On the Estimation of Demand Systems Through Consumption Efficiency

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Abstract. We consider a Bayesian implementation of a new approach to estimating Demand Systems. This approach, suggested by Varian (1990), is based on a generalization of Afriat's (1967) efficiency index. The model we propose leads to a very tractable posterior and predictive analysis, yet allows for interesting economic interpretations. We conduct a sensitivity analysis with respect to the prior in an application to annual aggregate U.S. consumption data, and conclude that the sample is quite informative. Average efficiency and expected budget shares are examined in some detail.

Keywords. Bayesian Methods, Budget Shares, Money-Metric Utility, Monte Carlo

JEL Classification System. C11, D12, C50

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1. Introduction

Varian (1990) proposes the use of the money-metric utility function[1] to estimate the parameters of a demand system. He arrives to that formulation from a parametric generalization of Afriat's (1967) efficiency index. The main motivation for such a novel approach is the utilization of a sensible norm of goodness of fit.

Let the utility function be characterized by the functional form $u(\cdot)$ and the parameter $\alpha \in R^q$. Suppose that, for $t = 1, \ldots, T$, we observe the pairs (x_t, p_t), where $x_t = (x_{1t}, \ldots, x_{nt})'$ is a vector of the n-good bundle chosen when prices are $p_t = (p_{1t}, \ldots, p_{nt})$; $m_t = p_t x_t$ will be the consumer's expenditure at t. We postulate that if x_t is not the optimal choice at prices p_t, it must provide a level of utility *close* to the optimal. The ordinal character of the utility function makes it difficult to operationalize this closeness concept. The most widespread practice in applied demand studies is to use a norm—usually a quadratic norm—on the goods' or the expenditure shares' space. This norm lacks any economic content as pointed out by Varian. Bundles providing similar levels of utility might be far away in the goods' space and vice-versa. Instead, we focus on a measure of efficiency based on money-metric utility.

We illustrate this in Figure 1, adapted from Varian (1990). While the optimal choice might be far from the observed choice in the goods space—*e.g.*, when using the Euclidean distance, $(d_1^2 + d_2^2)^{\frac{1}{2}}$—they might be close when a money metric is used instead. That is, the wasted expenditure—because of non-optimality—might be small enough to consider the non-optimal choice as 'sufficiently' close to the optimal.[2] (In Figure 1, the vertical distance labelled 'wasted expenditure' is measured in units of good 2 so it should be multiplied by p_2 to strictly represent expenditure.)

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[1] Denote by x the bundle of goods, by p the vector of prices, and by m the individual's income. Assuming the utility function $u(x)$, we define the indirect utility function as $v(p,m) \equiv \max_x\{u(x) : px \leq m\}$; the expenditure function as $c(p,u) \equiv \min_x\{px : u(x) \geq u\}$; and the money-metric utility function as $\psi(p,x) \equiv c(p,u(x))$. See Varian (1992) for the derivation and properties of these functions.

[2] Theil (1971) develops a theory of the second moments of the disturbances of behavioral equations based on the Hessian matrix of the criterion function. The idea being that if the second derivatives of the objective function evaluated at the extremum are close to zero, then the loss incurred when deviating from the optimum will not be considerable. Hence, large variances can be expected. In our Figure 1, flatter indifference curves would be associated with greater variability in the observed choices.

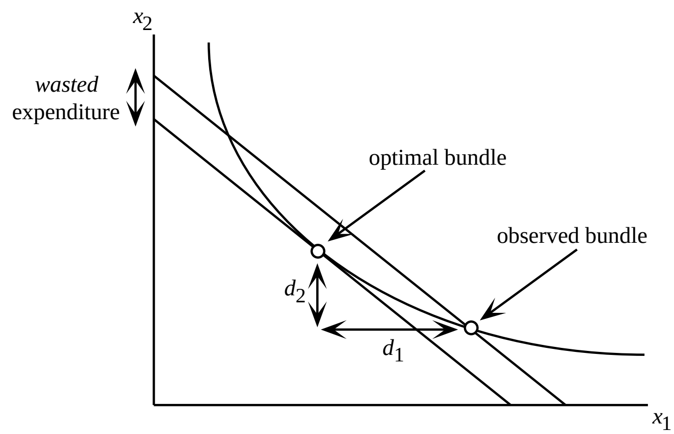

Figure 1. Different measures of goodness of fit

Bayesian methods of inference will be used to formally treat parameter uncertainty and actually derive posterior densities for the out-of-sample or "average" efficiencies under different prior assumptions. [Ley and Steel (1992) studied the behavior of within-sample efficiencies.] In addition, a predictive analysis of budget shares will be conducted. The numerical integration only requires simple importance sampling methods.

2. The Sampling Model

If $\psi(p_t, x_t)$ is the minimum expenditure required to achieve, at p_t, the utility level given by the chosen bundle, x_t, the *wasted* expenditure will be $m_t - \psi(p_t, x_t)$. Suppose that this wasted expenditure is *randomly proportional* to the *minimum* expenditure

$$m_t - \psi(p_t, x_t) = \psi(p_t, x_t)\eta_t, \tag{1}$$

where $\eta_t \geq 0$ is a random variable. Manipulating (1) and making explicit that the minimum expenditure is parametrized by some vector α, we can write the inverse efficiency measure

$$\frac{m_t}{\psi(p_t, x_t; \alpha)} = 1 + \eta_t \equiv e^{\varepsilon_t} \geq 1, \tag{2}$$

where $\varepsilon_t = \log(1 + \eta_t) \geq 0$. Taking natural logs, we can rewrite (2) more conveniently as

$$\log m_t - \log \psi(p_t, x_t; \alpha) = \varepsilon_t \geq 0. \tag{3}$$

An alternative specification that leads to the same sampling model starts by assuming that the wasted expenditure is randomly proportional to the *actual* expenditure, as opposed to the *minimum* expenditure. Equation (1) then needs to be replaced by

$$m_t - \psi(p_t, x_t) = m_t \nu_t,$$

where now the random variable ν_t must lie in $[0,1]$ with $\nu_t = 0$ representing full efficiency and $\nu_t = 1$ full inefficiency. Some manipulation leads to

$$\frac{m_t}{\psi(p_t, x_t; \alpha)} = \frac{1}{1-\nu_t} \equiv e^{\varepsilon_t} \geq 1,$$

which is equivalent to equation (2) with $\eta_t = \nu_t/(1+\nu_t)$.

In both cases, we obtain (3) where $\varepsilon_t \geq 0$ has some distribution defined over the positive real line. Basing the estimation of the relevant parameters on this equation offers several advantages over the methods traditionally employed in applied demand analysis. In addition to the motivation just presented in favor of using a norm with economic content, one only needs to impose homogeneity and concavity restrictions on $\psi(p_t, x_t; \alpha)$ to obtain estimates consistent with economic theory as opposed to the cross-equation restrictions imposed on a demand system to obtain a symmetric and negative-semidefinite Slutsky matrix which are far more difficult to implement in practice.

In demand analysis, the usual construction of a sampling model is through directly specifying an $(n-1)$-dimensional distribution on x_t, taking into account that total expenditure and prices are treated as given. Here we will let our formulation be guided by the fact that we expect choices to be close to optimal (in money-metric terms). Thus, we shall adopt a sampling distribution on $n-1$ elements of x_t, given total expenditure m_t, which is 'inspired' by a distribution on ε_t in (3).

In the Cobb-Douglas case with only three goods ($n = 3$), the normalized utility function is $u(x;\alpha) = x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3}$, with $\alpha_3 = 1-\alpha_1-\alpha_2$, and the minimum expenditure—*i.e.*, the money-metric utility function—becomes

$$\psi(p_t, x_t; \alpha) = \prod_{i=1}^{3} \left(\frac{p_{it} x_{it}}{\alpha_i} \right)^{\alpha_i}.$$

Exponentially distributed error terms[3] ε_t determine a certain distribution on $\log \psi(p_t, x_t; \alpha)$ given m_t, p_t and α. We shall now retain the same *functional form*

[3] The random variable $z > 0$ has a gamma distribution with parameters a and b if its density function is given by $f_\gamma(z|a,b) = z^{a-1} e^{-z/b} b^{-a} / \Gamma[a]$, with $\mathrm{E}[x] = ab$, and $\mathrm{Var}[x] = ab^2$. A gamma distribution with shape parameter $a = 1$ is also known as an exponential distribution.

for the sampling distribution of x_t, leading to the following likelihood function based on T observations:

$$\begin{aligned} p(\mathbf{x}|\mathbf{m},\mathbf{p},\alpha,\mu) &\propto \exp\left\{-T\log\mu - \frac{1}{\mu}\sum_{t=1}^{T}[\log m_t - \log\psi(p_t,x_t;\alpha)]\right\} \\ &\propto \prod_{t=1}^{T}\left(\frac{p_{1t}x_{1t}}{\alpha_1}\right)^{\frac{\alpha_1}{\mu}}\left(\frac{p_{2t}x_{2t}}{\alpha_2}\right)^{\frac{\alpha_2}{\mu}}\left(\frac{p_{3t}x_{3t}}{\alpha_3}\right)^{\frac{\alpha_3}{\mu}} \end{aligned} \tag{4}$$

where we have defined $\mathbf{x} = (x_1, \ldots, x_T)'$, $\mathbf{p} = (p'_1, \ldots, p'_T)'$ and $\mathbf{m} = (m_1, \ldots, m_T)'$. Define the budget shares $s_{it} = p_{it}x_{it}/m_t$ and let $\mathbf{s}$ be defined conformably to $\mathbf{x}$. Then,

$$p(\mathbf{s}|\mathbf{m},\mathbf{p},\alpha,\mu) \propto \prod_{t=1}^{T}(s_{1t})^{\frac{\alpha_1}{\mu}}(s_{2t})^{\frac{\alpha_2}{\mu}}(1-s_{1t}-s_{2t})^{\frac{\alpha_3}{\mu}} \tag{5}$$

which is a Dirichlet[4] $(\frac{\alpha_1}{\mu}+1, \frac{\alpha_2}{\mu}+1, \frac{\alpha_3}{\mu}+1)$ kernel. We then have:

$$\begin{aligned} p(\mathbf{x}|\mathbf{m},\mathbf{p},\alpha,\mu) = &\left(\frac{\Gamma(3+\frac{1}{\mu})}{\Gamma(\frac{\alpha_1}{\mu}+1)\Gamma(\frac{\alpha_2}{\mu}+1)\Gamma(\frac{\alpha_3}{\mu}+1)}\right)^T \\ &\times \prod_{t=1}^{T}\frac{p_{1t}p_{2t}}{m_t^2}\left(\frac{p_{1t}x_{1t}}{m_t}\right)^{\frac{\alpha_1}{\mu}}\left(\frac{p_{2t}x_{2t}}{m_t}\right)^{\frac{\alpha_2}{\mu}}\left(\frac{p_{3t}x_{3t}}{m_t}\right)^{\frac{\alpha_3}{\mu}}, \end{aligned} \tag{6}$$

where the first factor in the product over t is the Jacobian of the transformation from $\mathbf{s}$ to $\mathbf{x}$. The likelihood function in (6) was here derived for $n = 3$, but trivially extends to general n.

From the properties of the Dirichlet we can derive the expected shares:

$$\mathrm{E}[s_i|\mathbf{m},\mathbf{p},\alpha,\mu] = \frac{\alpha_i+\mu}{1+n\mu}. \tag{7}$$

If $\mu = 0$ —*i.e.*, a degenerate distribution on ε_t implying no wasted expenditure— then the expected share of good i's expenditure is given by α_i in the sampling as corresponds to the usual Cobb-Douglas assumptions. If, however, $\mu > 0$, then

[4] The vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)'$ with $\alpha_i > 0$, $\forall i$, and $\sum_{i=1}^n \alpha_i = 1$ has a Dirichlet distribution with parameters $c = (c_1, c_2, \ldots, c_n)'$ if its density function is given by $f_D^n(\alpha|c) = \Gamma(C)\prod_{i=1}^n[\alpha_i^{c_i-1}/\Gamma(c_i)]$, where $c_i > 0$, $\forall i$, and $C = \sum_{i=1}^n c_i$. The first two moments are given by $\mathrm{E}[\alpha_i] = c_i/C$, and $\mathrm{Var}[\alpha_i] = c_i(C-c_i)/(C^2(C+1))$.

the expenditure shares will tend to be more equal as they are linear combinations of α_i and $1/n$ with weights $1/n$ and μ.

It is important to stress that the sampling distribution in (6) was obtained solely on the basis of the particular functional form in (4). Thus, (6) is, in general, not derived in a probabilistic sense from a distribution on ε_t, but is merely *inspired* by the functional form associated with an exponential distribution on ε_t in (3). The formal starting point of our analysis, therefore, is the kernel in (4) or the sampling density in (6), rather than any distribution on ε_t.

The model introduced here could be viewed as a consumption counterpart to production or cost frontier models, where the frontier [here $\psi(p_t, x_t; \alpha)$] is deterministic in the sampling (*i.e.*, given α), but agents can do worse than the frontier due to inefficiency. Stochastic frontier models (*i.e.*, with symmetric measurement error on the frontier added) are treated in a Bayesian framework by van den Broeck *et al.* (1994) and Koop *et al.* (1994, 1995). However, this complication typically requires more sophisticated numerical techniques than the simple importance sampling used here. Koop *et al.* (1994, 1995) find Gibbs sampling very appropriate in this context.

3. Prior Densities

To complete the Bayesian model we need to specify a prior density on the parameters in (6). The complicated form of the likelihood function precludes an analytical analysis and we are obliged to follow a numerical approach. We shall only consider proper priors here which ensures the existence of proper posterior densities. However, an extensive sensitivity analysis will be conducted.

The prior on α will be of the Dirichlet form

$$p(\alpha) = f_D^3(\alpha|k), \tag{8}$$

and on the parameter of the exponential model we specify

$$p(\mu^{-1}|\alpha) = p(\mu^{-1}) = f_\gamma(\mu^{-1}|1, (-\log r^*)^{-1}). \tag{9}$$

The reason for choosing (9) is its analytical tractability and its associated ease in elicitation. In particular, if ε_t would be exponentially distributed, it would lead to such a marginal prior density for ε_t that the prior median of the efficiency $r_t = e^{-\varepsilon_t}$ is exactly equal to r^* [see van den Broeck *et al.* (1994)]. Even though the distribution implicitly assumed for ε_t through our sampling density in (6) is not exactly exponential, we find in our empirical application that it very closely corresponds to an exponential distribution with mean μ. Thus, for the purpose of prior elicitation, this interpretation of r^* is quite useful.

We still need to choose the hyperparameters of the priors in (8) and (9). For the Dirichlet prior in (8) we take the following values for k: $k = (3, 3, 3)$ which is elicited postulating mean expenditure shares of 33.3% for all goods with standard deviations of 14.91%. Alternatively, we take parameters $(1, 1, 1)$ and $(30, 30, 30)$ leading to standard deviations of 23.57% and 4.94%. This covers a wide range of prior beliefs, ranging from very noninformative to rather informative. We also perform a sensitivity analysis over μ using the values of 0.50, 0.70, 0.80, 0.90, 0.95 and 0.98 for the prior median efficiency r^* in (9). The latter is meant to cover all prior opinions that could reasonably be entertained concerning the particular application.

4. Posterior Results

We use U.S. aggregate consumption data of three groups of goods: durables, nondurables and services from 1947 to 1987.[5] This implies $n = 3$ and $T = 41$. Varian (1990) lists the data.

The posterior analysis is conducted using importance sampling Monte Carlo [see, *e.g.*, Geweke (1989)]. A product of a Dirichlet distribution on α and a gamma distribution on μ constitutes a very useful importance function in practice. The hyperparameters of this importance function are chosen on the basis of preliminary Monte Carlo runs. Variation coefficients of the weights in the final runs are always less than 3, indicating quite satisfactory numerical stability. All the results are based on 1,000,000 Monte Carlo replications.[6]

Table 1 records posterior moments of the model parameters for all combinations of the priors used. With the possible exception of the extreme value $r^* = 0.5$, the posterior distributions are quite robust to the choice of the prior hyperparameters.

4.1. Efficiency and Budget Shares

We are particularly interested in the posterior distribution of the efficiency measure given by

$$r_t = \frac{\psi(p_t, x_t; \alpha)}{m_t} = \prod_{i=1}^{3} \left(\frac{s_{it}}{\alpha_i} \right)^{\alpha_i} \in (0, 1] \tag{10}$$

[5] We are well aware that the i.i.d. assumption for ε is a bit suspicious for these time-series data. However, we believe that sophisticating the sampling model at this point might not add much credibility to this simple aggregate demand system. We plan more ambitious applications of this approach using cross-sectional data.

[6] The Fortran code is available from http://econwpa.wustl.edu. For our empirical example, the non-optimized code for a full model analysis executes at a rate of 66,000 drawings per minute on a Power Macintosh 8100/100, rendering far more challenging applications a practical possibility.

Table 1. Posterior Characteristics of the Parameters

	$r^* = 0.98$		$r^* = 0.95$		$r^* = 0.9$		$r^* = 0.8$		$r^* = 0.7$		$r^* = 0.5$	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
	Dirichlet Hyperparameters: $k = (1, 1, 1)$											
μ	0.0217	0.0037	0.0226	0.0038	0.0241	0.0041	0.0275	0.0047	0.0313	0.0053	0.0415	0.0074
α_1	0.1480	0.0085	0.1477	0.0087	0.1473	0.0090	0.1465	0.0096	0.1453	0.0104	0.1425	0.0120
α_2	0.4678	0.0117	0.4681	0.0120	0.4683	0.0123	0.4689	0.0132	0.4698	0.0144	0.4718	0.0164
α_3	0.3842	0.0114	0.3842	0.0116	0.3844	0.0120	0.3846	0.0128	0.3849	0.0139	0.3856	0.0159
	Dirichlet Hyperparameters: $k = (3, 3, 3)$											
μ	0.0217	0.0037	0.0225	0.0038	0.0241	0.0041	0.0274	0.0047	0.0313	0.0054	0.0413	0.0074
α_1	0.1486	0.0085	0.1484	0.0086	0.1480	0.0089	0.1472	0.0096	0.1463	0.0103	0.1439	0.0119
α_2	0.4673	0.0117	0.4676	0.0119	0.4679	0.0123	0.4685	0.0132	0.4691	0.0140	0.4708	0.0163
α_3	0.3840	0.0113	0.3841	0.0116	0.3841	0.0119	0.3843	0.0128	0.3846	0.0137	0.3853	0.0159
	Dirichlet Hyperparameters: $k = (30, 30, 30)$											
μ	0.0216	0.0036	0.0225	0.0038	0.0240	0.0041	0.0273	0.0047	0.0310	0.0054	0.0409	0.0072
α_1	0.1564	0.0083	0.1566	0.0084	0.1568	0.0086	0.1573	0.0092	0.1576	0.0098	0.1591	0.0112
α_2	0.4617	0.0114	0.4616	0.0115	0.4615	0.0120	0.4611	0.0127	0.4609	0.0136	0.4599	0.0155
α_3	0.3818	0.0110	0.3818	0.0113	0.3817	0.0117	0.3816	0.0124	0.3815	0.0132	0.3811	0.0151

which is a parametric generalization of Afriat's efficiency index (Varian, 1990). From (10) we can derive that full efficiency is obtained only if $s_{it} = \alpha_i$, $i = 1, 2$. Furthermore, combining this with (7), we see that we shall move towards full efficiency as μ tends to zero. The posterior density of an "average" efficiency measure can be examined by first drawing budget shares from the predictive distribution, which is the Dirichlet sampling distribution in (5) integrated out with respect to the parameters. Thus, for every drawing of (α, μ), we generate one set of shares from (5) and evaluate the efficiency of an unobserved period, say r_f, through (10). This produces drawings from the posterior distribution of r_f, from which density plots and moments can easily be computed. The first two posterior moments of r_f are grouped in Table 2. Note that Varian (1990) finds an average "wasted expenditure" of 2% using classical nonlinear least squares methods. Characteristics of the predictive distribution of the budget shares themselves could also be recorded in the process. However, in order to achieve even greater precision with a fixed number of drawings, we can average over the known conditional moments [as in (7)] and the conditional density function [the Dirichlet in (5)] to deduce the marginal predictive moments and density function. This feature, often associated with the Rao-Blackwell Theorem, was introduced in Gelfand and Smith (1990) and used in Koop *et al.* (1995) in a stochastic production frontier context. Table 2 clearly illustrates that neither the posterior distribution of r_f, nor the predictive distributions of the budget shares are affected by substantial changes in the prior. Figure 2 graphically contrasts the wide range of prior densities used with the relatively unchanged form of the posterior density of r_f. The corresponding prior efficiency is here presented under the simplifying assumption that ε_t is exponentially distributed with mean μ. In that case (which is empirically found to be a very close approximation to the actual distribution of ε_t) the marginal prior distribution of ε_t can be found by integrating out μ from the joint density of (ε_t, μ) which leads to an analytical expression.[7]

Table 2. Efficiency and Budget Shares

	$r^* = 0.98$		$r^* = 0.95$		$r^* = 0.9$		$r^* = 0.8$		$r^* = 0.7$		$r^* = 0.5$	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
	Dirichlet Hyperparameters: $k = (1, 1, 1)$											
r_f	0.9797	0.0204	0.9789	0.0211	0.9777	0.0224	0.9748	0.0252	0.9717	0.0279	0.9633	0.0362
s_1	0.1593	0.0523	0.1595	0.0533	0.1598	0.0549	0.1607	0.0584	0.1614	0.0619	0.1636	0.0704
s_2	0.4596	0.0712	0.4595	0.0725	0.4592	0.0747	0.4586	0.0792	0.4581	0.0839	0.4565	0.0948
s_3	0.3811	0.0694	0.3810	0.0707	0.3810	0.0728	0.3807	0.0772	0.3805	0.0818	0.3799	0.0924
	Dirichlet Hyperparameters: $k = (3, 3, 3)$											
r_f	0.9798	0.0203	0.9790	0.0212	0.9777	0.0222	0.9749	0.0252	0.9716	0.0279	0.9635	0.0360
s_1	0.1599	0.0523	0.1601	0.0533	0.1605	0.0550	0.1614	0.0584	0.1623	0.0622	0.1648	0.0705
s_2	0.4591	0.0711	0.4591	0.0724	0.4588	0.0747	0.4583	0.0791	0.4574	0.0839	0.4556	0.0947
s_3	0.3809	0.0693	0.3808	0.0706	0.3807	0.0728	0.3804	0.0771	0.3802	0.0818	0.3796	0.0923
	Dirichlet Hyperparameters: $k = (30, 30, 30)$											
r_f	0.9799	0.0203	0.9791	0.0208	0.9777	0.0224	0.9751	0.0251	0.9717	0.0286	0.9638	0.0357
s_1	0.1672	0.0532	0.1677	0.0542	0.1686	0.0559	0.1706	0.0596	0.1725	0.0634	0.1780	0.0724
s_2	0.4539	0.0709	0.4535	0.0722	0.4529	0.0743	0.4515	0.0788	0.4501	0.0834	0.4461	0.0940
s_3	0.3789	0.0691	0.3788	0.0703	0.3785	0.0724	0.3779	0.0768	0.3774	0.0813	0.3759	0.0916

From Table 2 the sample information clearly dominates the prior information we assume in (8) and (9).

4.2. Marginal Rate of Substitution

Finally, we can look at the posterior distribution of any transformation of the parameters. For instance, the marginal rate of substitution (MRS) between nondurables, x_2, and services, x_3, when equal amounts of both goods are consumed is given by α_2/α_3. We perform an informal test on the adequacy of the Cobb-Douglas utility function (which assumes constant expenditure shares) by splitting the sample into two groups of 20 observations—$t = 1, \ldots, 20$ and $t = 22, \ldots, 41$, omitting observation 21—and comparing the posterior distribution of this MRS. Figure 3 displays the prior and the two posterior densities of the MRS in the exponential model.

As expected from the evolving shares of nondurables and services on total expenditure along the years, the posterior densities lie very far apart. We take

[7] Under the exponential assumption, the marginal prior density for $r_t = e^{-\varepsilon_t}$ is $p(r_t) = \frac{-1}{r_t \log r^*}\left(1 + \frac{\log r_t}{\log r^*}\right)^{-2}$ for all years.

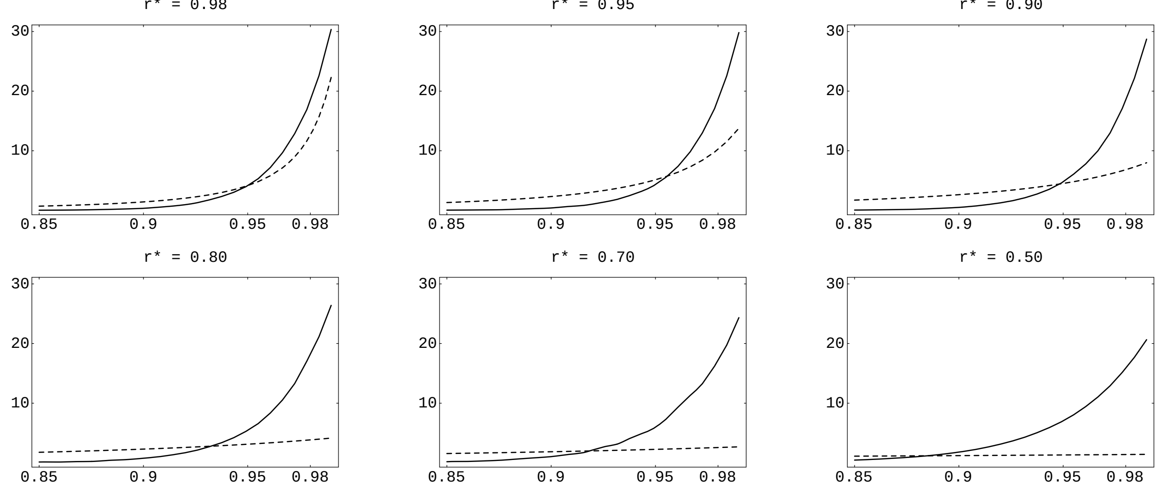

Figure 2. Prior and Posterior Densities of Efficiency: $k = (1, 1, 1)$
(Dashed: Prior; Solid: Posterior)

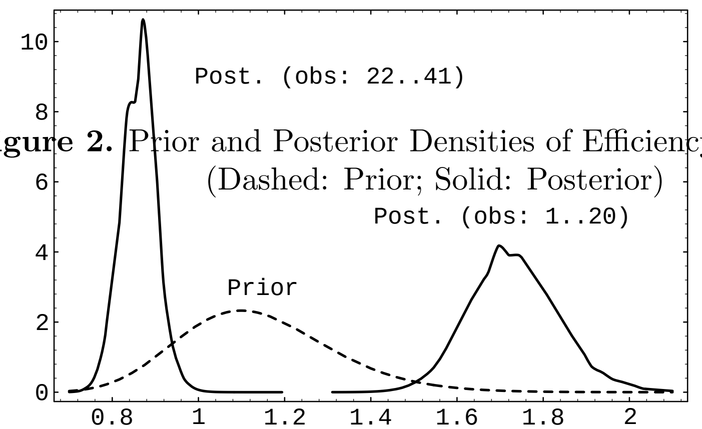

Figure 3. Prior and Posterior Densities of the MRS
($k = (1, 1, 1)$ and $r^* = 0.95$.)

this as clear evidence that the constant-share constraint imposed by the Cobb-Douglas functional form is at odds with these data. As this analysis is merely meant to illustrate efficiency-motivated demand analysis using Bayesian methods we shall, however, not explore more flexible functional forms in this paper. We leave that task for subsequent, more challenging, applications.

5. Concluding Remarks

In this paper we consider a new approach to estimating Demand Systems proposed by Varian (1990). The latter is based on a generalization of Afriat's efficiency index and focuses on using a norm with economic content. The goodness of fit is measured in terms of *economic* proximity in the sense that the distance between within-sample predicted demands and actual demands is determined by the expenditure wasted because of non-optimality. In the context of a Cobb-Douglas illustration, the use of this framework leads to the introduction of a very natural sampling distribution (namely Dirichlet for the budget shares) that automatically respects the nature of the data (budget shares being nonnegative and adding up to one). We then conduct a single-equation analysis using Bayesian methods under a wide range of prior assumptions. Simple importance sampling Monte Carlo proves to be very efficient for the required numerical integration. Results on quantities of economic interest, such as budget shares and efficiencies corresponding to unobserved (future) periods are immediately obtained. The model turns out to be very tractable from a statistical point of view, while possessing quite interesting economic implications.

References

Afriat, Sidney (1967): "The Construction of a Utility Function from Expenditure Data," *International Economic Review*, **8**:1 (February), 67–77.

van den Broeck, Julien, Gary Koop, Jacek Osiewalski and Mark F.J. Steel (1994): "Stochastic Frontier Models: A Bayesian Perspective," *Journal of Econometrics*, **61**:2 (April), 273–303.

Gelfand, Alan E. and Adrian F.M. Smith (1990): "Sampling-Based Approaches to Calculating Marginal Densities," *Journal of the American Statsitical Association*, **85**:2 (June), 398–409.

Geweke, John (1989): "Bayesian inference in econometric models using Monte Carlo integration," *Econometrica*, **57**:6 (November), 1317–1339.

Koop, Gary, Jacek Osiewalski and Mark F.J. Steel (1994): "Bayesian Efficiency Analysis With a Flexible Form: The AIM Cost Function," *Journal of Business and Economic Statistics*, **12**:3 (July), 339–346.

Koop, Gary , Mark F.J. Steel and Jacek Osiewalski (1995): "Posterior Analysis of Stochastic Frontier Models Using Gibbs Sampling," *Computational Statistics*, **10**:4 (December), 353-373.

Ley, Eduardo and Mark F.J. Steel (1992): "Bayesian Econometrics: Conjugate Analysis and Rejection Sampling," in *Economic and Financial Modelling using Mathematica*, ed. Hal R. Varian. New York: Springer-Verlag.

Theil, Henri (1971): "An Economic Theory of the Second Moments of Disturbances of Behavioral Equations," *American Economic Review*, **61**:1 (March), 190–194.

Varian, Hal R. (1990): "Goodness-of-Fit in Optimizing Models," *Journal of Econometrics*, **46**:1/2 (October/November), 125–140.

Varian, Hal R. (1992): *Microeconomic Analysis*. New York: W.W. Norton.