One-Factor-GARCH Models for German Stocks – Estimation and Forecasting –*

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Abstract

This paper presents theoretical models and their empirical results for the return and variance dynamics of German stocks. A factor structure is used in order to allow for a parsimonious modeling of the first two moments of returns. Dynamic factor models with GARCH dynamics (GARCH(1,1)-M, IGARCH(1,1)-M, Nonlinear Asymmetric GARCH(1,1)-M and Glosten-Jagannathan-Runkle GARCH(1,1)-M) and three different distributions for the disturbances (Normal, Student's t and Generalized Error Distribution) are considered. Out-of-sample forecasts for the stock returns based upon these models are computed. These forecasts are compared with forecasts based on individual GARCH(1,1)-M models, static factor models, naive, random walk and exponential smoothing forecasts.

Keywords: Dynamic Factors, GARCH, Asset Pricing, Forecasting
JEL classification: C32, G12

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1 Introduction

ARCH models first introduced by [Engle 1982] have been found to be very successful in describing the movement of stock returns and other financial time series. Especially their ability to model stock return variances and give forecasts for them is very appealing. These forecasts could be applied in option pricing, hedging and portfolio selection.
Various enhanced models most of which are named in [Hentschel 1995] performed sometimes even better than the plain vanilla ARCH model. A survey of the application of ARCH models for financial time series is given in [Bollerslev et al. 1992]. All those models have been used primarily to univariate time series of stock and bond returns. Multivariate models were used seldomly, mainly because a large number of parameters has to be estimated which could lead to estimation problems.
A new class of models which combines traditional asset pricing models, such as the CAPM and the APT and the new GARCH methodology in the multivariate case are Factor-GARCH models, first described by [Engle 1987]. They are applied to bond returns by [Engle et al. 1990] and to stock returns by [Ng et al. 1992] using GARCH(1,1)-M models with normally distributed disturbances for the factors.
This paper compares their GARCH(1,1)-M approach with the IGARCH(1,1)-M, the GJR-GARCH(1,1)-M, and the NGARCH(1,1)-M model using alternatively Student's t and Generalized Error distributions. In addition, these models are used to calculate multi-period out-of-sample forecasts of the conditional variances of the stocks and their corresponding weekly variance forecasts.
These models are compared with static factor models, a naive variance forecast based upon the historical average, random walk forecasts, an exponentially smoothed forecast, and forecasts based upon individual GARCH-M models for each stock.
The rest of the paper is organized as follows: The second part describes the theoretical basics of the models used. In the third part the data used for the analysis and the results from the estimations are presented. Section 4 describes the forecasting methodology and presents the empirical results for this application. In the closing paragraph a summary and an outlook for further research is given. Detailed tables showing characteristics of the data, the estimation, and forecast results as well as several graphs which visualize some properties of the time series and the different estimations and forecasts for one selected stock can be found in the appendix.

2 Theoretical basics

2.1 Dynamic factor models

Valuation models for assets are based upon the theory of economic behaviour in the situation of uncertainty. Valuation models for most kinds of assets rely in almost all cases solely on the **first two moments** of the return series, that is the means, variances and covariances. It is therefore necessary to model these moments in order to apply the asset pricing models. If one considers n assets and does not impose any restrictions on the model, one has to estimate $\frac{1}{2}(n^2+3n)$ parameters, i. e. n expected returns, n variances and $\frac{1}{2}(n^2-n)$ covariances. Therefore, one tries to introduce a restrictive structure such that the number of parameters to be estimated is significantly reduced without lowering the explanatory power of the model too much.
It is well known that the return series of different assets are correlated with each other, i. e. the assets follow **common influences** on their returns. This can be used to reduce the number of parameters to be estimated. Various forms of factor models such as the **Capital Asset Pricing Model (CAPM)** and the **Arbitrage Pricing Theory (APT)** are often used. The CAPM treats the correlation of individual assets with the market portfolio, i. e. the portfolio consisting of all stocks in the market with the weights according to the share of the assets in the whole market, as a measure for risk. The APT allows several factors to influence the return series of the assets. For a detailed discussion see [Ross 1976].
In general, factor models postulate that the return of an asset is composed as the sum of an **expected** and an **unexpected** part. The unexpected part of the return is assumed to consist of a **systematic** portion which cannot be diversified and an **unsystematic** portion which is specific to the single asset.
Economic theory states that there are common influences such as macroeconomic data which drive the returns of different assets. These are known as **factors**.
The systematic unexpected part of the return y_i ($i \in \{1,\ldots,n\}$) is assumed to follow a factor structure. The general model with K factors and n assets can be written in its **static** form as follows:

$$y_i = \underbrace{\overbrace{\mathrm{E}(y_i)}^{\text{expected part}}}_{\mu_i} + \overbrace{\underbrace{\sum_{k=1}^{K} \lambda_{ik} \cdot f_k}_{systematic} + \underbrace{\epsilon_i}_{unsystematic}}^{\text{unexpected part}} . \tag{1}$$

The f_k are named **factors** and the λ_{ik} which belong to them are known as **factor loadings**. In order to achieve reduction of the number of parameters the number of factors K should be much smaller than the number of assets n.
Different ways of identifying the factors are discussed in the literature. Some approaches use prespecified factors which base upon macroeconomic data such as the inflation rate. Other lines of research build factors which are linear combinations of the time series considered with prespecified weights derived from economic theory.
This paper presents a solution which is based upon **principal components analysis** following [Ng et al. 1992].

Because time series of returns are to be modelled and they are known to have a time-dependent behavior a dynamic factor model is adequate. Let there be n assets with returns at time $t = 1, \ldots, T$ given in the vector $\vec{y}_t$. Let $\mathcal{F}_{t-1}$ be the information set available at time t. Then define the expectation conditional on this set $\mathrm{E}(\cdot|\mathcal{F}_{t-1})$ as $\mathrm{E}_{t-1}(\cdot)$. The **conditional moments** are then denoted as follows[1]:

$$\begin{aligned} \mathrm{E}_{t-1}(\vec{y}_t) &=: \vec{\mu}_t \qquad (2)\\ \mathrm{Var}_{t-1}(\vec{y}_t) &=: \Omega_t. \qquad (3)\end{aligned}$$

The **dynamic factor model** is given by the following equation ($K < n$):

$$\vec{y}_t = \vec{\mu}_t + \sum_{k=1}^{K} \vec{\lambda}_k \cdot f_{kt} + \vec{\epsilon}_t. \qquad (4)$$

For all $t \in \{1, \ldots, T\}$ and for all $j, k \in \{1, \ldots, K\}, j \neq k$ it is assumed:

$$\begin{aligned} \mathrm{E}_{t-1}(f_{kt}) &= 0 \qquad (5)\\ \mathrm{Var}_{t-1}(f_{kt}) &= \theta_{kt}^2 \qquad (6)\\ \mathrm{Var}_{t-1}(\vec{\epsilon}_t) &= \Psi \qquad (7)\\ \mathrm{Cov}_{t-1}(f_{kt}, f_{jt}) &= 0 \qquad (8)\\ \mathrm{Cov}_{t-1}(f_{kt}, \epsilon_{it}) &= 0. \qquad (9)\end{aligned}$$

Hence, the factors have a time varying variance (6) and are uncorrelated with each other (8). The covariance matrix of the disturbance term is not time-varying.
The model of [ENGLE ET AL. 1990] also assumes conditional normality of the disturbances:

$$\vec{\epsilon}_t | \mathcal{F}_{t-1} \sim N(\vec{0}, \Psi). \qquad (10)$$

The **conditional covariance matrix** Ω_t of the returns $\vec{y}_t$ can be written as follows:

$$\Omega_t = \sum_{k=1}^{K} \vec{\lambda}_k \cdot \vec{\lambda}_k' \cdot \theta_{kt}^2 + \Psi. \qquad (11)$$

The conditional variances of the factors are the only time-varying parts of the covariance matrix.
The predicted values of the conditional variances of the factors can be used to **predict the covariance matrix** Ω_{t+s} through the following equation:

$$\mathrm{E}_t(\Omega_{t+s}) = \sum_{k=1}^{K} \vec{\lambda}_k \cdot \vec{\lambda}_k' \cdot \mathrm{E}_t(\theta_{k,t+s}^2) + \Psi. \qquad (12)$$

[1] The analysis uses conditional moments because they reflect the level of information an investor has at time of his decision. One assumes that the investors change their believes about the means and (co)variances in accordance with the new information they get in a period of time (see [BOLLERSLEV ET AL. 1988, PP 118F]). This reflects the assumption of efficient markets.

Since the hypothesized factors f_{kt} are **unobservable** they have to be replaced by proxies fp_{kt} which are by construction perfectly correlated with them. The factor value vectors $\vec{\alpha}_k$ obtained by Principal Components Analysis are used to build these portfolios $fp_{kt} := \vec{\alpha}_k' \vec{y}_t$ with $\mathrm{Var}_{t-1}(\vec{\alpha}_k' \vec{y}_t) = \theta_{kt}^2$. They are called **factor representing portfolio** (see [ENGLE ET AL. 1990, PP 216F]).
The risk premium $\vec{\mu}_t$ of the assets is priced through the following **valuation model** which resembles the APT formula:

$$\vec{\mu}_t = \sum_{k=1}^{K} \vec{\lambda}_k \cdot rp_{kt}. \tag{13}$$

$rp_{kt} := \vec{\alpha}_k' \vec{\mu}_t$ is the **risk premium** of the k-th factor portfolio which consists of a constant part cn_k and a time-varying part $\gamma_k \cdot \theta_{kt}^2$. This factor risk premium rp_{kt} is assumed to be a linear function of the conditional variance of the factor portfolio fp_{kt}:

$$rp_{kt} = cn_k + \gamma_k \cdot \theta_{kt}^2. \tag{14}$$

γ_k can be seen as a coefficient of **relative risk aversion** assuming constant preferences.

2.2 The GARCH family

The dynamics of each factor portfolio fp_{kt} is modelled as following a univariate xGARCH process. Two symmetric and two asymmetric xGARCH models were chosen from the wide range of available specifications (c. f. [HENTSCHEL 1995])[2].
The model described in the previous section together with this assumption of a xGARCH process for the factor dynamics is called a **k-Factor-GARCH** model. A main advantage of this class of Factor-GARCH models is that the covariance matrix Ω_t is guaranteed to be positive semidefinite without further assumptions (see [ENGLE ET AL. 1990, P 216]).
In the following, only one-factor models are considered and hence the index k is omitted. The different assumptions for the disturbance terms are presented in section 2.4.

GARCH-M

The GARCH(p,q)-in-Mean, shortly GARCH(p,q)-M-model, was introduced by [ENGLE ET AL. 1987]. The conditional variance is allowed to influence the conditional mean resulting in a time-varying risk premium (see equation 14). Setting the parameters p and q to one has been found to give a good adaption to most financial data. The GARCH(1,1)-M model for the factor portfolio has the following form:

$$\begin{aligned} fp_t &= cn + \gamma \cdot \theta_t^2 + u_t & (15)\\ \theta_t^2 &= \omega^2 + a \cdot u_{t-1}^2 + b \cdot \theta_{t-1}^2. & (16) \end{aligned}$$

[2]The Exponential GARCH (EGARCH) model of [NELSON 1991] was also used but severe estimation problems occured. This has also been noted by [FRACHOT 1995, P 230]. Especially the starting values for the numerical optimization were found to be very critical. For those estimations which converged to a stable solution prediction results were found to be worse than those of the xGARCH models described below. The predicted variance of a Factor-EGARCH model was in most cases much higher than the observed value. This could be explained by the exponential growth of the variance function in response to rising disturbances.

The sum of the GARCH-Parameters a and b is restricted to be less than 1 to guarantee a covariance stationary model (see [Bollerslev et al. 1992, pp 9f]). Furthermore, a, b and ω^2 need to be positive.

IGARCH-M

The sum of the estimated GARCH parameters a and b from the GARCH(1,1)-M model is often found to be close to 1, resulting in the phenomenon of **persistence in variance**, i. e. the occurence of shocks is relevant for the variance for the whole future[3]. The Integrated GARCH(p,q)-in-mean, shortly IGARCH(p,q)-M model, introduced by [Engle/Bollerslev 1986], takes this into account by setting the sum of the GARCH(1,1)-M-parameters a and b equal to one.
The IGARCH(1,1)-M model has the following form:

$$\begin{aligned} fp_t &= cn + \gamma \cdot \theta_t^2 + u_t \qquad (17) \\ \theta_t^2 &= \omega^2 + a \cdot u_{t-1}^2 + (1-a) \cdot \theta_{t-1}^2 . \qquad (18) \end{aligned}$$

The estimation is restricted in requiring that $0 < a < 1$ and $\omega^2 > 0$.

NGARCH-M

High frequency financial data has been found to exhibit the so-called **leverage effect** which means that negative innovations have a larger impact than positive innovations of the same size. Hence an asymmetric model may be called for. The Nonlinear Asymmetric GARCH(p,q) (NGARCH(p,q)) model was developed by [Engle/Ng 1993]. In contrast to the models described above it has an asymmetric news impact curve[4], i. e. the news impact curve is shifted to the right. The amount of shift is determined by the additional parameter β. Negative news has a larger impact on volatility than positive news if $\beta > 0$.
The NGARCH(1,1)-M model has the following form:

$$\begin{aligned} fp_t &= cn + \gamma \cdot \theta_t^2 + u_t \qquad (19) \\ \theta_t^2 &= \omega^2 + a \cdot \theta_{t-1}^2 \cdot \left(\frac{u_{t-1}}{\theta_{t-1}} - \beta \right)^2 + b \cdot \theta_{t-1}^2 . \qquad (20) \end{aligned}$$

a, b and ω^2 need to be positive, $a + b < 1$.

GJR-GARCH-M

The Glosten-Jagannathan-Runkle GARCH(p,q) (GJR-GARCH(p,q)) model (see [Glosten et al. 1993]) is another way of modeling asymmetry in the news impact curve.

[3] The same empirical phenomenon can also result from shifts in the unconditional variance (see [Lamoureux/Lastrapes 1990]). Since the time series used in our analysis is not very long this possibility is not taken into account here.

[4] The news impact curve introduced by [Engle/Ng 1993] plots the conditional variance for different values of the disturbance term which is interpreted as "news". The other parameters, especially the information from the previous periods, are held constant.

In this model the news impact curve is rotated. The form of the rotation is driven by the additional parameter c. The usual form, i. e. negative news have a larger impact on volatility than positive news, results if $c > 0$.
The GJR-GARCH(1,1)-M model has the following form[5]:

$$\begin{aligned} fp_t &= cn + \gamma \cdot \theta_t^2 + u_t & (22) \\ \theta_t^2 &= \omega^2 + a \cdot \theta_{t-1}^2 \cdot \left((1+c^2) \cdot \frac{u_{t-1}^2}{\theta_{t-1}^2} - 2c \cdot \left| \frac{u_{t-1}}{\theta_{t-1}} \right| \cdot \frac{u_{t-1}}{\theta_{t-1}} \right) + b \cdot \theta_{t-1}^2. & (23) \end{aligned}$$

a, b and ω^2 have to be positive.

2.3 Static Factor Model

In order to compare the performance of the Factor-xGARCH models presented above a static factor model without time-varying mean and variance is used. It has the following form:

$$\begin{aligned} fp_t &= cn + u_t & (24) \\ u_t | \mathcal{F}_{t-1} &\sim N(0; \omega^2). & (25) \end{aligned}$$

The constant variance ω^2 has to be positive.
For this model the Normal distribution was applied for the disturbance term.

2.4 Distributions of the disturbances

Most applications of xGARCH models use the **Gaussian (normal) distribution** assumption for the disturbances.
The log-likelihood function for a **conditionally Gaussian disturbance** u_t with variance θ_t^2 has the following form:

$$\ln L = \sum_{t=1}^{T} \left(-\frac{1}{2} \left(\ln(\theta_t^2) + \frac{u_t^2}{\theta_t^2} \right) + \ln \frac{1}{\sqrt{2\pi}} \right). \qquad (26)$$

The unconditional distribution of a xGARCH model with a disturbance which is conditionally normal is leptokurtic. Nevertheless, this leptokurtosis is not large enough to explain the leptokurtosis found in most financial data (c. f. [Bollerslev 1987, pp 544ff]). Therefore one should take this into account and use a conditionally leptokurtic distribution for the disturbance[6].
One alternative possibility is **Student's t distribution**. The degrees of freedom ν have to be estimated as an additional parameter. If ν approaches infinity the t distribution

[5] The original specification given in [Glosten et al. 1993, p 1787] is:

$$\theta_t^2 = \omega^2 + g_1 \cdot u_{t-1}^2 + g_2 \cdot u_{t-1}^2 \cdot I_{t-1} + b \cdot \theta_{t-1}^2 \qquad (21)$$

with $I_{t-1} = 1$ if $u_{t-1} > 0$ and 0 otherwise. This equals the given form if $g_1 = a \cdot (1+c)^2$ and $g_2 = 2a \cdot c^2$.

[6] Nevertheless, the Gaussian distribution is often used as a Quasi Maximum Likelihood (QML) method. The QML estimates are consistent and asymptotically normal if the conditional means and variances are correctly specified (see [Bollerslev/Wooldridge 1992]).

converges to a normal distribution. The lower limit for the degrees of freedom has been set to 3 in order to guarantee at least the existence of the variance.
The log-likelihood function for a **conditionally t-distributed error term** u_t with ν d. f. and variance θ_t^2 has the following form:

$$\ln L = \sum_{t=1}^{T} \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi}\sqrt{(\nu-2)\theta_t^2}} \cdot \left(1 + \frac{u_t^2}{\theta_t^2(\nu-2)}\right)^{-\frac{\nu+1}{2}}. \tag{27}$$

Another generalization of the normal distribution is the **Generalized Error Distribution (GED)**. It includes the normal distribution if the parameter ν has a value of 2. For degrees of freedom ν less than 2 a fat-tailed distribution results. The lower limit for ν is 0. If $\nu < 1$, the unconditional variance does not exist.
The log-likelihood function for a disturbance u_t following a **conditional GED** with ν d. f. and variance θ_t^2 has the following form:

$$\ln L = \sum_{t=1}^{T} \left(-\frac{1}{2} \left| \frac{u_t}{\theta_t \sqrt{2^{-\frac{2}{\nu}} \frac{\Gamma(\frac{1}{\nu})}{\Gamma(\frac{3}{\nu})}}} \right|^{\nu} - \frac{1}{2} \ln \theta_t^2 + \frac{\ln \nu}{\sqrt{2^{-\frac{2}{\nu}} \frac{\Gamma(\frac{1}{\nu})}{\Gamma(\frac{3}{\nu})}} \cdot 2^{1+\frac{1}{\nu}} \Gamma(\frac{1}{\nu})} \right). \tag{28}$$

2.5 Estimation methods

The first step in the empirical application of the Factor-xGARCH models was the extraction of the factors by means of Principal Components Analysis.
The Factor-xGARCH models were then estimated by a two-step approach. In the first stage, the parameters of the xGARCH(1,1)-M model for the factor dynamics were estimated using the **BHHH** algorithm.
The estimated conditional factor variances $\hat{\theta}_t^2$ and the estimated factor risk premia $\widehat{rp}_t = fp_t - \hat{u}_t$ were used as predetermined variables for estimating the second stage of the model, i. e. the **factor loading** and the mean value for the i-th stock through the following model:

$$\begin{aligned} y_{it} &= \mu_i + \lambda_i \cdot \widehat{rp}_{it} + \xi_{it} && (29) \\ \sigma_{it}^2 &= \lambda_i^2 \cdot \hat{\theta}_t^2. && (30) \end{aligned}$$

σ_{it}^2 is the conditional variance of ξ_{it} which is assumed to have the same type of distribution as u_t in the corresponding first stage.
An unrestricted **ML-procedure** using the **Gauss-Newton** algorithm was used in this step. The appropriate likelihood functions which have to be maximized can be found in section 2.4. As shown in [Lin 1992] this two-step estimation method is consistent and asymptotically efficient.

3 Data and estimation results

3.1 Description of the data

The results described in the following is based upon daily return series of 30 German stocks. These stocks were contained in the German stock index DAX at that time[7]. Their abbreviations used for trading and the full names are given in Table A1. The **continuously compounded rates of return were calculated by**[8]

$$y_{it} := \ln P_{it} - \ln P_{i,t-1} \tag{33}$$

with $P_{\cdot t}$ the closing prices of the stocks corrected for dividend payments and changes of the capital basis of the firms[9]. The data used covers the period from January 08, 1990 to May 31, 1994, resulting in 1082 return values.
Figure 1 shows the time series of the RWE stock returns[10]. The empirical phenomenon of **volatility clustering** can be observed, i. e. periods of large changes and periods of small changes of returns tend to cluster.
Table A2 shows the **excess kurtosis** and **skewness** of the return series. All time series are leptokurtic, i. e. the excess kurtosis is positive. The skewness is in most cases significantly different from 0, hence the assumption of a normal distribution seems not to be justified.
The **Kiefer-Salmon test** rejects the hypothesis of a normal distribution for all stocks at the 1% level.
It has the following test statistic for a time series $\vec{x}$:

$$\mathrm{KS}(\vec{x}) = \frac{T}{6} \cdot \mathrm{sk}(\vec{x})^2 + \frac{T}{24} \cdot \mathrm{ku}(\vec{x})^2 \tag{34}$$

with $\mathrm{sk}(\vec{x})$ being the skewness of $\vec{x}$ and $\mathrm{ku}(\vec{x})$ being the excess kurtosis of $\vec{x}$. The test statistic is asymptotically distributed as χ^2_2 under the null.
The **autocorrelations** of the return series and the squared return series are shown in Table A3. In most cases, autocorrelation is highly significant in the squared return series which is a sign for **ARCH effects**.
The **Ljung-Box statistic** for a $(T \times 1)$-vector $\vec{x}$ considering 40 lags has the following form (QLB40 results if $\vec{x}$ is chosen to be the squared series):

$$\mathrm{LB40}(\vec{x}) = T(T+2) \sum_{i=1}^{40} \frac{\mathrm{corr}(\vec{x}, \vec{x}_{-i})}{T-i} \tag{35}$$

[7]The composition of the DAX portfolio is changed sometimes according to major changes in the importance of the stocks for the market.

[8]This stems from the following relationship:

$$P_{it} = P_{i,t-1} \cdot e^{\ln P_{it} - \ln P_{i,t-1}}. \tag{31}$$

Hence, $\ln P_{it} - \ln P_{i,t-1}$ is a continuous rate of return. Furthermore, for small price changes, y_{it} approximately equals the relative return

$$y_{it} \approx \frac{P_{it} - P_{i,t-1}}{P_{i,t-1}} \tag{32}$$

[9]These time series were kindly provided by the Deutsche Finanzdatenbank (DFDB).

[10]This stock was chosen arbitrary for demonstrating the model.

with

$$\mathrm{corr}(\vec{x}, \vec{x}_{-i}) = \frac{\vec{x}'\vec{x}'_{-i} \cdot (T-1)}{\vec{x}'\vec{x} \cdot (T-i-1)} \tag{36}$$

The test statistic is distributed as χ^2_{40} under the null of no autocorrelation.
Figure 2 shows the **autocorrelation function** of the squared RWE return series. The high autocorrelation of the squared values even at high lags is clearly visible.
The ARCH effect can also be analysed by inspecting the values of an **ARCH-LM test** proposed by [ENGLE 1982]. For this reason, the following OLS-regression was carried out:

$$u_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + e_t \tag{37}$$

$$e_t \sim \text{i.i.d. } N(0; \sigma_e^2) \tag{38}$$

with $u_t = y_{it} - \mathrm{E}(y_i)$. The test statistic $T \cdot R^2$ with R^2 from the above regression is asymptotically distributed as χ^2_1 under the null of no ARCH. As can be seen in Table A3 the statistic is highly significant for most stocks and its results coincide with those of the QLB40 results in most cases.
As can be seen in Figure 3, the distribution of the returns of the RWE stock exhibits **fat tails**, i. e. the distribution has more weights in the tails and around the mean than the appropriate normal distribution which is also shown in this graph. Note that outliers are omitted in constructing the histograms.

3.2 Estimation results for the factor dynamics

Results from Principal Components Analysis

For the unconditional correlation matrix of the stock returns of the 30 German stocks a **Principal Components Analysis** was carried out using the program SPSS for Windows. One factor with a corresponding eigenvalue of 17.57 was found. The second largest factor had an eigenvalue of 0.96, the third largest of 0.86. Therefore, only one factor was used in the following. The factor explains in the static case 58.6 percent of the variance of the returns. Its factor score matrix is shown in Table A4. The weights for the individiual stocks are of similar magnitude.
This factor score matrix $\vec{\alpha}$ was used to build a factor portfolio $fp_t = \vec{\alpha}'\vec{y}_t$. Its statistical properties are shown in Table A5. The ARCH effect is significant and leptokurtosis is present. The time path of the factor portfolio is shown in Figure 4 and the ACF of the squared factor in Figure 5. In contrast to the stock RWE, the autocorrelations of the squared factor are not that high although the QLB40 statistic is highly significant. The histogram in Figure 6 shows severe deviations from the normal distribution.
This factor portfolio is supposed to follow one of the xGARCH models of section 2.2.
In the following, the estimation results for the different model specifications of the factor dynamics are given. Robust standard errors are given in parenthesis. Since the results differ not too much only the models applying the t distribution are reported. A comparison of the results is given below. Note that the factor was multiplied by 1000 in order to ease the estimation.

Static factor model (without GARCH)

$$
\begin{aligned}
fp_t &= \underset{(0.4545)}{0.1941} + u_t & (39)\\
\theta_t^2 &= \underset{(23.6468)}{222.9423} & (40)\\
u_t|\mathcal{F}_{t-1} &\sim N(0;\theta_t^2) & (41)
\end{aligned}
$$

GARCH(1,1)-M models, t distributed errors

$$
\begin{aligned}
fp_t &= \overset{(0.5996)}{0.2552} + \overset{(0.003)}{0.00174} \cdot\theta_t^2 + u_t & (42)\\
\theta_t^2 &= \overset{(3.0097)}{4.6147} + \overset{(0.0342)}{0.0823} \cdot u_{t-1}^2 + \overset{(0.0421)}{0.8966} \cdot\theta_{t-1}^2 & (43)\\
u_t|\mathcal{F}_{t-1} &\sim t_6(0;\theta_t^2) & (44)
\end{aligned}
$$

IGARCH(1,1)-M model, t distributed errors

$$
\begin{aligned}
fp_t &= \overset{(0.5553)}{0.2790} + \overset{(0.0023)}{0.0014} \cdot\theta_t^2 + u_t & (45)\\
\theta_t^2 &= \overset{(2.3243)}{2.8997} + \overset{(0.0428)}{0.0989} \cdot u_{t-1}^2 + (1-0.0989)\cdot\theta_{t-1}^2 & (46)\\
u_t|\mathcal{F}_{t-1} &\sim t_5(0;\theta_t^2) & (47)
\end{aligned}
$$

NGARCH(1,1)-M model, t distributed errors

$$
\begin{aligned}
fp_t &= \overset{(0.6050)}{0.4841} + \overset{(0.0032)}{-0.0006} \cdot\theta_t^2 + u_t & (48)\\
\theta_t^2 &= \overset{(1.5893)}{2.1846} + \overset{(0.0326)}{0.0685} \cdot \left(\frac{u_{t-1}}{\theta_{t-1}} - \overset{(0.2302)}{0.5387}\right)^2 + \overset{(0.0461)}{0.8901} \cdot\theta_{t-1}^2 & (49)\\
u_t|\mathcal{F}_{t-1} &\sim t_5(0;\theta_t^2) & (50)
\end{aligned}
$$

GJR-GARCH(1,1)-M model, t distributed errors

$$
\begin{aligned}
fp_t &= \overset{(0.5297)}{0.2483} + \overset{(0.0024)}{0.0012} \cdot\theta_t^2 + u_t & (51)\\
\theta_t^2 &= \overset{(1.8386)}{2.7960} + \overset{(0.0312)}{0.0786} \cdot\theta_{t-1}^2 \cdot \left((1+\overset{(0.1146)}{0.2438}{}^2) \cdot \frac{u_{t-1}^2}{\theta_{t-1}^2} - 2\cdot \overset{(0.1146)}{0.2438} \cdot \left|\frac{u_{t-1}}{\theta_{t-1}}\right| \cdot \frac{u_{t-1}}{\theta_{t-1}}\right) & (52)\\
&+ \overset{(0.0455)}{0.8887} \cdot\theta_{t-1}^2 &\\
u_t|\mathcal{F}_{t-1} &\sim t_5(0;\theta_t^2) & (53)
\end{aligned}
$$

Comparison of the estimation results

For all models the **standardized residuals**

$$\hat{u}_t^{st} := \frac{\hat{u}_t}{\hat{\theta}_t} \tag{54}$$

were computed and several statistical properties of them are given in Table A6 and A7. As can be seen by inspecting these tables, all xGARCH models for the factor dynamics can explain the ARCH-effects found in the factor, contrary to the Static factor model. The differences in the QLB40 and ARCH(1) test statistics for the different xGARCH specifications are quite small.

The fact that the standardized residuals of the Factor-xGARCH models show no further ARCH effects supports the idea that the factor explains these effects.

Table A8 shows that the NGARCH(1,1)-M-t model has the highest **log-likelihood** value and is thus the best model. But the differences to the other xGARCH models are not very large. Table A9 and A10 show the values of the **Akaike Information Criterion (AIC)** and the **Schwarz Information Criterion (SIC)**. The AIC also selects the NGARCH(1,1)-M-t model, followed by the GJR-GARCH(1,1)-M-t model while the SIC slightly favours the more parsimoniously parametrized IGARCH(1,1)-M-t model and the NGARCH(1,1)-M-t model ranking second place.

For several pairs of models a **likelihood ratio (LR)** test was carried out. If $\ln L$ is the log-likelihood of the general model and $\ln L_0$ the log-likelihood of a nested model, which is a restricted case of the general model, the LR test has the following statistic:

$$\mathrm{LR} = 2 \cdot (\ln L - \ln L_0). \tag{55}$$

This test statistic is asymptotically distributed as χ^2_m under the null where m is the number of restrictions.

Table 1: Likelihood Ratio tests for model restrictions

General model	Special model	Restrictions m	LR
GARCH(1,1)-M	Static factor model	3	103.27***
GARCH(1,1)-M-t	IGARCH(1,1)-M-t	1	3.20*
NGARCH(1,1)-M-t	NGARCH(1,1)-M	1	185.65***
NGARCH(1,1)-M-t	GARCH(1,1)-M-t	1	8.46***

* : parameter significant at the 10% level
** : parameter significant at the 5% level
*** : parameter significant at the 1% level

As can be seen, the differences between the likelihood values of the models compared are all significant which means that the factor displays dynamics in the variance, asymmetries are present and the t-distribution has a better fit than the normal distribution. Note the restriction of the IGARCH(1,1)-M-t model ($a + b = 1$) is only refused at the 10% level.

Because the estimation results from the different xGARCH models are quite close, only the results of the NGARCH(1,1)-M-t model are discussed. The standardized residuals $\hat{u}_t^{st}$ are plotted in Figure 7. Except for some rare peaks (the large peak in August 1991 stems from the Gorbachev crisis) the series seems to be without great regularities. Autocorrelations

in the squared standardized residuals are almost non-existent as can be seen in Figure 8. Figure 9 shows that the distribution of the standardized residuals of the NGARCH(1,1)-M-t model is closer to the appropriate normal distribution than that of the factor itself.
A method for comparing the different behaviour of the xGARCH-M models concerning the impact of news is to plot the **news impact curve** which graphs the influence of different values of news on the variance. Figure 10 shows this curve for the GARCH(1,1)-M-t, IGARCH(1,1)-M-t, NGARCH(1,1)-M-t and GJR-GARCH(1,1)-M-t model. As can be seen, the GARCH(1,1)-M and IGARCH(1,1)-M models have symmetric news impact curves while those of the NGARCH(1,1)-M-t and GJR-GARCH(1,1)-M-t models are asymmetric.

3.3 Estimation results for the factor loadings

The parameters of the second stage (factor loadings, mean values and, for the Student's t and GED-model, the degrees of freedom) are listed in Table A11 only for the NGARCH(1,1)-M-t factor model[11]. The mean values for the individual stocks differ quite distinctively in the different models although they are not significant in most cases, but the factor loadings of the models are very similar. The degrees of freedom for the xGARCH(1,1)-M-GED models lie between 0.90 and 1.62 and the degrees of freedom for the xGARCH(1,1)-M-t-models lie between 3.00 and 6.93, both results indicating significant deviation from normality.
Figure 11 shows the standardized residuals from the second stage of the Factor-NGARCH(1,1)-M-t model for the stock RWE.
Table A12 shows the statistical properties of these standardized residuals. Through looking at the QLB40 and ARCH(1) test statistics it can be seen that the GARCH-effects were captured for all but two and three stocks, respectively. The time path of the returns of the MET (Metallgesellschaft) cannot be explained by any of the nine models. This can be easily understood if one has the large loss in mind which MG Corp., the US branch of Metallgesellschaft, has sustained by speculative derivative deals resulting in major changes of its stock prices but not of the whole market, which is represented by the factor. Hence, the factor does not react sufficiently to this influence. The failure of the model for the SCH (Schering) stock is not so obvious. Somewhat surprising is the case of BAY (Bayer) and VIA (VIAG). Although their QLB40 statistics are not significant at any reasonable level, the ARCH(1) test statistic is highly significant, though both statistics are asymptotically equivalent (c. f. [Bollerslev et al. 1993, p 16]). The large values of excess kurtosis shows that the assumption of a non-Normal distribution for the disturbances is justified.
Figure 12 shows the autocorrelation function of the squared standardized residuals of the second stage of the Factor-NGARCH(1,1)-M-t model for the stock RWE. All autocorrelations are not significant. The distribution of the standardized residuals which is shown as a histogram in Figure 13 shows less deviation form the normal distribution than the distribution of the stock returns themselves.
As can be seen in Table A13, the likelihood values for the Factor-NGARCH(1,1)-M-t factor model are for all stocks but one (LHA) better than or equal to the appropriate values of the other models, although the differences are not very large.

[11] The results for the other models can be requested from the author.

4 Models for predictions

In many areas of finance accurate forecasts of stock price variances are needed. These forecasts are especially useful in application of stock valuation and option pricing models. Furthermore, it is interesting to study the behaviour of the Factor-xGARCH models not only in-sample but also out-of-sample. Rivaling forecasting models were used as benchmarks for the performance of the Factor-xGARCH models.

4.1 Theoretical basics

The models described above were used to predict the weekly variance[12] of the individual stocks. Due to the results reported in the empirical part of this paper only the t distribution was assumed for the disturbance term. Application of the Factor-xGARCH models for prediction purposes was done in several steps. First the parameters of the Factor-xGARCH models with dynamics given in section 2.2 and the static factor model 2.3 were estimated according to the methodology described in section 2.5. Then the one- through five-step-ahead out-of-sample forecasts of the conditional factor variance was computed as follows.

Forecasting of the conditional factor variance with factor models

The one- and multistep forecasts of the conditional factor variance $\hat{\theta}^2_{t+1} = \mathrm{E}_t(\theta^2_{t+1})$ can be computed as follows:

1. Static factor model
   One-step-ahead forecast:

$$\hat{\theta}^2_{t+1} = \hat{\omega}^2 \tag{56}$$

   s-step-ahead forecast ($s > 1$):

$$\hat{\theta}^2_{t+s} = \hat{\omega}^2 \tag{57}$$

2. GARCH(1,1)-M model
   One-step-ahead forecast:

$$\hat{\theta}^2_{t+1} = \hat{\omega}^2 + \hat{a} \cdot \hat{u}^2_t + \hat{b} \cdot \hat{\theta}^2_t \tag{58}$$

   s-step-ahead forecast ($s > 1$):

$$\hat{\theta}^2_{t+s} = \hat{\omega}^2 + (\hat{a} + \hat{b}) \cdot \hat{\theta}^2_{t+s-1} \tag{59}$$

3. IGARCH(1,1)-M model

   One-step-ahead forecast:

$$\hat{\theta}^2_{t+1} = \hat{\omega}^2 + \hat{a} \cdot \hat{u}^2_t + (1 - \hat{a}) \cdot \hat{\theta}^2_t \tag{60}$$

[12] 5 trading days are assumed to form a week.

s-step-ahead forecast ($s > 1$):

$$\hat{\theta}^2_{t+s} = \hat{\omega}^2 + \hat{\theta}^2_{t+s-1} \tag{61}$$

4. NGARCH(1,1)-M model
   One-step-ahead forecast:

$$\hat{\theta}^2_{t+1} = \hat{\omega}^2 + \hat{a} \cdot \hat{\theta}^2_t \cdot \left(\frac{\hat{u}_t}{\hat{\theta}_t} - \hat{\beta} \right)^2 + \hat{b} \cdot \hat{\theta}^2_t \tag{62}$$

   s-step-ahead forecast ($s > 1$):

$$\hat{\theta}^2_{t+s} = \hat{\omega}^2 + (\hat{a} \cdot (1 + \hat{\beta}^2) + \hat{b}) \cdot \hat{\theta}^2_{t+s-1} \tag{63}$$

5. GJR-GARCH(1,1)-M model
   One-step-ahead forecast:

$$\hat{\theta}^2_{t+1} = \hat{\omega}^2 + \hat{a} \cdot \hat{\theta}^2_t \cdot \left((1 + \hat{c}^2) \cdot \frac{\hat{u}^2_t}{\hat{\theta}^2_t} - 2\hat{c} \cdot \left| \frac{\hat{u}_t}{\hat{\theta}_t} \right| \cdot \frac{\hat{u}_t}{\hat{\theta}_t} \right) + \hat{b} \cdot \hat{\theta}^2_t \tag{64}$$

   s-step-ahead forecast ($s > 1$):

$$\hat{\theta}^2_{t+s} = \hat{\omega}^2 + (1 + \hat{c}^2 + \hat{b}) \cdot \hat{\theta}^2_{t+s-1} \tag{65}$$

Forecast of the conditional stock variances with a factor model

The third step consists in forecasting the individual conditional stock variances $\sigma^2_{i,t+s}$ using the following equation resulting from relation (30):

$$\hat{\sigma}^2_{i,t+s} = \hat{\lambda}^2_i \cdot \hat{\theta}^2_{t+s} \tag{66}$$

Forecast of the weekly stock variances

The last step in forecasting the weekly stock variances consists in summing up the individual daily conditional stock variances.
Furthermore, in order to compare the forecasts from the Factor-xGARCH models, a naive forecast based on the past observations, a random walk forecast and an exponentially smoothed forecast taking into account the last twelve observations of the weekly stock variances were also computed. Because univariate xGARCH models are used in stock variance prediction quite often, an individual GARCH(1,1)-M-t model was estimated for each stock and used to forecast the stock variances. The reason for choosing the simple GARCH(1,1)-M-t model instead of e. g. a GJR-GARCH(1,1)-M-t model lies in the computation time. The estimation of the individual GARCH(1,1)-M-t models and the forecast of conditional stock variances took already several days on a HP9000-715 workstation.

1. Observed stock variance

   The observed weekly variance of the stock i at time t is given as follows (y_{ij} is the daily return of stock i at day j):

$$\vartheta_{i,t}^2 = \sum_{j=t}^{t+4} (y_{ij} - \bar{y}_{i,t})^2 \tag{67}$$

   with

$$\bar{y}_{i,t} = \frac{1}{5} \cdot \sum_{j=t}^{t+4} y_{ij} \tag{68}$$

2. Naive forecast

   The naive forecast is simply the historical average using all available data:

$$\hat{\vartheta}_{i,t}^2 = 5 \cdot \frac{1}{t} \cdot \sum_{j=1}^{t} (y_{ij} - \bar{y}_{i,t})^2 \tag{69}$$

   with

$$\bar{y}_{i,t} = \frac{1}{t} \cdot \sum_{j=1}^{t} y_{ij} \tag{70}$$

3. Random walk forecast

   The random walk model for the stock return variance assumes that

$$\sigma_{i,t}^2 = \sigma_{i,t-1}^2 + \xi_{i,t}. \tag{71}$$

   Hence, the forecast for the weekly variance is simply 5 times the last observed daily variance:

$$\hat{\vartheta}_{i,t}^2 = 5 \cdot \sigma_{i,t-1}^2 \tag{72}$$

4. Exponential smoothing model

   The exponential smoothing model uses the last twelve observations of the stock variance as an exponentially weighted sum. The weighting exponent w is chosen as to minimize the mean squared error (MSE) in the observations preceding the forecasting period.

$$\hat{\vartheta}^2_{i,t} = 5 \cdot (1-w) \cdot \sum_{i=1}^{60} w^{i-1} \cdot \sigma^2_{i,t-i} \tag{73}$$

5. Individual GARCH-M forecast
   For each stock i an individual GARCH(1,1)-M-t model was estimated. The estimated parameters in equations (15) and (16) are used to predict the conditional variances $\theta^2_{i,t+s}$ through equation (58) and (59) and then the weekly stock variances $\vartheta^2_{i,t}$:

$$\hat{\vartheta}^2_{i,t} = \sum_{k=1}^{5} \hat{\theta}^2_{i,t+s} \tag{74}$$

6. Factor models
   The forecast of the weekly stock variance is simply the sum of the five predicted conditional stock variances.

$$\hat{\vartheta}^2_{i,t} = \sum_{k=1}^{5} \hat{\sigma}^2_{i,t+s} \tag{75}$$

4.2 Methods for comparing forecasting models

There are many methods which can be applied in order to compare concurring out-of-sample forecast models. One of the criterions often used is the **Mean Squared Error** or its square root. It takes into account the squared deviation of the forecasted variance from the observed variance.
The **Root Mean Squared Error (RMSE)** criterion using N predicted values has the following form:

$$\text{RMSE}(\hat{\vartheta}^2_{i\cdot}) = \sqrt{\sum_{t=1}^{N} \left(\hat{\vartheta}^2_{i,t} - \vartheta^2_{i,t}\right)^2} \tag{76}$$

Another way of measuring the performance of a forecast model is based upon the relative absolute deviation from the true value.
The **Mean Absolute Percent Error (MAPE)** was calculated according to the following formula:

$$\text{MAPE}(\hat{\vartheta}^2_{i\cdot}) = \sum_{t=1}^{N} \frac{|\hat{\vartheta}^2_{i,t} - \vartheta^2_{i,t}|}{\vartheta^2_{i,t}} \tag{77}$$

A measure robust against deviations from normality assumptions is the **Median Squared Error (MedSE)**. It is given by:

$$\text{MedSE}(\hat{\vartheta}^2_{i\cdot}) = \text{Median}\left(\hat{\vartheta}^2_{i,t} - \vartheta^2_{i,t}\right)^2 \tag{78}$$

These three measures were compared using a **performance index** which is inspired by the theory of decision making, i. e. the Savage-Niehans rule:

$$\text{Perf}_i = \sum_{j=1}^{n} \frac{\text{EC}_i - \min_i(\text{EC}_i)}{\min_i(\text{EC}_i)} \tag{79}$$

with EC being one of the error criterions described above and n being the number of stocks for which forecasts have been obtained. It can be interpreted as the relative loss of one specific model in forecasting accuracy compared to the model which turned out **ex post** to be the best model for stock j.
Finally, it is a convenient method to regress the observed variances on the predicted variances, i. e. to run the following OLS regression

$$\vartheta_{i,t} = v + w \cdot \hat{\vartheta}_{i,t} + \phi_{i,t} \tag{80}$$

and to test whether the constant v equals zero and the slope coefficient w equals one, resulting in an unbiased forecast.

4.3 Empirical forecasting results

The method of rolling estimation and forecasting was used in this study. The first 800 observations of the stock returns and the stock factor were used to estimate the parameters of the models, as described in section 2.5. Then 5 forecasts of the conditional variance of the factor were made, as shown in section 4. The forecasts of the weekly stock variance were obtained, along with the forecasts using the other prediction models. The observed variance of these 5 days was computed, too. Then the estimation time interval was shifted 5 days towards the future and the same procedure was repeated again. Through applying this method 40 times, 40 weekly stock variances were obtained for each stock. This procedure was carried out for all 9 different forecasting models.
The time path of the observed variance of the selected stock RWE along with the naive forecast and each of other 6 forecasting models can be found in Figures 14 through 19. The result of the Factor-IGARCH(1,1)-M-t model has been omitted because it is very similar to the Factor-GARCH(1,1)-M-t result. It can be seen that the Factor-GJR-GARCH(1,1)-M and Factor-NGARCH(1,1)-M models give smoother variance forecasts than the Factor-GARCH(1,1)-M and Individual GARCH(1,1)-M models. On the other hand, especially the huge variance in week 37 has almost no impact on the variance forecast of all Factor-xGARCH models. This is a clear sign that the large observed variance has occured only in the individual stock, but not in the whole market which is represented by the factor.
Since it is not very easy to judge the quality of the prediction models by visual inspection the three comparison criteria RMSE, MAPE and MedSE described above were computed for all 9 forecasting models. Their values are shown in Table A14, A15 and A16, respectively. The lowest and second lowest values of these criteria for the 30 stocks are distributed as follows:

Table 2: Performance of forecasting models
Number of stocks for which a model achieves the (second-)lowest RMSE/MAPE/MedSE

Model	NP	ES	NG	Mt	IMt	NMt	GMt	INt	RW
Lowest RMSE	0	20	0	0	1	6	2	0	1
Second lowest RMSE	1	5	0	2	0	8	8	6	0
Lowest MAPE	0	19	0	0	0	7	0	0	4
Second lowest MAPE	0	7	0	0	0	13	2	1	7
Lowest MedSE	0	21	0	1	0	7	1	0	0
Second lowest MedSE	0	7	0	0	1	14	4	1	3

NP = Naive forecast; ES = Exponential Smoothing
NG = Static factor; Mt = GARCH(1,1)-M-t
IMt = IGARCH(1,1)-M-t; NMt = NGARCH(1,1)-M-t
GMt = GJR-GARCH(1,1)-M-t; INt = Ind. GARCH(1,1)-M-t
RW = Random walk

As can be seen, the naive forecast and the Static factor model have the worst prediction quality. According to the RMSE, MAPE and MedSE criteria, the Exponential smoothing model performs best followed by the two asymmetric Factor-xGARCH models and the Random walk on the third place. The symmetric Factor-xGARCH models perform in many cases worse than the asymmetric ones.
Notably the individual GARCH(1,1)-M-t models for the 30 stocks perform worse than the Factor-xGARCH models. This is somewhat surprising because the individual GARCH(1,1)-M-t models should have more flexibility in modeling the dynamics of the stock returns. This shows that the correlation of the stock returns expressed by the common factor is a valuable information for forecasting. Hence, multivariate models outperform univariate ones.
The performance indices introduced in section 4.2 are shown in the following table.

Table 3: Performance index of forecasting model

Model	NP	ES	NG	Mt	IMt	NMt	GMt	INt	RW
RMSE	15.61	0.44	14.89	6.68	9.17	2.98	3.73	5.58	33.82
MAPE	62.78	2.67	60.40	28.12	31.62	10.15	18.77	27.73	17.40
MedSE	309.86	2.35	279.29	63.83	74.50	23.25	36.67	66.14	61.35

NP = Naive forecast; ES = Exponential Smoothing
NG = Static factor; Mt = GARCH(1,1)-M-t
IMt = IGARCH(1,1)-M-t; NMt = NGARCH(1,1)-M-t
GMt = GJR-GARCH(1,1)-M-t; INt = Ind. GARCH(1,1)-M-t
RW = Random walk

The first rank is occupied by the Exponential smoothing model for all criteria, the Factor-NGARCH(1,1)-M-t model comes always second. The Factor-GJR-GARCH(1,1)-M-t model ranks third place for the RMSE and MedSE criteria and the Random walk takes this place for the MAPE criterion. The naive forecast and the static factor model have about the same worse performance. The Factor-GARCH(1,1)-M-t, the Factor-IGARCH(1,1)-M-t model and the individual GARCH(1,1)-M-t model are similar in the results. When looking at the results for the Random walk model, it can be seen that the ranking depends crucially on the error criterion used. The Random walk is the worst model when applying the RMSE criterion but the third best when looking at the MAPE values.

Finally the regression of the observed variances on the forecasted variances was run. The results of the tests on the null hypothesis that the intercept is zero and the slope is one (acceptance of both nulls would indicate an unbiased forecasting model) are shown in the following table:

Table 4: Results from regression of variance forecasts
Number of stocks for which null hypothesis is not rejected at 5% level

Model	Intercept zero	Slope one
NP	7	7
ES	28	29
NG	9	8
Mt	24	16
IMt	20	7
NMt	17	17
GMt	25	23
INt	19	17
RW	1	1

NP	=	Naive forecast	ES	=	Exponential Smoothing
NG	=	Static Model	Mt	=	GARCH(1,1)-M-t
IMt	=	IGARCH(1,1)-M-t	NMt	=	NGARCH(1,1)-M-t
GMt	=	GJR-GARCH(1,1)-M-t	INt	=	Ind. GARCH(1,1)-M-t
RW	=	Random Walk			

It can be seen that none of the models gives unbiased forecasts for all 30 stocks but dynamic forecasting models perform better than static ones. The interpretation of the coefficient of determination R^2 of the regressions which can be found sometimes in the literature has not be done because it is dangerous to compare this measure if the forecasts are biased.

5 Conclusion

The series of returns from German stocks used in this paper are found to exhibit the usual characteristics of financial time series, i. e. leptokurtosis, volatility clustering and the leverage effect.
The data has been found to follow one significant factor determined by Principal Components Analysis. This factor has a time-varying variance which can be seen by the significant GARCH parameters in different xGARCH models. All xGARCH specifications have been found to capture the structure of the factor dynamics. The NGARCH(1,1)-M model with t distributed disturbances turned out to be among the best models.
Dynamic factor models with xGARCH dynamics, i. e. Factor-xGARCH models are able to capture the behaviour of the individual stock returns. Asymmetric Factor-xGARCH models have been found to perform better than symmetric ones. Hence, the ability of these models to capture asymmetric behaviour of the conditional variance and the assumption of a leptokurtic conditional distribution for the disturbances of the factor dynamic model are improvements over the Factor-GARCH(1,1)-M approach of [ENGLE ET AL. 1990] and [NG ET AL. 1992].
These Factor-xGARCH models can be used to forecast stock variances. The forecasts based upon this class of models are better than the naive and the static factor forecasts and show comparable performance to individual GARCH(1,1)-M forecasts. The Exponential smoothing forecast seems to be better in this case. The forecasts based on the Factor-GJR-GARCH(1,1)-M-t and the Factor-NGARCH(1,1)-M-t model showed the second lowest RMSE, MAPE and MedSE values in most cases. This might be caused by the ability of these models to capture asymmetries such as the leverage effect. A regression of the observed variances on their forecasts shows that no model gives unbiased predictions for all stocks. However, the Factor-xGARCH, individual GARCH(1,1)-M and the Exponential smoothing forecasts are better in this sense than the other models.
It can be concluded that Factor-xGARCH models give a good fit of the volatility of stock returns. They provide forecasts for the stock return volatilities which are much better than static volatility models and even outperform individual GARCH models while being easier to estimate than the latter. However, time series models without much economic theory behind, i. e. the Exponential smoothing model, seem to give more precise forecasts.
Further research will compare the relative performance of the models using different forecasting horizons. Furthermore, it will be investigated if it enhances the quality of the Factor-xGARCH models if more than one factor is used. Formal tests for the equivalence of forecast error statistics will be applied.

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A Tables

Table A1: List of stocks used

Shortcut	Name of company
ALV	Allianz AG Holding
BAS	BASF AG
BAY	Bayer AG
BHW	Bayerische Hypotheken- und Wechsel-Bank AG
BMW	Bayerische Motoren Werke AG
BVM	Bayerische Vereinsbank AG
CBK	Commerzbank AG
CON	Continental AG
DAI	Daimler-Benz AG
DBC	Deutsche Babcock AG
DBK	Deutsche Bank AG
DGS	Degussa AG
DRB	Dresdner Bank AG
HEN	Henkel KGaA
HFA	Hoechst AG
KAR	Karstadt AG
KFH	Kaufhof Holding AG
LHA	Deutsche Lufthansa AG
LIN	Linde AG
MAN	MAN AG
MET	Metallgesellschaft AG
MMW	Mannesmann AG
PRS	Preussag AG
RWE	RWE AG
SCH	Schering AG
SIE	Siemens AG
THY	Thyssen AG
VEB	VEBA AG
VIA	VIAG AG
VOW	Volkswagen AG

Table A2: Statistical properties of return series

Stock	Excess kurtosis[13]	Skewness[14]	KS test
ALV	4.87***	0.11	1059.7***
BAS	4.22***	0.13*	795.7***
BAY	4.14***	0.20***	771.0***
BHW	7.32***	−0.48***	2427.8***
BMW	5.97***	0.06	1589.6***
BVM	8.27***	−0.19***	3059.3***
CBK	9.44***	−0.70***	4066.6***
CON	2.57***	-0.10	295.5***
DAI	4.36***	-0.03	848.4***
DBC	5.64***	−0.16**	1425.1***
DBK	6.76***	−0.45***	2073.9***
DGS	2.99***	-0.05	399.4***
DRB	7.64***	−0.49***	2649.7***
HEN	10.90***	−1.01***	5482.6***
HFA	4.30***	0.04	822.8***
KAR	8.13***	−0.42***	2983.4***
KFH	4.99***	−0.44***	1143.4***
LHA	4.06***	-0.09	734.6***
LIN	8.52***	−0.45***	3275.5***
MAN	7.87***	−0.69***	2852.2***
MET	13.36***	−0.77***	8069.9***
MMW	10.52***	−0.73***	5038.1***
PRS	6.19***	−0.63***	1778.3***
RWE	6.78***	-0.04	2050.4***
SCH	3.36***	0.16**	508.1***
SIE	7.88***	0.07	2770.2***
THY	4.56***	-0.10	926.8***
VEB	7.61***	−0.33***	2601.7***
VIA	6.49***	-0.03	1877.2***
VOW	5.65***	-0.09	1423.4***

* : parameter significant at the 10% level
** : parameter significant at the 5% level
*** : parameter significant at the 1% level

KS test : Kiefer-Salmon test for deviation from normality

[13]Significance according to Kiefer-Salmon skewness test.
[14]Significance according to Kiefer-Salmon kurtosis test.

Table A3: Ljung-Box statistics and ARCH tests for stock returns

Stock	LB40	QLB40	ARCH(1)
ALV	40.01	141.12***	5.806**
BAS	43.76	386.42***	31.338***
BAY	50.46	320.60***	17.200***
BHW	34.25	88.66***	0.391
BMW	53.54*	108.61***	17.127***
BVM	44.44	53.74*	2.831*
CBK	51.38	42.68	0.462
CON	38.57	121.70***	23.471***
DAI	64.12***	421.87***	28.531***
DBC	46.25	60.54**	2.185
DBK	39.97	162.26***	4.026**
DGS	59.38**	95.91***	20.418***
DRB	62.45**	71.15***	2.071
HEN	38.82	72.46***	38.294***
HFA	45.38	181.81***	18.411***
KAR	36.13	24.88	5.459**
KFH	46.17	58.82**	7.707***
LHA	54.61*	101.07***	4.045**
LIN	59.17**	45.59	2.550
MAN	68.52***	70.69***	13.116***
MET	86.28***	228.72***	9.080***
MMW	40.63	46.36	4.445**
PRS	62.31***	64.36***	5.491**
RWE	57.67**	416.17***	32.560***
SCH	64.59***	257.08***	53.704***
SIE	37.65	173.44***	20.426***
THY	41.67	141.03***	5.891**
VEB	41.57	234.56***	11.078***
VIA	48.06	79.40***	26.330***
VOW	33.07	74.71***	1.256

* : parameter significant at the 10% level
** : parameter significant at the 5% level
*** : parameter significant at the 1% level

LB40 : Ljung-Box statistic of order 40 for levels
QLB40 : Ljung-Box statistic of order 40 for squared series
ARCH(1) : ARCH-LM test of order 1

Table A4: Factor score matrix

Stock	Factor score
ALV	0.04701
BAS	0.04564
BAY	0.04536
BHW	0.04489
BMW	0.04700
BVM	0.04542
CBK	0.04755
CON	0.02961
DAI	0.04971
DBC	0.03273
DBK	0.05178
DGS	0.03795
DRB	0.04859
HEN	0.03665
HFA	0.04418
KAR	0.04085
KFH	0.04183
LHA	0.03632
LIN	0.04278
MAN	0.04358
MET	0.03044
MMW	0.04479
PRS	0.04147
RWE	0.04747
SCH	0.03473
SIE	0.05127
THY	0.04545
VEB	0.04965
VIA	0.04308
VOW	0.04722

Table A5: Statistical properties of factor portfolio

Excess kurtosis	Skewness	LB40	QLB40	ARCH(1)
10.23***	−0.61***	34.582	74.794***	3.043*

* : parameter significant at the 10% level
** : parameter significant at the 5% level
*** : parameter significant at the 1% level

Table A6: QLB40 of the standardized residuals of the models (Static Model: 74.937)

Model/Distribution	Normal	t	GED
GARCH(1,1)-M	3.219	2.622	2.903
IGARCH(1,1)-M	6.967	2.257	2.395
NGARCH(1,1)-M	5.376	2.136	2.769
GJR-GARCH(1,1)-M	2.691	2.424	2.709

Table A7: ARCH(1)-LM-test for standardized residuals (Static Model: 3.043)

Model/Distribution	Normal	t	GED
GARCH(1,1)-M	0.120	0.033	0.067
IGARCH(1,1)-M	0.408	0.023	0.043
NGARCH(1,1)-M	0.016	0.026	0.015
GJR-GARCH(1,1)-M	0.136	0.052	0.084

Table A8: Mean Log-Likelihood of the models (Static Model: -4.12239)

Model/Distribution	Normal	t	GED
GARCH(1,1)-M	-4.07467	-3.98062	-4.00130
IGARCH(1,1)-M	-4.08875	-3.98210	-4.00426
NGARCH(1,1)-M	-4.06250	-3.97671	-3.99746
GJR-GARCH(1,1)-M	-4.06564	-3.97803	-3.99855

Table A9: AIC of the models[15] (Static Model: 8924.85)

Model/Distribution	Normal	t	GED
GARCH(1,1)-M	8827.59	8626.06	8670.81
IGARCH(1,1)-M	8856.06	8627.26	8675.22
NGARCH(1,1)-M	8803.26	8619.60	8664.50
GJR-GARCH(1,1)-M	8810.05	8622.46	8666.86

Table A10: SIC of the models[16] (Static Model: 8934.83)

Model/Distribution	Normal	t	GED
GARCH(1,1)-M	8852.52	8655.98	8700.73
IGARCH(1,1)-M	8876.00	8652.20	8700.15
NGARCH(1,1)-M	8833.17	8654.51	8699.41
GJR-GARCH(1,1)-M	8839.96	8657.36	8701.77

[15] $AIC = -2 \cdot lnL + 2m$

[16] $SIC = -2 \cdot lnL + ln(T) \cdot m$

Table A11: Results factor loading estimation (Factor-NGARCH(1,1)-M-t, t-distributed errors)

Stock	Mean μ	Loading λ	DF ν
ALV	-0.45457	1.05812***	5.19602***
BAS	-0.38282	0.94790***	5.42264***
BAY	-0.23026	0.93629***	5.61472***
BHW	-0.36202	1.03768***	3.58151***
BMW	0.01363	1.07084***	4.54312***
BVM	-0.11002	1.03730***	4.06087***
CBK	0.02260	1.01124***	5.06810***
CON	-0.28480	1.38245***	5.53417***
DAI	-0.32890	1.05766***	5.26043***
DBC	0.97738*	1.49445***	4.69141***
DBK	-0.19203	0.85087***	4.84287***
DGS	-0.62899	1.22012***	4.89392***
DRB	-0.30104	0.91895***	4.51877***
HEN	0.08056	0.84488***	4.89268***
HFA	-0.50330	1.07647***	4.33839***
KAR	-0.51097	1.03640***	5.62076***
KFH	-0.63113	1.19724***	5.21604***
LHA	-0.90326	1.80145***	4.32066***
LIN	-0.29167	0.87146***	4.66936***
MAN	-0.55913	1.16015***	5.14982***
MET	−1.22510**	1.79613***	3.00000***
MMW	-0.14050	1.19745***	5.37771***
PRS	-0.02579	1.15576***	4.67244***
RWE	-0.18621	0.87400***	4.72331***
SCH	-0.37474	1.02395***	3.54477***
SIE	-0.24284	0.80228***	6.82993***
THY	0.04851	1.17866***	5.94718***
VEB	0.28052	0.85659***	5.37859***
VIA	-0.23573	1.00565***	4.33846***
VOW	-0.08169	1.28524***	6.61800***

* : parameter significant at the 10% level
** : parameter significant at the 5% level
*** : parameter significant at the 1% level

Table A12: Statistical properties standardized residuals
factor loading estimation, Factor-NGARCH(1,1)-M-t, t-distributed errors

Stock	LB40	QLB40	Skewness[17]	Exc. kurtosis[18]	ARCH(1)
ALV	39.54215	4.39354	−0.86559***	13.50540***	0.134
BAS	40.55312	29.82837	0.10768	2.80955***	0.000
BAY	52.71681*	24.28001	−0.16896**	3.68037***	5.912***
BHW	40.09937	7.27202	−1.82246***	25.76410***	0.030
BMW	52.58479*	6.46013	−0.58515***	15.63910***	0.775
BVM	39.83552	4.27018	−1.75323***	23.60342***	0.037
CBK	38.09279	1.85565	−2.48237***	36.58589***	0.036
CON	31.75677	24.39184	−0.34046***	3.01537***	0.322
DAI	56.67138**	6.55948	−0.41079***	5.94012***	0.019
DBC	40.09937	3.60979	−1.39543***	19.89826***	0.016
DBK	29.74214	2.90164	−1.85385***	25.84894***	0.163
DGS	52.94045*	8.89750	−0.33477***	8.54075***	0.000
DRB	48.72749	2.15822	−1.88333***	28.22313***	0.059
HEN	28.38650	3.27622	−2.17299***	29.79454***	0.043
HFA	47.04126	17.39149	−0.22129***	6.76304***	1.888
KAR	35.87158	2.85271	−2.00459***	31.20259***	0.000
KFH	43.58506	8.14609	−1.37691***	18.68014***	0.000
LHA	54.19215*	16.33018	−0.86585***	7.74783***	0.284
LIN	36.06428	1.75896	−2.04702***	34.17286***	0.013
MAN	53.78918*	2.26598	−1.82287***	28.67962***	0.000
MET	87.11396***	205.41192***	−0.89386***	14.72232***	7.321***
MMW	35.09660	1.55784	−2.51988***	36.51544***	0.014
PRS	52.48710*	2.56944	−1.50198***	21.82999***	0.004
RWE	33.82457	4.66956	−1.10201***	12.03375***	0.120
SCH	81.36326***	96.79932***	−0.26504***	5.59170***	0.378
SIE	27.59364	2.77653	−1.51642***	22.77836***	0.103
THY	36.62039	5.41417	−0.91012***	10.69908***	0.119
VEB	32.70087	7.14285	−1.70868***	20.12457***	0.000
VIA	45.89977	17.05090	−0.40372**	12.39501***	9.563***
VOW	26.42871	4.03121	−1.18646***	14.62467***	0.127

* : parameter significant at the 10% level
** : parameter significant at the 5% level
*** : parameter significant at the 1% level

[17] Significance according to Kiefer-Salmon skewness test.
[18] Significance according to Kiefer-Salmon kurtosis test.

Table A13: Negative mean Log-likelihoods (*100) of the second stages

Stock	M	Mt	MG	IM	IMt	IMG	NM	NMt	NMG	GM	GMt	GMG	NG
ALV	408	402	403	409	402	403	408	402	403	409	402	403	413
BAS	395	391	392	397	391	392	395	391	392	395	391	391	402
BAY	395	390	391	397	391	391	395	390	391	395	390	391	401
BHW	403	393	394	400	393	394	402	393	393	404	393	394	405
BMW	410	401	402	412	402	403	410	401	402	410	401	402	414
BVM	405	396	398	404	397	398	404	396	397	405	396	397	408
CBK	407	397	399	407	398	399	405	397	398	406	397	399	408
CON	432	430	429	432	430	430	430	429	428	432	430	429	434
DAI	408	402	403	410	402	403	407	402	402	407	402	403	415
DBC	441	435	436	443	436	436	441	435	435	441	435	436	442
DBK	389	379	380	389	379	380	387	379	380	388	379	380	394
DGS	419	415	415	421	416	416	420	415	416	419	415	416	421
DRB	396	386	388	396	386	388	394	386	387	395	386	387	398
HEN	388	379	380	388	379	380	387	379	380	388	379	380	392
HFA	406	401	401	408	401	401	406	401	401	406	401	401	411
KAR	406	401	402	407	402	403	406	401	402	406	401	402	407
KFH	419	414	415	420	415	415	419	414	414	419	414	414	420
LHA	456	453	452	460	453	454	455	453	452	456	453	452	459
LIN	389	381	381	391	382	382	389	381	382	389	381	381	391
MAN	418	411	412	421	411	413	418	411	412	417	411	412	421
MET	456	440	441	461	440	441	461	440	441	456	440	441	456
MMW	424	416	416	427	415	417	423	415	416	423	415	416	427
PRS	417	409	411	419	410	411	418	409	411	417	409	411	420
RWE	391	382	383	392	381	383	391	382	383	391	382	383	400
SCH	396	390	390	401	390	391	401	391	392	397	391	391	398
SIE	382	376	378	383	376	378	382	376	378	382	376	378	389
THY	418	414	415	421	415	415	417	414	414	417	414	414	422
VEB	390	381	383	390	381	382	390	381	383	390	381	382	398
VIA	401	394	395	405	394	395	405	394	395	402	394	395	405
VOW	427	423	424	428	424	425	426	423	424	427	423	424	430

M	=	GARCH(1,1)-M	Mt	=	GARCH(1,1)-M-t	MG	=	GARCH(1,1)-M-GED
IM	=	IGARCH(1,1)-M	IMt	=	IGARCH(1,1)-M-t	IMG	=	IGARCH(1,1)-M-GED
NM	=	NGARCH(1,1)-M	NMt	=	NGARCH(1,1)-M-t	NMG	=	NGARCH(1,1)-M-GED
GM	=	GJR-GARCH(1,1)-M	GMt	=	GJR-GARCH(1,1)-M-t	GMG	=	GJR-GARCH(1,1)-M-GED
NG	=	Static Model						

Table A14: RMSE of variance forecast models

Stock	NP	ES	NG	Mt	IMt	NMt	GMt	INt	RW
ALV	795.02	476.93	761.97	553.93	590.30	505.56	507.61	564.89	1996.98
BAS	550.55	381.43	561.90	438.87	471.32	407.75	406.31	457.07	784.66
BAY	637.17	389.00	619.83	468.10	493.12	388.75	426.06	446.02	710.71
BHW	722.79	382.41	698.22	614.86	672.35	485.79	555.62	422.06	549.50
BMW	879.32	625.72	869.81	731.50	783.65	684.18	680.92	776.87	1644.18
BVM	689.05	364.17	676.85	531.66	581.93	393.70	449.57	383.48	813.91
CBK	702.39	399.00	677.44	508.10	550.18	391.21	432.93	406.50	831.68
CON	1263.00	843.66	1263.36	1096.12	1202.60	897.60	949.65	1252.65	1880.98
DAI	778.47	513.15	757.13	593.15	631.13	510.50	538.48	604.73	1136.48
DBC	1793.18	1706.18	1778.79	1650.68	1702.81	1820.43	1631.95	1815.62	1930.59
DBK	551.62	288.42	531.76	364.47	389.06	305.87	324.61	327.25	512.57
DGS	1045.96	1025.05	1061.36	1018.42	1049.47	1005.05	976.18	1182.26	3742.28
DRB	522.90	292.59	519.67	378.67	412.74	323.04	328.04	317.89	801.14
HEN	501.24	358.70	510.20	387.91	411.11	357.76	362.43	395.82	597.58
HFA	730.65	532.73	739.47	641.14	679.51	525.17	575.48	643.45	939.67
KAR	685.89	556.98	683.87	601.65	648.62	580.84	572.33	644.24	1296.42
KFH	879.35	735.03	864.52	786.44	842.50	722.98	726.34	882.39	2987.89
LHA	1905.73	1161.11	1995.01	1942.36	2227.23	1575.27	1660.78	1402.96	1470.87
LIN	528.19	292.60	512.22	405.09	444.74	327.42	351.36	500.54	753.71
MAN	865.37	622.77	853.56	728.18	782.71	688.01	673.77	799.90	1737.57
MET	8933.14	7898.74	8920.90	8844.17	8749.33	8929.08	8918.64	7739.20	6412.15
MMW	1144.23	780.19	1104.31	884.45	940.07	820.80	842.59	908.63	974.94
PRS	1006.67	867.56	977.76	914.32	950.10	898.26	889.46	965.48	1557.06
RWE	720.99	422.43	684.29	452.96	457.21	444.79	441.96	449.01	778.20
SCH	1464.67	1382.76	1477.55	1467.79	1444.09	1519.39	1503.75	1469.71	2117.95
SIE	506.14	260.62	479.60	317.72	342.07	290.42	291.88	326.92	739.01
THY	819.41	516.36	814.23	676.71	755.48	656.20	619.59	627.13	1056.95
VEB	658.03	339.39	623.41	423.39	442.48	355.49	382.65	375.68	450.82
VIA	1537.13	1575.24	1536.69	1504.55	1502.05	1611.73	1545.25	1610.81	1901.59
VOW	1045.41	639.89	1050.49	877.06	980.15	815.76	788.31	783.41	1204.35

NP	=	Naive forecast	ES	=	Exponential Smoothing
NG	=	Static Model	Mt	=	GARCH(1,1)-M-t
IMt	=	IGARCH(1,1)-M-t	NMt	=	NGARCH(1,1)-M-t
GMt	=	GJR-GARCH(1,1)-M-t	INt	=	Ind. GARCH(1,1)-M-t
RW	=	Random Walk			

Table A15: MAPE (%, rounded) of variance forecast models

Stock	NP	ES	NG	Mt	IMt	NMt	GMt	INt	RW
ALV	408	125	392	221	229	153	190	190	224
BAS	261	149	263	170	179	129	149	199	181
BAY	1181	393	1141	628	631	428	512	637	279
BHW	474	108	461	342	357	222	304	206	181
BMW	477	160	472	244	249	201	216	287	349
BVM	372	76	365	236	250	136	188	153	136
CBK	309	78	297	199	215	105	155	148	156
CON	546	280	541	453	489	340	375	501	266
DAI	363	165	355	232	243	150	190	237	219
DBC	361	164	349	251	269	173	213	330	332
DBK	425	91	410	230	239	160	196	182	153
DGS	217	127	222	168	183	117	142	214	174
DRB	335	108	332	200	209	129	165	174	287
HEN	330	97	331	185	195	152	165	214	203
HFA	271	112	269	192	205	107	153	217	155
KAR	224	96	222	161	177	125	139	198	171
KFH	569	272	552	398	422	280	328	443	381
LHA	375	149	393	421	477	294	335	258	131
LIN	618	209	601	441	475	267	373	500	228
MAN	320	117	316	190	201	155	164	262	134
MET	138	125	141	108	116	91	93	163	167
MMW	440	95	424	228	239	146	187	246	197
PRS	509	249	475	358	384	218	293	404	342
RWE	809	203	748	379	385	258	333	320	232
SCH	207	271	203	154	163	135	141	336	134
SIE	289	60	273	129	136	101	111	127	158
THY	207	77	203	133	146	105	115	128	104
VEB	531	96	500	264	274	167	214	190	124
VIA	365	178	351	211	219	156	177	253	157
VOW	273	114	274	205	225	148	171	185	154

NP	=	Naive forecast	ES	=	Exponential Smoothing
NG	=	Static Model	Mt	=	GARCH(1,1)-M-t
IMt	=	IGARCH(1,1)-M-t	NMt	=	NGARCH(1,1)-M-t
GMt	=	GJR-GARCH(1,1)-M-t	INt	=	Ind. GARCH(1,1)-M-t
RW	=	Random Walk			

Table A16: Median Squared Error (*1/1000) of variance forecasting models

Stock	NP	ES	NG	Mt	IMt	NMt	GMt	INt	RW
ALV	694.07	81.84	623.68	189.08	226.81	73.56	150.07	147.39	96.06
BAS	261.08	95.26	249.18	111.68	113.39	92.09	108.14	116.29	195.64
BAY	402.23	55.26	326.30	88.92	73.34	45.66	58.60	97.13	177.04
BHW	541.70	28.91	463.37	210.28	248.74	121.33	157.57	89.01	140.28
BMW	713.73	77.78	654.88	189.86	202.72	97.71	120.13	295.39	368.22
BVM	453.00	25.86	447.16	192.82	176.58	84.94	121.79	98.70	53.13
CBK	459.11	34.87	406.88	164.69	183.50	74.01	123.71	118.48	178.39
CON	1626.27	247.46	1559.17	627.87	702.75	294.51	361.72	1137.44	561.19
DAI	533.66	85.77	509.66	149.77	157.05	133.28	157.94	180.97	243.57
DBC	2258.61	390.88	2143.13	1202.04	1589.40	484.39	837.53	1766.24	700.65
DBK	327.60	8.09	307.74	70.93	85.18	42.42	44.63	45.68	64.45
DGS	757.92	228.04	784.17	380.56	413.00	213.59	269.06	513.47	308.81
DRB	224.83	31.86	235.39	57.31	51.98	54.70	54.64	68.25	125.33
HEN	228.89	30.59	229.98	67.90	83.37	31.30	38.83	82.56	102.21
HFA	426.13	59.57	407.39	216.95	216.46	50.91	95.53	226.10	132.01
KAR	467.70	78.59	453.47	123.14	167.20	71.07	80.33	303.28	333.39
KFH	772.84	165.18	786.89	329.59	370.09	275.83	339.94	626.29	692.39
LHA	3400.80	414.90	3689.87	2378.16	2917.24	1199.34	1462.62	1354.79	1076.80
LIN	303.19	25.13	288.48	61.81	69.78	56.55	57.33	176.11	51.68
MAN	811.80	137.99	812.40	173.65	216.02	62.10	125.16	281.21	189.18
MET	916.73	532.86	921.28	513.16	650.15	567.28	580.37	1123.83	1210.55
MMW	1216.83	106.15	1034.30	259.70	326.75	134.20	237.42	364.27	198.36
PRS	822.16	203.97	757.66	318.66	310.83	209.66	270.38	365.17	271.24
RWE	480.67	15.03	348.23	70.48	68.79	28.94	45.49	41.56	41.96
SCH	244.79	226.59	236.52	189.74	192.90	182.83	163.73	213.14	249.74
SIE	285.72	8.60	257.44	28.52	36.77	29.73	25.25	32.58	51.90
THY	631.69	100.33	597.59	137.55	133.12	124.01	160.34	231.70	245.33
VEB	410.50	14.01	354.03	85.53	87.21	42.70	39.27	40.17	36.68
VIA	408.45	82.43	343.04	165.38	177.21	89.42	135.13	192.57	111.50
VOW	1121.52	202.68	1099.28	333.80	402.18	273.63	230.29	498.67	446.36

NP = Naive forecast; ES = Exponential Smoothing
NG = Static Model; Mt = GARCH(1,1)-M-t
IMt = IGARCH(1,1)-M-t; NMt = NGARCH(1,1)-M-t
GMt = GJR-GARCH(1,1)-M-t; INt = Ind. GARCH(1,1)-M-t
RW = Random Walk