Batch Queues with Choice of Arrivals:
Equilibrium Analysis and Experimental Study

William E. Stein
Texas A&M University

Amnon Rapoport
University of Arizona
and
Hong Kong University of Science and Technology

Darryl A. Seale
University of Nevada Las Vegas

Hongtao Zhang
Hong Kong University of Science and Technology

Rami Zwick
Hong Kong University of Science and Technology

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Please address all correspondence to

Amnon Rapoport
University of Arizona
Department of Management and Policy
405 McClelland Hall
P.O. 210108
Tucson, AZ 85721
Phone (O): 520-621-9325
Fax (O): 520-621-4171
Email: amnon@u.arizona.edu

Abstract

We study both theoretically and experimentally the decisions players make in two queueing games with batch service. In both games, players are asked to independently decide when to join a discrete-time queue to receive service, or they may simply choose not to join it at all. Equilibrium solutions in pure and mixed strategies are constructed for two games where balking is prohibited and where it is allowed. They are then tested experimentally in a study that varies the game type (balking vs. no balking) and information structure (private vs. public information) in a 2×2 between-subject design. With repeated iterations of the stage game, all four experimental conditions result in aggregate, but not individual, behavior approaching mixed-strategy equilibrium play.

1. Introduction

This paper reports the results of a study of the decisions players make whether to join a queue, and if so at what time to arrive when they are served in batches that have a fixed and commonly known capacity. Examples of such queueing systems are common in transportation markets. Long-distance ferries can carry a maximum number of cars at a time, and the duration of the trip and departure times are usually commonly known. Interurban buses and intra-campus shuttles can carry a maximum number of passengers at a time, and the length and departure times are also fixed and commonly known (Glazer & Hassin, 1987). Yet another example is of a department store announcing that a fixed number *s* of prizes, all of the same value, will be distributed to the first *s* customers who arrive at the store on the morning after Thanksgiving. In all examples, she typically also has the option of staying out of the queue. If she joins the queue, the duration of her wait depends on her arrival time, the arrival times of the other customers, the capacity of the server, and the queue discipline. If the service operates several times a day with fixed time intervals between service starts, then the length of her wait also depends on the frequency of service. Chaudhry and Templeton (1983) discuss batch queueing with endogenously determined arrivals. In a recent and excellent reference, Hassin and Haviv (2003) survey the equilibrium behavior of customers in several queueing systems.

To illustrate the problem a passenger faces in batch queueing systems, consider a long-distance ferry that departs once a day at a predetermined time, say 12:00. Assume that the ferry can transport at most *s* passengers, that all *n* potential passengers are symmetric in terms of their cost structure (cost of waiting and utility of successfully completing service), and that they are served according to a first-come first-served (FCFS) queue discipline. Consider two passengers, A and B, who arrive at the queue at 11:00 and 11:45, respectively. Although passenger A must wait an hour until departure, and incur the associated cost of waiting, he is more likely to find fewer than *s* passengers ahead of him and therefore embark on the ferry. And although passenger B must wait a shorter period of 15 minutes only until departure, and therefore incur a considerably smaller cost of waiting, she is more likely than A to find *s* or more passengers ahead of her. Consequently, she is less likely than passenger A to embark on the ferry and receive the utility associated with successful completion of the trip. There are two variants of this example that, in general, result in different decision behavior. In one of them the queue length is fully *observable* upon arrival, and in the other it is *unobservable* (see Chapters 2 and 3 of Hassin & Haviv, 2003). We consider both cases under the assumption that reneging (leaving the queue after joining it) is not permitted.

Batch service queueing systems have been studied extensively in the operations research literature. These studies mostly assume an infinite stream of identical customers who arrive according to a Poisson process at a service facility with a single server that serves them in batches. The size of each batch has a (possibly infinite) limit. A fixed service fee is incurred as well as waiting cost that is an increasing function of the waiting time. Batch queueing systems with random service times have been studied by Bailey (1954), Chaudhry and Templeton (1983), Deb and Serfozo (1973), Mehdi (1975), Neuts (1967), and others. Batch queueing systems with deterministic service times have been discussed by Barnett (1973), Chaudhry and Templeton (1983), Kosten (1973), and Weiss (1979) among others. These studies are prescriptive in nature trying to establish optimal service policies or computing efficient control limits (Glazer & Hassin 1987). They leave no room for the customer to make decisions, as the arrival pattern is typically (see exceptions below) assumed to be exogenously determined.

Our approach and goals differ from these studies in three major respects. First, our goal is descriptive not prescriptive. We wish to identify and explain whatever behavioral regularities emerge when financially motivated subjects have to decide independently and simultaneously whether to join a queue, and if so at what time to arrive. For this purpose, we create two batch queueing games in the laboratory, derive and fully characterize the equilibrium solutions for these two games, and then compare observed to predicted behavior in a fully controlled environment. Since it is unlikely that subjects will adhere to equilibrium play in a single-shot queueing game, we iterate the stage game a large number of times to determine the effects of experience, if any, on behavior. Second, we focus on finite rather than infinite populations and on discrete-time rather than continuous-time systems. This is mandated in part by experimental considerations. The third point of departure is that we study batch queueing systems where players are required to decide independently whether to join the queue, and if so at what time to arrive, without making additional assumptions about inter-arrival times or the distribution of number of customer arrivals.

Related Literature

Assuming that arrival time is a choice variable, four previous studies are most closely related to the work reported in our study. Holt and Sherman (1982) presented a simple model of waiting-line auctions, formulated as a noncooperative game with incomplete information. In their model, a fixed number s of identical prizes are awarded simultaneously to n players ($s<n$) at a specified time on a FCFS basis, one prize to each of the s successful customers. Customers who queue for the prizes choose their waiting times independently through their arrival at the queue. Early arrivals are associated with cost of waiting, assumed to be linear, and their usefulness depends on the arrival times of the other customers. Holt and Sherman consider a more general case than

ours by assuming differential opportunity costs of time with each customer knowing her own time value for the prize and all customers having identical subjective beliefs about the time values of prizes for the other claimants. This allows them to construct a pure-strategy equilibrium solution in which players with higher time values for the prize arrive earlier.

In another related theoretical paper, Glazer and Hassin (1987) consider queueing systems in which customers are served in batches. In their system, service begins at fixed pre-determined times, regardless of the number of customers at the queue. There are fixed time intervals between two successive service starts that are called *cycles*. Unlike the model of Holt and Sherman, Glazer and Hassin assume that all customers are identical. They also assume a FCFS queue discipline and batches of fixed capacity. In their model, if the queue length exceeds the server capacity s at a scheduled service time, exactly s customers are served and the remaining customers have to wait for the beginning of the next cycle. The model studied by Glazer and Hassin differs for the one we present below in several important respects. First, they assume that customers arrive from a very large population, whereas the number of customers in our model is finite, relatively small, and commonly known. Second, they assume continuous time, whereas the strategy space in our model is discrete. Third, although arrival times are a choice variable in their model, they assume that the *number* of customer arrivals during a cycle follows a Poisson distribution with a known mean. In contrast, our model makes no such assumption.

The two other studies are experimental, focusing on the description and explanation of observed behavioral regularities in queues. The first study by Rapoport, Parco, Stein, and Seale (2004) examines queueing systems with fixed opening and closing times, finite population size, FCFS queue discipline, single server, deterministic service time, no balking nor reneging, and linear waiting costs. All the game parameters are assumed to be commonly known. The second study by Seale, Parco, Stein, and Rapoport (2004) is similar in its design with the major exceptions of 1) introducing a positive (rather than zero) payoff for staying out, 2) allowing for early arrivals before the service station opens up, 3) providing information (in two of the four conditions) about the behavior of each member of the population at the end of each trial, and 4) manipulating congestion by increasing (in two of the four conditions) the fixed service time so that not all players join the queue in equilibrium with no waiting even if pre-play coordination is allowed. The data analysis in both studies is driven by the Nash equilibrium solution. However, unlike the present study where players are served in batches with capacity that is smaller then the population size, in both studies of Rapoport et al. and Seale et al. service time is positive and customers are served one at a time. Both studies have generated consistent and replicable patterns of behavior in queues that are accounted for quite well on the aggregate level by a symmetric mixed-strategy equilibrium solution. In contrast, individuals display a wide variety of arrival profiles that defy a simple classification. A few subjects stick to the same arrival time possibly with a few switches. On the other extreme are subjects who keep switching their arrival times on almost every trial. The remaining subjects fall between these two extremes. The frequencies of staying out decisions are also distributed unevenly across the subjects. There is evidence for learning across iterations that is manifested in a steady decline in frequency of switches across iterations. These results suggest that equilibrium analysis of our two batch queueing games may prove useful in accounting for the aggregate, but not individual, results observed in the present study.

The rest of the paper is organized as follows. Section 2 presents the two batch queueing games and characterizes their equilibrium solutions, and Section 3 presents the experimental design and the results. Section 4 summarizes the main conclusions.

2. The Batch Queueing Games

The Model

The batch queueing game is presented to our subjects as a choice of arrival to a ferry that departs once a day at a pre-determined and commonly known time (12:00). Making their decisions independently, players have the option of staying out and deriving a fixed payoff. The assumptions of the game are as follows:

Service Time: The ferry departs at a fixed time (12:00 in our study). Service time is 0.

Calling Population: The calling population is finite of size n.

Arrival Pattern: Decisions are made in discrete time (5-minute intervals in our experiment). Each player must decide independently whether to join the queue, and if so at what time to arrive.

Tie-Breaking Rule: If multiple players arrive at the same time interval, their order of arrival is determined randomly with equal probabilities. Once ties are resolved, the players are informed immediately of the outcome.

Reneging and Balking: Reneging is prohibited. We study two games (conditions), one (Condition WB) that allows for balking and the other (Condition NB) that prohibits it.

Queue Discipline: FCFS.

Number of Servers: One.

Service capacity: s ($s<n$),

Payoff Structure: Assume that balking is prohibited. Then, each player who joins the queue is charged 1) a fixed entry fee f, and 2) a variable cost c per minute of waiting in the queue until the ferry departs. If he completes the service successfully (i.e., embarks on the ferry), he receives a fixed reward r. Each player who stays out of the queue receives a fixed payoff g ($g<r$-f). The resulting payoff function, the same for each player i (i=1, … , n), takes the form:

$$H_i = \begin{cases} g, & \text{if player } i \text{ stays out of the queue} \\ -f - cw_i, & \text{if player } i \text{ waits } w_i \text{ minutes without completing service} \\ r - f - cw_i, & \text{if player } i \text{ waits } w_i \text{ minutes and completes service} \end{cases}$$

In the payoff function above, w_i is the time (in minutes) player i waits for the departure of the ferry. The waiting cost c, entry fee f, and reward r are assumed to be positive, whereas the payoff g for staying out can be positive or negative.

Next, assume that balking is allowed. In our experiment balking is enforced if s or more players precede player i in the queue. In this case, a player does not incur the variable waiting cost unless he is assured of completing the service successfully. The payoff function, which is the same for each player, takes the form:

$$H_i = \begin{cases} g, & \text{if player } i \text{ stays out of the queue} \\ -f, & \text{if player } i \text{ is assured of not completing service} \\ r - f - cw_i, & \text{if player } i \text{ waits } w_i \text{ minutes and completes service} \end{cases}$$

Pure and Mixed Equilibrium Solutions

Symmetric mixed-strategy equilibria. Because the players are symmetric, we only focus on symmetric mixed-strategy equilibria. Assume that arrivals are only possible at equally spaced time points $\{0,1,...,T\}$ where 0 is the initial time that queueing is permitted and T corresponds to the departure of the ferry. In out experiment, T=12:00, 0 corresponds to 11:00, and time points are spaced 5 minutes apart. (Any arrival before 11:00 would be certain to yield a payoff smaller than g.) Let $p_0,...,p_T$ be the equilibrium probabilities of arrival for each player at the various times. Set $p_{out} = 1-(p_0 + ... + p_T)$ to be the probability a player chooses to stay out of the queue. Since we want to distinguish between arrivals strictly before time t versus at time t, we define $F(t)$ as the probability of arriving strictly before t: $F(t) = p_0 + ... + p_{t-1}$. Now distinguish one of the n players; we will compute the expected payoff for this player for each t and use the results to solve for the equilibrium probabilities $p_0,\ldots,p_T$.

Since the other n-1 players all choose their arrival times independently from the same distribution, we have the following results:

$$P(i \text{ arrivals before time } t) = \binom{n-1}{i} F(t)^i \left[1-F(t)\right]^{n-1-i}$$

$$P(j \text{ arrivals at } t \mid i \text{ arrivals before } t) = \binom{n-1-i}{j} h_t^j (1-h_t)^{n-1-i-j}$$

where

$$h_t = p_t \big/ (1-p_0-\ldots-p_{t-1}) = p_t \big/ (p_t+\ldots+p_T+p_{out}) .$$

h_t is the probability of arriving at t given an arrival before t did not occur.

Suppose that there are i arrivals before the designated player and j others arrive at the same time he does. Then there are only $s-i$ available spaces on the ferry for him and the j others. The probability that this player obtains a ride on the ferry is denoted $a(i,j)$ and given by

$$a(i,j) = \begin{cases} 0, & \text{if } s-i \le 0 \\ (s-i)\big/(j+1), & \text{if } 0 < s-i < j+1 \\ 1, & \text{if } s-i \ge j+1 \end{cases}$$

The probability that the designated player is Accepted on the ferry if he arrives at time t is:

$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1-i} P(\text{Accepted at } t \mid j \text{ at } t, i \text{ before } t) P(j \text{ at } t, i \text{ before } t)$$

$$= \sum_{i=0}^{n-1}\sum_{j=0}^{n-1-i} a(i,j) P(j \text{ at } t \mid i \text{ before } t) P(i \text{ before } t)$$

$$= \sum_{i=0}^{n-1}\sum_{j=0}^{n-1-i} a(i,j) \binom{n-1-i}{j} h_t^j (1-h_t)^{n-1-i-j} \binom{n-1}{i} F(t)^i \left[1-F(t)\right]^{n-1-i}$$

$$= \sum_{i=0}^{n-1} \binom{n-1}{i} F(t)^i \left[1-F(t)\right]^{n-1-i} \sum_{j=0}^{n-1-i} a(i,j) \binom{n-1-i}{j} h_t^j (1-h_t)^{n-1-i-j}$$

If a player arrives at time $t \in \{0,1,...,T\}$, he faces a wait of $5(T-t)$ minutes until departure of the ferry. The expected payoff E_t to the designated player is therefore:

$$E_t = \begin{cases} -f + P(Accepted)r - 5(T-t)c, & \text{if No Balking} \\ -f + P(Accepted)\left[r - 5(T-t)c\right], & \text{if Balking Allowed} \end{cases}$$

Now assume that the parameters have been chosen so that the equilibrium solution has $p_{out} > 0$. If $E_t < g$, then the player is better off not arriving at time *t* and instead staying out. If $E_t > g$, then there is no advantage in staying out so we must have $p_{out} = 0$. Therefore, if $p_{out} > 0$ then $E_t = g$ for all *t* such that $p_t > 0$ and $E_t \leq g$ when $p_t = 0$. We can use this to numerically search for p_t that makes $E_t = g$ and repeat for each larger value of *t*. This is very easy and quick. If it turns out that our initial conjecture that $p_{out} > 0$ is not true, we will discover that fact by noting that we were unable to construct a probability distribution as a solution, *i.e.*, the sum of the values of p_t will exceed 1. (At this point the procedure will stop since the term h_t will be negative or undefined.) We then follow the procedure below.

If the parameter values have been chosen such that $p_{out} = 0$ in the equilibrium, then the solution proceeds in an iterative fashion. We first guess a number *E*, greater than or equal to *g*. Starting at $t = 0$, search for p_t that makes $E_t = E$. After applying the solution procedure for all *t*, either (i) we obtain a solution with $p_{out} > 0$, (ii) a solution with $p_{out} = 0$, or (iii) no solution as described previously. If (i), then we repeat the search with a smaller conjectured value of *E*. If (iii), then we repeat the search with a larger conjectured value for *E*. We never achieve case (ii) except by luck. But we can approximate the solution since we will have narrowed *E* down to an arbitrarily small interval using cases (i) and (iii).

In our experiment, n=20, f=40, c=4 (per minute), r=340, T=12:00, and g=60. Table 1 presents the symmetric mixed-strategy equilibria for the values of n, f, c, r, and g above and values of s ranging from 5 to 19. The equilibrium strategies for the case where balking is prohibited are displayed in the top panel of Table 1; those for the case where balking is allowed are shown in the bottom panel. Note that for all values of s, the earliest arrival time in equilibrium is 11:05 (with an associated payoff of 80 if service is obtained). Entering the queue at 11:00 and receiving service yields a payoff of 60 that is the same as the one for staying out. Table 1 shows that when balking is prohibited (top panel), as s increases from 5 to 18 the probability of entering at 11:05 increases from 0.180 to 0.952 whereas the probability of staying out decreases from 0.700 to 0.048. The sum of these two probabilities increases from 0.880 when s=5 to 1.000 when s=18. In particular, when s=14, as in our experiment, in equilibrium each player should enter the queue at 11:05 with probability 0.702, stay out with probability 0.236, and enter at times 11:45, 11:50, 11:55 and 12:00 with relatively small probabilities that do not exceed 0.025. When s=19, all the n players should arrive at 12:00.

When balking is allowed (bottom panel), the pattern is similar with smaller probabilities of entering the queue after 11:05. However, if $9 \leq s \leq 16$ (and, in particular, s=14), then players should either enter at 11:05 or stay out but never enter after 11:05. Comparison of the equilibrium solutions for balking allowed and balking prohibited shows that for all server capacity values, if they opt to join the queue players are expected to do so earlier when balking is allowed.

--Insert Table 1 about here—

Table 1 presents the equilibrium probabilities for fixed values of n, r, f, g, and c. We conducted additional computations to determine the effects of the reward r and waiting cost c on the solution. Our results show that the uniqueness of the symmetric mixed-strategy equilibrium is in general not guaranteed. However, for the parameter setting that we study the symmetric mixed-strategy equilibrium is unique. Varying the per-minute waiting cost c (which is equivalent to varying the interval length in our discrete-time model), while holding the other parameter values fixed, has yielded the following results. When balking is prohibited and c is varied between 2.5 and 20, the shape (and uniqueness) of the equilibrium solution remains the same, with a peak at the earliest profitable arrival time, another peak at staying out, and possibly small entry probabilities at other arrival times toward the ferry's departure time T. If $c \geq 20.6$, then multiple equilibria appear, the dominant one is when all n players arrive at T. The probability of staying out is not monotone in the waiting cost. Similar results obtain when balking is allowed. Varying the reward value r while keeping all other parameter values fixed shows that, whether balking is allowed or prohibited, as r decreases the earliest profitable arrival time moves toward T but the overall shape of the equilibrium solution is not altered. The major conclusion from these computations is that for a wide range of values of r and c the overall shape of the equilibrium solution tested in our experiment is unchanged.

Asymmetric pure strategies The batch queueing game also has asymmetric pure-strategy equilibria. These are characterized below for the case n=20, g=60, r=340, f=40, T=12:00, and c=4 as before.

(a) *Balking Allowed*

Suppose that k players arrive at 11:05 and n-k stay out. For this to be an equilibrium, we must have $k \geq s$ since otherwise more could arrive at 11:05, embark on the ferry with certainty, and receive a payoff exceeding g. For $k \geq s$, the probability is s/k of embarking on the ferry. Therefore, we require: $-f + (s/k)(r - 55c) \geq g$. If $k<n$, so that some players are staying out, we also require: $-f+s(r-55c)/(k+1) \leq g$ so that deviating from staying out to entering at 11:05 is not

beneficial. We require that $k \geq s+1$ to prevent one person moving from 11:05 to a later arrival time and still being able to get on the ferry with certainty. With the numerical values as specified above, these 3 inequalities reduce to $s+1 \leq k \leq 1.2s \leq k+1$. In particular, if $s=14$, then $16 \leq k \leq 16.8 \leq k+1$ so that k must be 16. If $1.2s$ is an integer, there may be two values of k satisfying these inequalities e.g., if $s=10$ then $k=11$ or 12.

If $s \geq 17$, then $1.2s>n$ so that $k=20$ at 11:05 and no one stays out. In addition, there are other equilibria (which do not occur for $s<17$). By direct verification we can show that all 20 players arriving at certain times is an equilibrium. If $s=17$, then 11:05 is the only such time. If $s=18$, then all 20 players can arrive at 11:05, 11:10, 11:15, 11:20, 11:25 with arrival at 11:25 Pareto dominating all other strategies If $s=19$, then all times from 11:05 to 12:00 are in equilibrium for the arrival of all 20 players.

(b) *Balking Prohibited*

We search again for an equilibrium with k players arriving at 11:05 $(t=1)$ and n-k staying out. If this is an equilibrium, then (i) we must have the expected payoff at 11:05 satisfy $E_1 \geq g$, (ii) if $k+1$ arrived at 11:05 then $E_1 \leq g$ in order to prevent the k+1st person from arriving, and (iii) $k-1 \geq s$ so that if one of the k players would try to move to a later time the ferry would already be full from the $k-1$ at 11:05 and so this move would not be advantageous.

These three conditions are: $-f+(s/k)r-55c \geq g$, $-f+sr/(k+1)-55c \leq g$, and $k-1 \geq s$. They reduce to $s+1 \leq k \leq 17s/16 \leq k+1$. If $s=14$, then $15 \leq k \leq 14.875$ and so we have no pure-strategy solution. We see that we must have $s+1 \leq 17s/16$ or there is no possible solution. This reduces to $s \geq 16$. With $s=16$ players, the only solution to $s+1 \leq k \leq 17s/16 \leq k+1$ is

$k = 17$. In fact, if 16≤s≤18, then $k = s + 1$ is the only solution. If $s = 19$, we have additional equilibria (for a total of 12) which are the same as in the Balking Allowed case.

3. Experiment

Method

Subjects. Two hundred and forty subjects in roughly equal proportions of males and females participated in the experiment. All the subjects volunteered to take part in a group decision making experiment with payoff contingent on performance. All of them were undergraduate or graduate students at the Hong Kong University of Science and Technology (HKUST). The subjects were divided into twelve equal-size groups, three groups in each of four different experimental conditions (see below). Each group participated in a single session that lasted, on average, 90 minutes and included 60 iterations (trials) of the stage game.

Design. We used a 2×2 *information structure* (no information vs. full information) by *game type* (balking allowed vs. balking prohibited) between-subject experimental design with three independent groups in each condition. Hereafter, we shall refer to these four experimental treatments as NINB (no information, no balking), NIWB (no information, with balking), FINB (full information, no balking), and FIWB (full information, with balking). The game type factor was introduced for studying the effects of balking allowed vs. balking prohibited on behavior. The information structure factor was introduced for studying the effects of information on the dynamics of play across iterations. Each stage game was iterated 60 times in a self-paced experiment. In the two "full information" conditions FINB and FIWB, at the end of each trial subjects were presented with a computer screen showing the number of players who joined the queue at 5-minute intervals, the cumulative distribution of arrival times, the number of subjects who opted to stay out of the queue, the subject's payoff for the trial, and his cumulative payoff from trial 1. In the two “no information” conditions NINB and NIWB, at the end of the trial each subject was only informed of his payoff for the trial and his cumulative payoff. In the two “no balking” conditions NINB and FINB, the variable waiting cost was deducted whether or not service was completed, whereas in the two “with balking” conditions the variable waiting cost was incurred only if service was completed. The subject instructions for condition FINB are presented in the Appendix; the others are similar with the appropriate changes.

The values of n, s, g, r, f, and c were commonly known. With the ferry departing at 12:00, subjects were allowed to join the queue at or after 10:00. Arrival times were restricted to 5-minute intervals (i.e., 10:00, 10:05, … , 12:00) in order to reduce the number of pure strategies so that all could easily be displayed on the PC screen in the full information conditions.

Payoffs were stated in terms of a fictitious currency called “francs.” At the end of the session, the cumulative payoffs across the 60 trials were computed and converted into money at the rate 40 francs=HK$1.00 (US$1.00=HK$7.78). Individual payoffs ranged between 0 and HK$130 with mean payoff of HK$54.00.

Procedure. The experiment was conducted in a spacious computer laboratory at HKUST that includes 60 networked PC terminals. Upon arrival at the laboratory, the subjects were seated as far away from one another as possible and provided with sets of instructions that they read at their own pace. Any form of communication between them was strictly forbidden. Questions asked about the game (there were only a few) were privately answered by the experimenter.

Each trial was structured in the same way. Once the trial number was displayed on the PC, each subject was asked to choose whether to join the queue. If opting to do so, he was further asked to choose his arrival time. The decisions were made anonymously and independently. Identification of individual subjects was not possible. No time pressure was imposed. Once all the group members typed in their decisions, a “Results” screen was displayed informing the subject of 1) his decision, 2) success or failure in embarking on the ferry, 3) subject’s payoff for the trial, and 4) subject’s cumulative payoff. Subjects in the full information conditions were also informed of how many players joined the queue at each time interval. This information was displayed both as discrete and cumulative distributions. Hence, information on the latest arrival time (on the just completed trial) that still allowed players to embark the ferry was easily accessible. Three examples were provided in the instructions (see Appendix) to illustrate alternative outcomes of the game and explain the computation of the subject’s payoff.

Results

The experimental design that we have used, where the same *n* players interact repeatedly over time, raises a statistical issue that has to be confronted upfront. Under this design, where the player’s decision at a given trial might affect the decisions of the other players on subsequent trials, the appropriate statistical unit is the group. With only three data points for each of our four conditions, statistical comparisons of groups within condition are not possible. The same is true about the comparison of aggregate and mixed-strategy equilibrium play. When *n* is large, as in our study, gathering data from 10-15 20-player groups for each condition is economically infeasible. Notwithstanding this problem, in the rest of the analysis players (rather than repeated decisions of the same player) are assumed to be independent. In partial justification of this assumption, we note that as *n* becomes very large in non-cooperative *n*-person games the effect of any particular player on the population becomes negligible. With *n*=20 and no possibility to establish reputation, considering the group as a population is a reasonable assumption.

Group Differences. The three groups in each condition were compared to one another in terms of the individual mean waiting time in the queue. For each subject separately, we computed the mean waiting time across all the trials in which he joined the queue. Naturally, frequencies of joining the queue varied across subjects. The null hypothesis that the mean waiting times (computed across the individual measures) were the same for all the three groups was tested by a one-way ANOVA. It could not be rejected in all the four conditions: F(2,57)=2.21, 1.09, 3.11, and 0.06 for conditions FIWB, FINB, NIWB, and NINB, respectively, $p>0.05$. Consequently, the data were aggregated across the three groups in each of the four conditions.

Aggregate Decisions on Trial 1. When making their decisions on trial 1, the subjects have had no opportunity to experience the game or observe previous decisions and outcomes of the other group members. Therefore, from the perspective of previous information on trial 1 the group members are symmetric. Whatever differences are observed in their decisions must be attributed to individual differences in risk attitude, "home made" beliefs about behavior in batch queues, deliberate attempts to randomize that result in different realizations of identical strategies, or some combination of the above. Figure 1 exhibits the proportions of arrival time and staying out decisions on trial 1. The results are displayed separately by condition. Each figure summarizes the decisions of 3×20=60 subjects. Inspection of the distributions suggests that, despite the difference in game type, they are quite similar to one another. Clearly, none of the four figures supports equilibrium play. In all of them, arrival frequencies are distributed over the entire arrival time range. We observe no mode at 11:05, fewer arrivals after 11:45 than between 11:00 and 11:40, and a small but non-negligible proportion of players staying out. All four figures show similar proportions of arrivals before 11:00. Arriving before 11:00 is clearly irrational as it yields a payoff smaller than the one associated with staying out. Not all the subjects might have realized this fact on trial 1.

Neither of the two experimental factors had a significant effect on the distributions of decisions displayed in Fig. 1. Comparison of conditions FINB and FIWB could not reject the null hypothesis of equality of the two frequency distributions of arrival time and staying out decisions ($\chi^2(14)=11.33$, $p>0.6$). Similar results were obtained in comparing conditions NINB and NIWB ($\chi^2(14)=13.21$, $p>0.4$), conditions FINB and NINB ($\chi^2(14)=17.13$, $p>0.1$), and conditions FIWB and NIWB ($\chi^2(14)=16.64$, $p>0.3$).

--Insert Fig. 1 about here—

Aggregate Decisions on Trial 60. Using the same format as Fig. 1, Fig. 2 displays the corresponding proportions of arrival time and staying out decisions on the final trial of the session. The effect of the experience gained in repeatedly playing the game with the same group members for 60 trials is quite dramatic. In qualitative agreement with the mixed-strategy equilibrium solution, all four distributions in Fig. 2 have two major peaks, one at 11:05 and the other at the staying out category. There are relatively many arrivals near 11:05 at 11:00 and 11:10, hardly any arrivals between 11:15 and 11:55, and ten percent of arrival decisions at 12:00 only in the two "no balking" conditions FINB and NINB. The proportions of staying out decisions in these two "no balking" conditions are nearly twice as large as the proportions of staying out decisions in the two "with balking" conditions. The results in Fig. 2 suggest a significant effect of game type but hardly any effect of the information structure.

--Insert Fig. 2 about here--

Aggregate Decisions across Trials. Figure 3 exhibits the proportions of arrival time and staying out decisions computed across all the 60 trials. Each subplot is based on 3600 observations. In addition to the observed proportions, each subplot also displays the mixed-strategy equilibrium probabilities (see the probability values under the *s*=14 columns in the two panels of Table 1).

Although the observed and predicted distributions have the same shape with a major peak at 11:05 and a second peak at the staying out category, and both display practically no weight on arrival between 11:25 and 12:00, they show consistent discrepancies between actual and predicted behavior. Most importantly, in all four conditions the observed proportion of arrival at 11:05 is considerably smaller than predicted. The difference is larger in the two "with balking" conditions NIWB and FIWB than in the two "no balking" conditions FINB and NINB. The subplots show that the proportions of arrival "spilled over" from 11:05 to the neighboring time intervals 11:00 and 11:10, and to a lesser extent to 11:15. Consequently, this deviation had only a small effect on the actual payoffs. Another consistent difference between observed and predicted decisions is a smaller proportion of staying out decisions than predicted. Although, as predicted, the subjects stayed out more often when balking was prohibited than when it was allowed, in all four conditions they stayed out less than expected. Similar results were reported by Rapoport et al. (2004) in their experimental study of queueing systems without batches.

--Insert Fig. 3 about here--

Comparison of Figs. 1 and 3 shows that entering before 11:00 practically disappeared with experience; the subjects learned very quickly to avoid irrational decisions. Comparison of Figs. 2 and 3 suggests that entering at 12:00 in the two "no balking" conditions on trial 60 with a probability about 0.1 might have been an end effect. Additional analyses not reported here show that the frequency of entering at 12:00 increased steadily in the last 6-7 trials in both "no balking" conditions. Arrivals at exactly 11:00 are a puzzle, as the payoff for the trial *conditional* upon successful service is the same as the one *guaranteed* by staying out (60 "francs" in each case). But there is no guarantee that arrival at 11:00 will result in successful service. Arrivals at 11:00 cannot be attributed to errors of computation in the early trials because Fig. 2 shows even higher proportions of entering at 11:00 on trial 60. A possible explanation might be in terms of the “demand characteristics” of the task. Many subjects might prefer joining the queue at 11:00 and thereby actively participate in the game (in the sense that their decisions may affect the outcome of players who join the queue later) and earn 60 “francs” almost surely rather than staying out of the queue and earning the same amount without having the same effect. As one of the subjects said at the end of the session, “staying out is no fun.”

Statistical tests confirm our earlier observations. Using the subject (rather than decision) as the unit of analysis, the one-sided Kolmogorov-Smirnov (K-S) test rejected the null hypothesis of no difference between the observed and predicted cumulative distributions in three of the four conditions (D=0.275, 0.246, and 0.465 for conditions FIWB, NINB, and NIWB, respectively, $p<0.05$ in each case). It could not reject the null hypothesis in condition FINB (D=0.151, $p>0.05$). As we show later, the discrepancies between observed and predicted frequencies of joining the queue and staying out decisions decreased over iterations and in any case their effect on the actual payoffs was minor.

Individual Decisions. Patterns of aggregate behavior displayed in Fig. 3 tell us very little about individual behavior. There is the theoretical possibility that each subject randomizes his decisions with fixed probabilities that may or may not correspond to the mixed-strategy equilibrium probabilities. In contrast, the experimental results reported by Rapoport et al. and Seale et al. provide no support for any form of mixing. In fact, they show considerable individual differences, albeit in a different and possibly more difficult game for the subjects, which defy simple categorization. The individual results in the present study are in complete agreement with those reported in the earlier studies. Figure 4 exhibits the individual decisions (arrival time and staying out decisions) of all the twenty member of group 1 in condition FINB. We refer to them as *individual profiles*. As the individual profiles in other groups of the same condition and all groups in the other three conditions exhibit similar variability between subjects, they are not displayed here. Subject numbers from 1 through 20 are listed above each individual profile. The horizontal axis presents the trial number from 1 through 60, whereas the vertical axis shows the arrival time on a scale from 11:00 (bottom) to 12:00 (top). A short vertical line that extends below 11:00 indicates staying out. Absence of a bar indicates arrivals from 10:00 to 10:55 (there are very few of these). For example, subject 3 arrived at 11:05 on 50 of the 60 trials, at 11:10 on 8 trials, at 11:00 on a single trial, and at 11:15 also on a single trial (trial 2). For a second example, subject 1 stayed out on trial 1, subject 2 stayed out on trial 6, and subject 4 stayed out a total of 6 trials (8, 11, 12, 13, 15, and 16).

--Insert Fig. 4 about here--

Figure 4 shows substantial differences between individual profiles and no support for mixing. It shows that eleven of the twenty subjects stayed out of the queue no more than five times. Most of the staying out decisions are due to only four subjects in the group, namely, subjects 9, 13, 18, and 20. In contrast, four subjects (1, 3, 10, 12) never stayed out. About half of the subjects (2, 3, 4, 5, 6, 12, 14, 17, and 19) joined the queue no later than 11:10 on the majority of the trials, whereas others (subjects 16 and 18) tended to join the queue late on a substantial proportion of the trials. In addition, we observe large individual differences in the incidence of switching decisions between trials. For example, subjects 2, 3, and 17 switched their decisions very infrequently. Other subjects (e.g., subjects 8, 10, 18, and 20) tended to switch their decisions, on average, every 2-3 trials.

We tested the null hypothesis that the subject's 60 decisions in a session constitute a random sequence generated by probabilities p_1, p_2, … , p_j ($p_1+p_2+\ldots+p_j=1$) that may differ from one subject to another but remain fixed across iterations. The hypothesis was tested on the individual level using a bootstrap run test for all 60 trials. For each subject's overall proportions of choosing each strategy, we simulated 10,000 sequences, calculated the theoretical number of runs, and compared it to the observed number of runs. The hypothesis could not be rejected for only 24 of the 240 subjects. In most cases it was rejected because of two few observed runs. Additional analyses not reported here show strong sequential dependencies that are mediated in part by the information condition.

Figure 5 organizes the same individual decisions into frequency distributions, collapsing the decisions across trials and therefore providing no information about the dynamics. Here, the horizontal axis portrays the pure strategies (13 arrival times and a single staying out strategy), whereas the vertical axis portrays the frequencies. Similarly to Fig. 4, Fig. 5 shows a wide variety of individual frequency distributions, some (subjects 9, 13, and 20) are indeed, bimodal, but the majority are not. A major feature of Fig. 5 is that, with the possible exception of the individual profiles of subjects 8 and 14, none of the twenty individual frequency distributions resemble the aggregate distribution for condition FINB in Fig. 3. Therefore, whatever explanation is invoked to account for the aggregate results it cannot also be invoked to explain the variety of individual decision profiles.

--Insert Fig. 5 about here—

<u>Dynamics</u>. As noted by Rapoport et al., it is by now well recognized in population biology and market economics that consistent and replicable patterns of animal and human behavior may exist at the aggregate level while being totally absent at the individual level. The major challenge is to explain how events at these two different levels of organization—individual and group—are related (Gordon, 1999) by a theory that unifies them. Although we make no such attempt in the

present paper, we describe below selected features of the dynamics of aggregate play in our study that may contribute to the construction of such theory. In particular, we focus on the 1) change across trials between the observed and predicted decisions, 2) change across trials in the rate of switches of decisions between successive trials, and 3) change in mean payoff over time.

Recall that there are fourteen strategies, namely, thirteen arrival times and a single staying out strategy. Denote a strategy by *j* and its relative frequency within a group on trial *t* by p_{jt}. Denote the mixed-strategy equilibrium probability of strategy *j* by p^*_j. For each group and each trial, we calculated the sum of squared differences $d_t = \sum_{j=1}^{14} (p_{jt} - p_j^*)^2$ as a measure of difference between observed and predicted decisions. Figure 6 displays the 5-trial moving average of the d_t scores computed over the three groups in each condition. The results are shown separately for each condition. For all four conditions, Fig. 6 shows that the mean d_t scores decrease across trials. They start at about 0.8 and drop to some value between 0.05 and 0.2, depending on the condition. The two functions for the “no information” conditions NIWB and NINB slowly increase for the first fifteen trials or so and then decline. In contrast, the two functions for the “full information” conditions FIWB and FINB decline sharply almost from the beginning of the session, reaching the value of 0.2 after about 15 trials. This is the first indication we have that, possibly not surprisingly, trial-to-trial information about the decisions and outcomes of all group members dramatically facilitates learning in the direction of equilibrium play during the early part of the session. The difference between the two information conditions practically disappears in the last 5-7 trials.

--Insert Fig. 6 about here—

Moving to the second measure of change in behavior, for each trial *t*, *t*=1, 2, …, 59, we computed the number of switches in individual decisions between trials *t* and *t*+1, v_t. Any change in decision from joining the queue to staying out, from staying out to joining the queue, or from joining the queue at two different times was counted as a switch. The magnitude of the switches is ignored in this analysis. Across all three groups in a given condition, the number of switches could take any value between 0, if no player switched his decision, to 60 if all switched their decisions. Figure 7 exhibits the 5-trial running average of the v_t scores. It shows that, depending again on the condition, between 68 percent and 80 percent of the subjects switched their decisions between successive trials in the first 5-6 trials of the session. These percentages dropped slowly but steadily reaching 32 percent in the two "no information" conditions and 43 percent in the two "full information" conditions on the final trial. The percentages of switches at the end of the session are still high, as they are in the study of Rapoport et al. (2004). There is no indication that with considerably more trials subjects would have reached zero percent of switching, where every subject is playing a pure strategy. But this possibility cannot be ruled out.

--Insert Fig. 7 about here—

Using the same format as Figs. 6 and 7, Fig. 8 displays the 5-trial moving average of the subjects' payoffs. For all four conditions, the expected payoff is 60. Figure 8 shows that toward the end of the session, the mean payoff in all four conditions approached the theoretical value, with condition NINB outperforming the three other conditions. It suggests that game type had an effect on the time course of the mean payoff. When balking was allowed in conditions FIWB and NIWB, the mean payoff at the first ten trials exceeded the theoretical value of 60, and then it declined steadily to a value about 57. When balking was prohibited, mean payoffs decreased for the first 35 trials or so, and then steadily recovered. This difference in the time course of the mean payoff reflects the difference between the "no balking" game where players have to wait until the ferry departs and possibly sustain heavy losses if they do not receive service to the considerably less risky “with balking” game where no waiting cost is ever incurred and a subject can lose no more than 40. Taken together, the results exhibited in Figs. 6 and 8 provide evidence of slow convergence to mixed-strategy equilibrium play. Together with Fig. 7, they suggest that any adaptive learning model aspiring to account for the consistent patterns of behavior on the individual and aggregate levels must allow for convergence of behavior to equilibrium as well as the effects of game type and information structure.

--Insert Fig. 8 about here—

4. Conclusions

The present study examines *decentralized* decision making in two different batch queueing games in which the number of players is finite, relatively small, and commonly known, the strategy space is discrete, the queue discipline is FCFS, the stage game is iterated in time, reneging is prohibited, and the arrival time is a decision variable. In both games, where balking is allowed and where it is prohibited, players are given the option of staying out of the queue. Characteristic of both games is that the equilibrium solutions are inefficient; they do not achieve social optimality (Naor, 1969; Hassin & Haviv, 2003) in the sense of maximizing the overall social welfare of the n players. In both games, social optimality is achieved when s players enter the queue at exactly 12:00 and n-s players stay out. This results in group payoff of $s(r\text{-}f)+(n\text{-}s)g$. In contrast, expected group payoff under mixed-strategy equilibrium play is ng. In a different context, that of routing traffic to optimize the performance of a congested network, Papadimitriu (2001) and subsequently Roughgarden and Tardos (2002) refer to the inefficiency inherent in a selfishly defined solution as the “price of anarchy.” In the context of our present study this price (compare 5120 to 1200) is substantial.

Our results show that when decisions are decentralized with *n*=20 players in a group, players cannot avoid paying this price. We observe no evidence for successful attempts to maximize overall social welfare. Rather, mean payoffs converge to expected payoff under mixed-strategy equilibrium play (Fig. 8). Observed arrival time and staying out decisions on the aggregate level are also accounted for quite well by the mixed-strategy equilibrium solution (Fig. 3 and particularly Fig. 6). These results are consistent with those reported earlier by Rapoport et al. and Seale et al., who studied different discrete-time queueing games with symmetric players and negative externalities in which service time is a positive constant, customers are served by a single server one at a time, and the option of staying out is available. The mixed-strategy equilibrium solutions for the games studied by Rapoport et al. and Seale et al. are quite different from the ones constructed in the present study (Table 1). In the study by Seale et al., where early arrivals before the service station opens are allowed, equilibrium arrival times are distributed more or less evenly over most of the strategy space. When early arrivals are prohibited in the study by Rapoport et al., the distribution of arrival times and staying out decisions under mixed-strategy equilibrium play is less transparent: there is a relatively high probability of arrival just when the service station opens, followed by zero probability of arrival over a short segment of the strategy space, and then arrival probabilities that slowly decrease over time as players approach the closing time of the station. In contrast, with the exception of small probabilities of arrival between 11:45 and 12:00 (Table 1) in the present study, under mixed-strategy equilibrium play players either arrive at 11:05 or stay out of the queue. Despite these sharp differences in the equilibrium solutions for the three queueing studies, we observe very similar patterns of behavior: aggregate behavior converging to the mixed-strategy equilibrium solution, heterogenous patterns of individual behavior that do not support equilibrium play and defy categorization, and switches in arrival time and staying out decisions from one trial to another whose frequency declines slowly over iterations.

There is one experimental question and one theoretical issue that remain unresolved. The individual profiles (see, e.g., Fig. 4) indicate considerable differences in both the frequency and magnitude of switching. Will players eventually converge to pure-strategy play so that with experience different players either join the queue at possibly different times or stay out? Answering this question requires iterating the stage game for a considerably larger number of trials than in the present study, possibly in the hundreds, over several experimental sessions. The theoretical challenge is to construct a learning model that accounts simultaneously for both the individual and aggregate patterns of results. A preliminary attempt in this direction has been made by Bearden, Rapoport, and Seale (2004), who proposed and subsequently tested a reinforcement-type adaptive learning model for the experimental results reported by Rapoport et al. and Seale et al.

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Appendix

Instructions for an Experiment on "Decision Making in Queues"

Welcome to the experiment on decision making in queues. The instructions for this experiment are quite simple. If you follow them carefully and make good decisions, you may earn a considerable amount of money. The money will be paid to you, in cash, at the end of the experiment. A research foundation has provided funds to support this experiment. If you have any questions before or during the experiment, please raise your hand and someone will come to assist you.

We assure you that the data we collect during the course of this experiment will be held in strict confidence. Anonymity is guaranteed; information will not be reported in any manner or form that allows associating names with individual players.

Description of Task

This experiment has been designed to study how individual members of a group decide ***if*** and ***when*** to join a queue when communication among them is not possible. Situations like that are quite common. Imagine that you are one of 20 people interested in boarding a ferry that leaves exactly at 12:00 pm (noon). Whereas there are **20** members in the group, whom we call *players*, the capacity of the ferry is only **14**. The ferry operator allows potential passengers to line up early and then boards passengers at 12:00 in a first-come first-served manner.

There are costs and payoffs associated with your decision:

- If you attempt to board the ferry and *are successful*, then you will earn a reward of **340** francs (francs are an experimental currency that will exchanged for HK dollars at the end of the experiment). However, it costs you **40** francs for joining the queue, and **4** francs for every minute that you have to wait in line.

- If you attempt to board the ferry but *are not successful* because 14 or more group members precede you in line, then you will earn no reward. However, you will still have to pay the 40 francs for attempting to board the ferry and 4 francs for every minute that you have to wait. *Please note that once you join the queue you are not allowed to leave it. Rather, you have to wait in line until the ferry departs at 12:00 pm.*

- Alternatively, you may decide not to join the queue. In this case, you are guaranteed **60** francs; you do not pay the 40 francs for joining the queue or any cost for waiting.

Ties. It is quite possible that several players arrive at the same time thereby creating a tie. In the case of ties – if two or more players choose the same entry time – the computer will break them randomly to determine the order of arrival. For example, supposing that no player arrives before 11:15 and 16 players arrive at exactly 11:15. In this case, the computer will choose randomly 14 of the 16 entrants to board the ferry. The other two will stay in line. For another example, supposing that 2 players arrive at 11:05, 3 players arrive at 11:10, and 12 players arrive at 11:30. In this case, after breaking the three different ties, the 5 players arriving before 11:30 and 9 of the

12 players arriving at 11:30 will board the ferry. These 9 players will be chosen randomly from the 12 players who arrived at 11:30. The other 3 players (as well as players who might be arriving after 11:30) will have to wait in line and pay the waiting cost.

Example #1: Supposing that you decide to join the queue at 11:10. At 12:00 the ferry operator informs you that you are one of the first 14 people in line who are allowed to board. Your earnings are computed as follows (see the table below). In this example, you earned the reward of 340 francs for successfully boarding the ferry (and reaching your destination), you lost 40 francs for joining the queue, and you lost additional 200 francs (50 minutes x 4 francs per minute) for arriving at 11:10, 50 minutes prior to departure. Your net payoff is, therefore, 100.

Earnings – Example #1	
You earned reward of:	340
Less cost of joining queue:	(40)
Less cost of waiting:	(200)
Net earnings:	100

Alternatively, were you to choose not joining the queue, you would have earned 60 francs.

Example #2: Supposing that you decide to join the queue at 11:40. At 12:00 the ferry operator informs you that you are ***not*** one of the first 14 people in line and, therefore, will ***not*** be allowed to board. Your earnings are computed as follows (see the table below): In this example, you did not earn a reward because you did not successfully board the ferry. Additionally, you incurred the cost of joining the queue (40 francs) plus the cost of waiting (80 francs = 20 minutes x 4 francs per minute) for arriving at 11:40. Your net loss is 120.

Earnings – Example #2	
You earned reward of:	0
Less cost of joining queue:	(40)
Less cost of waiting:	(80)
Net earnings:	(120)

Alternatively, were you to choose not joining the queue, you would have earned 60 francs.

Example #3: Supposing that you decide to join the queue at 11:40. Assume that 6 players joined the queue before 11:40 and 10 players (including yourself) joined the queue at exactly 11:40. The computer will choose randomly 8 of these 10 players to board the ferry (together with the 6 players who joined the queue before 11:40). If you are one of the 14 players chosen to board the ferry, your earning are computed as follows:

Earnings – Example #3	
You earned reward of:	340
Less cost of joining queue:	(40)
Less cost of waiting:	(80)
Net earnings:	220

If you are of the two players not allowed to board the ferry, your earnings are computed as in Example #2 above.

Summary of Experiment

This is the type of decision you will be asked to make during the course of the experiment. The same game that was described and illustrated in three examples above will be repeated for 60 trials (repetitions) over a computer network. The same 20 group members will participate in all 60 trials. Each trial is identical except for the decisions made by the group members. Thus, the following payoff and cost parameters will remain the same over the course of the experiment:

Values of Parameters	
Number of members in the group:	20
Capacity of the ferry:	14
Reward for boarding the ferry:	340
Cost of joining the queue:	40
Cost of waiting (per minute):	4
Payment for not joining the queue:	60

At the beginning of each trial the computer will remind you of these costs and payoffs, and then ask you to decide whether to join the queue or stay out. If you decide to join the queue, you must choose an entry time, **in five-minute intervals**, between 10:00 am and 12:00 (e.g., 10:00, 10:05, … , 12:00). Ties in entry time will be broken randomly by the computer to determine the first 14 players to board the ferry.

We expect the experimental session to take approximately two hours.

Information at the End of the Trial

After all the 20 members of the group have made their decisions independently of one another, the computer will inform you of the results for the trial:

- You will be presented with a computer screen that shows the number of players who joined the queue at each five-minute interval. You will also be shown the cumulative distribution of the total number of entries through each five-minute interval.

- You will be presented with the number of players who decided not to join the queue at this trial.

- The computer will also show you how your earnings for the trial were computed, as well as your cumulative earnings from trial 1.

Payment

At the end of the experiment, we will pay you in cash for your cumulative earnings, at the rate of 40 francs = HK$1.0. To collect your earnings, you will be asked to complete and sign a receipt with your name, email address, and student ID number. We will be happy to answer any questions you may have concerning the experiment.

If you wish to participate in this experiment, please be kind enough to sign the consent form on your desk.

If you have no questions, please wait until each of the 20 players has completed reading the instructions. Trial 1 will start after the experimenter announces the beginning of the session.

Good luck.

Table 1. Symmetric mixed-strategy equilibria for balking/no balking where *n*=20, *f*=40, *c*=4 (per minute), g=*60*, r=*340*, and *s*=5, … , 19*.

BALKING PROHIBITED

Server capacity

Time	*s=5*	*s=6*	*s=7*	*s=8*	*s=9*	*s=10*	*s=11*	*s=12*	*s=13*	*s=14*	*s=15*	*s=16*	*s=17*	*s=18*	*s=19*
11:05	.180	.233	.288	.345	.402	.461	.520	.580	.641	**.702**	.764	.827	.889	.952	0
11:25	.025	.013	.004	0	0	0	0	0	0	**0**	0	0	0	0	0
11:30	.005	.018	.033	.021	.008	0	0	0	0	**0**	0	0	0	0	0
11:35	.024	.012	0	.012	.026	.028	.013	0	0	**0**	0	0	0	0	0
11:40	.005	.018	.031	.021	.007	.006	.021	.030	.013	**0**	0	0	0	0	0
11:45	.024	.013	.001	.012	.026	.028	.013	.003	.019	**.024**	.003	0	0	0	0
11:50	.005	.019	.032	.021	.008	.006	.021	.030	.012	**.006**	.025	.005	0	0	0
11:55	.026	.014	.002	.014	.028	.029	.013	.003	.020	**.025**	.003	.020	0	0	0
12:00	.008	.022	.036	.024	.009	.008	.023	.032	.014	**.007**	.026	.006	.022	0	1.000
Stay Out	.700	.638	.578	.531	.486	.435	.377	.322	.282	**.236**	.179	.142	.089	.048	0

BALKING ALLOWED

Server capacity

Time	*s=5*	*s=6*	*s=7*	*s=8*	*s=9*	*s=10*	*s=11*	*s=12*	*s=13*	*s=14*	*s=15*	*s=16*	*s=17*	*s=18*	*s=19*
11:05	.260	.324	.389	.454	.518	.583	.647	.711	.774	**.837**	.899	.960	1.00	0	0
11:25	0	0	0	0	0	0	0	0	0	**0**	0	0	0	1.00	0
11:35	.005	0	0	0	0	0	0	0	0	**0**	0	0	0	0	0
11:40	.011	0	0	0	0	0	0	0	0	**0**	0	0	0	0	0
11:45	.003	.016	0	0	0	0	0	0	0	**0**	0	0	0	0	0
11:50	.010	0	.007	0	0	0	0	0	0	**0**	0	0	0	0	0
11:55	.002	.011	.006	0	0	0	0	0	0	**0**	0	0	0	0	0
12:00	.009	0	.005	.007	0	0	0	0	0	**0**	0	0	0	0	1.00
Stay Out	.700	.649	.593	.539	.482	.417	.353	.289	.226	**.163**	.101	.040	0	0	0

* Only times (rows) with positive entry probability for some *s* values are included.

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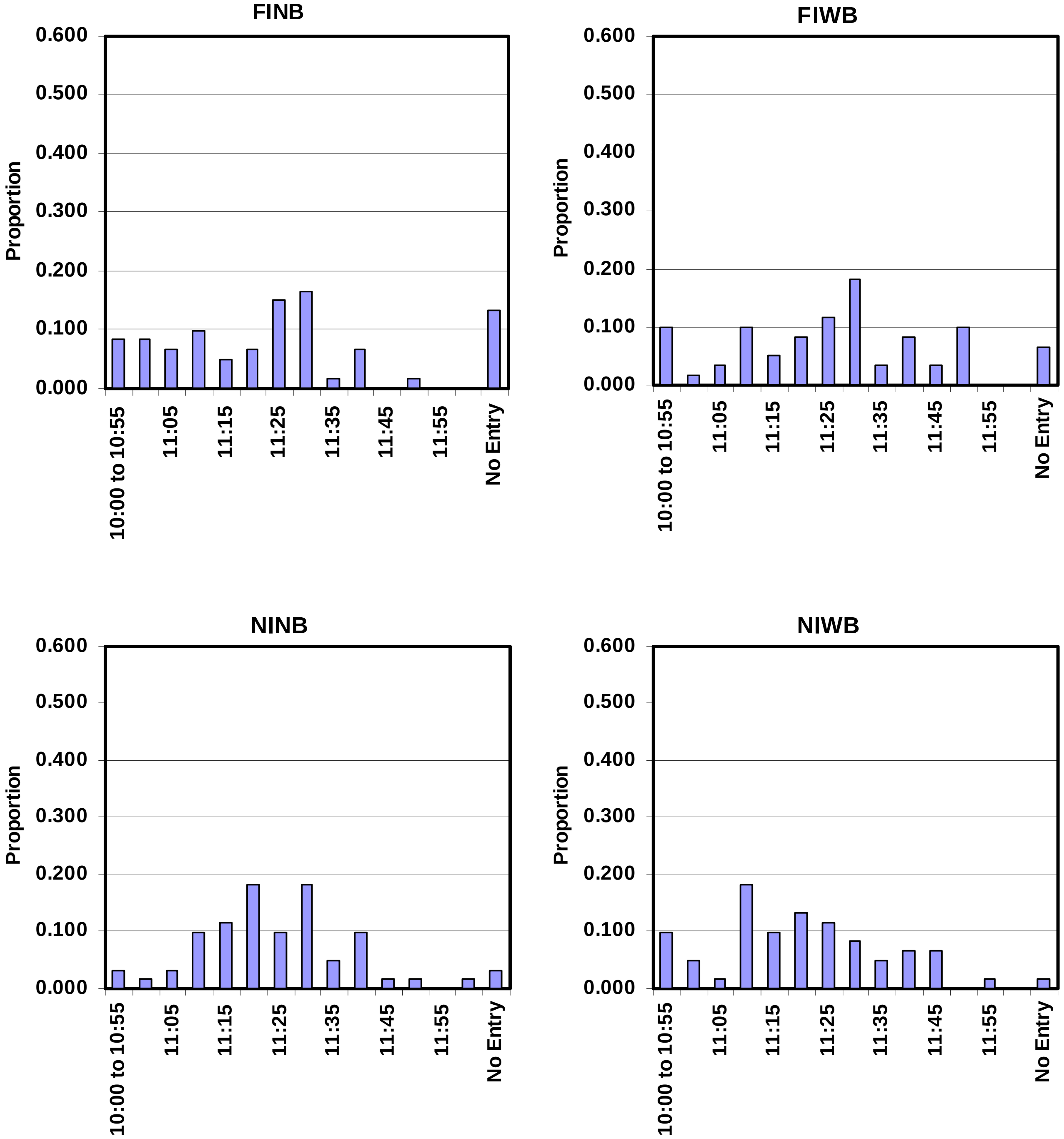

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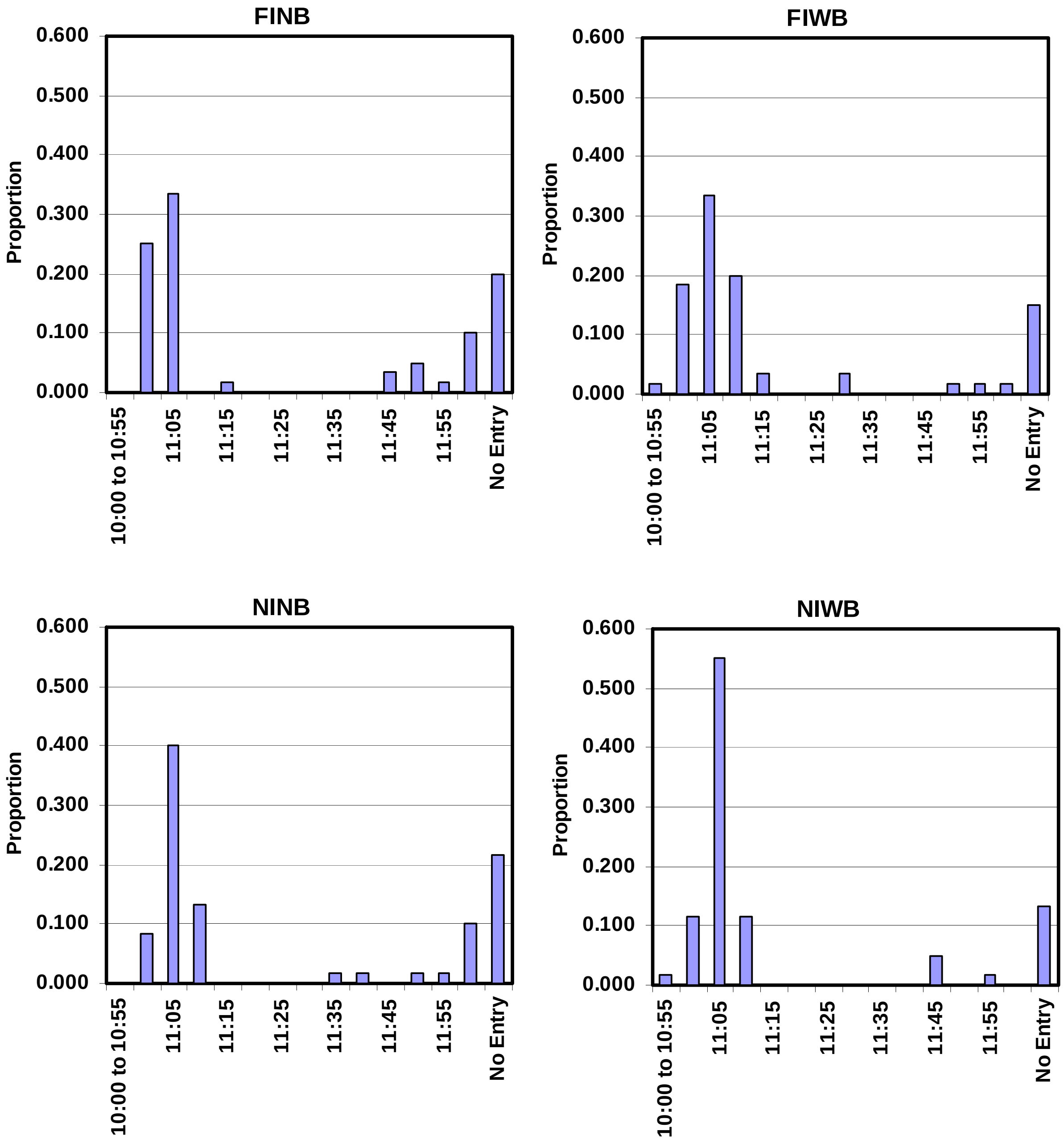

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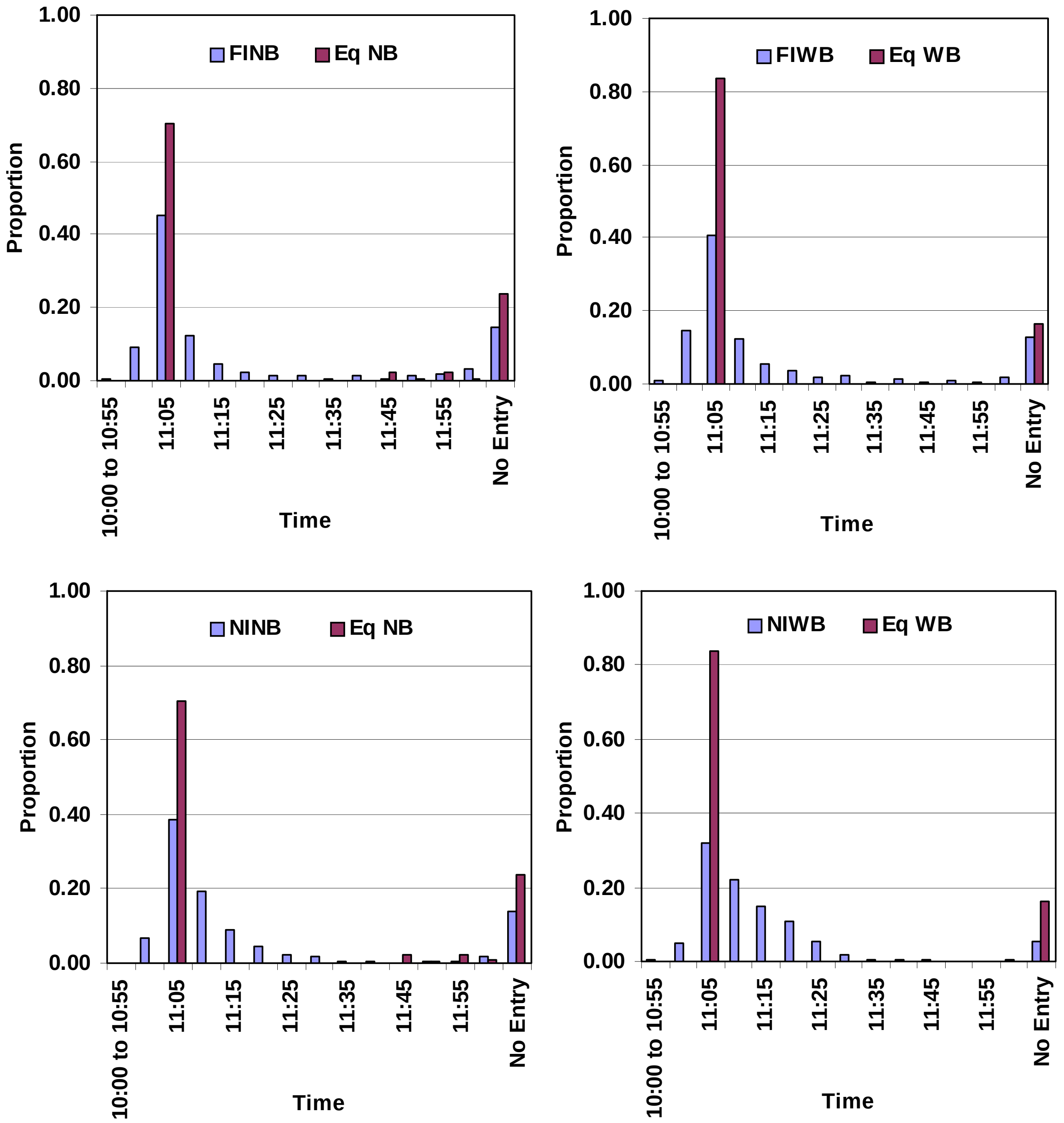

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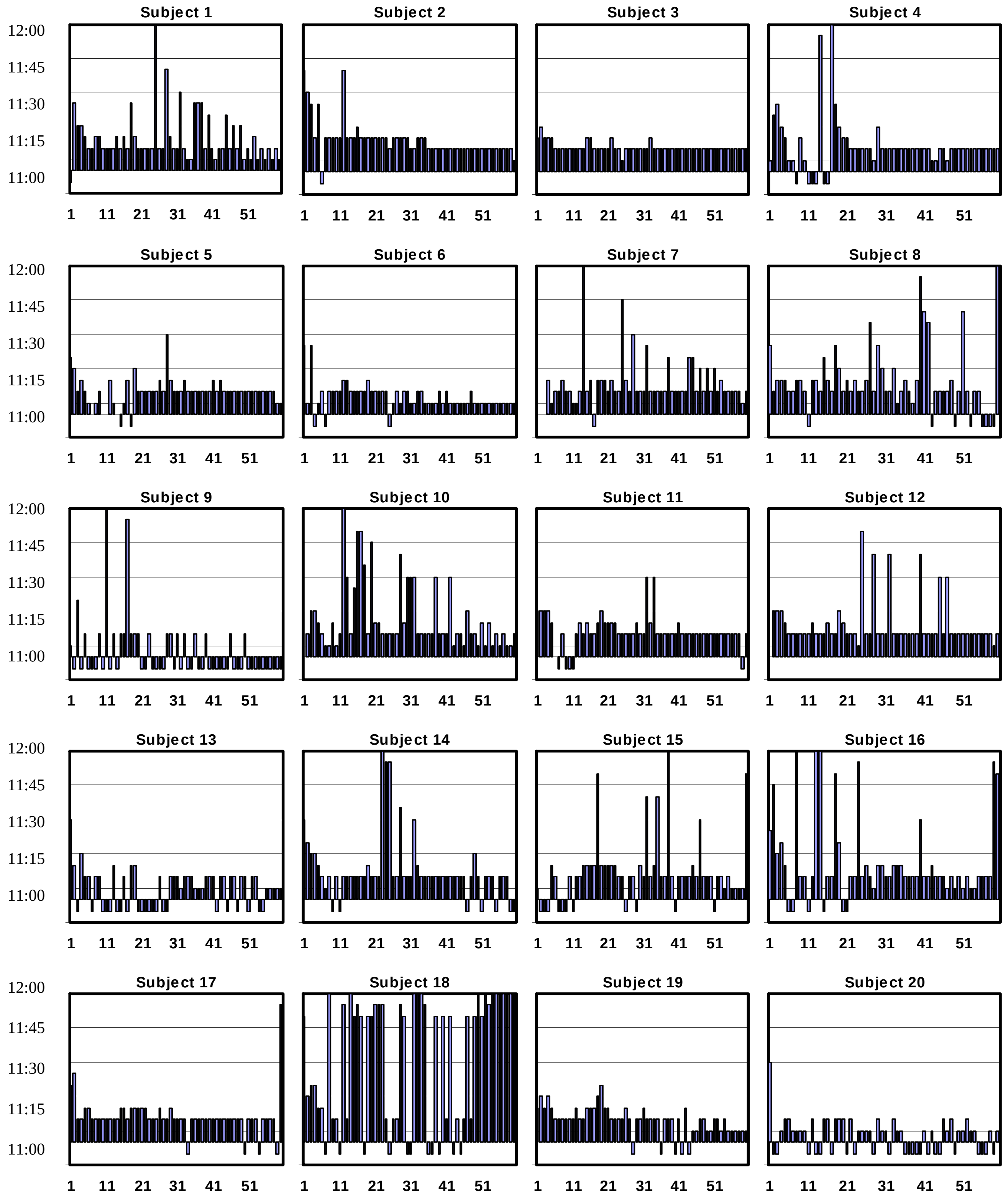

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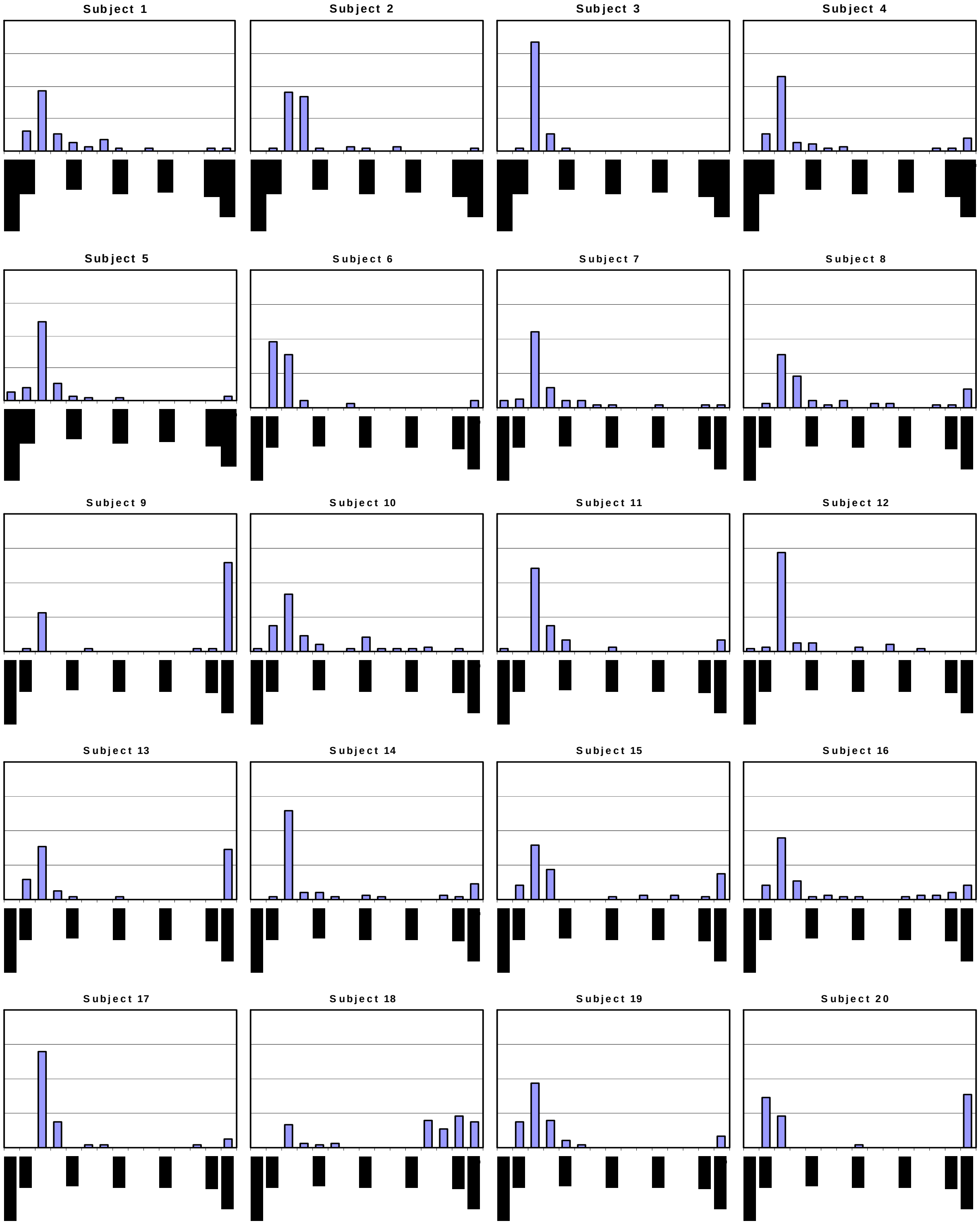

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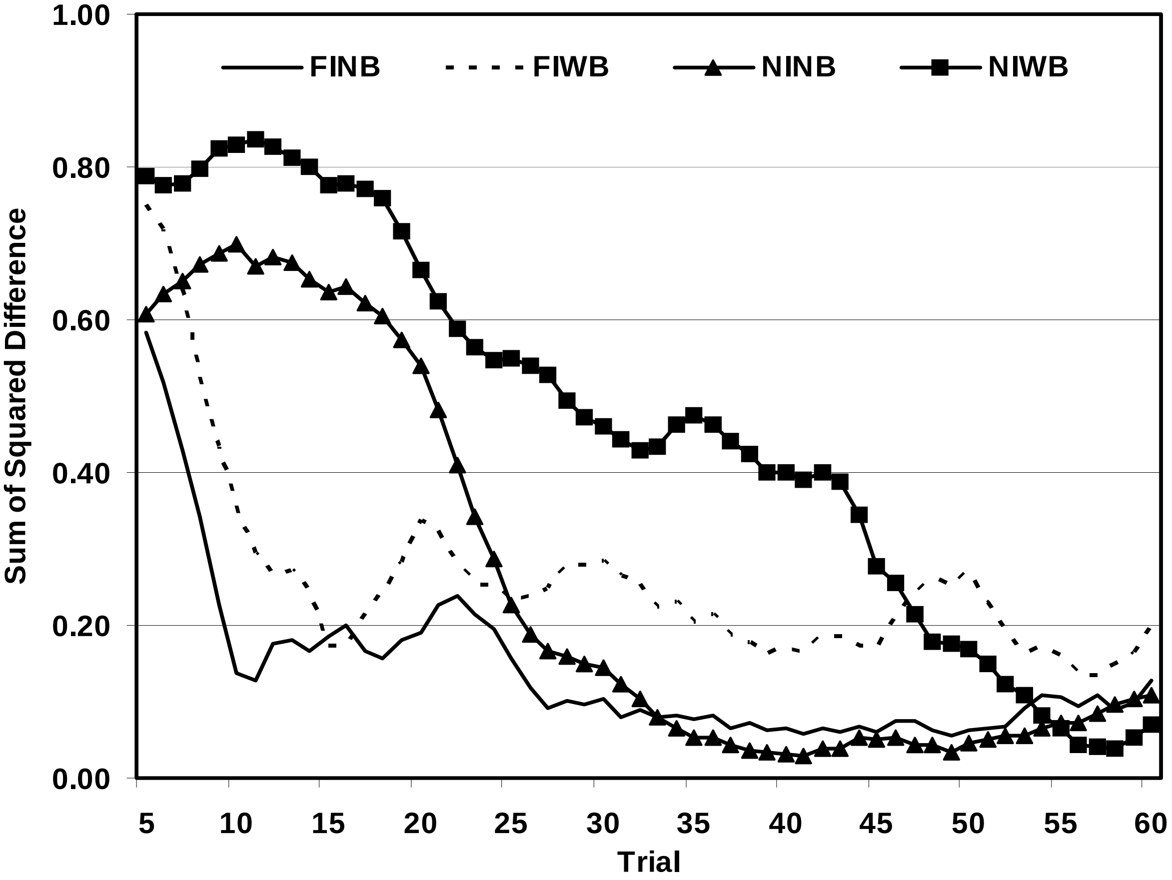

Fig. 7. Five-trial moving average of frequency of switches between decisions.

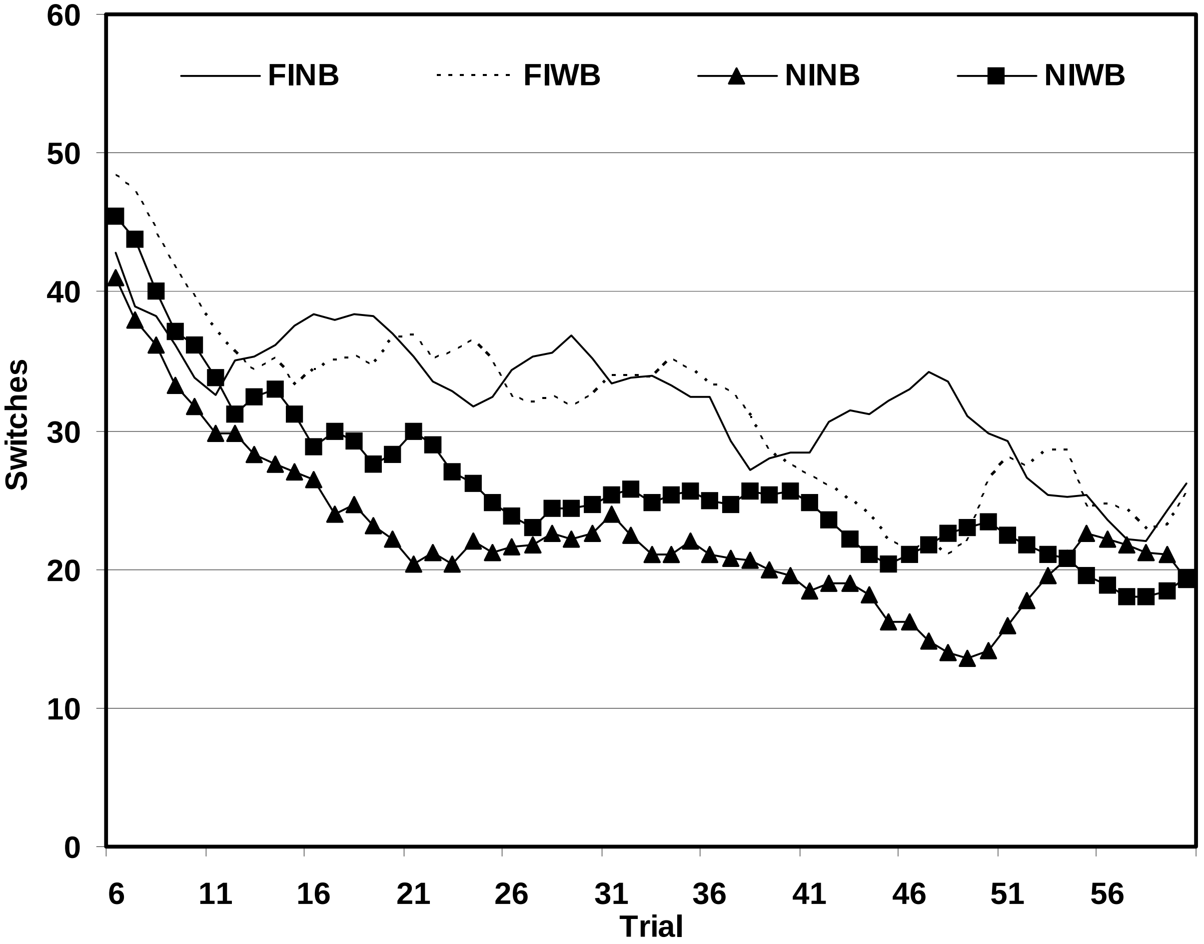

Fig. 8. Five-trial moving average of mean observed payoffs.