Forecasting Market Shares Using VAR and BVAR Models: A Comparison of their Forecasting Performance

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This paper develops a Bayesian vector autoregressive model (BVAR) for the leader of the Portuguese car market to forecast the market share. The model includes five marketing decision variables. The Bayesian prior is selected on the basis of the accuracy of the out-of-sample forecasts. We find that our BVAR models generally produce more accurate forecasts of market share. The out-of-sample accuracy of the BVAR forecasts is also compared with that of forecasts from an unrestricted VAR model and of benchmark forecasts produced from univariate (e.g., Box-Jenkins ARIMA) models. Additionally, competitive dynamics of the market place are revealed through variance decompositions and impulse response analyses.

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INTRODUCTION

Multiple time series models have been proposed, for some time, as alternatives to structural econometric models in economic forecasting applications. In the Marketing field, applications of multiple time series models (Transfer functions, Intervention and VARMA models) include for example Aaker, Carman and Jacobson (1982), Adams and Moriarty (1981), Ashley, Granger and Schmalensee (1980), Bass and Pilon (1980), Bhattacharyya (1982), Franses (1991), Grubb (1992), Hanssens (1980a, 1980b), Heyse and Wei (1985), Heuts and Bronckers (1988), Jacobson and Nicosia (1981), Krishnamurthi, Narayan and Raj (1986), Leone (1983, 1987), Lui (1987), Moriarty and Salamon (1980), Moriarty (1985), Sturgess and Wheale (1985), Takada and Bass (1988), and Umashankar and Ledolter (1983).

One class of multiple time series models which has received much attention recently is the class of Vector Autoregressive (VAR) models. VAR models constitute a special case of the more general class of Vector Autoregressive Moving Average (VARMA) models. Although VAR models have been used primarily for macroeconomic models, they offer an interesting alternative to either structural econometric (market share) or univariate (e.g., Box-Jenkins ARIMA or exponential smoothing) models for problems in which simultaneous forecasts are required for a collection of related microeconomic variables, such as industry and firm sales forecasting. The use of VAR models for economic forecasting was proposed by Sims (1980), motivated in part by questions related to the validity of the way in which economic theory is used to provide *a priori* justification for the inclusion of a restricted subset of variables in the "structural" specification of each dependent variable [1]. Sims (1980) questions the use of the so called "exclusionary and identification restrictions". Such time series models have the appealing property that, in order to forecast the endogenous variables in the system, the modeller is not required to provide forecasts of exogenous explanatory variables; the explanatory variables in an econometric model are typically no less difficult to forecast than the dependent variables. In addition, the time series models are less costly to construct and to estimate. This does not imply, however, that VAR models necessarily offer a parsimonious representation for a multivariate process. While it is true that any stationary and invertible VARMA process has an equivalent representation as a VAR process of possibly infinite order (see, for example, Fuller, 1976), it is usually the case that the VAR representation will not be as parsimonious as the corresponding VARMA representation, which includes lags on the error terms as well as on the variables themselves. Despite this lack of parsimony, and the additional uncertainty imposed by the use of a finite-order VAR model as an approximation to the infinite-order VAR representation, VAR models are of interest for practical forecasting applications because of the relative simplicity of their model identification and parameter estimation procedures, superior forecasting performance, compared with those associated with structural and VARMA models [2]. Brodie and De Kluyver (1987) have reported empirical results in which simple "naive" market share models (linear extrapolations of past market share values) have produced forecasts as accurate as those derived from structural econometric market share models.

The number of parameters to be estimated may be very large in VAR models, particularly in relation to the amount of data that is typically available for business forecasting applications. This lack of parsimony may present serious problems when the model is to be used in a forecasting application. Thus, the use of VAR models often involves the choice of some method for imposing restrictions on the model parameters: the restrictions help to reduce the number of parameters and / or to improve their estimability. One such method, proposed by Litterman (1980), utilizes the imposition of stochastic constraints, representing prior information, on the coefficients of the vector autoregression. The resulting models are known as Bayesian Vector Autoregressive (BVAR) models.

In this paper, we develop a Bayesian vector autoregressive model (BVAR) for the leader of the Portuguese car market, for the period 1988:1 through 1993:6 using monthly data. The rationale for the choice of a multiple time series technique is twofold. Structural models of market share, as surveyed in Cooper and Nakanishi (1988), for example, tend to be based on a number of generalizations about the effectiveness and relative importance of advertising, price, and other elements of the marketing mix, with little emphasis being placed on the correct determination of exogeneity assumptions and on the appropriate dynamic model specification. Secondly, and presumably partly because of this, the forecasting performance of such models has been poor compared to that of time series models: for such evidence, see Brodie and De Kluyver (1987), Danaher and Brodie (1992) and Brodie and Bonfrer (1994). Out-of-sample one-through twelve-months-ahead forecasts are computed for the leader's market share and their accuracy is evaluated relative to that of forecasts from an unrestricted VAR model and from best-fitting univariate ARIMA models.

The paper is organized as follows. The next section briefly describes the VAR and the BVAR modelling methodologies. The third section describes the data base used and the rational behind the choice of the variables. The fourth section presents the forecasting strategy, the main empirical results, and illustrate the use of impulse response analysis and variance decompositions.We conclude with a section on the limitations of our research and possible extensions.

VAR AND BVAR MODELLING

The theory underlying VAR models has its foundation in the analysis of the covariance stationary linearly regular stochastic time series Y_t . We assume here that Y_t is $(n \times 1)$ in dimension, i.e., $Y_t' = (Y_{1t}, \ldots Y_{nt})$. By Wold's decomposition theorem Y_t possesses a unique one-sided vector moving-average representation which, assuming invertibility, gives rise to an infinite-ordered vector autoregressive representation (VAR). In empirical work it is assumed that Y_t can be approximated arbitrarily well by the finite p-th ordered VAR :

$$Y_t = \sum_{k=1}^{p} B_k Y_{t-k} + e_t \tag{1}$$

where e_t is a zero-mean vector of white noise processes with positive definite contemporaneous covariance matrix and zero covariance matrices at all other lags; and the B_k 's are $(n \times n)$ coefficient matrices with elements b_{ijk} . This approximation assumption holds, in fact, if Y_t is covariance-stationary linearly-regular process. Equation (1) can be used to generate the forecast $f_{t,h}$ at time t of Y_{t+h} , with subsequent forecast error $e_{t,h} = Y_{t+h} - f_{t,h}$ and error variance-covariance matrix $\mathrm{V_h} = \mathrm{E}(\mathrm{e_{t,h}} \cdot \mathrm{e'_{t,h}})$. Granger and Newbold (1986, ch. 7) show that the optimal (in terms of minimizing the quadratic form associated with $\mathrm{V_h}$) h period ahead forecast $f_{t,h}$ of Y_{t+h} made at time t is

$$f_{t,h} = \sum_{k=1}^{p} B_k f_{t,h-k} \tag{2}$$

where $f_{t,h-k} = Y_{t-(k-h)}$ for $k = h, h+1, \ldots, p$, and B_k 's are the coefficient matrices in equation (1).

Equation (1) is the prototype for all of the variations of VAR 's mentioned hereafter.The several approaches differ primarily in terms of one or more of the following considerations: (1) transformation of the data and the inclusion of non-random deterministic variables; (2) determination of the maximum lag length p ; (3) specification of non-zero elements of the coefficient matrices B_k , $k = 1, \ldots p$; (4) estimation of the coefficients. We will now briefly describe each approach and highlight its distinguishing features.

The unrestricted VAR

In a VAR with n variables there is an individual equation for each variable. For the unrestricted case there are p lags for each variable in each equation. For example, the equation for the ith variable is

$$Y_{it} = \sum_{k=1}^{p} b_{i1k} Y_{1,t-k} + \ldots + \sum_{k=1}^{p} b_{ink} Y_{n,t-k} + e_{it} \quad . \tag{3}$$

As in the problem of seemingly unrelated regressions, when the right-hand-side variables are the same in all equations the applications of OLS equation by equation is justified. The coefficient estimates are maximum likelihood estimates conditioned on the initial observations, and under a variety of alternative assumptions on the Y 's and e 's , are consistent, asymptotically efficient, and asymptotically normally distributed (Litterman, 1980). The unrestricted VAR has been used extensively by Sims (1980), and in the initial stages of the model building by Caines, Keng and Sethi (1981), Tiao and Box (1981), and Tiao and Tsay (1983).

In terms of data transformations, Tiao and Box (1981), and Tiao and Tsay (1983) recommend against differencing each individual series to achieve stationarity. Acording to Tiao and Tsay, differencing is not only unnecessary when considering several series jointly, but will lead to unnecessary complexity in the model. Neither Tiao and Box nor Tiao and Tsay make recommendations on the use of instantaneous data transformations or deterministic trend components. In empirical work they log the data but do not include time trends. Maximum lag length is determined in each case by using a slight variation of the likelihood ratio statistic and testing the null hypothesis that $B_k = 0$ for sucessive lag lengths.

The main problem with the unrestricted VAR is the large number of free parameters that must be estimated. Since the number of parameters increases quadratically with the number of variables, even moderately sized systems can become highly over-parameterized relative to the number of data points. This overparameterization [3] results in multicollinearity and loss of degrees of freedom that can lead to inefficient estimates and large out-of-sample forecasting errors.While estimation of such a highly parameterized system will provide a high degree of fit to the data, the out-of-sample forecasts can be very poor in terms of mean square error. Because of these problems researchers have suggested imposing various types of parameter restrictions on VAR models. Several types of these restrictions are described in the literature One solution is to exclude insignificant variables / lags based on statistical tests. An alternative approach to overcome overparameterization is to use a Bayesian VAR model as described in Litterman (1980), Doan et al. (1984), Todd (1984),Litterman (1986), and Spencer (1993) .

The Bayesian VAR

The Bayesian approach starts with the presumption that the given data set does not contain information in every dimension. This means that by fitting an overparameterized system some coefficients turn out to be non-zero by pure chance. Since the influence of the corresponding variables is just accidental and does not correspond to a stable relationship inherent in the data, the out-of-sample forecasting performance of such models deteriorates quickly. The role of the Bayesian prior can therefore be described as prohibiting coefficients to be non-zero " too easily ". Only if the data really provide information will the barrier raised by the prior be broken through.

In an attempt to reduce the dimensionality of VAR 's, Litterman (1980) applied Bayesian techniques directly to the estimation of the VAR coefficients. His procedure generates a shrinkage type of estimator similar in many respects to the ridge and Stein estimators. Since there is a ridge regression analogy to the BVAR, it is not surprising that BVAR solved the multicollinearity problem. As is well known, from a Bayesian standpoint shrinkage estimators can be generated as the posterior means associated with certain prior distributions. While Litterman 's estimator can be justified as a posterior mean, the economic content of the prior information is not strong. Litterman's prior gives us a middle ground between the extremes of usual structural specification (strong unbelievable priors) and unrestrictedVARs (no priors).

To demonstrate Litterman 's procedure consider the ith equation of the VAR model (3) :

$$Y_{it} = d_{it} + \sum_{k=1}^{p} b_{i1k} Y_{1,t-k} + \ldots + \sum_{k=1}^{p} b_{ink} Y_{n,t-k} + e_{it} \ , \qquad (4)$$

where d_{it} is the deterministic component of Y_{it} and can include the constant, trend, and dummies. Litterman 's prior is based on the belief that a reasonable approximation of the behavior of an economic variable is a random walk around an unknown, deterministic component. For theith equation the distribution is centered around the specification

$$Y_{it} = d_{it} + Y_{i,t-1} + e_{it} \ . \qquad (5)$$

The parameters are all assumed to have means of zero except for the coefficient on the first lag of the dependent variable, which has a prior mean of one. All equations in the VAR system are given the sameforme of prior distribution.

In addition to the priors on the means, the parameters are assumed to be uncorrelated with each other (the covariances are set equal to zero) and to have standard deviations which decrease the further back they are in the lag distributions. The standard deviations of the prior distribution on the lag coefficients of the dependent variable are allowed to be larger than for the lag coefficients of the other variables in the system. Also, since little is known about the distribution of the deterministic components, a "flat" or uninformative prior giving equal weight to all possible parameter values is used. In equation form the standard deviation of the prior distribution for the coefficient on lag k of variable j in equation i is

$$s_{ijk} = \begin{cases} \dfrac{g}{k^d} & \text{if} \quad i = j \\ \dfrac{g \cdot w \cdot \hat{s}_i}{k^d \cdot \hat{s}_j} & \text{if} \quad i \neq j \end{cases} . \tag{6}$$

In equation (6) $\hat{s}_j$ is the estimated standard error of residuals from an unrestricted univariate autoregression on variables j . Since the standard deviations of lag coefficients on variables other than the dependent variables are not scale invariant, the scaling factor $\hat{s}_i / \hat{s}_j$ is used. This ratio scales the variables to account for differences in units of measurement and thus enables specification of the prior without consideration of the magnitudes of the variables. The term g " the overall tightness of the prior " is the prior distribution standard deviation of the first lag of the dependent variable. A tighter prior can be produced by decreasing the value of g . The term d " the decay parameter " is a coefficient which causes the prior standard deviations to decline in a harmonic manner. The prior can be tightened on increasing lags by using a larger value for d . The parameter w " the relative tightness " is a tightness coefficient for variables other than the dependent variables. Reducing its value, i.e. decreasing the interaction among the different variables, tightens the prior[4].

This prior is referred to as the " Minnesota prior " due to its development at the Federal Reserve Bank of Minneapolis and the University of Minnesota. Note that the prior distribution is symmetric [5]. The same prior means and standard deviations are used for each independent variable in each equation and across equations, and the same priors are used for each dependent variable across equations.

The BVAR model is estimated using Theil 's (1971) mixed-estimation technique that involves supplementing data with prior information on the distributions of the coefficients. For each restriction on the parameter estimates, the number of observations and degrees of freedom are increased by one in an artificial way. The loss of degrees of freedom due to overparameterization associated with a VAR model is therefore not a problem with the BVAR model. The incorporation of the prior information in the estimation of theith equation is accomplished by re-writing equation (4) as

$$Y_i = XB_i + e_i \tag{7}$$

where Y_i is the vector of observations on Y_{it} ; X is the matrix of deterministic components and observations on all lags of variables; B_i is the vector of coefficients on deterministic components and lags of variables; and e_i is the $(T \times 1)$ residual vector. The estimator suggested byLitterman is

$$\hat{B}_i = (X'X + h_i R_i' R_i)^{-1} (X'Y_i + h_i R_i' r_i) \tag{8}$$

where R_i is a diagonal matrix with zeros corresponding to the deterministic components and elements $\left(g/s_{ijk}\right)$ corresponding to the kth lag of variable j , j = 1,...n ; r_i is a vector of zeros and a one corresponding to the first lag of the dependent variable i ; and $h_i = \hat{s}_i^2 / g^2$. $\hat{B}_i$ is immediately seen to be a version of Theil 's mixed estimator where $R_i B_i = r_i + v_i$ and v_i is distributed $N\left(0,\ g^2 I\right)$.

To apply Litterman 's procedure one must search over the parameters g, d, and w until some predetermined objective function is optimized. The objective function can be the out-of-sample mean-squared forecast error, or some other measure of forecast accuracy. Doan, Litterman and Sims (1984) suggest minimizing the log determinant of the sample covariance matrix of the one-step-ahead forecast errors for all the equations of the BVAR. In a forecasting comparison such as ours, a portion of the sample must be withheld to determine the parameters g, d, and w ; while the remainder of the sample is used with the selected model to generate out-of-sample forecasting statistics for comparison purposes.

DATA

The data base used for this study is a monthly time series sample of market share, and 5 marketing variables, for the period 1988:1-1993:6 in the car market in the Portugal. The marketing variables include such variables as relative price, major media advertising expenditures (TV, radio,and newspaper), and an Age variable for the leader brand in the car market. The portuguese car market consists of twenty five imported car brands, but the top six account on average for 82.3% of the total market, with a standard deviation of 4.75%. The leader is a general brand, presented in all segments of the market and represents on average 16.8% of the total market, with a standard deviation of 3.56%.

The time series variables used in this study are defined as following:

$MS1_t$ = market share of the leader brand,

$A1_t$ = relative age of the leader brand,

$P1_t$ = relative price of the leader brand,

$TVS1_t$ = TV advertising expenditures in shares of the leader brand,

$RS1_t$ = Radio advertising expenditures in shares of the leader brand, and

$PS1_t$ = Press(newspaper and magazine) advertising expenditures in shares of the leader brand.

The data on $MS1_t$ are calculated from the monthly new automobile registrations. These data are published by the Portuguese General Directorate of Transports. Figure 1 shows that our brand is not successful in increasing its market share. This series seems to be stationary and does not present seasonal fluctuations.

The variable $A1_t$ represents the differents models (versions) offered by the brand in all segments of the market. It measures the time in market (in months) of the most representative models of each segment. This variable represents the models life cycle of the brand, and can be seen as a strategic marketing variable[6]. It was obtained as follows:

-- for the leader, we measure the time in market after the launching of the most representative model of each market segment [7], i.e, the model with the highest share of the segment;

-- for the competing brands, we calculate the simple average age of the most representative model of each segment;

-- to obtain the brand average age and / or of their competitors, we calculate the weighted average age for the models choosen on each segment.The weights are given by the relative importance of each segment on total demand (S1+S2+S3+S4);

-- the relative age, named A1, is then calculated as the ratio between the weighted average age of the brand and the weighted average age of their competitors.

Pricing decision by our brand is measured by the relative price defined as a price index which is calculated by dividing our brand's average price by an average price of ours competitors. The variable $P1_t$ is obtained following the steps just described for $A1_t$. The weights are the same, and the price of each model is the consumers price (all taxes included) of the most representative model of each segment. The price data are published on a monthly basis and are recorded from the ' Guia do Automóvel ' [8]. Figure 1 plots the relative price and the relative age of our leading brand. The overall trend of $A1_t$ is upward, and seems negatively related with the market share series. The price figure appears to be consistent with the declining market position of our brand.

The data on $TVS1_t$, $RS1_t$, and $PS1_t$ are expressed as shares of total advertising expenditures by media ($us/industry$) and were obtained from ' Sabatina ' [9]. Figure 1 indicates that major media advertising has been used extensively by our brand, and has fluctuated widely over the observed time period.

A preliminary examination suggested that these variables are all stationary [10]. This dataset is available from the author on request.

(Figure 1 about here)

EMPIRICAL APPLICATION

Models selected

Three classes of models are included in our empirical comparisons [11], each class being represented by one or more specific models. The classes are univariate Box-Jenkins, unrestricted VAR, and BVAR. The usual criteria, e.g. stationarity, autocorrelation, and partial autocorrelation functions, significance of coefficients, and the Akaike Information Criterion, are used to select the best models.

All variables are measured in logs to handle nonstationarity in variance, i.e., heteroscedasticity.

The best-fit ARIMA model for MS1 is as follows:

$$\underset{(0.03)^a}{(1-0.916B)}\, MS1_t = \underset{(0.06)^a}{-1.898} + (1+\underset{(0.07)^a}{0.814B})e_t \quad ,$$

standard errors are in parentheses and ^{a} means significant at the 0.01 level. As Montgomery and Weatherby (1980, p. 306) note: ' The Box-Jenkins approach uses inefficient estimates of impulse reponses weights which are matched against a set of anticipated patterns, implying certain choices of the parameters ... the analyst's skill and experience often play a major role in the success of the model building effort '[12].

To determine the optimal lag length of the unrestricted VAR [VAR(U)] we have employed the likelihood ratio test statistic (LR), suggested by Sims(1980) [13]. Given the number of observations, we have considered a maximum lag of nine and then tested downwards. The LR test supports the choice of six lags. Our six-variable VAR system with a lag length of 6 and 37 parameters (including the constant) in each equation, was then efficiently estimated in levels (so that it is comparable to the BVAR model), using least squares estimators [14]. A summary of these estimation results is provided by the critical levels of F-tests of the hypothesis that all lags of a particular variable in each equation are zero. These critical levels are reported in table 1. An examination of table 1 immediately reveals that the relative age of our brand (A1) has a significant effect on MS1, P1 and TVS1. There is some indication that relative age movements precede those on market share, relative price and TV advertising expenditures. TV advertising expenditutes has seen to have a significant effect not just on market share, which is perfectly predicted by the marketing theory of advertising effects on sales, but also on press advertising expenditures (PS1). This later relation means that there is evidence relating the Press advertising expenditures with TV actions.

(Table 1 about here)

In the class of BVAR models, the variables are specified in levels because as pointed out by Sims et al. (1990, p. 360) ' ... the Bayesian approach is entirely based on the likelihood function, which has the same Gaussian shape regardless of the presence of nonstationarity, [hence] Bayesian inference need take no special account of nonstationarity '[15] . The models are estimated with six lags on each variable[16].

The 'optimal' Bayesian prior was selected by minimising, for the period 1992:1 to 1992:12, the RMSEs statistics for one-to six-months-ahead forecasts. In a first step we assume a symmetric prior, i.e., $f(i,j) = w$, $i \neq j$ [BVAR(S)], then we relax this assumption to take in account a more general interaction between the variables [BVAR(G)] (see footnote 5).

Specification of Marketing priors

In our search for the symmetric prior we have considered three values for w : 0.25, 05, 075. For example, w = 0.5 means that, in the market share equation of our BVAR system, all right-hand-side variables, except the lagged market share, enter with a weight of 50%. For the parameter g we have assumed a relatively ' loose ' value of 0.3 and a 'tight ' value of 0.1. We set the harmonic lag decay, d , to 1 as recommended by Doan, Litterman and Sims (1984) . This has given us six alternative specifications. The best values according to our criterion function [17] were obtained for g = 0.3, w = 0.5 and d = 1 .

As an alternative to the symmetric prior, general tightness parameters are specified equation-by-equation. This means that ours prior beliefs, based on marketing theory and / or management experience, could be incorporated in a more precisely way. To specify the general prior we have to define g , d , and the inter-series tightness parameters , $f(i,j)$. The initial values of overall tightness, g , and harmonic lag decay, d , are set at 0.2 and 1, respectively. These are recommended in Doan (1992). The initial values for $f(i,j)$, the relative weights of variable j in equation i, are set from the information provided by table 1, marketing theory and author's beliefs. The BVAR model is initially estimated with these parameters and the average of the RMSEs for one to six months ahead is recorded for each variable for the period 1992:1 to 1992:12. The parameters in the prior are changed sequentially and after some initial search, the best values were obtained for d = 1, g = 0.15 and

$$f(i,j) = \begin{bmatrix} eq. \ddots v. & MS1 & A1 & P1 & TVS1 & RS1 & PS1 \\ MS1 & 1 & .8 & .5 & .4 & .1 & .1 \\ A1 & .5 & 1 & .1 & .1 & .1 & .1 \\ P1 & .5 & .5 & 1 & .1 & .1 & .1 \\ TVS1 & .4 & .5 & .1 & 1 & .1 & .1 \\ RS1 & .4 & .1 & .1 & .4 & 1 & .1 \\ PS1 & .4 & .1 & .1 & .5 & .1 & 1 \end{bmatrix}.$$

The tightness parameters represent the authors' subjective priors on the tightness of the coefficients associated with lagged values of variables listed in the right-hand columns. Of course, these priors do not represent much more than a single person's opinion and are certainly not thought to reflect substantive opinion--which of course would be an interesting extension. Substantive experts (subject matter experts) may possess prior information on the center of the distribution of coefficients on lagged variables as well. While this information may prove difficult to elicit, the potential for forecasting improvement makes further work along this line important.

Evaluation of accuracy

The accuracy of the out-of-sample forecasts for 1993:1 to 1994:6 is measured by the RMSE and the Theil U statistics for one-to twelve-months-ahead forecasts. If A_t denotes the actual value of a variable, and F_t the forecast made in period t , then the RMSE and the Theil statistic are defined as follows:

$$RMSE = \left\{ \sum_{j=1}^{k} \left(A_{t+j+k} - F_{t+j+k} \right)^2 / N \right\}^{0.5}$$

$$U = RMSE(\text{model}) / RMSE(\text{random walk})$$

where $k = 1, 2, \ldots 12$ denotes the forecast step and N is the total number of forecasts in the prediction period.

The U statistic is the ratio of the RMSE for the estimated model to the RMSE of the simple random walk model which predicts that the forecast simply equals the most recent information. Hence if U < 1, the model performs better than the random walk model without drift; if U > 1, the random walk outperforms the model. The U statistic is therefore a relative measure of accuracy and is unit-free (Bliemel, 1973). The forecasted value used in the computation of the RMSE and U statistics is the level (in logarithm) of the market share, so these statistics can be compared across the different models.

The RMSE and the U statistics are generated using the Kalman filter algorithm in RATS. The models are estimated for the initial period 1988:1 to 1992:12. Forecasts for up to twelve months ahead are computed. One more observation is added to the sample and forecasts up to twelve months ahead are again generated and so on. Based on the out-of-sample forecasts, RMSEs and the Theil U statistics are computed for one-to twelve-months-ahead forecasts. The relevant comparison is on "out-of-sample" forecasts.

The RMSEs and the U statistics for $MS1$ for the four models discussed above are reported in table 2. The table also reports the average of the RMSEs and of the U statistics for the one-,three-,six- and twelve-months-ahead forecasts, and summaries of tests of significant differences among the RMSEs mesures. The conclusions from table 2 are as follows:

(1) RMSEs versus Theil U statistics: The RMSEs and the Theil U statistics do not follow a consistent pattern with an increase in the forecast horizon.

(2) BVAR versus univariate ARIMA models: BVAR models produce more accurate forecasts (for all horizons) than the corresponding ARIMA model[18].

(3) BVAR versus the unrestricted VAR models: In all cases, BVAR(G) outperforms the unrestricted VAR model. The comparison between BVAR(S) and VAR(U) produces the same conclusion; BVARs are always superior for all forecasting periods.

(4) BVAR(G) versus BVAR(S): BVAR(G) offers the best out-of-sample forecast performance, except at lag twelve where its RMSEs (U's) are not significantly lower than those of the symmetric prior. Without regard to significance, the general prior yields the lowest RMSEs (U's). This is not surprising since the prior for the model was selected on the basis of minimisation of the average of one-to six-month-ahead RMSEs.

The results, in general, show that there are gains from using a BVAR approach to forecasting. BVAR models produce more accurate forecasts than the alternative forecasts.

Finally, like other authors (Hafer and Sheehan, 1989) we found that the accuracy of the forecasts is sensitive to the specification of the priors. If the prior is not well specified, an alternate model such as an unrestricted VAR or an ARIMA model may perform better.

(Table 2 about here)

Performance of alternative models in predicting turning points

While the BVAR models, in general, produce the most accurate forecasts, another way to evaluate the performance of alternative models is to examine their ability in predicting turning points, which are of key managerial significance. This topic is of vital importance, as statistical methods can perform extremely well in terms of forecasting overall levels, and yet still perform poorly in the prediction of turning points. Unfortunately, the question of turning points does not easily lend itself to quantitative analysis due to difficulties in, firstly, defining turning points; and, secondly, knowing when a given method has adequately predicted a turn. Moreover, there are various types of turn (e.g., those which are seasonal, cyclical, due to saturations in trends, etc.) which need to be dealt with.

We focus on the performance of the BVAR models relative to that of the unrestricted VAR and the univariate ARIMA models. Figure 2 shows that, in spite of the unrestricted VAR model generally correctly predict the direction of change, BVAR models are superior and do very well in predicting turning points. As such, BVAR models can be recognized as important to the planning function in the organization. As the growth in the formalization of the budgeting process continues, the strategic role of accurate forecasting methods as the 'budget cornerstone' will continue to be emphasized (Dalrymple, 1987).

(Figure 2 about here)

Besides generating excellent baseline forecasts, BVAR models can also be used to study the effects of movements in one variable on movements in others using impulse response analysis and variance decompositions.

Impulse response analysis

Impulse responses are the time paths of one or more variables as a function of a one-time shock to a given variable or set of variables. Impulse responses are the dynamic equivalent of elasticities. For example, in a static multiplicative interaction model of the form $ms = a \cdot p^b$ price elasticity $\left[e = (dms/dp)(p/ms) = b \right]$ is constant. However, in a dynamic system, changes in market share in one month are a function of changes in price over several months. The net effect must be represented as a convolution sum. The graphic representation of this sum provides a complete description of the dynamic structure of the model (it is incorrect to directly interpret estimated coefficients for the variables of a BVAR model).

To illustrate, figure 3a shows the impulse responses of MS1 to a shock to each one of the five marketing variables of the system. The results show that over the sample period an unexpected increase in A1 produces a longer decrease (over 6 months) of MS1 that is not totally recuperated after 12 months, i.e., A1 has a negative permanent impact on MS1. The MS1 responses to shocks on advertising expenditures (TV, Radio and Press) are positive, lagged and varying in magnitude. TV advertising effects begin after 3 months, but seems to rest for a period longer than that of Radio and Press. Our findings are consistent with the theory of the cumulative advertising effects on sales (e.g., Palda, 1964) and, even the measurement and duration ["the p implied duration interval" in Clarke's (1976) terminology] of these effects are easy to calculate. Surprisingly, the response of the MS1 to a price shock does not appear to be significant. A plausible explanation for this behavior is that our brand does not compete on a price basis and even if it charges its prices, these increases will be associated with the lauching of new models (versions).

The responses of A1 to a positive shock on MS1 and on P1 are given in Fig. 3b. The results show that an unexpected increase in market share or on prices generates a fast decrease on A1. Positive shocks on sales or on prices would probably stimulate the introduction of new models in market (A1 decrease).

Fig 3c shows that prices react negatively to an increase in market share. This result is spurious and does not make any sense in light of marketing theory.

Figure 4 (a, b, c) show that all three advertising variables react positively (increase its shares) to an unexpected increase in market share. However, the impact is more important in Radio advertising expeditures than in the others. Our findings confirm the existence of a feedback relationship between sales (market share) and advertising expenditures.

The effect of interaction among the advertising variables is one of the issues that deserves to be investigated in this analysis. For example, while an increase in TV advertising is followed one month later by increases in Radio and Press advertising expenditures, reenforcing the major role of TV advertising, an increase in Press advertising seems to produce an opposite reaction. The impact of RS1 on TVS1 and PS1 does not appear to be significant in either direction. We show evidence that confirm the existence of interactions between the advertising variables and that the nature of these interactions vary accordingly with the variable that produced the reaction.

(Figure 3 about here)

(Figure 4 about here)

Variance decompositions

Variance decompositions give the proportion of the h-periods-ahead forecast error variance of a variable that can be attributed to another variable. The pattern of the variance decomposition also indicates the nature of Granger causality among the variables in the system, and as such, can be very valuable in making at least a limited transition from forecasting to understanding. If innovations in $A1_t$ result in unexpected fluctuations in $MS1_t$, then information on $A1_t$ would be useful in predicting $MS1_t$. In interpreting these variance decompositions, one should bear in mind Runkle's (1987) criticism that the implicit confidence intervals attached to both variance decompositions and impulse responses functions are often so large as to render precise inferences impossible.

The Choleski decompositions are examined for the BVAR(G) model. The variables are ordered in the following sequence - $A1_t, MS1_t, P1_t, TVS1_t, RS1_t, PS1_t$. This ordering is based partly on our prior belief that changes in $A1_t$ promote a response in $MS1_t$ and $P1_t$, which then, through the created expectations, alters $TVS1_t, RS1_t, PS1_t$, and partly on the timing of the availability of data to management. For instance, information on prices and market shares is released before that on advertising expenditures[19].

The variance decompositions for our six variables model for the period 1988:1 to 1993:6 are reported in table 3. For each variable in the left-hand column, the percentage of the forecast error variance for one, six and twelve months ahead that can be attributed to shocks in each of the variables in the remaining columns is reported. Each row sums to 100% (ignoring rounding errors) since all the forecast error variance in a variable must be explained by the variables in the model. If a variable is exogenous in the Granger sense, i.e. if other variables in the model are not useful in predicting it, a large proportion of that variable's error variance should be explained by its own innovations. How large is 'large'? According to Doan (1992), in a six-variable model such as ours, 50% is quite high. If another variable is useful in explaining a left-column variable, that useful variable will explain a positive percentage of the prediction error variance. In practice, it is difficult to distinguish between a variable that has no predictive value and one that has little predictive value. Some conclusions, however, can be derived by comparing the magnitudes.

Table 3 shows that at a forecast horizon of 12 months, only 35.6% of the forecast error variance in the MS1 is explained by its own innovations supporting the assumption that MS1 is not exogenous, and other variables like A1, TVS1, and PS1 can be equally useful in forecasting MS1. Market share is extremely stable from one month to the next -- none of the other variables figure at all in its 1-month ahead forecast. Moreover, longer term forecasts of market share are heavily influenced by "age", TV advertising and not much at all by price. The exogenous behaviour of A1 seems to be reflected in the 64.2% error variance explained by its own innovations. The results for the price variable vary with the horizon of forecasting. For instance, the market share variable seems more important at shorter horizons (1 to 6), while the "age" variable becomes the largest contributor for longer horizons (12 months).

There are some interesting media differences in the advertising variable. TV advertising seems much more "exogenous" than the other media advertising (46.79% vs. 29.01% and 34%). Long term forecasts of TV advertising are rather more explained by innovations in A1 than on MS1. However, for RS1 and PS1 it is the MS1 innovations which help explain most of the forecast error variance attributable to other innovations.

(Table 3 about here)

LIMITATIONS AND EXTENSIONS

In this paper, we demonstrate the utility of VAR and BVAR methodologies as a marketing tool that fulfils two requirements: it forecasts market share, and it provides insights about the competitive dynamics of the marketplace. Here are compared the forecasting accuracy of BVAR with several traditional approaches. Using data, we establish that BVAR is a superior forecasting tool compared to univariate ARIMA and VAR models. Because BVAR uses few degrees of freedom and is easy to identify, it satisfies the practical requirements as a marketing forecasting tool. Finally, using impulse response functions and variance decompositions, we illustrate that BVAR provides important insights for the marketing manager.

Although BVAR is a promising and reliable forecasting tool, certain limitations should be pointed out. First, BVAR models are highly reduced forms. Structural interpretations based on the signs and magnitudes of estimated parameters should be avoided. Hypothesis about effects should be tested using impulse response analysis.

Second, the accuracy of the forecasts is sensitive to the specification of the prior. If the prior is not well specified, an alternate model such as an unrestricted VAR or an ARIMA model may perform better. Third, the prior that is selected on the basis of some objective function (e.g. the Theil's U) for the out-of-sample forecasts may not be "optimal " for beyond the period for which it was selected.

This model, like all time series models, is best suited for stable environments (e.g. wide-sense stationary processes) where sufficient number of observations are available. Thus, BVAR is not a new product model and its forecasts may be unreliable in markets characterized by frequent new entries or drop-outs.

We propose several extensions of this initial application of BVAR to forecast brand market shares. First, the bayesian approach can be improved by putting more structure based on marketing theory into the prior thereby abandoning the symmetric treatment of all variables. This would make the approach more bayesian in spirit since the prior can now reflect better the a priori beliefs of the investigator. On the other hand the greater flexibility makes it more difficult to find the optimal forecasting model. Second, the inclusion of the contemporaneous values of some variables in some equations (using, e. g., a Wold causal ordering) may result in improved forecasting accuracy due to a simpler model specification. Third, we used one specification of each model to forecast over the entire testing period. More frequent specifications (e.g., a time-varying parameter BVAR model) would no doubt improve accuracy. An important problem deals with how often a model should be reespecified. Finally, we used single equation procedures to estimate all models. Forecasting accuracy may improve by estimating all equations in each model simultaneously and exploiting the information in the cross-equation residual covariance matrix.

Our experience with BVAR models suggests that it is a robust, reliable marketing forecasting tool.

Footnotes:

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Table 1. Multivariate Granger causality tests: Critical levels of F-statistic testing hypothesis that all lags of indicated right-hand-side variable are zero.

Variable Equation	MS1	A1	P1	TVS1	RS1	PS1
MS1	*	*	0.33	*	0.18	0.12
A1	0.73	**	0.44	0.42	0.7	0.51
P1	0.41	*	*	0.59	0.48	0.61
TVS1	0.47	*	0.44	*	0.11	0.22
RS1	0.34	0.13	0.41	0.09	*	0.36
PS1	0.56	0.12	0.24	*	0.28	*

Notes: * and ** indicate numbers less that 0.05 and 0.01.

When we speak of "Granger causality", we are really testing if a particular variable precedes another and not causality in the sense of cause and effect. The term causality as used here follows Granger's (1969) temporal definition: a variable X_t Granger-causes another variable Y_t, if given information of both X_t and Y_t, the variable Y_t can be better predicted in the mean square error sense by using only past values of X_t than by not doing so. For a recent summary on testing for causality in multivariate time series models, see Cromwell et al. (1994).

Table 2. Accuracy of out-of-sample forecasts (1993:1-1994:6)

Month ahead	Accuracy Statistic	N	ARIMA (1)	VAR(U) (2)	BVAR(S) (3)	BVAR(G) (4)
1	U	18	0.979	0.916	0.813	0.744
	RMSE		0.189	*0.175 a*	*0.155 a,b*	*0.142 a,b,c*
3	U	16	0.846	0.807	0.761	0.655
	RMSE		0.175	0.167	*0.158 a*	*0.136 a,b,c*
6	U	13	0.665	0.596	0.501	0.481
	RMSE		0.174	*0.156 a*	*0.136 a,b*	*0.124 a,b,c*
12	U	7	1.13	1.06	0.523	0.508
	RMSE		0.209	*0.195 a*	*0.137 a,b*	*0.133 a,b*
Average	U		0.905	0.845	0.649	0.597
Average	RMSE		0.187	0.173	0.146	0.134

Notes: N is the number of observations. The RMSEs and the U statistics are reported for the Log MS1. 'Average' is the average of the one-, three-, six- and twelve-months-ahead RMSEs and the U statistics.

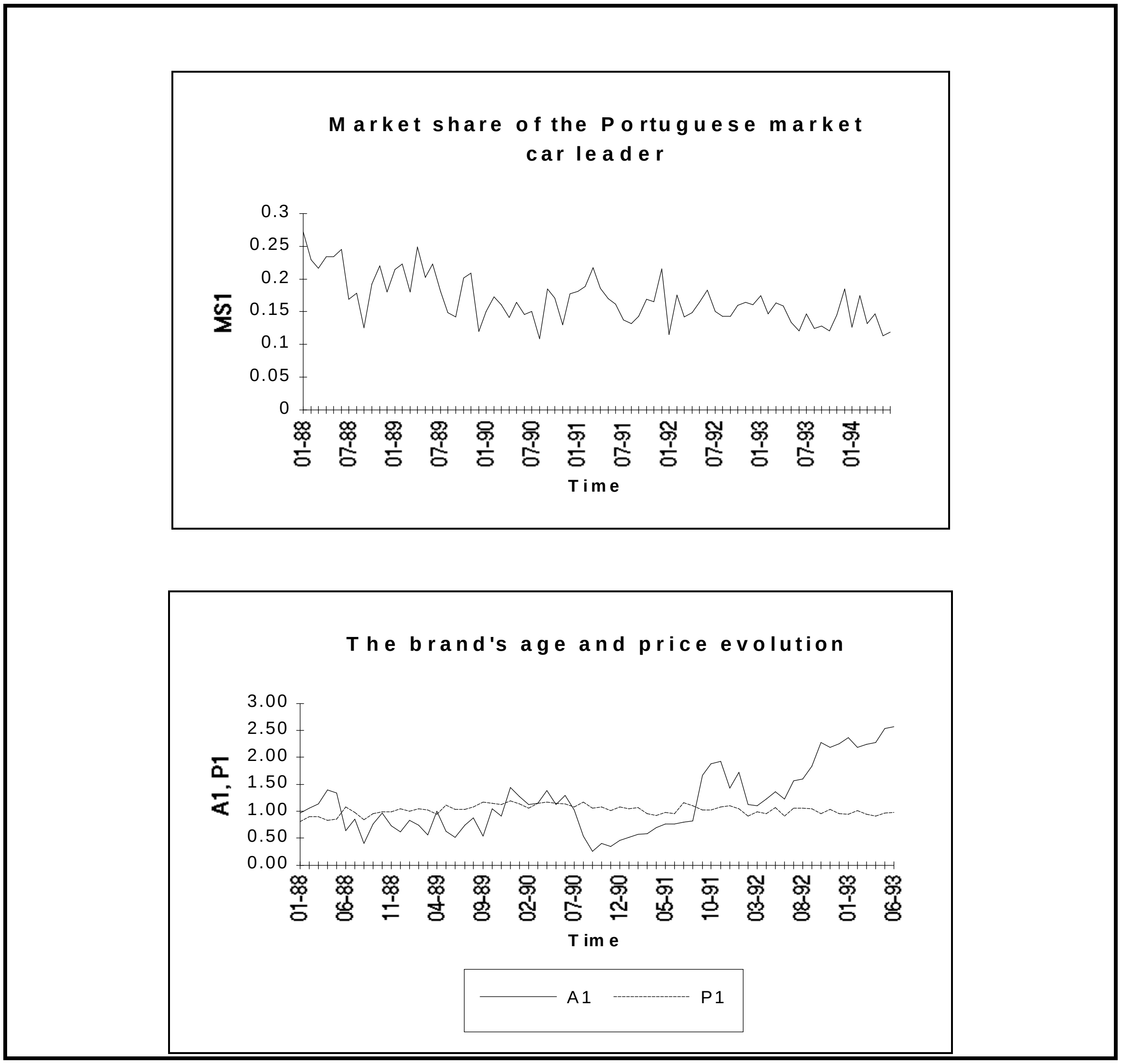
A test of significant differences in root mean squared errors is carried out following the procedure given in Brandt and Bessler (1983). The "*a*" signifies that the RMSE of model *i* is significantly lower (at the 5% significance level) than that of model *1*, *i = 2,3, or 4*; *b* signifies the RMSE of model *i* is significantly lower than that of model *2*, *i = 1,3,4*; *c* and *d* are defined in an analogous fashion.

Table 3. Variancedecompositions

Variable	Step	A1	MS1	P1	TVS1	RS1	PS1
A1	1	94.13	5.87	0	0	0	0
	6	75.83	6.95	8.09	3.74	2.45	2.94
	12	64.2	14.08	12.85	4.06	1.34	3.45
MS1	1	0	100	0	0	0	0
	6	29.03	51.01	3.87	3.61	6.61	6.43
	12	31.9	35.6	3.91	15.8	4.97	7.8
P1	1	9.37	24.86	65.76	0	0	0
	6	7.59	16.25	52.67	9.55	8.96	4.96
	12	17.68	11.69	40.44	12.06	11.88	6.21
TVS1	1	9.62	0.54	5.94	83.88	0	0
	6	21.63	9.34	4.56	56.87	3.9	3.68
	12	21.05	9.7	7.75	46.79	4.79	9.9
RS1	1	2.5	27.64	0.32	1.22	68.3	0
	6	17.45	21.79	1.18	14.52	36.05	8.99
	12	13.91	20.15	9.49	18.23	29.01	9.19
PS1	1	0	10.47	0	12.16	4.4	72.96
	6	15.38	10.3	5.42	19.01	10.14	39.75
	12	13.36	19.98	8.56	15.17	8.91	34

Note: Entries in each row are the percentages of the variance of the forecast error for each variable indicated in the rows that can be attributed to each of the variables indicated in the column headings. The decompositions are reported for one-, six- and twelve-month horizons.

Figure 1


Market share of the Portuguese market car leader
MS1
0.3
0.25
0.2
0.15
0.1
0.05
0
01-88
07-88
01-89
07-89
01-90
07-90
01-91
07-91
01-92
07-92
01-93
07-93
01-94
Time
The brand's age and price evolution
A1, P1
3.00
2.50
2.00
1.50
1.00
0.50
0.00
01-88
06-88
11-88
04-89
09-89
02-90
07-90
12-90
05-91
10-91
03-92
08-92
01-93
06-93
Time
A1
P1

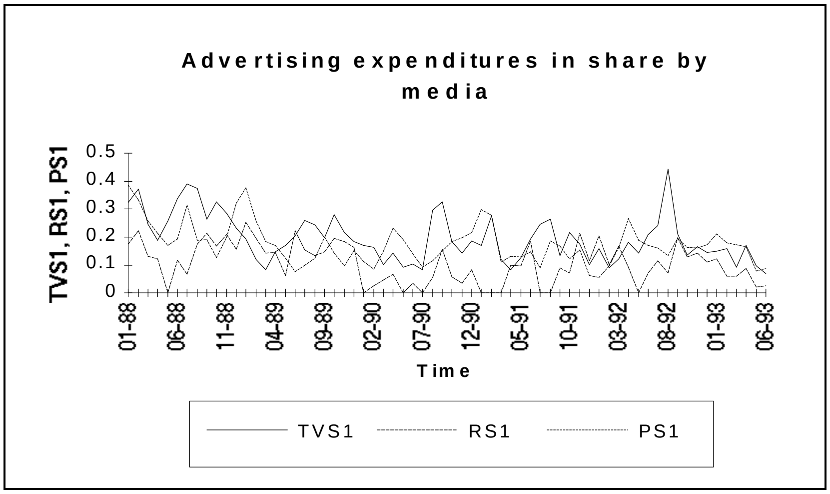

Advertising expenditures in share by media
TVS1, RS1, PS1
0.5
0.4
0.3
0.2
0.1
0
01-88
06-88
11-88
04-89
09-89
02-90
07-90
12-90
05-91
10-91
03-92
08-92
01-93
06-93
Time
TVS1
RS1
PS1

Figure 2

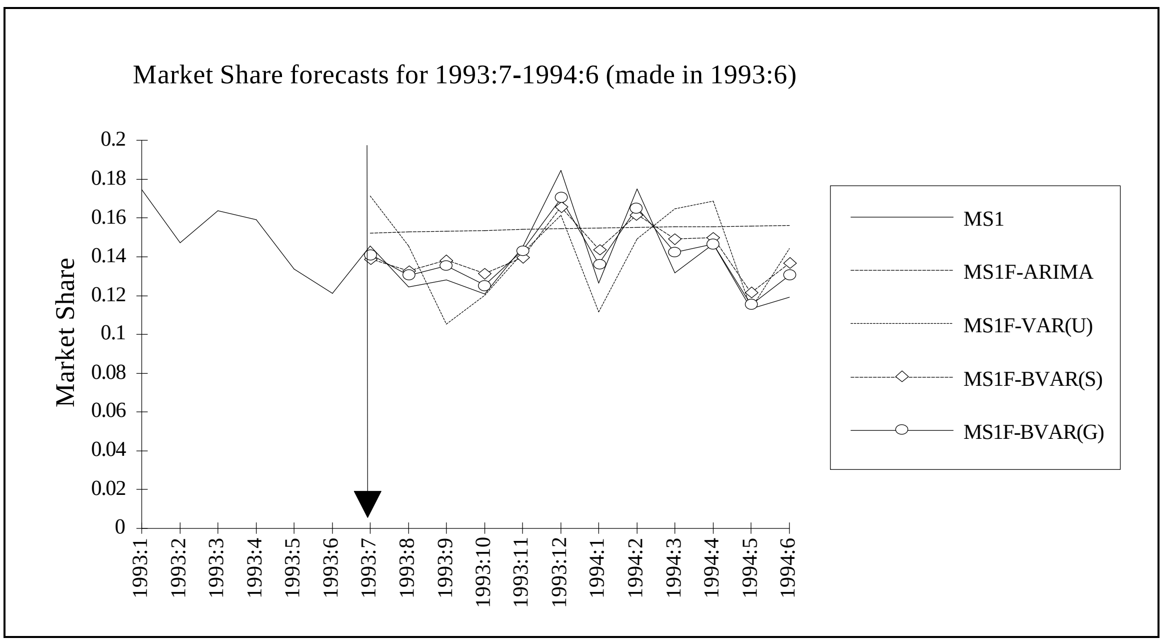
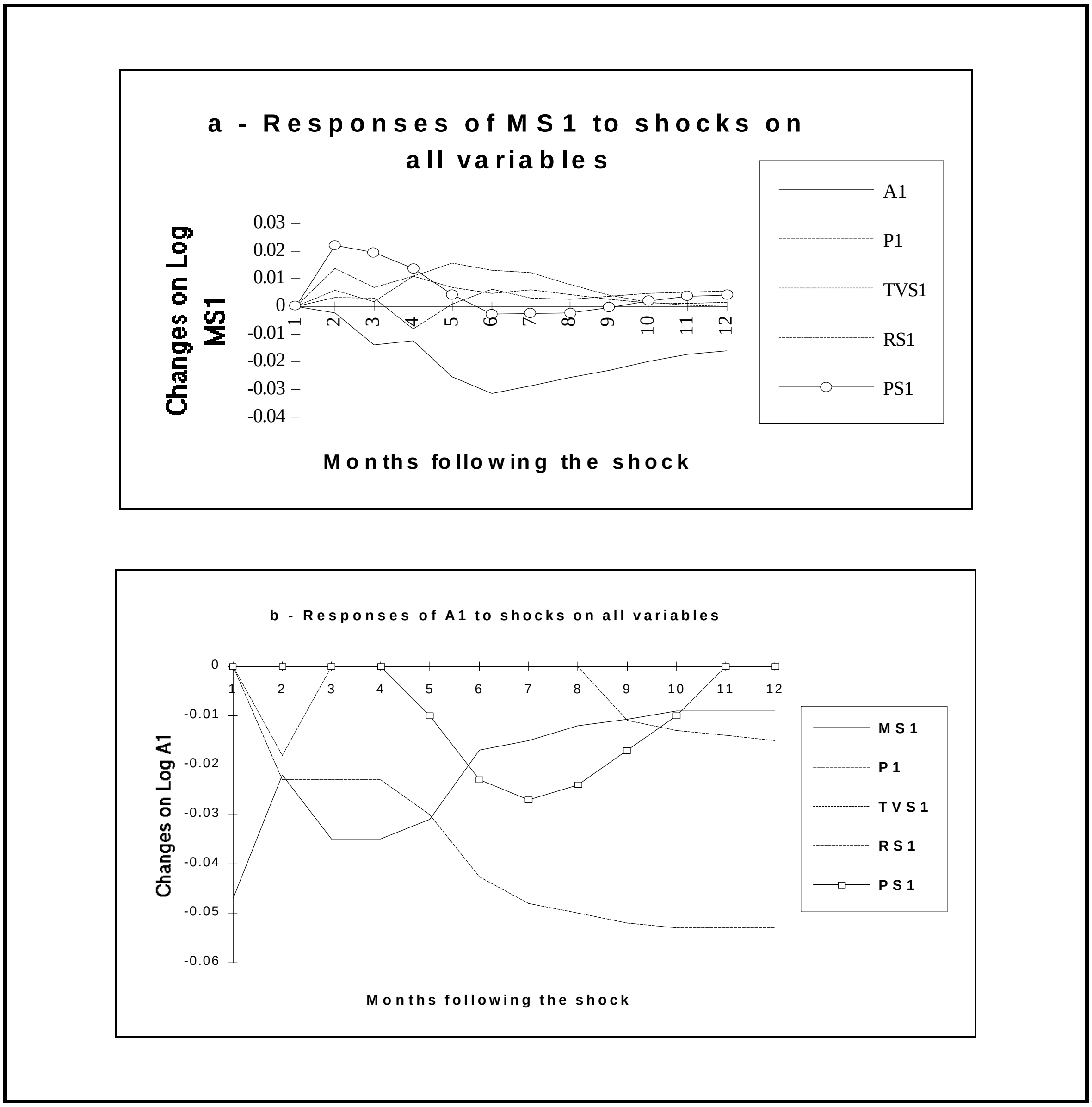

Market Share forecasts for 1993:7-1994:6 (made in 1993:6)
Market Share
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1993:1
1993:2
1993:3
1993:4
1993:5
1993:6
1993:7
1993:8
1993:9
1993:10
1993:11
1993:12
1994:1
1994:2
1994:3
1994:4
1994:5
1994:6
MS1
MS1F-ARIMA
MS1F-VAR(U)
MS1F-BVAR(S)
MS1F-BVAR(G)

Figure 3


a - Responses of MS1 to shocks on all variables
Changes on Log MS1
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
1 2 3 4 5 6 7 8 9 10 11 12
Months following the shock
A1
P1
TVS1
RS1
PS1
b - Responses of A1 to shocks on all variables
Changes on Log A1
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
1 2 3 4 5 6 7 8 9 10 11 12
Months following the shock
MS1
P1
TVS1
RS1
PS1

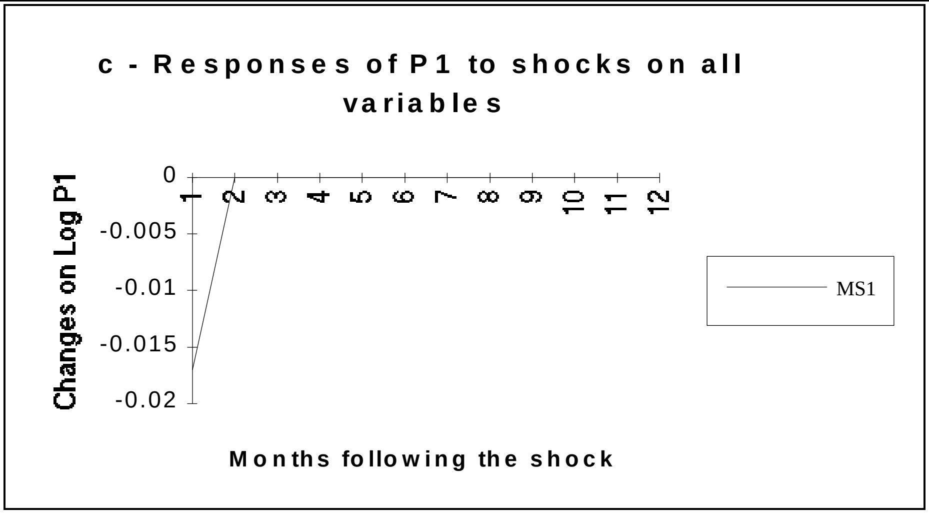
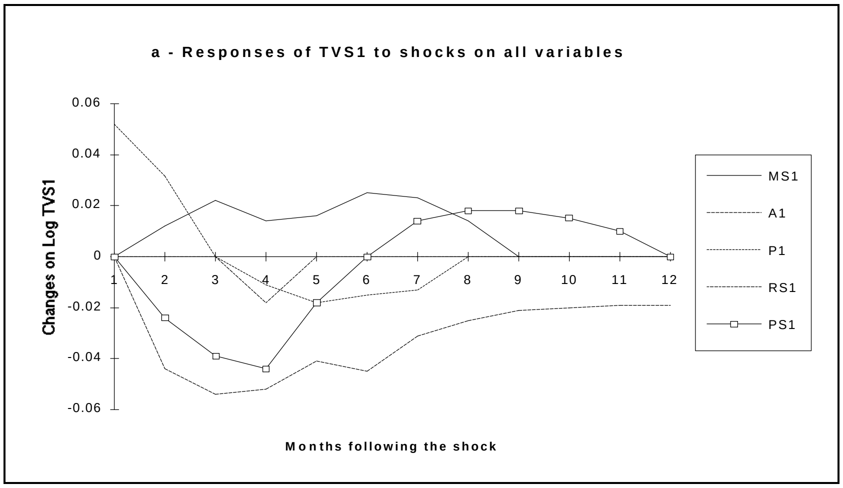
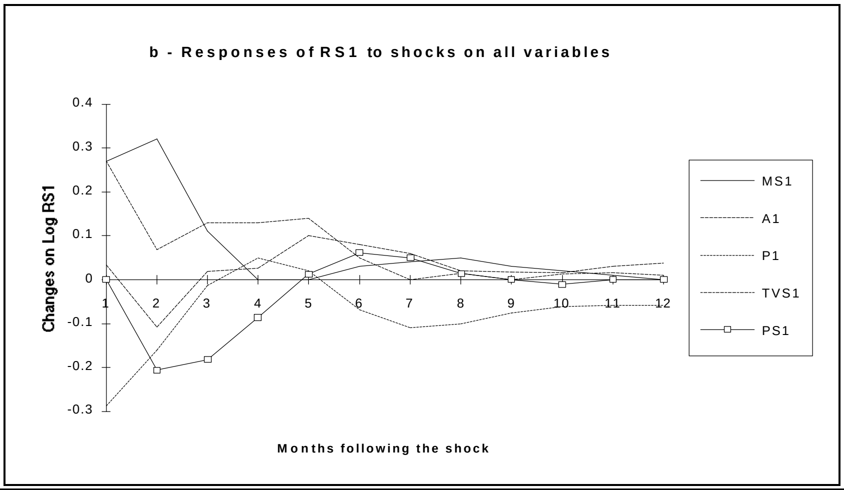
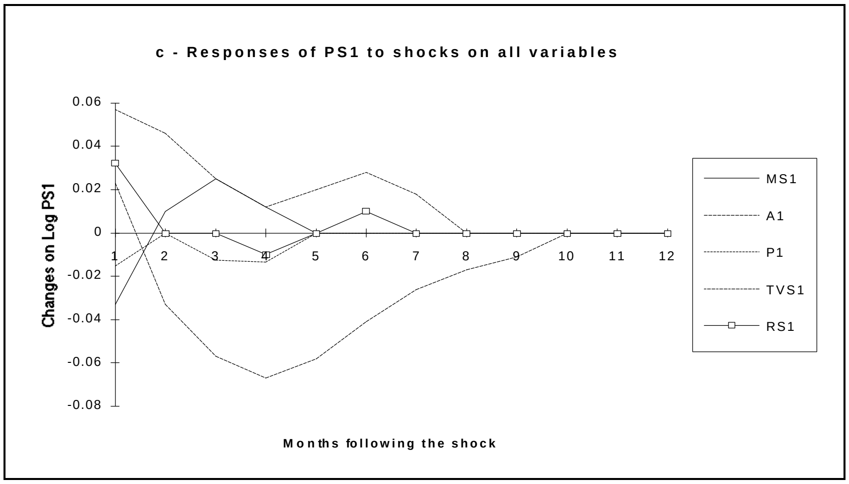

c - Responses of P1 to shocks on all variables
Changes on Log P1
0
-0.005
-0.01
-0.015
-0.02
1 2 3 4 5 6 7 8 9 10 11 12
Months following the shock
MS1

Figure 4


a - Responses of TVS1 to shocks on all variables
Changes on Log TVS1
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
1
2
3
4
5
6
7
8
9
10
11
12
Months following the shock
MS1
A1
P1
RS1
PS1


b - Responses of RS1 to shocks on all variables
Changes on Log RS1
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
1
2
3
4
5
6
7
8
9
10
11
12
Months following the shock
MS1
A1
P1
TVS1
PS1



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[1] In Marketing these arguments are "mutatis mutandis" also valid. The lack of generally accepted theory about aggregate market response and marketing mix competition means there is little a priori reason to support or reject any of a number of plausible model specifications.

[2] Very few VARMA analyses of higher-dimensional time series (e.g., models with more than four series) are reported in the marketing literature. The wider class of vector ARMA models were not considered because there was little evidence of moving average components and because both identification and estimation of such models are relatively complicated. For a recent summary of the specification of vector ARMA models, seeTiao and Tsay (1989).

[3] An alternative to a vector autoregressive model is a simultaneous equations structural model. However, there are limitations to using structural models for forecasting since projected values of the exogenous variables are needed for this purpose. Further, Zellner (1979) and Zellner and Palm (1974) show that any linear structural model can be expressed as a VARMA model, the coefficients of the VARMA model

being combinations of the structural coefficients. Under certain conditions, a VARMA model can be expressed as a VAR model and a VMA model. A VAR model can therefore be interpreted as an approximation to the reduced form of a structural model.

[4] To illustrate, if $g = 0.2$, the standard deviation of the first own lag in each equation would be 0.2 since $1^d = 1$, $d > 0$. The standard deviation of all other lags equals $\frac{0.2 \cdot w \cdot \hat{\sigma}_i}{k^d \cdot \hat{\sigma}_j}$. For k = 1 through 6 and $d = 1$, $k^{-1} = 1, \ 0.5, \ 0.33, \ 0.25, \ 0.2, \ 0.16$, respectively, showing the decreasing influence of longer lags. The value of the parameter w would determine the importance of variable j relative to variable i in the equation for variable i ; a higher w allows for more interaction by setting a higher a priori standard deviation for cross effects. For instance, $w = 0.5$ implies that relative to variables i , variable j has a weight of 50%. A tighter prior can be produced by decreasing g , and / or increasing d , and / or decreasing w .

[5] Doan, Litterman and Sims (1984) also have considered another type of prior , known as " general ". In a general prior the interaction among the variables led to the specification for the weighting matrix $f(i, j)$ given by:

$$f(i,j) = \begin{cases} 1 & \text{if } i = j \\ f_{ij} & \text{if } i \neq j \ \left(0 < f_{ij} < 1\right) \end{cases}$$

If $f(i, j) = w$, $i \neq j$ the prior becomes symmetric.

[6] LAMBIN and DOR (1989) use the same variable in their analysis of the Belgian car market.

[7] We apply a pseudosegmentation method based on the horse power , and followed by the Portuguese Trade Automobile Association. This pseudosegmentation creates four segments: S1 (lower), S2 (lower-middle), S3 (upper-middle), and S4 (upper).

[8] The oldest and the most read Portuguese car magazine.

[9] The Portuguese firm that records monthly the advertising expenditures by media and by brand. These advertising expenditures represents only ' official or contractual prices '. We know in the industry that prices are frequently lower.

[10] We recognise that for a VAR system to be valid (in Sims' sense), following Engle and Yoo (1987), it must contain all the important variables in a system (i.e., a co-integrating set). However, we have followed the current widespread practice of using differenced variables.

[11] In all computations we have used the RATS program (RATS 386, version 4.02).

[12] For other criticisms of ARIMA, see Chatfield and Prothero, 1973; Hillmer and Tiao, 1982; Prothero and Wallis, 1976.

[13] If AR(m) is the unrestricted VAR and AR(l) the restricted one, where m and l are the respective lags, then the LR statistic for testing AR(l) against AR(m) is given by

$$LR = (T - c)(\ln|\Omega_l| - \ln|\Omega_m|)$$

where T is the number of observations, c is the correction factor which is equal to the number of regressors in each equation in AR(m), and Ω is the covariance matrix of residuals of AR(l) and AR(m) respectively. The statistic LR is asymptotically distributed as $\aleph^2$ with $k^2(m-l)$ degrees of freedom, where k is the number of regressions.

[14] The final specification of unrestricted VAR is not presented here due to space limitations (RATS program generates a lote of output). However, these can be obtained from the author on request.

[15] See also Sims (1988) for a discussion on Bayesian skepticism on unit root econometrics.

[16] Longer lags (up to nine) were also tried but the substantive results were unchanged.

[17] Instead of preselecting some values we could select the Bayesian parameters by minimising the following function:

$$Min\ U(g,w,d) = \sum_{i=1}^{n} \sum_{h=1}^{H} u_{ih}$$

where u_{ih} is the Theil U for time-series i h-forecast steps ahead. Because the functional relationship is highly nonlinear, numerical methods must be used to minimise U. In particular, we could use a grid search over the arguments {g, w, d}.

[18] Improvement relative to an ARIMA model should be viewed positively. If a model performs as well (or even almost as well) as the univariate time series model and is able to capture the interseries relationships, then perhaps it has something to offer for applied decision making.

[19] In an alternative ordering the positions of the RS1 and PS1 variables were switched. Neither of these reorderings affected the main results.