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Egalitarian vs. Proportional Profit Sharing Rules

in Multi-Level Collective Action Problems

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Abstract

We consider a class of multi-level competitions that include within-group conflict for a public good embedded in a between-group competition for an exogenously determined prize. The group's probability of wining the prize depends on how contributions to the provision of a public good by its members compare to the contributions by members of the competing group. The profit sharing rule for dividing the prize among members of the winning group is either egalitarian or proportional to the individual members' contributions. Under appropriate parameterization, zero contribution in the within-group competition for the public good provision is no longer the equilibrium strategy. Moreover, in equilibrium contributions are higher under the proportional profit sharing rule than the egalitarian profit sharing rule. In general, our experimental results support the equilibrium predictions. Intergroup competition substantially reduces free riding, and contributions under the proportional profit sharing rule considerably exceed contributions under the egalitarian profit sharing rule. A simple reinforcement learning model accounts for the dynamics of play on the aggregate but not individual level.

1. Introduction

Theoretical and experimental research on interactive decision making behavior has mostly focused on within-group (intragroup) conflicts. Rational choice theories in economics and political economy, often driven by the conceptual framework of game theory, and descriptive models in social psychology and experimental economics with their emphasis on social dilemmas and public good provision have often ignored, or at best underplayed, the fact that in many social and organizational situations groups as well as their constituents interact with other groups and often operate within the more general framework of between-group (intergroup) competition. For example, when communities compete with one another to secure federal subsidies or some other form of support, their individual members—who are asked to contribute time, money or effort—are simultaneously engaged in both intergroup conflict and intragroup competition. These two levels of conflict are present whenever groups or organizations that compete with one another for some prize cannot be adequately modeled as unitary players. Rather, they are often composed of individual members or subgroups that must determine their actions independently of one another while explicitly considering both levels of competition and trying to strike a balance between the demands each places on their actions. Stripping players of the wider social context facilitates the formal analysis of the interactive decision situation and allows for tractability of the model. However, it comes at a price of reducing the model's realism and decreasing its ecological validity.

The last twenty years or so have witnessed a growing number of attempts in such diverse fields as economics, public choice theory, marketing, and social psychology to examine multi-level collective action problems that allow for the simultaneous occurrence of between-group and within-group conflicts. Although we make no attempt to review this literature, we note in particular the work of Katz, Nitzan, and Rosenberg (1990), Nitzan (1991), Lee (1995), Baik and Lee (1997), and Baik and Shogren (1995) in public choice theory; Rabbie (1982), Tajfel (1982), and in particular Bornstein and his collaborators (Bornstein & Rapoport, 1988, Bornstein, Winter, & Goren, 1996, Bornstein & Gneezy, 1998, Rapoport & Bornstein, 1987) in social psychology; Amaldoss et al. (2000) in marketing; and Hausken (1995a, 1995b) in economics.

These streams of research differ from one another on several dimensions. In the present study we focus on a special class of multi-level competitions that share the following features:

(1) There are several groups, each facing a possibly different within-group conflict of the Prisoner's Dilemma (PD) kind.

(2) The groups compete with one another for a single exogenously determined and divisible prize.

(3) Only a single group wins the prize.

(4) The prize is divided among members of the winning group. The profit sharing rule for dividing the prize is commonly known before individual players are asked to make their decisions.

(5) Members of each group decide independently on their level of contribution, which, in turn, affect the outcome of both the within-group conflict and between-group competition.

For examples of multi-level competitions that can appropriately be modeled by these assumptions see, for example, Amaldoss et al. (2000) and many of the papers by Bornstein and his collaborators.

To broadly classify collective action problems of this type, Rapoport and Amaldoss (1999) have proposed three major dimensions. The first concerns the *payoff structure* that underlies the within-group conflict. The second is the *contest rule* that determines the outcome of the between-group competition. The third concerns the profit sharing or *distribution rule* that stipulates how the prize is to be divided among members of the group that wins the between-group competition.

With regard to the first dimension, the interactive decision situation that we examine here assumes that players within each of the competing groups are symmetric and that their strategy spaces are continuous. With the appropriate parameterization, this gives rise to a within-group conflict of the PD kind. It differs, though, from the traditional PD game in which the strategy spaces are binary.

The contest rule that we employ is the one most commonly used in public choice theory. Proposed and studied by Tullock (1967, 1980), this rule compares rent seeking activity—or group contribution in our study—to purchasing tickets in order to win a lottery. Winners are chosen probabilistically, so that the greater the rent-seekers' expenditure compared to her competitors, the greater her probability of winning the prize (rent). Patent races between alliances of firms investing in R&D, competitions among communities seeking to secure governmental concessions, or military conflicts in which nations expend budgets for military armament, are often invoked to support Tullock's probabilistic contest rule.

Whereas the payoff structure and contest rule are held fixed in our experimental investigation, the major independent variable of the present study, which we manipulate experimentally in a between-subjects design, is the distribution rule. We compare to each other the egalitarian (equality) and proportional (equity) profit sharing rules, which have received much attention from psychologists, economists, and philosophers. These are by far the two most commonly used rules in practice to allocate money or other divisible commodities like land, food, and water. The former rule stipulates that the prize be shared equally among members of the winning group regardless of the level of their contributions. The latter rule stipulates that the prize be divided among the members of the winning group in proportion to their individual contributions.

The rest of the paper is organized as follows. Section 2 presents the model and the equilibrium solutions for the two distribution rules under study. In equilibrium, the individual contribution under the proportional distribution rule exceeds the individual contribution under the egalitarian rule. Whereas this result is not particularly surprising, the magnitude of the effect is: in equilibrium the ratio of the individual contributions between the proportional and egalitarian rules increases linearly in the total number of competing individuals. Section 3 describes the experimental method. The results of the experiment are presented in Section 4. The focus of the analysis is on testing the (static) equilibrium predictions and the observed trends in contribution decisions across multiple iterations of the game due to learning. Section 5 concludes with a brief summary and discussion of the results.

2. The Model

Denote the number of groups that compete for the prize by n $(n \geq 2)$, the number of players i in group k $(i = 1, 2, \ldots, n)$ by $m(k)$, and the total number of players across all the n groups by N $(N = m(1) + m(2) + , \ldots, m(n))$. Assume that each of the N players has the same budget (endowment in our experiment) that we denote by e. The strategy space is continuous; each member i of group k can invest (contribute) any fraction of her endowment. Denote the individual contribution by x_{ik} $(0 \leq x_{ik} \leq e)$, the total investment of group k by X_k $(X_k = \sum x_{ik})$, and the total investment of the n groups by X $(X = \sum X_k)$.

The Payoff Structure. In determining how much to contribute, each player simultaneously considers the effects of her decision on the within-group conflict and the between-group competition. Recall that the within-group conflict is modeled as a PD game. In particular, following the literature in economics on the Voluntary Contribution Mechanism (see, for example, Davis & Holt, 1993; Ledyard, 1995), each player *i* of group *k*, by investing x_{ik}, generates a (local, i.e. within-group) public good proportional to the level of her contribution. Let g_k denote the public good that group *k* generates if each of its members contributes her entire endowment *e*. Then, the public good that group *k* actually generates is given by $g_k\,[X_k/m(k)e]$. This term equals to zero if each player in group *k* contributes nothing, and to g_k if each contributes *e* (as, in this case, $X_k = \sum e = m(k)e$). Since individual contributions are assumed to be irrecoverable, the payoff of player i in the within-group conflict is given by

$$(e - x_{ik}) + g_k \frac{X_k}{m(k)e} \qquad (1)$$

The Contest Rule. Moving from the within-group conflict to the between-group competition, it is assumed that the groups compete for a prize (rent) that we denote by *S*. Denote the probability that group *k* wins the between-group competition by Π_k. Then, the probabilistic contest rule used to determine the winning group k is given by

$$\Pi_k = X_k/(X_1 + X_2 + \ldots + X_n), \qquad k = 1, 2, \ldots, n \qquad (2)$$

Equations (1) and (2) together imply that by increasing her contribution x_{ik}, player *i* of group *k* both increases the size of the public good associated with the within-group conflict and the probability that her group will win the between-group competition. However, by doing so she does not necessarily increase her profit. Recall that the within-group conflict has the PD property. Therefore (see Rapoport & Amaldoss, 1999), if $0 < g_k \leq m(k)e$, then the equilibrium solution for player *i* of group *k* in the within-group conflict is to contribute nothing (i.e., $x_{ik} = 0$). Put differently, if the within-group conflict is not embedded in the between-group competition, then the equilibrium solution—that has been studied extensively in multiple experiments using the Voluntary Contribution Mechanism (VCM) (see Ledyard, 1995, and Davis & Holt, 1993, for a review)—is to contribute nothing. We test this prediction in Section 4. Our interest, however, is not primarily in testing the VCM but, rather, in embedding it in a between-group competition as a structural mechanism designed to reduce, if not completely eliminate, free riding.

The Distribution Rule. Having specified the payoff structure of the within-group conflict and the contest rule for determining the outcome of the between-group competition, we next describe the distribution rule used to divide the prize *S* among members of the winning group. Denote the distribution rule for group *k* by f_k:

$$f_k = x_{ik}^c/(x_{1k}^c + x_{2k}^c + \ldots + x_{m(k)k}^c), \qquad 0 \leq c \leq \infty$$

Noting that the same distribution rule applies to all groups and therefore simplifying the notation, we can write this rule as

$$f = x_{ik}^c/X_k^c \tag{3}$$

where $X_k^c = \sum x_{ik}^c$. The parameter *c* in Eq. (3) determines a family of profit sharing rules for distributing the prize *S*. It is basically an incentive mechanism that can be varied to determine the individual contribution. If $c = 0$, then each member of the winning group receives an equal share $1/m(k)$ of the prize. This is the egalitarian profit sharing rule that we alluded to before. If $c = 1$, then each member of the winning group receives the fraction x_{ik}/X_k of the prize. This is the proportional profit sharing rule mentioned earlier. Other incentive mechanisms are generated by values of c in the interval [0, ∞).

Recall that each player contributes to the resolution of the two-level interaction (within-group and between-group). Denote the expected payoff of player i of group k, having contributed x_{ik}, by V_{ik}. Then, combining the terms in Equations (1) through (3), we have

$$V_{ik} = (\frac{X_k}{X})\left[(e - x_{ik}) + g_k\left(\frac{X_k}{m(k)e}\right) + S\left(\frac{x_{ik}{}^c}{X_k{}^c}\right)\right] + \left(\frac{X - X_k}{X}\right)\left[(e - x_{ik}) + g_k\left(\frac{X_k}{m(k)e}\right)\right] \quad (4)$$

The first term in square brackets is player i's payoff if her group wins the between-group competition, and the second term in square brackets is her payoff if her group does not. Each of these terms is multiplied in turn by the probability that her group either wins the competition (X_k/X) or loses it $(X - X_k)/X$.

The present experiment considers a special case of this model with two groups of equal size and the same within-group conflict. Thus, $n = 2$, $g_1 = g_2 \equiv g$, and $m(1) = m(2) \equiv m$. Therefore, $N = 2m$ and $X = Nx_{ik}$. Substituting these terms into Eq. (4), solving, and setting $x_{11} = x_{21} .. = x_{m1} = x_{12} = x_{22} = .. x_{m2} = x_i$, the equilibrium solutions under the egalitarian and proportional rules, denoted by $x_i{}^*(Eg)$ and $x_i{}^*(Pr)$, are given by (Rapoport & Amaldoss, 1999):

$$x_i{}^*(Eg) = \frac{S}{N^2(1 - \frac{2g}{Ne})} \quad (5)$$

and

$$x_k{}^*(\mathrm{Pr}) = \frac{S(N-1)}{N^2(1 - \frac{2g}{Ne})} \quad (6)$$

In particular, $x_i*(Pr) = (N - 1) [x_i*(Eg)]$.

3. Method

Subjects. Forty-eight participants, all University of Arizona undergraduates, took part in the experiment. All the participants volunteered to take part in a two-hour computer-controlled experiment on interactive decision making with payoff contingent on performance. Individual earnings, in addition to a $5.00 show-up fee, ranged between $17.75 and $26.25.

Procedure. The participants (players) were divided into three separate conditions each including sixteen subjects. The players in Condition VCM only participated in a public good experiment under the regular VCM where they were asked to divide their endowment between a private account and a group account, and where groups did not interact. This condition served both as a baseline for the other two conditions and replication of previous experiments on public good provision that involved a considerably smaller number of rounds. The sixteen players in Condition EG participated in a bi-level competition in which the VCM was embedded in a between-group competition with an egalitarian distribution rule. Condition PR only differed from Condition EG by implementing the proportional rather than egalitarian sharing rule.

The experiment was conducted at the University of Arizona Economic Science Laboratory. The sixteen players of each condition were seated in cubicles separated from one another by partitions. Each player was provided with a hard copy of the instructions, pocket calculator, as well as pen and paper to take notes. All the players had to pass a computerized pre-experimental quiz to ensure their understanding of the instructions. The quiz presented the players with single-round choices of a hypothetical group of $m = 4$ players, who contributed different amounts to a public account. The actual players were then asked to calculate the payoffs of these fictitious players. In the quiz, one of the fictitious players contributed nothing, one contributed her entire endowment, and the other two contributed intermediate amounts. This illustrated to the actual players that free riding could be individually advantageous and that group payoff was maximized if all four group members contributed their entire endowment. To ensure their understanding of the between-group competition, subjects were asked to calculate the payoffs of the fictitious group members both for the case where the hypothetical group won the competition and for a case where it did not. The experiment started only after all the players in the session successfully passed the quiz. Thus, understanding of the game was rendered common knowledge.

Each session included $T = 80$ identical rounds and lasted about two hours. At the beginning of each round (trial), the sixteen players were randomly assigned to four different groups each including $m = 4$ players. This randomized-group design was used to prevent reputation building. Thus, on each trial a player had no information about the other three members of her group. In Condition VCM, the four groups played independently of one another, whereas in Conditions PR and EG the four groups were divided into two sets of two groups each ($N = 8$) with the two groups in each set competing for the prize. Table 1 presents the experimental design and the parameter values for all three conditions.

--Insert Table 1 about here—

After all the N players submitted their contribution decisions, the computer totaled the individual contributions in each group. In Conditions EG and PR, but not VCM, the contest rule (Eq. 2) was used to determine the winning group, and then either the egalitarian or proportional distribution rule was invoked to distribute the prize. At the end of each round, the players were informed of the total contribution of their group, and, in Conditions EG and PR, the total contribution of the competing group, the associated probabilities of winning the competition, and who the winning group was. Players were also informed of their own payoff for the round, broken down by its source (private account, public account, intergroup competition) and their cumulative payoff. Information about individual contributions was not disclosed, nor did the players receive any information about the contributions of members of other groups.

Parameter Values. The group size $m = 4$ was chosen to allow comparison of Condition VCM with previous VCM experiments that included four-player groups (e.g., Gunnthorsdottir, Houser, McCabe & Ameden, 2001, Isaac & Walker, 1998). The marginal per capita rate of substitution (MPCR) between the public and private account in the VCM was set at $g/me = 0.5$ because this value has been commonly used in earlier VCM experiments (e.g., Andreoni, 1995, 1988; Gunnthorsdottir et al., 2001). The prize value was set at $S = 208$ to ensure considerable separation between the equilibrium solutions for Conditions EG and PR (see bottom row of Table 1). Note that when $S = 208$, the resulting equilibrium solutions are interior with about equal distance of the equilibrium point solution for Condition EG from the lower bound of zero tokens and for Condition PR from the upper bound of 50 tokens. Interior Nash equilibria allow players room to deviate both above and below the predicted contribution thereby not forcing the distributions of individual contributions to be skewed (see Isaac & Walker, 1998; Laury & Holt, 1998).

4. Results

In equilibrium, individual contribution in Condition PR should exceed the one in Condition EG, which, in turn, should exceed the individual contribution in Condition VCM. These qualitative implications are clearly supported by our findings. The left-hand panel of Fig.

1 displays the mean contributions per trial. The results are exhibited for each condition separately. The three empirical functions are already separated from one another in the first few trials, and later on they never overlap. Table 2 presents the means and standard deviations of the individual contributions by blocks of ten trials. These results, too, are presented for each condition separately. They suggest smaller variability in Condition PR than in the other two conditions, and the existence of learning trends in Conditions VCM and EG, but not Condition PR. We shall explore these learning trends below.

--Insert Fig. 1 and Table 2 about here--

Static Analysis

Condition VCM. The results of Condition VCM—a symmetric four-player PD game with continuous strategy space—replicate the results observed in earlier VCM experiments (see reviews in Davis & Holt, 1993; and Ledyard, 1995), Initially, mean contributions are about 50% of the endowment. Then, they decline in the direction of the equilibrium prediction of zero contribution. This is evident from inspecting Fig. 1a and Table 2; no statistical tests are required to show that, based on the mean contributions of sixteen players, the down trend is statistically significant.

The right-hand panel of Fig. 1 exhibits the median contributions (rather than means) per round of play. Comparing Figs. 1a and 1d shows that the medians are in general smaller than the means. This result is simply due to the fact that the distributions of individual contributions per trial are almost always positively skewed, rendering the median a more representative measure of the central tendency of the results. Exhibiting the proportion of players who contributed zero in Condition VCM, Fig. 2a shows that with experience the number of players adhering to the equilibrium prediction slowly increased.

--Insert Fig. 2 about here—

Condition EG. Under the egalitarian distribution rule, mean contributions per round also started at about 50% of the endowment (Fig. 1b) and then declined slowly. We computed for each player separately her mean contribution in the first ten trials (block 1) and last ten trials (block 8) and compared them by a paired t-test. The comparison yielded a significant difference between the means ($p < 0.05$) of the first and last block. Figure 1b as well as Table 2 indicate that mean contributions per trial stabilized about 21 tokens in the last twenty trials. This value is consistently higher than the equilibrium prediction of 6.5, thereby providing evidence for considerable over-contribution in this condition. Figure 2b shows that the proportion of players contributing zero hardly changed across trials and exceeded 0.20 on only two of the eighty trials.

Condition PR. Figure 1c shows that under the proportional distribution rule mean contributions started at about 36 and remained high and stable across all eighty trials. Similarly to Condition VCM, the distributions of individual contributions per trial were skewed but in a negative rather than positive direction. Figure 1f shows that the median contributions were stabilized quite early in the session at around 45 tokens. The pattern of results in Fig. 1c is very similar to the one reported by Nalbantian and Schotter (1997), who found that, when the equilibrium contribution was close to the upper bound of the strategy space, mean contribution levels were stable over trials but somewhat below the equilibrium level. Figure 2c shows that the proportion of players who contributed zero under this condition was quite small, seldom exceeding 0.05.

Taken together, the results of Conditions VCM and PR provide strong support to the equilibrium solution on the aggregate level. The median contributions at the end of the session are close and often at the point equilibrium predictions. The deviations of mean contributions from equilibrium play are minor and mostly due to skewed error distributions. These reflect in part the closeness of the equilibrium predictions to the boundaries of the strategy spaces—0 in Condition VCM and 50 in Condition PR. Leaving not enough room to err, these boundaries force the distributions of individual contributions in Conditions VCM and PR to be positively or negatively skewed, respectively. Decision errors have been identified by other studies of public good provision as a major source of over-contribution and under-contribution, depending on the position of the equilibrium point prediction with respect to the boundaries of the individual strategy space (Isaac & Walker, 1998; Laury & Holt, 1998; Andreoni, 1995).

In contrast to Conditions VCM and PR, the players in Condition EG exhibited a substantial and significant degree of over-contribution. On the average, the players contributed between three to four times as much as the equilibrium prediction. Similar results were reported by Amaldoss et al. (2000) in their low-reward condition. In both studies, there is a significant over-contribution under the egalitarian rule and a slow but significant trend across rounds in the direction of equilibrium play.

Individual Differences. Despite the support for equilibrium play on the *aggregate* level, we find no support for it on the *individual* level. In this regard, our results are similar to the ones reported by Rapoport, Seale, Erev and Sundali (1998), Rapoport, Seale and Winter (2000), and Rapoport, Seale, and Winter (in press) in their study of market entry games as well as other experiments on tacit coordination supporting equilibrium play on the aggregate but not individual level. Whatever strategies individual players used to determine their contributions, they cannot be accounted for by the pure-strategy equilibrium that allows no differences between players and, of equal importance, no differences between trials for the game. Results reported by Gunnthorsdottir (2001, Ch. 4) indicate that the aggregate analyses disguise the considerable individual differences in all three experimental conditions. Moreover, they also conceal substantial within-player differences in the contribution decisions across rounds of play. Many players exhibited sharp oscillations moving from zero contribution in one round to contribution of their entire endowment on another. The block means of the within-player standard deviations of contributions, though slightly decreasing across blocks of trials, varied between 6 or 7 and 12. See Figure 3. Such oscillations are apparently not unusual. Figure 3 also shows that oscillations in the first ten rounds in present study are similar in magnitude to oscillations in a comparable ten-round VCM experiment conducted by Gunnthorsdottir et al. (2001).

--Insert Fig. 3 about here—

Dynamic Analysis

Recent years have seen a marked shift in the attempt to rationalize equilibrium play in terms of players' introspection and common knowledge to the idea that equilibrium play is *learned* through repeated play. When relatively inexperienced individuals participate in a repeatedly iterated game with the level of complexity exhibited in the present study they do not and cannot reason their way to equilibrium by sheer speculation. Rather, "they watch, listen and do" (Smith, 1990, p. 12 – 13). Explanations of adaptive learning in iterated noncooperative games have become a major topic of research in experimental economics and related disciplines. Belief-based (e.g., Fudenberg & Levine, 1998), reinforcement-based (e.g., Roth & Erev, 1995), Bayesian learning (Jordan, 1991), hierarchy-of-player type-based (e.g., Stahl, 1999), and hybrid models (e.g., Camerer & Ho, 1999) that integrate both belief- and reinforcement-based models have been proposed and tested with various sets of data (see, e.g., Cox, Schahat, & Walker 2001; Erev & Roth, 1998; Sarin & Valid, 2001; Feltovich, 2000). Depending on their familiarity with these models, the type of explanation they seek, and the particular features of the payoff structure and experimental design, researchers today have a large body of competing models to choose from. This is the case because, despite heated debates and a few comparative studies of subsets of these models (e.g., Feltovich, 2000), the superiority of one model over the others has not been ascertained.

To account for the dynamics of play in the present study and the patterns of results displayed in Fig. 1, we focus on a simple reinforcement-based learning model. Although our choice is to some extent arbitrary, we mention briefly several reasons for its support. First, the structure of our experiment seems to support this choice. The players in each of the three experimental conditions were randomly assigned to groups on each round. This prevented them from establishing reputation or acquiring beliefs about the behavior of specific players. Whatever learning took place in the experiment, it occurred on the *population* level. This assignment method seems to rule out hierarchy-of-player type-based models that require fixed-group designs. Second, belief-based models seem to be ruled out by the structure of each of the rounds of play as they would have required a level of cognitive complexity unlikely to be achieved in practice. In particular, they would have required that a player calculates the payoffs from the within-group conflict for each of her strategies, given other members' contributions. In Conditions EG and PR she would also have to calculate for each of her strategies the expected value of her share in the lottery used to determine the winning group while taking into account the total contribution of her group and the competing group. A third reason in support of this choice is a recent and similar study by Amaldoss and Rapoport (2001) on multi-level competitions with a probabilistic contest rule. Amaldoss and Rapoport applied the hybrid EWA model of Camerer and Ho (1999) to their data, and found that a simple reinforcement-based version best accounted for their results.

Recall that the strategy space included 51 strategies, namely, all the integers in the interval [0, 50]. Selten (1997) has suggested the prominence hypothesis according to which players tend to state their choices in multiples of five. Our data largely support his hypothesis. In all three conditions, our players most often chose contributions in multiples of five (Fig. 4). Consequently, we discretized the strategy space to a manageable number of eleven strategies. The strategy "0" consisted of contributions in the interval [0, 2], the strategy "5" consisted of contributions in the interval [3, 7], and so on, with the strategy "50" consisting of contributions in the interval [48, 50]. The learning model described and tested below operates on these strategies.

--Insert Fig. 4 about here—

A Reinforcement-Based Learning Model The learning model builds on the original study of Roth and Erev (1995) that, in turn, was inspired by earlier extensive studies of reinforcement-based learning in psychology (e.g., Bush & Mosteller, 1955). This is a probabilistic model assuming that player *i*'s probability of playing strategy *j* at trial *t* is a function of the reinforcement this strategy received in previous trials. We only present a brief description of this three-parameter model; for additional justification and details see Roth and Erev (1995) and Erev and Roth (1998).

Denote the reinforcement that player *i* receives from playing strategy *j* ($j = 1, 2, \ldots, J$) on trial *t* ($t = 1, 2, \ldots, T$) by $V_{ijt} - V_{ijt}^{min}$. The first term is the actual payoff received on trial *t* and the second is the minimum of all possible payoffs associated with playing strategy *j*. With the value of the MPCR set at 0.5 in our experiment, and noting that V_{ijt}^{min} does not depend on the particular trial number, we can write

$$V_{ijt}^{min} = V_{ij}^{min} = (e - x_{it}) + 0.5x_{it}.$$

The learning model includes three parameters to be estimated from the data: they are supposed to capture the strength of the initial propensity *S(1)*, generalization of reinforcement ε, and forgetting □. Let $S_i(1)$ denote the experience of player *i* gained before participating in the experiment, and assume that her propensity of choosing strategy *j* at trial $t = 1$ is given by

$$P_{ij}(1) = S_i(1)/J, \qquad (7)$$

where J ($J = 11$ in our case) is the number of strategies. The associated probability of choosing strategy *j* at trial $t = 1$ is given by

$$\Pi_{ij}(1) = [P_{ij}(1)]/S_i(1). \qquad (8)$$

Following Roth and Erev, we disregard individual differences in the initial propensities by assuming that $S_i(1) = S(1)$ for *all i.* Let q_{ij} represent an individual's sum of all past reinforcements for a given strategy *j* up to and including trial *t-1,* including the initial *S(1)/J.* If player *i* chose strategy *j* on trial *t*, assume that

$$\Pi_{ij}(t+1) = \frac{q_{ij} + (V_{ijt} - V_{ij}^{\min})}{\sum q_{ij} + (V_{ijt} - V_{ij}^{\min})} \qquad (9)$$

while the probability of choosing a strategy *j* <u>not</u> selected on trial *t* is given by

$$\Pi_{ij}(t+1) = \frac{q_{ij}}{\sum q_{ij} + (V_{ijt} - V_{ij}^{\min})} \qquad (10)$$

These equations imply that the probability of player i choosing some strategy j at trial $t + 1$ is the sum of past reinforcements received from playing strategy j divided by the sum of all the reinforcements player i received from playing all J strategies.

Strategies are assumed to have a natural ordering, as in our experiment, and the reinforcement that strategy j receives is assumed to spill over to neighboring strategies. This is captured by a generalization parameter ε $(0 \leq \varepsilon \leq 1)$ and the assumption that if strategy j is chosen on trial t it is reinforced with probability $(1 - \varepsilon)(V_{ijt} - V_{ij}^{min})$ while each of its two neighboring strategies $j - 1$ and $j + 1$ in the natural ordering is reinforced with probability $(\varepsilon/2)(V_{ijt} - V_{ij}^{min})$. This rule is modified in a natural way for the two extreme strategies "0" $(j = 1)$ and "50" $(j = 11)$. To account for recency effects, the parameter δ $(0 \leq \delta \leq 1)$ is introduced. It is supposed to reflect the player's rate of "forgetting" previous reinforcements.

Let q_{ji} represent player i's sum of all past reinforcements for a given strategy j up to and including trial $t-1$, including the initial propensity $S(1)/J$, and discounted at each trial by the forgetting parameter δ. Then, assuming that player i chose strategy j at trial t, her probability of choosing strategy j at time $t + 1$ is given by

$$\Pi_{ij}(t+1) = \frac{(1-\delta)q_{ji} + (1-\varepsilon)(V_{ijt} - V_{ij}^{\min})}{(1-\delta)\Sigma q_{ji} + (V_{ijt} - V_{ij}^{\min})} \tag{11}$$

Note that the probabilistic learning model described above is formulated on the individual level. However, the same parameter values $S(1)$, ε, and δ are assumed to hold for all players, thereby allowing for no individual differences. Our results that show considerable individual differences clearly falsify the model. However, it is still useful to test the descriptive power of the model on the aggregate data. Indeed, despite this major drawback, this is the common practice for testing learning models.

Individual players' expected contribution per trial were calculated by multiplying the value of each of the *J* strategies by the choice probability for that strategy. Then, the mean expected contributions were taken over all players within a given session. To test the *ex-post* descriptive power of the model, a three-dimensional grid was systematically searched to find the values of the three parameters *S(1)*, ε, and δ that jointly minimize the mean squared deviation between mean observed contributions per trial and the ones predicted by the model. Table 3 presents the best fitting parameter estimates for each condition separately and the associated mean square deviation between observed and predicted results.

--Insert Table 3 about here—

Table 3 indicates no generalization of the reinforcement received by strategy *j* to neighboring strategies in each of the three conditions. This result is surprising given the evidence for generalization in previous tests of the Roth-Erev model (e.g. Roth & Erev, 1995; Erev & Roth, 1998). We take this result as evidence against the basic assumption of the model that reinforcement is generalized to neighboring strategies. This is particularly the case in our study as the strategy space is cardinal and the ordering of the 11 strategies from "0" to "50" is quite natural. On the other hand, this finding reduces the number of free parameters from 3 to 2. The difference between Conditions EG and PR with respect to the estimate of *S(1)* is also puzzling, as we had no reason to expect a difference between the two conditions in terms of past experience of players participating in essentially the same multi-level competition.[1]

[1] The parameters reported in Table 3 are the ones that minimize the distance between the model and the data. Note, however, that forcing the parameters from one condition upon the other two conditions changes the fit measures very little.

Despite these shortcomings, the learning model accounts for the aggregate data quite well. With only two parameters in Conditions VCM and EG and a single parameter in Condition PR, it tracks the major trends in the mean contributions very closely. Figure 5 compares the predicted contributions to the observed mean contributions for each condition separately. As might be expected (Roth & Erev, 1995), the theoretical results exhibit less variability between trials. However, they track the experimental results very well. In particular, they track the spikes in the aggregate data, particularly in Conditions VCM and EG. This is due to the relatively high estimates of the forgetting parameter δ. Further, the model accounts simultaneously for the general down trends in Conditions VCM and EG and essentially the flat learning curve in Condition PR.

--Insert Fig. 5 about here—

5. Discussion

The major finding of the present study is that embedding independent within-group social dilemmas of the PD kind in a between-group competition alleviates to a large degree the free-riding problem. We note that the mean contribution by all the N players in Condition PR—about 320—exceeds the value of the prize S (S = 208) by about 60%. Even in Condition EG, it is only slightly less than the prize value. However, rendering the different groups interdependent in this way cannot be construed as a non-structural mechanism for solving social dilemmas (e.g., Dawes, 1980) because the prize for the winning group-—regardless of its method of distribution—is exogenously determined. Rather than modifying the payoff structure, most of the solutions for social dilemmas have focused on changing the utility functions of the players by appeals to altruism, moral considerations, non-binding commitments, face-to-face interaction, etc. However, the finding of high contribution levels in Conditions EG and PR, in contrast to Condition VCM, gives rise to the hypothesis that partial alleviation of free riding in within-group conflicts may be achieved even with smaller prize values *relative* to the players' endowment and the MPCR value. This is a topic for further experimentation. Evidence in support of this hypothesis comes from organizations that have succeeded in reducing free riding by having separate departments engage in a competition for what is often a nominal prize (e.g., commendation, a token monetary prize).

A second major finding is that the profit sharing rule matters provided it is commonly known before individual players make their independent contributions. If replicated with a larger number of competing groups with symmetric or asymmetric players (see Amaldoss & Rapoport, 2001), this finding has potential policy implications (e.g., by having United Way campaigns introduce competition between academic departments to increase the level of individual contributions, and hierarchical organizations re-structuring their incentive mechanism to increase productivity). However, drawing policy implications from this study should be approached with caution. First, replications of this study are warranted with either randomized-group or fixed-group designs. When players are re-assigned to groups on each round, as in the present study, the independent unit of analysis is the session not the group. Therefore, strictly speaking, the present study only has a single independent unit of n = 16 in each of the three conditions. Second, in drawing policy implications one has to ensure that the incentives of the competing groups are aligned with the global goals of the organization as group competition has the potential of becoming intense and of having unexpected consequences. (e.g., Diab, 1970; Sherif, 1966). Thirdly, the advantage of the proportional over the egalitarian distribution rule is strongly qualified by the fact that the former requires close monitoring of individual decisions whereas the latter does not. Monitoring is often impossible and frequently prohibitively expensive. We note that the effects of costly monitoring may be examined experimentally by introducing costs of monitoring that are to be deducted from the individual payoffs of the winning group when the distribution rule is proportional.

The third major result is the sharp difference between the performance of the models on the aggregate and individual levels. It is, of course, important to know whether the dynamics of play are better accounted for by a reinforcement-based or belief-based learning model, or that one distribution rule is significantly more effective in eliciting contributions than another. However, if a more fundamental understanding of human behavior in interactive decision making situation is desired, individual differences cannot be pushed under the rug of aggregate statistics. A complete understanding of interactive decision making cannot be achieved without understanding the individual behavior of the interacting players. In the present study we observe patterns of behavior that on the aggregate level are asymptotically accounted for by the equilibrium solution for two of the three conditions and different trends across trials in the three conditions that are accounted for by the same learning model. However, although both the equilibrium solution and learning model are formulated on the individual level, they fail to account for the individual data. More imaginative experimental designs for studying individual differences are warranted that rather than bringing large groups of participants to relatively brief single-session experiments focus on studying smaller groups of interacting participants across different tasks in multi-session experiments

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APPENDIX A

Group Decision Making Experiment

Instructions

You are about to participate in a group decision experiment that involves <u>two</u> types of interaction: WITHIN-group interaction and BETWEEN-group interaction.

Your payoff for each round of the experiment will therefore come from two sources, the within-group interaction and the between-group interaction. The payoff will depend on your decision, the decisions made by other members of *your* group, and the decisions made by the members of *another* group.

Payoffs from the experiment are given in tokens, which will be converted to US dollars at the end of the experiment. There will be many decision rounds, all structured in exactly the same way. Below are the rules of the game that determine your payoff for each round.

<u>Rules of the game and computation of payoffs.</u>

Before *each* round, each participant in the experiment will be randomly assigned to a group of <u>four</u>. Your group will then interact with another group of four to which members have also been randomly assigned. At the beginning of each round, each participant will receive an investment capital of 50 tokens. They then must decide how much of it to invest. You may invest any number of tokens between 0 and 50. The remaining part of your investment capital is yours to keep.

<u>1. Within-group interaction.</u>

With their investments, members of each group of four generate a *within-group payoff.* The within-group payoff is paid out to all individual members of a group and is the same for all of them. The size of the within-group payoff is determined by the following ratio:

$$\frac{\textbf{Group's total investment}}{\textbf{Group's total investment capital}}$$

Which amounts to:

$$\frac{\textbf{Group's total investment}}{\textbf{4 x 50}} = \frac{\textbf{Group's total investment}}{\textbf{200}}$$

As investments may not exceed investment *capital*, this ratio can never be larger than 1. The ratio can also not be smaller than 0. In order to compute the within-group payoff to each group member, this ratio is multiplied by 100 tokens.

Therefore, your earnings from the within-group interaction will be computed as follows:

$$\left(\textbf{Your investment} - \textbf{Your investment capital (50 tokens)}\right) + \left[\left(\frac{\textbf{Group's total investment}}{\textbf{200}}\right) \times \textbf{100}\right]$$

The expression in the first parenthesis is the number of tokens you keep after making your investment. The expression in the second parenthesis is the within-group payoff.

You can see from this formula that the within-group payoff to each member reaches its maximum of 100 tokens if <u>all</u> group members contribute their <u>entire</u> investment capital. It follows that if all group members contribute their entire investment capital, each group member earns 100 tokens from the within-group interaction. If all group members contribute nothing, each earns 50 tokens from the within-group interaction, because each kept his or her investment capital for him/herself.

<u>2. Between-group interaction with another group</u>.

As mentioned earlier, at each round, your group will be randomly paired with another group of four. Either your group or the other group will get a *between-group payoff* of 208 tokens. In each round, only one of the two groups can obtain this payoff.

I.) A group's probability of obtaining the between-group payoff. The computer will assign the between-group payoff either to your group or to the other group, *via a lottery*. This means that a group can never guarantee itself the between-group payoff. However, by increasing your investment, you can increase your group's *probability* of obtaining this payoff. A group's probability of obtaining the between-group payoff is calculated according to the following formula:

$$\textbf{Probability of Group A getting the between-group payoff} = \frac{\textbf{Group A's total investment}}{\textbf{Group A's total investment + Group B's total investment}}$$

Of course, the two probabilities (of Group A getting the between-group payoff and of Group B getting the between-group payoff) always sum up to one.

II.) Allocating the between-group payoff (in case your group obtained that payoff). Members of the group that was awarded the between-group payoff will receive a share of the between-group payoff of 208 tokens, *in proportion* to their individual investments. The exact calculation of a group member's share in the between-group payoff is as follows

$$\textbf{Individual's share in between-group payoff} = \left(\frac{\textbf{Individual's investment}}{\textbf{Group's total investment}}\right) \times \mathbf{208}$$

In other words, a group member's share in the between-group payoff is in exact proportion to his/her contribution to the group's total investment.

3. Computing total earnings per round.

Your total earnings per round are the sum of your earnings from the *within*-group interaction and from the *between*-group interaction (in the event your group was awarded the between-group payoff). That is, total earnings are computed according to the following formula:

$$\textbf{Total earnings per round} = \left(\textbf{Within-group earnings}\right) + \left(\textbf{Individual's share in between-group payoff (if any)}\right)$$

4. General:

I.). Group membership. To remind you, the composition of your group and of the other group changes before each round, because each participant in the experiment will be randomly re-assigned to a *new* group of four.

II.). Numbers. When making investment decisions, use whole numbers. Earnings in tokens will be rounded to the nearest whole number.

III.) Anonymity. All decisions during the experiment are anonymous. You will never know the identity of other members of your group or of the other group. Also, you will only receive information about the total group investment but not about individual investments.

IV.) Managing your decision task. Before the actual experiment, you will take a computer quiz to assure that you understand the payoff computations. You may refer back to the instructions at any time during the quiz and during the experiment, and you may take notes at any time. Calculator use is allowed throughout.

V.) Experimental earnings. A the end of the experiment, tokens will be converted to US$ at a rate of 100 tokens = ____ US$. You will be asked to sign a receipt for the money and complete a brief questionnaire before leaving the laboratory.

If you have any questions, please raise your hand and an experimenter will assist you.

Attached: An example of earnings computations for one round, for members of a hypothetical group of four. NOTE: The numbers (hypothetical investments) in this example are not identical to the numbers in the computer quiz which you are about to take.

Table 1

Experimental Conditions and Game Parameters

	Experimental condition		
	Regular VCM No group competition (baseline)	VCM with intergroup competit□on Condition EG	VCM with intergroup competit□on Condition PR
Group size	m = 4	m = 4	m = 4
# participants	16	16	16
Endowment per trial	e = 50	e = 50	e = 50
Maximum amount of public good a group can generate	g = 100	g = 100	g = 100
Prize	-----	S = 280	S = 208
# decision rounds	80	80	80
Nash equilibrium contribution	$x_i = 0$	$x_i = \frac{S}{(N)^2 * \left(1 - \frac{2g}{Ne}\right)}$	$x_i = \frac{S\ (N\text{-}1)}{(N)^2 * \left(1 - \frac{2g}{Ne}\right)}$
Nash equilibrium contribution for parameters used	0 tokens	6.5 tokens	45.5 tokens

Table 2

Mean Individual Contributions per Block and First and Final Rounds

Block No.	Condition VCM		Condition EG		Condition PR	
	Mean	SD	Mean	SD	Mean	SD
1	20.90	18.67	29.64	17.22	41.24	10.65
2	15.71	18.59	26.52	16.69	40.37	13.11
3	13.12	16.05	24.09	16.03	37.53	15.80
4	14.67	16.29	23.54	17.39	38.93	14.32
5	13.57	16.31	24.34	16.95	38.52	15.52
6	12.51	15.84	25.22	16.91	38.79	14.07
7	9.86	14.94	21.48	16.56	38.98	14.40
8	8.66	14.94	21.14	16.04	40.11	13.96
Round 1	25.62	16.01	26.56	18.50	37.38	10.90
Round 80	6.31	13.30	21.62	16.99	41.88	11.38
Mean across rounds	13.62	16.84	24.49	16.88	39.31	14.06

Table 3

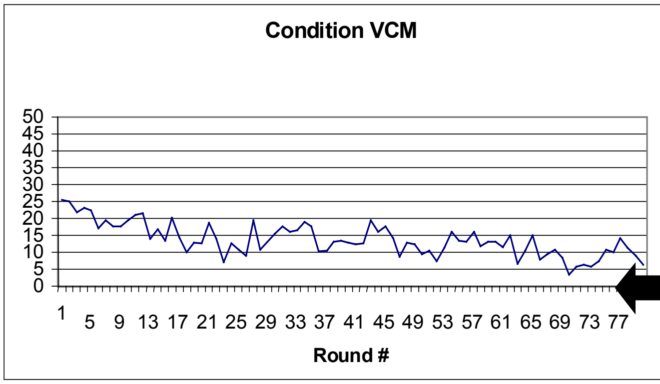
Parameter Estimates and Goodness of Fit Measures by Condition

Parameter	Condition VCM	Condition EG	Condition PR
S(1)	10.00	10.00	0
ε	0.00	0.00	0.00
δ	0.31	0.40	0.22
MSD	3.49	2.62	3.37

Figure 1

Mean and median contributions per round

Means **Medians**



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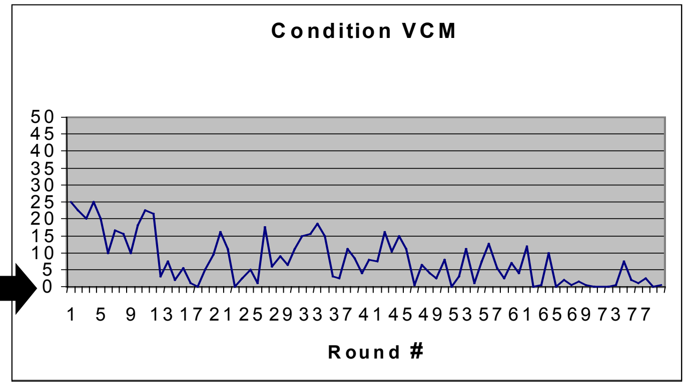
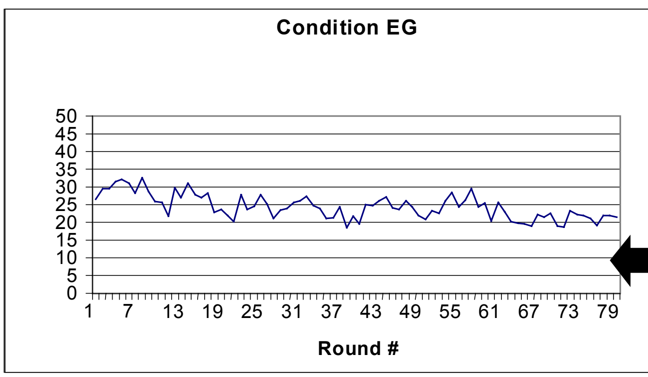


1d



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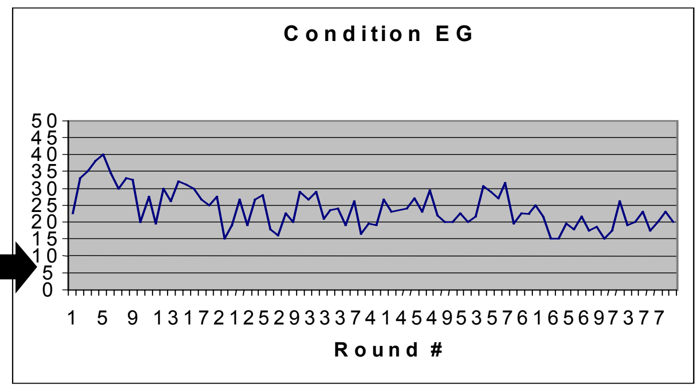

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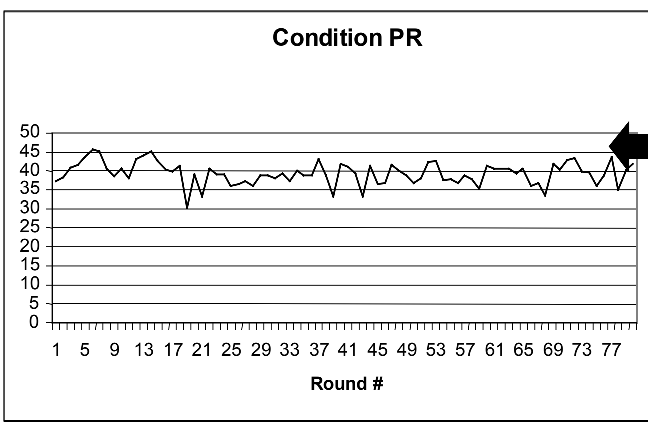

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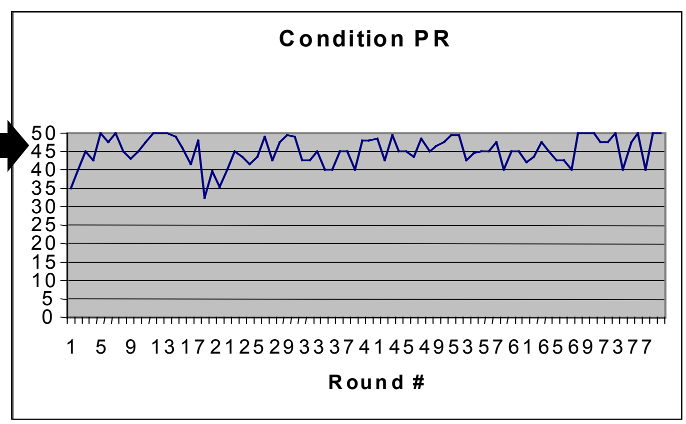

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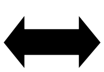
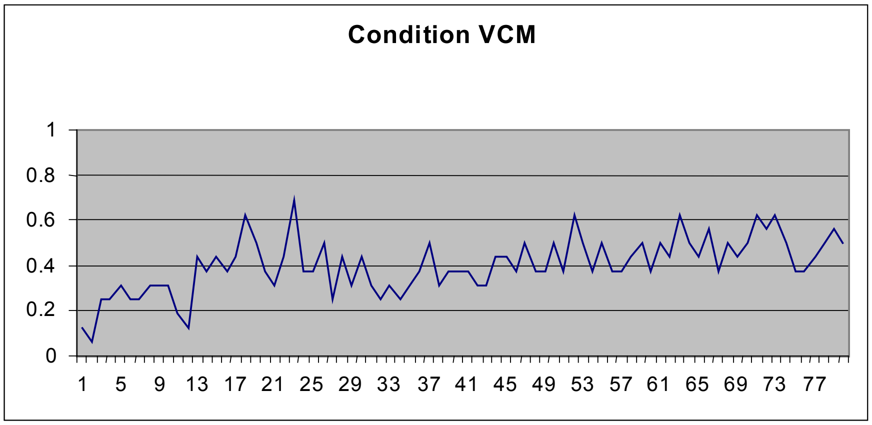
 NE contribution

Figure 2

Percentages of non-contributors by round



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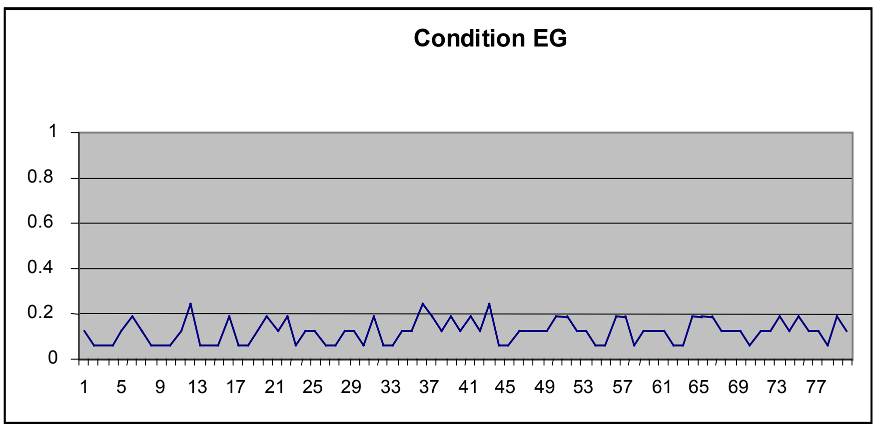

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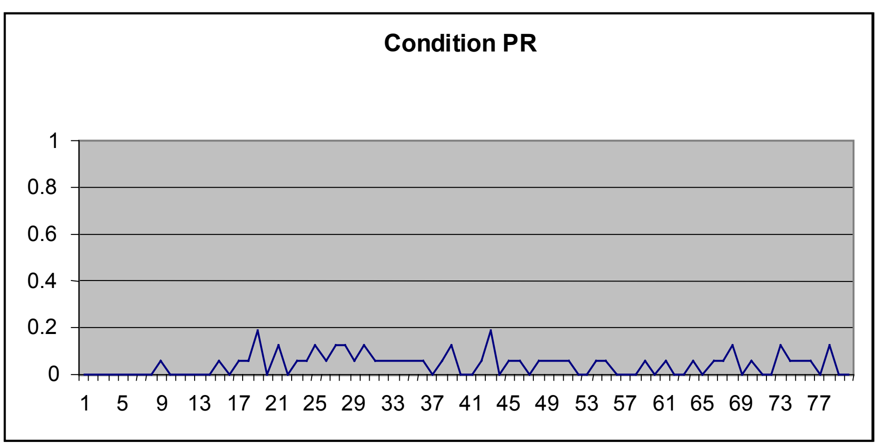
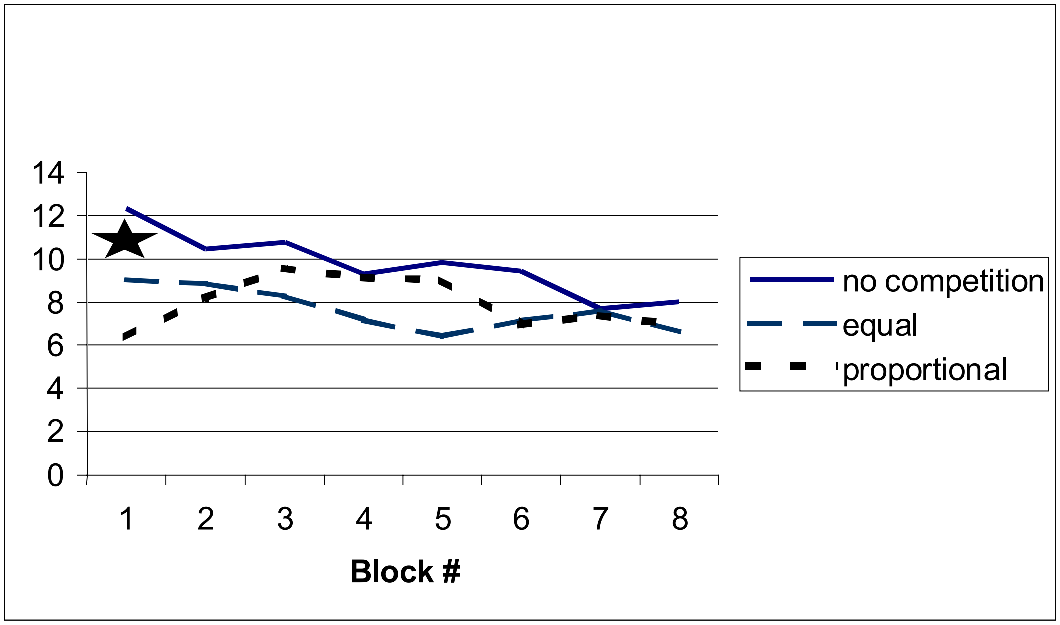


2c

Figure 3

Mean standard deviations of subjects by block



The average standard deviation of individual subjects over ten rounds in the Gunnthorsdotti study:

10.46 (adjusted for differing size of endowment between the two studies)

Figure 4

Strategy choice by frequency, over players, trials and conditions

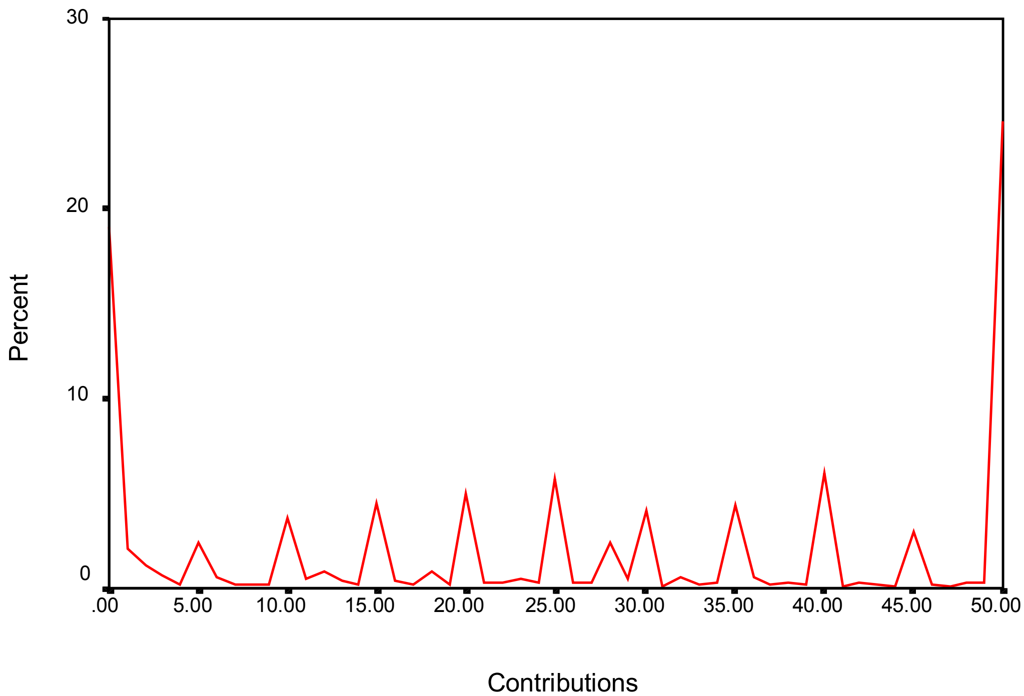
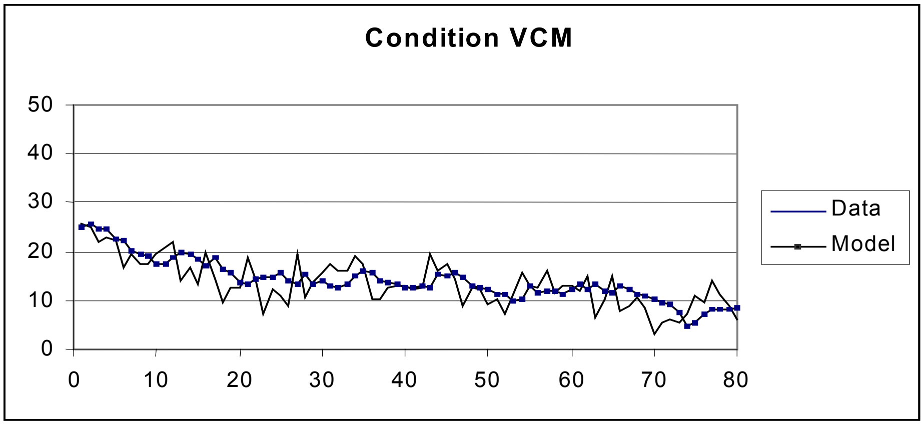


Figure 5

Observed mean contributions compared with contributions predicted by the learning model



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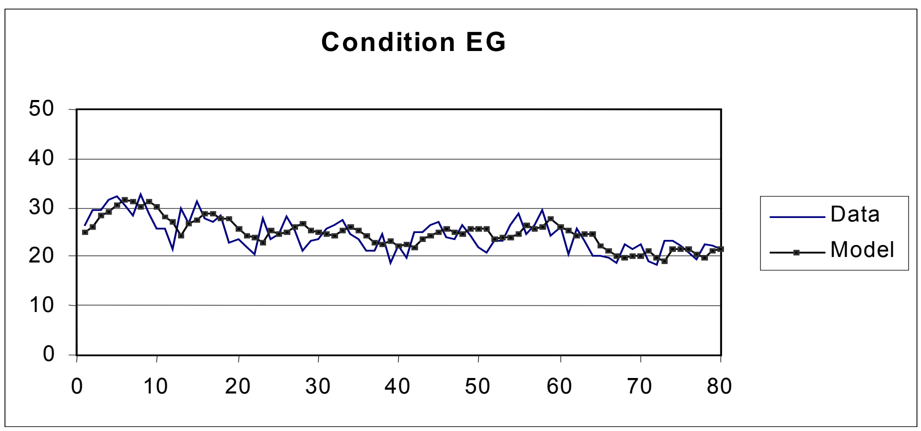

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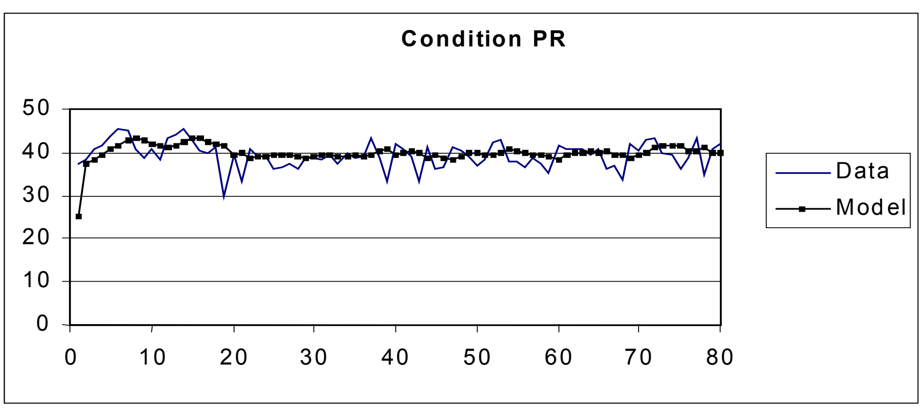

5c