INTRODUCTION

In this paper, we use Canadian cross-section micro-data to formulate a system of household budget-share equations to test some fundamental concepts of the theory of consumer demand behavior. According to Pollak and Wales (1978), a complete system of demand equations is considered to be "theoretically plausible" if it is obtained from a "well-behaved" utility function, i.e., fundamental concepts of consumer demand be satisfied. These fundamental concepts relate to the demand equations being homogeneous of degree zero in prices and total expenditure (or income)[1], and the implied Slutsky matrix being symmetric and negative semi-definite. It is important to test these restrictions since their rejection might imply that the data does not support either the theory of utility maximization behavior or the particular functional form chosen to be estimated [Berndt, Darrough and Diewart (1977)]. In this paper we are mainly concerned with looking at the homogeneity[2] and symmetry restrictions[3] using a third-order translog model.

Many applied demand studies have tested these concepts of consumer demand theory. Barten (1967) used pre-and postwar data for Holland to test the regularity conditions. He rejected symmetry and homogeneity restrictions although a similar study done by Barten using Canadian data did not lead to clear rejection. Lluch (1971) conducted similar study with Spanish data, using variations over provinces rather than time as his basis, and found that homogeneity was rejected; though once homogeneity was imposed symmetry could not be rejected as an additional constraint. Similar results were found by Deaton (1972) for the United Kingdom using a nine-commodity classification with data going back to the beginning of the century. He failed to reject symmetry restrictions apart from its homogeneous content. Parks (1969) used Swedish data and failed to reject the symmetry restrictions although he did not conduct any tests for homogeneity restrictions. Barten (1977), Deaton (1986) and Blundell (1988) used aggregate, time-series data to estimate systems of expenditure or budget-share equations such as the Rotterdam systems of Barten (1969), the translog system of Christensen, Jorgenson, and Lau (1975) and the "Almost (AI) Ideal Demand System" of Deaton and Muellbauer (1980). Most of these studies ended up rejecting the regularity conditions but according to Nicol (1989) these results could have been influenced by misspecification since these systems represented approximation to the unknown functional form. Nicol (1987) presented a homogeneity and symmetry unconstrained variant of the "third-order translog" representing a third-order approximation to an indirect utility function, and found that the regularity conditions are rejected. These results were important since they demonstrated that standard restrictions implied by the theory of consumer behavior could be rejected. However, as pointed out by Nicol (1987), the fact that the exact aggregation conditions under a time-series data were imposed *a priori* the use of aggregate, time-series data may not be appropriate to test the regularity conditions. To avoid this problem, Nicol (1989) presented a similar model using an aggregate, cross sectional data but once again ended up rejecting the regularity conditions.

In this paper, we also test the statistical importance of third-order terms. Many studies such as the one done by Strauss (1982), and Barnes and Gillingham (1984) used cross-sectional data to estimate an expenditure demand system quadratic in form. However, these studies estimated functions, which were approximations to the true, unknown functional forms, meaning that the results of these tests may have been influenced by the adequacy of these approximations. One recent exception has been Hayes (1986), who used a cross-sectional data to examine the significance of the third-order terms over the lower order terms, and found the third-order terms to be statistically significant. The only problem with this study was that Hayes failed to test the regularity conditions, i.e., she tested the statistical significance of the third-order terms using the third-order translog model where symmetry and homogeneity restrictions were already imposed. Nicol (1989) presented a similar model but did not impose the regularity conditions to test the significance of the third-order terms and found that the third-order terms were significant.

In the next section we describe the data used to estimate and test our model. Then we present an outline of the model. Next, the results of estimation and test results are discussed. Finally, we summarize our findings and discuss further areas of research.

Expenditure Data

The data used in our paper are of two types: price and expenditure data. Expenditure data are drawn from the Canadian Family Expenditure Survey [FAMEX] micro-data files for the years 1969, 1974, 1978, 1982, 1984, 1986, 1990, 1992 and 1996. These data files provide information on a large number of expenditure categories for the various

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1 This restriction is also known as homogeneity restrictions (or conditions).
2 According to Brown and Deaton (1972), the homogeneity restrictions follow from the condition that proportional changes in all prices and money income leave the choice of commodities unchanged.
3 Homogeneity and Symmetry restrictions are also known as regularity conditions.

years. Table 1 provides total sample sizes for each FAMEX survey year. These data sets include demographic characteristics of the sample such as household composition, employment status, regional location and housing characteristics.

Table 1: Observations in various years

Years	1969	1974	1978	1982	1984	1986	1990	1992	1996
Observations	15,140	6,630	9,356	10,938	4,792	10,356	4,569	9,492	10,417

These micro-data files contain extensive characteristics on the sample units. It is thus important that we only select the required observations i.e., the sub-samples from each micro-data file based on certain criterion, which corresponds to the type of study undertaken in our paper. This is done because we require certain demographic composition and specific physical location in choosing the spending units for our model.

Regional Characteristics

A major consequence of using the expenditure data is related to the availability of price data - the price data is only available for 10 major "Census Metropolitan Areas" (CMA's) for Canada,[4] but the spending units of our expenditure data have a distribution pattern valid across Canada (rural and urban areas).[5] Thus we have to select the observations from our expenditure data for which the price data are available. Given the available price data, we select spending units from our expenditure data on households. Using the intercity indices for the various years included in our study, we construct regional price indices for each expenditure category. Households are then matched to one of the 14 price vectors,[6] depending on which cross-section and region of residence the household is drawn from. This leads to a reduction in spending units. Also, by including the spending units from five geographic regions, we are able to incorporate regional effects, which can serve as important determinants of household expenditure behavior.

Demographic Composition

Many studies in empirical demand analysis have found that demographic variables can be important determinants of consumer demand. For example, Nicol (1985) found that demographic variables influenced demand when using micro data due to the constantly changing demographic composition of the population. According to Pollak and Wales (1978), the inclusion of demographic variables are desirable in the analysis of household consumption since there are systematic differences in the consumption behavior of households with different demographic characteristics. Our model contains exact aggregation restrictions, which means that any violation of exact aggregation could influence the outcome of hypothesis testing. One way to reduce the importance of these influences would be to extract households with common demographic characteristics. Barnes and Gillingham (1984) found that the demographic variables such as household type, housing tenure, marital status, and number of offspring have a significant impact on demand behavior. To avoid imposing these strong seperability restrictions, we attempt to establish controls on these factors by selecting spending unit comprising married couples of a certain age group only. We can further subdivide the data sets into household homeowners with mortgages *[1M, 2M]* and households who are renters *[1R, 2M]*. This would mean that we would get four data sets, namely *1R, 1M, 2R* and *2M* as shown in table 2, where *R* stands for renters and *M* stands for mortgagees. Thus each data set represents common demographic characteristics.

Table 2: Observations in data sets 1R, 1M, 2R and 2M

Data Set	1969	1974	1978	1982	1984	1986	1990	1992	1996	Total
1R	689	424	478	583	273	483	217	374	300	3,821
1M	286	162	380	419	168	357	189	336	396	2,693
2R	1,696	658	641	655	290	537	245	403	301	5,426
2M	1,629	749	1,439	1,363	483	1,089	525	970	926	9,173
Total	4,300	1,993	2,938	3,020	1,214	2,466	1,176	2,083	1,923	21,113

Expenditure Categories

The data sets described in the previous sub-section provide information on expenditures on many categories of goods and services. We would like to have as many categories of goods and services included as possible, but we are

4 These include Vancouver, Edmonton, Regina, Winnipeg, Toronto, Ottawa, Montreal, St.John, Halifax and St.John's.

5 This is because the expenditure data is available for 5 regions (Atlantic, Quebec, Ontario, Prairies and British Columbia) compared to the price-data which is only available for a selected number of cities representing each of these 5 regions.

6 These represent 14 expenditure categories: Food, Household operation, Furnishings and floor coverings, Clothing, Automobile purchase, Gasoline, Other automobile operation, Public transit, Personal care services, Personal care supplies, Recreation Reading and Education, Tobacco and Alcoholic beverages.

once again limited by the availability of price data in the choice of our categories. Also, there are a limited number of expenditure categories for which the data reflecting the inter-regional price differences is available. For example, the category of 'shelter' is not available with the expenditure categories, nor are the appropriate price data available for this category. We exclude such expenditures from all expenditure categories. We use total expenditure as a proxy to the income variable since it serves as a better representation of permanent income because expenditure figures are obtained by the optimization of lifetime budget by the household. In total, we end up with fourteen spending categories which all add up to the "income variable."

Price data

Price data are available for ten major "Census Metropolitan Areas"(CMA) for Canada which represents the five regions included in our study. The price data are available in index number form, which means we need to determine the appropriate price index for our model. The fourteen spending categories each contain a price vector, which includes the price of the fourteen composite goods. This means that we have two cross-sections of spending units living at different geographical areas at nine different points in time. Thus, the index base we use incorporates both geographical and temporal factors. This is important since we are interested in observing how the expenditure behavior of the spending units fluctuates (positively or negatively) due to relative shifts in prices. We thus end up with five price vectors each representing the five regions allocated to the spending units living in a geographic region over the nine years. Each price vector is matched to a geographic region (where the spending unit lives) and contains fourteen elements each representing the specific spending categories. The Statistics Canada publication "Consumer Prices and Price Indexes"[7] is used to extract the raw price indices needed to construct the price vectors for the respective years being studied. The base year used is the inter-city weighted average retail price for each category of goods and services is September 1978. In years where 1978 is not the base year, we perform arithmetic manipulations[8] to establish 1978 as the base year. The prices are then normalized to unity at their mean value, which is a standard practice used when dealing with functional flexible forms such as ours.

Price Aggregation

The price vectors have fourteen elements, one for each of the expenditure categories, which are then matched with the spending units in our expenditure data. The price indices for the different CMA's are then used to create regional[9] price indices. We create regional prices because only regional rather than city locations of households are available from FAMEX, which is the source of expenditure data used in this paper. These regional or geographic price indices are created using the following equation

$$\log (p^{m}_{j}) = w_1 \log(p^{m}_{1,j}) + w_2 \log(p^{m}_{2,j}) + \ldots + w_k \log(p^{m}_{k,j}) \quad (1)$$

where $m = 1,...,5$ representing the five geographical regions; $j = 1,...,14$ representing the fourteen expenditure categories; and $\Sigma^{k}_{i} w_i = 1$, where w_i refer to the weights. The weights are calculated by taking the ratio of the population of city i to the total sum of the populations of cities in the geographic region for which we have a price index. Thus $1,..,k$ indexes cities within a geographic region, such as the Prairies, Ontario, or the Atlantic provinces.

Aggregation of the price and expenditure data

Since our model deals with estimating budget-share equations, the number of estimable parameters in a budget-share system is directly related to the number of expenditure categories. This is because the price of each expenditure category can be used as an explanatory variable in each of the budget share equations contained in our system of equations. The large number of observation in our data set also means that we need to impose across-equation restrictions when estimating the complete budget share system. The relatively large number of observations and the high correlation between some of the categories also makes it desirable for us to aggregate the categories. The expenditure items are grouped into three aggregate categories: durable goods, non-durable goods, and services. Items included in each of these categories are shown in table 3.[10] To construct budget-share equations for our model, we sum

7 Statistics Canada, Catalogue No. 62-010.
8 Detailed explanation of the arithmetic manipulation used in constructing the price indices is available from the author on request.
9 Price indices for 5 regions are -- British Columbia using Vancouver's prices, Prairie region using Edmonton, Regina and Winnipeg prices, Ontario region using Ottawa and Toronto's prices, Quebec region using Montreal prices and the Atlantic region using St.John's, Halifax and St.John prices.
10 It could be possible to create four instead of three aggregate categories namely durable, semi-durable, non-durable and services, but since we are looking at the higher order systems, we decide to use three categories only. See Nicol (1985) for an explanation on this point.

expenditures for each spending unit or observation and then deflate them by total expenditures. The mean budget shares for each of the aggregate expenditure categories and the mean total expenditure by family reference size are shown in table 4.

Table 3: Expenditure items included in aggregate categories

Aggregate categories	Expenditure items
Durable goods	Furnishings, recreation, automobile purchase, other automobile operation
Non-durable goods	Food, alcoholic beverages, gasoline, tobacco, clothing, floor coverings, reading, personal care supplies
Services	Personal care services, public transportation, education, household operation

Table 4: Mean budget shares and total expenditures by family reference size

Aggregate categories	Durable	Non-durable	Services	Mean total expenditures
1	0.33152	0.50840	0.16008	$14,681
1R	0.31608	0.52495	0.15897	$12,772
1M	0.33291	0.50226	0.16483	$16,538
2	0.28775	0.54847	0.16378	$15,010
2R	0.26219	0.57635	0.16146	$11,469
2M	0.30287	0.53197	0.16515	$17,104

It is necessary to aggregate the fourteen price indices for each geographic regions and cross-section into three categories. The procedure to compute the weighted regional means is similar to the one outlined in the previous subsection, shown as

$$log(p^m{}_j) = w_1\, log(p^m{}_{1,j}) + w_2\, log(p^m{}_{2,j}) + ... + w\_l\, log(p^m{}_{l,j}) \tag{2}$$

where $m = 1,...,5$ representing the five geographical regions and $j = 1,2,3$ representing the three expenditure categories and $\Sigma^k{}_i\, w_i = 1$, and w_i are the weights. The weights are calculated by taking the ratio of expenditures on particular goods in an expenditure category to total expenditures on that category by all spending units in the cross-section. Table A (Appendix A) shows an example for one of the years included in our study.

MODEL

A third-order translog can be interpreted as having the same level of approximation to the indirect utility function as the quadratic expenditure system. To understand this concept better, we define a basic budget share equation as

$$\omega^*{}_n = \{p_n\, q_n\}/\{M\} = \omega^*{}_n\, [ln(p_1),....,ln(p_n),\, ln(M)]; \qquad \forall n = 1,....,N \tag{3}$$

where p_n refers to the price of n goods, and q_n refers to the quantity of n goods and M refers to total expenditure. Here, the indirect utility function for household k can be written as

$$lnV_k = \phi(ln\, P_1,.....,ln\, P_N); \qquad \forall k=1,...,K, \text{ and } P = (P_1,....,P_N) = p_1/M_k,...., p_N/M_k \tag{4}$$

Here P is a $(1\; X\; N)$ price vector of normalized prices of N goods and services. The third order Taylor approximation of lnV_k at $(p^0,M^0{}_k) = \iota^T = (1,...,1)^T$ leads to the third-order[11] translog shown as

$$lnV_k = \phi^0 + \sum_{i=1}^{N} \phi_i\, lnP_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \phi_{ij}\, lnP_i\, ln\, P_j + \frac{1}{6}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{m=1}^{N} \phi_{ijm}\, lnP_i\, ln\, P_j\, lnP_m \tag{5}$$

where $k=1,....,K$ households and $\phi^0, \phi^I, \phi_{ij}, \phi_{ijm}$ represents the evaluation of ϕ, its first, second, and third derivatives respectively at the point of expansion, $(p^0,M^0{}_n) = \iota^T$.

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[11] Throughout this paper ι is used to refer to a unit vector of appropriate dimension.

The logarithmic form of Roy's identity, shown as

$$w_{nk} = \frac{\partial ln\ V / \partial ln\ P_i}{\Sigma_k\ ln\ V / ln\ P_k}\ ; \qquad \forall n = 1,....,N,\ \forall k = 1,....,K \tag{6}$$

can be used to find the n th expenditure share equation.[12] In order to do so, we apply equation (6) to equation (5) to obtain the n th budget-share equation for household k, shown as

$$w^*_{nk} = \frac{p_n\ q_n\ (p,\ M_k)}{M_k} = \frac{\phi_n + \Sigma^N_{j=1}\ \phi_{nj}\ ln\ P_j + \frac{1}{2}\ \Sigma^N_{j=1}\ \Sigma^N_{m=1}\ \phi_{njm}\ ln\ P_jP_m}{\Sigma^N_{i=1}\ [\ \phi i + \Sigma^N_{j=1}\ \phi_{ij}\ ln\ P_j + \frac{1}{2}\ \Sigma^N_{j=1}\ \Sigma^N_{m=1}\ \phi_{ijm}\ ln\ P_jP_m]} \tag{7}$$

$\forall\ n \neq j \neq m = 1,....,K$ for all n and k.

Relaxing Homogeneity

In the previous section, we normalized the prices by equating $P = (P_1,...,P_N) = p_1/M_k,...., p_N/M_k$, which lead to the imposition of homogeneity. Homogeneity restrictions[13] imply that the demand function is a function of relative prices and real total expenditures. Nicol (1993) stated that when using data at the micro-level, the amount of heterogeneity in household's budget shares are much greater than seen with time-series data. This requires the use of homogeneous household groups in estimating the demand models than are normally used for given number of households. We need to relax the homogeneity conditions in order to generate *market* demand functions consistent with individual utility maximization behavior [Berndt, Darrough and Diewart (1977)]. Relaxing the homogeneity restrictions also leads to unrestricted estimation, which can yield significant explanatory powers [Barnes and Gillingham (1984)]. If we ease the homogeneity constraint, we can transform equation (7) to a homogeneity unconstrained third-order translog, which can be written more compactly in matrix form shown as

$$w^{*i}_n = \frac{\alpha_n + B_n\ ln\ P^T + \gamma_n\ ln\ M + L_n\ p^T\ ln\ M + ln\ pC_n\ ln\ p^T + \mu_n(ln\ M)^2}{\iota^T\ [\alpha_n + B\ ln\ P^T + \gamma\ ln\ M + L_n\ ln\ p^T\ M + \mu_n(ln\ M)^2] + \Sigma_m\ ln\ PC^T_m\ ln\ p^T} \tag{8}$$

with appropriate redefinition of parameters and omission of k for notation purposes. The parameter vectors and matrices in equation (8) can be interpreted as being the derivative of the indirect utility function to various orders, with respect to lnP_N and lnM evaluated at ι. Here, B_n and C_n are matrices of dimension (3×3) and (3×6) respectively, and can be interpreted as having elements that are combinations of second derivatives of ϕ, evaluated at a point of approximation. α, L, γ are vectors containing the elements α_n, L_n, γ_n respectively. α_n, L_n, γ_n are vectors of dimension (3×1), (3×1), and (3×1) respectively.

Although equation (8) is a reduced form of equation (7), it is difficult to estimate since numerically it is non-linear in parameters.[14] Some restrictions are needed in order to make it linear in form. We do this to simplify the estimation process. In order to do so, we need to impose few restrictions[15] on equation (8). The first set of restriction required is

$$\iota^T\alpha = 1 \tag{9}$$

This restriction imposes the normalization requirement, which is always required in translog-type systems, since the equations are homogeneous of degree zero in parameters. This restriction is important because if the equations are left with having homogeneity of degree zero, then it will be impossible to estimate their parameters. The second set of restrictions as shown below are needed to define the price parameters empirically

$$\iota^T\ B = 0;\ \Sigma_m\ C_m = 0; \qquad \forall m = 1,.....,N \tag{10}$$

These restrictions ensure that the translog model can be exactly aggregated. The final set of restrictions as shown below Reduces the equation to a linear form

$$\iota^T\gamma = 0;\ \iota^T L = 0;\ \iota^T\ \mu = 0 \tag{11}$$

12 In the theory of demand, price p and expenditure M are exogenous, and quantities demanded are endogenous. Thus if we use Roy's identity we can find demand functions specifying quantity as a function of prices and expenditure, which leads to a reduced form of equation.
13 Simmons and Weiserbs (1979) rejected the homogeneity restrictions using Christensen, Jorgensen and Lau's (1975) data.
14 This problem can be further aggravated due to the large data set being used.
15 These restrictions are also known as adding-up conditions.

After imposing all of the restrictions [equation (9) to (11)], we obtain a linear budget-share equation system,[16] which fulfills the "Fundamental theorem of exact aggregation" put forward by Jorgenson, Lau, and Stroker (1982).[17]

$$w^{*i}_n = \alpha_n + B_n \ln P^T + \gamma_n \ln M + L_n p^T \ln M + \ln pC_n \ln p^T + \mu_n(\ln M)^2 \quad (12)$$

Here $n=1,2,3$ denoting the three expenditure categories: durable, nondurables and services. The above system of equations is estimated using six data sets representing household couples with no children, and households with children respectively ($i = 1, 1M, 2, 2M, 2R$). Thus in total, we end up with six equations which form our model. The estimated parameters are shown in tables B.1-B.6 in Appendix B. We further add a stochastic component ε_n to each equation of the system. This stochastic term[18] helps us in incorporating the random errors in household utility maximizing plans and heterogeneity across agents in our model.

$$w^*_n = w^{*i}_n + \varepsilon_n \quad (13)$$

Equation (13) represents systems of equations, which are all linear in form and where each equation of each system contains identical regressors. This is substantiated by the fact that $\varepsilon = (\varepsilon, ..., \varepsilon_N)^T \sim N(0, \Omega)$, which means that the error term is normally distributed[19] with zero means and Ω covariance matrix, where $rank\ (\Omega) \sim N\text{-}1$. We can use ordinary least square (OLS) method to estimate the model,[20] since all the three equations are linear, have identical regressors and do not contain cross-equation restrictions. We can then use the residuals from $N\text{-}1 = 2$ equations to estimate the log-likelihood and the test statistics corresponding to the computed likelihood ratios. The third equation can be computed by the adding-up condition.

EMPIRICAL ANALYSIS AND TESTING

There is a considerable body of literature available on testing the significance of the third order estimation, and on testing the symmetry[21] and homogeneity conditions on models similar to ours.[22] Christensen Jorgenson and Lau (1975) tested the theory of demand without imposing the assumption of homotheticity and additivity as part of the maintained hypothesis. They performed a series of tests and concluded that for either direct or indirect series of tests, the theory of demand was inconsistent with the evidence for the double logarithmic demand systems.[23] According to them, if the theory of demand was valid, then the double logarithmic form for the system of demand equations would imply that the utility function is linear logarithmic. Although Deaton (1986), found some variations in results regarding the symmetry and homogeneity conditions, he found enough evidence to reject both the restrictions. The evidence was stronger for homogeneity, with less evidence against symmetry over and above the restrictions due to homogeneity. But the problem with above studies was that they all used time-series data, which according to Nicol (1989) can have misspecified the model, affecting the test results. Strauss (1982) and Barnes and Gillingham (1984) used cross-sectional data similar to ours to estimate a quadratic expenditure system and found that higher than first order terms were statistically significant explanatory terms. Hayes (1986) also estimated a similar third-order translog using cross-sectional data and found that the third-order terms were significant but ended up imposing homogeneity and symmetry restrictions without testing them. This is avoided in this paper since we do test for homogeneity and symmetry restrictions. Nicol (1989) used a model similar to ours with Canadian household data for years 1978 and 1982 and performed tests on the significance of third order terms and homogeneity and symmetry restrictions. He estimated log-likelihoods and computed likelihood ratio test statistics and found that third-order terms were statistically important explanatory variables. He also ended up rejecting homogeneity and symmetry restrictions separately and jointly for the third-order translog.

16 Budget share equation refers to a relationship expressing the proportion of expenditure out of income on three expenditure categories (durable, nondurables and services) as a function of all prices and income

17 This theorem deals with a unique solution to the simultaneous estimation of demographic and price effects in demand systems.

18 The adding-up condition implied with the addition of a stochastic term leads to the imposition of over-identifying restrictions.

19 The error term is normally distributed because $\Sigma_n w_n = 1$ and $\Sigma_n \varepsilon_n = 0$.

20 See Zellner (1962) for a discussion on the significance of OLS estimation.

21 Also known as Slutsky condition.

22 According to Deaton (1986), these tests show that homogeneity conditions are strongly rejected, whereas symmetry conditions are rarely rejected over and above the restrictions embodied in homogeneity.

23 For a detailed review of tests of the theory of demand, see Brown and Deaton (1972).

Testing the significance of third order model

According to Simmons and Weiserbs (1979) and Nicol (1985), the omission of higher order terms can lead to misspecifications in models such as translog and Rotterdam systems.[24] In order to obtain the first-order model,[25] we need to impose the following restrictions on equation (13)

$$L_n = 0;\ C_n = 0;\ \gamma_n = 0;\ \forall n = 1,\ldots\ldots,N \tag{14}$$

The test we use to determine the significance of the third order terms is called the "Likelihood Ratio (LR) Test. The results are presented in Table 5. The LR ratio test statistic is calculated as twice the difference of the maximum of the log-likelihood function with and without restrictions and the test statistic is asymptotically distributed as χ^2 with the degrees of freedom equal to the number of independent restrictions imposed

$$2[\ln L\,(\hat{O}) - \ln L\,(\tilde{O})] \sim^{A} \chi^2(q) \tag{15}$$

where $\ln L\,(\hat{O})$ and $\ln L(\tilde{O})$ are estimates of log-likelihood under the null (third-order) and the alternative hypothesis (lower-order), respectively and q is the number of restrictions or degrees of freedom of the test. The null hypothesis (which states that the second and third-order terms are zero) is strongly rejected at a significance level of 0.001 for all the data sets. We find that the upper-tail probability values relating to the test statistics are approximately zero, meaning that rejection of the null hypothesis[26] would even occur at lower significance levels. We also find that the third-order estimation leads to an increase in the likelihood values proving that the third-order terms are significant.[27]

Table 5: Tests for the exclusion of third-order terms

Data Set	Third order	Lower order	Test statistic	D.F	Critical Values
1	11,878.7	11,554.2	649.0	20	45.32
1R	6,902.3	6,780.6	243.5	20	45.32
1M	5,033.8	4,946.2	175.3	20	45.32
2	28,193.3	27,781.4	823.8	20	45.32
2R	10,673.7	10,606.9	133.6	20	45.32
2M	17,612.0	17,255.5	713.0	20	45.32

Note: The critical value used is 45.32 at α level of 0.001 and 20 degrees of freedom. The significance level chosen for testing is 0.001, which according to Davidson and Mackinnon (1993), is a conservative significance level in order to control Type I error (making the mistake of rejecting a null hypothesis when it is true). The only flaw this might create is Type II error (accepting the null hypothesis when it is false) due to the various restrictions leading to a decrease in power. However, Nicol (1989) found that strong rejections of hypothesis would still occur under similar test conditions.

Testing homogeneity

Several researchers such as Byron (1970), Christensen, Jorgenson, and Lau (1975), Deaton (1974) and Nicol (1989) have indicated that their data rejected homogeneity conditions using models similar to ours. Homogeneity conditions mainly reflect the assumptions about individual choice behavior and relates to the fact that the demand equations are each homogeneous of degree zero in total expenditure M, and prices p, or, for a positive α shown as

$$q = q(\alpha M,\ \alpha p) \tag{16}$$

This means that proportional changes in all prices and money income leave the choice of commodities unchanged. To test the homogeneity restrictions, we impose the following homogeneity constraints on equation (14)

$$\iota^T B_n = -\gamma_n,\ L_n = -2\iota^T C_n, \iota^T C_n^{\iota} = \mu,\ \forall n = 1,\ldots,N \tag{17}$$

The results of the LR test as shown in table 6. We find that homogeneity is rejected for all the data sets for the model. This is because the given test statistics for all the data sets (except data set *1M*) exceeds the critical value of the test statistic which is 25.2 at a significance level of 0.005 with 10 degrees of freedom.

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24 See Theil (1965) and Barten (1967).
25 Unlike the higher-order terms, first order translog is only a first-order Taylor series approximation to the unknown budget-share system.
26 The null hypothesis states that the third-order terms are not significant.
27 See Pollak and Wales (1992) for a similar study.

Table 6: Test of homogeneity restrictions of the third-order translog

Data Set	Unrestricted	Restricted	Test statistic	D.F
1	11,878.70	11,862.80	31.80	10
1R	6,902.31	6,888.56	27.50	10
1M	5,033.84	5,024.65	18.38	10
2	28,193.30	28,164.70	57.20	10
2R	10,673.70	10,659.70	28.00	10
2M	17,612.00	17,581.10	61.80	10

Testing symmetry

To test the symmetry restrictions on our model without imposing homogeneity restrictions, we require the following restrictions on equation (13)

$$B_{nj} = B_{jn};\ C_{njm} = C_{nmj} = C_{jnm},\ \forall n \neq j \neq m \neq 1,...,N \qquad (18)$$

We find that symmetry is rejected at a significance level of 0.005 for all data sets. This is because the given test statistics for all the data sets (except data set *2R*) exceeds the critical value of the test statistic which is 29.8 at a significance level of 0.005 with 13 degrees of freedom as shown in table 7.

Table 7: Test of symmetry of the third-order translog

Data Set	Unrestricted	Restricted	Test statistic	D.F
1	11878.70	11788.30	180.80	13
1R	6902.31	6879.71	45.20	13
1M	5033.84	4997.66	72.36	13
2	28193.30	28136.80	113.00	13
2R	10673.70	10667.50	12.40	13
2M	17612.00	17579.40	65.20	13

Testing symmetry and homogeneity

In the previous sections we imposed homogeneity and symmetry conditions separately. These restrictions are not independent, we can combine the restrictions to impose the homogeneity and symmetry restrictions. We find that a joint hypothesis of homogeneity and symmetry is rejected at a significance level of 0.005 (critical value of test-statistic is 40.0) for all the data sets (except for data set *2R*) as shown in table 8.

Table 8: Test of homogeneity and symmetry of the third-order translog

Data Set	Unrestricted	Restricted	Test statistic	D.F
1	11,878.70	11,813.1	131.20	20
1R	6,902.31	6,864.72	75.18	20
1M	5,033.84	4,994.23	79.22	20
2	28,193.30	28,063.60	259.40	20
2R	10,673.70	10,661.50	24.40	20
2M	17,612.00	17,487.90	248.20	20

Testing violation of homoskedasticity

Heteroskedasticity is a violation of one of the assumptions of "Classical Linear Regression Model" (CLRM) called homoskedasticity. Prais and Houthakker (1955) found that while analyzing family spending patterns, there was a greater variation in expenditure on certain commodity groups among high-income families compared to the low-income families due to the greater discretion allowed by higher incomes. Greene (1993), also stated that data aggregation which is common while analyzing household consumption similar to our model could also cause heteroskedasticity. In our model, we study the same cross-sectional unit (married households in Canada) over time while dealing with the household units at a given point, which means that we could encounter the problem of hetroskedasticity.

Thomas (1985) stated that the assumption of homoskedasticity is most likely to be violated when there is a large variation in the sizes of *n* values of the explanatory variable. To explain this concept, he put forward the following example. Let nX refer to disposable incomes (total expenditure M in our case) of a cross-section of households, some with incomes close to subsistence levels, others with much higher incomes. The consumption expenditure of the low-income group is not likely to depart from their mean values $E(X)$ due to the fact that they do not possess the savings to indulge in higher than average expenditures (nor are they likely to spend below average since that would refer to consuming below subsistence levels). This means that the disturbances for the low-income households will be small leading to smaller variances. On the other hand, high-income households who have the savings to have higher than average expenditures will have larger disturbances leading to bigger variances. Thus in general, the error terms ε_i will not tend to be constant but will tend to vary directly with the size of the explanatory variable in cross-sectional studies like our dealing with a large variations in the sample households.

Heteroskedasticity could be a serious problem affecting the validity of our tests and statistical inferences. Although some studies such as the one put forward by Golderberger and Gamaletsos (1970) neglected this type of disturbance in covariance matrix, they pointed out that this only reduced the efficiency of the parameters and did not distort their expectations. But recent studies have found that heteroskedasticity can have far reaching consequences on the hypothesis testing of the model. It can mean that the test results we have obtained so far might not be correct (i.e., robust) [Nicol (1989)]. It thus becomes highly necessary for us to test for the heteroskedastic errors and devise some process to deal with the errors. According to Nicol (1989), this problem of heteroskedasticity can take the form of $\sigma^2_{nk} = \sigma^2_n(\ln M_k)^2$ for the third-order estimation. We use Goldfeld-Quandt test to test for heteroscedastic errors.

Goldfeld-Quandt test

Goldfeld-Quandt test [Goldfeld and Quandt (1965)] can be used to find out if *a priori* information regarding a particular independent variable causing heteroskedastic errors exists i.e., the heteroskedastic variance is related to one of the explanatory variables in the regression model. We wrote a SHAZAM program to conduct this test and found that there was an evidence of heteroskedastic errors in the model. We obtain a p-value of 0.01451, which mean that we can reject the assumption of homoscedasticity at a significance level of 0.05. The disadvantage of using this test is that if the form of heteroskedasticity is specified wrong, then the test will not be effective. However, we know the form of heteroskedasticity, which means that we are not affected by this problem.

Heteroskedastic Consistent Covariance Matrix Estimator (HCCME)

Since our model suffers from heteroskedastic errors, the need arises to estimate the variance-covariance matrix of the OLS estimator consistently, which would allow for correct hypothesis tests and asymptotically valid inferences. It is possible to employ an HCCME,[28] even when not much is known about the form of the skedastic function [Davidson and Mackinnon (1993)]. This idea of HCCME was put forward by White (1980),[29] who claimed that an appropriate estimator of the variance-covariance matrix could be obtained even if the model suffers from heteroskedasticity of an unknown form. The conventional variance-covariance matrix of the OLS estimator $\hat{\beta}$

$$V(\hat{\beta}) = \sigma^2(X^TX)^{-1} \tag{19}$$

However, if the model suffers from heteroskedastic errors, then $\mu \sim N(0,\Omega)$, where Ω ($n \times n$ matrix) is the variance-covariance matrix of the vector of error terms μ. Because of hetroskedastic errors, the variance-covariance matrix of $\hat{\beta}$ is significantly different from the conventional estimator, and can be shown as

$$V(\hat{\beta}) = \frac{(X^TX)^{-1}X^T}{\Omega X(X^TX)^{-1}} \tag{20}$$

In general, heteroskedasticity is the assumption that Ω is some diagonal matrix instead of $\sigma^2 I$ found in homoskedastic case. The normality condition under heteroskedasticity is shown as $\hat{\beta}_n \sim N(\beta_0, (X'X)^{-1}X'\Omega X\,(X'X)^{-1}$ instead of $\hat{\beta}_n \sim N[\beta_0, \sigma\,(X'X)^{-1}]$. Since Ω is an $n{\times}n$ matrix with n unknown parameters in the case of heteroskedasticity, we can still find estimators of $V\hat{\beta}$ using the HCCME method. Here the matrix Ω with n unknown parameters is not estimated but rather a fixed matrix is estimated shown as.

$$(\hat{X}^T\hat{X})^{-}1(\hat{X}^T\Omega\hat{X})\,(\hat{X}^T\hat{X})^{-1} \tag{21}$$

28 According to Davidson and Mackinnon (1993), one must be cautious when using HCCME when `n' is not large since there is a great deal of evidence that HCCME is unreliable in small samples. Since we have a fairly large sample, we can safely discount this problem.
29 Precursors of White's paper include Eicker (1967) and Hinkley (1977).

FINAL RESULTS

To find out whether the test results put forward in tables 6-7 are robust to heteroskedastic errors, the implied hypothesis is tested again allowing the data to adjust for heteroscedastic errors using HCCME method. This is done by estimating with a 'Broyden-Fletcher-Goldfarb-Shano' (BFGS) algorithm as implemented in the nonlinear method in SHAZAM (White, 1978). We use the "Wald test" [Wald (1943)] or the "Wald χ^2 test" which is one of the three "Classical" test statistics,[30] to perform the various tests using heteroskedastic consistent variances. We first test for the significance of the third-order terms and then test for the regularity conditions. In order to do this we use Generalized Moments of Mean (GMM) method to obtain consistent (i.e., heteroskedastic consistent) parameters estimates. We use Wald test instead of the original Log-Likelihood test because the Wald test is the most appropriate test to be used with GMM estimation.[31] It is computationally the most attractive test compared to the other classical tests. This is because the Wald test statistic requires the calculation of only the unrestricted model whereas Langrange multiplier principle requires the estimation of the restricted and the unrestricted model and the Likelihood-ratio test requires both the restricted and the unrestricted estimates of β to be calculated. Wald test constructs a test statistic based on unrestricted parameters estimates and an estimate of the unrestricted covariance matrix. Davidson and Mackinnon (1993) provides an example (on the assumption that the restrictions are linear)[32] of the Wald-test. The Wald test statistic can also be shown as

$$[R\hat{\beta} - d]^T \, [R\hat{\beta} - d] \, [R\hat{V}\hat{\beta}\}) \, R^T]^{-1} \sim \chi^2(d); \text{ where } \hat{V}\hat{\beta} \cong \sigma^2 \, (\hat{X}^T\hat{X})^{-1} \qquad (22)$$

Equation (22) is asymptotically normally distributed as a central $\chi^2(q)$, where q refers to the degrees of freedom. Here $\hat{V}[\hat{\beta}]$ refers to an asymptotically valid estimator $V[\hat{\beta}]$, which would be the HCCME in our case. Under Wald test, if the null hypothesis is true, then $R\beta$ should be close to $R\beta=d$. Thus if we obtain a value of $R\hat{\beta} - d$ which if far from zero, then there is some evidence against the null hypothesis. We repeat the tests using HCCME and present the results in table (9)-(12).

Table 9: Wald test of the significance of third-order translog using HCCME

Data 1	Data 1R	Data 1M	Data 2	Data 2R	Data 2R	Data 2M
χ^2	703.48	146.70	183.38	367.17	122.89	341.50
p-value	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

We find that the test statistic reported by the table 9 exceeds the critical value of 45.32 at significance level of 0.001. This means that we can reject the null hypothesis that the third-order terms can be excluded, i.e., the null hypothesis which states that the third-order terms are not significant, is rejected. The reported p-values of 0.00000 for all the data sets also means that we can reject the null-hypothesis at a significance level lower than 0.001 for all the data sets.

Table 10: Wald test of the homogeneity restriction of the third-order translog using HCCME

Data 1	Data 1R	Data 1M	Data 2	Data 2R	Data 2R	Data 2M
χ^2	7.83	2.69	2.93	7.44	6.67	8.21
p-value	0.01998	0.26078	0.23050	0.02422	0.03554	0.01646

Table 10 shows that we fail to reject the null hypothesis that homogeneity restrictions hold, for all data sets using third-order translog model at a significance level of 0.001.

Table 11 Wald test of symmetry restrictions of the third-order translog using HCCME

Data 1	Data 1R	Data 1M	Data 2	Data 2R	Data 2R	Data 2M
χ^2	135.43	31.89	23.06	53.95	7.93	63.84
p-value	0.00000	0.00042	0.01051	0.00000	0.63547	0.00000

Looking at table 11, we fail to reject the null hypothesis that symmetry restrictions hold, for data sets *1M* and *2R* using third-order model at a significance level of 0.001, but reject the null hypothesis for data sets *1, 1R, 2* and *2M.*

30 The three classical tests are Wald test, Langrange Multiplier test, and Likelihood Ratio test.
31 Davidson and Mackinnon (1993) stated that both these tests were "asymptotically equivalent" meaning that if the sample size was large and the hypothesis being tested was either true or almost true, then the test statistics of the same null hypothesis based on these two tests would yield the same results.
32 The restrictions shown in the above equation are linearly distributed for simplicity reasons.

Table 12: Wald test of the homogeneity and symmetry restrictions of the third-order translog using HCCME

Data 1	Data 1R	Data 1M	Data 2	Data 2R	Data 2R	Data 2M
χ^2	135.90	35.77	23.33	79.21	17.69	87.98
p-value	0.00000	0.00035	0.02507	0.00000	0.12546	0.00000

Looking at table 12, we find that we fail to reject the null hypothesis that homogeneity and symmetry restrictions hold (joint hypothesis), for data sets *1M* and *2R* using third-order model at a significance level of 0.001, but reject the null hypothesis for data sets *1, 1R, 2* and *2M*.

CONCLUSION

In this paper, we estimated a third-order translog system of demand equations using Canadian cross-sectional micro-data using family size stratification. We then test this model for the significance of the third-order terms and the significance of the homogeneity and symmetry restrictions (these are also known as regularity conditions. We find that the third-order terms are significant and the regularity conditions are rejected for the third-order translog model for most of the data sets. As mentioned before, these regularity conditions forms the basic properties of the theory of demand and their rejections might imply problems with the data not being able to support the theory of demand or the functional form used in the model not being the correct form. But our model suffers from heteroscedastic errors and we use HCCME to deal with this problem. Using the HCCME, we once again test the model for the significance of third order terms and the regularity conditions. The third-order terms are once again found to be significant. But for the regularity conditions, we obtain different results than before. The homogeneity restrictions are rejected for the third-order translogs for all the data sets. Symmetry restrictions for the third-order translog are rejected for all data sets except for data sets 1M and 2R. A joint hypothesis test for homogeneity and symmetry restrictions for the third-order translog reveals that the null hypothesis is rejected for all data sets except for *1M* and *2R*

Thus our study reveal mixed results in testing the regularity conditions. In regard to the significance of the third-order terms our test results agree with the previous studies done such as the one done by Nicol (1989), i.e., third-order terms are found to be significant. There can be a number of reasons why we reject the regularity conditions for some of the data sets. As mentioned before, it is important to test these regularity conditions, since their rejection might imply that our data does not support either the theory of utility maximization or the particular functional form chosen by us to be estimated is not the right one. Rejection of the regularity conditions can be caused by a number of reasons. Nicol (1989) stated that rejection of regularity conditions can occur if we approximate the unknown demand functions with first-order approximation, which can lead to problems in model specification giving us wrong test results. Nicol (1991) also stated that the rejection of the regularity conditions could possibly occur even at a general approximation. This means that higher than the third-order approximations might be the right form of functional form to use. But using higher than the third-order approximations can increase the number of parameters which can complicate the computational process. The one way in which we can deal with this problem is by finding out the "neighbourhood" of observation space where the regularity conditions can hold for all data sets as pointed out by Nicol (1989). Future research needs to be done in this area. Another plausible reason for the rejections of the regularity conditions for some of the data sets could be the violation of the aggregation conditions due to the adding-up restrictions. This could potentially influence the regularity conditions. Nicol (1989) points out to the studies done by Browning (1987), which suggested that the representative household should be conditioned on the number of demographic variables. This area needs to be researched more to see that if the regularity conditions are directly influenced by not correctly imposing the aggregation conditions.

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APPENDIX A

Table A: Geographically Aggregated Price Indices on Fourteen Expenditure Categories for Five Regions (1996) with 1978 as the base year

Expenditure Category	Atlantic	Quebec	Ontario	Prairies	British Columbia
Food	222.8	212.1	215.1	211.3	227.1
Household Operation	242.4	230.9	244.9	214.1	235.6
Household Furnishings	205.7	197.0	198.6	191.0	199.0
Automobile Purchase	264.2	253.2	254.1	244.6	253.2
Clothing	215.4	210.6	212.7	207.1	210.6
Gasoline	266.7	276.6	257.7	245.6	274.0
Other Automobile Operation	243.9	252.2	306.9	218.7	365.6
Public Transportation	390.7	375.7	409.0	356.4	379.6
Health	306.7	299.0	290.8	272.1	257.1
Personal Care Services	236.7	294.7	247.9	247.9	258.2
Personal Care Supplies	204.4	195.6	198.9	193.1	193.7
Recreation, Reading, Education	263.8	245.2	262.4	241.3	255.3
Tobacco	490.7	319.1	320.4	532.5	555.5
Alcohol	325.8	311.5	316.6	283.8	345.8

APPENDIX B

TABLE B.1: Unrestricted third-order translog, data set 1

Parameters	*Estimates*	*Standard Errors*	*t-statistic*
α_1	-0.3526	0.2723	-1.2946
B_{11}	0.2969	0.3150	0.9425
B_{12}	-0.2425	0.2806	-0.8641
B_{13}	0.2498	0.2370	1.0538
γ_1	-0.0716	0.0598	-1.1977
L_{11}	-0.0852	0.0230	-2.8448
L_{12}	0.0394	0.0291	1.3526
L_{13}	-0.0587	0.0244	-2.4044
C_{111}	-0.1384	0.0674	-2.0545
C_{112}	0.0966	0.2709	0.3565
C_{113}	0.5699	0.1976	2.8839
C_{122}	-0.2384	0.1447	-1.6482
C_{133}	-0.2758	0.1039	-2.6551
C_{123}	0.2492	0.1463	1.7033
μ_1	0.0197	0.0033	5.8915
α_2	1.4182	0.2367	5.9911
B_{21}	0.0708	0.2738	0.2585
B_{22}	0.1150	0.2439	0.4713
B_{23}	-0.2858	0.2060	-1.3874
γ_2	-0.0295	0.0520	-0.5671
L_{21}	0.0340	0.0260	1.3068
L_{22}	-0.0225	0.0253	-0.8875
L_{23}	0.0542	0.0212	2.5542
C_{211}	0.0797	0.0586	1.3614
C_{212}	0.1338	0.2355	0.5682
C_{213}	-0.6176	0.1718	-3.5956
C_{222}	0.0653	0.1258	0.5190
C_{233}	0.2856	0.0903	3.1619
C_{223}	-0.1558	0.1272	-1.2249
μ_2	-0.0104	0.0029	-3.5676
α_3	-0.0657	0.1450	-0.4527
B_{31}	-0.3677	0.1677	-2.1919
B_{32}	0.1275	0.1494	0.1494
B_{33}	0.0361	0.1262	0.2859
γ_3	0.1011	0.0319	3.1748
L_{31}	0.0512	0.0159	3.2090
L_{32}	-0.0169	0.0155	-1.0913
L_{33}	0.0045	0.0130	0.3457
C_{311}	0.0587	0.0359	1.6357
C_{312}	-0.2304	0.1443	-1.5970
C_{313}	0.0478	0.1052	0.4537
C_{322}	0.1732	0.0770	2.2478
C_{333}	-0.0097	0.0553	-0.1754
C_{323}	-0.0934	0.0779	-1.1990
μ_3	-0.0093	0.0018	-5.2396

Table B.2: Unrestricted third-order translog, data set 1R

Parameters	*Estimates*	*Standard Errors*	*t-statistic*
α_1	-0.4139	0.4389	-0.9428
B_{11}	2.8656	0.4060	7.0588
B_{12}	-1.7673	0.3683	-4.7986
B_{13}	-0.5452	0.3507	-1.55
γ_1	-0.1117	0.1046	-1.0672
L_{11}	-0.3218	0.0421	-7.6471
L_{12}	0.1563	0.0412	3.7917
L_{13}	0.0531	0.0389	1.3674
C_{111}	-0.2367	0.0802	-2.9523
C_{112}	0.3836	0.3280	1.1694
C_{113}	0.1578	0.2438	0.6470
C_{122}	0.0750	0.1694	0.4427
C_{133}	0.0532	0.1260	0.4226
C_{123}	-0.3019	0.1712	-1.7641
μ_1	0.0245	0.0063	3.8702
α_2	1.1706	0.3913	2.9913
B_{21}	-1.0571	0.3620	-2.9206
B_{22}	1.6200	0.3284	4.9338
B_{23}	-0.8000	0.3126	-2.5591
γ_2	0.0492	0.0933	0.5272
L_{21}	0.1389	0.0375	3.7026
L_{22}	-0.1449	0.0367	3.9432
L_{23}	0.0737	0.0346	2.1290
C_{211}	0.1183	0.07146	1.6551
C_{212}	-0.3387	0.2924	-1.1582
C_{213}	-0.0580	0.2174	-0.2669
C_{222}	-0.1195	0.1510	-0.7913
C_{233}	-0.1196	0.1123	-1.0653
C_{223}	0.4027	0.1526	2.6392
μ_2	-0.0159	0.0057	-2.8112
α_3	0.2433	0.2345	1.0375
B_{31}	-1.8086	0.2169	-8.3400
B_{32}	0.1473	0.1967	0.7487
B_{33}	1.3452	0.1873	7.1819
γ_3	0.0625	0.0625	1.1179
L_{31}	0.1829	0.0225	0.0225
L_{32}	-0.0114	0.0220	-0.5170
L_{33}	-0.1268	0.0207	-6.1132
C_{311}	0.1184	0.0428	2.7646
C_{312}	-0.0449	0.1752	-0.2561
C_{313}	-0.0998	0.1303	-0.7658
C_{322}	0.0445	0.0905	0.4919
C_{333}	0.0664	0.0673	0.9868
C_{323}	-0.1008	0.0914	-1.1024
μ_3	-0.0086	0.0034	-2.5534

TABLE B.3: Unrestricted third-order translog, data set 1M

Parameters	*Estimates*	*Standard Errors*	*t-statistic*
α_1	-3.6652	0.6001	-6.1080
B_{11}	-1.8303	0.5915	-3.0941
B_{12}	1.6662	0.5278	3.1568
B_{13}	-0.9110	0.4967	-1.8343
γ_1	0.7490	0.1395	5.3691
L_{11}	0.2121	0.0641	3.3094
L_{12}	-0.2267	0.0591	-3.8327
L_{13}	0.0862	0.0554	1.5551
C_{111}	-0.5476	0.1206	-4.5422
C_{112}	0.4426	0.4422	1.0009
C_{113}	0.2326	0.3185	0.7303
C_{122}	0.1305	0.2402	0.5435
C_{133}	-0.0189	0.1730	-0.1094
C_{123}	-0.1778	0.2609	-0.6813
μ_1	-0.0311	0.0082	-3.7856
α_2	2.5762	0.5085	5.0662
B_{21}	1.2075	0.5013	2.4088
B_{22}	-0.7676	0.4473	-1.7161
B_{23}	-0.0508	0.4209	-0.1207
γ_2	-0.3111	0.1182	-2.6316
L_{21}	-0.1419	0.0543	-2.6127
L_{22}	0.1364	0.0501	2.7225
L_{23}	0.0050	0.0470	0.1067
C_{211}	0.3721	0.1022	3.6417
C_{212}	-0.3797	0.3748	-1.0132
C_{213}	-0.0798	0.2699	-0.2958
C_{222}	-0.0667	0.2035	-0.3276
C_{233}	0.0706	0.1466	0.4813
C_{223}	-0.0155	0.2211	-0.0670
μ_2	0.0068	0.0070	0.9839
α_3	2.0890	0.3232	6.4639
B_{31}	0.6228	0.3186	1.9549
B_{32}	-0.8987	0.3186	1.9549
B_{33}	0.9618	0.2675	3.5958
γ_3	-0.4379	0.0751	-5.8286
L_{31}	-0.0702	-0.0702	-2.0338
L_{32}	0.0902	0.0318	2.8327
L_{33}	-0.0912	0.0299	-3.0551
C_{311}	0.1756	0.0649	2.7037
C_{312}	-0.0629	0.2382	-0.2642
C_{313}	-0.1528	0.1715	-0.8907
C_{322}	-0.0639	0.1294	-0.4936
C_{333}	-0.0516	0.0932	-0.5542
C_{323}	0.1932	0.1405	1.3750
μ_3	0.0242	0.0044	5.4809

TABLE B.4: Unrestricted third-order translog, data set 2

Parameters	*Estimates*	*Standard Errors*	*t-statistic*
α_1	0.9813	0.1230	7.9796
B_{11}	0.0086	0.2217	0.0386
B_{12}	-0.5015	0.1913	-2.6216
B_{13}	0.6113	0.1988	3.0748
γ_1	-0.3373	0.0257	-13.1200
L_{11}	-0.0125	0.0244	-0.5128
L_{12}	0.0378	0.2158	1.7529
L_{13}	-0.0741	0.2230	-3.3218
C_{111}	-0.2166	0.3641	-5.9490
C_{112}	0.3190	0.1366	2.3348
C_{113}	0.0808	0.1063	0.7603
C_{122}	0.0118	0.0710	0.1657
C_{133}	0.0808	0.6219	1.2996
C_{123}	-0.2066	0.8634	-2.3923
μ_1	0.03091	0.0014	21.935
α_2	0.0141	0.1084	0.1304
B_{21}	0.0997	0.1954	0.5102
B_{22}	0.6494	0.1687	3.8508
B_{23}	-0.7976	0.1753	-4.5502
γ_2	0.2853	0.0227	12.5870
L_{21}	0.0014	0.0215	0.0650
L_{22}	-0.0460	0.0190	-2.4186
L_{23}	0.0836	0.0197	4.2521
C_{211}	0.1236	0.0321	3.8504
C_{212}	-0.3528	0.1205	-2.9291
C_{213}	0.0763	0.0937	0.8145
C_{222}	-0.0119	0.0626	-0.19029
C_{233}	-0.1211	0.0549	-2.2084
C_{223}	0.2017	0.0761	2.6492
μ_2	-0.0268	0.0012	-21.5090
α_3	0.0045	0.0671	0.0676
B_{31}	-0.1082	0.1210	-0.8949
B_{32}	-0.1479	0.1044	-1.4169
B_{33}	0.1862	0.1085	1.7163
γ_3	0.0520	0.0140	3.7089
L_{31}	0.0111	0.0133	0.8346
L_{32}	0.0082	0.0118	0.6950
L_{33}	-0.0095	0.0122	-0.7822
C_{311}	0.0930	0.0199	4.6810
C_{312}	0.0338	0.0746	0.4535
C_{313}	-0.1571	0.0580	-2.7089
C_{322}	0.0001	0.03876	0.0038
C_{333}	0.0038	0.0339	1.1860
C_{323}	0.0049	0.0471	0.1041
μ_3	-0.0042	0.0008	-5.4462

TABLE B.5: Unrestricted third-order translog, data set 2R

Parameters	*Estimates*	*Standard Errors*	*t-statistic*
α_1	0.4827	0.3904	1.2363
B_{11}	-0.0529	0.3256	-0.1625
B_{12}	-0.0098	0.2873	-0.0340
B_{13}	0.0262	0.2865	0.0915
γ_1	-0.2118	0.0940	-2.2546
L_{11}	0.0066	0.0364	0.1804
L_{12}	-0.0119	0.0325	-0.3671
L_{13}	-0.0151	0.0324	-0.4665
C_{111}	-0.1033	0.0492	-2.1001
C_{112}	0.0663	0.1761	0.3763
C_{113}	0.0559	0.1368	0.4083
C_{122}	0.1356	0.0929	1.4600
C_{133}	0.1029	0.0847	1.2143
C_{123}	-0.2262	0.1227	-1.8437
μ_1	0.0227	0.0057	3.9623
α_2	0.6690	0.3612	1.8521
B_{21}	0.1120	0.3013	0.3718
B_{22}	0.4291	0.2658	1.6145
B_{23}	-0.4403	0.2651	-1.6609
γ_2	0.1298	0.0869	1.4930
L_{21}	-0.0094	0.0337	-0.2794
L_{22}	-0.0232	0.0300	-0.7711
L_{23}	0.0487	0.0300	1.6222
C_{211}	0.0380	0.0455	0.8358
C_{212}	-0.0414	0.1629	-0.2542
C_{213}	-0.0224	0.1266	-0.1766
C_{222}	-0.1921	0.0859	-2.2358
C_{233}	-0.1246	0.0784	-1.5893
C_{223}	0.2850	0.1135	2.5103
μ_2	-0.0173	0.0053	-3.2788
α_3	-0.1516	0.2245	-0.6743
B_{31}	-0.0591	0.1876	-0.3151
B_{32}	-0.4193	0.1655	-2.5343
B_{33}	0.4141	0.1651	2.5088
γ_3	0.0821	0.0541	1.5160
L_{31}	0.0028	0.0210	0.1355
L_{32}	0.0351	0.0187	1.8759
L_{33}	-0.0335	0.0187	-1.7958
C_{311}	0.0652	0.0283	2.3033
C_{312}	-0.0248	0.1014	-0.2450
C_{313}	-0.0335	0.0789	-0.4252
C_{322}	0.0565	0.0535	1.0567
C_{333}	0.0217	0.0488	0.4447
C_{323}	-0.0588	0.0707	-0.8314
μ_3	-0.0053	0.0033	-1.6121

TABLE B.6: Unrestricted third-order translog, data set 2M

Parameters	*Estimates*	*Standard Errors*	*t-statistic*
α_1	1.0959	0.1567	6.9931
B_{11}	0.1360	0.3377	0.4028
B_{12}	-1.041	0.2751	-3.7852
B_{13}	1.0064	0.2946	3.4158
γ_1	-0.3652	0.0308	-11.8450
L_{11}	-0.0351	0.0369	-0.9516
L_{12}	0.0955	0.0309	3.0861
L_{13}	-0.1144	0.0329	-3.4811
C_{111}	-0.3007	0.0512	-5.8677
C_{112}	0.5155	0.1976	2.6096
C_{113}	0.1078	0.1536	0.7018
C_{122}	-0.0714	0.1021	-0.6986
C_{133}	0.0926	0.0876	1.0572
C_{123}	-0.2504	0.1192	-2.1011
μ_1	0.0329	0.0016	20.5830
α_2	-0.1427	0.1349	-1.0578
B_{21}	0.1559	0.2906	0.5367
B_{22}	0.8525	0.2368	3.6004
B_{23}	-1.0087	0.2536	-3.9785
γ_2	0.3146	0.0265	11.8570
L_{21}	0.0014	0.0317	0.0435
L_{22}	-0.0679	0.0267	-2.5492
L_{23}	0.1032	0.0283	3.6499
C_{211}	0.1867	0.0441	4.2329
C_{212}	-0.5904	0.1700	-3.4729
C_{213}	0.1565	0.1322	1.1839
C_{222}	0.1041	0.0879	1.1843
C_{233}	-0.1487	0.0754	-1.9731
C_{223}	0.1972	0.1025	1.9231
μ_2	-0.0281	0.0014	-20.4720
α_3	0.0497	0.0252	1.9703
B_{31}	0.0374	0.0391	0.9562
B_{32}	-0.1889	0.0608	-3.1073
B_{33}	0.0818	0.0448	1.8257
γ_3	0.0331	0.0037	8.9674
L_{31}	0.0003	0.0023	0.0139
L_{32}	0.0133	0.0037	3.5595
L_{33}	-0.0029	0.0028	-1.043
C_{311}	0.0109	0.0272	0.4002
C_{312}	-0.0288	0.0816	-0.3532
C_{313}	-0.0167	0.0678	-0.2469
C_{322}	0.1050	0.0581	1.8073
C_{333}	0.0135	0.0369	0.3659
C_{323}	-0.0606	0.0585	-1.0355
μ_3	-0.0026	0.0002	-11.5820