The Ebbinghaus effect and the implications of net learning for the performance of production systems

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Abstract

A simple Ebbinghaus model suggests that the policy implications of gross learning for the performance of production systems can be misleading. The rates of net learning tend to be transitory such that knowledge accumulation and diffusion processes thereof take longer to bear fruit than conventional learning models would indicate. Consequently, continuous retooling and retraining of production systems are necessary conditions for offsetting the effects of forgetting on gross learning.

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1.Two related problems

For a long time now learning models have been valuable tools for economic analysis and econometric modeling of the performance of production systems such as factories (Wright, 1936; Jordan, 1966; Nanda and Adler, 1977; Hafiz, 1979; Belkaoui, 1986; Young, 1991; Lucas, 1993; Benhabib and Spiegel, 1994; Solow, 1997, Ariforic, Bullard, and Duffy, 1997; Jackson, 1998). Even so, the two major strands of existing literature suffer two related problems. The first problem is that nearly all early learning models, including Arrow (1962), emphasize mainly learning by doing. In so doing they only show indirectly how forgetting affects learning. The emphasis assumes that the average production system tends to retain more and longer from practice than from theory. Under this more realistic than predictive assumption, however, the implications of gross learning for knowledge building and diffusion can be misleading.

Current learning models do attempt to accommodate forgetting. For example, Benkard (1999; 2000) analyzes the dynamics of learning with forgetting in commercial aircraft production and finds that unit costs do not decline everywhere with learning as the conventional learning model predicts. To determine the lowest cost of acquiring technology, Gagnon and Sheu (2000) propose a nonlinear program, which depends on learning and forgetting characteristics of the personnel using the acquired technology. Ash and Smith-Daniels (1999) use simulation experiments to assess the impact of learning, forgetting, and relearning on system proficiency. Jaber and Bonney's (1997) present comparative net learning curves as a sequential rather than simultaneous "learn-forget" curves. While these four and similar studies adjust learning for forgetting directly, they are too involved and inaccessible to policy makers. That is the second problem.

2. Learning processes

This letter first presents a conventional gross learning model. Second, it experiments with the gross model by adjusting it for forgetting. Third, it compares the rates of change and growth of gross and net learning, (Gorfein and Hoffman, 1987; Estes, 1987; Murdock, 1987; Goldstein, Lay, and Schneider, 1998). The experiment assumes that forgetting encompasses failure to retrieve, differential recall among factors, fading memory traces, distortions of, and interference with, memory (Catania, 1998). The assumption is made in order to advance a commonsensical idea that not all that is learned is retained (Friedberg, 2000). In addition, of all that is retained, not all is recalled; and not all that is recalled is applied.

2.1 Gross learning ≡ learning without, or with very little, forgetting

Let Q(t) represent a gross learning curve of a conventional (non-forgetting) production system such that

$$Q(t) = Q^*(1 - e^{-kt}), Q(0) = 0, \qquad (1)$$

where Q(t) is (cumulative) output produced weekly, output level Q* is the maximum level of performance of which the system is capable, ***k*** is a positive constant, and ***t*** is time in weeks. The rate at which system performance improves over time is

$$Q'(t) = k(Q^* - Q(t)) = kQ^* e^{-kt}. \qquad (2)$$

The relative rate of growth of Q(t) becomes

$$rQ(t) = Q'(t)/Q(t) = ke^{-kt}[1-e^{-kt}]^{-1}. \qquad (3)$$

Clearly (1)-(3) assumes that all that is learned is *permanently retained and always usefully retrieved* for application, which can be misleading.

2.2 The Ebbinghaus effect: Net learning ≡ learning with significant forgetting

Suppose that q(t) is Q(t) adjusted for forgetting such that the net learning experience is

$$q(t) = [Q(t) - \alpha Q(t)]e^{-\lambda t} + \alpha Q(t) = Q(t)[\beta e^{-\lambda t} + \alpha], \qquad (4)$$

where $\beta = 1-\alpha$ is the share of learning that is retained and usefully applied, α is what is forgotten, and $\lambda < k$ is a positive constant. Then in terms of Q*

$$q(t) = \beta Q^* e^{-\lambda t} - \beta Q^* e^{-\lambda t} e^{-kt} = \beta Q^* e^{-\lambda t}[1-e^{-kt}]. \qquad (5)$$

In other words, net learning is target Q* discounted by the pure forget effect $(Q^* e^{-\lambda t})$, and the combined effect of the interaction between the forces of retention and forgetting $(Q^* e^{-kt} e^{-\lambda t})$, both evaluated at the rate $\beta = 1 - \alpha$. Consequently, the ratio of *what is actually retained and usefully retrieved to what is learned and potentially retrievable* ($pq : Q$) is

$$pq:Q = q(t)/Q(t) = e^{-\lambda t}\beta - \alpha. \tag{6}$$

Eq. (6) is the proper Ebbinghaus model for forgetting. Now, from (4) if we set

$f(t) = e^{-\lambda t}\beta - \alpha,$ then

$$q(t) = Q(t)f(t). \tag{7}$$

In that case the rate of change of q(t) becomes

$$q'(t) = Q'(t)f(t) + f'(t)Q(t) = Q'(t)[e^{-\lambda t}\beta + \alpha] - \lambda\beta Q(t)e^{-\lambda t}. \tag{8}$$

Alternatively,

$$q'(t) = Q'(t)[pq:Q] - \lambda\beta Q^* e^{-\lambda t}[1 - e -^{-kt}], \tag{9}$$

where $q'(t)$ is the marginal rate of net learning (MRNL), $Q'(t)$ is the marginal rate of gross learning (MRGL), and pq: Q = Q(t)/q(t) is the ratio of net to gross learning.

We have already met $\beta Q^* e^{-\lambda t}[1 - e^{-kt}]$ in (5) above. The term $\lambda\beta Q^* e^{-\lambda t}[1 - e^{-kt}]$ is the *Ebbinghaus effect* in its two components of which $\lambda\beta Q^* e^{-\lambda t} < 0$ is the marginal rate of pure forgetting (MROF), and $\lambda\beta Q^* e^{-\lambda t}e^{-kt} > 0$ is the marginal rate of the interaction between the competing forces of retention and retrieval, and those of forgetting (MRIR). Thus,

$$MRNL = (MRGL * pq : Q) - (MROF \pm MRIR). \tag{10}$$

Note that the arithmetic sign of MRIR depends on which one of the competing forces dominates. Thus, the relative growth rate of q(t), rq(t), is

$$rq(t) = q'(t)/q(t) = ke^{-kt}[1-e^{-kt}]^{-1} - \lambda\beta e^{-\lambda t} = rQ(t) - \lambda\beta e^{-\lambda t}, \tag{11}$$

where $\lambda\beta e^{-\lambda t}$ is the growth rate of forgetting. The proportion of $q'(t)$ to $Q'(t)$ is

$$pq':Q' = q'(t)/Q'(t) = pq:Q - \lambda\beta e^{-\lambda t}[1-e^{-kt}]. \tag{12}$$

In this case $\lambda\beta e^{-\lambda t} < 0$ and $\lambda\beta e^{-\lambda t}e^{-kt}$ represent the ratios of net to gross learning, and the share of the interaction effect between full retention and retrieval, and full forgetting, respectively.

3. An illustrative Experiment

For an illustrative example, take two learning systems seeking to improve weekly performance in producing a homogenous good. Let the first system assume (1), and the second follow (4). Further suppose that the second system utilizes only β = 0.85 of its learning capacity and forgets at a constant rate of λ = 0.05. Assume that desired Q* = 500 units per week, and k = 0.20, which means that both systems are of normal intelligence (0

≤ λ < k). The question is: on their way to Q*, what are some of the key differences in performance between the two systems? **Put Figs 1 and 2 here**

Figure 1 shows both Q(t) and q(t) increasing, but at decreasing rates of change [$Q'(t), q'(t'')$] depicted in Figure 2, and of growth [$rQ(t), rq(t)$] displayed in Figure 3. While Q(t) approaches Q* asymptotically, q(t) peaks in the ninth week of production at only 289 units. At that same point in time the first system had already learned to produce 417 units per week. Over time the $Q(t) - q(t)$ weekly difference increases from 128 units at the peak of the forgetting system to 367.4 units at the bottom of it in the 40th week. In terms of margins, $Q'(t) - q'(t)$ is largest by the 17th week when $Q'(t) = 3.3\%$ and $q'(t) = -7.1\%$. Corresponding declining rQ(t) and rq(t) are given in Figure 3, and $pq : Q$ and $pq' : Q'$ are in Figure 4. **Put Figs 3 and 4 here**

4. Concluding implications

The preceding example indicates that gross and net learning processes lead to different rates of change and growth, and therefore to different policy implications. Although Q'(t) is falling over time the forces of retention and retrieval dominate, or at the least remain stronger than, those of forgetting such that Q(t) approaches Q* asymptotically. By contrast q(t) is optimized in the ninth week of production such that the by the 40th week the forgetting system produces about what both systems produced in the third and fourth weeks.

If these results can be sustained by real data, they indicate that beyond a certain point an intelligent system can neither forget nor learn any further, because one neither forgets what one does not already know, nor learn more than one's capacity allows. They also suggest that forgetting, like learning of which it is apart, is a cumulative process. Hence, when the effects of forgetting dominate those of retention, a forgetting system comes to depend on innovations external to learning in order to sustain itself. This seems to suggest that when based on gross learning system performance is not sustainable for two reasons. First, dependency on learning what is soon to be forgotten stifles system innovations and reduces its performance. Second, the net learning effects on system performance are of a short duration, implying that the knowledge fund from net learning accumulates and spreads (diffuses) slower than gross learning models would indicate. These implications seem to explain how growth rates have differed across economies seemingly equally exposed to the same learning experience. As such they recommend continuous relearning in order to offset the Ebbinghaus effect and provide better predictions of system performance than possible by gross learning models. Incidently, this recommendation is consistent with Solow's (1997) re-interpretative reading of Arrow (1962), that "continuous improvement of products and processes is … the most important source of increased productivity, …[which] distinguishes successful firms and nations from unsuccessful firms and nations" (p.24). In other words, the gains from comparative advantage are sustainable only through net learning.

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