in [31, 49]$,
 $b_1 = x - 1$ if $x < 31$ 2nd bid: $b_2 = b_1 + 5$ if $x > 49$, $b_2 = b_1$ otherwise3rd bid: $b_3 = b_2 + 5$ if $x > 49$, $b_3 = b_2$ otherwise4th bid: $b_4 = .6x$ if $x > 49$, $b_4 = .9x$ otherwise

Average auction payoff 13.38

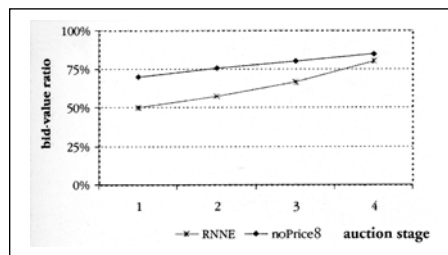
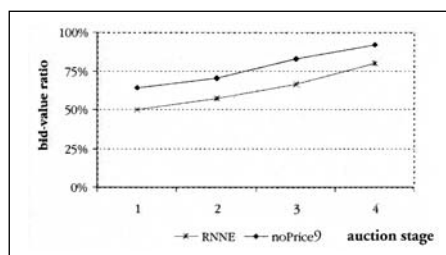


Table A4 NO PRICE continued

Computerized-No-Price 9 - PC 6

1st bid: $b_1 = .4x$ if $x > 80$, $b_1 = .5x$ if $x \in [60, 79]$,
 $b_1 = .7x$ if $x \in [40, 59]$, $b_1 = .8$ if $x < 40$
2nd bid: $b_2 = .5x$ if $x > 75$, $b_2 = .6x$ if $x \in [50, 74]$,
 $b_2 = .8x$ if $x \in [30, 49]$, $b_2 = .9$ if $x < 30$
3rd bid: $b_3 = .6x$ if $x > 80$, $b_3 = .7x$ if $x \in [60, 80]$,
 $b_3 = .85x$ if $x \in [30, 59]$, $b_3 = .95$ if $x < 40$
4th bid: $b_4 = .75x$ if $x > 80$, $b_4 = .85x$ if $x \in [60, 79]$,
 $b_4 = .9x$ if $x \in [30, 59]$, $b_4 = .95$ if $x < 30$

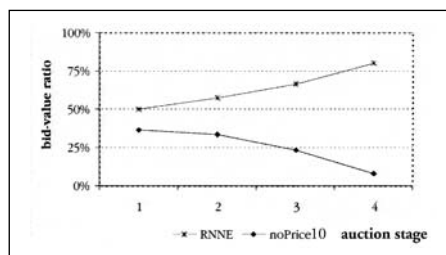
Average auction payoff 13.26



Computerized-No-Price 10 - PC 15 **incomplete**

1st bid: $b_1 = 40$ if $x > 40$
2nd bid: $b_2 = 45$ if $x > 45$
3rd bid: $b_3 = 50$ if $x > 50$
4th bid: $b_4 = 70$ if $x > 70$
(the program applies a zero bid where not indicated)

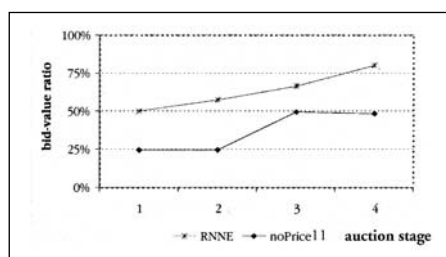
Average auction payoff 12.94



Computerized-No-Price 11 - PC 8

1st bid: $b_1 = .25x$
2nd bid: $b_2 = .25x$
3rd bid: $b_3 = .5x$
4th bid: $b_4 = .5x$ if $x > 49$, $b_4 = x - 10$ if $x < 50$

Average auction payoff 12.73



Computerized-No-Price 12 - PC 4

1st bid: $b_1 = x - 40$ if $x > 50$, $b_1 = x - 10$ if $x < 51$
2nd bid: $b_2 = x - 30$ if $x > 50$, $b_2 = x - 5$ if $x < 51$
3rd bid: $b_3 = b_2$
4th bid: $b_4 = 50$ if $x > 50$, $b_4 = x - 5$ if $x < 51$

Average auction payoff 12.65

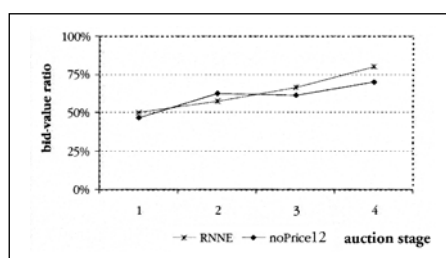
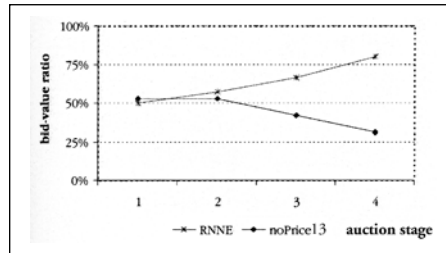


Table A4 NO PRICE continued

Computerized-No-Price 13 - PC 16

1st bid: $b_1=40$ if $x>40$, $b_1=x-10$ if $x<41$
 2nd bid: $b_2=42$ if $x>42$, $b_2=x-10$ if $x<43$
 3rd bid: $b_3=x-30$ if $x>65$, $b_3=x-20$ if $x<66$
 4th bid: $b_4=x-20$ if $x>65$, $b_4=x-15$ if $x<65$

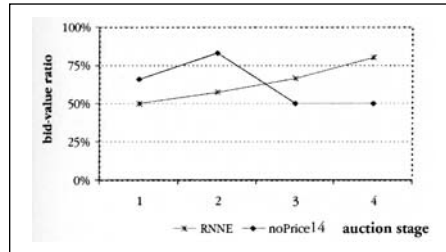
Average auction payoff 11.95



Computerized-No-Price 14 - PC 14

1st bid: $b_1 = x(1/16,60\%;1/16,61\%;\dots;1/16,75\%)$ if $x<91$, $b_1=50$ if $x>90$
 2nd bid: $b_2 = x(1/36,65\%;1/36,66\%;\dots;1/36,100\%)$
 3rd bid: $b_3 = (1/x,1;1/x,2;\dots;1/x,x)$
 4th bid: $b_4 = (1/x,1;1/x,2;\dots;1/x,x)$

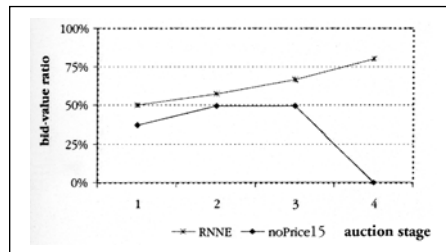
Average auction payoff 10.33



Computerized-No-Price 15 - PC 2incomplete

1st bid: $b_1=x-50$ if $x>74$, $b_1=x-25$ if $x \in [50,74]$, $b_1=35$ if $x \in [36,49]$, ? otherwise
 2nd bid: $b_2=.5x$
 3rd bid: $b_3=.5x$
 4th bid: $b_4=?$
 (the program applies a zero bid where not indicated)

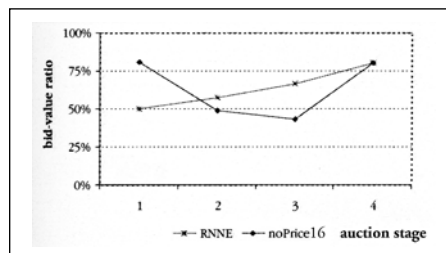
Average auction payoff 8.92



Computerized-No-Price 16 - PC 11

1st bid: $b_1=x-5$
 2nd bid: $b_2=x-10$
 3rd bid: $b_3=x-(.2,20;.2,21;\dots;.2,24)$ if $x>24$,
 $b_3=x-5$ if $x<25$
 4th bid: $b_4=x-(.5,2;.5,3)$

Average auction payoff 3.04



Las Instrucciones

Acabas de participar en 100 rondas de subastas secuenciales. La Segunda Parte consiste simplemente en especificar una estrategia que tu piensas es la que paga mejor en esa misma subasta.

Para todos los valores entre 0 y 100 debes describir

1. la puja para el primer artículo.
2. la puja para el segundo artículo, en caso de que no ganes el primero.
3. la puja para el tercer artículo, en caso de que no ganes un anterior.
4. la puja para el cuarto artículo, en caso de que no ganes un anterior.

Después de cada venta se revelará el precio a que se haya vendido el artículo.

La Ganancia

En total participarán 48 estudiantes en esa parte (vosotros y 32 en la Universidad de York en Inglaterra).

En todas las posibles combinaciones de grupos de ocho vas a jugar con tu estrategia siempre contra las estrategias de 7 otros.

En cada grupo tu estrategia se empleará en 1000 rondas de subastas secuenciales.

Aquel participante que haya especificado la estrategia que acumula más ganancias en todas las rondas en las que participe ganará 10000-15000 Pesetas (el equivalente a 50 Libras)

Si ganas tú, te avisaremos por teléfono o por correo electrónico. La lista de los rangos (incluyendo el resultado de las ganancias acumuladas) se podrá consultar a partir de la segunda semana de 2001 en el despacho 2P07.

Nombre:

Teléfono:

Correo electrónico:

Número del participante:

Mi Estrategia

Mi puja para el primer artículo (my bid for the first item)

Mi puja para el segundo artículo (en caso de que no hayas ganado el primero)

Mi puja para el tercer artículo (en caso de que no hayas ganado ninguno de los anteriores)

Mi puja para el cuarto artículo (en caso de que no hayas ganado ninguno de los anteriores)

ERRATUM

In the paper “Bidding Strategies of Sequential First Price Auctions Programmed by Experienced Bidders” by Tibor Neugebauer (*Cuadernos de Economía* 75(3): 153-184, 2004) the following corrections should be made:

On page 156 in the 3rd sentence of section 2, the index to the bid and the auction stage should be notated κ instead of k . The revised sentence is as follows.

Buyers demand exactly one lot each, such that they submit sealed bids $b_{\kappa} \in [0,1]$ at every auction stage $\kappa = \{1, \dots, k\}$ as long as their demand is not met.

On page 157 in the numerator in the first line of eq. (3) $n-k$ should read $n-\kappa$. The revised equation is as follows.

$$\begin{aligned} E[p_{\kappa}] &= E[b_{\kappa}(x_{(\kappa)})] = E\left[x_{(\kappa)} \frac{n-k}{n-\kappa+1}\right] \\ &= E[x_{(\kappa)}] \frac{n-k}{n-\kappa+1} = \frac{n-\kappa+1}{n+1} \frac{n-k}{n-\kappa+1} \\ &= \frac{n-k}{n+1} \end{aligned} \quad (3)$$

These corrections are only typographical in nature, and none of the results or conclusions of the paper are affected.