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FURTHER INVESTIGATION OF THE UNCERTAIN UNIT ROOT IN GNP

by

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Abstract: This paper adopts a different approach to the study of the persistence of U.S. GNP. First, this paper uses a more powerful version of the ADF test developed by Elliot, Rothenberg and Stock (1992). Second, we also examine the results from a unit root test that has trend stationarity as the null (Kwiatkowski et al., 1992). Third, simulated critical values generated from plausible trend stationary and difference stationary models for GNP data are used, in order to minimize the possible biases induced by nuisance parameters in finite samples. The ability of these two tests to discriminate against plausible alternatives is evaluated using alternative-specific rejection frequencies. Fourth, to evaluate the implication of extending the time span of the data on the ability to make clear inferences regarding the presence of unit roots, we examine both post-war quarterly data and a longer annual series spanning the period 1869 to 1986. For quarterly data, these two unit root tests do not provide a definite conclusion regarding the existence of a unit root in GNP data, thereby confirming Rudebusch's (1993) results. In contrast, when analyzing annual data over the 1869-1986 period, we obtain very sharp results: The unit root null is rejected, while the trend stationary null is not. Moreover, the alternative-specific power for the trend stationary null test is fairly high. We conclude that with a longer span of data, one can obtain strong evidence of trend stationarity in per capita GNP. JEL categories: C22, E32.

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1. Introduction

Output persistence is one of the most debated issues in macroeconomics. In the wake of the seminal work by Nelson and Plosser (1982), a large literature testing for unit roots was spawned, including Stock and Watson (1986), Perron and Phillips (1987), Campbell and Mankiw (1987) and Evans (1989), to name only a few studies which have failed to reject the presence of a unit root in GNP. Recently, concern has arisen regarding the low power of conventional unit root tests, such as the augmented Dickey-Fuller (ADF) test, and consequently, the apparent finding of a unit root in GNP data using these tests. For instance, Christiano and Eichenbaum (1990), Stock (1991), Rudebusch (1992, 1993), and DeJong et al. (1992) show that the ADF test has low power to differentiate between the trend and difference stationary properties of GNP.

This paper adopts a different approach to the study of the persistence of U.S. GNP. First, instead of the standard ADF test, we use the ADF-GLS$^{\tau}$ test of Elliot, Rothenberg and Stock (1992). These authors show that the modified ADF test is more powerful than the original ADF test and is approximately uniformly most power invariant.

Second, the commonly used ADF test has the unit root, or I(1), process as the null hypothesis. In addition to the aforementioned power consideration, the use of ADF tests also gives the unit root specification the benefit of a doubt. In particular, we reject the unit root specification only if there is strong evidence against it. To account for this asymmetric treatment, we also examine the results from a unit root test that has trend stationarity, or I(0), as the null. The test employed is the KPSS test developed by Kwiatkowski et al. (1992).

Third, simulated critical values generated from plausible trend and stationary models for GNP data are used to minimize the possible biases induced by nuisance parameters in finite samples. The ability of these two tests to discriminate against a plausible alternative is evaluated using alternative-specific rejection frequencies.

Fourth, to evaluate the implication of extending the span of the data on the ability to make clear inferences regarding the presence of unit roots, we examine both post-war quarterly data and a longer annual series spanning the period 1869 to 1986. For quarterly data, these two unit root tests do not provide a definite conclusion regarding the existence of a unit root in GNP data. Using null-specific critical values, neither the trend nor difference stationary null hypotheses can be rejected. However, we also observe that the alternative-specific power of both tests are low, so that no unambiguous conclusions can be made. Hence we confirm the Rudebusch (1993) results for this data set. In contrast, when analyzing annual data over the 1869-1986 period, we obtain very sharp results: The unit root null is rejected, while the trend stationary null is not. Moreover, the alternative-specific power for the test with a trend stationary null is fairly high. We conclude that with a longer span of data, one can obtain strong evidence of trend stationarity in per capita GNP.

The outline of the paper is as follows. In Section 2, the methodology is described. Empirical results are presented in Section 3. Section 4 concludes.

2. Methodology

2.1 Overview

First, we identify the most plausible trend stationary ARMA and ARIMA representations for the GNP data. Then, both the estimated trend stationary ARMA process and the estimated difference stationary ARIMA process are used to generate the empirical distributions of the ADF-GLS$^{\tau}$ and the KPSS tests. The empirical distribution of the ADF-GLS$^{\tau}$ (KPSS) statistic computed from the estimated difference stationary ARIMA (trend stationary ARMA) process provides the null-specific critical values to test the unit root (trend stationary) null against the trend stationary (unit root) alternative. Information on the ability of these two tests to reject the plausible alternative dynamic specification is given by the other two empirical distributions. The size-adjusted power of the test is then obtained using the null-specific critical value.

If the ADF-GLS$^{\tau}$ rejects the unit root null and the KPSS test fails to reject the stationary null, this result is considered strong evidence in favor of a trend stationary specification for the GNP data. If, in contrast, the ADF-GLS$^{\tau}$ fails to reject while KPSS rejects, we obtain strong evidence in support of a difference stationary GNP process. If both tests fail to reject their respective null hypotheses, we then conclude that the data do not contain sufficient information to discriminate between the difference and trend stationary hypotheses. A more complicated situation occurs when both tests reject their respective null hypotheses. This outcome may indicate that the underlying data generating mechanism is more complex than that captured by standard linear time series models.

2.2. Identification

The first step is to identify and estimate the ARMA and ARIMA processes which best describe the respective trend and difference stationary hypotheses. For the first case, various ARMA processes are fitted to the data,

$$y_t = \mu + \beta t + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t \ , \qquad \textbf{(1)}$$

where $\{y_t\}$ is log real per capita GNP. The final specification is chosen from models with the lag parameters p and q ranging from 0 to 5 using the Schwarz (1978) Information Criterion (SIC). As long as the true lag parameters are less than 5, the SIC will select the true model with probability one in large samples (Hannan, 1980). The Box-Ljung statistic is used to insure that there is no significant serial correlation in the residuals of the selected model specification.

For the second case, the relevant series is first-differenced, and then an ARMA process is fit to the differenced series,

$$(1-L)y_t = \mu + \sum_{i=1}^{p} \phi_i (1-L) y_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t \quad \textbf{(2)}$$

where L is the lag operator. The same selection criterion is applied. We label the selected specifications as the TS and DS models.

2.3. The ADF-GLS$^\tau$ Test

The ADF-GLS$^\tau$ test is carried out using the following regression[1]:

$$(1-L)y_t^\tau = a_0 y_{t-1}^\tau + \Sigma_{j=1}^{p} a_j (1-L) y_{t-j}^\tau + \omega_t \quad \textbf{(3)}$$

where y_t^τ, the locally detrended data process under the local alternative of $\bar{\alpha}$, is given by

$$y_t^\tau = y_t - \tilde{\beta}^{\prime} z_t \quad \textbf{(4)}$$

with $z_t = (1,t)'$ and ß being the regression coefficient of $\tilde{y}_t$ on $\tilde{z}_t$, for which

$$(\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_T) = (y_1, (1-\bar{\alpha}L)y_2, ..., (1-\bar{\alpha}L)y_T)$$
$$(\tilde{z}_1, \tilde{z}_2, ..., \tilde{z}_T) = (z_1, (1-\bar{\alpha}L)z_2, ..., (1-\bar{\alpha}L)z_T).$$

The ADF-GLS$^\tau$ test statistic is given by the usual t-statistic

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[1] The lag parameter p in equation (3) is chosen on the basis of the SIC. Hall (forthcoming) shows that the use of lag selection criteria such as the SIC can improve both the size and the power of conventional unit root tests.

testing $a_0 = 0$ against the alternative of $a_0 < 0$ in regression (4). Elliot, Rothenberg and Stock (1992) recommend that the parameter $\bar{c}$, which defines the local alternative through $\bar{\alpha} = 1 + \bar{c}/T$, be set equal to -13.5. Critical values for the ADF-GLS$^{\tau}$ test statistic are provided by Elliot, Rothenberg, and Stock (1992; Table 1) using the Monte Carlo method. These authors explicitly derive the asymptotic power envelope and show that the ADF-GLS$^{\tau}$ test can achieve a substantial gain in power over conventional unit-root tests.

For this exercise, we generate null specific critical values using the selected DS specification. If the ADF-GLS$^{\tau}$ statistic exceeds the null specific critical value, then we reject the difference stationary null. If the test fails to reject the null, then it is important to assess the "size-adjusted" power of the test. This can be done, given an empirical size of a test, by inspecting the empirical distribution of the TS model, and calculating the proportion of times the ADF-GLS$^{\tau}$ test statistic exceeds the null specific critical value. Both the null specific critical value and the alternative-specific power are generated based on 10,000 replications of the relevant process.

2.3. The KPSS test

To examine the dynamic properties of GNP in a symmetric manner, we also apply the KPSS test to test the trend stationary null hypothesis against the unit root alternative. The procedure assumes that the time series is the sum of a deterministic trend, a random walk, and a stationary error. It is a Lagrange Multiplier test for the null hypothesis that the error variance in the random walk component of the series is zero.

To conduct the test, we first obtain the residual e_t from the regression of y_t on a constant and a trend. The KPSS $\hat{\eta}_t$ statistic is given by

$$\hat{\eta}_t = T^{-2}\sum S_t^2/s^2(\ell) \tag{5}$$

where S_t is the partial sum process defined by

$$S_t = \sum_{i=1}^{t} e_i , \quad t=1,2,...,T. \tag{6}$$

$$s^2(\ell) = T^{-1}\sum_{t=1}^{T} e_t^2 + 2T^{-1}\sum_{s=1}^{\ell} w(s,\ell) \sum_{t=s+1}^{T} e_t e_{t-s} \tag{7}$$

and $s^2(\ell)$ is the serial correlation and heteroskedasticity consistent variance estimator given by
$w(s,\ell)$ is an optimal weighting function corresponding to the choice of a spectral window.[2]

From the simulated empirical distribution of the TS model, we obtain the null specific critical values. For each critical value, we can then obtain the DS alternative-specific power. One complication is that ℓ in equation (7) is a choice parameter to be determined. Following KPSS' suggestion we focus on the ℓ8 rule, which sets $\ell=\text{INT}[8(T/100)^{1/4}]$, although we also tabulate the results for ℓ0, ℓ4, ℓ12 rules.

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[2] We use a Bartlett window, $w(s,\ell) = 1 - s/(\ell+1)$, as suggested by KPSS.

3. Empirical Results

Data on quarterly U.S. GNP in 1987$ and total population from CITIBASE are used to construct the real per capita output series. The data span 1948:1-1993:2. For the TS model (see equation 1), an ARMA(2,0) with constant and trend was selected. For the DS model (equation 2), an ARIMA(1,1,0) process with constant is selected by SIC.[3] These model estimates are reported in Table 1. In both cases, the Ljung-Box Q statistics indicate insignificant serial correlation in the residuals. It is interesting to note that the largest characteristic root of the ARMA(2,0) process is approximately 0.91, which is substantially less than unity. Both of the estimated models closely resemble those obtained by Rudebusch (1993), so our results are not specific to the data set we used.

We first examine the characteristics of the unit root test. The simulated critical values for the 10%, 5% and 1% marginal significance levels (MSLs) are reported in the top part of Panel A of Table 2. These critical values are quite similar to those tabulated in Elliot, Rothenberg and Stock (1992). In the bottom part of Panel A, the size-adjusted power for each MSL is reported, assuming the given trend stationary ARMA(2,0) alternative hypothesis.

The ADF-GLS$^{\tau}$ statistic is -2.3401, which is larger than the 10% critical value; hence we fail to reject the null hypothesis of a unit root in per capita GNP. The statistic is computed from a lag

[3] This model is also selected by the Akaike Information Criterion (AIC). In general, the models reported are chosen by both AIC and SIC.

2 specification selected by the SIC. This is the same lag structure identified in Table 1. Using the 10% critical value, the alternative-specific power is less than 50%.

We now turn our attention to viewing the trend stationary null test (see Panel B of Table 2). The finite sample critical values in the top half of Panel B differ depending upon the window size used. In the bottom half of Panel B are the associated alternative-specific levels of power. The null specific critical values are quite different from those asymptotic critical values reported in KPSS (1992). For example, consider the KPSS $\ell 8$ statistic (following KPSS' suggestion). The null specific critical values are given as .235, .269, .330 for the 10%, 5%, 1% MSLs. The actual $\hat{\eta}_t$ statistic is .198 so we fail to reject the trend stationary null. However, if the KPSS (1992) asymptotic critical values (which are .119, .146, .176 for the 10%, 5%, 1% MSLs respectively) are used, then we would reject at the 1% level. This contrast in results is indicative of the sensitivity of this procedure to nuisance ARMA parameters in finite samples and the consequent importance of adjusting for finite sample biases.

As mentioned above, the KPSS $\ell 8$ test statistic fails to reject the trend stationary null at the 10% significance level; the corresponding size-adjusted power is again around 50%. The non-rejection result is robust across different ℓ specifications.

In sum, the ADF-GLS$^{\tau}$ test cannot reject the unit root null hypotheses while the KPSS test does not reject the trend stationary null. We consider this outcome as evidence of the low power of the tests. Hence, the quarterly per capita GNP data, which has a span of about 40 years, appear uninformative with regard to the presence or absence of a unit root.

We now turn our attention to the annual data. This data set extends from 1869-1986[4]. An ARIMA(1,1,0) specification is selected as the DS model, while a ARMA(2,0) specification is selected for the TS model. The model estimates are reported in Table 3. The Box-Ljung statistic, again, indicates a satisfactory fit.

The null-specific critical values and alternative specific power are presented in Table 4. Consistent with the previous case, the ADF-GLS$^{\tau}$ test appears to be more robust to nuisance ARMA parameters than the KPSS test. However, the test results are quite different from those obtained from the quarterly data. The two tests combined together provide strong evidence of a trend stationary GNP series. For this long historical annual data, the ADF-GLS$^{\tau}$ rejects the unit root null at 1% while the KPSS test fails to reject the trend stationary alternative. The alternative-specific and size-adjusted power levels are 90% for the ADF-GLS$^{\tau}$ test, and 79% for the KPSS test. The relatively high power for the tests further reinforces the test result.

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[4] This series represents a combination of work by Kuznets (1961), Kendrick (1961), Gallman (1986) for the period 1869-1908, while the Department of Commerce (1986) series is used for the period 1909-1928 period. These data are reported in Department of Commerce (1973). Finally, the post 1928 data are from the National Income and Product Accounts. All the alternative measures of GNP (i.e., Romer and Balke-Gordon) are the same for these years. This series is the one most commonly used in historical analyses of US economic growth.

4. Concluding Remarks

This paper reports the results of a study where the issue of a unit root in GNP is examined from two perspectives -- from the unit root null as well as from the trend stationary null. We have taken advantage of the latest econometric technology, including the more powerful unit root test developed by Elliot, Rothenberg and Stock (1992), as well as the Kwiatkowski, Phillips, Schmidt and Shin (1992) trend stationarity test. Data-specific empirical distributions are used to mitigate possible finite sample biases. We also explore the sensitivity of the results to data length. The recently developed tests do not help determine the presence or absence of a unit root in post-war quarterly per capita GNP data; nor do they agree on the degree of persistence in the data. However, when the time span is extended to about 120 years, both the tests support a trend stationary specification for the GNP data. Thus, the availability of long time span data seems to be a more important factor than the number of observations in discerning the unit root property of GNP data (see also Shiller and Perron, 1985; Perron, 1989).

It is known that the autoregressive parameter estimates reported in Tables 1 and 3, while consistent, are biased. Following Rudebusch (1993), we "correct" the bias and repeat our exercise.[5] The test results, which are summarized in Tables A1 and A2, are

[5] Note that the analytically-derived bias applies to the estimator, which is a random variable. Using this estimated bias to adjust the estimate, which is a realization, may push the estimate either toward or away from the true parameter

essentially the same as those reported using uncorrected estimates.

To check the robustness of the trend stationarity result, we also applied the same procedure to the historical data series which are reported in, for example, Romer (1989) and Balke and Gordon (1983), both also extended to 1986. Similar evidence in support of trend stationarity is obtained. However, when the Nelson and Plosser (1982) GNP series -- which spans the period from 1909 to 1970 -- was used, neither the ADF-GLS$^{\tau}$ nor the KPSS tests reject their respective null hypotheses. Consequently, we cannot conclude from the Nelson and Plosser sample whether GNP has a unit root.[6] This further emphasizes the importance of the time span of data, as well as appropriate testing procedures, in the study of persistence in GNP.

[6] These empirical results are available from the authors upon request.

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TABLE 1
Time Series Representations
for Quarterly GNP Per Capita
1948.1-1993.2

Variable	DS Spec. ARIMA(1,1,0)	TS Spec. ARMA(2,0) + c + t
Constant	0.00278899 (0.000770347)	-0.23778 (.0090145)
time		.223039
(x1000)		(.0885330)
ϕ_1	0.374522 (0.069314)	1.34558 (0.069228)
ϕ_2		-0.396972 (0.069438)
SER	0.0095194	0.0093880
Q(10)	7.65	7.29
Q(20)	15.78	13.90
Roots	1.0000 0.3745	.9087 0.4368

Notes: Dependent variable is log real per capita GNP. ϕ_i is the estimate of the i-th order autoregressive coefficient. time (x1000) is the coefficient on time, multiplied by 1000. SER is the standard error of regression. Q(j) is the Ljung-Box Q statistic for serial correlation of the 1st to j-th residuals. "Roots" are the roots of the AR polynomial.

TABLE 2
Empirical Size and Corresponding Alternative-Specific Power for Quarterly GNP Per Capita Data

A. ADF-GLS$^{\tau}$ Test

Model Specific Null Hypothesis Critical Values ARIMA(1,1,0)

MSL	10%	5%	1%	Actual
c.v.	-2.6550	-2.95865	-3.52233	-2.3401

Alternative Hypothesis Specific Power ARMA(2,0) + c + t

MSL	10%	5%	1%
Power	47.08%	27.99%	7.47%

B. KPSS Test

Model Specific Null Hypothesis Critical Values ARMA(2,0) + c + t

MSL	10%	5%	1%	Actual
ℓ0 c.v.	1.804647	2.097019	2.707968	1.388
ℓ4 c.v.	0.401141	0.466301	0.588308	0.310
ℓ8 c.v.	0.234544	0.268777	0.330261	0.198
ℓ12 c.v.	0.189155	0.215171	0.258809	0.154

Alternative Hypothesis Specific Power ARIMA(1,1,0)

MSL	10%	5%	1%
ℓ0 power	55.07%	44.51%	27.07%
ℓ4 power	52.68	42.22	25.64
ℓ8 power	49.16	39.15	23.29
ℓ12 power	46.55	36.51	21.06

Notes: In the top portion of panel A, each entry indicates the finite critical value corresponding to the indicated marginal significance level (MSL) for the simulated difference stationary null hypothesis. In the bottom half of panel A is the empirical power associated with each MSL, for the specific simulated trend stationary alternative. In the top portion of panel B, each entry indicates the finite critical value corresponding to the indicated marginal significance level (MSL) for the simulated trend stationary null hypothesis; there are four entries corresponding to the selected window size. In the bottom half of panel B is the empirical power associated with each MSL, for the specific simulated difference stationary alternative.

TABLE 3
Identification of Time Series Representations for Annual GNP Per Capita
1869-1986

Variable	DS Spec. ARIMA(1,1,0)	TS Spec. ARMA(2,0) + c + t
Constant	0.013847 (0.005885)	0.152346 (0.035245)
time		3.51831
(x1000)		(0.870371)
ϕ_1	0.212839 (0.091087)	1.10918 (0.089592)
ϕ_2		-0.316050 (0.089579)
SER	0.061398	0.057953
Q(10)	13.68	8.18
Q(20)	19.47	14.0
Roots	1.0000 0.2128	0.5959 ± .0921i

Notes: Dependent variable is log real per capita GNP. ϕ_i is the estimate of the i-th order autoregressive coefficient. time (x1000) is the coefficient on time, multiplied by 1000. SER is the standard error of regression. Q(j) is the Ljung-Box Q statistic for serial correlation of the 1st to j-th residuals. "Roots" are the roots of the AR polynomial.

TABLE 4
Empirical Size and Corresponding Alternative-Specific Power for Annual Per Capita Data

A. ADF-GLS$^{\tau}$ Test

Model Specific Null Hypothesis Critical Values ARIMA(1,1,0)

MSL	10%	5%	1%	Actual
c.v.	-2.69919	-2.97186	-3.56684	-4.1139

Alternative Hypothesis Specific Power ARMA(2,0) + c + t

MSL	10%	5%	1%
Power	99.83%	99.14%	89.77%

B. KPSS Test

Model Specific Null Hypothesis Critical Values ARMA(2,0) + c + t

MSL	10%	5%	1%	Actual
ℓ0 c.v.	0.624936	0.755073	1.075817	0.367
ℓ4 c.v.	0.180516	0.216334	0.296904	0.105
ℓ8 c.v.	0.141946	0.166709	0.212922	0.084
ℓ12 c.v.	0.132770	0.151083	0.184657	0.083

Alternative Hypothesis Specific Power ARIMA(1,1,0)

MSL	10%	5%	1%
ℓ0 power	86.57%	79.32%	60.38%
ℓ4 power	79.12	69.54	47.27
ℓ8 power	68.79	56.32	37.70
ℓ12 power	56.83	46.43	29.15

Notes: In the top portion of panel A, each entry indicates the finite critical value corresponding to the indicated marginal significance level (MSL) for the simulated difference stationary null hypothesis. In the bottom half of panel A is the empirical power associated with each MSL, for the specific simulated trend stationary alternative. In the top portion of panel B, each entry indicates the finite critical value corresponding to the indicated marginal significance level (MSL) for the simulated trend stationary null hypothesis; there are four entries corresponding to the selected window size. In the bottom half of panel B is the empirical power associated with each MSL, for the specific simulated difference stationary alternative.

TABLE A1
"Bias-Corrected" Estimates of Time Series Representations for Quarterly and Annual GNP Per Capita

Variable	Quarterly		Annual	
	DS Spec. ARIMA(1,1,0)	TS Spec. ARMA(2,0)	DS Spec. ARIMA(1,1,0)	TS Spec. ARMA(2,0)
ϕ_1	0.3865	1.3599	0.2273	1.1309
ϕ_2		-0.3912		-0.3033
Roots	1.000 0.3865	0.9467 0.4132	1.000 0.2273	0.6939 0.4370

Notes: Dependent variable is log real per capita GNP. ϕ_i is the estimate of the i-th order autoregressive coefficient. These "bias-corrected" parameter estimates correspond to the estimates in Table 1 (quarterly) and in Table 2 (annual). "Roots" are the roots of the AR polynomial.

TABLE A2
Empirical Size and Corresponding Alternative-Specific Power for "Bias-Corrected" Estimates

Quarterly GNP Per Capita Data

A. ADF-GLS$^{\tau}$: Model Specific Null Hypothesis CV's ARIMA(1,1,0)

MSL	10%	5%	1%	Actual
c.v.	-2.65530	-2.95647	-3.52909	-2.3401

B. KPSS: Model Specific Null Hypothesis CV's ARMA(2,0) + c + t

MSL	10%	5%	1%	Actual
ℓ0 c.v.	2.360699	2.703636	3.319882	6.117
ℓ4 c.v.	0.509052	0.580839	0.702728	1.385
ℓ8 c.v.	<u>0.284223</u>	<u>0.321969</u>	<u>0.379719</u>	<u>0.760</u>
ℓ12 c.v.	0.222049	0.249863	0.289487	0.572

Annual Per Capita Data

A. ADF-GLS$^{\tau}$: Model Specific Null Hypothesis CV's ARIMA(1,1,0)

MSL	10%	5%	1%	Actual
c.v.	-2.65530	-2.95647	-3.52909	-4.1139

B. KPSS: Model Specific Null Hypothesis CV's ARMA(2,0) + c + t

MSL	10%	5%	1%	Actual
ℓ0 c.v.	0.679675	0.820156	1.159068	0.367
ℓ4 c.v.	0.190247	0.227432	0.311466	0.105
ℓ8 c.v.	<u>0.146556</u>	<u>0.171589</u>	<u>0.217396</u>	<u>0.084</u>
ℓ12 c.v.	0.134832	0.153459	0.187172	0.083

Notes: In panel A, each entry indicates the finite critical value corresponding to the indicated marginal significance level (MSL) for the simulated difference stationary null hypothesis. In panel B, each entry indicates the finite critical value corresponding to the indicated marginal significance level (MSL) for the simulated trend stationary null hypothesis; there are four entries corresponding to the selected window size.