

Technology Modeling: Curvature is not Sufficient for Regularity

by

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June 24, 1999

1. Introduction

Recently there has been a growing tendency to impose curvature, but not monotonicity, on specifications of technology. This practice is especially common with the currently popular generalized McFadden (also called generalized quadratic) model. We believe that this practice of overlooking monotonicity should not be taken lightly. Regularity requires satisfaction of both the curvature and the monotonicity conditions. Without both satisfied, the second order conditions for optimizing behavior fail, duality theory fails, and the specification should be viewed as compromised in a very serious manner. An earlier practice with "flexible functional forms" was to impose neither monotonicity nor curvature. In that earlier literature, ex post verification of monotonicity and curvature was rare. Hence the newer convention of at least taking curvature seriously is to be welcomed as a step in a positive direction. See, e.g., Diewert and Wales (1987,1988, 1995). But the damage done by violations of monotonicity, even with satisfaction of curvature, should not be underestimated. We explore that issue in this paper.

One productive manner in which this issue could be investigated would be to set the model's parameters at various levels that are consistent with plausible elasticities and then produce the regular region within which the model satisfies monotonicity, conditionally upon imposition of curvature. Experiments of that sort have been done with the earlier flexible functional forms by Caves and Christensen (1980), Barnett, Lee, and Wolfe (1987), and Barnett and Lee (1985).¹ Such experiments have not been conducted with the newer generalized

¹Other relevant research can be found in Guilkey, Lovell, and Sickles (1983) and Wales (1977).

McFadden model. While we believe that such experiments should be conducted, we believe that a logically prior investigation would display the isoquants and investigate regularity of technologies estimated with actual data. If problems are revealed by that econometric investigation, then the more complicated experiment becomes warranted. In this paper, we provide the results of an exploration of properties of the model conditional upon estimation of the parameters.

Since we condition our investigation upon estimation of the parameters, rather than upon a large range of possible settings of the parameters, we believe that estimation in a manner consistent with the state of the art is appropriate, and the state of the art currently could be viewed as generalized method of moments (GMM) estimation of Euler equations for production under risk. This paper applies an approach to the estimation of technology parameters when financial assets are included among either the inputs or outputs of the technology and explores both the monotonicity and curvature properties of the resulting technology. The relevant technologies are those of the financial intermediaries that produce inside money as output services and the nonfinancial firms that demand financial services as inputs to production technology. Virtually every firm in the economy falls into one of those two groups. The need for GMM estimation results from the inclusion of monetary assets in the model, with interest paid at the end of the period and thereby unknown at the start of the period.

The specification of technology that we use is the currently popular generalized McFadden (generalized quadratic) model. That model can be estimated subject to imposition of global curvature, but monotonicity can be imposed only at one point, since the existence of a bliss point with the quadratic specification renders imposition of global monotonicity impossible. We impose curvature globally, but we impose monotonicity only at a point. We do check for monotonicity at each data point.

2. Financial Intermediaries

One of the recent approaches to modeling financial intermediaries is to model them as profit maximizing neoclassical multiproduct firms, which produce financial services, such as

demand deposits and time deposits, as outputs by employing financial and non financial factors as inputs. Early work that used this approach was based on the assumption of perfect certainty. See Hancock (1985, 1987, 1991), Barnett (1987), and Barnett and Hahm (1994). Barnett and Zhou (1994a) extended this approach to the case of uncertainty. Barnett, Kirova, and Pasupathy (1995) introduced capital accumulation and relaxed the assumption of "no retained earnings." They also rigorously nest exact supplied monetary output aggregates within the transformation function of the financial intermediary and report the behavior of the resulting exact monetary aggregate.

The resulting model can be viewed as a step in the direction of exploring technological change and economies of scale and scope in financial intermediation in a manner that is invariant to central bank policy intervention, and in a manner that can produce inside money aggregates that are consistent with the theory that produced the policy invariant Euler equations. In our current investigation of the regularity properties of the model, we use Barnett, Kirova, and Pasupathy's (1995) model and estimates as an illustration of the implications of the generalized Mc Fadden model under state-of-the-art application and estimation.

2.1. Financial Firm's Production Model

The theoretical model builds on the ground work set forth by Barnett and Zhou (1994a) in their adaptation and application of Hancock's (1991) specification of the variable profit function. The financial firm uses real resources such as labor, capital and other material inputs, plus monetary input in the form of cash in the production of the services of the produced liabilities. The output of the firm in Barnett, Kirova, and Pasupathy's (1995) application consists of demand deposits and time deposits, which are liabilities to the firm.

Let Y_t be the real balances of the asset (loan) portfolio, $y_{i,t}$ the real balances of the i th produced account (liability) type, C_t the real balances of cash holdings, $z_{j,t}$ the quantity of j th real input (including labor), and K_t the quantity of capital stock of the financial firm at time t . In the model, $y_{i,t}$ constitute the outputs of the financial firm, while C_t , $z_{j,t}$, and K_t are the inputs. Let R_t be the portfolio rate of return, which is unknown at the beginning of period t , and

let $h_{i,t}$ be the holding cost per dollar of the i th liability. All financial transactions are contracted at the beginning of the period. Interests on the deposits are paid at the end of the period. The cost per unit of the j th real input, $w_{j,t}$, is incurred at the beginning of the period. Let $P_{K,t}$ be the cost of capital and P_t be the general price index, which is used to deflate nominal to real terms.

Variable profits (net of investment expenditure), π_t , at the beginning of period t , can be represented by

$$\begin{aligned} \pi_t = & (1 + R_{t-1})Y_{t-1}P_{t-1} - Y_tP_t + C_{t-1}P_{t-1} - C_tP_t \\ & + \sum_{i=1}^I [y_{i,t}P_t - (1 + h_{i,t-1})y_{i,t-1}P_{t-1}] - \sum_{j=1}^J w_{j,t}z_{j,t} - P_{K,t}I_t \end{aligned} \quad (2.1)$$

The first two terms in the above equation represent the change in variable profits from rolling over the loan portfolio during period t . The third and fourth terms represent the change in the nominal value of excess reserves. The fifth term represents the change in the firm's variable profits from the change in the issuance of produced financial liabilities. The sixth term constitutes payments for real inputs, and the last term is the expenditure on investments.

Portfolio investment, Y_t , is constrained by total available funds. The constraint is given by

$$Y_tP_t = \sum_{i=1}^I [(1 - k_{i,t})y_{i,t}P_t] - C_tP_t - \sum_{j=1}^J w_{j,t}z_{j,t} - P_{K,t}I_t \quad (2.2)$$

where $k_{i,t}$ is the required reserves ratio on the i th produced liability. Equation (2.5) implies that the total deposits $\sum_{i=1}^I y_{i,t}P_t$ are allocated to required reserves, excess reserves, payment for all real inputs used in production, investment in capital, and investment in loans.

The time to build approach is adopted to model capital dynamics. Capital accumulation based on this approach is given by:

$$K_t = I_{t-1} + (1 - \delta)K_{t-1} \quad (2.3)$$

where the depreciation rate δ is a constant and is assumed to be given. Gross investment at time $t-1$, I_{t-1} , becomes productive only in period t . Substituting equations (2.2) and (2.3) into equation (2.1) to eliminate investment in loans and investment in capital goods, we get the variable profits at time t to be

$$\begin{aligned} \pi_t = & \sum_{i=1}^I [(1 + R_{t-1})(1 - k_{i,t-1}) - (1 + h_{i,t-1})] y_{i,t-1} P_{t-1} + k_{i,t} y_{i,t} P_t \\ & - R_{t-1} C_{t-1} P_{t-1} - (1 + R_{t-1}) \sum_{j=1}^J w_{j,t-1} z_{j,t-1} \\ & + (1 - \delta)(1 + R_{t-1}) K_{t-1} P_{K,t-1} - (1 + R_{t-1}) K_t P_{K,t-1} \end{aligned} \quad (2.4)$$

The financial firm maximizes the expected value of the discounted intertemporal utility of its variable profits stream, subject to the firm's technological constraint. The firm's optimization problem is then given by:

$$\begin{aligned} \text{Max } E_t \left[\sum_{s=t}^{\infty} \left(\frac{1}{1 + \mu} \right)^{s-t} U(\pi_s) \right] \\ \text{s.t. } \Omega(y_{1,s}, \dots, y_{I,s}, C_s, z_{1,s}, \dots, z_{J,s}, K_s) = 0 \quad \forall s \geq t \end{aligned} \quad (2.5)$$

where E_t is the expectation at time t , μ is the subjective rate of time preference, U is the utility function, π_s is the variable profit at time s , and Ω is the transformation function.

The transformation function, Ω , is convex in its arguments. The derivatives of Ω with respect to the inputs and outputs are respectively given by:

$$\frac{\partial \Omega}{\partial C_s} \leq 0, \quad \frac{\partial \Omega}{\partial K_s} \leq 0, \quad \frac{\partial \Omega}{\partial z_{j,s}} \leq 0 \quad \forall j = 1, \dots, J \quad (2.6)$$

and

$$\frac{\partial \Omega}{\partial y_{i,s}} \geq 0 \quad \forall i = 1, \dots, I. \quad (2.7)$$

To derive the Euler equations using the Bellman method, we must select state and control variables in a manner that will transform the decision problem to be in Bellman form. The financial firm is assumed to behave competitively both in the input and the output markets. Hence R_s , P_s , $P_{K,s}$, $k_{i,s}$, and $w_{j,s}$ cannot be controlled by the firm. Let \mathbf{w}_s be the vector of all state variables and \mathbf{u}_s be the vector of all control variables during period s . We define the vector \mathbf{w}_s to contain $y_{i,s-1} \forall i$, C_{s-1} , K_{s-1} , R_{s-1} , R_s , $k_{i,s}$, $h_{i,s-1} \forall i$, $w_{j,s} \forall j$, $P_{K,s-1}$, $P_{K,s}$, P_{s-1} , and P_s . We define the vector \mathbf{u}_s to contain $y_{i,s} \forall i$, $z_{j,s} \forall j$ and K_s . Let Λ_s defined by, $\Lambda_s = (R_s, k_{i,s}, h_{i,s-1} \forall i, w_{j,s} \forall j, P_{K,s}, P_s)$, be a subset of \mathbf{w}_s . We assume that Λ_s follows a first-order Markov process, with transition equations given by the conditional distribution $F(\Lambda_{s+1}/\Lambda_s)$. Hence the transition equations, which represent the evolution of the state, for the vector of exogenous state variables (R_{s-1} , R_s , $k_{i,s}$, $h_{i,s-1} \forall i$, $w_{j,s} \forall j$, $P_{K,s-1}$, $P_{K,s}$, P_{s-1} , P_s) are implicitly defined by $F(\Lambda_{s+1}/\Lambda_s)$. The remaining transition equations are defined by the obvious time shifts between some of the elements that appear simultaneously, but with time shifts, in the control and state vectors and by the technological transformation function, Ω , used to produce the transition equation of the remaining state variable, C_{s-1} . Making these substitutions and changes in notation, it can be shown that the problem now is in Bellman form.²

We now specify the utility function, U , to be in the class of functions exhibiting Hyperbolic Absolute Risk Aversion (HARA). The HARA class functions can be represented by:

$$U(\pi_t) = \frac{1-\rho}{\rho} \left(\frac{h}{1-\rho} \pi_t + d \right)^\rho \quad (2.8)$$

where ρ , h and d are parameters to be estimated.

²For similar examples of this derivation procedure, see Sargent (1987, ch. 1).

Using Bellman's method and the Benveniste and Scheinkman equation, we obtain the following set of Euler equations:

$$E_t \left\{ \frac{\partial U}{\partial \pi_t} (\pi_t) k_{i,t} P_t + \frac{1}{1+\mu} \frac{\partial U}{\partial \pi_t} (\pi_{t+1}) P_t \left[R_t \frac{\partial \Omega / \partial y_{i,t}}{\partial \Omega / \partial C_t} + [(1+R_t)(1-k_{i,t}) - (1+h_{i,t})] \right] \right\} = 0$$

$\forall y_{i,t}, i = 1, \dots, I.$

(2.9)

$$E_t \left\{ \frac{\partial U}{\partial \pi_t} (\pi_{t+1}) \left[R_t P_t \frac{\partial \Omega / \partial z_{j,t}}{\partial \Omega / \partial C_t} - (1+R_t) w_{j,t} \right] \right\} = 0$$

(2.10)

$\forall z_{j,t}, j = 1, \dots, J.$

$$E_t \left\{ \frac{\partial U}{\partial \pi_t} (\pi_t) [(1+R_{t-1}) P_{K,t-1}] - \frac{1}{1+\mu} \frac{\partial U}{\partial \pi_t} (\pi_{t+1}) \left[R_t P_t \frac{\partial \Omega / \partial K_t}{\partial \Omega / \partial C_t} + (1-\delta)(1+R_t) P_{K,t} \right] \right\} = 0$$

(2.11)

where $\frac{\partial U}{\partial \pi_t} = h \left(\frac{h}{1-\rho} \pi_t + d \right)^{\rho-1}$

(2.12)

Since closed form algebraic solutions rarely exist for Euler equations, we can solve only numerically for $(y_{1,t}, \dots, y_{I,t}, C_t, z_{1,t}, \dots, z_{J,t}, K_t)$ from the system of Euler equations and the transformation function, Ω . The parameter estimation can be done through the estimation of Euler equations under rational expectations by using Hansen and Singleton's Generalized Methods of Moments (GMM) estimation.

2.2 Output Aggregation

The financial firm's outputs consist of demand deposits and time deposits. The financial firm's output of demand and time deposits are important in determining the level of inside money in the economy. In this section, we find the aggregation-theoretic exact quantity output aggregate that measures the firm's produced service flow. Relative to the money markets, our aggregation in this case is on the supply side. In a later section below, when we investigate monetary service factor demand by nonfinancial manufacturing firms, we will be producing

demand side monetary aggregates, as also is relevant to consumer demand for monetary services.

Generating the exact quantity aggregate consists of first identifying the components over which aggregation is admissible and then determining the aggregator function defined over the identified components.³ The first step determines the existence of an exact aggregate, and the second step produces that aggregate in the manner that is consistent with microeconomic theory. The second step cannot be applied unless the first step succeeds in identifying the existence of an admissible cluster of components. The condition for the existence of an admissible component group is *blockwise weak separability*. In accordance with the definition of weak separability, a component grouping is admissible if and only if the group can be factored out of the rest of the economy's structure through a subfunction. Then the economic structure can be represented in the form of a composite function, with the goods in the separable block being the only goods in the inner function of the structure. If this condition is satisfied, an exact quantity aggregate exists over the goods in the block, and the aggregator function that produces the exact quantity aggregate over those goods is the inner ("category") function itself. Without weak separability, no such inner function exists and hence no aggregate exists.

Let $\mathbf{y} = (y_{1t}, \dots, y_{It})'$ be the firm's output vector, and let $\mathbf{x} = (z_{1,t}, \dots, z_{J,t})'$ be the input vector, so that transformation function can be written as $\Omega(\mathbf{y}, \mathbf{x}) = 0$. An exact supply side aggregate exists over all of the firm's outputs if and only if \mathbf{y} is weakly separable from \mathbf{x} within the function Ω . In accordance with the definition of weak separability, there then exist two functions H and y_0 such that

$$\Omega(\mathbf{y}, \mathbf{x}) = H(y_0(\mathbf{y}), \mathbf{x}),$$

where the output aggregator function, $y_0(\mathbf{y})$, is a convex function of \mathbf{y} .⁴ Although weak separability alone is sufficient for the existence of an aggregate, a considerable (although

³For details, refer to Barnett (1980).

⁴This property of the aggregator function is explained in Barnett (1987).

unnecessary) simplification is available if we also assume that $y_0(\mathbf{y})$ is linearly homogeneous in \mathbf{y} .⁵

The weak separability condition on functional structure is equivalent to the following restriction:⁶

$$\frac{\partial}{\partial x_k} \left(\frac{\partial \Omega(\mathbf{y}, \mathbf{x}) / \partial y_i}{\partial \Omega(\mathbf{y}, \mathbf{x}) / \partial y_j} \right) = 0 \quad i \neq j.$$

If we can test for weak separability of the transformation function and then estimate the resulting aggregator function $y_0(\mathbf{y})$, we obtain the econometrically estimated exact output aggregate. The related literature on statistical index numbers, such as Divisia and Laspeyres, seeks to produce nonparametric approximations that can track the level of $y_0(\mathbf{y})$ over time without the need to estimate the parameters of the aggregator function y_0 itself.

In this paper the econometric estimate of the aggregator function is obtained by estimating the Euler equations using the generalized method of moments (GMM) technique.

2.3 Testing for weak separability

The conventional parametric approach to testing weak separability is adopted in this paper, since weak separability is a strictly nested null hypothesis within our parametric specification of technology. To minimize the biases that can be produced from specification error, we use a flexible functional form for technology.⁷ Unfortunately flexible functional forms need not satisfy the regularity conditions imposed by economic theory, including the

⁵If the function y_0 is not assumed to be linear homogeneous, then the aggregator function is no longer y_0 but rather becomes the distance function. The growth rate of the distance function is equal to that of y_0 only under the assumption of linear homogeneity, so that our use of y_0 as the aggregator function under the assumption of linear homogeneity is a strictly nested special case of the more general result, which is straightforward and will not be considered further in this paper. See Barnett (1980) for details.

⁶See Leontief (1947a, 1947b).

⁷The form of flexibility that we use is called Diewert-flexibility or second-order flexibility. See Barnett (1982) for the definition and its connection with other definitions of parametric flexibility. The newer concept of global or Sobolev flexibility (see Barnett, Geweke, and Wolfe (1991a,b)) is beyond the scope of this paper.

monotonicity and curvature conditions. Hence we must consider methods for testing and imposing those conditions, at least locally, as well as methods for testing and imposing global blockwise weak separability of the technology in its outputs. For existence of aggregator functions, the weak separability must be global. Hence we must test and impose weak separability globally. We use the Generalized McFadden functional form to specify the technology of the firm. That specification, which also was used in the case of stochastic choice by Barnett and Zhou (1994a), was originated by Diewert and Wales (1991), who also originated the Generalized Barnett functional form. That latter model was applied by Barnett and Hahn (1994) in the perfect certainty case, but has not yet been adapted to the case of stochastic choice.

We assume that the transformation function, Ω , is linearly homogeneous. Instead of specifying the form of the full transformation function Ω , and then imposing weak separability in \mathbf{y} , we directly impose weak separability by specifying $H(y_0, \mathbf{x})$ and $y_0(\mathbf{y})$ separately⁸. The specification for Ω is then obtained by substituting $y_0(\mathbf{y})$ into $H(y_0, \mathbf{x})$. Since $y_0(\mathbf{y})$ and $H(y_0, \mathbf{x})$ are both specified to be flexible, the full technology Ω is flexible, subject to the separability restriction.

The function H is specified to be the symmetric generalized McFadden functional form

$$H(y_0, \mathbf{x}) = a_0 y_0 + \mathbf{a}'\mathbf{x} + \frac{1}{2} [y_0, \mathbf{x}'] \bar{\mathbf{A}} \begin{bmatrix} y_0 \\ \mathbf{x} \end{bmatrix} / \boldsymbol{\alpha}'\mathbf{x}, \quad (2.13)$$

where $\boldsymbol{\alpha}'\mathbf{x} \neq 0$, and a_0 , $\mathbf{a}' = (a_1, \dots, a_n)$, and $\bar{\mathbf{A}}$ are parameters to be estimated. The matrix $\bar{\mathbf{A}}$ is $(n+1) \times (n+1)$ and symmetric. The vector $\boldsymbol{\alpha}' = (\alpha_1, \dots, \alpha_n)$ contains all fixed nonnegative constants, which are chosen by the researcher. The matrix $\bar{\mathbf{A}}$ is partitioned as follows:

$$\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A} \end{bmatrix},$$

⁸For the more general form of the model which does not include the imposition of weak separability, refer to Barnett and Zhou (1994a).

where A_{11} is a scalar, \mathbf{A}_{12} is a $1 \times n$ row vector, \mathbf{A}_{21} is an $n \times 1$ column vector, and \mathbf{A} is an $n \times n$ symmetric matrix. Since $\bar{\mathbf{A}}$ is symmetric, we have $\mathbf{A}_{12} = \mathbf{A}'_{21}$.

Let $(y_0^*, \mathbf{x}^*) \neq \mathbf{0}$ be the chosen point about which the functional form is locally flexible.

Within the class of linearly homogeneous transformation functions, the specification given above is not parsimonious, and hence we can impose further restrictions on the model without losing local flexibility.⁹ We impose the following restrictions, which reduces the number of free parameters in our specification to the minimum required number to maintain local flexibility.

$$\boldsymbol{\alpha}'\mathbf{x}^* = 1, \quad (2.14)$$

$$A_{11}y_0^* + \mathbf{A}_{12}\mathbf{x}^* = 0, \quad (2.15)$$

$$\mathbf{A}'_{12}y_0^* + \mathbf{A}\mathbf{x}^* = \mathbf{0}_n, \quad (2.16)$$

Solving (2.18) and (2.19) for A_{11} and \mathbf{A}_{12} , and then substituting into (2.19) results in

$$\begin{aligned} H(y_0, \mathbf{x}) = & a_0y_0 + \mathbf{a}'\mathbf{x} + \frac{1}{2}(\boldsymbol{\alpha}'\mathbf{x})^{-1}\mathbf{x}'\mathbf{A}\mathbf{x} \\ & - (\boldsymbol{\alpha}'\mathbf{x})^{-1}\mathbf{x}'\mathbf{A}\mathbf{x}(y_0/y_0^*) + \frac{1}{2}(\boldsymbol{\alpha}'\mathbf{x})^{-1}\mathbf{x}'\mathbf{A}\mathbf{x}^*(y_0/y_0^*)^2, \end{aligned} \quad (2.17)$$

which is flexible at (y_0^*, \mathbf{x}^*) .¹⁰

The aggregator function $y_0(\mathbf{y})$ is specified as:

$$y_0(\mathbf{y}) = \mathbf{b}'\mathbf{y} + \frac{1}{2}\mathbf{y}'\mathbf{B}\mathbf{y}/\boldsymbol{\beta}'\mathbf{y}, \quad (2.18)$$

where $\mathbf{b}' = (b_1, \dots, b_m)$ and the $m \times m$ symmetric matrix \mathbf{B} contain the parameters to be estimated, while $\boldsymbol{\beta}' = (\beta_1, \dots, \beta_m)$ is the vector of fixed nonnegative constants chosen by the researcher. As similarly done above with H , we can impose the following restrictions without losing local flexibility:

⁹A flexible functional form is parsimonious, if it has the minimum number of parameters needed to maintain flexibility. Diewert and Wales (1988) show that a parsimonious specification of a flexible functional form, which provides second order approximation to any arbitrary function with n variables, must have $1+n+n(n+1)/2$ free parameters.

¹⁰See Diewert and Wales (1987) for the proof of flexibility.

$$\beta' \mathbf{y}^* = 1, \quad (2.19)$$

$$y_0^* = \mathbf{b}' \mathbf{y}^* \quad (2.20)$$

$$\mathbf{B} \mathbf{y}^* = \mathbf{0}_m \quad (2.21)$$

Substituting (2.18) into (2.17), we get the following flexible functional form for $\Omega(\mathbf{y}, \mathbf{x})$, which satisfies the weak separability condition

$$\begin{aligned} \Omega(\mathbf{y}, \mathbf{x}) &= H(y_0(\mathbf{y}), \mathbf{x}) \\ &= a_0 \left(\mathbf{b}' \mathbf{y} + \frac{1}{2} (\beta' \mathbf{y})^{-1} \mathbf{y}' \mathbf{B} \mathbf{y} \right) + \mathbf{a}' \mathbf{x} + \frac{1}{2} (\alpha' \mathbf{x})^{-1} \mathbf{x}' \mathbf{A} \mathbf{x} \\ &\quad - (y_0^* \alpha' \mathbf{x})^{-1} \mathbf{x}' \mathbf{A} \mathbf{x} \left(\mathbf{b}' \mathbf{y} + \frac{1}{2} (\beta' \mathbf{y})^{-1} \mathbf{y}' \mathbf{B} \mathbf{y} \right) \\ &\quad + \frac{1}{2} (y_0^* \alpha' \mathbf{x})^{-1} \mathbf{x}' \mathbf{A} \mathbf{x} \left(\mathbf{b}' \mathbf{y} + \frac{1}{2} (\beta' \mathbf{y})^{-1} \mathbf{y}' \mathbf{B} \mathbf{y} \right)^2. \end{aligned} \quad (2.22)$$

The neoclassical curvature conditions require $\Omega(\mathbf{y}, \mathbf{x})$ and $y_0(\mathbf{y})$ to be convex functions. Monotonicity requires that $\partial\Omega/\partial\mathbf{y} \geq \mathbf{0}$ and $\partial\Omega/\partial\mathbf{x} \leq \mathbf{0}$. Convexity of $H(y_0, \mathbf{x})$ and $y_0(\mathbf{y})$ requires the matrices \mathbf{A} and \mathbf{B} to be positive semidefinite¹¹. Global convexity of $\Omega(\mathbf{y}, \mathbf{x})$ further requires the condition

$$\frac{\partial H(y_0, \mathbf{x})}{\partial y_0} \geq 0. \quad (2.23)$$

Positive semidefiniteness of the matrices \mathbf{A} and \mathbf{B} can be imposed without loss of flexibility by substituting

$$\mathbf{A} = \mathbf{q} \mathbf{q}' \quad (2.24)$$

and

$$\mathbf{B} = \mathbf{u} \mathbf{u}', \quad (2.25)$$

¹¹See Diewert and Wales (1987).

where \mathbf{q} is an $\mathbf{n} \times \mathbf{n}$ lower triangular matrix and \mathbf{u} is an $\mathbf{m} \times \mathbf{m}$ lower triangular matrix. Although global convexity can be imposed on the functions $H(\mathbf{y}_0, \mathbf{x})$ and $y_0(\mathbf{y})$ separately, global convexity of the composite function $\Omega(\mathbf{y}, \mathbf{x})$ cannot be imposed without damaging the flexibility of the function. Similarly, global monotonicity with respect to the inputs and outputs cannot be imposed on the transformation function $\Omega(\mathbf{y}, \mathbf{x})$ without destroying the flexibility of the functional form.

The method of squaring technique can be used to obtain local monotonicity of $\Omega(\mathbf{y}, \mathbf{x})$ at $(\mathbf{y}^*, \mathbf{x}^*)$. The first derivatives of $\Omega(\mathbf{y}, \mathbf{x})$ are

$$\begin{aligned} \frac{\partial \Omega}{\partial \mathbf{y}} = & \mathbf{a}_0 \left[\mathbf{b} + \frac{1}{2} \left(2(\beta' \mathbf{y})^{-1} \mathbf{B} \mathbf{y} - (\beta' \mathbf{y})^{-2} \beta \mathbf{y}' \mathbf{B} \mathbf{y} \right) \right] \\ & - (\mathbf{y}_0^* \alpha' \mathbf{x})^{-1} \mathbf{x}^* \mathbf{A} \mathbf{x} \left[\mathbf{b} + \frac{1}{2} \left(2(\beta' \mathbf{y})^{-1} \mathbf{B} \mathbf{y} - (\beta' \mathbf{y})^{-2} \beta \mathbf{y}' \mathbf{B} \mathbf{y} \right) \right] \\ & + (\mathbf{y}_0^{*2} \alpha' \mathbf{x})^{-1} \mathbf{x}^* \mathbf{A} \mathbf{x} \left[\mathbf{b} + \frac{1}{2} \left(2(\beta' \mathbf{y})^{-1} \mathbf{B} \mathbf{y} - (\beta' \mathbf{y})^{-2} \beta \mathbf{y}' \mathbf{B} \mathbf{y} \right) \right] \left(\mathbf{b}' \mathbf{y} + \frac{1}{2} (\beta' \mathbf{y})^{-1} \mathbf{y}' \mathbf{B} \mathbf{y} \right) \end{aligned} \quad (2.26)$$

$$\begin{aligned} \frac{\partial \Omega}{\partial \mathbf{x}} = & \mathbf{a} + \frac{1}{2} \left(2(\alpha' \mathbf{x})^{-1} \mathbf{A} \mathbf{x} - (\alpha' \mathbf{x})^{-2} \alpha \mathbf{x}' \mathbf{A} \mathbf{x} \right) - \\ & - \left[(\mathbf{y}_0^* \alpha' \mathbf{x})^{-1} \mathbf{A} \mathbf{x}^* - (\mathbf{y}_0^* \alpha' \mathbf{x})^{-2} \mathbf{y}_0^* \alpha \mathbf{x}' \mathbf{A} \mathbf{x}^* \right] \left(\mathbf{b}' \mathbf{y} + \frac{1}{2} (\beta' \mathbf{y})^{-1} \mathbf{y}' \mathbf{B} \mathbf{y} \right) \\ & - \frac{1}{2} (\mathbf{y}_0^{*2} \alpha' \mathbf{x})^{-2} \mathbf{y}_0^{*2} \alpha \mathbf{x}' \mathbf{A} \mathbf{x}^* \left(\mathbf{b}' \mathbf{y} + \frac{1}{2} (\beta' \mathbf{y})^{-1} \mathbf{y}' \mathbf{B} \mathbf{y} \right)^2 \end{aligned} \quad (2.27)$$

At $(\mathbf{y}^*, \mathbf{x}^*)$ the value of the derivatives reduces to

$$\frac{\partial \Omega}{\partial \mathbf{y}} = \mathbf{a}_0 \mathbf{b} \quad (2.28)$$

and

$$\frac{\partial \Omega}{\partial \mathbf{x}} = \mathbf{a}. \quad (2.29)$$

Imposing monotonicity on (2.31) and (2.32) results in

$$\frac{\partial \Omega}{\partial \mathbf{y}}(\mathbf{y}^*, \mathbf{x}^*) = \mathbf{a}_0 \mathbf{b} \geq \mathbf{0} \text{ and } \frac{\partial \Omega}{\partial \mathbf{x}}(\mathbf{y}^*, \mathbf{x}^*) = \mathbf{a} \leq \mathbf{0} \quad (2.30)$$

The transformation function $\Omega(\mathbf{y}, \mathbf{x})$ defined by (2.22) and restricted to satisfy equations (2.14), (2.19)-(2.21), (2.23)-(2.25), and (2.30) is flexible, locally monotone at $(\mathbf{y}^*, \mathbf{x}^*)$ and globally regular, subject to the weak separability condition. Local monotonicity is verified empirically at each point within the data.

Testing for weak separability and estimating the parameters of the transformation function can be done by Hansen and Singleton's generalized method of moments (GMM) method. Substituting the functional form given by equation (2.22) into the system of Euler equations, we obtain the structural model, which is a system of integral equations. The GMM estimator of the parameters of such a nonlinear rational expectations system is asymptotically efficient and normally distributed under very weak conditions¹². We test for weak separability using Hansen's asymptotic χ^2 statistic to test for no overidentifying restrictions. Since equation (2.22) was derived after imposing weak separability, (2.22) can be substituted into the Euler equations to impose the null hypothesis of weak separability. If the test of no overidentifying restrictions is rejected, then we reject the null hypothesis of weak separability. That rejection in turn would imply that no aggregator function exists over all of the outputs of the firm.

2.4. Empirical Application

We apply our approach to estimating the technology of commercial banks. The outputs of that aggregated financial firm in our application consist of demand deposits and time deposits.¹³ Demand deposits and time deposits account for the major portion of the fund-providing functions of the bank's balance sheet¹⁴. The inputs used in the production process include both financial and nonfinancial inputs. The financial input in the form of cash is excess reserves. The nonfinancial inputs includes labor, materials, and physical capital. The output

¹²For proof, see Hansen (1982), and for detailed discussion of GMM estimation, see Hansen (1982), Hansen and Singleton (1982), and Newey and West (1987).

¹³As shown in Debreu (1959), perfect competition alone is a sufficient condition for the existence of a representative firm. Since we are assuming perfect competition in this paper, no additional assumptions are implicit in our use of a representative neoclassical firm to model aggregate banking behavior.

¹⁴"Demand deposits and time deposits ``occupy almost 80% of total portfolio activities of the average financial firm" (Barnett and Hahm (1994)).

vector is given by $\mathbf{y}' = (D_t, T_t)$ and the input vector is $\mathbf{x}' = (C_t, L_t, M_t, K_t)$, where D_t is demand deposits, T_t is time deposits, C_t is excess reserves, L_t is labor input, M_t is material inputs, and K_t is capital.

In our empirical application we use the power utility function, which is a nested special case of the general class of HARA utility functions, $U(\pi_t)$, given by equation (2.8). We use this simplification, since the available sample size does not permit the use of the more general form. The power utility function is obtained by setting $\mathbf{d} = \mathbf{0}$, and by imposing the restriction $0 < \rho < 1$, in equation (2.8). The power utility function is then represented by:

$$U(\pi_t) = \frac{\pi_t^\rho}{\rho} \quad (2.31)$$

and the derivative of $U(\pi_t)$ is given by

$$\frac{\partial U(\pi_t)}{\partial \pi_t} = \pi_t^{\rho-1} \quad (2.32)$$

Using equations (2.9) - (2.11) and equation (2.32), the Euler equations are

$$E_t \left\{ (\pi_t)^{\rho-1} k_{1,t} P_t + \frac{1}{1+\mu} P_t (\pi_{t+1})^{\rho-1} \left[R_t \frac{\partial \Omega / \partial D_t}{\partial \Omega / \partial C_t} + (1+R_t)(1-k_{1,t}) - (1+h_{1,t}) \right] \right\} = 0 \quad (2.33)$$

$$E_t \left\{ (\pi_t)^{\rho-1} k_{2,t} P_t + \frac{1}{1+\mu} P_t (\pi_{t+1})^{\rho-1} \left[R_t \frac{\partial \Omega / \partial T_t}{\partial \Omega / \partial C_t} + (1+R_t)(1-k_{2,t}) - (1+h_{2,t}) \right] \right\} = 0 \quad (2.34)$$

$$E_t \left\{ (\pi_{t+1})^{\rho-1} \left[R_t P_t \frac{\partial \Omega / \partial L_t}{\partial \Omega / \partial C_t} - (1+R_t)w_{1,t} \right] \right\} = 0. \quad (2.35)$$

$$E_t \left\{ (\pi_{t+1})^{\rho-1} \left[R_t P_t \frac{\partial \Omega / \partial M_t}{\partial \Omega / \partial C_t} - (1+R_t)w_{2,t} \right] \right\} = 0. \quad (2.36)$$

$$E_t \left\{ (\pi_t)^{\rho-1} \left[(1+R_{t-1})P_{K,t-1} - \frac{1}{1+\mu} (\pi_{t+1})^{\rho-1} \left[R_t P_t \frac{\partial \Omega / \partial K_t}{\partial \Omega / \partial C_t} + (1-\delta)(1+R_t)P_{K,t} \right] \right] \right\} = 0 \quad (2.37)$$

where $h_{1,t}$ and $h_{2,t}$ are respectively the holding costs of demand deposits and time deposits, $k_{1,t}$ and $k_{2,t}$ are respectively the required reserves ratio on demand and time deposits, and $w_{1,t}$ and $w_{2,t}$ are respectively the prices of labor and material inputs. The derivatives of Ω with respect to the various inputs and outputs are given by equations (2.26) and (2.27).

Before using Hansen's χ^2 statistic to test for weak separability in outputs, we have to choose the fixed constants and the center of local approximation. We choose $y_0^* = 1$, $\mathbf{y}^* = (1,1)$, and $\mathbf{x}^* = (1,1,1,1)$ as the center of approximation. To locate the center within the interior of the observations, we rescale the data about the midpoint observation so that the rescaled data becomes $\tilde{x}_i^t = x_i^t / x_i^{t^*}$ for $i = 1,2,3,4$ and $\tilde{y}_i^t = y_i^t / y_i^{t^*}$ for $i = 1,2$, where t^* is the midpoint observation. Each price vector is correspondingly rescaled by multiplying by the midpoint observation. This rescaling of prices leaves the dollar expenditure on various goods unaffected by the rescaling of the corresponding quantity.

The fixed nonnegative constants are chosen such that

$$\alpha_i = \frac{|\bar{\tilde{x}}_i|}{4 \sum_{j=1}^4 |\bar{\tilde{x}}_j|} \quad \forall i = 1,2,3,4 \quad (2.38)$$

and

$$\beta_i = \frac{|\bar{\tilde{y}}_i|}{2 \sum_{j=1}^2 |\bar{\tilde{y}}_j|} \quad \forall i = 1,2, \quad (2.39)$$

where $\bar{\tilde{x}}_i$ and $\bar{\tilde{y}}_i$ are the sample means of \tilde{x}_i and \tilde{y}_i respectively. The α_i and β_i thus chosen satisfy restrictions (2.14) and (2.19) respectively.

Equation (2.20) implies $b_1 + b_2 = 1$, which is imposed through the substitution $b_1 = 1 - b_2$. There are also inequality restrictions to be imposed. The monotonicity condition (2.30) implies $b_i \geq 0$ for $i = 1,2$, and hence from equation (2.20) we also have $b_i \leq 1 \quad \forall i = 1,2$. Combining these two conditions and the mathematical identity $\sin^2 \theta + \cos^2 \theta = 1$, we have the substitution $b_1 = \sin^2 \theta$, and $b_2 = \cos^2 \theta$, where the parameter θ must now be estimated.

We further normalize $a_0 = 1$, since $\Omega(\mathbf{y}, \mathbf{x}) = 0$. The monotonicity condition (2.30) implies that $a_i \leq 0$ for $i = 1, 2, 3, 4$. We impose that restriction by replacing a_i by $-\tilde{a}_i^2$ for $i = 1, 2, 3, 4$, and estimating \tilde{a}_i . The convexity conditions are imposed by replacing the matrices \mathbf{A} and \mathbf{B} by the matrices $\mathbf{q}\mathbf{q}'$ and $\mathbf{u}\mathbf{u}'$ respectively, where the lower triangular matrices \mathbf{q} and \mathbf{u} are given by

$$\mathbf{q} = \begin{bmatrix} q_{11} & 0 & 0 & 0 \\ q_{21} & q_{22} & 0 & 0 \\ q_{31} & q_{32} & q_{33} & 0 \\ q_{41} & q_{42} & q_{43} & q_{44} \end{bmatrix} \text{ and } \mathbf{u} = \begin{bmatrix} u_{11} & 0 \\ u_{21} & u_{22} \end{bmatrix}. \quad (2.40).$$

Equation (2.21) implies

$$\begin{bmatrix} u_{11} & 0 \\ u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} u_{11} & u_{21} \\ 0 & u_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (2.41)$$

which, when solved, produces the restrictions $u_{22} = 0$ and $u_{21} = -u_{11}$.

These relationships reduce the matrix \mathbf{B} to

$$\mathbf{B} = u_{11}^2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

Following these substitutions, the parameters that remain to be estimated within technology are θ , u_{11} , \mathbf{q} , and $\tilde{\mathbf{a}} = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4)$. In addition the subjective rate of time discount μ and the risk aversion parameter ρ must be estimated.

The data used for estimating the model was mainly obtained from the Federal Reserve Bank Functional Cost Analysis (FCA) Program. Data on the National Average Banks for the years 1966-1992 was used in the estimation. Labor inputs consist of two groups: managerial and non-managerial. Data on expenditure and quantity for the two categories of labor were obtained from FCA. Material inputs are divided into three categories: printing and stationery, telephone and telegraph, and postage, freight and delivery. Physical capital is made up of structures (bank buildings), furniture and equipment, and computers. Data on expenditure on the various types of

material inputs and physical capital were obtained from the FCA, while the corresponding price indices were obtained from the *Survey of Current Business*. A quantity aggregate and the corresponding price aggregate must be constructed for each of the three nonfinancial inputs¹⁵.

Data on the nominal quantity of demand deposits and time deposits, net interest rate on demand deposits and time deposits, and the bank's portfolio rate of return were obtained from the FCA¹⁶ data set. The required reserves ratio was obtained from the Federal Reserve *Bulletin*. Nominal dollar balances of all financial goods were converted to real balances by deflating the nominal balances using Fisher's ideal price index¹⁷.

2.7. Results

The parameter estimates were obtained by estimating the system of Euler equations ((2.33)-(2.37)) using the GMM estimation procedure on mainframe TSP (version 7.02). This estimation process allows for heteroskedasticity and autocorrelation in the disturbance terms. We specified a second order moving average serial correlation. Bartlett kernels were specified for the kernel density. Discount window rate, federal funds rate, composite bond rate, lagged value of excess reserves, lagged value of interest paid on time deposits, and constant were chosen as instruments. During estimation, to ensure that $0 < \rho < 1$, we replace ρ by $\sin^2(\hat{\rho})$

¹⁵For details on data and data construction, refer to Hancock (1991) and Barnett and Hahn (1994).

If data on expenditure and price for the individual categories of each input are available, we first construct the Divisia price aggregate and then find the corresponding quantity aggregate using Fisher's factor reversal test. If on the other hand we have data on expenditure and quantity, we first construct the Divisia quantity aggregate and then find the corresponding price aggregate.

An alternative data source is the Call Report Data. However that data base is very expensive, and we did not feel that the cost was warranted for the objectives of this particular project. In later research on economies of scale and scope, we do believe that the Call Report Data will be needed.

¹⁶Net interest rate is the interest paid on the deposit plus FDIC insurance premium paid minus service charges earned.

¹⁷The Fisher ideal price index is used as a proxy for the *true cost of living index*. The Fisher ideal price index is equal to the geometric mean of the Laspeyres and Paasche index, where we approximate the former from Bureau of Labor Statistic's consumer price index (CPI) and the latter from Commerce Department's GDP implicit price deflator (IPD). Data on the CPI and IPD are from the Citibank Database.

and estimate $\hat{\rho}$. Similarly, to rule out the possibility of getting negative values for the subjective rate of time preference, μ , we replace μ by $\tilde{\mu}^2$ and estimate $\tilde{\mu}$.

The parameter estimates are reported in table 2.1. The test for weak separability of monetary assets, using Hansen's χ^2 test for no over-identifying restrictions, was conducted by Barnett, Kirova, and Pasupathy's (1995) and showed that weak separability could not be rejected. This implies the existence of an output aggregator function. Plots of the resulting exact monetary aggregate along with superimposed plots of the simple sum and Divisia monetary aggregate are provided by Barnett, Kirova, and Pasupathy's (1995).

Using the parameter estimates, we tested the functional form to see if the regularity conditions, namely monotonicity and curvature conditions, were satisfied at all data points. This exercise revealed that monotonicity conditions were satisfied at all points within the output aggregator function: i.e., the condition $\frac{\partial y_0}{\partial \mathbf{y}} \geq 0$ was satisfied at all data points. Within the transformation function, Ω , the condition $\frac{\partial \Omega}{\partial y_i} \geq 0, \forall i$ was satisfied at all points. There were no violations of the monotonicity conditions with respect to the outputs. For inputs, the condition, $\frac{\partial \Omega}{\partial \mathbf{x}} \leq 0$, was tested for each of the four inputs at all data points. It was found that this condition was satisfied everywhere in the case of cash and labor, while it was violated at two data points in the case of materials and at four data points in the case of capital. The curvature condition for the composite function, $H(y_0(\mathbf{y}), \mathbf{x})$, was satisfied globally, since it was found that the condition $\frac{\partial H(y_0, \mathbf{x})}{\partial y_0} \geq 0$ was satisfied at all data points.

2.5 Consequences of Regularity Violations

As stated in the concluding portion of the previous section, the functional form failed to satisfy some of the regularity conditions that are required by economic theory of a valid transformation function. In this section, we provide further details of this problem.

As discussed in the previous section, we could not reject the null hypothesis of no over-identifying restrictions and hence could not reject weak separability in outputs. The weak separability of outputs from inputs within the transformation function implies that the equation $H(y_0(\mathbf{y}), \mathbf{x}) = 0$ can be solved to represent $y_0(\mathbf{y})$ as a function of the inputs \mathbf{x} , provided the conditions imposed by the implicit function theorem are satisfied. In our case this condition reduces to verifying that $\frac{\partial H(y_0, \mathbf{x})}{\partial y_0} \neq 0$ at all data points. We found that this condition was indeed satisfied at all data points. Hence, we proceeded to solve the equation, so that the model can be used to obtain the elasticities of substitution between the inputs and outputs of the production technology.

We found that we were in fact able to find an explicit solution to the equation. The function H , specified as an adaptation of the symmetric McFadden functional form is given by (from equation 2.17):

$$H(y_0, \mathbf{x}) = a_0 y_0 + \mathbf{a}' \mathbf{x} + \frac{1}{2} (\boldsymbol{\alpha}' \mathbf{x})^{-1} \mathbf{x}' \mathbf{A} \mathbf{x} - (\boldsymbol{\alpha}' \mathbf{x})^{-1} \mathbf{x}' \mathbf{A} \mathbf{x}^* (y_0 / y_0^*) + \frac{1}{2} (\boldsymbol{\alpha}' \mathbf{x})^{-1} \mathbf{x}' \mathbf{A} \mathbf{x}^* (y_0 / y_0^*)^2 \quad (2.42)$$

where (y_0^*, \mathbf{x}^*) is the point of local flexibility.

These points were chosen as $y_0^* = 1$ and $\mathbf{x}^* = (1, 1, 1, 1)$ during estimation. At this point using the representation given in equation (2.42), equation $H(y_0(\mathbf{y}), \mathbf{x}) = 0$, can be rewritten as a quadratic equation in y_0 . The quadratic equation is given by:

$$\left[(\boldsymbol{\alpha}' \mathbf{x})^{-1} \mathbf{x}' \mathbf{A} \mathbf{x}^* \right] y_0^2 + 2 \left[a_0 - (\boldsymbol{\alpha}' \mathbf{x})^{-1} \mathbf{x}' \mathbf{A} \mathbf{x} \right] y_0 + 2 \mathbf{a}' \mathbf{x} + (\boldsymbol{\alpha}' \mathbf{x})^{-1} \mathbf{x}' \mathbf{A} \mathbf{x} = 0 \quad (2.43)$$

We found that both roots to the equation (2.43) existed. Hence, there were two values for the production possibility surface y_0 , which implies that y_0 was in fact a correspondence and not a function, as it should be. We believe that the problem of multiple solutions has arisen due to the violations of regularity conditions reported earlier. We further found that for part of the sample (1976 - 1992), the term

$$\left(a_0 - (\alpha'x)^{-1} x^* \mathbf{A}x \right)^2 - (\alpha'x)^{-1} x^* \mathbf{A}x \left(2\mathbf{a}'x + (\alpha'x)^{-1} x' \mathbf{A}x \right)$$

had negative values. Since, this term appears under the square root in the solution to equation (2.43), the roots to the equation had complex values at these sample points.

To study this rather strange behavior of the functional form further, we tried to plot the isoquants. We once again encountered the phenomenon of multiple solutions. We found that there were two sets of isoquants for every pair of inputs. The isoquants between labor and capital, and between materials and capital entered the complex plane.

3. Manufacturing Firms

As discussed above, the supply side model produced in section 2 is an extension of the model previously produced and estimated by Barnett and Zhou (1994a), and an analogous model on the consumer demand side has been produced and estimated by Poterba and Rotemberg (1987) and by Barnett, Hinich, and Yue (1991). But this modern approach to modeling, with nested demand-side monetary aggregator functions and GMM estimation of Euler equations, had not been attempted for manufacturing firms prior to Barnett, Kirova, and Pasupathy (1995). However, in the perfect certainty case, relevant theory is available in Barnett (1987) and a positive contribution to dynamic modeling of firm demand for money, although without nested exact quantity aggregation, has been made by Robles (1993). The potential importance of this under-researched area has been emphasized by Drake and Chrystal (1994). We now provide Barnett, Kirova, and Pasupathy's (1995) model of a manufacturing firm that employs a monetary asset portfolio as inputs. We assume rational expectations under risk, and we investigate the existence of an exact aggregation-theoretic monetary asset input aggregate.

There is no unanimous agreement among economists about the specific role that money plays in the production process. But regardless of the explicit role of money in the operation of a manufacturing firm, a derived production function always exists that absorbs that motive into the firm's technology, even if no direct role exists for money inside the factory's production

activities.¹⁸ When we enter the monetary asset portfolio into the firm's technology as factors of production, the technology should be understood to be that derived technology of the firm, and not necessarily the physical technology of the factory.

3.1 The Model

Barnett, Kirova, and Pasupathy's (1995) model is based on Barnett's (1987) monetary aggregation-theoretic approach, extended to include uncertainty and capital accumulation. Perfect competition in all markets and risk neutrality of the firm are assumed. The objective of the firm is to maximize the expected discounted value of its future variable profit flow, subject to its production technology. The firm uses L_t real units of labor, K_t real units of capital goods, a vector $\boldsymbol{\varepsilon}_t$ of monetary assets, and a vector \mathbf{x}_t of other variable inputs as factors of production in producing y_t real units of output quantities during period t . The firm's technology is given by the production function:

$$y_t = F(L_t, K_t, x_{1t}, \dots, x_{Nt}, \varepsilon_{1t}, \dots, \varepsilon_{Jt}) \quad (3.1)$$

The production technology is assumed to be concave in its arguments in accordance with the properties of a neoclassical production function. In addition, the following monotonicity conditions hold:

$$\begin{aligned} \partial F / \partial x_{nt} &\geq 0, \quad \forall n = 1, \dots, N, \\ \partial F / \partial \varepsilon_{jt} &\geq 0, \quad \forall j = 1, \dots, J, \\ \partial F / \partial L_t &\geq 0 \quad \text{and} \quad \partial F / \partial K_t \geq 0. \end{aligned} \quad (3.2)$$

¹⁸It is with respect to that derived technology that Fischer (1974) concluded that "there is a well-defined sense in which real money balances may be said to be a factor of production". Fischer illustrates that conclusion with two types of models: a Baumol-Tobin inventory model, in which the firm holds real balances because it is cheaper to hold them temporarily than to buy bonds, and a vending machine model in which it is expensive for the firm to be short of cash. A re-interpretation of the Baumol-Tobin model by regarding transaction costs as costs of hiring labor brings in an interaction between production and financial decisions of the firm and justifies putting money as an input factor in the production function.

Let y_t be real output with a price p_t^y , x_{nt} be the n th variable input with a price $p_t^{x_n}$, where $n=1,\dots,N$, and w_t be the wage rate of labor L_t . The j th component of the real balances of monetary assets, held by the firm in period t is ϵ_{jt} , where $j = 1, \dots, J$. Real balances ϵ_t are defined to equal nominal balances divided by p_t^* , the true cost of living index. The return on holding nominal money balances of type j is r_{jt} and is paid to the firm at the end of the period.

As the firm operates over time, it retains part of the earnings and uses them to finance its expansion and development. It is assumed that there exist markets for new and used capital. Capital accumulation is given by:

$$K_t = (1 - \delta)K_{t-1} + I_t \quad (3.3)$$

where I_t is gross investment and δ is the physical depreciation rate of capital. Investment becomes productive instantaneously, but capital installation is costly to the firm. Thus the total costs of purchasing and installing I_t are given by $p_t^I [I_t + C(I_t)]$ where p_t^I is the price of capital goods and $C(I_t)$ is a convex function, representing the costs of adjustment associated with installing capital.¹⁹

Extending Barnett's (1987, eq. 4.3) formula to include q theory capital dynamics, the firm's variable profits during period t are:

$$\pi_t = y_t p_t^y - \sum_n x_{nt} p_t^{x_n} - w_t L_t + \sum_j \left[(1 + r_{jt-1}) p_{t-1}^* \epsilon_{jt-1} - p_t^* \epsilon_{jt} \right] - p_t^I [I_t + C(I_t)] \quad (3.4)$$

where

$$I_t = K_t - (1 - \delta)K_{t-1} \quad (3.5)$$

The first term represents revenues from production during period t . The second term is the cost of other variable inputs x_t , and the third term represents costs of labor. The fourth term in the equation represents the flow of funds from rolling over the firm's portfolio of monetary assets, where the first part of the term is the nominal value of the monetary asset portfolio, available at

¹⁹Note that our notation for the price of capital differs from that used in section 2. We here use the superscript I in the role used by K in section 2, since we shall wish to reserve the notation p_t^K below for the user cost price of capital services.

the beginning of the period as a result of last period holdings. The last term is the total cost of purchasing and installing capital during the period.

Summing over each period's discounted profit flow and substituting (3.5) into (3.4) to eliminate I_t , we obtain the intertemporal profit flow function:

$$\begin{aligned}
\Pi_t &= \sum_{s=t}^{\infty} \frac{1}{\mu_s} \pi_s \\
&= \sum_{s=t}^{\infty} \frac{1}{\mu_s} \left[y_t p_t^y - \sum_n^N x_{ns} p_s^{x_n} - w_s L_s + \sum_j^J \left[(1 + r_{js-1}) p_{s-1}^* \epsilon_{js-1} - p_s^* \epsilon_{js} \right] \right. \\
&\quad \left. - p_s^I [K_s - (1 - \delta) K_{s-1} + C] \right] \tag{3.6}
\end{aligned}$$

where μ_s is a discount factor, defined as

$$\mu_s = \begin{cases} 1 & \text{when } s = t \\ \prod_{a=t}^{s-1} (1 + R_a) & \text{when } s > t \end{cases}$$

and R_a is the rate of return on the firm's capital.

Regrouping the terms with common time subscripts in (3.6) gives the following form of the intertemporal profit flow function:

$$\begin{aligned}
\Pi_t &= \sum_{s=t}^{\infty} \left\{ \sum_i^I y_s \frac{p_s^y}{\mu_s} - \sum_n^N x_{ns} \frac{p_s^{x_n}}{\mu_s} - \frac{w_s L_s}{\mu_s} - \sum_j^J \left[\frac{p_s^*}{\mu_s} - \frac{(1 + r_{js}) p_s^*}{\mu_{s+1}} \right] \epsilon_{js} \right. \\
&\quad \left. - \frac{p_s^I}{\mu_s} [K_s + C] + \frac{p_{s+1}^I (1 - \delta) K_s}{\mu_{s+1}} \right\} \tag{3.7}
\end{aligned}$$

In equation (3.7) the contribution of monetary assets and depreciated capital from period $t-1$ is ignored, since it is fixed in period t and does not affect the variable profit flow in period t .

Define $\bar{p}_s^y = \frac{p_s^y}{\mu_s}$, $\bar{p}_s^{x_n} = \frac{p_s^{x_n}}{\mu_s}$, $\bar{w}_s = \frac{w_s}{\mu_s}$, $\eta_{js} = \frac{p_s^*}{\mu_s} - \frac{(1 + r_{js}) p_s^*}{\mu_{s+1}}$, $\bar{p}_s^I = \frac{p_s^I}{\mu_s}$,

and $p_s^K = \frac{p_s^I}{\mu_s} - \frac{p_{s+1}^I(1-\delta)}{\mu_{s+1}}$. Substituting these definitions into (3.7) gives another expression

for the intertemporal profit function:

$$\Pi_t = \sum_{s=t}^{\infty} \left\{ \sum_i^I y_s \bar{p}_s^y - \sum_n^N x_{ns} \bar{p}_s^{x_n} - \bar{w}_s L_s - \sum_j^J \eta_{js} \varepsilon_{js} - p_s^K K_s - \bar{p}_s^I C \right\} \quad (3.8)$$

It is assumed that the manufacturing firm chooses the levels of output and real and monetary factors of production to maximize the expected discounted intertemporal profit flow, subject to its production technology. Under the assumption of complete markets, perfect competition, and risk neutrality of the owners of the firm, the problem can be presented as the following dynamic choice problem:

$$\begin{aligned} \text{Max } E_t \left\{ \sum_{s=t}^{\infty} \frac{1}{\mu_s} \left[\sum_i^I y_s p_s^y - \sum_n^N x_{ns} p_s^{x_n} - w_s L_s + \sum_j^J \left[(1+r_{j,s-1}) p_{s-1}^* \varepsilon_{j,s-1} - p_s^* \varepsilon_{js} \right] \right. \right. \\ \left. \left. - p_s^I [K_s - (1-\delta)K_{s-1} + C] \right] \right\} \end{aligned} \quad (3.9)$$

$$\text{subject to } y_t = F(x_{1s}, \dots, x_{Ns}, L_s, K_s, \varepsilon_{1s}, \dots, \varepsilon_{Js}) \quad \forall s \geq t$$

where E_t denotes expectations, given the information available at period t .

The first order conditions of this stochastic optimal control problem can be derived by applying Bellman's dynamic programming method. To do so, the state and control variables must be selected in a manner such that the decision is in Bellman form. The prices $p_s^y, p_s^{x_n}, p_s^*, p_s^I, w_t$ and R_s are random stochastic processes that are not controllable by the firm. The monetary asset yield $r_{j,s-1}$, and p_{s-1}^* are nonstochastic and are taken as given by the firm. The control variables are $x_{ns} \forall n, \varepsilon_{js} \forall s, L_s$ and K_s , while the selected state variables for period s are $K_{s-1}, \varepsilon_{j,s-1} \forall j, p_s^y, p_s^{x_n} \forall n, p_s^I, p_s^*, p_{s-1}^*, R_s, w_s, r_{j,s-1} \forall j$.

Define w_t to be the vector of all state variables, and define u_t to be the vector of all control variables. Let Λ_s be the subset of state variables defined by $\Lambda_s =$

$\left\{ p_s^y, p_s^{x_n} \forall n, p_s^I, p_s^*, p_{s-1}^*, R_s, w_s, r_{js-1} \forall j \right\}$. We assume that Λ_s follows a first-order Markov process, with transitions governed by the conditional distribution function $F(\Lambda_{s+1} | \Lambda_s)$. This conditional distribution function defines implicitly the transition equation for the state variables included in Λ_s . The remaining transition equations are defined by the obvious time shifts between some of the elements that appear simultaneously, but with time shifts, in the control and state vectors. Making these substitutions and changes in notation, it can be shown that the problem now is in Bellman form.

The dynamic decision problem now can be put in the familiar Bellman form:

$$\begin{aligned} \text{Max } E_t \left\{ \sum_{s=t}^{\infty} \frac{1}{R_s} \pi_s(\mathbf{w}_s, \mathbf{u}_s) \right\} & \quad (3.10) \\ \text{subject to } \mathbf{w}_{s+1} = g(\mathbf{u}_s), \quad s \geq t & \end{aligned}$$

where $\pi_s(\mathbf{w}_s, \mathbf{u}_s)$ is given by equation (3.4) and g is the vector of all transition equations for the state variables. Using Bellman's method and the Benveniste and Scheinkman equation, the Euler equations are found to be

$$E_t \left\{ \frac{\partial \pi_t(\mathbf{w}_t, \mathbf{u}_t)}{\partial \mathbf{u}_t} + \frac{1}{1 + R_t} \frac{\partial g'}{\partial \mathbf{u}_t} \frac{\partial \pi_{t+1}(\mathbf{w}_{t+1}, \mathbf{u}_{t+1})}{\partial \mathbf{w}_{t+1}} \Big|_{\mathbf{w}_t} \right\} = \mathbf{0} \quad (3.11)$$

Substituting the quadratic function $\frac{\gamma I_t^2}{2}$ for the cost of adjustment function $C(I_t)$ and replacing the corresponding symbols with the actual state and control variables and transition equations, we obtain the following system of Euler equations:

$$E_t \left\{ p_t^y \frac{\partial F}{\partial x_{nt}} - p_t^{x_n} \right\} = 0 \quad \forall n = 1, \dots, N \quad (3.12)$$

$$E_t \left\{ p_t^y \frac{\partial F}{\partial L_t} - w_t \right\} = 0 \quad (3.13)$$

$$E_t \left\{ p_t^y \frac{\partial F}{\partial \varepsilon_{jt}} - p_t^* \frac{(R_t - r_{jt})}{1 + R_t} \right\} = 0 \quad \forall j = 1, \dots, J \quad (3.14)$$

$$E_t \left\{ p_t^y \frac{\partial F}{\partial K} - p_t^I \left[1 + \gamma(K_t - (1 - \delta)K_{t-1}) \right] - \frac{p_{t+1}^I}{1 + R_t} \left[1 - \delta + \gamma(1 - \delta)(K_{t+1} - (1 - \delta)K_t) \right] \right\} = 0 \quad (3.15)$$

The result is a system of $J+N+2$ nonlinear equations. A solution exists for $(x_{1t}, \dots, x_{Nt}, L_t, K_t, \varepsilon_{1t}, \dots, \varepsilon_{Jt})$ to these Euler equations augmented by the production function (3.1). But in practice a closed form algebraic solution rarely exists, so that only a numerical solution can be produced. Nevertheless, with a parametric specification of the technology, GMM can be used to estimate the parameters from the Euler equations themselves.

3.2 Demand-Side Monetary Aggregation and Weak Separability

By estimating the parameters of the Euler equations by GMM, we can investigate properties of technology, such as returns to scale. If the firm's monetary inputs are weakly separable from output, we also can investigate the resulting exact demand-side monetary aggregate.

The approach to identifying and generating an exact theoretical demand-side monetary aggregate for a manufacturing firm is described by Barnett (1987) in the case of perfect certainty. We extend his approach to the case of risk. The procedure involves two steps. First one tests the existence condition for an exact aggregate. The existence condition is blockwise weak separability of the monetary assets in the production function from the firm's other factor inputs. Under this existence condition, it becomes possible to factor those monetary assets as a subfunction out the rest of the firm's structure.

Define $\mathbf{y}_0 = (x_{1t}, \dots, x_{Nt}, L_t, K_t)'$ and $\mathbf{z} = (\varepsilon_{1t}, \dots, \varepsilon_{jt})'$. If \mathbf{z} is weakly separable from \mathbf{y}_0 , then there exist functions H and z_0 such that

$$F(\mathbf{y}_0, \mathbf{z}) = H(\mathbf{y}_0, z_0(\mathbf{z})),$$

where z_0 is the aggregator function over the monetary asset inputs \mathbf{z} . The weak separability condition is mathematically equivalent to:

$$\frac{\partial \left(\frac{\partial F / \partial z_i}{\partial F / \partial z_j} \right)}{\partial \phi} = 0 \quad \text{for } i \neq j,$$

where ϕ is any of the components of \mathbf{y}_0 . Under that condition, the marginal rate of substitution between any two different monetary assets is independent of the levels of any of the other nonmonetary inputs. An additional restriction which is usually imposed on the aggregator function is linear homogeneity in the components.²⁰

If the existence condition is satisfied, we can progress to the second step, which is estimation of the aggregator function $z_0(\mathbf{z})$. Clearly the first step must precede the second step, since the aggregator function estimated in the second step does not exist unless the hypothesis of weak separability tested in the first step is accepted.

3.3 Flexible functional form specification and regularity conditions

As in the case of the financial intermediary in section 2, we specify the technology of the firm to be Diewert and Wales's (1991) symmetric generalized McFadden flexible functional form, but now with exact nested input aggregation for financial assets rather than exact nested output aggregation²¹. Hence the null hypothesis of exact monetary input aggregation is imposed directly on the production function F by specifying separately flexible functional forms for $H(\mathbf{y}_0, z_0)$ and $z_0(\mathbf{z})$, where $z_0(\mathbf{z})$ is nested into $H(\mathbf{y}_0, z_0)$ to assure the desired weakly separable structure for F . It is further assumed that the production function is linearly homogeneous in its components. The extension to nonconstant returns to scale is a subject for future research.

Define H to be the symmetric generalized McFadden functional form

²⁰ As discussed in section 2.2, the extension to nonhomogeneous z_0 requires the introduction of distance functions, which is beyond the current scope of this paper. Regarding that possible extension, see Barnett (1987, sections 7.1 and 8.1).

²¹ The alternative specification of Diewert and Wales's (1991) symmetric generalized Barnett flexible functional form could be the subject for future research. For an application of that specification to banks, see Barnett and Hahm (1994).

$$H(\mathbf{y}_0, z_0) = a_0 z_0 + \mathbf{a}' \mathbf{y}_0 + \frac{1}{2} \begin{bmatrix} z_0 & \mathbf{y}_0' \end{bmatrix} \bar{\mathbf{A}} \begin{bmatrix} z_0 \\ \mathbf{y}_0 \end{bmatrix} / \alpha' \mathbf{y}_0 \quad (3.16)$$

with $\alpha' \mathbf{y}_0 \neq 0$, where a_0 , $\mathbf{a}' = (a_1, \dots, a_n)$, and $\bar{\mathbf{A}}$ are parameters to be estimated. The matrix $\bar{\mathbf{A}}$ is symmetric $(n+1) \times (n+1)$. The vector $\alpha' = (\alpha_1, \dots, \alpha_n)$ contains fixed nonnegative constants selected by the researcher. The linear homogeneity of H in \mathbf{y}_0 and z_0 is obtained by dividing H by $\alpha' \mathbf{y}_0$.

To conform with the partitioning of the elements of H , the matrix $\bar{\mathbf{A}}$ is partitioned as follows:

$$\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A} \end{bmatrix},$$

where \mathbf{A}_{11} is a scalar, \mathbf{A}_{12} is a $1 \times n$ row vector, \mathbf{A}_{21} is a $n \times 1$ column vector, and \mathbf{A} is $n \times n$ symmetric matrix. Since $\bar{\mathbf{A}}$ is symmetric, we have that $\mathbf{A}_{12} = \mathbf{A}'_{21}$.

Let $(\mathbf{y}_0^*, z_0^*) \neq 0$ be the point about which the functional form is locally flexible. Because of the linear homogeneity assumption, the parameters in the functional form (3.16) are $n+1$ more than are needed for a parsimonious flexible functional form²². Hence more restrictions can be imposed without compromising the Diewert-flexibility property. We therefore impose

$$\alpha' \mathbf{y}_0^* = 1, \quad (3.17)$$

$$\mathbf{A}_{11} z_0^* + \mathbf{A}_{12} \mathbf{y}_0^* = 0, \quad (3.18)$$

and

$$\mathbf{A}'_{12} z_0^* + \mathbf{A} \mathbf{y}_0^* = \mathbf{0}_n, \quad (3.19)$$

where $\mathbf{0}_n$ is an n -dimensional vector of zeros. Under the above restrictions the number of parameters is reduced to $1+n+n(n+1)/2$, which is the minimum number of free parameters needed to maintain flexibility.

Solving (3.18) and (3.19) for \mathbf{A}_{11} and \mathbf{A}_{12} , and substituting into (3.16), we get

²²See analogous details in section 2 above.

$$\begin{aligned}
H(\mathbf{y}_0, z_0(\mathbf{z})) &= a_0 z_0 + \mathbf{a}' \mathbf{y}_0 + \frac{1}{2} (\boldsymbol{\alpha}' \mathbf{y}_0)^{-1} \mathbf{y}_0' \mathbf{A} \mathbf{y}_0 \\
&\quad - (\boldsymbol{\alpha}' \mathbf{y}_0)^{-1} \mathbf{y}_0^{*'} \mathbf{A} \mathbf{y}_0 (z_0 / z_0^*) + \frac{1}{2} (\boldsymbol{\alpha}' \mathbf{y}_0)^{-1} \mathbf{y}_0^{*'} \mathbf{A} \mathbf{y}_0^* (z_0 / z_0^*)^2.
\end{aligned} \tag{3.20}$$

Similarly define the monetary aggregator function $z_0(\mathbf{z})$ to be

$$z_0(\mathbf{z}) = \mathbf{b}' \mathbf{z} + \frac{1}{2} \mathbf{z}' \mathbf{B} \mathbf{z} / \boldsymbol{\beta}' \mathbf{z} \tag{3.21}$$

with the parameters satisfying

$$\boldsymbol{\beta}' \mathbf{z}^* = 1, \tag{3.22}$$

$$z_0^* = \mathbf{b}' \mathbf{z}^*, \tag{3.23}$$

$$\mathbf{B} \mathbf{z}^* = \mathbf{0}_m, \tag{3.24}$$

and $\boldsymbol{\beta}' \mathbf{z} \neq 0$. The vector $\mathbf{b}' = (b_1, \dots, b_m)$ and the symmetric $m \times m$ matrix \mathbf{B} are parameters to be estimated. The vector $\boldsymbol{\beta}' = (\beta_1, \dots, \beta_m)$ is a vector of fixed nonnegative constants, and the point $\mathbf{z}^* \neq \mathbf{0}$ is the point about which equation (3.21) is locally flexible.

Substituting (3.21) into (3.16) yields

$$\begin{aligned}
F(\mathbf{y}_0, \mathbf{z}) &= H(\mathbf{y}_0, z_0(\mathbf{z})) \\
&= a_0 \left(\mathbf{b}' \mathbf{z} + \frac{1}{2} (\boldsymbol{\beta}' \mathbf{z})^{-1} \mathbf{z}' \mathbf{B} \mathbf{z} \right) + \mathbf{a}' \mathbf{y}_0 + \frac{1}{2} (\boldsymbol{\alpha}' \mathbf{y}_0)^{-1} \mathbf{y}_0' \mathbf{A} \mathbf{y}_0 \\
&\quad - \left(z_0^* \boldsymbol{\alpha}' \mathbf{y}_0 \right)^{-1} \mathbf{y}_0^{*'} \mathbf{A} \mathbf{y}_0 \left(\mathbf{b}' \mathbf{z} + \frac{1}{2} (\boldsymbol{\beta}' \mathbf{z})^{-1} \mathbf{z}' \mathbf{B} \mathbf{z} \right) \\
&\quad + \frac{1}{2} \left(z_0^{*2} \boldsymbol{\alpha}' \mathbf{y}_0 \right)^{-1} \mathbf{y}_0^{*'} \mathbf{A} \mathbf{y}_0^* \left(\mathbf{b}' \mathbf{z} + \frac{1}{2} (\boldsymbol{\beta}' \mathbf{z})^{-1} \mathbf{z}' \mathbf{B} \mathbf{z} \right)^2
\end{aligned} \tag{3.25}$$

which by construction satisfies weak separability in monetary asset inputs.

Neoclassical curvature conditions require $F(\mathbf{y}_0, \mathbf{z})$ to be a concave function in all of its arguments, increasing in inputs. The monetary aggregator function $z_0(\mathbf{z})$ should be concave and monotonically increasing. Suppose further that

$$\frac{\partial H(\mathbf{y}_0, z_0)}{\partial z_0} \geq 0. \tag{3.26}$$

Then $F(\mathbf{y}_0, \mathbf{z})$ is globally concave, when $H(\mathbf{y}_0, z_0)$ is concave in (\mathbf{y}_0, z_0) and $z_0(\mathbf{z})$ is concave in \mathbf{z} .

Diewert and Wales (1987) prove that $H(\mathbf{y}_0, \mathbf{z}_0)$ is a globally concave function if and only if the matrix \mathbf{A} is negative semidefinite, and $z_0(\mathbf{z})$ is a globally concave function if and only if the matrix \mathbf{B} is negative semidefinite. Negative semidefiniteness of the matrix \mathbf{A} and of the matrix \mathbf{B} can be imposed without destroying flexibility by the substitution

$$\mathbf{A} = -\mathbf{q}\mathbf{q}' \quad (3.27)$$

and

$$\mathbf{B} = -\mathbf{u}\mathbf{u}' \quad (3.28)$$

where \mathbf{q} is a lower triangular $n \times n$ matrix and \mathbf{u} is a lower triangular $m \times m$ matrix.

Following the substitution of (3.27) and (3.28), we estimate \mathbf{q} and \mathbf{u} .

Monotonicity restrictions are imposed locally at the point of flexibility. The first derivatives of (3.25) with respect to $(\mathbf{y}_0, \mathbf{z}_0)$ are

$$\begin{aligned} \frac{\partial F}{\partial \mathbf{y}_0} = & \mathbf{a} + \frac{1}{2} \left[2(\alpha' \mathbf{y}_0)^{-1} \mathbf{A} \mathbf{y}_0 - (\alpha' \mathbf{y}_0)^{-2} \alpha \mathbf{y}_0' \mathbf{A} \mathbf{y}_0 \right] \\ & - \left[(z_0^* \alpha' \mathbf{y}_0)^{-1} \mathbf{A} \mathbf{y}_0^* - (z_0^* \alpha' \mathbf{y}_0)^{-2} z_0^* \alpha \mathbf{y}_0'^* \mathbf{A} \mathbf{y}_0 \right] \left(\mathbf{b}' \mathbf{z} + \frac{1}{2} (\beta' \mathbf{z})^{-1} \mathbf{z}' \mathbf{B} \mathbf{z} \right) \\ & - \frac{1}{2} \left(z_0^{*2} \alpha' \mathbf{y}_0 \right)^{-2} z_0^{*2} \alpha \mathbf{y}_0'^* \mathbf{A} \mathbf{y}_0^* \left(\mathbf{b}' \mathbf{z} + \frac{1}{2} (\beta' \mathbf{z})^{-1} \mathbf{z}' \mathbf{B} \mathbf{z} \right)^2 \end{aligned} \quad (3.29)$$

and

$$\begin{aligned} \frac{\partial F}{\partial \mathbf{z}} = & a_0 \left[\mathbf{b} + \frac{1}{2} \left(2(\beta' \mathbf{z})^{-1} \mathbf{B} \mathbf{z} - (\beta' \mathbf{z})^{-2} \beta \mathbf{z}' \mathbf{B} \mathbf{z} \right) \right] \\ & - (z_0^* \alpha' \mathbf{y}_0)^{-1} \mathbf{y}_0'^* \mathbf{A} \mathbf{y}_0 \left[\mathbf{b} + \frac{1}{2} \left(2(\beta' \mathbf{z})^{-1} \mathbf{B} \mathbf{z} - (\beta' \mathbf{z})^{-2} \beta \mathbf{z}' \mathbf{B} \mathbf{z} \right) \right] \\ & + (z_0^{*2} \alpha' \mathbf{y}_0)^{-1} \mathbf{y}_0'^* \mathbf{A} \mathbf{y}_0^* \left[\mathbf{b} + \frac{1}{2} \left(2(\beta' \mathbf{z})^{-1} \mathbf{B} \mathbf{z} - (\beta' \mathbf{z})^{-2} \beta \mathbf{z}' \mathbf{B} \mathbf{z} \right) \right] \\ & * \left(\mathbf{b}' \mathbf{z} + \frac{1}{2} (\beta' \mathbf{z})^{-1} \mathbf{z}' \mathbf{B} \mathbf{z} \right) \end{aligned} \quad (3.30)$$

Similarly the first derivative of the aggregator function (3.21) with respect to \mathbf{z} is

$$\frac{\partial z_0}{\partial \mathbf{z}} = \mathbf{b} + (\beta' \mathbf{z})^{-1} \mathbf{Bz} - \frac{1}{2} (\beta' \mathbf{z})^{-2} \beta \mathbf{z}' \mathbf{Bz} \quad (3.31)$$

Evaluating these derivatives at the chosen point of local flexibility $(\mathbf{y}_0^*, \mathbf{z}^*)$ yields

$$\frac{\partial F}{\partial \mathbf{y}_0} (\mathbf{y}_0^*, \mathbf{z}^*) = \mathbf{a} , \quad (3.32)$$

$$\frac{\partial F}{\partial \mathbf{z}} (\mathbf{y}_0^*, \mathbf{z}^*) = a_0 \mathbf{b} \quad (3.33)$$

and

$$\frac{\partial z_0}{\partial \mathbf{z}} (\mathbf{z}^*) = \mathbf{b} . \quad (3.34)$$

At present, we assume that the firm produces only one output, which is the first element of the vector \mathbf{y}_0 . Imposing the neoclassical monotonicity conditions, equations (3.32), (3.33), and

(3.34) imply the following constraints on the parameters

$$a_0 \geq 0, \quad a_i \geq 0 \quad \text{for } i=1, \dots, n \quad \text{and} \quad \mathbf{b} \geq \mathbf{0}. \quad (3.35)$$

Under imposition of (3.35), the monotonicity conditions are satisfied locally at $(\mathbf{y}_0^*, \mathbf{z}^*)$.

Substituting the functional form defined by (3.25) into the Euler equations (3.12)-(3.15), provides a structural dynamic system of nonlinear integral equations to be estimated. Our parameter restrictions assure valid global curvature, local monotonicity, and global weak separability in the monetary asset portfolio employed by the firm.

As with the financial firm, we use GMM to estimate the technology of the representative manufacturing firm, and we use Hansen's χ^2 test to test the imposed assumption of weak separability in monetary assets, although those assets now are inputs rather than outputs. If the test of no overidentifying restrictions is rejected, then the null hypothesis of weak separability is rejected.

3.4 Data and Empirical Application

The model is applied empirically to the aggregate US manufacturing sector with data for the period 1949-1988. Real input resources include capital, labor, and materials. Monetary inputs include two types of assets: cash on hand and in US banks and US government securities.

Using equations (3.12)-(3.15), the system of Euler equations to be estimated becomes

$$E_t \left\{ p_t^O \frac{\partial F}{\partial L_t} - p_t^L \right\} = 0 \quad (3.36)$$

$$E_t \left\{ p_t^O \frac{\partial F}{\partial M_t} - p_t^M \right\} = 0 \quad (3.37)$$

$$E_t \left\{ p_t^y \frac{\partial F}{\partial C_t} - p_t^* \frac{(R_t - r_t^C)}{1 + R_t} \right\} = 0 \quad (3.38)$$

$$E_t \left\{ p_t^y \frac{\partial F}{\partial S_t} - p_t^* \frac{(R_t - r_t^S)}{1 + R_t} \right\} = 0 \quad (3.39)$$

and

$$E_t \left\{ p_t^O \frac{\partial F}{\partial K_t} - p_t^K \left[1 + \gamma(K_t - (1 - \delta)K_{t-1}) \right] - \frac{p_{t+1}^K}{1 + R_t} \left[1 - \delta + \gamma(1 - \delta)(K_{t+1} - (1 - \delta)K_t) \right] \right\} = 0 \quad (3.40)$$

where K_t is capital services, L_t is labor, M_t is materials, C_t is cash on hand and in banks, S_t is government securities, δ is the rate of capital depreciation, p_t^* is the true cost-of-living index, and R_t is the rate of return to capital. The prices of output, capital, labor and materials respectively are p_t^O , p_t^K , p_t^L , p_t^M , while the rates of return on cash and securities respectively are r_t^C and r_t^S .

In accordance with this notation, we can write $\mathbf{y}'_0 = (K_t, L_t, M_t)$ and $\mathbf{z}' = (C_t, S_t)$. The center of local approximation is selected to be the point at which $y_0^* = (1,1,1)$, $z_0^* = 1$ and $\mathbf{z}'^* = (1,1)$. To assure that the center of approximation is located within the interior of the observations, we rescale the data on all quantities about a chosen data point such that $\tilde{y}_i^t = y_i^t / y_i^{t^*} \quad \forall i = 1,2,3$ and $\tilde{z}_i^t = z_i^t / z_i^{t^*} \quad \forall i = 1,2$, where t^* represents the year of the chosen data point, $y_i \quad \forall i = 1,2,3$ are the elements of \mathbf{y}_0 , and $z_i \quad \forall i = 1,2$ are the elements of \mathbf{z} .²³ To prevent dollar revenues and expenditures from being altered by our data normalization, each price is rescaled by multiplication by the corresponding quantity at the chosen data point.

The fixed nonnegative constants α_i and β_i are selected such that

$$\alpha_i = \frac{|\bar{y}_i|}{\sum_{j=1}^3 |\bar{y}_j|} \quad \forall i = 1,2,3 \quad (3.41)$$

and

$$\beta_i = \frac{|\bar{z}_i|}{\sum_{j=1}^2 |\bar{z}_j|} \quad \forall i = 1,2 \quad (3.42)$$

where \bar{y} and \bar{z} are the sample means of \tilde{y} and \tilde{z} respectively. With our data, we find $\alpha_1=0.26$, $\alpha_2=0.42$, $\alpha_3=0.32$, $\beta_1=0.56$ and $\beta_2=0.44$. Note that the constants α_i and β_i satisfy equations (3.17) and (3.22), as is required.

Equation (3.23) implies that $b_1+b_2=1$, and the monotonicity condition (3.35) requires that $b_1 \geq 0$ and $b_2 \geq 0$. Therefore it follows that $b_i \leq 1$ for $i=1,2$. Combining these constraints permits us to replace b_i by

$$b_1 = \sin^2 \varphi \text{ and } b_2 = \cos^2 \varphi. \quad (3.43)$$

We then estimate φ .

We normalize $a_0=1$ to satisfy the monotonicity condition (3.35). The monotonicity conditions (3.35) are imposed by the substitution $a_1 = \tilde{a}_1^2$, $a_2 = \tilde{a}_2^2$, and $a_3 = \tilde{a}_3^2$, where $\tilde{a}_i \quad \forall i = 1,2,3$ are new parameters to be estimated. The curvature conditions are imposed by replacing the matrices \mathbf{A} and \mathbf{B} by $-\mathbf{q}\mathbf{q}'$ and $-\mathbf{u}\mathbf{u}'$ respectively, where the lower triangular

²³In our estimation the data point at which quantities are set to unity is chosen to be 1982.

matrices \mathbf{q} and \mathbf{u} are as in equations (2.40) of section 2, but with the dimension of \mathbf{q} being 3×3 here, instead of 4×4 . Equation (3.24) then becomes as in equation (2.41) of section 2.

Solving (2.41) for u_{21} and u_{22} , we get $u_{21} = -u_{11}$ and $u_{22} = 0$. Substituting for these parameters in (3.28) yields the following form of the matrix \mathbf{B}

$$\mathbf{B} = u_{11}^2 \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \quad (3.44)$$

The parameters to be estimated are φ , u_{11} , γ , the vector $\tilde{\mathbf{a}}' = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3)$, and the matrix \mathbf{q} .

The data comes primarily from two sources. Data on output and factor inputs in US Manufacturing for the period 1949-1988 is acquired from the Division of Multifactor Productivity of the Bureau of Labor Statistics. The data consists of quantity and price Törnqvist indices. Capital input is defined as the flow of services from physical assets, which include equipment, structures, inventories, and land. Labor input is defined as the paid hours of all persons engaged in the sector. Materials input consists of all commodity inputs exclusive of fuel inputs.

The source of data on money balances held by manufacturing firms is the *Quarterly Financial Report for Manufacturing, Mining, and Trade Corporations* for the period 1949-1988. To convert to real units, we deflate the nominal balances by the Fisher ideal index approximation to the true cost-of-living index, computed as in section 2 above. The rates of return on cash on hand and in banks and government securities are from the City Bank database. We use the 6-month commercial paper yield as the rate of return on cash on hand and in banks and the 3-month Treasury-bill rate as the rate of return on government securities. The reason for the non-zero rate of return for cash on hand and in banks is that it does not consist solely of currency. Cash on hand and in banks in our data source is defined to be the sum of manufacturing sector holdings of currency, demand deposits, and time deposits. Separate data on such holdings of currency are available only for the most recent observations.

We use an external nominal bond rate for the rate of return on capital, R_t , since data was unavailable to compute an internal rate of return²⁴. Data on Moody's Baa bond rate is obtained from the City Bank database.

3.5 Results

The model is estimated using the GMM estimator in the TSP mainframe version 7.02. The following variables are used as instruments in the estimation procedure: a constant, total US population, and lagged values of the prices of capital and materials, of the rate of return on cash and securities, and of the Moody's Baa bond rate. The results are robust to heteroscedasticity and autocorrelation.

The GMM estimates of the manufacturing firm's production function are reported in Table 3.1. We tested weak separability of monetary assets from the other variables in technology. We ran that test using Hansen's χ^2 test for no overidentifying restrictions in the model with weak separability imposed through the structure of the model. The test statistic is $F=TQ$, where $T=40$ is the sample size and $Q=.35$ is the value of the objective function in the GMM estimation. The test statistic is distributed as a χ^2 with $e-f$ degrees of freedom, where $e=35$ is the number of orthogonality conditions and $f=12$ is the number of parameters. The calculated test statistic is 14, and the critical value at the 10% level of significance is 32.01. Hence we cannot reject the hypothesis of weak separability of monetary assets. The resulting monetary aggregator function was used to produce plots of the estimated exact monetary aggregate, along with the corresponding simple sum and Divisia monetary aggregate, and those plots are displayed in Barnett, Kirova, and Pasupathy (1995).

In this paper, we use the parameter estimates to explore the regularity properties of the manufacturing firm's production technology. Monotonicity of the monetary aggregator function was imposed only at the point of local flexibility, and checked at all data observations. No

²⁴For a rigorous discussion of theoretical and empirical issues concerning the measurement of the rate of return on capital, see Harper, Berndt and Wood (1989).

violations were observed at our data points. Concavity of the monetary aggregator function was imposed globally.

Monotonicity of the composite production function F was imposed at only one point, and checked at all data points after estimation. Monotonicity conditions were satisfied at all points for capital, for labor, and for materials. Violations of monotonicity were observed for the components of the monetary aggregate. The production function was not found to be increasing in cash and in securities at 9 observations in the beginning of the sample period. These violations of monotonicity of F implied violations of monotonicity of H in the monetary aggregate, $z_0(\mathbf{z})$. Concavity of the function H , defined to include capital, labor, materials, and the monetary aggregate as inputs, was imposed globally. However, the concavity of the composite production function F , defined to include each of the monetary components as inputs in addition to capital, labor, and materials, is contingent upon the monotonicity of H in the monetary aggregate, $z_0(\mathbf{z})$. The violations of monotonicity of H with respect to the monetary aggregate translated into violations of the neoclassical curvature conditions of the composite production function F . Recalling that $F(\mathbf{y}_0, \mathbf{z}) = H(\mathbf{y}_0, z_0(\mathbf{z}))$, it is evident that even if z_0 and H are globally concave and even if z_0 satisfies monotonicity everywhere, a violation of monotonicity of H in $z_0(\mathbf{z})$ can produce a violation of concavity of F . Concavity of F was violated at 9 data points where the monotonicity condition (3.26)

$$\frac{\partial H(\mathbf{y}_0, z_0)}{\partial z_0} \geq 0$$

did not hold.

To visualize the impact of regularity violations on the characteristics of production we plotted the isoquants of each pair of inputs over a range that covers our data observations. See Figures 1-10. Suppose that ridge lines are drawn onto the isoquant plots. There are two possible ridge lines. One connects points at which the isoquants are vertical, and the other ridge line connects points at which the isoquants are horizontal. Consider any point outside of the region

bounded by the ridge lines. At any such point there is an isoquant that goes through that point. Move along that isoquant, from the point to the nearest ridge line. Along that path, output will remain constant by the definition of an isoquant, but the quantity of both factors will decrease until the ridge line is reached. Hence by moving along that path, output and revenue remain constant, but factor employment and factor costs are decreased. With constant revenue and lower cost, profits are increased. Therefore, no firm will ever operate outside of the area bounded by the ridge lines, since that would imply inefficient overemployment of all factors, and behavior inconsistent with the profit-maximization objectives of the firm. Within the region bounded by the ridge lines, if output is held constant, the only way to decrease the employment of one factor will be to increase the employment of the other factor of production.

Our plots demonstrate that violations of efficient behavior within the region of our data could occur for almost all pairs of inputs. In fact within our data region, there exist points lying outside of the region bounded by the ridge lines for all pairs of inputs, except for materials and securities, and for cash and securities. Hence the observed violations of regularity suggest that there are serious limitations to the use of the Generalized McFadden parametric specification of technology, and illustrate that even with curvature imposed globally for the aggregator functions nested within technology and also with curvature imposed globally for the technology defined over aggregates, the composite function produced by substituting the aggregator functions into the technology within which the aggregator functions are nested can violate curvature----because of violations of monotonicity. In short, violations are by no means a trivial matter that easily can be ignored by those emphasizing satisfaction of curvature, since violations of monotonicity can induce violations of curvature of composite functions.

4. Conclusions

We find that the generalized McFadden (or generalized quadratic) specification of technology produces disturbing violations of monotonicity, even when curvature is imposed globally. We encountered that problem both with the manufacturing firm model and the financial intermediary

model. In addition, we find that the violations of monotonicity create very serious problems, including induced violations of curvature in composite functions defined by weakly separable nesting of aggregator functions within the firm's technology---even when curvature has been imposed globally for all of the component functions defining the composite function. In short, imposing curvature without monotonicity, while perhaps to be preferred to the prior common practice of imposing neither, is not satisfactory. Monotonicity is much too important to be overlooked.

Research on models permitting imposition of both curvature and monotonicity remains at an early stage and has so far had little impact on the literature on production modeling. While a difficult literature, we believe that research on models permitting flexible imposition of true regularity---i.e. both monotonicity and curvature---should expand.²⁵

²⁵Contributions to that literature include the generalized Barnett model of Diewert and Wales (1987) and the approaches to imposing regularity proposed by Terrell (1995a,b), Gallant and Golub (1984), and Koop, Osiewalski, and Steel (1994).

Table 3.1

GMM Estimates of Parameters of Manufacturing Firm's Technology

<u>Parameter</u>	<u>Estimate</u>	<u>t-Ratio</u>
γ	.73	.86
ϕ	61.82	288,478
\tilde{a}_1	.23	7.37
\tilde{a}_2	.41	63.41
\tilde{a}_3	.51	167.83
u ₁₁	.02	16.98
q ₁₁	.68	27.53
q ₂₁	.37	24.82
q ₂₂	.94	66.98
q ₃₁	.11	1.92
q ₃₂	-.23	-6.73
q ₃₃	-.002	-.001

Table 2.1:
GMM Estimates of Parameters of Financial Firm's Technology

<u>Parameter</u>	<u>Estimate</u>	<u>t-Ratio</u>
θ	58.05	839.3
\tilde{a}_1	.039	2.52
\tilde{a}_2	.230	9.11
\tilde{a}_3	.030	.06
\tilde{a}_4	.034	.05
$\tilde{\rho}$	260.8	2.90
$\tilde{\mu}$.259	2.35
u_{11}	-.532	-2.54
q ₁₁	0.098	-5.06
q ₂₁	.326	.80
q ₃₁	.444	1.08
q ₄₁	-.479	-1.30
q ₂₂	-1.586	-5.08
q ₃₂	1.755	4.29
q ₃₃	-1.764	-6.24
q ₄₂	-.298	-1.59
q ₄₃	1.914	6.26
q ₄₄	1.163	9.26

Figure 1: Isoquants of Capital vs. Labor

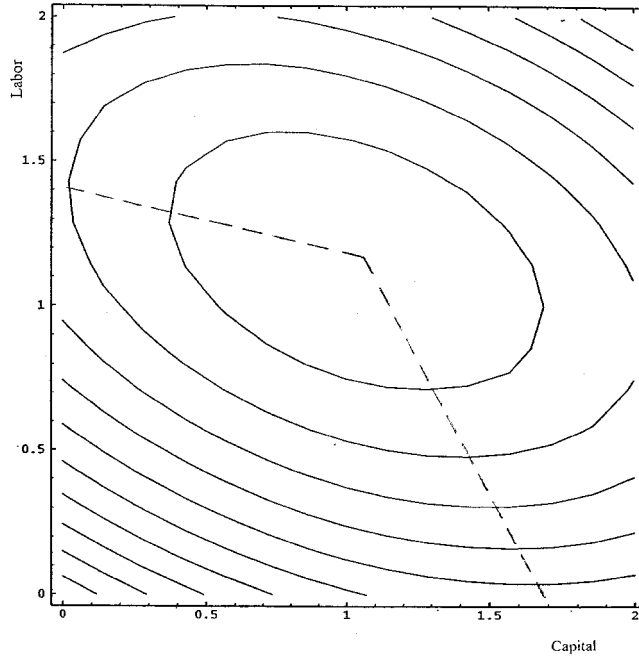


Figure 2: Isoquants of Capital vs. Materials

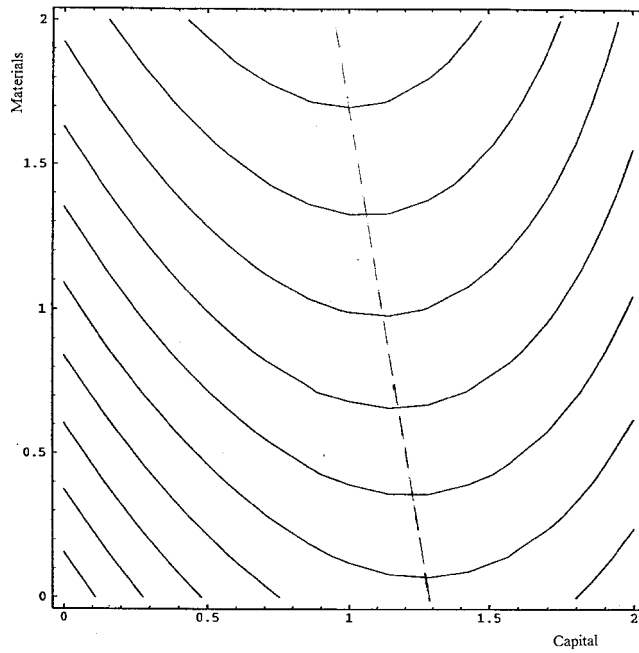


Figure 3: Isoquants of Capital vs. Cash

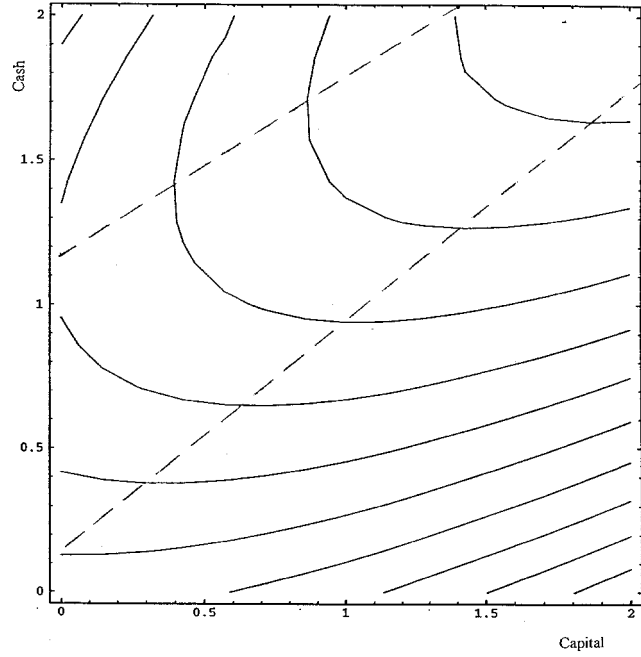


Figure 4: Isoquants of Capital vs. Securities

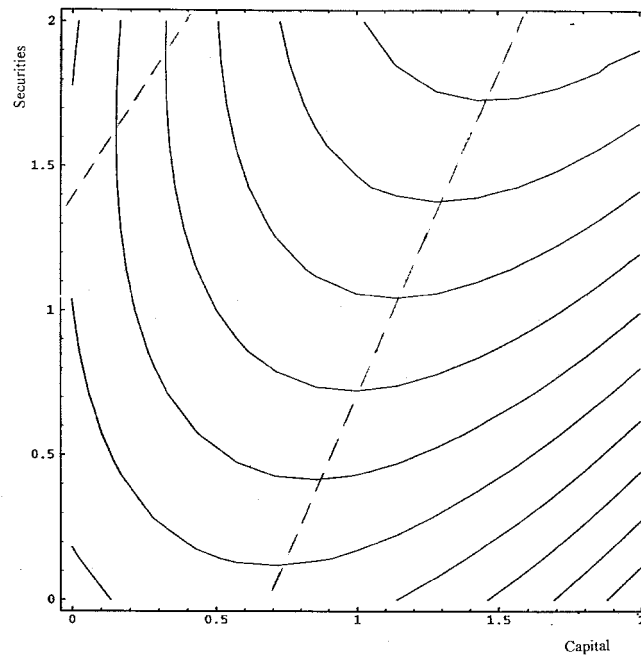


Figure 5: Isoquants of Labor vs. Materials

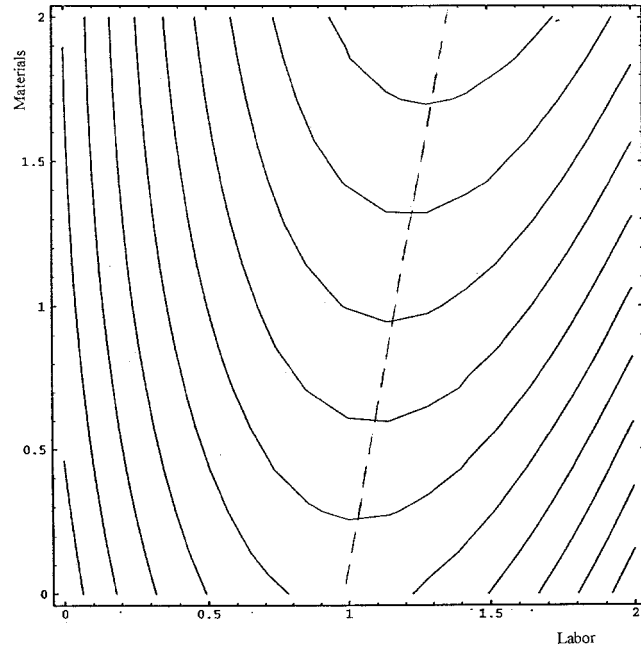


Figure 6: Isoquants of Labor vs. Cash

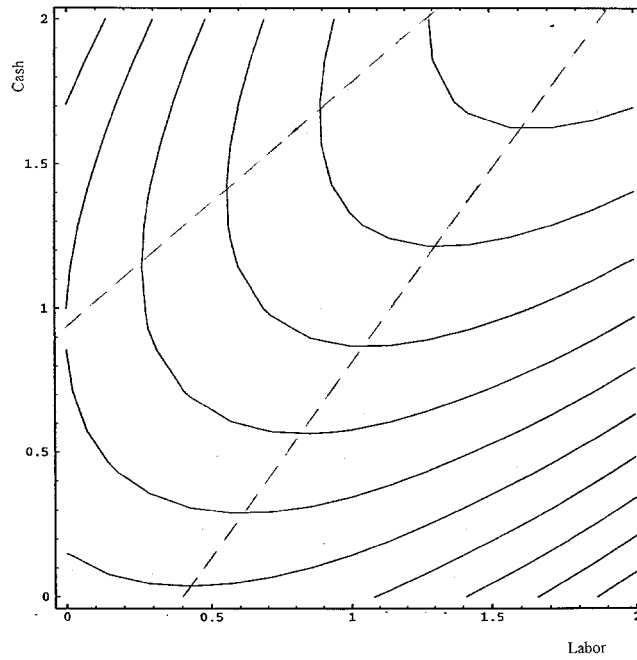


Figure 7: Isoquants of Labor vs. Securities

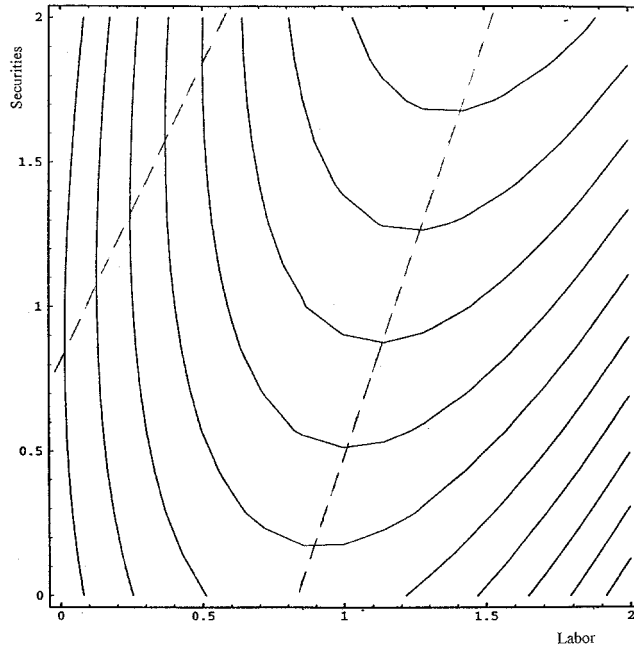


Figure 8: Isoquants of Materials vs. Cash

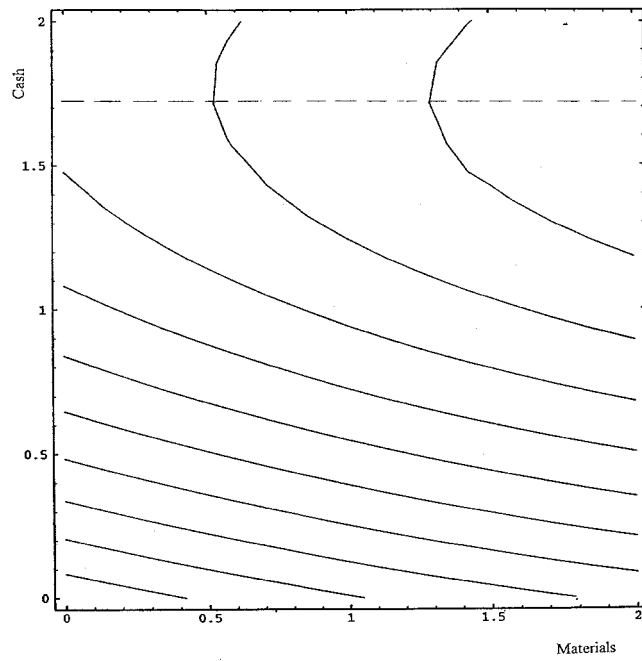


Figure 9: Isoquants of Materials vs. Securities

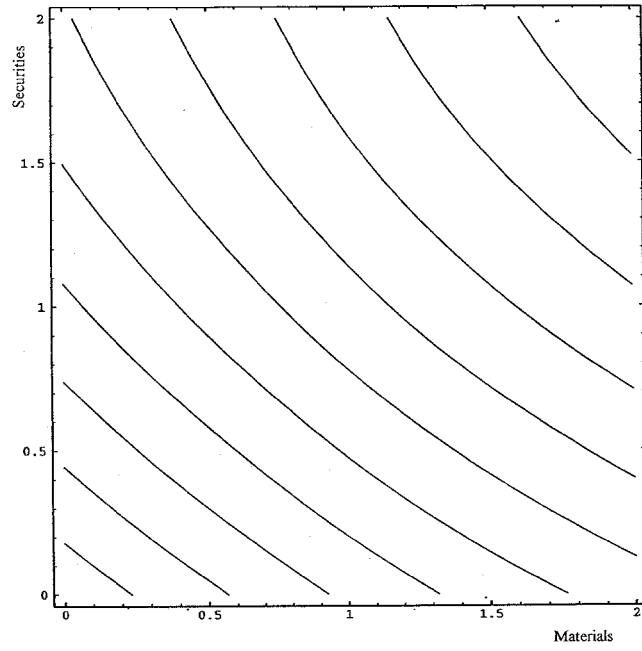
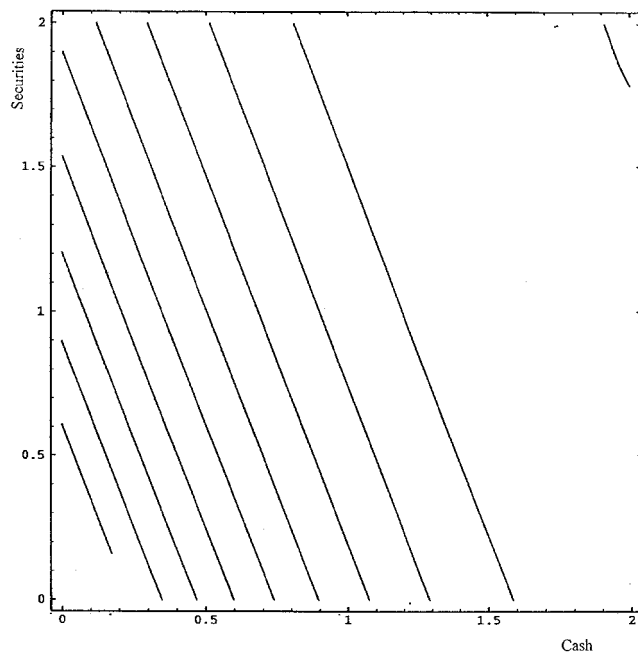


Figure 10: Isoquants of Cash vs. Securities



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