Centre de recherche sur l'emploi et les fluctuations économiques (CREFÉ)
Center for Research on Employment and Economic Fluctuations (CREFE)

Université du Québec à Montréal

Cahier de recherche/Working Paper No. 40

The Liquidity Effect: Testing Identification Conditions Under Time-Varying Conditional Volatility*

Michel Normandin
Département des sciences économiques and CREFE, Université du Québec à Montréal

Louis Phaneuf
Département des sciences économiques and CREFE, Université du Québec à Montréal

May 1996

Normandin: Phone number: (514) 987-3000 ext. 6816, e-mail: r32454@nobel.si.uqam.ca
Phaneuf: Phone number: (514) 987-3000 ext. 3015, e-mail: phaneuf.louis@uqam.ca

* We would like to thank René Garcia, Alain Guay, Alain Paquet and Gregor Smith for their useful suggestions. Normandin acknowledges financial support from SSHRC, while Normandin and Phaneuf acknowledge financial support from FCAR.

Abstract: In the recent SVAR literature, the liquidity effect has been studied by imposing a variety of identifying restrictions required under the assumption that the SVAR fundamental disturbances are homoscedastic. Using typical SVAR processes, we first show that this assumption is not supported by the data and that the SVAR residuals are not characterized by common conditional scedastic structures. Under time-varying conditional volatility of residuals, we are then able to formally test typical sets of restrictions that have been imposed in previous studies. Our results indicate that to obtain a well characterized liquidity effect, one must measure monetary policy shocks as innovations in the Federal funds rate.

Keywords: Conditional Heteroscedasticity, Structural Vector Autoregressive Processes, Recursive System, Simultaneous System, Short-Run Elasticities, Dynamic Responses.

JEL classification: C32, E52

1. Introduction

Does monetary policy work its way in the short run through a negative relationship between money and interest rates? Following Sims' (1986) pioneering work, many researchers have investigated this question through the estimation of structural vector autoregressive (SVAR) processes. Among the major findings are the following. If monetary policy shocks are measured by innovations to the monetary base or M1, the answer is no [Leeper and Gordon (1992), Christiano and Eichenbaum (1992), and Eichenbaum (1992)]. If instead they are measured by innovations in non borrowed reserves [Christiano and Eichenbaum (1992), and Eichenbaum (1992)], or by innovations in the portion of non borrowed reserves which is orthogonal to total reserves (hereafter, adjusted non borrowed reserves) [Strongin (1995)], the answer is yes. Finally, if as suggested by McCallum (1983), Bernanke (1990), and Bernanke and Blinder (1992), monetary policy is more closely associated with innovations in the Federal funds rate, the answer is also affirmative [Leeper and Gordon (1992), Sims (1992), and Gordon and Leeper (1994)].

Generally, the dynamic impacts of changes in monetary policy (the impulse responses) have been identified through Choleski factorizations. The way this procedure has been typically applied has two main implications. First, the short-run elasticities of output with respect to monetary variables are null. Second, the money supply is assumed to be either perfectly inelastic in the short run if monetary policy shocks are measured by innovations in a monetary aggregate, or perfectly elastic if they are measured by innovations in the Federal funds rate. Hence, these restrictions imply that the money market is not simultaneous, and also that there is no simultaneity between the money and the goods markets. Needless to say, the output, interest rate, and money responses required to gauge the liquidity effect can be misleading if these restrictions on short-run elasticities are invalid. Unfortunately, because the SVAR are exactly identified, these restrictions cannot be tested.

Gordon and Leeper (1994) follow a different approach. They decompose the SVAR into two subsystems. The first subsystem implies simultaneity on the money market: money demand is negatively sloped and money supply is positively sloped. The second subsystem however is recursive, so that the short-run elasticities of output with respect to monetary variables remain null. Hence, their approach is semi simultaneous and has the advantage that the extreme restrictions usually imposed on the elasticity of money supply are relaxed.

Our paper begins by questioning the validity of an assumption that has been imposed throughout the recent SVAR literature on the liquidity effect, namely, that the SVAR fundamental disturbances are conditionally homoscedastic. Instead, we first test the presence of time-varying conditional volatility in the residuals of a variety of SVAR processes intended to cover most of the recent approaches that have been followed to measure the liquidity effect.

The first two systems we work with are four-variable SVAR processes. Both include output, aggregate prices and the Federal funds rate, but differs from each other by the inclusion of M1 and total reserves, respectively. The third SVAR incorporates five variables: output, aggregate prices, total reserves, non borrowed reserves and the Federal funds rate. For all the SVAR specifications we find substantial evidence against conditional homoscedasticity in the SVAR fundamental disturbances. Moreover, we show that these disturbances are not characterized by common conditional scedastic structures. Based on these findings, and following Sentana (1992), and King, Sentana and Wadhwani (1994), we are then able to formally test the Choleski restrictions, and the restrictions imposed in the semi simultaneous approach of Gordon and Leeper.

Apart from allowing us to test these sets of restrictions, our procedure has one more important advantage: it restores the simultaneity within the money market and between the money and the goods markets. The fact that our approach is fully simultaneous is, we believe, a crucial step in the estimation of the liquidity effect as the ultimate concern in searching for it is to find whether the Fed, through implementation of its monetary policy, can alter output in the short run. Thus, by restoring simultaneity, we can relax the extreme assumptions about the short-run elasticities of money supply and output. Of course, this can have a significant impact on the liquidity effect.

Briefly, our main findings are summarized as follows. We test the restrictions associated with the Choleski decompositions under conditional homoscedasticity of the SVAR fundamental disturbances and find that they are overwhelmingly rejected by the data. These results are robust to alternative definitions of the monetary policy shocks. They raise serious doubts about earlier findings reported in the SVAR literature on the liquidity effect.

Relying on the evidence about time-varying conditional volatility, we relax the assumption of homoscedastic SVAR fundamental disturbances and test more directly the Choleski and Gordon-Leeper restrictions. With heteroscedastic SVAR fundamental disturbances, we show that the definition of monetary policy shocks is important. The Choleski restrictions continue to be strongly rejected by the data when monetary policy shocks are measured as innovations in adjusted non borrowed reserves. However, we are unable to reject the Choleski restrictions when monetary policy shocks are measured as innovations in either M1, the total reserves or the Federal funds rate. The Gordon and Leeper restrictions also cannot be rejected.

The final step of our empirical investigation is to look at the dynamic impacts of changes in monetary policy on output, the monetary aggregates and the Federal funds rate under time-varying conditional volatility. By imposing the Choleski or Gordon-Leeper restrictions and measuring unanticipated expansionary monetary policy shocks by positive innovations in either M1 or total reserves, we find that the Federal funds rate rises for a few months, and then declines, whereas output persistently declines, contrary to what one expects under the liquidity effect.

Our impulse response analysis also confirms that the well characterized liquidity effect found by others in response to a positive innovation in adjusted non borrowed reserves under the Choleski restrictions reflects mainly the imposition of invalid restrictions. Under the Gordon-Leeper restrictions which we were unable to reject, the decrease in the Federal funds rate is small and lasts only four or five months. With all the restrictions relaxed, the nominal interest rate response following a shock to non borrowed reserves is even weaker and statistically insignificant.

Finally, our findings most supportive of the liquidity effect under time-varying conditional volatility of SVAR residuals are those obtained with the assumption that an unanticipated expansionary monetary policy is associated with negative innovations in the Federal funds rate. Combining the Federal funds rate with either $M1$ or the non borrowed reserves, we find evidence of a strong liquidity effect: output, $M1$ and the non borrowed reserves all increase sharply and persistently following a negative shock to the Federal funds rate. For example, the estimated $M1$ response lasts about 72 months, whereas the output response is positive and persists for roughly 50 months. Interestingly, we also find that the output responses display a hump shape following a shock to the Federal funds rate. These findings hold both under the Choleski and the Gordon-Leeper restrictions. Overall, our study provides strong evidence that measuring monetary policy as shocks to the Federal funds rate is a necessary condition to generate the liquidity effect.

The next section of the paper discusses econometric issues related to the identification and estimation of the SVAR processes under conditional homoscedasticity and conditional heteroscedasticity of the residuals. In section 3, we discuss the SVAR specifications, analyze the conditional scedastic structure of the SVAR residuals, and estimate the short-run elasticities. In section 4, we test the Choleski and Gordon- Leeper restrictions. The impulse response functions analysis is the object of section 5. Sections 6 contains a summary and conclusions.

2. Econometric issues

2.1 The SVAR process and the liquidity effect

It is assumed that the economy is characterized by the following SVAR process:

$$AZ_t = B(L)Z_{t-1} + \xi_t, \tag{1}$$

where the $(N \times 1)$ vector Z_t is partioned as $Z_t = (Z_{1t}Z_{2t})^{'}$. Here, Z_{1t} represents the contemporaneous non-policy variables and Z_{2t} corresponds to the current monetary policy instrument. Similarly, the $(N \times 1)$ vector ξ_t is partioned as $\xi_t = (\xi_{1t}\xi_{2t})^{'}$, where ξ_{1t} contains the fundamental disturbances in the non-policy variables and ξ_{2t} is the monetary policy shock.

It is standard practice to assume fundamental disturbances that are unconditionally and conditionally orthogonal. Therefore, both the $(N \times N)$ unconditional covariance matrix $E(\xi_t\xi_t^{'}) = \Gamma$, and the conditional covariance matrix $E_{t-1}(\xi_t\xi_t^{'}) = \Gamma_t$ are diagonal. Moreover, the scale of the unobservable fundamental disturbances is determined (without loss of generality) by fixing the unconditional variances to unity, so that $\Gamma = I$ —where I is the identity matrix. When $\Gamma_t = I$, the fundamental disturbances are conditionally homoscedastic. This assumption has been uniformly imposed to evaluate the liquidity effect. In contrast, when $\Gamma_t \neq I$, the fundamental disturbances are conditionally heteroscedastic.

The $(N \times N)$ matrix A captures the contemporaneous interactions between the variables included in Z_t. The short-run elasticities can be recovered from these interactions. Zero-type restrictions may be incorporated in A. Finally, the $(N \times N)$ matrix polynomial $B(L)$ of order τ reflects the dynamic interactions between current and lagged variables. As usual, $B(L)$ is unrestricted.

In computing the impulse response functions, it does not matter whether or not the SVAR residuals are conditionally homoscedastic. The impulse response functions are given by the moving average representation of the SVAR process:[1] Note that (2) involves an infinite matrix polynomial. Of course, a truncated version of this matrix polynomial must be used when the system is non stationary.

$$Z_t = [I - A^{-1}B(L)]^{-1}A^{-1}\xi_t. \tag{2}$$

The responses of non-policy variables (Z_{1t}) to the monetary policy shock (ξ_{2t}) can be computed from these functions. The liquidity effect is examined by looking at the responses of monetary non-policy variables and output to a one unconditional standard deviation shock in the monetary instrument.

The M-rule measures an unanticipated expansionary monetary policy by a positive shock in a monetary aggregate. The liquidity effect then consists in a well characterized negative response of the nominal interest rate and a well defined positive response of output following the positive monetary shock. The R-rule is such that an unanticipated expansionary monetary policy corresponds to a negative innovation in the Federal funds rate. The liquidity effect is defined as well characterized positive responses of the monetary aggregate and output to a negative shock to the Federal funds rate.

2.2 Identification

Of course, the impulse responses given by (2) have an economic interpretation only if A and $B(L)$ are identified. To investigate the identification conditions, we have to note first that the unrestricted reduced form of the SVAR process (1) corresponds to the VAR process:

$$Z_t = C(L)Z_{t-1} + \nu_t, \tag{3}$$

where the $(N \times N)$ matrix polynomial $C(L)$ of order τ is unrestricted and the $(N \times 1)$ vector ν_t corresponds to the statistical innovations. Also, the $(N \times N)$ unconditional covariance matrix $E(\nu_t\nu_t^{'}) = \Sigma = A^{-1}A^{-1'}$ and the $(N \times N)$ conditional covariance matrix $E_{t-1}(\nu_t\nu_t^{'}) = \Sigma_t = A^{-1}\Gamma_t A^{-1'}$ can be non diagonal. Clearly, $\Sigma_t = \Sigma$ holds under conditional homoscedasticity. In contrast, we have $\Sigma_t \neq \Sigma$ under conditional heteroscedasticity.

[1] 1

As long as A is identified, $B(L) = AC(L)$ is exactly identified. However, under conditional heteroscedasticity, $A\nu_t = \xi_t$ is observationally equivalent (up to unconditional second moments) to $A^*\nu_t = \xi_t^*$ —where $A^* = Q^{-1'}A, \xi_t^* = Q\xi_t$,and Q is an $(N \times N)$ arbitrary orthogonal matrix—since the unconditional covariance matrix $\Sigma = (A^*)^{-1}(A^*)^{-1'} = A^{-1}A^{-1'}$ remains unchanged.

Hence, some restrictions are needed on A. One way to set these restrictions is to use Dunn's (1973) sufficient exact identification conditions. These conditions are obtained by imposing that $N \times (N-1)/2$ of the N^2 elements in A are null, so that the only admissible orthogonal matrices Q are I and its square roots. Then, A is locally identifiable up to column sign changes. This local identifiability can trivially be transformed into global identifiability by fixing arbitrarily the sign of one non-zero element in each column of A.

These identification conditions have been used systematically to evaluate the liquidity effect. For example, Sims (1986, 1992), Christiano and Eichenbaum (1992), and Strongin (1995) have applied Choleski factorizations — A is a lower triangular matrix with positive elements on the diagonal. The Choleski factorizations imply that the SVAR process (1) is a recursive system, that the short-run elasticities of output with respect to monetary variables are null, and that the money supply is either perfectly inelastic in the short run under the M-rule, or perfectly elastic under the R-rule. Clearly, from equation (2), imposing such arbitrary restrictions may alter the short-run responses of the non-policy variables to monetary policy shocks, and therefore any conclusions regarding the liquidity effect may be misleading if the restrictions are invalid. Moreover, these restrictions cannot be tested since the SVAR process (1) with Choleski factorizations is exactly identified.

Gordon and Leeper (1994) use a different set of zero-type restrictions. More precisely, A is a non triangular matrix such that the SVAR process (1) is composed of two subsystems. The first subsystem implies that the money market is simultaneous. The second subsystem, which includes output, is recursive.

In contrast, under conditional heteroscedasticity, Sentana (1992) has shown that a sufficient condition (up to column sign changes) to identify every element in A is that no common conditional scedastic structure characterizes two or more of the N fundamental disturbances.[1] Once again, this local identifiability can

[1] 2

trivially be transformed into a global one by fixing arbitrarily the sign of one non-zero element in each column of A. This is done by imposing that the diagonal elements of A are positive.

This implies, among other things, that there is at most one fundamental disturbance which is conditionally homoscedastic. Furthermore, Sentana's condition does not depend on a particular parameterization of conditional heteroscedasticity.

The intuition behind Sentana's result is as follows. It is certainly true that for the time period $t_0, A\nu_{t_0} = \xi_{t_0}$ is observationally equivalent (up to conditional second moments) to $A^*_{t_0}\nu_{t_0} = \xi^*_{t_0}$ —where $A^*_{t_0} = Q^{-1'}\Gamma_{t_0}^{-1/2}A$ and $\xi^*_{t_0} = Q\Gamma_{t_0}^{-1/2}\xi_{t_0}$ —since the conditional covariance matrix $\Sigma_{t_0} = (A^*_{t_0})^{-1}(A^*_{t_0})^{-1'} = A^{-1}\Gamma_{t_0}A^{-1'}$ remains unchanged. But, unlike the conditional homoscedastic case, this observational equivalence does not hold for all $t \neq t_0$ because A^*_t, which is a time-varying matrix (since $\Gamma_t \neq I$), is not equal to A, which is a time-invariant matrix for all time period. This implies that the N^2 elements incorporated in A are all uniquely defined.

By relaxing the assumption of conditional homoscedasticity, the SVAR process (1) becomes a completely simultaneous system. Therefore, the restrictions usually imposed on the short-run elasticities are unnecessary. Put differently, these restrictions become over identified, and as such, can be tested. Moreover, one can verify whether rejecting these restrictions (assuming that they are rejected) has an important impact on the liquidity effect. To do so, the unrestricted matrix A simply is substituted in equation (2) in order to derive the Federal funds rate, monetary aggregate and output responses.

2.3 Empirical procedure

The first step of our empirical procedure is to determine the lag structure τ of the VAR process (3). The VAR process is estimated by Ordinary Least Squares (OLS) for several lag lengths. Sims, Stock and Watson (1990) have shown that the OLS estimators are consistent (and possibly super-consistent) even for variables that are characterized by a unit root nonzero drift in their univariate representation and whether or not the variables are cointegrated. Then, for each lag length, a Heteroscedasticity-Robust Gauss Newton Regression (HRGNR) procedure is used to test against the kth order serial correlation of the statistical innovations. This procedure allows for the possibility of conditional heteroscedasticity of unknown form [Davidson and Mackinnon (1993)]. This is important given that $\Sigma_t \neq \Sigma$ if $\Gamma_t \neq I$, that is, the fundamental disturbances are conditionally heteroscedastic. Finally, the relevant lag structure τ is the one for which no detectable serial correlation is found for all k's.

Given this lag structure, the statistical innovations ν_t are generated. Then, they are used to verify if the fundamental disturbances have common conditional variances. This can be checked by testing whether all the possible linear combinations involving n (where $n = 2, \ldots, N$) statistical innovations are uncorrelated in the squares with the instruments included in the information set. In such a case, there is no common conditional scedastic structure, that is each structural innovation is characterized by a specific conditional variance [Engle and Susmel (1993)].

In the absence of common conditional variances, Sentana's identification conditions hold, and it is possible to estimate the N^2 elements contained in A. We use the generated statistical innovations and assume that they are conditionally Gaussian in order to construct the sample log likelihood (ignoring the constant term):

$$L = -(1/2) \sum_{t=\tau+1}^{T} \log \mid (A^{-1}\Gamma_t A^{-1'}) \mid -(1/2) \sum_{t=\tau+1}^{T} \nu_t^{'}(A^{-1}\Gamma_t A^{-1'})^{-1}\nu_t, \tag{4}$$

where $A^{-1}\Gamma_t A^{-1'} = \Sigma_t$. Moreover, as explained above, there is a non trivial advantage in estimating the matrix A to explicitly recognize the existence of conditional heteroscedasticity. For this reason, we use the following specification:

$$\Gamma_t = (I - \Delta_1 - \Delta_2) + \Delta_1 \bullet \xi_{t-1}\xi_{t-1}^{'} + \Delta_2 \bullet \Gamma_{t-1}, \tag{5}$$

where $\bullet$ denotes the element-by-element matrix multiplication, the constrained intercept terms ensure that the unconditional covariance matrix is $\Gamma = I$(section 2.1), $\Delta_j = diag(\delta_{j1}, \ldots, \delta_{jN})$(where$j = 1, 2$), and $\Gamma_t = diag(\gamma_{11,t}, \ldots, \gamma_{NN,t})$. Therefore, equation (5) implies that the ith fundamental disturbance is conditionally homoscedastic if $\delta_{ji} = 0$(where$j = 1, 2$). Moreover, equation (5) states that the conditional heteroscedasticity of the ith fundamental disturbance is characterized by (i) a univariate ARCH(1) process if $0 < \delta_{1i} < 1$ and $\delta_{2i} = 0$, and (ii) a univariate GARCH(1,1) process if $\delta_{ji} > 0$ and $(\delta_{1i} + \delta_{2i}) < 1$(where $j = 1, 2$).[1] These restrictions ensure that the conditional variances are positive in all periods.

[1] 3

As is well known, ARCH(1) and GARCH(1,1) processes have been useful to describe the time-varying conditional volatility of many macroeconomic and financial time series.

Following King, Sentana, and Wadhwani (1994), we maximize equation (4) subject to (5) over the parameters involved in the matrices A and Δ_j(where $j = 1, 2$). To this end, we use the simplex and the BHHH algorithms. Interestingly, from the estimates involved in the matrix A, we can test the validity of the restrictions which have been imposed on the short-run elasticities, such as the zero elasticities of output with respect to the monetary variables, and the perfect elasticity or inelasticity of money supply.

Finally, if these restrictions are rejected, it is possible to evaluate the economic consequences on the liquidity effect. We simply have to compare the responses of output, the Federal funds rate and the monetary aggregate induced by the unrestricted matrix A under the M-rule and the R-rule and those obtained under various sets of zero- type restrictions. For completeness, Monte Carlo experiments (with 10,000 draws) which amend Sims and Zha's (1995) bayesian procedure in order to take into account the scedastic structure (5) are performed. Then, the appropriate quantile is chosen to construct the (possibly asymmetric) 95% probability intervals (see the appendix).

3. SVAR specifications and conditional time-varying volatility

3.1 SVAR specifications

For the purpose of estimation, we work with three basic SVAR processes. They are estimated using U.S. monthly data over the sample period 1959:1 to 1994:9.[1] We have also experimented over the subsample period 1979:11 to 1994:9. The results are the same as those found for the full sample, except that there is less precision in the estimates over the subsample.

The first two SVAR include four variables: the log of industrial production, Y_t, the log of the consumer price index, P_t, the Federal funds rate, R_t, and the log of a monetary aggregate, which we take to be either $M1_t$ or total reserves, TR_t. The four-variable SVAR that includes M1 have been used by Sims (1992), Eichenbaum (1992) and Christiano and Eichenbaum (1992), among others. They have shown that replacing M1 by the monetary base yields very similar results, so for the sake of brevity we experiment only

[1] 4

with M1.[2] See also Leeper and Gordon (1992).

The inclusion of total reserves instead of M1 follows from the work of Gordon and Leeper (1994), and the suggestion in Pagan and Robertson (1995). The five-variable SVAR includes Y_t, P_t, TR_t, R_t, and the log of non borrowed reserves, NBR_t. Christiano and Eichenbaum (1992) argue that the non borrowed reserves are more directly under the control of the Federal Reserve Open Market Committee (FOMC) than broader monetary aggregates like M1 or the monetary base. In the same vein, Strongin (1995) proposes a measure of monetary policy based on the non borrowed reserves net of the accommodation of the demand for reserves.

3.2 Time-varying conditional volatility of SVAR residuals

For the three basic SVAR processes just described, the lag structure is determined by applying the HRGNR procedure for $\tau = 1, \ldots, 24$ and $k = 1, 3, 6, 12$(see section 2.3). This procedure reveals that the relevant lag length is $\tau = 18$ for the three SVAR processes. Next, we investigate the conditional scedastic structure of the statistical disturbances by applying artificial regressions, in which each squared statistical residual (generated for the relevant lag structure) is regressed on its own lagged values—for up to 12 lags.

We begin by examining the persistence of the conditional volatility of the statistical innovations in the variables that belong to the three SVAR processes. Table 1 suggests that each statistical innovation seems to be characterized by a specific conditional scedastic structure. More precisely, the degree of persistence of the conditional volatility associated with the statistical innovations in R_t substantially exceeds that related to the statistical residuals of P_t, which in turn seems to be larger than that associated with the statistical disturbances in Y_t —regardless of the specification of the SVAR process used. Moreover, the degree of persistence of the conditional volatility of the monetary aggregate depends on the definition used. For example, the ranking (in decreasing order) of the degree of persistence corresponds to $NBR_t, M1_t$, and TR_t. In fact, TR_t is the only variable for which the statistical innovations seem to be conditionally homoscedastic.

Taken together, these results suggest that there is potentially no common conditional volatility characterizing the structural innovations. This is formally tested by verifying whether some linear combinations

[2] 5

of n (where $n = 2, \ldots N$) statistical innovations are correlated in the squares with q lags of the n statistical disturbances and their cross-products, where $q = 1, 2, 4$(section 2.3). Empirically, the p-value of not rejecting the orthogonality hypothesis is always smaller than 1% — for the three SVAR specifications as well as for all the values of n and q. This means that there is no common conditional volatility which characterizes the structural innovations.

Consequently, invoking Sentana's identification condition, we can jointly estimate all the elements contained in A (and thus, impose no restriction on the short-run elasticities) and the GARCH parameters captured in $\Delta_j (j = 1, 2)$. The estimated GARCH processes are presented in Table 2a. The first column shows the results for $Z_t = [Y_t, P_t, M1_t, R_t]^{'}$. We detect conditional heteroscedasticity in the four structural innovations included in the SVAR. In the second column, we experiment with $Z_t = [Y_t, P_t, TR_t, R_t]^{'}$ and find that three of the four innovations in the SVAR are conditionally heteroscedastic. In the third column, we analyze $Z_t = [Y_t, P_t, TR_t, NBR_t, R_t]^{'}$ and find that four of the five innovations are conditionally heteroscedastic. Finally, Table 2b suggests that the univariate GARCH(1,1) processes involved in (5) represent adequate specifications for the conditional volatility of all the structural innovations included in each of the three SVAR specifications.

3.3. Implied short-run elasticities under conditional time-varying volatility

We now examine the short-run elasticities implied by the three aforementioned SVAR processes under time-varying conditional volatility of residuals. We base the computation of these elasticities on the unrestricted SVAR processes. The results are reported in Table 3.

Consider first the elasticities implied by the four-variable SVAR process comprising $Y, P, M1$ and R in the first column of Table 3. We find that the short-run elasticity of $M1$ with respect to R is negative and statistically significant at the 5% level, whereas the short-run elasticity of R with respect to $M1$ is positive and significant at the 10% level. Therefore, these elasticities imply that money demand is negatively sloped and money supply is positively sloped as long as monetary policy is measured by innovations in R. Then, as assumed by Gordon and Leeper (1994), the money market is simultaneous. Moreover, the short-run elasticities of output with respect to $M1$ and R are not statistically different from zero. This also is consistent with the restriction imposed by Gordon and Leeper that the goods market is not simultaneously related to the money market. Finally, we find that monetary policy does not significantly respond contemporaneously to output, and that money demand does not statistically depend on current income. In contrast, the Choleski restrictions imply a perfectly elastic short-run money supply under the R-rule ($\epsilon_{r,m} = 0$) and a perfectly inelastic short-run money supply under the M-rule ($\epsilon_{m,r} = 0$), both of which are not supported by the data. However, this is the only Choleski restriction that is found to be invalid using $M1$. If TR replaces $M1$ in the four-variable SVAR, we find that no elasticity is statistically significant. Therefore, it is difficult to infer any conclusion from these elasticities for the four-variable SVAR process involving TR.

With the five-variable SVAR that includes Y, P, TR, NBR and R, we find that the short-run elasticity of NBR to R is negative and statistically significant at the 1% level. However, the elasticity of R with respect to NBR is not statistically different from zero. These computed elasticities suggest that the money market is not simultaneous, money demand being negatively sloped and short-run money supply being perfectly elastic if monetary policy is measured by shocks to R. Furthermore, our results indicate that the goods market is not contemporaneously related to the money market, that money demand is not affected contemporaneously by income, and that monetary policy does not contemporaneously respond to output. Overall, these findings suggest that under R-rules the Choleski restrictions are a good description using the five- variable SVAR process.

4. Testing identification restrictions

The absence of common conditional scedastic structures in the structural innovations of each SVAR process allows us to perform a variety of log likelihood-ratio tests to verify the validity of the Choleski and Gordon and Leeper restrictions. These restrictions which we have discussed earlier are summarized in Table 4.

4.1 Choleski restrictions and homoscedastic SVAR residuals

We begin by the joint test of the Choleski restrictions. For the sake of comparison with the recent literature on the liquidity effect, we initially impose that the SVAR fundamental disturbances are conditionally homoscedastic.[1] Under conditional homoscedasticity, Gordon and Leeper restrictions cannot be tested

[1] 6

because there are less than $N(N-1)/2$ zero-restrictions, so that the system is not identified (see Table 4).

This approach has been followed by Sims (1986,1992), Leeper and Gordon (1992), Christiano and Eichenbaum (1992), and Strongin (1995), among others. The test results are reported in Table 5.

M-rules

The first set of Choleski restrictions is based on the SVAR process $Z_t = [Y_t, P_t, M1_t, R_t]'$. This particular Wold ordering means that both $M1_t$ and R_t do not affect output contemporaneously. Also, in setting the growth rate of $M1$, the FOMC responds to the current period values of real output and the price level, but not to the current period Federal funds rate. It follows that the money supply is constrained to be perfectly inelastic. The second process is one where TR_t replaces $M1_t$. Hence, we also test the set of Choleski restrictions implied by the following ordering of the SVAR process $Z_t = [Y_t, P_t, TR_t, R_t]'$, in combination with the restriction that the SVAR fundamental disturbances are conditionally homoscedastic.[1] Christiano and Eichenbaum (1992) have shown that the liquidity effect is not very sensitive to alternative orderings.

The third set of Choleski restrictions is based upon the following ordering of the five-variable SVAR process $Z_t = [Y_t, P_t, TR_t, NBR_t, R_t]'$. Strongin (1995) argues that the major difficulty in measuring monetary policy shocks by innovations in any monetary aggregate, including non borrowed reserves, is that a significant proportion of the variance in the aggregate reflects mainly the Federal Reserve's accommodation of innovations in the demand for reserves rather than policy-induced supply innovations. He argues that a better measure of money supply shocks can be obtained by having total reserves immediately precede non borrowed reserves in the standard Choleski decompositions. The monetary policy shock is then measured as the innovation in non borrowed reserves. The log likelihood-ratio tests of the Choleski restrictions under conditional homoscedasticity of the SVAR residuals are reported in the first line of Table 5. Each of the three sets of restrictions is rejected by the data at the 1% level.

R-rules

We also test the Choleski restrictions jointly with the conditional homoscedasticity of the SVAR re-

[1] 7

siduals for the class of R-rules. Then, the orderings of the three SVAR processes are $Z_t = [Y_t, P_t, R_t, M1_t]'$, $Z_t = [Y_t, P_t, R_t, TR_t]'$, and $Z_t = [Y_t, P_t, R_t, TR_t, NBR_t]'$, respectively. These orderings imply that R_t responds to current output and prices, but not to current monetary aggregates. Therefore, the money supply is assumed to be perfectly elastic. They also imply that the monetary aggregates and the Federal funds rate have no contemporaneous impact on output. The results of the log likelihood-ratio tests can be found in the third line of Table 5. Again, the three sets of restrictions are overwhelmingly rejected by the data.[1] Even though the orderings of the variables for the M-rule and the R-rule alter the matrix A under Choleski factorizations (see Table 4), it does not affect the unconditional matrix $\Sigma = A^{-1}A^{-1'}$. Therefore, under conditional homoscedasticity, the log-likelihood functions (4) (and thus the log-likelihood ratio tests) obtained under the M-rule and the R-rule are the same.

Taken together, these test results clearly indicate that the Choleski restrictions and the assumption of conditional homoscedasticity of the SVAR residuals which have been systematically imposed to study the liquidity effect are jointly invalid. Since we have found strong evidence of conditional heteroscedasticity in the SVAR residuals, these rejections may primarily reflect invalid restrictions about the scedastic structure of the SVAR residuals and not the rejection of the Choleski restrictions per se. The next step is therefore to test the various sets of restrictions with heteroscedastic SVAR residuals.

4.2 Testing restrictions with heteroscedastic SVAR residuals

By relaxing the assumption of homoscedastic SVAR residuals, we are able to test more directly the Choleski restrictions which have been imposed to identify the impacts of monetary policy shocks. The orderings for the class of M-rules and the class of R- rules are the same as before. We also test the semi simultaneous approach restrictions of Gordon and Leeper (1994).

Choleski restrictions under M-rules and R-rules

The log likelihood-ratio tests of the Choleski restrictions for the M-rules appear in the second line of Table 5. Here, while we are unable to reject (at all conventional levels) these restrictions for the SVAR processes involving M1 or total reserves, we reject at the 5% level the Choleski restrictions associated with the SVAR process that includes adjusted non borrowed reserves. In the fourth line of Table 5, one can

[1] 8

find the test results of Choleski restrictions for the class of R-rules.[1] The orderings of the variables under Choleski factorizations for the M-rule and the R- rule alter the matrix A as well as the conditional covariance matrix $\Sigma_t = A^{-1}\Gamma_t A^{-1'}$. Consequently, the log-likelihood functions (4) (and thus the log-likelihood ratio tests) are different for the M-rule and the R-rule.

These restrictions are never rejected.

Gordon and Leeper semi simultaneous approach restrictions

We also test the restrictions imposed in the semi simultaneous approach of Gordon and Leeper (1994). As mentioned before, the novel aspect of their approach is that the simultaneity of the money market is restored. Therefore, one does not have to impose extreme assumptions about short-run money supply. However, they impose that output and aggregate prices do not respond to current innovations in monetary aggregates and in the Federal funds rate, but only to past innovations in these variables. This amounts to break the simultaneity between the goods market and the money market. These tests are reported in the fifth line of Table 5. The Gordon and Leeper restrictions cannot be rejected under the M-rule and the R-rule.[1] The log-likelihood functions (4) and the log-likelihood ratio tests are similar under the Gordon-Leeper restrictions for the M-rule and the R-rule with conditional heteroscedasticity because the restrictions included in A are the same (see Table 4), so that the conditional covariance matrices $\Sigma_t = A^{-1}\Gamma_t A^{-1'}$ are the same under the two rules.

To summarize the evidence reported in this section, we have shown that the Choleski restrictions are rejected if monetary policy shocks are measured by innovations in adjusted non borrowed reserves, and that this is true whether SVAR residuals are homoscedastic or heteroscedastic. Measuring monetary policy shocks either by $M1$, total reserves or the Federal funds rate, the Choleski restrictions were rejected with homoscedastic SVAR fundamental disturbances but not with time-varying conditional volatility of residuals. The Gordon and Leeper restrictions also were not rejected.

5. Impulse response functions and the liquidity effect

We now move to the final step of our empirical investigation which consists in examining the liquidity

[1] 9

[1] 10

effect through the impulse response functions implied by the SVAR processes estimated under conditional heteroscedasticity of the SVAR fundamental disturbances.

5.1. Impulse responses with M-rules

In Figures $1a, 1b$ and 1c, the liquidity effect, if any, is defined as the positive output response and the negative Federal funds rate response following a one positive unconditional deviation shock to $M1_t, TR_t$ and NBR_t, respectively. The error bands correspond to the 95% probability intervals. Each figure has three columns. The first column displays the impulse response functions obtained by imposing the Choleski restrictions. The impulse responses under the Gordon-Leeper restrictions are presented in the second column. In the third column, one can find the impulse responses computed when all restrictions are relaxed, that is, with a fully simultaneous approach.

The first column in Figure 1a shows that by imposing the Choleski restrictions, a positive innovation in $M1_t$ is followed by a persistent decrease in output and a rise in the Federal funds rate for roughly a year. If instead one imposes the Gordon and Leeper restrictions, the output response remains persistently negative whereas the Federal funds rate response is positive for about a year. The last column where all restrictions are relaxed delivers the same message. As Figure 1b shows, replacing $M1$ by TR produces similar results, except that the output and Federal funds rate responses following a positive shock to TR generally are statistically insignificant. Therefore, measuring monetary policy shocks by innovations in $M1_t$ or TR_t does not yield the liquidity effect.

The impulse responses displayed in Figure 1c are those computed following a positive innovation in NBR_t. The impulse responses presented in the first column are those estimated when the invalid Choleski restrictions are imposed. Output persistently increases and the Federal funds rate declines for about ten months.

The impulse responses in the second column are those corresponding to the Gordon- Leeper restrictions which we are unable to reject. Following a positive shock to NBR_t, output increases for about 42 months. The Federal funds rate decreases for roughly four months. Moreover, the effect on the Federal funds rate is actually quite small. In the third column all restrictions are relaxed. The effects on output and the Federal funds rate are even weaker. In fact, the Federal funds rate response is statistically insignificant.

Therefore, the impulse response functions displayed here show that defining monetary policy shocks as innovations in $M1_t, TR_t$ or NBR_t does not produce meaningful liquidity effects. Hence, we can conclude from these findings that the liquidity effects found by others using Choleski restrictions with M-rules result from invalid restrictions. Of course, this raises the question of whether measuring monetary policy shocks by innovations in the Federal funds rate can generate the liquidity effect. We turn to this question next.

5.2. Impulse responses with R-rules

In Figures $2a - 2c$, unanticipated expansionary monetary policy is measured by a one negative unconditional deviation shock to R_t. The results are striking. In Figure $2a$, R is combined with $M1$. Imposing the Choleski or Gordon-Leeper restrictions which we found to be valid, not only do we find that following a negative shock to the Federal funds rate $M1$ increases, but also that the rise in $M1$ is persistent lasting 72 months or more. The output response generally is positive and lasts roughly 50 months. Interestingly, the output response exhibits some kind of a hump shape with the peak taking place little after the twentieth month. The impulse responses displayed in the last column of Figure 2a where all restrictions are relaxed are very similar to those of the first two columns. This simply reflects that the Choleski or Gordon-Leeper restrictions cannot be rejected.

In Figure $2b$, TR replaces $M1$. Here, following a negative innovation in R_t, total reserves rise. However, the TR response is not statistically significant, so one can hardly speak of a liquidity effect. The output response still is positive and persistent. One also notes that the output response is hump shaped.

In Figure $2c$, R is combined with NBR. Here, we find that the positive NBR response lasts between 25 and 30 months depending on the specific case considered, although it is estimated less precisely than the $M1$ response. The output response generally is positive and quite persistent, lasting roughly 45 months.

To summarize the results presented in this section, we have shown first that measuring monetary policy shocks by innovations in a monetary aggregate does not give rise to well characterized liquidity effects. Assuming instead that unanticipated monetary policy is measured by innovations in the Federal funds rate, we have found a persistent liquidity effect when R is combined with $M1$ or NBR, although it is estimated with more precision when R is combined with $M1$. However, when TR is used with R, the TR response is statistically insignificant.

6. Conclusion

Is the liquidity effect an important transmission mechanism of monetary policy? We have shown in this paper that the answer to this question is affirmative if monetary policy shocks are measured by innovations in the Federal funds rate. The results reported here clearly indicate that under time-varying conditional volatility, the restrictions which have been usually imposed are either rejected by the data or cannot generate a well characterized liquidity effect when monetary policy shocks are measured as innovations in monetary aggregates. In contrast, with monetary policy shocks measured as innovations in the Federal funds rate, Choleski and the Gordon and Leeper restrictions cannot be rejected and monetary policy has strong and lasting effects on output, and on monetary aggregates such as $M1$ and the non borrowed reserves. Explaining the strength and persistence of the liquidity effect that we have found empirically, certainly poses a challenge to macroeconomists interested in models of the monetary transmission.

Appendix: Bayesian Posterior Probability Intervals

This appendix amends Sims and Zha's (1995) procedure to construct the bayesian posterior probability intervals for impulse responses when there is conditional heteroscedasticity ($\Gamma_t \neq I$, so that $\Sigma_t \neq \Sigma$). To do so, it is first useful to rewrite the VAR process (3) as:

$$Z = (I \otimes X)\Phi + \nu, \tag{a.1}$$

where Z is the $(NT \times 1)$ vector of dependent variables, I is the $(N \times N)$ identity matrix, X is the $(T \times N\tau)$ matrix of predetermined variables, Φ is the $(N^2\tau \times 1)$ vector of coefficients, and ν is the $(NT \times 1)$ vector of statistical innovations—where T denotes the sample size. Also, Θ is defined as the $(S \times 1)$ vector containing the S_1 non-zero elements of A and the S_2 non-zero elements of $\Delta_j (j = 1, 2)$ —where $S = (S_1 + S_2), S_1 \leq N^2$, and $S_2 \leq 2N$.

As is well known, Φ follows asymptotically a normal posterior distribution. Also, the posterior distribution of Θ is approximated by a normal. Thus, making the Monte Carlo draws directly on these posterior distributions yield:

$$\Theta = \Theta + W_\Theta R_\Theta, \tag{a.2}$$

$$\Phi = \Phi + W_\Phi R_\Phi, \tag{a.3}$$

where $\hat{\Theta}$ and $\hat{\Phi}$ are consistent estimates (described in section 2.3) of Θ and Φ, respectively. Also, W_Θ and W_Φ are the $(S \times S)$ and $(N^2\tau \times N^2\tau)$ matrices corresponding to the Choleski factorizations of the covariance matrices of $\Theta(V_\Theta)$ and $\Phi(V_\Phi)$, respectively. Finally, R_Θ and R_Φ are the $(S \times 1)$ and $(N^2\tau \times 1)$ vectors of standard normal draws.

In our application, it is easy to compute W_Θ from the estimate of V_Θ since S is relatively small. In contrast, it is more challenging to calculate W_Φ because V_Φ involves products of large matrices. Fortunately, W_Φ can be rewritten as:

$$W_\Phi = \sum_{i=1}^{N} (\Pi_i \otimes W_{\mathrm{Xi}}) \tag{a.4}$$

where Π_i is $a(N \times N)$ matrix, where the ith column corresponds to the ith column of A^{-1} and the other columns represent vectors of zeros. Also, W_{Xi} is the $(N\tau \times N\tau)$ matrix of the Choleski decomposition of $(X^{'}X)^{-1}(X^{'}\Gamma_i X)(X^{'}X)^{-1}, \Gamma_i =$diag$(\gamma_{\mathrm{ii,1}}, ..., \gamma_{\mathrm{ii,T}})$, and $\gamma_{\mathrm{ii,t}}$ is the conditional variance of the ith fundamental innovation in period t —given by equation (5). Interestingly, equation $(a.4)$ involves exclusively matrices of tractable sizes. Moreover, the $(N\tau \times N\tau)$ matrix $(X^{'}X)^{-1}$ is automatically computed by most softwares. Also, the $(N\tau \times N\tau)$ matrix $(X^{'}\Gamma_i X)$ is easy to calculate since the (j,k) element of this matrix is simply $\sum_{t=1}^{T} X_{jt}X_{kt}\gamma_{\mathrm{ii,t}}$. Finally, it can be verified that $(a.4)$ implies that:

$$V_\Phi = \sum_{i=1}^{N}[(\Pi_i\Pi_i^{'}) \otimes (X^{'}X)^{-1}(X^{'}\Gamma_i X)(X^{'}X)^{-1}], \qquad (a.5)$$

$$= [(I \otimes X)^{'}(I \otimes X)]^{-1}(I \otimes X)^{'}\Omega(I \otimes X)[(I \otimes X)^{'}(I \otimes X)]^{-1}, \qquad (a.6)$$

where $\Omega = E_{t-1}(\nu\nu^{'})$ is formed of $N^2(T \times T)$ diagonal submatrices. This reflects the idea that the statistical disturbances (ν_t) are innovations— *i.e.* they can be contemporaneously correlated, but are serially uncorrelated.

Therefore, equations $(a.2)$ and $(a.3)$ allow one to perform the Monte Carlo draws directly on the posterior distributions of Θ and Φ. Substituting the draws for Θ and Φ in equation (2) produces the simulated posterior distribution for impulse responses. Moreover, choosing the appropriate quantiles of this distribution yields the 95% posterior probability intervals. These intervals can be asymmetric, which reflects the asymmetry in the posterior distribution of the impulse responses or in the sampling distribution of their estimates. Finally, note that this Monte Carlo experiment is similar to Sims and Zha's (1995) procedure when $S_1 \leq N(N+1)/2$ and $S_2 = 0$.

References

Bernanke, Ben S. "On the Predictive Power of Interest Rates and Interest Rate Spreads." *New England Economic Review* (Nov.-Dec. 1990) : 51 – 68.

Bernanke, Ben S. and Blinder, Alan S. "The Federal Funds Rate and the Channels of Monetary Transmission." *A.E.R.* 82 (September 1992) : 901 – 21.

Christiano, Lawrence J., and Eichenbaum, Martin. "Identification and the Liquidity Effect of a Monetary Policy Shock." In *Business Cycles, Growth and Political Economy,* edited by Alex Cukierman, Zvi Hercowitz and Leonardo Leiderman. Cambridge: MIT Press, 1992 : 335 – 70.

Davidson, Russell, and Mackinnon, James. *Estimation and Inference in Econometrics.* New York: Oxford University Press, 1993.

Dunn, J.E. "A Note on a Sufficiency Conditions for Uniqueness of a Restricted Factor Matrix." *Psychometrika* 38(1973) : 141 – 43.

Eichenbaum, Martin. "Comments on 'Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy' by Christopher Sims." *European Econ. Rev.* 36 (June 1992) : 1001 – 11.

Engle, Robert F., and Susmel, Raul. "Common Volatility in International Equity Markets." *Journal of Bus. and Econ. Stats.* (April 1993) : 167 – 176.

Gordon, David B., and Leeper, Eric M. "The Dynamic Impacts of Monetary Policy: An Exercise in Tentative Identification." *J.P.E.* 102 (December 1994) : 1228 – 47.

King, Mervyn, Sentana, Enrique, and Wadhwani, Sushil. "Volatility and Links Between National Stock Markets." *Econometrica* 62 (July 1994) : 901 – 33.

Leeper, Eric M., and Gordon, David B. "In Search of the Liquidity Effect." *J. Monetary Econ.* 29 (June 1992) : 341 – 69.

McCallum, Bennett. "A Reconsideration of Sims' Evidence Concerning Monetarism." *Economics Letters* 13(2 – 31983) : 167 – 71.

Pagan, Adrian R., and Robertson, John C. "Resolving the Liquidity Effect." *Fed. Reserve Bank of St. Louis Rev.* 77 (May/June 1995) : 33 – 54.

Sentana, Enrique. "Identification of Multivariate Conditionally Heteroskedastic Factor Models." Working paper, London, Eng.: London School of Economics, Dept. of Econ. and Financial Markets Group, 1992.

Sims, Christopher A. "Are Forecasting Models Usable for Policy Analysis?" *Fed. Reserve Bank of Minneapolis Q. Rev.* 10 (Winter 1986) : 2 – 16.

———."Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy." *European Econ. Rev.* 36 (June 1992) : 975 – 1000.

Sims, Christopher A., Stock, James H., and Watson, Mark W. "Inference in Linear Time Series Models with Some Unit Roots." *Econometrica* 58 (January 1990) : 113 – 44.

Sims, Christopher A., and Zha, Tao. "Error Bands for Impulse Responses". Manuscript. Atlanta, Georgia: Federal Reserve Bank of Atlanta, September 1995.

Strongin, Steven. "The Identification of Monetary Policy Disturbances: Explaining the Liquidity Puzzle." *J. Monetary Econ.* 35 (June 1995) : 463 – 98.

Table 4
Restrictions on A

M-Rule

	Choleski	*Gordon-Leeper*	*Variables*
4-Var. SVAR	$\begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$	$\begin{bmatrix} Y \\ P \\ M1 \text{ or } TR \\ R \end{bmatrix}$
5-Var. SVAR	$\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$	$\begin{bmatrix} Y \\ P \\ TR \\ NBR \\ R \end{bmatrix}$

R-Rule

	Choleski	*Gordon-Leeper*	*Variables*
4-Var. SVAR	$\begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$	$\begin{bmatrix} Y \\ P \\ M1 \text{ or } TR \\ R \end{bmatrix}$
5-Var. SVAR	$\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$	$\begin{bmatrix} Y \\ P \\ TR \\ NBR \\ R \end{bmatrix}$

Under the Choleski restrictions, the orderings are for the M-rule: $(i) Z_t = [Y_t, P_t, M1_t, R_t]'$, $(ii) Z_t = [Y_t, P_t, TR_t, R_t]'$, and $(iii) Z_t = [Y_t, P_t, TR_t, NBR_t, R_t]'$, while they are for the R-rule: $(iv) Z_t = [Y_t, P_t, R_t, M1_t]'$, $(v) Z_t = [Y_t, P_t, R_t, TR_t]'$, and $(vi) Z_t = [Y_t, P_t, R_t, TR_t, NBR_t]'$. With the Gordon-Leeper restrictions, the SVAR are: $(vii) Z_t = [Y_t, P_t, M1_t, R_t]'$, $(viii) Z_t = [Y_t, P_t, TR_t, R_t]'$, and $(ix) Z_t = [Y_t, P_t, TR_t, NBR_t, R_t]'$.