METRON – *International Journal of Statistics*
1993, vol. LI, n. 1-2, pp. 83-98

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A fresh look at the topical interest of the Gini concentration ratio (**)

CONTENTS: 1. Introduction. — 2. Considerations on the topical interest of the Gini index. — 3. Some final remarks. References. Summary. Riassunto. Key words and phrases.

1. INTRODUCTION

Querying the topical interest of the R Gini concentration ratio 79 years after its appearance in literature (Gini, 1914) could seem rather useless and a little out of place.

However, answering such a query is really less simple and obvious than we assume and it should be thought about carefully.

The various, sometimes complex, aspects should be taken into account, the temporal evolution for example which, in our opinion, is worthy of a particular mention.

Some authors maintain that the concentration ratio dates back to before 1914 as they may be misled by the fact that the index can be expressed as relative mean difference which was introduced into statistics as a variability measure by Gini in 1912.

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(**) Extended version of the paper presented at the 49th Session of the I.S.I. (Florence, Aug. 25-Sept. 2, 1993) during the Selected Topic Meeting ST10-The continuing impact of Gini and Pareto on statistical analysis of inequality.

The majority of the bibliographic references until 1990 quoted here is in Giorgi (1990). Further references are specified at the end of the paper and constitute an *integration* and an *up-date* of the bibliography on the Gini index which recently appeared in this journal (Giorgi, 1990).

Nevertheless, reference should not be made to this year as the concentration ratio is unequivocally proposed by Gini in the article "On the measure of the concentration and variability of characters" presented at the meeting of the Royal Venetian Institute for Science, Letters and Arts on 29 March 1914.

On the other hand it is Gini himself who, in 1930, at the 29th Session of the International Statistical Institute held in Tokyo, in his "Remarks on Prof. L. von Bortkiewicz's paper" maintains (Gini, 1931, p. 305) *"... in 1914 I proposed the concentration ratio showing contemporarily the relations between this index and the Lorenz curve and the mean difference"*.

2. Considerations on the topical interest of the Gini index

The Gini index has always provoked contrasting opinions and it is the link with the mean difference which directly or sometimes indirectly drags it into an old argument about the use of some variability and concentration indices, and about the paternity of some of the related theoretical results. On this point we can very briefly say that it begins with the objection made by Bresciani Turroni (1916) concerning Gini's preference (1914) for the mean difference rather than other summary indices. To be honest Gini's position is not absolute, but relative to some cases and variables. He considers it particularly suitable for studying income and wealth.

More recently, the use of the mean difference [1] and therefore of the concentration ratio in the study of income inequality is considered particularly attractive by certain authors like, for example, Sen (1973, p. 31) and Kakwani (1980a) because it is *"a very direct measure of income difference, taking note of differences between every pair of incomes"*. Among other things, it also allows for interesting interpretations; Sen's (1973, p. 33), for example, in terms of depression (in fact in every pairwise comparison, the individual with the lower income can become depressed for having discovered he has the lower

[1] An interesting and new use of the mean difference is proposed by Yitzhaki and other scholars who adopt it together with the Gini covariance and the Gini correlation to define the so-called Gini method (cf. Yitzhaki-Olkin, 1991; Schechtman-Yitzhaki, 1987). This method has been extended to concentration curves (Yitzhaki-Olkin, 1991) and to the Gini regression (Olkin-Yitzhaki, 1992; Yitzhaki, 1993).

one. If we hypothesize that this depression is in proportion to the difference in incomes, then the average of all the depressions deriving from all possible pairwise comparisons leads to the concentration ratio) and Pyatt's (1976), which is very original in terms of a statistical game (based on the expected gain of having the opinion of being someone better off).

Pietra (1917) can also be included in the above-mentioned quarrel. Commenting on an article by Ricci (1916) he argues, in a different way, his preference for the mean difference and harshly contests some affirmations as well as the paternity of some of the results.

The argument [2] is taken up again at the 19th Session of the International Statistical Institute held in Tokyo in 1930 when a paper by Bortkiewicz (1931a), reproposing some results on variability and concentration measures already reached by Italian scholars, provokes an outcry as Gini (1931a,b), Pietra (1931a,b) and Savorgnan (1931) claim that these results were first obtained by the Italian Statistical School.

Behind the interventions of the Italian statisticians Gini's wise direction may be caught sight of. On other occasions he also uses his collaborators to reply harshly to any oponent. This seems to show that there was a general intolerance of criticism, particularly direct or indirect criticism of the concentration ratio.

In an attempt to prevent or at least to subdue possible criticism, on several occasions, Gini integrates his published papers with comments or notes made by himself or by his pupils. This can be seen from the numerous additions made to the two editions of his *"Memories of Statistic Methodology"* (1939, 1955) in which some of his principle scientific papers can be found.

In the light of this, we can understand Benedetti's words better (1980a, p. 2). Remembering an episode in his academic life he says that his *"career risked ending at the time of its birth"* when, at the beginning of the 1950s in a conversation with Gini, he showed him the results of some of his research which demonstrated how the concen-

[2] The quarrel continues into subsequent years and even if it directly or indirectly still concerns the concentration ratio, it also extends to new areas such as the discordance between concentration indices or variability indices (e.g. D'addario, 1934; Mortara, 1935; Castellano, 1935; Pietra, 1935a,b, 1937).

tration ratio is similar to many other analagous indices inasmuch as it is a particular case of a more general formula.

Going back now to the quarrel it probably would have been less fiery and more objective if, as Frosini maintains (1986, p. 374), they had established the *"general operative definitions regarding distributions rather than single indices"* and furthermore *"the biggest reason for the confusion was the inextricable mixture of variability and concentration concepts"*.

We wanted to briefly draw attention to this dispute because we feel that it can help to understand Gini's personality which undoubtedly had a decisive influence, at least initially, on the success of his most famous index, both within the Italian Statistical School and elsewhere.

The development of the theory and the exploration of the application potentiality of the R index appear in literature almost at the same time. Infact Gini immediately stimulates his collaborators, particularly those who are more familiar with mathematics, into dealing with the most complex formal aspects. This is due to the fact that although Gini was gifted with an intelligence and intuition that his ex-collaborators and pupils define, still today, as being *"genial"*, he had had an economic-juridical education. Infact he graduated in law (from Bologna University in 1905) and therefore did not have all the necessary requirements for strictly theoretical-mathematical research.

Therefore, Pietra (1915) defines the Lorenz curve in the continuous case and, in this context, he introduces the graduation function which is better known in the international field as the inverse of the cumulative distribution function and is rediscovered severals decades later by other scholars. He also shows, within the Lorenz diagram, the geometrical interpretation of the concentration ratio expressed in terms of relative mean difference.

In turn, Gini (1930, 1932) determines the correction factors of his index in the case of upper and lower bounded variables and variables bounded at both ends, while Galvani (1932) establishes the transformations to apply to the parametric equations of the Lorenz curve in the three above-mentioned cases, so that the R index can always be expressed from a geometrical point of view as the relation between the concentration area and the theoretical area of the triangle of maximum concentration.

On the other hand, in order to single out a link between Gini's coefficient and other concentration indices, other researchers algebraically elaborate the original formulae by expressing them in terms of mean values (e.g. De Vergottini, 1940; Amato, 1947; Fortunati, 1955) or they deduce such measures from a same general expression (e.g. Amato, 1948, p. 509; Giaccardi, 1950a,b; De Vergottini, 1950; Benedetti, 1980a).

In order to reduce the inconvenience of the concentration ratio of assuming the same value for different distributions because the intersecting Lorenz curves enclose the same concentration area, some indices of asymmetry of the Lorenz curve (Giurovich, 1959; Zanardi, 1963) or some algorithms, are proposed in order to add more information to the *R* index (Hagerbaumer, 1977; Patimo, 1977, 1980).

Very briefly then this is the evolution of the *R* index up until the death of Gini in 1965. However, for some time before the concentration ratio had failed to stir up any further interest, even among Italian statisticians, and with the death of its bravest defender, it seems destined to fall into oblivion.

Maybe this is what would have happened if an article by Atkinson had not appeared in the Journal of Economic Theory in 1970. The article gives rise to one of those quarrels which periodically characterize the concentration ratio and which decisively share in its topical interest. The contribution made by Atkinson could paradoxically be remembered as a stimulating factor for the evolution and renewal of the concentration ratio rather than as a proposal for a definite abbandon of its use, which was Atkinson's primary intention.

The reason for the quarrel lies in the position taken by Atkinson who firmly criticizes Gini's index and some other conventinal inequality summary measures because they do not rank income distributions according to strictly concave social utility functions. He also makes a sharp distinction between objective measures (directly based on the observed modalities and including the traditional inequality indices) and normative measures (based on functions chosen to describe the economic welfare and their properties). In truth, this sharp distinction is not at all justified, as has recently been shown (Giorgi, 1984a; Giorgi & Pallini, 1985; Muliere, 1987).

Another criticism of the concentration ratio (Atkinson, 1970, p. 256-257) is that of attributing more weight to transfers of income near the modal value of the distribution rather than at the tails. This idea is

not shared by everyone, and, in particular, it is contested by Dagum (1986, p. 391) who maintains that such a criticism does not take into account that *"a transfer of income changes the shape of distribution"*.

Yet, if Atkinson had paid due attention to Gini's reply to Dalton (1920) in 1921 in the Economic Journal, he may not have been so rigid in his conclusions and probably the concentration ratio would no longer have attracted so much attention. In fact Gini (1921, p. 124) points out to Dalton that the aim of his index and those proposed by other Italian authors *"is to estimate, not the inequality of economic welfare but the inequality incomes and wealth, independently of all hypotheses as to the functional relations between these quantities and economic welfare of individuals"*.

However, in spite of what has just been said, the debate equally shows how forced Atkinson's conclusions are as they are deduced in a utilitarian context, the limits of which are known and clearly pointed out by Sen (1972, p. 344). He maintains that *"the utilitarian approach fails precisely because of its lack of concern with distribution of welfare levels and such it should be a particularly poor approach to use for judging the appropriateness of inequality measures"*. In other words *"the trouble with this approach is that maximizing the sum of individual utilities is supremely unconcerned with the interpersonal distribution of that sum"* (Sen, 1973, p. 16).

Other authors (e.g. Sheshinski, 1972; Kats, 1972) consider the constraints laid down by Atkinson (1970) on the social welfare function to be highly restrictive and in particular Sen (1973a,b) shows how the additive condition is rather objectionable and how the strict concavity is too strong a condition.

Sen (1974) also attempts to set the R index into a welfare context by proposing some axioms.

The debate arising from Sen's proposal is very extensive and complex and it is unthinkable to give even a brief summary here.

However we would like to recall an interesting axiomatic approach to the concentration ratio and to welfare measurement considered by Pyatt (1985), who shows how even the Lorenz diagram can be interpreted in terms of individual and social welfare measure.

Dagum (1990a) also occupies an important place in this debate since he shows, among other things, how the interpretation of the social welfare underlying the concentration ratio agrees with social values, unlike the social welfare function considered by Atkinson

(1970) which is based on individualistic utility functions, the specification of which occurs without considering the income of the other members of society.

However, even if with all possible caution, the welfare formulation is in some way received, then so are possible interpretations and extensions which were unthinkable for Gini and for many other scholars of that period. Therefore, given the close link between welfare and poverty, Sen (1976) uses the concentration ratio to construct a poverty index, and other scholars follow in this direction (e.g. Takayama, 1979; Thon, 1979; Kakwani, 1984b; P.K. Sen, 1986; Dagum *et al.*, 1988).

Among the most interesting topics discussed by scholars we should not forget the estimation of the concentration ratio for grouped data which makes particular reference to the interval estimate between the upper and lower bounds [3].

Now, there are so many extensions and interpretations of the Gini index that unfortunately we must confine ourselves to consider only those which, in our opinion, can improve the understanding of its evolution, its topical interest and the corresponding debate.

Some aspects of reliability theory (Chandra and Singpurwalla, 1981), of the life tables (Hanada, 1983; Silber, 1992) and of the occupational segregation (Siber, 1989c) are interpreted in terms of concentration ratio.

The extension to the multivariate case is the most interesting and initially it is carried out by Lunetta (1972a,b) and Taguchi (1972-73) who autonomously and independently propose a two-dimensional version of the concentration ratio. Subsequently Taguchi (1981) arrives at a fascinating multivariate version of the R index (see also De Simoni, 1979) and in further studies (Taguchi, 1988, 1989a) on some characteristics considered previously, he obtains interesting results of a high theoretical-formal content. Recently, the Japanese scholar has defined (Taguchi, 1991) a general method for the multivariate concentration study in which the Gini index plays an important role. Furthermore, in re-elaborating certain aspects of his previous articles, he (Taguchi *et al.*, 1993) succeeds in making complex

[3] For the numerous bibliographic references on this topic see Giorgi (1990, pp. 188-190).

statistical-mathematical results comprehensible to practitioners and scholars not directly involved in the subject.

The concentration ratio is also proposed as a test for homogeneity of variances (Girone, 1974; Cicchitelli, 1979) and as a (scale-free) goodness-of-fit test for the exponential distribution (Gail & Gastwirth, 1978). The exponential test is found to be identical to the one based on *"total time on test transform"* as shown by Chandra & Singpurwalla (1981).

The proposals of decomposition according to income sources and according to groups or sub-populations also contribute to making the concentration ratio a more modern instrument of analysis.

There are various decompositions by sources (e.g. Rao, 1969; Fei *et al.*, 1978 (in the case of grouped data the latter should be used with caution as stressed by Pyatt *et al.*, 1980); Fields, 1979a,b; Lerman & Yitzhaki, 1985).

With regard to the decomposition by groups or sub-populations, the concentration ratio is criticized because it is not additively decomposable in the analysis of variance sense (within + between) even if it is not at all clear why an inequality measure should necessarily be additively decomposable.

Infact Mehran (1975b) shows that if the R index is not decomposable in the within + between sense, it is in the within + interaction + between sense and he interprets the interaction between two groups as a measure of the extent of income domination of one group over the other, apart from the difference between their mean incomes. He therefore shows, as do Nygard & Sandström (1981, p. 313), that the new algorithm $(w+i+b)$ supplies added information in respect of the traditional within + between one. By using different procedures Kakwani (1984), Yitzhaki (1988a) and Yitzhaki & Lerman (1989) also obtain a decomposition of the concentration ratio in three parts. The applications made by these authors also show that these decompositions allow for a better clarification of some of the aspects related to the inequality existing between the various groups. Furthermore Lambert and Aronson (1993) have recently used a geometrical approach to show operational interpretation for all components of overall inequality, providing *"a solid understanding of the residual term as a sub-area of the Lorenz diagram"*. In our opinion this reinforces Mehran's results (1975b) of an explicative superiority of the Gini index in respect of the additively decomposable indices.

There are many other papers [4] concerning some particular approaches and interpretations, modifications, extensions as well as criticism and comparisons of the decomposition methods of Gini's coefficient, including the interesting and original ones by Pyatt (1976) and by Frosini (1989a,b,c).

The study of the sampling characteristics [5] also plays an important part in keeping alive the interest in the concentration ratio. This aspect has been dealt with quite deeply and systematically in relatively recent times, let's say the last 25 years, as scholars initially paid more attention to the descriptive aspects of the Gini index rather than the inferential ones. This is partly due to the fact that the income inequality in both developed and developing economies has become so important that it is considered to be one of the most important features of a country.

In this context, one of the first themes dealt with concerns the determination of the sampling variance of the concentration ratio in both the discrete case and in the continuous case. In the latter the studies are carried out with reference to particular populations such as the Pareto, the rectangular, the exponential, the lognormal and the Burr.

Another subject for research is the determination of the exact sampling distribution for both the natural estimator (for rectangular and exponential populations see Girone, 1971; Cicchitelli, 1976; Gail & Gastwirth, 1978) and for the maximum likelihood estimators (for the exponential and the Pareto, see Moothathu, 1985a,b) of the R index.

Another aspect which continues to attract scholars is the asymptotic behaviour of the sampling estimators. Hoeffding (1948) is the first to study this aspect and subsequently other researchers use different procedures to obtain the same result (e.g. Cucconi, 1965; Dall'Aglio, 1965; Goldie, 1977; Sendler, 1978; Sandström, 1983, 1987; Giorgi & Pallini, 1990).

However, the use of the asymptotic properties of the sampling estimators for the construction of the confidence intervals of the R

[4] There is so much literature on this topic that it is impossible to give a brief summary in this paper. Readers who are interested should see Giorgi (1990, pp. 192-194) for the bibliography and Giorgi (1986, 1992) for a critical survey.

[5] cf. Giorgi (1990, pp. 190-192) for more bibliographic references and Giorgi (1988, 1992) for further study on the subject.

index depends prevalently on the speed of convergence to the limit distribution of the estimators. This means that information is required on the minimum sample size from which the above-mentioned properties are verified. This topic is studied further by Giorgi & Pallini (1987, 1990) who use a Berry-Essen type bound to single out within the Gini family, the factors which can influence the above speed of convergence (cf. also Giorgi & Provasi, forthcoming).

Recently, new and interesting research material has been derived from the influence function used, for example, by Monti (1991) to study the robustness of the concentration ratio, to obtain its resistent version to the outlying observations (Monti, 1992) and to approximate its small sample bias (Monti, 1993).

Finally, in concluding this short survey, we hope to have succeeded in showing, on the basis of scientific literature, the considerable interest which the concentration ratio arouses still today at both a theoretical and empirical level. It is sufficient to note that, on the basis of a recent *"bibliographic portrait"* (Giorgi, 1990), about 75% of the scientific work on the Gini index (apart from trivial applications) is dated from 1970 onwards.

Therefore this can only suggest an affirmative reply to the query mentioned at the beginning of the paper.

3. Some final remarks

From what we have said, we do not want to convey the wrong idea that we are firm defenders of the Gini concentration index. On the contrary, from a careful observation of the picture portrayed, it is possible to deduce that we feel that it is an inequality index which is similar to many others with all its strengths and weaknesses.

However, we certainly find it difficult to share the proposals put forward by some authors to completely and definitely abbandon the R index as it can often be used as an alternative to another index which in turn only has its own strengths and weaknesses. Sometimes the strengths and weaknesses are different from those of the Gini index and therefore the use of one index does not necessarily exclude the other. Infact, *"the use of each of them is completely legitimate when we ask of one index what it can tell us and then their coexistence gives us more and more information"* (Castellano, 1935, p. 131).

Nevertheless, even this case has its exception. In fact, for example, it is rather difficult to understand the proposal of Theil's index as a measure of income and welfare inequality, since it is based on entropy which shows how *"the perfect equality is achieved when there is maximum entropy, i.e. maximum disorder, whereas perfect equality can only be thought of as the outcome of maximizing society order, hence SW"* (Dagum, 1983, p. 36).

What country or what government could nowadays accept a similar social organization?

Those who would like to definitely abbandon the concentration ratio should probably follow new and completely different ways, which, at present, do not seem to be in sight.

Finally, we could maintain that the concentration ratio seems to be of topical interest because of its conceptual and applicative simplicity, several interesting characteristics, new interpretations and extensions, and useful decomposition proposals. The interest in the Gini index can also be attributed to the link which joins it to the Lorenz curve and the mean difference, allowing the results of the last two to be extended to the first.

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A fresh look at the topical interest of the Gini concentration ratio

SUMMARY

The author outlines the main reasons why the Gini concentration ratio is still of topical interest 79 years after its first appearance in literature. Numerous bibliographic references are supplied as an integration and up-date of a bibliographic portrait of the Gini index which appeared in this journal (Giorgi, 1990).

Un recente sguardo all'attualità del rapporto di concentrazione

RIASSUNTO

L'Autore evidenzia le principali cause che contribuiscono, 79 anni dopo la sua comparsa in letteratura, all'attualità del rapporto di concentrazione di Gini. Sono forniti numerosi riferimenti bibliografici come integrazione ed aggiornamento di un precedente ritratto bibliografico dell'indice di Gini apparso su questa rivista (Giorgi, 1990).

KEY WORDS AND PHRASES

Evolution, strengths and weaknesses, topical interest of the Gini concentration ratio.