ESTIMATION OF DYNAMIC DECISION MODELS WITH CORNER SOLUTIONS:

A MODEL OF PRICE AND INVENTORY DECISIONS

BY

VICTOR AGUIRREGABIRIA[1]

Department of Economics

University of Western Ontario

London, Ontario, N6A 5C2. Canada

This paper proposes and applies a method of moments to estimate dynamic decision models with corner solutions. The method extends previous results by Hotz and Miller (1993) and Pakes (1994), and it allows for unobserved state variables affecting both the continuous choice (interior solution) and the discrete choice (interior solution / corner solution). The method is applied to estimate a model of price and inventory decisions using data of individual goods from a supermarket chain. The estimation shows that lump-sum ordering costs and lump-sum price adjustment costs are significant but ordering costs are quantitatively more important. Numerical solutions of the model show that, in that context, the interaction between price and inventory decisions can explain the relatively high frequency of nominal price reductions observed in the data.

KEYWORDS: Dynamic decision models, limited dependent variables, (S,s) price and inventory models, lump-sum adjustment costs.

JEL No's: C34, C35, D92, E31, L81

1. INTRODUCTION

Since the seminal paper by Hansen and Singleton (1982) the most common approach to estimate dynamic decision models has been to construct sample counterparts of the orthogonality conditions provided by the stochastic Euler equations and use them to estimate the parameters of interest by the Generalized Method of Moments (GMM). The main advantage of this strategy is that it permits to identify and estimate the structural parameters without having to solve the model. However, this method has several limitations when the characteristics of the model imply a strictly positive probability of corner solutions. This is the case in many dynamic models which have gained increasing theoretical and empirical interest during the last years, e.g., models of irreversible investment, labour demand with fixed hiring and firing costs, (S,s) inventory and pricing models, or consumption of durable goods, between others.

The standard Euler equations used in Hansen and Singleton approach are the combination of marginal conditions of optimality at two consecutive periods. Nevertheless, for the class of models that I consider in this paper, the marginal conditions do not hold with probability one because there are some periods where corner solutions occur. In this context Hansen and Singleton approach has three main limitations. First, the selection of the subsample of observations where marginal conditions of optimality occur at two consecutive periods, say t and t+1, implies conditioning to the event that the solution at t+1 will be interior. Provided that this information is unknown at period t this approach suffers of selection bias, and in general this bias enters with unknown form. The second problem is that not all the structural parameters can be identified using only the subsample of interior solutions. For example, the identification of fixed adjustment cost parameters requires one to exploit the information contained in the corner solution observations. Finally, another limitation is that, even if we are not interested in the estimation of fixed adjustment costs, the discrete choice can contain a lot of information about the other parameters of interest.

Therefore, when the proportion of corner solutions in the sample is relatively large, an estimator based only on moments from the Euler equations can be very inefficient.

Under some regularity conditions the optimal decision rule in this type of models is unique and it can be characterized by a function defining the optimal interior solution and a condition defining when it is optimal to choose an interior solution or the corner solution. In the context of a model of capital investment Pakes (1994) has defined sufficient conditions to obtain Euler equations when the model has both continuous and discrete control variables, or when there are binding constraints. Section 2.2 shows how this approach can be easily extended to the case where corner solutions are the result of discontinuity or/and not differentiability of the utility function.

The decision of being in a corner solution against being in an interior solution can be represented as a dynamic discrete choice problem. Hotz and Miller (1993) present a proposition showing that in a dynamic discrete choice model the expected conditional choice value functions can be written as known functions of the state variables, discrete choice probabilities, and transition probabilities of the state variables. Therefore, if choice and transition probabilities can be estimated in a first stage, the structural parameters of the discrete choice model can be estimated without solving the dynamic programming model at each iteration in the search for the parameters of interest.

This paper combines the results of Pakes and Hotz and Miller in the context of dynamic decision models with corner solutions. Their results are extended in two directions. First, I consider a model where there are unobserved state variables affecting both the continuous choice (interior solution) and the discrete choice (interior solution / corner solution). This extension introduces three potential problems: (1) unobservable state variables may enter in the Euler equations; (2) if unobservable state variables affecting the continuos choice are correlated with those affecting the discrete choice, one has to take into account the selection bias that arises when selecting the subsample of interior solutions to construct sample counterparts of the Euler equations; and (3) Hotz and Miller proposition should be extended to this context. In Section 2 I present sufficient conditions that allow to overcome these three problems. These conditions refer to the stochastic properties of the unobservable state variables and the form in which they enter in the utility function.

The second extension that this paper presents is related to the computation of the conditional discrete choice value functions. When using Hotz and Miller approach the value functions can be a very complicated function of the choice and transition probabilities and the parameters of interest. This is the case when the range of variation of the state variables or the time horizon of the problem are relatively large. Hotz et al (1994) propose a simulation based estimator to overcome this problem. Here I propose an alternative approach. If observables and structural parameters are multiplicatively separable in the one-period utility function, the conditional choice value functions can be written as the product of a vector of functions of the parameters of interest and a vector of functions of the choice and transition probabilities. Therefore, once choice and transition probabilities have been computed in a first stage, the functions of these incidental parameters that appear in the conditional value functions become data which do not have to be re-calculated during the estimation of the parameters of interest.

In Section 2.4 I define a sequential GMM estimator of the structural parameters that exploits the moment conditions from the Euler equations and from the optimal discrete choice. The computational simplicity of this method permits the estimation of dynamic structural models with corner solutions and with a large number of state variables. This is an advantage over methods based on nested solution-estimation algorithms whose computational burden makes impractical to estimate models with many state variables.

I have applied this method to estimate a dynamic decision model of price and inventory decisions in a supermarket. Sticky prices, cross-section price dispersion, and the propagation of real shocks in the economy have been explained theoretically using models of price and inventory decisions with lump-sum adjustment costs. The micro and macroeconomic implications of these models depend on the structure of ordering costs and price adjustment costs. However, there have been very few empirical work estimating fixed and variable adjustment costs in structural models of price or inventory decisions. One reason has been the econometric problems associated to the estimation of dynamic structural models with corner solutions. Another important reason has been the lack of appropriate datasets with information at the level of individual firms and individual goods and with high sample frequency.[2]

Some predictions of the stationary (S,s) pricing model developed by Sheshinski and Weiss (1977, 1983) have been rejected in several empirical studies using micro data sets.[3] In the dataset used in this paper, nominal price reductions are too frequent to be consistent with a simple (S,s) pricing policy during a period of relative high inflation (around 7% annual rate). In fact, for most of the goods in the sample, the frequency of reductions in nominal selling prices is around 20 percentage points larger than the frequency of reductions in nominal cost prices (i.e., price paid to manufacturers). The paper presents reduced form and structural form evidence of how part of these nominal price reductions are the result of the interaction of price and inventory decisions.

Section 3 presents a model of joint price and inventory decisions where the firm faces lump-sum adjustment costs both when placing orders and changing prices. Numerical solutions of the model show that, if lump-sum ordering costs are larger than fixed costs of changing prices, nominal price reductions tend to occur when the firm places orders, even if the inflation rate is relatively large. This model is estimated using a dataset from a supermarket chain. This dataset contains monthly information (at the level of individual goods) about stocks, orders, sales, selling prices and cost prices. The estimation of the structural model shows that lump-sum ordering costs are much larger than fixed price adjustment costs. Moreover, the role of price decisions in the reduction of lump-sum ordering costs explains why nominal price increases tend to occur before orders are placed, and nominal price reductions just when orders are placed. It can explain more than 60% of the observed nominal price reductions.

2. MOMENT CONDITIONS IN DYNAMIC DECISION MODELS WITH CORNER SOLUTIONS.

2.1. A Dynamic Decision Model with Corner Solutions

Consider a sequential decision problem where at each period t an individual chooses a Dx1 vector of decision variables, d_t, in order to maximize the expected value of its intertemporal utility. The problem at period t is [4]

$$
\begin{aligned}
&\text{Max}_{d_t} \quad E_t \sum_{j=0} \beta^j \, u(s_{t+j}, d_{t+j}) \\
&\text{subject to:} \quad d_t \in \Gamma(s_t) \\
&\qquad\qquad\qquad p(s_{t+1} | s_t, d_t)
\end{aligned}
\tag{1}
$$

where s_t is the vector of state variables at period t; $\Gamma(.)$ is the correspondence of feasible choices; p(.) is the transition probability function for the state variables; β is the discount factor; u(.) is the one-period utility function; and E_t is the expected value conditional to the information available at period t. A particular decision maker is characterized by the set of primitives $\{u,\beta,\Gamma,p\}$. Rust (1992) has proved that these primitives are nonparametrically unidentified. Therefore, for the moment, I will consider a parametric specification of the primitives, i.e., u(.), $\Gamma(.)$ and p(.) are known functions up to the vector of parameters

θ (that includes β).

Consider that the usual framework is extended to allow for discontinuity and non-differentiability of the utility function. For the sake of simplicity consider that there is only one decision variable. The function u(.) is discontinuous with respect to the control variable d_t, at one point, that without loss of

$$(2) \qquad u(s_t, d_t, \theta) = D_t^0 \, u^0(s_t;\theta) + D_t^1 \, u^1(s_t, d_t;\theta) + D_t^2 \, u^1(s_t, d_t;\theta)$$

generality is zero. In addition, u(.) can be asymmetric around the discontinuity point.

where $D^0_t = I(d_t = 0)$, $D^1_t = I(d_t < 0)$, and $D^2_t = I(d_t > 0)$ where I(A) is the indicator for the event A. The conditional choice utility functions have the following form,

$$(3) \qquad \begin{aligned} u^0(s_t;\theta) &= u^0(x_t, \omega_t;\theta) + \epsilon_t^0 \\ u^j(s_t, d_t;\theta) &= u^j(x_t, \omega_t, d_t;\theta) + \epsilon_t^j \quad \text{for } j = 1,2 \end{aligned}$$

where x_t is the vector of state variables observed by the econometrician; and $\epsilon_t = (\epsilon^0_t, \epsilon^1_t, \epsilon^2_t)$ and ω_t are state variables unobserved by the econometrician. This specification allows for unobserved state variables that affect the marginal utilities, ω_t, and unobserved state variables that affect absolute utility but not marginal utility. This framework encompasses a large number of dynamic decision models with corner solutions, e.g., (S,s) inventory and pricing models, labour demand with fixed firing and hiring costs, consumption of durable goods, or irreversible investment, between others.

In order to guarantee the existence and uniqueness of the solution to this problem I make the following assumptions.

ASSUMPTION 1: The observed state variables x_t have two components: exogenous state variables,

z_t, and a endogenous state variable, k_t. The unobserved state variables ω_t and ϵ_t are weakly exogenous. The conditional choice transition probability of the state variables factors as

$$p(dx_{t+1}, d\omega_{t+1}, d\epsilon_{t+1} \mid x_t, \omega_t, \epsilon_t, d_t) = q_\epsilon(\epsilon_{t+1} \mid x_{t+1})\ q_\omega(\omega_{t+1} \mid x_{t+1})\ I\{k_{t+1} = \rho_K k_t + d_t\}\ q_z(z_{t+1} \mid z_t)$$

where $I(.)$ is the indicator function; $|\rho_K|$ is lower than 1 and $q_z(.)$, $q_\epsilon(.)$, and $q_\omega(.)$ are continuously differentiable functions.

ASSUMPTION 2: The conditional choice utility functions are bounded and continuously diffenrentiable in ω, x and d, and strictly increasing in k. And $u^1(.)$ and $u^2(.)$ are strictly concave in (k, d).

The model defined by equations (1) and (3) and assumptions 1 and 2 has a unique solution where the optimal decision rule is

$$(4)\quad \delta(s_t) = \begin{cases} 0 & \text{if } u^0(s_t) + \epsilon_t^0 + v^0(x_t) > u^j(s_t, \delta_t^j) + \epsilon_t^j + v^j(x_t) \quad \text{for } j = 1, 2 \\ \delta^1(x_t, \omega_t) & \text{if } u^1(s_t, \delta_t^1) + \epsilon_t^1 + v^1(x_t) > u^j(s_t, \delta_t^j) + \epsilon_t^j + v^j(x_t) \quad \text{for } j = 0, 2 \\ \delta^2(x_t, \omega_t) & \text{otherwise} \end{cases}$$

where $\delta^1(x_t, \omega_t)$ and $\delta^2(x_t, \omega_t)$ are the optimal interior solutions under a negative and a positive adjustment, respectively.

$$(5)\quad \delta^1(x_t, \omega_t) = \arg\max_{d<0} \left\{ u^1(x_t, \omega_t, d_t) + \beta \int V(x_{t+1}, \omega_{t+1}, \epsilon_{t+1})\, p(ds_{t+1} \mid x_t, d_t) \right\}$$

and

$$(6)\quad \delta^2(x_t, \omega_t) = \arg\max_{d>0} \left\{ u^2(x_t, \omega_t, d_t) + \beta \int V(x_{t+1}, \omega_{t+1}, \epsilon_{t+1})\, p(ds_{t+1} \mid x_t, d_t) \right\}$$

where $V(x_t, \omega_t, \epsilon_t)$ is the value function,

$$(7)\quad V(x_t, \omega_t, \epsilon_t) = \max_{j \in \{0,1,2\}} \left\{ u^j(x_t, \omega_t, \delta^j[x_t, \omega_t]) + \epsilon_t^j + v^j(x_t) \right\}$$

and $v^j(x_t)$ are the conditional choice value functions.

(8) $$v^j(x_t) = \beta \int V(x_{t+1}, \omega_{t+1}, \epsilon_{t+1})\, p(ds_{t+1} | x_t, D_t^j = 1)$$

In this model we could estimate θ using a nested solution-estimation algorithm where, at each iteration in the search for the parameter estimates, the $v^j(.)$ functions are obtained numerically as fixed points of the contraction mapping provided by the system of functional equations (7) and (8). These nested solution-estimation algorithms have been successfully implemented in the estimation of dynamic discrete choice models by Wolpin (1984), Pakes (1986), Rust (1987), Eckstein and Wolpin (1989) or Das (1992) between others. However, when the number of state variables is greater than two or three this method is impractical even on the most sophisticated of computing equipment. Alternatively, I propose a method that does not require one to solve the model at each iteration. This method exploits moment conditions for both the optimal interior solutions and for the optimal discrete choice corner solution versus interior solution.

2.2. Euler Equations and Moment Conditions for the Continuous Choice

In this Section I follow the same approach that Pakes (1994) to obtain the Euler equation of the previous problem. Consider that at period t there is an interior solution, say $d_t = \delta^1(s_t)$, and define τ_t as the number of periods until the next interior solution.

(9) $$\tau_t = \min\{ \tau : d_{t+\tau} > 0 \text{ and } \delta(s_{t+\tau}) \neq 0 \}$$

Now, consider a decision rule $\{d^*(s_{t+n}, \epsilon_{t+n}, \alpha) : n \geq 0 \}$ that is an alternative to the optimal decision rule δ(.). The starred decision rule is defined as follows,

$$d^*(s_t;\alpha) = \begin{cases} 0 & \text{if } u^0(s_t) + \epsilon_t^0 + v^0(x_t) > u^j(s_t, d_t^{*j}) + \epsilon_t^j + v^j(x_t) \quad \text{for } j = 1,2 \\ d^{*1}(x_t, \omega_t; \alpha) & \text{if } u^1(s_t, d_t^{*1}) + \epsilon_t^1 + v^1(x_t) > u^j(s_t, d_t^{*j}) + \epsilon_t^j + v^j(x_t) \quad \text{for } j = 0,2 \\ d^{*2}(x_t, \omega_t; \alpha) & \text{otherwise} \end{cases}$$

where

$$(10) \qquad d^{*j}(s_{t+n};\alpha) = \begin{cases} \delta^j(x_{t+n},\omega_{t+n}) - \alpha & \text{if } n = 0 \\ \delta^j(x_{t+n},\omega_{t+n}) + \rho_K^{\tau_t}\,\alpha & \text{if } n = \tau_t \\ \delta^j(x_{t+n},\omega_{t+n}) & \text{otherwise} \end{cases}$$

and α is any real value close to zero. Given that $u^j(.)$, $\delta^j(.)$ and $v^j(.)$ are continuous functions, and $|\rho_K|<1$, for α close to zero the path of discrete choices in the starred decision rule will be the same that the discrete choices in the optimal decision rule. In addition, it is easy to verify that the path of endogenous state variables under the starred policy deviates from the path in the optimal decision rule only between t and $t+\tau_t$. Thus, the decisions generated by the starred rule are the optimal decisions at every period except at periods t and $t+\tau_t$.

The difference between the discounted expected stream of utilities under the optimal and the starred policies should have its maximum at α=0. Given that it is a differentiable function its derivative must be 0at α=0. This marginal condition of optimality is the Euler equation.

$$(11) \qquad E_t\left[\frac{\partial u_t^j}{\partial d_t} + \sum_{n=1}^{\tau_t-1} \beta^n \rho_K^{n-1} \frac{\partial u^j_{t+n}}{\partial k_{t+n}} + \beta^{\tau_t}\rho_K^{\tau_t-1} \sum_{j=1}^{J} D^j_{t+\tau_t}\left[\frac{\partial u^j_{t+\tau_t}}{\partial k_{t+\tau_t}} - \rho_K \frac{\partial u^j_{t+\tau_t}}{\partial d_{t+\tau_t}} \right] \right] = 0$$

Assume, for the moment, that the econometrician observes all the state variables affecting the continuous choice (i.e., ω_t =0 for any t). In that case equation (11) provides a set of orthogonality conditions that can be used to construct a GMM estimator of some of the parameters in θ. Notice that equation (11) shows why using the standard Euler equations for the subsample of observations with τ_t =1 can lead to inconsistent estimates. Equation (11) can be written as

$$\sum_{n=1}^{\infty} p(\tau_t = n | s_t)\; E(\Psi[s_t, d_t, \ldots, s_{t+\tau_t}, d_{t+\tau_t}] \;| s_t, \tau_t = n)$$

Conditioning to the event "τ_t=1" the left hand side of this expression becomes equal to the left hand side in a standard Euler equation (i.e., model without corner solutions). But in a model with corner solutions this expression is no longer equal to zero. It is equal to $(-1/p(\tau_t=1 | s_t)) \sum_{j=2}^{\infty} p(\tau_t=j | s_t)$ $E(\Psi(s_t,d_t,...,s_{t+j},d_{t+j}) | s_t,\tau_t=j)$, that is an unknown function of s_t. Therefore, the estimation of the parameters of interest using the standard Euler equations for the subsample of interior solutions with $\tau_t=1$ will yield inconsistent estimates. The larger $p(\tau_t>1 | s_t)$ the larger the bias of these estimates.

In a longitudinal dataset for the last interior solution of each individual we do not observe the number of periods until the next interior solution (i.e., τ). Therefore, these observations cannot be included in the estimation. The effect of this censoring on our parameter estimates depends on the type of sample we have. The effect will be negligible in long panels with frequent interior solutions for each individual. However, the censoring may bias our estimates when the sample presents a small number of interior solutions per individual. The most simple approach to deal with this problem is to estimate the model using only those years for which there are enough subsequent number of periods such that τ is observed for any (i,t) (see Olley and Pakes [1991]).

So far I have considered that there not exist unobserved state variables, ω_t. However, there are many applications where unobserved state variables enter not additively in the Euler equations. A possible solution is to integrate over the unobserved state variables using a specification of $q_\omega(\omega_t|x_t)$. An alternative approach is to use an additional structural equation that provides a relationship between ω_t an a set of observable variables. This approach can be useful in many applications.

ASSUMPTION 3: There is a vector of observed variables y_t and a structural equation that relates the unobserved state variables with the observables y_t , x_t and d_t

$$\omega_t = \omega(x_t , y_t , d_t ; \gamma)$$

where $\omega(.)$ is known up to the vector of parameters γ.

Example: Consider a dynamic model where a firm decides its demand of several inputs. Output, that is neither a decision nor a state variable, depends on the amount of inputs but also on some variables that are unobservable for the econometrician, e.g., a technological shock. In this context, if output is observable for the econometrician the production function provides the $\omega(.)$ function in assumption 3. If all the parameters of the production function are identified in the Euler equation the orthogonality between x_t and ω_t is not required. Otherwise one should assume that some observed state variables are orthogonal to the technological shock.

In some cases the vectors of parameters θ and γ can be identified using only the Euler equations. In general that will not be the case and we will have to exploit moment conditions from the equation $\omega(.)$. Notice that both y_t and d_t depend on ω_t and therefore they cannot be used as instrumental variables in the GMM estimation.

2.3 Moment Conditions for the Optimal Discrete Choice

There are two main reasons why it is important to exploit the discrete choice information in a dynamic decision model with corner solutions. First, there may be some parameters of interest not entering in the Euler equations. That is the case of lump-sum adjustment costs or any other parameters capturing discontinuous jumps in the utility function. And second, but not less important, the discrete choice can contain very much information about all the parameters in the model. In fact, if corner solutions are very frequent in our dataset the discrete choice will contain a lot of information about most of the parameters. Therefore, ignoring the information in the discrete choice will imply a significant loss of efficiency in our estimates. In this section I will obtain an expression for the conditional choice value functions, $v^j(x_t)$, in terms of observed variables, structural parameters and some incidental parameters. Using this expression I will construct moment conditions for the optimal discrete choice that will allow to estimate (θ,γ) using a two stages procedure.

Let D^{jO}_t be the indicator of the event “alternative j is optimal at period t”. Using equation (4) the optimal discrete choice can be written as

$$
(12) \qquad D_t^{jO} = 1 \quad \Leftrightarrow \quad j = \arg\max_{k \in \{0,1,2\}} \left\{ \; u^k(x_t, \omega_t, \delta_t^k) + \epsilon_t^k + v^k(x_t) \; \right\}
$$

where $\delta^k_t = \delta^k(x_t, \omega_t)$. By definition, we can write $v^j(x_t)$ as the expected discounted stream of future utilities if alternative j were chosen at period t.

$$
(13) \qquad \begin{aligned}
v^j(x_t) &= \sum_{n=1}^{\infty} \beta^n \int \sum_{k=0}^{2} D_{t+n}^{kO} \Big[u^k(x_{t+n}, \omega_{t+n}, \delta_{t+n}^k) + \epsilon_{t+n}^k \Big] q_{\epsilon\omega}(d\epsilon_{t+n}, d\omega_{t+n} | x_{t+n}) \, f_x(dx_{t+n} | x_t, D_t^j = 1) \\
&= \sum_{n=1}^{\infty} \beta^n \int \sum_{k=0}^{2} P^k(x_{t+n}) \Big[Eu^k(x_{t+n}) + E\epsilon^k(x_{t+n}) \Big] \; f_x(dx_{t+n} | x_t, D_t^j = 1)
\end{aligned}
$$

where the $P^j(x_t)$ functions are the *Conditional Choice Probabilities*,

$$
(14) \qquad P^j(x_t) \;\equiv\; p(D_t^{jO} = 1 | x_t) \;=\; \int I[j = \arg\max_{k \in \{0,1,2\}} \{ u^k(x_t, \omega_t, \delta_t^k) + \epsilon_t^k \}] \;\; q_{\epsilon\omega}(d\epsilon_t, d\omega_t | x_t)
$$

and $Eu^j(x_t)$ and $E\epsilon^j(x_t)$ are

$$
(15) \qquad Eu^j(x_t) \;=\; E(\, u^j(x_t, \omega_t, \delta_t^j) \,|\, x_t, D_t^{jO} = 1 \,)
$$

and

$$
(16) \qquad E\epsilon^j(x_t) \;=\; E(\, \epsilon_t^j \,|\, x_t, D_t^{jO} = 1 \,)
$$

Assume that the observed state variables can be discretized in M cells, i.e., $x_t \in X = \{x^1, x^2, ..., x^M\}$. We can define the following vectors of choice probabilities and matrices of transitions probabilities.

$$P^j = (P^j[x^1], \ldots, P^j[x^M])' \qquad \text{for } j = 0, 1, 2$$

$$F^j = (F^j[x^1], \ldots, F^j[x^M])' \qquad \text{for } j = 0, 1, 2$$

$$F = (F[x^1], \ldots, F[x^M])'$$

where

$$F^j[x^m] = (p[x_{t+1} = x^1 | x_t = x^m, D_t^j = 1], \ldots, p[x_{t+1} = x^M | x_t = x^m, D_t^j = 1])' \qquad \text{for } j = 0, 1, 2$$

$$F[x^m] = (p[x_{t+1} = x^1 | x_t = x^m], \ldots, p[x_{t+1} = x^M | x_t = x^m])'$$

I also define the vectors of conditional expected utilities,

$$Eu^j = (Eu^j[x^1], \ldots, Eu^j[x^M])' \qquad \text{for } j = 0, 1, 2$$

$$E\epsilon^j = (E\epsilon^j[x^1], \ldots, E\epsilon^j[x^M])' \qquad \text{for } j = 0, 1, 2$$

Using this notation we can write expression (13) as

$$\text{(16)} \quad \begin{aligned} v^j(x_t) &= \sum_{n=1}^{\infty} \beta^n \; F^j(x_t)' \; F^{n-1} \left[\sum_{k=0}^{2} P^k * (Eu^k + E\epsilon^k) \right] \\ &= \beta \; F^j(x_t)' \; (I_M - \beta F)^{-1} \left[\sum^{2} P^k * (Eu^k + E\epsilon^k) \right] \end{aligned}$$

where * is the element-by-element (Hadamard) product. Notice that F, F^j, and $\dot{P}$ contain incidental parameters that can be estimated using a sample of x_t and d_t. By Hotz and Miller theorem (1993) $E\epsilon^k(x_t)$ is a function of the conditional choice probabilities. The form of this function depends on the probability distribution of the ϵ's. For example, if the ϵ's are iid distributed with a double exponential distribution, $E\epsilon^k(x_t) = c - \ln P^k(x_t)$, where c is the Euler constant.

Using assumption 3 the conditional choice utility functions can be writen as

$$U^j(x_t, y_t, d_t; \theta, \gamma) = u^j(x_t, \omega(x_t \; y_t, d_t; \gamma), d_t; \theta)$$

ASSUMPTION 4: *The conditional choice utility functions, $U^j(x_t, y_t, d; \theta, \gamma)$, are multiplicatively* *separable in observables and structural parameters.*

$$U^j(x_t, y_t, d_t; \theta, \gamma) = h(\theta,\gamma)' g^j(x_t, y_t, d_t)$$

where $h(\theta,\gamma)$ and $g^j(x_t, y_t, d_t)$ are Lx1 vectors of known functions.

This assumption holds in most currently estimated dynamic decision models. Under assumption 4 we have that:

$$(17) \qquad Eu^j(x_t) = h(\theta,\gamma)' \; E(g^j[x_t, y_t, d_t] \mid x_t, D^j_t = 1)$$

where $E(g^j[x_t, y_t, d_t] \mid x_t, D^j_t = 1)$ depends on x_t, and on moments of y_t and d_t conditional to x_t. Therefore, under assumption 4 the conditional choice value functions are

$$(18) \qquad v^j(x_t) = h(\theta,\gamma)' \, \Pi^j_g(x_t;\mu) + \Pi^j_\epsilon(x_t;\mu)$$

where μ is the set of incidental parameters

[illegible]

Π^j_g and Π^j_ϵ have the following expressions

$$(19) \qquad \begin{aligned} \Pi^j_g(x_t,\mu) &= \beta \; F^j(x_t)' \; (I_M - \beta F)^{-1} \left[\sum_{k=0}^{2} P^k * G^k \right] \\ \Pi^j_\epsilon(x_t,\mu) &= \beta \; F^j(x_t)' \; (I_M - \beta F)^{-1} \left[\sum^{2} P^k * E\epsilon^k \right] \end{aligned}$$

and G^k is the MxL matrix $\{ E(g^k[x_t, y_t, d_t] \mid x_t = x^m, D^k_t = 1) : m=1,...,M \}$.

Finally, we can write the optimal discrete decision as,

$$(20) \qquad D^{jO}_t = 1 \;\; \Leftrightarrow \;\; j = \arg\max_{k \in \{0,1,2\}} \{ \; h(\theta,\gamma)' [g^k(x_t, y_t) + \Pi^k_g(x_t,\mu)] + \Pi^k_\epsilon(x_t,\mu) + \epsilon^k_t \; \}$$

Given a fixed value of β and reduced form estimates of the incidental parameters in μ, the Π^j functions become data that will not change during the estimation of θ and γ. Therefore, equation (20) provides a simple expression to estimate θ and γ.

2.4 Joint Estimation of the Model

The estimation of the model can be represented in terms of four sets of moment conditions.

$$E[\ W_1(x_t)\ m(x_t, d_t, y_t, \ldots, x_{t+\tau_t}, d_{t+\tau_t}, y_{t+\tau_t}\ ;\ \theta, \gamma)\ ] = 0 \quad \text{for } d_t > 0$$

$$E[\ W_2(x_t)\ \omega(x_t, d_t, y_t\ ;\ \gamma)\ ] = 0 \quad \text{for any } t$$

$$E[\ W_3(x_t)\ (D_t - P(x_t, y_t\ ;\ \theta, \gamma, \mu)\ ] = 0 \quad \text{for any } t$$

$$E[\ W_4(x_t)\ \phi(D_t, x_t\ ;\ \mu)\ ] = 0 \quad \text{for any } t$$

The first set of moment conditions comes from the Euler equations and they hold only for the subsample of interior solutions. The second set of conditions come from the structural equation $\omega(.)$. In the third set we have the moment conditions from the optimal discrete choice. Finally, the last set represents the moment conditions for the estimation of the incidental parameters. Notice that the model has a sequential structure that permits to estimate the structural parameters using a two stage procedure. First, we can estimate γ and μ using the second and fourth sets of moment conditions. In a second stage θ is estimated using the first and third set of moment conditions and the previous estimates of γ and μ. When γ and μ are just-identified in the first stage the expression of asymptotic covariance-matrix of the estimate of θ is straightforward using the approach proposed by Newey (1984 and 1994).

3. A MODEL OF PRICE AND INVENTORY DECISIONS
WITH FIXED ADJUSTMENT COSTS

3.1 Profit Function and Transition Probabilities

Consider a retailer that sells N different goods. At each period t the policy-maker decides the vectors of selling prices, $(p_{1t},..., p_{Nt})$, and new orders, $(q_{1t}, ..., q_{Nt})$, to maximize the discounted stream of expected current and future real profits. The one-period real profit, u_t, is the sum of the profits associated with each good, i.e., $u_t = \Sigma^N_{i=1} u_{it}$. The real profit that the firm obtains from selling any good i is equal to the real value of sales, minus ordering costs, storage costs and price adjustment costs.

$$(21) \quad u_{it} = (1+\pi)^{-t} \Big(p_{it} E_t y_{it} - c_{it} q_{it} - \eta^{(1)}_{it} I(q_{it}>0) - SC(s_{it};\alpha) - AC(p_{it}, p_{it-1};\phi) - \eta^{(2)}_{it} I(\Delta p_{it} \neq 0) \Big)$$

where π is the aggregate inflation rate (assumed constant); $E_t y_{it}$ are expected sales of good i at period t (in physical units) conditional to the information set at period t; c_{it} is the nominal cost price that supermarket pays to the manufacturer of good i; $\eta^{(1)}_{it}$ is a lump-sum ordering cost; SC(.) is the storage cost function that depends on the stock level at the beginning of the period, s_{it}, and on the vector of parameters α; AC(.) are variable price adjustment costs where ϕ is a vector of parameters; $\eta^{(2)}_{it}$ is a lump-sum price adjustment cost, and $\Delta p_{it} = p_{it} - p_{i,t-1}$.

Sales at period t are equal to demand only if the stock during the period is larger than demand. Therefore, one of the reasons for holding inventories in this model is to avoid stockouts. The data set used to estimate this model contains information on whether there has been an excess demand during the month.[5] This information allows to identify the dependence of sales with respect to the stock level and the importance of the stockout avoidance motive of holding stocks. The sales equation is

$$(22) \qquad y_{it} = (1 - I_{it})\, y^*_{it} + I_{it} \Big(s_{it} + (1-\lambda)(y^*_{it} - s_{it}) \Big)$$

where I_{it} is the indicator of the event "stockout of good i at period t"; y^*_{it} is the demand of good i at period t; and λ is the parameter that measures the proportion of demand that is lost when a stockout occurs. The demand of good depends on the real price of the good at this firm and at other firms selling good i.

(23) $$y_{it}^{*} = p_{it}^{-\gamma_{1i}} \left(m_{it}c_{it}\right)^{\gamma_{2i}} \exp\{\gamma_{0i} + \gamma_{0t} + a_{it}\}$$

where (γ_{0i}, γ_{1i}, γ_{2i}) are parameters associated to the demand of good i; m_{it} is the average markup of good i at period t in other stores; γ_{0t} is an "aggregate" shock affecting the demand of all the goods in this supermarket; and a_{it} is a good-specific demand shock that is unobservable to the firm at period t.

Given the previous specification of the profit function we can write the problem for good i in terms of the decision variables (p_{it}/c_{it} , q_{it}) and the vector of state variables (s_{it}, $p_{i,t-1}/c_{it}$, c_{it}, m_{it}, $\eta^{(1)}{}_{it}$, $\eta^{(2)}{}_{it}$). All these state variables are observed by the firm at the beginning of period t. From the point of view of the econometrician the vector x_{it} = (s_{it}, $p_{i,t-1}/c_{it}$, c_{it}) represents observed state variables, and (m_{it}, $\eta^{(1)}{}_{it}$, $\eta^{(2)}{}_{it}$) are unobservables. The variable sales and the stockout indicator are observables but they are not state variables (they are the variables y_t in Assumption 3 of Section 2).

The transition law for the stock is: $s_{i,t+1} = s_{it} + q_{it} - y_{it}$. Although the firm in our dataset is not a price-taker in its relationship with manufacturers the characteristics of the negotiation about cost prices make realistic to consider cost prices as exogenous state variables. Every year (around June) the supermarket bargains with each supplier the conditions that will hold during the next twelve months: cost price, discounts, trade promotions, possibility of returns, etc. Although in the final contract a cost price is fixed, the manufacturer keeps the right to change it if variations in manufacturing costs or in competition occur during the year. These changes usually occur in our dataset. Therefore, once the contract has been singned by the two parts (in some cases the negotiation does not end in an agreement) its conditions become predetermined or exogenous for the monthly or weekly decisions about orders and prices during the next twelve months. In addition, I assume that these decisions do not affect the cost price for the next year. This additional assumption seems also realistic for our dataset. Conversations with managers of the firm show that the taugher part of the negotiation is on the percentage of trade promotions that will be paid by the manufacturer, and on the form of payment (1 month, 2 months or 3 months after delivery), but not on cost prices that tend to be the same for all the large supermarkets chains in the area. Therefore, I assume cost prices are strictly exogenous state variables following a Markovian stochastic process.

The markup of good i in other supermarkets is, $m_{it} = m_i \, \zeta_{it}$ where ζ_{it} depends on aggregate shocks in the demand of good i, and therefore it is exogenous from the point of view of an individual firm. Thus, the demand of good i can be written as

$$y^*_{it} = p_{it}^{-\gamma_{1i}} \, c_{it}^{\gamma_{2i}} \, \exp\{\gamma^*_{0i} + \gamma_{0t} + \omega_{it}\} \tag{24}$$

where $\gamma^*_{0i} = \gamma_{0i} + \gamma_{2i} \ln(m_i)$ and $\omega_{it} = a_{it} + \gamma_{2i} \ln(\zeta_{it})$.

Finally, lump-sum adjustment costs are

$$(1+\pi)^{-1}\eta^{(j)} = \eta^{(j)} + \epsilon^{(j)} \tag{25}$$

where $\epsilon^{(j)}_{it}$ is a time and good specific component of the lump-sum cost j, that is independently distributed over (i,t) with zero mean. Thus, the conditional transition probability function of the state variables is

$$\begin{aligned} & p(dx_{i,t+1}, d\omega_{i,t+1}, d\epsilon_{i,t+1} | x_{it}, \omega_{it}, \epsilon_{it}) = \\ & q_\epsilon(d\epsilon_{i,t+1}) \; q_\omega(d\omega_{i,t+1}) \; I(s_{i,t+1} = s_{it} + q_{it} - y_{it}) \; q_c(dc_{i,t+1} | c_{it}) \end{aligned} \tag{26}$$

3.2 Numerical Solution of the Model

In order to obtain a better understanding of the model and its empirical implications I have solved it numerically. For the sake of computational simplicity the model has been solved under the following assumptions: (1) Aggregate inflation and cost price inflation are identical and deterministic, i.e.,

$c_{t+1}=(1+\pi)c_t$; (2) the demand is linear and has the form $y^*_t = \gamma_0 + \gamma_1 (p_t / c_t) + \omega_t$, where ω_t is iid $N(0, \sigma^2_\omega)$; and (3) lump-sum costs are $(1+\pi)^{-t} \eta^{(j)}_t = \eta^{(j)}$ for any t. Under these assumptions the decision variables are $(q_t , p_t/c_t)$ and the state variables are $(s_t , p_{t-1}/c_{t-1})$. In this simplified version of the model uncertainty comes from the unknown future demand shocks. Storage costs are quadratic in the stock level and variable price adjustment costs have not been considered. Thus, the one period profit function is

$$u_t = \frac{p_t}{c_t} E_t \min\left(s_t + q_t \,;\, \gamma_0 + \gamma_1 \frac{p_t}{c_t} + a_t \right) - q_t - \eta^{(1)} I(q_t > 0) - \alpha_1 s_t - \alpha_2 s_t^2 - \eta^{(2)} I\left(\frac{p_t}{c_t} \neq \frac{p_{t-1}}{(1+\pi) c_{t-1}} \right)$$

where α_1 and α_2 are the parameters of the storage cost function.

The following parameters have been fixed in all the numerical solutions: $\gamma_0 = 10$, $\gamma_1 = -5$, $\sigma^2_\omega = 1$, $\lambda=1$, $\pi = 0.10$, $\beta = 0.99$, and $\alpha_1 = 0.2$. The dynamic programming problem has been solved by the method of successive approximations to the value function using the Bellman's equation. The markup p/c has been discretized in 101 cells between 1.00 and 2.00, and stock and orders in 101 cells between 0 and 10. Figures 1 to 5 present the optimal supply and markup decisions as functions of the state variables in three different models: (1) $\eta^{(1)} > 0$ and $\eta^{(2)} = 0$; (2) $\eta^{(1)} = 0$ and $\eta^{(2)} > 0$; and (3) $\eta^{(1)} > 0$ and $\eta^{(2)} > 0$.

Figures 1 and 2 present the optimal supply (orders plus stock) and markup decision rules when there are lump-sum ordering costs but not price adjustment costs. The parameter $\eta^{(1)}$ has been fixed at 1.0, that is approximately the average real value of sales in one period. The model has been solved both with linear and quadratic storage costs. The optimal ordering decision is (S,s) where the lower threshold is 1.2 and the upper threshold 4.6. The optimal markup decision is more interesting. For values of the stock associated to positive orders (i.e., s <1.2) the optimal markup is 1.45, and it is invariant with the stock level at the beginning of the period. However, for values of the stock larger than the lower threshold the optimal markup is a decreasing function of the stock. Therefore, the markup is reduced when orders are placed and it increases during the period between two orders. This result holds both for $\alpha_2 = 0$ and $\alpha_2 > 0$. The explanation of this result is that price decision plays and important role in the reduction of lump-sum ordering costs. When the stock level approaches to the lower threshold it is optimal to increase the markup in order to reduce the probability of positive orders at the next period. Simulations of the decision and state variables from the previous numerical solution show an ordering frequency of 0.28, and a frequency of nominal price adjustemnts equal to one, where 79% are positive nominal price adjustments and 21% negative adjustments. The average change in markup is asymmetric: 0.18 for positive adjustments and -0.06 for negative ones. Therefore, even with an inflation rate of 10%, nominal prices are reduced in 21% of the periods.

Figure 3 and 4 present the optimal supply, (s+q), and markup as functions of the markup at the previous period when $\eta^{(1)} = 0$ and $\eta^{(2)} = 1$. Now, the stock at the beginning of the month is not a state variable (the one-period real profit can be writen in terms of s_t+q_t, p_t/c_t and p_{t-1}/c_{t-1}). The optimal markup follows an (S,s) decision rule. When markup at the previous period is above the lower threshold the optimal supply depends with negative sign on that markup. This effect operates through the negative effect of p_{t-1}/c_{t-1} on the expected sales at period t. Simulations from this solution show that the frequency of nominal price changes is now 0.75, and all price adjustments are postive. The average change in markup when there is a nominal price change is now 0.41.

The third version of the model is probably the most interesting. Now there are lump-sum adjustment costs both for placing orders and for changing nominal prices. The solution of this model depends crucially on the relative magnitudes of the two types of lump-sum costs and on the degree of economies of scale in these two costs (i.e., to what extent the lump-sum cost of adjusting the two variables is lower than the sum of the two individual lump-sum costs). Figure 5 explains the optimal decision rules when $\eta^{(1)} = 1.0$,

$\eta^{(2)}$=0.3, and there are not economies of scale in adjusting the two variables. The selection of this case has been made in order to show how the interaction of the two decisions can make nominal price reductions optimal even under high inflation and important fixed price adjustment costs.

In Figure 5 the point (s^*, m^*) represents the optimal supply and markup when orders are placed and prices are changed simultaneously. It is also the optimal decision in a model without lump-sum costs. The curve m(s) represents the optimal markup when only nominal prices are adjusted (no orders). Around the point (s^*, m^*), m(s) is a decreasing function. The frontier of the ovale A represents the combination of stock levels and markups that provide the same intertemporal profit that the profit derived from (s^*, m^*) given that there has been an adjustment in the two variables, i.e., $Fr(A) = \{ (s,m): V(s,m) = V(s^*, m^*) - \eta^{(1)} - \eta^{(2)} \}$. Points inside A imply an intertemporal profit larger than $V(s^*, m^*) - \eta^{(1)} - \eta^{(2)}$, and points outside A the opposite. The frontier of ovale B is, $Fr(B) = \{ (s,m): V(s,m) = V(s^*, m^*) - \eta^{(2)} \}$. The arrows show the dynamics of stocks and markup in this model. If orders are not placed and nominal price does not change, stock and markup decrease overtime. When these variables are inside ovale B the optimal decision is doing nothing. But when these variables are outside B but inside A it may become optimal to change the nominal price, but not placing orders. If that is the case, optimal markup will be larger than m^*. Finally, when the two variables arrive to a point outside the ovale A the optimal decision will be to make supply and markup equal to (s^*, m^*). In some cases this dynamics will imply reductions in nominal price.

Simulations of this version of the model show that nominal price reductions can be optimal in this context. In particular, the frequency of positive nominal price adjustments is 0.76 and the frequency for negative ones is 0.11. Ordering frequency is 0.26. When nominal price reductions occur these are always associated to positive orders. These nominal reductions become optimal because markup increases "too much" during the period between two orders. As the stock level drops down, markup increases in order to decrease the probability of placing orders (i.e., to decrease expected ordering costs).

3.3 Euler Equations and Optimal Discrete Choice

In this section I will present the equations that will be used in the estimation of the structural parameters of this model: Euler equations and optimal discrete choice. Consider that at period t orders for good i have been placed and define τ^q_{it} as the number of periods until the next order of good i. Analogously consider that at period t there has been a change in the nominal price of good i and define τ^p_{it} as the number of periods until the next change in the price of good i. Applying expression (11) to this model we can obtain the following Euler equation for orders

$$(27)\qquad E_t\left(-c_{it} - \sum_{n=1}^{\tau^q_{it}} \beta^n \frac{\partial SC_{i,t+n}}{\partial s_{i,t+n}} + \beta^{\tau^q_{it}} (1 - I_{i,t+\tau^q_{it}})\, c_{i,t+\tau^q_t} + \beta^{\tau^q_{it}} \lambda\, I_{i,t+\tau^q_{it}}\, p_{i,t+\tau^q_t} \right) = 0$$

and the Euler equation for the price decision is

$$(28)\qquad E_t\left(y_{it}(1-\gamma_1[1-\lambda I_{it}]) - \frac{\partial AC_{it}}{\partial p_{it}} + \sum_{n=1}^{n} \beta^n y_{i,t+n}(1-\gamma_1[1-\lambda I_{i,t+n}]) + \sum_{n=1}^{\tau^p_{it}} \beta^n \frac{\partial SC_{i,t+n}}{\partial s_{i,t+n}} \frac{y_{i,t+n}}{p_{it}} \gamma_1[1-\lambda I_{i,t+n}] + \beta^{\tau^p_{it}} \frac{\partial AC_{it+\tau^p_{it}}}{\partial p_{it}} \right) = 0$$

The discrete choice problem can be defined in terms of two indicator variables:: D^q=I(q >0) and $D^{\Delta p}$=I(Δp>0) - I(Δp<0). Based on two indicators we can define the six possible discrete choice alternatives: { (j,k) : j = 0,1 and k=-1,0,1 }. The optimal discrete choice can be written as in equation (20) where now

[illegible]

and

$$g^{jk}(x_{it}, y_{it}) = \left(p_{it}\, y_{it} \;;\; -c_{it}\, q_{it} \;;\; -s_{it} \;;\; -s_{it}^2 \;;\; -|\Delta \ln p_{it}| \;;\; -|\Delta \ln p_{it}|^2 \;;\; -D^q_{it} \;;\; -|D^{\Delta p}_{it}| \right) \mid D^q_{it} = j \text{ and } D^{\Delta p}_{it} = k \;)$$

where quadratic specifications for the storage costs and variable price adjustment costs functions have been considered. Using these $g^{j,k}$ vectors the $\Pi^{j,k}_g$ vector has also 8 components

$$\Pi^j_g = (\Pi^j_{py} , \Pi^j_{og} , \Pi^j_{s} , \Pi^j_{s^2} , \Pi^j_{Alpp} , \Pi^j_{A^2} , \Pi^j_{Dc} , \Pi^j_{Dp})$$

4. DATA AND PRELIMINARY EVIDENCE

The model in Section 3.1 has been estimated using a data set from the central store of a supermarket chain operating in northern Spain. Some characteristics of this supermarket chain are described in Appendix 1. In this section I will describe the data set. The estimation has been performed using a subsample of 534 goods that have been sold in the supermarket chain at each month during the period January-1990 to May-1992 (29 months).

The data set contains the following information for each good and month: name and description, weight of an item, cost price (i.e., price paid to the manufacturer), selling price (i.e., price to which outlets sell the good), trade promotions, sales or shipments from the central store to the outlets, orders from the central store to manufacturers, stock in the central store at the beginning of the month, and orders from outlets to central store which were not attended due to a stockout in the central store. All prices are measured in pesetas per item, and they are monthly averages. Quantities are measured in number of items.

4.1 Descriptive Analysis of Price and Inventory Decisions

In this sample the average monthly rate of inflation in cost prices is 0.46%, that implies a 5.15% annual inflation rate. The annual aggregate inflation rate during this period was 7.3% in 1990, 6.9% in 1991, and 6.1% in 1992 (Retail Price Index). The classification of the goods in products and groups of products has followed the criterium used by the firm in its database. Goods are classified in 77 *products* (groups of goods that differ only in their brands, sizes and/or qualities), and these in 7 large groups.

Table 1 presents some descriptive statistics obtained for all the 534 goods and for each of the seven groups. Table 2 shows the same statistics but for the 77 products. Columns 2 to 4 present statistics related to the inventory policy, and the statistics in columns 5 to 10 refer to the price decision. Column 2 presents the average number of months between two consecutive orders. For the whole sample, orders are placed every 1.377 months. Column 3 presents the ratio between the stock level when orders are placed and average sales. This statistic is a proxy for the lower threshold in a (S,s) inventory model. Another statistic related to this lower threshold is the frequency of stockouts, in column 4. This probability is relatively large for most of the goods, but it is important to keep in mind that a stockout in the central store will not always imply stockouts in the outlets of the supermarket chain.

In column 5 the average markup for the whole sample is 16.6%, that shows the low price-cost margins in this industry. Column 6 presents the average number of months between two consecutive price changes. For each group this statistic is lower than the average number of months between two orders (column 2). This evidence holds when we look at each of the 534 goods, and it is an important preliminary evidence in favour of lump-sum ordering costs larger than lump-sum price adjustment costs. We have shown in Section 3.2 how in that context the interplay of inventory and pricing policies can explain nominal price reductions even with relatively large rates of inflation.

Columns 7 and 8 split the frequency of price changes in positive and negative ones. It is remarkable that the frequency of negative price changes is relatively large in a period of positive inflation. Part of these nominal price reductions can be the result of decreasing cost prices. For that reason columns 9 and 10 present the frequency of increases and reductions in cost prices, respectively. Comparing columns 8 and 10 we can see that for each of the seven groups (and for almost evey product in Table 2) there is a diffrence of around 20 percentage points between the frequency of selling price reductions and that for cost prices. In other words, 60% of all the reductions in nominal selling prices cannot be expalined by decreasing cost prices.

It is plausible to think that at least part of these "cuts" in selling prices are the result of changes in the degree of competition in the market. However, there are at least three reasons to suspect that war of prices cannot explain all (even most) of these nominal price reductions. First, these reductions are homogenously distributed over time. For 26 of the 28 months for which changes in prices can be obtained, the proportion of goods with nominal reductions in selling prices is between one standard deviation around the mean (i.e., between 0.301 and 0.423). Second, looking at each of the 77 products, there is not evidence of synchronization in price reductions for goods within the same product. In particular, for most of the products (even for those where competition between supermarkets tends to be taugher: milk, olive oil, detergents or carbonated soft drinks) the proportion of goods with price reductions is between 10% and 50% at almost each month. It means that, for price wars to be explaining nominal price reductions, the supermarket should decide to decrease its markup only for particular brands, and the selection of these brands is not always the same.

A third reason against an explanation of the nominal reductions based on changing competition is the evidence about the state dependence of price reductions with respect to the stock level. Table 3 presents evidence about this state dependence. The average markup for the whole sample is 0.166 but it is the result of different markups for different stock levels. When the stock level decreases between two orders, the markup increases. This increment is always around 3 percentage points, whatever the number of periods between the two consecutive orders. When orders are placed selling price tend to be reduced and the cycle starts again. This is exactly the path of real prices that our theoretical model predicts when lump-sum ordering costs are larger than lump-sum price adjustment costs.

4.2 State Dependence of Price and Inventory Decisions

Table 3 presents more evidence about the state dependence of price and inventory decisions. Discrete and continuous decision variables are regressed on the state variables $\ln(s_{it})$, $\ln(p_{i,t-1}/c_{it})$, and $\ln(c_{it})$. In order to control for spurious state dependence due to unobserved heterogeneity I have included individual *good-dummies* both in the Probit models and in the linear selection equations. Given that the number of observations per good is relatively large (i.e., 28) and the number of goods is not very large (i.e., 534), this method yields accurate estimates and the *incental parameters bias* should be small.

Estimates in columns 1 and 2 present a state dependence of the orders decision that is completely consistent with an (S_t,s_t) inventory policy where the lower threshold depends on current cost price, and S_t-s_t depends on current cost price and markup at the beginning of the month. In the price adjustment probit models (columns 3 and 5) the markup at the beginning of the month is the state variable with larger explanatory power and it has the expected effect. The larger p_{t-1}/c_t the lower the probability of a positive price adjustment and the larger the probability of a negative one. The stock level has a significant effect on the probability of both types of nominal price adjustments. When the stock is large (just after an order) the firm reduces the selling price. But when the stock starts to decrease price increases become more likely. Again, this evidence is consistent with the version of our theoretical model where there are lump-sum adjustment costs in placing orders and changing nominal prices, but the first type of costs are quantitatively more important than the second.

Columns 4 and 6 present the regression equations for markup when there are positive and negative price adjustments, respectively. I have controlled for selection bias *ala Heckman*. The markup at the beginning of the month has a positive and significant effect on the optimal markup during the month, both under positive and negative price adjustments. Given that we have controlled for unobserved heterogeneity, the significant effect of markup at the beginning of the month might be the result of variable price adjustment costs. The stock level has a negative and significant effect on the optimal markup when there is a positive adjustment, but not when price adjustment is negative. This result can also be explained by the theoretical model. Price increases tend to occur between two orders. At these periods the stock level determines the supply during the month and affects the optimal markup. Price reductions tend to occur when orders are placed. In that case the stock at the beginning of the month has a negligible effect on the supply during the month, and so it does not affect the optimal markup.

5. ESTIMATION OF THE DYNAMIC DECISION MODEL

This section presents estimates of the optimal discrete choice in the structural model. Section 5.1 and Appendix 2 describes the estimation of the incidental parameters, and Section 5.2 presents estimates for structural parameters. Finally, Section 5.3 shows some measures of the goodness of fit of the model.

5.1 Estimation of the Incidental Parameters

To estimate the econometric model derived from the optimal discrete choice, using the method described in Section 2, one has to obtain estimates for the vectors $\Pi^{j,k}{}_{g}$ and $\Pi^{j,k}{}_{\epsilon}$. These vectors depend on the incidental parameters μ, that is, conditional choice and transition probabilities and conditional expectations for orders, selling prices and sales.

The range of variation of sales, stocks, orders and prices is different for the different goods in the sample. However, it is straightforward to write the model in terms of the state variables (s_{it}/s_i, $p_{i,t-1}/c_{it}$, c_{it}/c_{i0}) and the decision variables (q_{it}/s_i , p_{it}/c_{it}), where s_i is the average stock for good i, and c_{i0} is the initial value for the cost price (see Appendix 2). All these variables have very similar empirical distributions for the different goods.

Incidental parameter estimates have been obtained using a Kernel method. The reason for using this type of nonparametric estimator is that a simple cell frequency estimator provides inefficient estimates in those cells of the state space where there are few observations. Hotz et al (1994) have shown that the finite sample properties of the CCP and CCS estimators are clearly improved when using Kernel estimates of the incidental parameters. These estimates have been obtained at percentiles 10, 30, 50, 70 and 90 of each of the state variables, what implies a space of 125 cells for the observed state variables (i.e., 5x5x5).

Notice that without imposing any of the restrictions that the structural model implies the number of transition probabilities to be estimated is 74,400 (i.e., 124x125x4). However, as most structural models, the model in Section 3 imposes a large number of restrictions on the transitions probabilities.

(a) Cost prices are strictly exogenous.

$$p(x_{i,t+1} \mid x_{it}, D^{q}_{it}= j \,;\, D^{\Delta p}_{it}=k) = p(s_{i,t+1}, p_{it}/c_{it} \mid x_{it}, D^{q}_{it}= j \,;\, D^{\Delta p}_{it}=k)\; p(c_{i,t+1} \mid c_{it})$$

It reduces the number of probabilities to 14,425.

(b) We can write $p(s_{i,t+1}, p_{it}/c_{it} \mid x_{it}, D^{q}_{it}= j \,;\, D^{\Delta p}_{it}=k)$ as the product of two transition probabilities:

$$p(s_{i,t+1} \mid p_{it}/c_{it}, s_{it}, D^{q}_{it}= j)\; p(p_{it}/c_{it} \mid s_{it}, p_{i,t-1}/c_{i,t-1}, c_{it}, D^{q}_{it}= j \,;\, D^{\Delta p}_{it}=k)$$

When there are not orders, the first transition probability (stocks) is a block upper-triangular matrix. In addition, it depends on whether orders are placed or not, but conditional to p/c it does not depend on whether prices are changed. The second transition probability (markup) is known for $D^{\Delta p}_{it}=0$ (no change in prices).

Taking into account (a) and (b) the total number of transition probabilities to be estimated is 1,175. It is a feasible problem given the 15486 good-month observations in the sample. Details about the estimation of the incidental parameters are described in Appendix 2.

Table 5 presents descriptive statistics for the estimated Π values entering in the intertemporal profit function. These values have been obtained for different values of β, 0.985, 0.990, 0.995 and 0.999.

Although the values of $\Pi^{j,k}g_{it}$ and $\Pi^{j,k}\epsilon_{it}$ change very much for different values of β, the differences ($\Pi^{j,k}g_{it}$- $\Pi^{00}g_{it}$) and ($\Pi^{j,k}\epsilon_{it}$- $\Pi^{00}\epsilon_{it}$) and the structural parameter estimates are very robust to changes in β. Table 5 shows how the differences in profits associated to each choice alternative do not only arise from differences in the current profits. There are important intertemporal effects. Thus, the difference between the expected values of future variable ordering costs when orders are placed and when they are not is between US$600 and US$900, when the stock level is at percentile 10 (see the fourth panel). The fifth panel shows the expected value of current and future sales minus current and future variable ordering cost. If they represented all the relevant costs and revenues, placing orders and changing prices would be the optimal choice for values of the stock even above percentile 50. If that were the case, the estimates in this panel would predict that D_q=1 and D_p≠0 would have a sample frequency equal to 72.6%. However, this alternative represents 53.17% of the observed choices in the sample. Therefore, there may be other costs associated to placing orders or/and changing prices. Using the estimates in this panel we can also obtain that for almost 5% of the observations the optimal choice would be not placing orders. Thus, even if there were not lump-sum adjustment costs, the "irreversibility" of the inventory investment would explain around one fourth of the observed infrequency in placing orders.

The last two panels in Table 5 show the expected and discounted value of the number of times orders will be placed (panel six) and prices will be changed (panel seven) in the future. They also show important intertemporal effects of the current decisions. Thus, for example, placing orders when the stock is lower than percentile 30 implies a saving in future ordering costs (variable and fixed) approximatelly equal to 25% of the average ordering costs of one month.

5.2 Estimation of the Discrete Choice

Table 6 presents 3-stage GMM estimates of the structural parameters based on the moment conditions for the optimal discrete choice. There are five sets of moment conditions, one for each choice alternative (except one that is redundant). In the second and third stages estimates are obtained using as weighting matrix the inverse of the estimated disturbance covariance matrix. The assumptions for the estimation of this covariance matrix are: (1) no correlation across choice alternatives; (2) no correlation across different goods; (3) homoscedasticity over time; and (4) autocorrelation up to second order. I allow for conditional between-goods heteroscedasticity, and different covariance matrices for each choice alternative. The instrumental variables are the three state variables at t and t-1, and their square and cubic powers.

Notice that, without exploiting the continuous information from the Euler equations, it is possible to obtain accurate estimates for most of the structural parameters. In fact, this empirical application is an example of how the discrete choice can contain more behavioural information than the continuous choice. In this case the reason for that result is that, conditional to the discrete choice, orders and markup do not have very much time variability. Results from the estimation of the Euler equations can be found in Aguirregabiria (1995).

The specification for the storage cost function is $\alpha_1 \, (s_{it} / \bar{s}_i) + \alpha_2 \, (s_{it} / \bar{s}_i)^2$, and for variable price adjustment costs, $\phi_1 \, |\Delta \ln p_{it}| + \phi_2 \, |\Delta \ln p_{it}|^2$. The overidentifying restrictions cannot be rejected with a 30% significance level. The quadratic term in the storage cost function is clearly not significant. The estimate of α_1 says that when the stock increases in an amount equivalent to 100% of its mean value, storage costs increase in US$ 31.86. The linear term in the variable price adjustment costs is significant but quantitatively not very important. The quadratic term is not significant.

Probably the most interesting estimates are those of the lump-sum adjustment costs. Notice that these parameters are identified not just as the constant terms associated to the intertemporal profit of each alternative. They are also associated to the values $\Pi^{j,k}_{Dq}$ and $\Pi^{j,k}_{Dp}$. Lump-sum ordering costs are significant and quantitatively very important. They are around US$ 290, that amounts 4% of the mean value of monthly sales of a good in the sample. Fixed price adjustment costs are also significant but quantitatively not very important (US$ 54, that is 0.8% of the mean value of monthly sales). The estimation allows for economies of scale, that is, it does not impose $\eta^{(3)} = \eta^{(1)} + \eta^{(2)}$. Although point estimates show that $\eta^{(3)}$ is lower than $\eta^{(1)} + \eta^{(2)}$, the null hypothesis $\eta^{(3)} = \eta^{(1)} + \eta^{(2)}$ cannot be rejected even with large significance levels.

5.3 Goodness of Fit of the Model

The test of overidentifying restrictions gives a too general idea about the fit of the model. A better picture of its goodness of fit is given by the percentage of correct out-of-sample predictions. Table 7 presents the relative frequencies at each of the cells that result from crossing actual choices and predicted choices. I also compare the fit of the structural model with that from the Kernel estimates of the conditional choice probabilities. The two models have been estimated using the 534 goods during the first 20 months of the sample. The predicted choices have been obtained for the sample of 534 goods during the last 7 months of the sample.

First, both the reduced form and the structural model provide very good accurate predictions for the decision of placing orders. However, the goodness of fit is not very good for the decision about prices, specially for the reduced form model. In particular, the reduced form tends to underpredict the decision of not changing prices. This is also true for the structural model, but the improvement with respect to the reduced form is important.

Finally, predictions of the structural model has been obtained fixing lump-sum ordering costs to zero. In that case the model only predicts a 5% of observations where orders are not placed. Therefore, even without lump-sum ordering costs, the irreversibility of inventory investment explains 25% of the no adjustments. The remaining 75% is the result of lump-sum ordering costs.

6. CONCLUSIONS

In this paper I have presented a method to estimate dynamic structural models with corner solutions. The main advantage of this method is that alternative specifications of the structural model can be estimated with a very small additional computational cost. The method has been applied to estimate a model of joint price and inventory decisions with fixed adjustment costs. Lump-sum costs associated to placing orders and changing prices are significant, but the first one is quantitatively more important. In this context, the theoretical model predicts that nominal price reductions can be optimal even when the inflation rate is high. This result explains why in our dataset nominal price reductions are so frequent, and they tend to be associated to positive orders. The large lump-sum ordering costs make optimal increasing markup when the stock is low, in order to reduce ordering frequency and orderng costs. When orders are placed it becomes optimal to reduce the markup and, in some cases, it implies a nominal reduction. Finally, the irreversibility of the inventory investment (i.e., $q \geq 0$) can explain 25% of the infrequency in placing orders. The remaining 75% is the result of lump-sum ordering costs.

APPENDIX 1.

The Supermarket Chain

The supermarket chain operates only in the basque region of Spain and consists of around 60 outlets. Some of these outlets are owned and others are franchises. These outlets have very different sizes; most of them are small groceries but there are also big supermarkets. The supermarket industry in this area is characterized by the existence of a clear leader who captures more than fifty percent of the market. Our firm is one of the five most important supermarket chains in the region in terms of market share. This firm has always been characterized by its specialization in low and average income customers.

The central store is also the headquarters of the firm: it orders new deliveries from suppliers, stores the goods, sends to the outlets the orders that they have made, and decides upon selling prices. The relationship between the central store and the outlets is the following. At the beginning of each week the central store sends to every outlet a list of available goods and their respective selling prices. In order to ask for new deliveries an outlet has to communicate with the central store by modem. If the goods are available the orders will be delivered to the outlet within the next twenty-four hours. If the central store runs out of some good which an outlet has ordered the situation is communicated to the outlet.

The Data Set

The firm has maintained a data base since January 1990. Some information is recorded daily but after one month only monthly data are kept. I consider the period from January 1990 to May 1992 (29 months). During this period 8742 goods were sold by the supermarket chain and the raw data set contains information for all these individual goods. However, many of these goods are not stored by the firm. For these goods, the central store makes orders to the manufacturers and they (using their own trucks) deliver the orders to the different outlets of the supermarket chain. From the whole set of goods 4966 are stored by the central store.

Each good has been sold by the central store for a particular period of time. Some goods have been sold for the whole sample period (29 months), but most of them have been sold for shorter periods (even just one month). When the policy maker decides that a good will not be sold in the future there will not be orders for that good. Thus, after placing last order, the decision of not placing an order is not the result of an optimal inventory policy, but the result of the decision that the good will not be sold in the future. Therefore, only observations between the first and the last order can be considered. This restriction implies that the information from many goods which were ordered just in one period cannot be used. Finally, in order to have enough number of observations per good, only those goods which have been sold during the whole sample period are used (534 goods). It implies a total of 15,486 good-month observations.

APPENDIX 2

Estimation of the Incidental Parameters

The one-period profit function of the model in Section 3.1 can be wtitten as,

$$u_{it} = s_i \frac{c_{it}}{c_{i0}} \left(\frac{p_{it}}{c_{it}} \frac{y_{it}}{s_i} - \frac{q_{it}}{s_i} \right) - \alpha_1 \left(\frac{s_{it}}{s_i} \right) - \alpha_2 \left(\frac{s_{it}}{s_i} \right)^2 - \phi_1 \left(\ln\frac{p_{it}}{c_{it}} - \ln\frac{p_{it-1}}{c_{it}} \right) - \phi_2 \left(\ln\frac{p_{it}}{c_{it}} - \ln\frac{p_{it-1}}{c_{it}} \right)^2$$

$$- \eta^{(1)} I\left(\frac{q_{it}}{s_i} > 0 \right) - \eta^{(2)} I\left(\frac{p_{it}}{c_{it}} \neq \frac{p_{i,t-1}}{c_{i,t-1}} \right)$$

where s_i is the average stock of good i, and c_{i0} is the cost price of good i at the initial period. Therefore, the model can be defined in terms of the state variables, $x_{it} = (s_{it}/s_i,\ p_{i,t-1}/c_{it},\ c_{it}/c_{i0})$, and the decision variables $(p_{it}/c_i,\ q_{it}/s_i)$. In our sample these variables have very similar empirical distributions for the different goods.

All the incidental parameters have been estimated using Kernel methods. Thus, the estimators for the three components of the transition probabilities are,

$$\hat{F}_c(c'|c) = \frac{\sum_{i=1}\sum_{t=2} K_2([c_{it}, c_{i,t-1}] - [c', c]\,; b)}{\sum_{i=1}^{N}\sum_{t=2}^{T} K_1(c_{it-1} - c\,; b)}$$

$$\hat{F}_p(p'|x, D^j{=}1) = \frac{\sum_{i=1}^{N}\sum_{t=2}^{T} D_{it}^{j}\, K_4([p_{it}, x_{it}] - [p', x]\,; b)}{\sum_{i=1}^{N}\sum_{t=2}^{T} D_{it}^{j}\, K_3(x_{it} - x\,; b)}$$

$$\hat{F}_s(s'|x, D^j{=}1) = \frac{\sum_{i=1}^{N}\sum_{t=2}^{T} D_{it}^{j}\, K_4([s_{it+1}, x_{it}] - [s', x]\,; b)}{\sum^{N}\sum^{T} D_{it}^{j}\, K_3(x_{it} - x\,; b)}$$

where $K_q(.;\, b)$ is the pdf of the q-variate normal distribution, where the q variables are not correlated and they have standard deviation equals to b. The conditional choice probabilities are obtained using the same type of estimator. For the conditional expectations $E(p/c\ y/s \mid x, D)$ and $E(q/s \mid x, D)$ the estimator is the Nadaraya-Watson Kernel estimator. The bandwith parameter has been chosen to maximize a criterium of "out of sample" goodness of fit. In particular, conditional choice probabilities have been estimated using observations for the first 20 months, and the goodness of fit has been evaluated using the last 10 months. The bandwith that maximizes the number of correct predictions was 0.03.

Computing the Π's values requires to have a discrete range of variation for the state variables. For that reason the previous estimators have been evaluated at 125 different values for the vector of state variables x. These 125 cells correspond to 5 cells for each stae variable, i.e., percentiles 10, 30 50, 70 and 90.

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TABLE 1

BASIC STATISTICS
Means January 1990 to May 1992
7 Groups

	(1) Number of goods	(2) Average τ^q	(3) Ratio E(s|q>0) over E(x)	(4) Freq. I=1	(5) Markup	(6) Average τ^p	(7) Freq. Δ p>0	(8) Freq. Δ p<0	(9) Freq. Δc > 0	(10) Freq. Δc < 0
All goods	534	1.377	0.599	0.293	0.166	1.294	0.441	0.362	0.209	0.138
1. Health & Beauty	48	1.428	0.728	0.270	0.242	1.301	0.443	0.339	0.198	0.086
2. Cleaning	69	1.307	0.589	0.313	0.168	1.288	0.449	0.351	0.228	0.131
3. Pharmacy	9	2.083	0.953	0.260	0.278	1.299	0.472	0.353	0.195	0.061
4. Others (Not Food)	13	1.352	0.556	0.312	0.279	1.187	0.447	0.403	0.186	0.103
5. Soft Drinks	29	1.173	0.397	0.370	0.184	1.282	0.423	0.379	0.213	0.218
6. Food	277	1.336	0.577	0.302	0.162	1.326	0.432	0.360	0.211	0.147
7. Alcoholic Drinks	89	1.531	0.642	0.237	0.104	1.215	0.466	0.380	0.191	0.130

TABLE 2
DESCRIPTIVE STATISTICS FOR THE 77 PRODUCTS
Means January 1990 to May 1992

Freq. Freq. $\Delta c < 0$	(1) Number of goods	(2) Average	(3) τ^q over E(x)	(4) Ratio E(s|q>0)	(5) Freq. I=1	(6) Markup	(7) Average τ^p	(8) Δ p>0	(9) Freq. Δ p<0	(10) Freq. Δc > 0
All goods	534	1.377	0.599	0.293	0.166	1.294	0.441	0.362	0.209	0.138
1. Health & Beauty	48	1.428	0.728	0.270	0.242	1.301	0.443	0.339	0.198	0.086
101. Eau Cologne	2	2.000	0.661	0.172	0.277	1.288	0.410	0.375	0.155	0.103
102. Deodorant	2	1.386	1.112	0.137	0.190	1.110	0.446	0.446	0.207	0.086
103. Soap & Shampoo	1	1.484	0.803	0.282	0.216	1.309	0.439	0.371	0.193	0.103
104. Hair spray	6	1.339	0.545	0.287	0.270	1.275	0.482	0.315	0.235	0.097
105. Beauty creams	9	1.407	0.644	0.310	0.233	1.241	0.472	0.333	0.238	0.100
106. Tooth-paste	3	1.201	0.585	0.367	0.174	1.182	0.547	0.309	0.287	0.138
107. Shaving creams	3	1.254	0.525	0.264	0.236	1.459	0.404	0.285	0.138	0.046
108. Shaving blades	8	1.193	0.622	0.314	0.227	1.381	0.383	0.330	0.168	0.065
109. Bath sponges	5	1.867	1.197	0.124	0.355	1.352	0.421	0.321	0.138	0.034
2. Cleaning	69	1.307	0.589	0.313	0.168	1.288	0.449	0.351	0.228	0.131
201. Detergent	12	1.423	0.752	0.201	0.103	1.190	0.455	0.404	0.216	0.158
202. Bleach	12	1.062	0.377	0.428	0.243	1.390	0.428	0.297	0.244	0.135
203. Softeners	7	1.204	0.369	0.369	0.066	1.128	0.474	0.418	0.305	0.217
204. Dishwashers	5	1.376	0.465	0.351	0.079	1.150	0.528	0.357	0.241	0.165
205. Cleaning sprays	15	1.251	0.561	0.331	0.163	1.278	0.466	0.347	0.230	0.131
206. Floor Polish	7	1.311	0.632	0.231	0.212	1.580	0.403	0.285	0.202	0.064
207. Scourers	4	1.160	0.525	0.379	0.217	1.324	0.428	0.321	0.215	0.112
208. Dishcloths	2	1.636	1.711	0.206	0.267	1.250	0.410	0.410	0.172	0.069
209. Brooms	1	1.037	0.751	0.241	0.215	1.190	0.500	0.285	0.103	0.000
210. Insecticides	3	2.200	0.881	0.183	0.288	1.275	0.380	0.392	0.184	0.011
211. Kitchen paper	1	1.529	0.452	0.448	0.197	1.227	0.464	0.357	0.138	0.138

3. Pharmacy	9	2.083	0.953	0.260	0.278	1.299	0.472	0.353	0.195	0.061
301. Bandages	4	3.050	1.414	0.146	0.289	1.199	0.508	0.330	0.181	0.043
302. Sanitary napkins	2	1.361	0.458	0.431	0.356	1.182	0.464	0.392	0.224	0.138
303. Diapers	3	1.276	0.667	0.298	0.213	1.511	0.428	0.357	0.195	0.034
4. Others (Not Food)	13	1.352	0.556	0.312	0.279	1.187	0.447	0.403	0.186	0.103
401. Aluminium paper	6	1.528	0.615	0.241	0.241	1.131	0.452	0.434	0.167	0.126
402. Plastic objects	2	1.303	0.394	0.413	0.368	1.273	0.357	0.446	0.224	0.103
403. Batteries	4	1.190	0.528	0.293	0.280	1.253	0.455	0.348	0.190	0.076
404. Stick gum	1	1.037	0.633	0.620	0.332	1.080	0.571	0.357	0.207	0.069
5. Soft Drinks	29	1.173	0.397	0.370	0.184	1.282	0.423	0.379	0.213	0.218
501. Fruit Juices	10	1.086	0.411	0.320	0.199	1.374	0.417	0.357	0.190	0.186
502. Water	3	1.041	0.363	0.471	0.172	1.280	0.404	0.357	0.241	0.161
503. Carb. drink(2 L)	5	1.181	0.316	0.455	0.172	1.215	0.414	0.407	0.207	0.275
504. Carb. drink (small)	11	1.285	0.430	0.351	0.180	1.230	0.438	0.392	0.229	0.238

TABLE 2 (Cont.)

	# Goods	τ(q)	s/x	I	Markup	τ(p)	Δp>0	Δp<0	Δc>0	Δc<0
6. Food	277	1.336	0.577	0.302	0.162	1.326	0.432	0.360	0.211	0.147
601. Sugar	4	1.217	0.546	0.327	0.119	1.414	0.366	0.366	0.138	0.026
602. Coffee	12	1.140	0.431	0.327	0.103	1.751	0.345	0.386	0.178	0.212
603. Instant Coffee	10	1.471	0.597	0.234	0.164	1.152	0.442	0.428	0.203	0.203
604. Malta	6	1.072	0.504	0.241	0.196	1.566	0.386	0.285	0.132	0.029
605. Tea	2	1.563	1.174	0.137	0.204	1.132	0.428	0.464	0.103	0.069
606. Olive Oil	7	1.337	0.513	0.413	0.036	1.113	0.500	0.382	0.281	0.207
607. Sunflower Oil	4	1.249	0.373	0.474	0.052	1.140	0.428	0.455	0.233	0.310
608. Canned fish	11	1.415	0.486	0.376	0.142	1.183	0.448	0.415	0.185	0.172
609. Dry cod	3	1.239	0.649	0.275	0.157	1.387	0.511	0.226	0.161	0.011
610. Canned veget.	8	1.383	0.566	0.362	0.148	1.122	0.486	0.410	0.198	0.151
611. Marmalade	9	1.334	0.488	0.295	0.184	1.152	0.456	0.424	0.207	0.153
612. Beans &Lentils	9	1.095	0.423	0.448	0.156	1.313	0.472	0.353	0.234	0.123
613. Rice	4	1.677	0.713	0.189	0.071	1.360	0.410	0.357	0.198	0.172
614. Olives	6	1.207	0.574	0.281	0.156	1.357	0.404	0.333	0.167	0.161
615. Pickles	4	1.181	0.478	0.336	0.272	1.434	0.339	0.348	0.172	0.129
616. Biscuits	23	1.277	0.455	0.295	0.161	1.210	0.468	0.371	0.220	0.162
617. Cakes	5	1.088	0.269	0.462	0.191	1.436	0.400	0.321	0.186	0.117
618. Candies	11	1.373	0.782	0.257	0.249	1.351	0.435	0.321	0.188	0.081
619. Chocolate	30	1.333	0.550	0.259	0.162	1.398	0.409	0.346	0.190	0.132
620. Toasted Bread	6	1.198	0.434	0.379	0.204	1.263	0.452	0.369	0.190	0.103
621. Flour	2	1.633	0.506	0.120	0.221	1.385	0.357	0.375	0.138	0.034
622. Pasta	14	1.743	1.252	0.179	0.150	1.186	0.548	0.298	0.300	0.110
623. Instant soups	11	1.875	0.612	0.203	0.119	1.151	0.525	0.350	0.329	0.135
624. Instant Rice	5	1.505	0.642	0.200	0.164	1.811	0.328	0.228	0.193	0.138
625. Dried fruits	6	1.315	0.877	0.160	0.211	1.181	0.488	0.345	0.270	0.109
626. Pets food	15	1.564	0.821	0.197	0.234	1.717	0.340	0.273	0.138	0.094
627. Salt	3	1.412	0.546	0.229	0.239	2.071	0.285	0.190	0.092	0.022

628. Mayonnaise	9	1.176	0.426	0.371	0.171	1.082	0.460	0.464	0.238	0.210
629. Vinager	2	1.120	0.545	0.413	0.222	1.554	0.375	0.285	0.207	0.138
630. Eggs	3	1.000	0.148	0.712	0.143	1.272	0.345	0.440	0.218	0.184
631. Butter, Margar.	6	1.012	0.401	0.362	0.146	1.196	0.434	0.398	0.264	0.207
632. Cheese	11	1.211	0.488	0.322	0.152	1.152	0.451	0.425	0.263	0.225
633. Milk	6	1.025	0.302	0.574	0.093	1.240	0.380	0.428	0.241	0.293
634. Powder Milk	5	1.295	0.674	0.255	0.143	1.555	0.428	0.264	0.234	0.089
635. Sausages	3	1.013	0.491	0.402	0.195	1.221	0.369	0.440	0.230	0.241
636. Foie-gras	2	1.275	0.453	0.413	0.221	1.337	0.446	0.285	0.275	0.138
7. Alcoholic Drinks	89	1.531	0.642	0.237	0.104	1.215	0.466	0.380	0.191	0.130
701. Cognac	10	1.655	0.588	0.255	0.051	1.096	0.546	0.378	0.210	0.083
702. Whisky	4	1.286	0.465	0.336	0.046	1.040	0.544	0.419	0.267	0.129
703. Other Spirits	12	1.497	0.717	0.189	0.069	1.113	0.482	0.422	0.204	0.138
704. Sherry	5	1.354	0.661	0.179	0.129	1.524	0.400	0.285	0.145	0.062
705. Champagne	9	1.652	0.569	0.218	0.071	1.078	0.517	0.386	0.207	0.098
706. Vermouth	4	1.202	0.367	0.318	0.089	1.155	0.473	0.401	0.250	0.138
707. Sidra	3	1.144	0.279	0.252	0.125	1.280	0.428	0.345	0.184	0.057
708. Beer	12	1.244	0.493	0.318	0.149	1.365	0.419	0.324	0.193	0.169
709. Wine (Regular)	15	1.196	0.457	0.324	0.129	1.135	0.504	0.383	0.209	0.204
710. Wine (Quality)	18	2.154	1.055	0.113	0.122	1.319	0.400	0.402	0.134	0.103

TABLE 3

THE BEHAVIOUR OF MARKUPS BETWEEN TWO ORDERS

Subsample of observations with $\tau^q_{it} = 1$ (# Observations = 9365)	Mean Markup	Std. Deviation
Period 0 (when orders are placed)	0.1656	0.1059
1 month after (next orders)	0.1641	0.1053

Subsample of observations with $\tau^q_{it} = 2$ (# Observations = 1322)	Mean Markup	Std. Deviation
Period 0 (when orders are placed)	0.1517	0.1053
1 month after	0.1838	0.1024
2 months after (next orders)	0.1595	0.1054

Subsample of observations with $\tau^q_{it} = 3$ (# Observations = 383)	Mean Markup	Std. Deviation
Period 0 (when orders are placed)	0.1480	0.1202
1 month after	0.1724	0.1260

2 months after	0.1810	0.1125
3 months after (next orders)	0.1511	0.1125

Subsample of observations with $\tau^{q}_{it}= 4$ (# Observations = 155)

	Mean Markup	Std. Deviation
Period 0 (when orders are placed)	0.1367	0.1147
1 month after	0.1727	0.1516
2 months after	0.1825	0.0940
3 months after	0.1814	0.0944
4 months after (next orders)	0.1523	0.1139

TABLE 4

STATE DEPENDENCE OF DECISION VARIABLES

Fixed Effects Models (a)

	(1) Probit $q_t>0$	(2) Regression $E(\ln q_t \mid q_t>0)$	(3) Probit $\Delta p_t>0$	(4) Regression $E(\ln(p_t/c_t) \mid \Delta p_t>0)$	(5) Probit $\Delta p_t<0$	(6) Regression $E(\ln(p_t/c_t) \mid \Delta p_t<0)$
$\ln S_t$	-1.1844 (0.0252)	-0.0763 (0.0062)	-0.0462 (0.0082)	-0.0006 (0.0001)	0.0450 (0.0084)	-0.0001 (0.0001)
$\ln(P_{t-1}/C_t)$	-1.1164 (0.8638)	2.8622 (0.4213)	-64.417 (1.1886)	0.2229 (0.0127)	54.154 (1.1399)	0.3630 (0.0203)
$\ln C_t$	-1.2969 (0.2126)	-2.6220 (0.1128)	-1.0751 (0.1758)	-0.0334 (0.0018)	-0.5364 (0.1750)	0.0165 (0.0027)
Heckman's lambda		0.0859 (0.0336)		0.0099 (0.0005)		-0.0066 (0.0008)
# observ.[(b)]	12544	9107	14952	6608	14952	5421

LRI [c]	0.3429		0.2465		0.2157	
R^2		0.7705		0.8037		0.6599
Within-Goods R^2		0.1048		0.3786		0.2742

Standard errors in parentheses
(a) All the estimations contain good-specific dummies (fixed effects).
(b) Estimations in columns 1 and 2 were performed excluding 86 goods for which orders are placed at each period or at each period minus 1.
(c) LRI is the Likelihood Ratio Index, 1 - (ln L(θ^*) - ln L(θ^0)), where θ^* is the vector of estimates and L(θ^0) is the likelihood when all the parameters except the constant term are zero.

TABLE 5
Nonparametric Estimates for Some Components of the Intertemporal Profit Function

(Differences with respect the mean for choice alternattive: $D_q = 0$ & $D_p = 0$)
$\beta = 0.985$

Variable	Percentile for s	Choice Alternative			Percentile for p/c	Choice Alternative		
		$D_q = 1$ $D_p = 0$	$D_q = 0$ $D_p \neq 0$	$D_q = 1$ $D_p \neq 0$		$D_q = 0$ $D_p = 0$	$D_q = 0$ $D_p \neq 0$	$D_q = 1$ $D_p \neq 0$
E(py|x,D) US$0,000	10	1.81	-0.16	1.98	10	0.92	0.01	1.48
	30	1.45	-0.11	1.81	30	0.94	0.02	1.43
	50	1.09	-0.09	1.46	50	1.01	-0.06	1.22
	70	0.57	-0.01	0.91	70	1.05	-0.13	1.05
	90	0.06	0.00	0.41	90	1.06	-0.21	1.39
E(cq|x,D) US$0,000	10	2.44	0.00	2.70	10	1.83	0.00	2.05
	30	2.01	0.00	2.32	30	1.89	0.00	2.08
	50	1.91	0.00	2.07	50	1.93	0.00	2.11
	70	1.75	0.00	1.84	70	2.07	0.00	2.17
	90	1.74	0.00	1.70	90	2.13	0.00	2.22

$\beta\,\Pi^{jk}_{py}$ US$0,000	10	0.27	-0.15	0.13	10	0.14	-0.16	0.12
	30	0.29	-0.12	0.18	30	0.25	-0.12	0.12
	50	0.21	0.02	0.12	50	0.16	0.02	0.13
	70	0.13	0.01	0.11	70	0.23	0.01	0.12
	90	0.02	0.00	0.05	90	0.14	0.01	0.10
$\beta\,\Pi^{jk}_{cq}$ US$0,000	10	-0.67	-0.10	-0.92	10	-0.38	0.00	-0.44
	30	-0.58	-0.25	-0.67	30	-0.37	0.00	-0.44
	50	-0.42	0.01	-0.55	50	-0.35	-0.11	-0.45
	70	-0.13	0.02	-0.16	70	-0.33	-0.10	-0.46
	90	0.04	0.00	0.05	90	-0.34	-0.11	-0.46

TABLE 5
Nonparametric Estimates for Some Components
of the Intertemporal Profit Function

(Continued)

Variable	Percentile for s	Choice Alternative			Percentile for p/c	Choice Alternative		
		$D_q = 1$ $D_p = 0$	$D_q = 0$ $D_p \neq 0$	$D_q = 1$ $D_p \neq 0$		$D_q = 0$ $D_p = 0$	$D_q = 0$ $D_p \neq 0$	$D_q = 1$ $D_p \neq 0$
$E(py \mid x,D) - E(cq \mid x,D) + \beta\,\Pi^{jk}_{px} - \beta\,\Pi^{jk}_{cq}$	10	0.31	-0.34	0.33	10	-0.39	-0.15	-0.01
	30	0.31	0.02	0.34	30	-0.33	-0.10	-0.09
	50	-0.19	-0.08	0.06	50	-0.41	0.07	-0.48
	70	-0.92	-0.03	-0.66	70	-0.46	-0.02	-0.54
	90	-1.70	0.00	-1.29	90	-0.59	-0.09	-0.28
$\beta\,\Pi^{jk}_{Dq}$	10	-0.27	0.00	-0.29	10	-0.14	0.00	-0.15
	30	-0.23	0.00	-0.25	30	-0.16	-0.01	-0.16
	50	-0.20	-0.08	-0.21	50	-0.16	-0.03	-0.16
	70	-0.05	-0.03	-0.06	70	-0.14	-0.04	-0.17
	90	0.00	0.00	0.00	90	-0.15	-0.03	-0.17

$\beta\ \Pi^{jk}_{\mid Dp \mid}$	10	0.00	0.00	0.00	10	0.00	0.00	0.01
	30	0.00	0.01	0.00	30	0.00	-0.01	0.02
	50	0.00	0.01	0.02	50	0.00	0.00	0.03
	70	0.00	-0.01	0.04	70	0.00	0.01	0.03
	90	0.01	-0.02	0.05	90	0.01	0.00	0.02

TABLE 6

GMM Estimates of the Structural Parameters using Moment Conditions from the Optimal Discrte Choice

$\beta = 0.985$

observations = 14418

	Estimate	Asymp. s.e
$\alpha 1$	31.858	5.979
$\alpha 2$	0.003	0.040
$\phi 1$	2.401	0.610
$\phi 2$	0.098	0.070
$\eta 1$	288.585	21.688
$\eta 2$	54.350	10.917
$\eta 3$	323.681	24.001

Test of Overidentifying Restrictions

$\chi 2 = 112.06$; d.o.f = 105 ; p-value = 0.3007

Instrumental Variables
(s_{it}/s_i) ; $(s_{i,t-1}/s_i)$; (c_{it}/c_{i0}) ; $(c_{i,t-1}/c_{i0})$; $(c_{i,t-2}/c_{i0})$; $(p_{i,t-1}/c_{it})$; $(p_{i,t-2}/c_{i,t-1})$, and their square and cubics

All the structural parameters are measured in 1990 US$

TABLE 7
Out of Sample Goodness of Fit

Estimations with 534 goods and 20 month
Predictions with 534 goods and 7 months

%		Reduced Form (Kernel) Model		Structural Model		Total Actual Choices
Orders		Predicted Choice		Predicted Choice		
		q = 0	q > 0	q = 0	q > 0	
Actual Choice	q = 0	14.64	12.30	18.82	8.12	26.94
	q > 0	5.37	67.69	5.93	67.13	73.06
Total Pred. Choices		20.01	79.99	24.75	75.25	100.00
% Correct Preds.		82.33		85.95		

Change in Price given Orders=0	Predicted Choice			Predicted Choice			Total Actual Choice
	$\Delta p<0$	$\Delta p=0$	$\Delta p>0$	$\Delta p<0$	$\Delta p=0$	$\Delta p>0$	

Actual	Δp< 0	9.73	1.59	14.00	8.93	3.51	9.89	25.33
Choice	Δp= 0	9.83	2.18	10.43	2.39	8.84	11.21	22.44
	Δp>0	10.02	1.79	40.42	11.11	3.74	40.38	52.23
Total Pred. Choices		29.58	5.56	64.85	22.43	16.09	61.48	100.00
% Correct Preds.			52.34			58.15		
Change in Price given Orders>0		Predicted Choice			Predicted Choice			Total Actual Choice
		Δp<0	Δp=0	Δp>0	Δp<0	Δp=0	Δp>0	
Actual	Δp< 0	22.64	1.90	12.08	23.09	3.58	9.95	36.62
Choice	Δp= 0	11.86	1.76	8.60	9.46	6.85	5.91	22.22
	Δp>0	12.63	1.06	27.47	12.11	2.69	26.36	41.16
Total Pred. Choices		47.13	4.72	48.15	44.66	13.12	42.22	100.00
% Correct Preds.			51.87			56.30		

MODEL WITH LUMP-SUM ORDERING COSTS BUT WITHOUT LUMP-SUM PRICE ADJUSTMENT COSTS

$$\gamma_0 = 10 \; ; \; \gamma_1 = -5 \; ; \; \sigma^2_{\ a} = 1.0 \; ; \; \alpha_1 = 0.2 \; ; \; \alpha_2 = 0.0 \; ; \; \pi = 0.10 \; ; \; \beta = 0.99/(1+\pi) \; ;$$
$$\eta^{(1)} = 1.0 \; ; \; \eta^{(2)} = 0.0$$

Figure 1
Optimal Supply as
a Function of the Stock

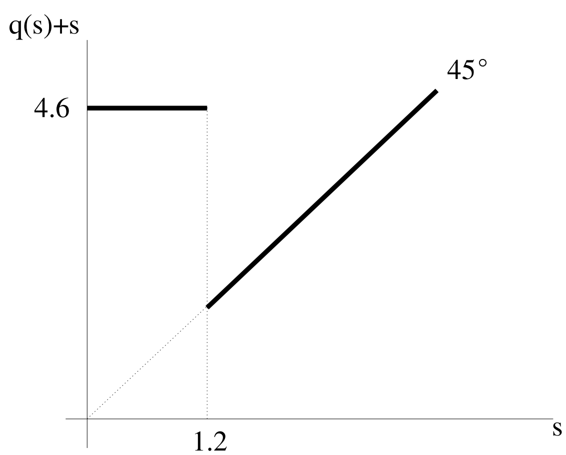
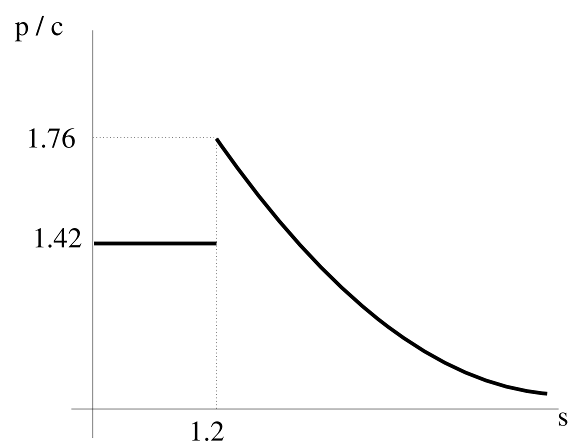


Figure 2
Optimal Markup
as a Function of the Stock



MODEL WITH LUMP-SUM PRICE ADJUSTMENT COSTS

BUT WITHOUT ORDERING COSTS

$\gamma_0 = 10$; $\gamma_1 = -5$; $\sigma^2_a = 1.0$; $\alpha_1 = 0.2$; $\alpha_2 = 0.0$; $\pi = 0.10$; $\beta = 0.99/(1+\pi)$;
$\eta^{(1)} = 0.0$; $\eta^{(2)} = 1.0$

Figure 3
Optimal Supply as a Function of Markup at the previous period

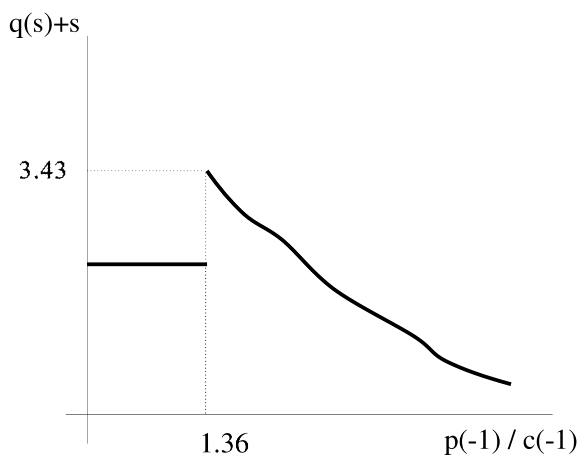
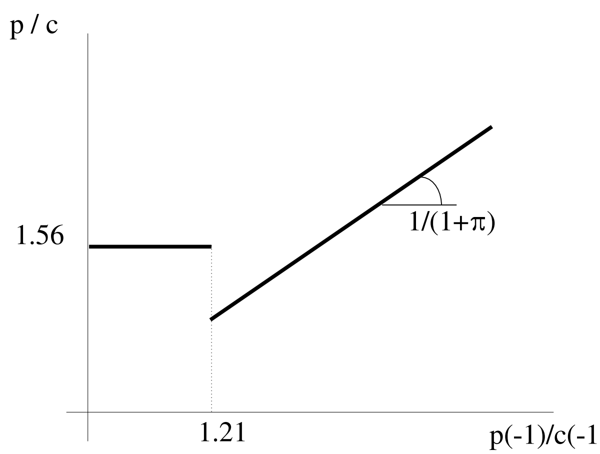


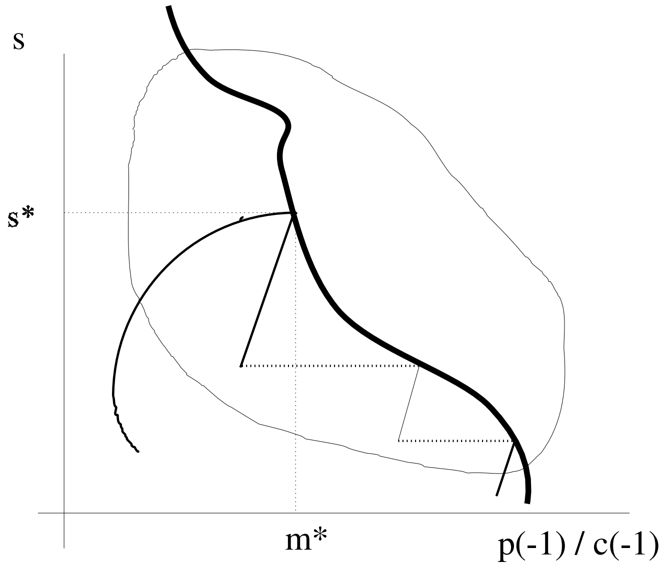
Figure 4
Optimal Markup as a Function of Markup at the previous period



MODEL WITH LUMP-SUM ORDERING COSTS AND LUMP-SUM PRICE ADJUSTMENT COSTS

$\gamma_0 = 10$; $\gamma_1 = -5$; $\sigma^2_a = 1.0$; $\alpha_1 = 0.2$; $\alpha_2 = 0.0$; $\pi = 0.10$; $\beta = 0.99/(1+\pi)$;
$\eta^{(1)} = 1.0$; $\eta^{(2)} = 0.3$

Figure 5



Footnotes

1. This paper is a revised version of Chapter 3 of my Ph.D dissertation. I am grateful to Bob Miller, Cesar Alonso-Borrego, Tim Bresnahan, Andreas Hornstein, Angelo Melino, Albert Marcet and Costas Meghir for their comments. I also wish to acknowledge my thesis advisor Manuel

Arellano for many helpful suggestions and discussions. I would like to thank the firm Sebastian de la Fuente S.A. for providing the dataset used in this study. Financial support and facilities provided by CEMFI are gratefully acknowledged.

2. Up to the best of my knowledge Slade (1994) is the only empirical work that has previously estimated the structure of price adjustment costs in individual firms.

3. In the stationary (S,s) model the real price erosion between two price changes (i.e., S-s) increases with the inflation rate. However, Cecchetti (1986) and Lach and Tsiddon (1992) observe that when inflation raises the frequency of price changes increases "too fast", such that it is only compatible with almost no change in S-s. Kashyap (1991) has found that small price increases are commonly observed within inflationary periods, which is inconsistent with the stationary (S,s) pricing model.

4. Problem (1) might seem restrictive because it is Markovian, with time-separable preferences and infinite horizon. However, it is always possible to write any non-stationary, time non-separable and finite horizon problem using expression (1) if we redefine the set of state variables (see Rust [1994]). For example, one could include time as a state variable.

5. This data set, described in Section 4 and Appendix 1, comes from the central store of a supermarket chain. In this dataset a stockout is defined as the situation when an outlet asks to the central store for a good that is out of stock in the central store.