Estimating a Life Cycle Model with Unemployment and Human Capital Depreciation

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Abstract

I estimate a life cycle model of consumption choice with unemployment risk. Employed individuals face the risk of losing their job. Unemployed agents receive job random offers of different quality, which they can accept or reject. Following the loss of a job and during unemployment, an agent's productivity declines.

Using micro data, I estimate the structural model for Germany, the UK, and the US following the method of simulated moments approach of Duffie and Singleton.

The estimated model is used to perform policy simulations that highlight the relationship between the unemployment insurance scheme and the unemployment rates of different age groups.

JEL Classification: C51, D1, E2, J24, J31, J38, J64

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1. Introduction

This paper presents an estimation of a life-cycle model with unemployment. A partial-equilibrium model with search unemployment and stochastic job matches is developed. Workers are characterised by individual human capital endowments that improve while an agent has a job and decline during periods of unemployment. There is a government that collects taxes and social security contributions, provides unemployment insurance and social assistance, and runs a public pension system. The model is too complicated to be solved analytically, but it can be simulated. Hence, a simulation approach is employed to estimate the model using German, British, and American micro data. For each of these countries, the specific institutional arrangements are modelled, increasing the level of realism for the estimation and allowing a detailed analysis of the incentives created by the various policies implemented in the three countries.

This model extends the standard lifecycle framework in two ways. First, by modelling search unemployment explicitly rather than simply assuming an income process with a positive probability of zero realisations, it is possible to obtain the income process endogenously. Thus, the effect of policies on wages and the unemployment rate can be analysed.

Second, the presence of an individual human capital stock that depends on the employment history may have important effects on the labour market outcome in the presence of an unemployment insurance system. Pissarides (1992) shows that in a search model, if skills decline during unemployment, shocks that affect the labour market have a persistent effect on the unemployment rate. If for some reason the average unemployment duration increases, then the average skill of the unemployed is lower. Firms then find it less attractive to create jobs thus enforcing an perpetuating the increase of unemployment spell durations. Ljungqvist and Sargent (1998) present a model in which workers' skills increase stochastically during employment and decline during unemployment. They show that in such an economy, the introduction of an unemployment benefit that is tied to the last wage earned can have a strong effect on the resulting unemployment rates. The authors also introduce economic "turbulence" into their model, which is defined as a stochastic decrease of a worker's skill level after job loss. While an increase of turbulence only has minor effects on the unemployment rate if there is no unemployment compensation, it has a strong negative effect in the presence of a generous unemployment insurance. If workers who have lost a large share of their former productivity can earn a social security income based on their last wage, their probability of finding a job that is as attractive as the benefit may be rather small. Ljungqvist and Sargent argue that this effect may help to explain the rise of the unemployment rates in Europe over the past decades.

This paper is laid out as follows. In section two, the model is described. Section three provides a brief discussion of the tax and benefit systems in Germany, the United Kingdom, and the United States, as far as they are modelled. The estimation procedure and the data are explained in the part four. Section five presents the estimation results and discusses some central properties of the simulated economies. Some simulations are conducted in section six. Section seven concludes.

2. The Model

Consider an economy that is populated by agents who live for L periods. Agents can be either employed or unemployed during the first L_r-1 periods of their lives, and are retired thereafter. In each period j_0 of their lives, they maximise their expected lifetime utility, a time-separable function of consumption

$$\max \mathrm{E}_{j_0}[\sum_{j=j_0}^{L} \beta^{j-j_0} u_j(c_j) + \beta^{L-j_0} v(a_{L+1})], \tag{1}$$

subject to a budget constraint that must hold with probability one. Here, u_j is the period utility function with the usual properties, c_j the consumption in period j, β is the discount factor, and v(.) captures a bequest motive.[1] E_{j_0} is the mathematical expectation operator conditional on all information available in period j_0.

Let a_j be the assets owned at the beginning of period j and y_j the net income (including all kinds of benefits, but excluding interest) received in the same period, then the period j_0 budget constraint for the remaining life span becomes

$$a_{j_0} + \sum_{j=j_0}^{L} R^{j_0-j}(y_j - c_j) = R^{j_0-L} a_{L+1}, \tag{2}$$

[1] For better readability, I leave out the index for the agent, i.e. I write c_j instead if $c_{i,j}$ for the consumption of individual i in period j.

where r is the risk-free interest rate, R=1+r, and a_{L+1} is a non-negative bequest. The budget constraint is required to hold for each individual.

Agents start their lives as unemployed workers. Workers who hold a job in period j earn a gross wage

$$w_j = w(m_j, q_j, q) \equiv \chi m_j (q_j + q) \tag{3}$$

which is a function of the job-specific "match quality" m_j and the current individual skill level, consisting of a variable component q_j and a fixed personal component q. χ is just a scaling parameter. Employees never quit, but face the risk of being laid off with probability λ per period.

Unemployed workers receive job offers at the Poisson rate ω. So each period, after making their consumption decision, these agents may be presented with zero, one, or several job offers, each characterised by its match quality m, sampled form a distribution $G(m)$. Agents consider the best offer and decide whether to accept or to reject the job.

As mentioned, the wage earned during employment depends on a variable individual skill component. These skills q_j depend on the employment history of the agent: during unemployment, they deteriorate by a factor 1-δ_u per period, and while employed, a worker enjoys a skill increase by the factor 1+δ_e. Immediately after being fired, q_j declines by 1-δ_f.

The period net income y_j results from the institutional framework considered. In general, it may be a function of an agent's entire current situation and history. A central assumption underlying the model is that any risk stemming from employment and income uncertainty must be borne by the individual, i.e. there is no way of insuring against fluctuations of y_j except saving. After retirement, income ceases to random in any respect, thus for agents aged L_r or above, the individual consumption choice problem is deterministic.

A recursive formulation of the agents' decision problem making use of Bellman's approach can be written as

$$v_j(a_j, s_j, q_j, m_j, \Omega_j) = \max_{a_{j+1}, \underline{m}} \{u_t(c_j) + \beta E_j v_{j+1}(a_{j+1}, s_{j+1}, q_{j+1}, m_{j+1}, \Omega_{j+1})\} \tag{4}$$

where the maximisation is subject to the budget constraint

$$a_{j+1} = R(a_j + y_j - c_j)\,,\; a_{L+1} \geq 0\,. \tag{BC}$$

The value function $v_j(.)$ represents the highest discounted lifetime value an agent of age j can achieve in the situation characterised by the function's parameters. $s_j \in \{e,u,r\}$ stands for the current employment status (employed, unemployed, retired) and Ω_j is meant to capture all relevant information about the individual employment history required to compute the net income y_j. $\underline{m}$ is the minimum match quality required to make a job acceptable to an unemployed agent.[2]

Under standard regularity conditions the agent's decision problem is well-defined and has the usual properties: The optimal savings choice requires smoothing the marginal utility of consumption over time according to the Euler equation

$$u_j'(c_j) = R\beta E_j[u_{j+1}'(c_{j+1})]\,. \tag{5}$$

Depending on the parameters R and β, some agents may behave like buffer-stock savers (see Deaton (1991) and Carroll (1997)). In a world of perfect foresight they would choose a consumption level above their current resources, borrow money and repay it later in their lives. But with income uncertainty and the requirement to repay any debt with probability one, the agents' ability to borrow is practically limited. Consequently, they choose a certain target-amount of savings (the buffer-stock), that is an optimal compromise between the unachievable long-run consumption profile and the necessity to self-insure against unforeseen income fluctuations. Moreover, in the absence of extraordinarily generous pension payments or other benefits, agents approaching the retirement age L_r are likely to accumulate assets in order to provide for the final periods of their lifetime, when they cannot earn labour income any more.

The optimal job-acceptance rule implies a match threshold $\underline{m}$ that depends on the current individual situation. If the best job offered to an unemployed agent in a certain period exhibits a match quality above $\underline{m}$, it is accepted, otherwise the agent chooses to remain unemployed.

[2] In general, a more sophisticated job acceptance rule than this may be required. For the qovernment policies modelled below, however, the formulation using a match threshold $\underline{m}$ suffices.

Ceteris paribus a faster skill decline δ_u during unemployment or a higher value of a given job (higher δ_e or lower λ) imply a lower value of $\underline{m}$.

In order to be able to simulate the model, three aspects need to be specified: the period utility function u_j, the distribution of match quality of job offers, and the net income $y_j = y(j, a_j, s_j, w_j, \Omega_j)$.

The period utility function is chosen to exhibit the constant relative risk aversion property. As shown by Attanasio and Weber (1995), stochastic life cycle models match observed behaviour better if the influence of demographic variables on the individual choice is accounted for. Thus, I allow the period utility function to depend on the current family size f_j of the household. For the relative risk aversion $\gamma > 0$ and a preference parameter $\phi \geq 0$, u_j is then defined as

$$u_j(c) := u(c, f_j) = \begin{cases} f_j^{\phi} \ln c & \text{if } \gamma = 1 \\ f_j^{\phi} \dfrac{c^{1-\gamma} - 1}{1-\gamma} & \text{otherwise} \end{cases}. \qquad (6)$$

This formulation is a special case of the period utility function

$$u(c, z_j) = e^{z_j \theta} \frac{c^{1-\gamma} - 1}{1-\gamma}$$

used by Attanasio and Weber (1995), Attanasio et al. (1999), Cagetti (2002), and French (2004), among others. These authors employ several demographic state variables compiled in the vector z_j and a parameter vector θ of the same dimension. Choosing $z_j = \ln f_j$ to be the logarithm of family size and $\theta = \phi$ yields function (6).

I assume that agents do not derive any utility from leaving bequests, i.e. $v(a_{L+1}) = 0$.

The distribution of the match quality of job offers is assumed to be log-normal with mean one.

$$\ln m \sim N(-\tfrac{1}{2}\sigma_m^2, \sigma_m^2) \qquad (7)$$

As mentioned above, an unemployed agent receives offers at a constant rate ω. Therefore, the number of job offers $k = 0,1,2,...$ per period follows a Poisson distribution. Because these offers are considered simultaneously, an agent will clearly only take into account the best offer he has got during the current period. The probability density function of the best offer conditional on there being at least one job opportunity can be shown to be

$$f^*(m \mid k > 0) = \frac{\omega}{1 - e^{-\omega}} f(m) e^{-\omega(1-F(m))}, \tag{8}$$

where $f(m)$ and $F(m)$ are the p.d.f. and the c.d.f. of the individual offers.

Finally, the net income function y must be defined. By mapping all relevant information about an individual on the net resource flow (excluding interest payment, but including e.g. taxes on interest), this function fully captures any influence the (welfare) state has on an agent. Since y is deterministic and known to individuals, changing its properties will generally alter the agents' decision problem and thus the optimal program chosen.

The simplest case is an economy without any taxes or benefits, where the net income function is simply

$$y(j, a_j, s_j, w_j, \Omega_j) = \begin{cases} w_j & \text{if } s_j = e \\ 0 & \text{otherwise} \end{cases}, \tag{9}$$

i.e. an agent receives exactly the wage paid by his employer if he has a job and gets nothing if he is unemployed or retired. Reality is more complex, of course. The specifications of y used in the estimation exercises are designed to capture important elements of the tax and social security systems of the countries under consideration. The institutional details modelled for each country are described in detail in the following section.

3. Institutions

For a simulation of household life cycle behaviour to match the data, it is essential to capture the incentives created by the tax and social security systems. While proportional consumption taxation does not affect the decisions of the households in this model, income taxes are highly non-linear and therefore distort both consumption/saving behaviour and the decision whether or not to accept a job of a certain productivity. The same is true for payroll taxes, for which it does not matter whether they are (formally) paid by employers or employees. The various transfers that may be available to workers often do not only depend on current income and employment status, but also on rather complicated eligibility criteria, age dependent means criteria, and the like.

In this section, I briefly sketch these institutions as far as they are modelled for Germany, the United Kingdom, and the United States. For all three countries, the model reflects the situation of the year 1997.[3] The tax and benefit systems are summarised in the appendix.

3.1. Germany

3.1.1. Taxes and Contributions

The 1997 German income tax differs from its British and American counterparts in being a piecewise quadratic function. No tax is paid on incomes below DM 12,095 per annum. For higher incomes, the marginal tax rate increases from an initial rate of 25.9%, following a piecewise linear function. The highest marginal tax rate of 53% is reached at a taxable income of DM 120,042. The total income tax due is increased by a "solidarity surtax" initially introduced to help finance the German re-unification.

Married couples can choose to file together and apply a "splitting" tariff, which is defined as twice the tax due on the average income of the couple. Since the tax schedule is highly progressive, this option is always attractive.

All labour and capital income and a certain (age dependent) percentage of social security pensions are in principal subject to tax. There are however relatively high standard deductions for labour income (DM 2,000), capital income deductions and saver's allowances (DM 100 and DM 6,000, respectively), and an old age allowance of 40% of all non-labour income up to a maximum of DM 3,720, which is available to persons aged 65 or above. If filing together, a couple can claim all these allowances twice.

Moreover, there are allowances designed to reduce taxable income approximately by the amount of payroll taxes, and for each child, the tax base is reduced by DM 3,456.

The contribution-based German social insurance system consists of four branches, a health insurance provided by a large number of mostly independent insurers, an unemployment

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[3] Two aspects of the institutional environment that I do not model for any of the three countries are housing benefits and early (or deferred) retirement. The reasons for excluding housing related benefits are the strong regional dispersion of rents and house prices and the complexity of the benefits available (e.g. assistance for rent payments, government housing programmes, investment subsidies for home owners). Also, modelling the decision of when to retire (see French (2000)) is beyond the scope of this paper, although assessing the importance of early retirement programmes in the context of this model would be an interesting extension.

insurance, a pay-as-you-go pension system, and a nursing insurance first introduced in 1995. The contributions are collected as payroll taxes paid by both employers and employees at identical rates. These rates are 6.60% (on average) and 0.85% up to an income ceiling of DM 6,150 per month for health and nursing insurance, and 3.25% and 10.15% up to DM 8,200 for unemployment and pension insurance[4], respectively. Thus, total social security contributions could amount to more than 41% of the gross labour income[5] in 1997.

3.1.2. Benefits

All parents are entitled to child benefits. These benefits amount to DM 220 per child and month for the first two children, DM 300 for the third and DM 350 for the fourth and any further child.

There are two kinds of benefits for unemployed workers. After losing their job, they can usually claim unemployment compensation[6] for a certain period of time. Those who run out of unemployment compensation or for some other reason do not meet the eligibility criteria are often entitled to means tested unemployment assistance[7].

Unemployment compensation is based on the insurance principle. It is available to any unemployed worker who has been paying contributions for at least twelve month during the three years preceding the job loss. The minimum benefit duration is six month. It is extended by another two month for each additional trimester of contribution payment during the last seven years, up to an age-dependent maximum. Persons aged 41 or less can never claim unemployment compensation for more then a year, for older workers the maximum duration increases to 18 months (42 to 43 years), 22 months (44 to 48 years), 26 month (49 to 53 years), and 32 month (54 years and above). These age limits, which are used for the simulation, were valid throughout the 1990ies, but were increased by three years in mid 1997.

The level of unemployment compensation is 60% of the last net wage earned (67% if the claimer has at least one dependent child). As contributions are paid on earnings below DM

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[4] The pension insurance tax rate of 10.15% goes for blue and white collar workers, who enjoy the same common conditions in spite of (formally) being insured by distinct institutions. Mining workers are covered by yet another public insurance scheme and pay slightly lower contributions.

[5] In order not to confuse the concept of the "gross wage" as defined by tax laws with the economic "gross wage" introduced above, I use this term for the productivity of a job (equivalently the cost of labour to the employer, $w_j = w(m_j, q_j, q) \equiv \chi m_j (q_j + q)$) only, whereas gross labour income refers to gross wage minus the employer's share of the social security contributions. All tax rates relate to this latter definition.

[6] Arbeitslosengeld

[7] Arbeitslosenhilfe

8,200 only, however, the benefit cannot exceed the amount that corresponds to this wage ceiling.

Needy unemployed persons who are not (or no longer) entitled to unemployment compensation are usually eligible to receive unemployment assistance. Unemployment assistance is a means-tested benefit of unlimited duration paid out of the general tax revenue. The replacement rate is fixed at 53% (57% if there is at least one child). Any income earned reduces the benefit. There are also limits to the personal wealth, which are however quite generous compared to those of social assistance. Apart from excluding a car, an “adequate” house and certain other goods from the means test, the unemployed is allowed to have savings of DM 1,000 per year of age (twice as much if he or she is married) to provide for retirement. As long as personal wealth exceeds the level specified by the means test, no unemployment assistance is paid. Unemployed workers do not bear the cost of social insurance.

The subsistence level of income guaranteed by the welfare state is provided as social assistance. Apart from supporting individuals in exceptional situations, a service that is not relevant for our purpose, social assistance provides a minimum income to persons in need. The benefit is computed as need minus own resources, i.e. it is reduced by one DM for each DM earned. The need of a household depends on the number and age of its members. In 1997, a household head was entitled to DM 534 per month on average (the actual figure slightly differs between the federal states). For any further adult, this figure is increased by 80% of that amount, for children by an age-dependent amount, 65% on average.

Social assistance is subject to a rather strict means test. The wealth limit is DM 2,500 (4,500 the claimer is 60 or older) for the household head, another DM 1,200 for his or her spouse, plus DM 500 for any further person living in the household.

None of the benefits described thus far is taxable.

3.1.3. The Pension System

Most Germans participate in the public pay-as-you-go pension system for blue and white collar workers. The basic concept is similar to the American “social security”, although closer to an insurance in providing pensions more closely related to the total amount of contributions paid.

While paying their contributions, workers earn “remuneration points” reflecting their relative position among the among the group of insured persons. A year of contributions corresponding to the average income (DM 52,143 in 1997) is worth one point. With an income ceiling of DM 8,200 per month for pension contributions a maximum of DM 8,200 x 12/DM 53,143=1.85 points could be earned in 1997.

In some cases, points are awarded without contribution payments, e.g. for maternity leave, unemployment, or military service. They are either fixed in value or are calculated in a complicated manner upon retirement as an "average value".[8]

Old age pensions are simply calculated as the number of points earned during working life multiplied by a "pension value" that is adjusted every year and in 1997 stood at DM 40.51 per month. Thus, a retiree who had earned exactly the average income for 45 years received DM 40.51 x 45=DM 1,822.95 per month as a pension.

State pensions are only partially taxed. For a 65 year old person, 27% of the pension are subject to income tax. This share drops further with any year of age down to only 2% for people older than 96. In combination with the various allowances this implies that only very few pensioners pay income tax.

3.2. The United Kingdom

3.2.1. Taxes and Contributions

The British income tax has only three tax brackets. All income above the basic allowance of £ 4,045 per annum (the basic allowance is increased to £ 5,220 and £ 5,400 for persons older than 65 and 75, respectively) is taxable. As the income tax is strictly individual, couples always file separately. The 1997 marginal tax rates are 20% on the first £ 4,100, 23% on the next £ 22,000 (only 20% on capital income), and 40% on the rest. Married couples receive a tax credit of 15% of £ 1,830 (£ 3,185 or £ 3,225 if older than 65 or 75, respectively).

Family credit is a means tested tax credit for working people with low income. It is available only to households with savings below £ 8,000 (£ 12,000 if the recipient is older than 60 years). For individuals working at least 30 hours a week, it is payable at a rate of £ 58,20 plus an age-dependent amount for each dependent child, £ 16.66 on average. Family credit is decreased by 70 p for each pound the net income exceeds £ 77.15 per week. This net income consists of wages, benefits, and imputed capital income form savings above £ 3,000 (£ 6,000 if older than 60), reduced by income and social security taxes.

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[8] To reduce the number of state variables required to describe the "pension account" of an individual to one, the following slightly simplified rules are used to update the number of points P_t earned until period t:

if employed: $P_t = P_{t-1} + \min\{\text{gross labour income}, \text{12xDM 8,200}\} / (\text{DM 52,143})$

if unemployed: $P_t = P_{t-1} + 0.8 \min\{\text{last gross labour income}, \text{12xDM 8,200}\} / (\text{DM 52,143})$,

where $\overline{P}_{t-1}$ is the average number of points earned per year so far.

Both employers and employees pay social security contributions, but at different rates. These contributions are collected to pay for unemployment compensation and the basic state pension. Only employees who earn at least £ 62 per week participate in the social insurance programmes. Their contributions amount to 2% of the first £ 62 earned and 10% on the next £ 403. Employers' contributions are a certain percentage of the total gross labour income. The (average) social insurance tax rate increases in several steps from 3% for wages less than £ 110 per week to 10% for wages above £ 220.

There is also a tax levied by the local governments (the council tax). This tax is assumed to be collected at a flat rate of £ 12.95 per week. Council tax benefit compensates low income households.

3.2.2. Benefits

Tax free child benefits are available to all households. They are paid at £ 11.05 per week for the first and £ 9 for any further child.

Like in Germany, there a two kinds of unemployment benefits. Unemployment compensation, called contribution-based jobseeker's allowance in Britain, is available for a maximum of six month to unemployed workers actively trying to find a job who meet the contribution conditions. Those who do not qualify for unemployment compensation may be eligible for means-tested unemployment assistance, referred to as income-based jobseeker's allowance.

To qualify for unemployment compensation, an unemployed worker must have paid contributions on income amounting to at least fifty times the weekly social security threshold income of £ 62.

Both benefits are paid at the same flat rate. Singles receive £ 49.15 per week (£ 38.90 if younger than 25), couples get £ 77.15. The benefit is increased by an age-dependent amount for each dependent child (£ 20.97 on average), and there is a family premium of £ 10.80. The benefit is usually reduced penny by penny for any earnings above a threshold of £ 5 per week.

The wealth limits for unemployment assistance are the same as for family credit: no benefit is available if assets exceed £ 8,000 (£12,000 if older than 60), and an income is assumed from savings above £ 3,000 (£ 6,000 for the aged).

There is also a social assistance programme in the UK, which is called income support. It is designed for persons working part-time and for those who need not be available for work. The benefits and means-test are the same as for unemployment assistance.

3.2.3. The Pension System

The 1997 public pension system in the UK is composed of two programmes: The basic state pension is designed as a minimum retirement income that is financed by the usual social security taxes. In addition to that, there is the state earnings-related pension scheme (SERPS)[9] that is suited to maintain the standard of living after retirement. As employees were encouraged to contract out of SERPS in favour of an occupational pension scheme or an individual pension plan, this programme is not modelled here.

The basic state pension is available to anybody who has paid enough social security contributions. To receive a full basic state pension, which was paid at a flat rate of £ 64.70 in 1997, it is usually required to have paid contributions during at least 90% of one's working life. For shorter contribution periods, the pension is reduced proportionally. Basic state pensions amounting to less than 25% of the full level are not paid. For married couples, the basic state pension of a spouse is either derived from her own or her partner's contribution, whatever yields a higher pension. A spouse's pension derived from a full basic state pension is worth £ 38.70 per month.[10]

3.3. The United States

In the United States, state and local governments have decidedly more power to shape the tax and social security framework than in the two other countries considered. As a consequence, it is impossible to ignore state and local policies when evaluating the incentives that drive employment decisions of individuals, and difficult to model institutions representative for the country as a whole.

I address these difficulties by modelling the institutions present in one certain region, including federal and state programmes and policies.[11] Following OECD (1999), I attempt to capture the 1997 tax and benefit structure relevant for Michigan "represent[ing] a typical manufacturing region."[12]

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[9] This system has been changed in 2002. SERPS is now called state second pension.

[10] For the simulation, I assume that everybody receives the full basic state pension. This significantly reduces the computational complexity of the problem. This simplification is justified by the observation that with a simulated working life of 45 years, a person would have to fail to pay contributions for 4½ years (i.e. be unemployed for such a long time) for the pension to fall below its maximum level.

[11] No local taxes are modelled.

[12] ibid., country chapter United States

3.3.1. Taxes and Contributions

The most important American tax is the federal income tax. Couples can choose between being taxed jointly or separately, where those filing a joint return enjoy somewhat lower marginal tax rates on intermediate (joint) incomes and higher deductions from the tax base.[13]

Taxable income is computed by subtracting from the sum of wages, capital income, and taxable benefits several exemptions and deductions. The standard deductions amount to $ 4,150 for singles and $ 6,900 for couples ($ 5,150 and $ 8,500, respectively, if over 65 years old). Taxable income is further reduced by $ 2,650 for each household member if gross income[14] is below a threshold of $ 121,200 ($ 181,800 for couples filing jointly). Above this level, the exemption is reduced by 1% per $ 1,250.

In 1997, there were five tax brackets with marginal tax rates ranging from 15% to 39.6%. For joint returns, the lower tax brackets are wider than for individual returns. In both cases, however, the highest rate of 39.6% is applied to income above $ 271,050.

To increase work incentives for low wage earners, an earned income credit (EIC) has been introduced. For low incomes, starting from zero, EIC first increases linearly with gross income, then remains constant for a certain income range, and finally decreases. Both the level of the credit and the relevant income ranges depend on the number of children in the household. For two or more children, its maximum of $ 3,656 (one child: $ 2,210, no children: $ 332) is available at annual incomes between $ 9,140 and $ 11,950 (on child: $ 6,500 to $ 11,950, no children: $ 3,340 to $ 5,450).

Michigan state tax is mainly based on the definitions and figures used to calculate the federal income tax. Its tax base is the gross income (excluding taxable social security benefits) minus exemptions. Like with the federal tax, exemptions are granted for household members ($ 2,500 per person). Further exemptions of $ 900 are available for the filer's or his spouse's being older then 65 years and if unemployment compensation amounts to at least 50% of gross income. The tax rate is 4.4%.

The American social insurance programme comprises old age, survivors, and disability insurance (OASDI) and hospital insurance (HI). Although "Social Security" officially stands for the whole OASDI programme, the term is often used in reference to the old age pension system only. HI is part of the Medicare system.

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[13] Because only the household head earns labour income in the simulated model, it is assumed that couples always file a joint return, as this is very likely to result in a lower tax in this case.

[14] The legal term is "adjusted gross income", AGI, which is defined as gross income minus several deductions not discussed here.

Both OASDI and HI are financed by payroll taxes, paid by both employers a employees at the same rates. The HI tax is collected at a rate of 1.45% on total wage income, OASDI contributions are paid at a rate of 6.2% on the first $ 65,400 p.a. Thus, social insurance contributions can add up to as much as 15.3% of gross labour income.

There is also a contribution-based federal unemployment insurance system in the United States. Contributions are paid by employers only. The usual tax rate is 6.2% levied on the first $ 7,000 of wage earnings per year. It can however be reduced to 0.8% in States that run their own approved unemployment insurance programme, which all states do. To discourage excessive hire and fire, states usually impose variable insurance tax rates depending on an "experience rating" of companies that reflects the labour turnover observed in the past. In the case of Michigan, these tax rates could vary between 0 and 10% in 1997, with a maximum tax base of $ 9,500 per annum. The average unemployment insurance tax rate in 1997 was 4.4%.[15]

3.3.2. Benefits[16]

To qualify for unemployment compensation, a minimum employment record of two quarters within the past year is required, with a wage corresponding to at least an annual income of $ 3,084. The benefit duration is six months, and the replacement rate is 50% of the former gross salary, subject to lower and upper bounds of $ 87 and $ 300 per week, respectively. Unemployment benefits are taxable. There is no unemployment assistance.

Among the federal and state programmes that could be seen as part of a social assistance concept, only SSI and food stamps are of major relevance in the present context. SSI (supplemental security income) provides a minimum income to the aged, blind, and disabled. Only the first of these groups exists in the model. Persons aged 65 or above who pass the means test can apply for SSI. The benefit is calculated as the federal benefit rate ($ 484 per month for single, $ 726 for couples in 1997) minus any income above $ 20 per month. The means test specifies an asset limit of $ 2,000 for single households and $ 3,000 for couples, but excludes certain goods like an owner-occupied house, household goods, and life insurances up to certain value.

The food stamps programme has been created to guarantee adequate nutrition to low-income households. All households with resources below $ 2,000 ($ 3,000 if at least one household member is older than 60 years) who meet the income criteria may apply for this benefit. There

[15] see 1998 Green Book, Section 4.

[16] The specification of the benefit amounts and conditions is mainly based on OECD (1999)

are two figures needed to determine eligibility and the benefit, “gross income”, which is basically monthly household cash income, and on “net income”, computed as cash income minus several deductions, like a standard deduction of $ 134 plus 20% of gross earnings and certain child-care expenses. If both income figures are below certain limits that depend on household size (e.g. $ 893 and $ 671 for single-person households), the household is eligible to receive food stamps. The benefit amount is then determined as a maximum monthly allotment, which is depends on the number of persons in the household (e.g. $ 125, $ 230, and $ 329 for one, two, and three persons, respectively, in 1997) reduced by 30% of the “net income”.

Any food stamps granted do not affect eligibility for or the amount of any other benefits. Neither SSI nor food stamps are taxable.

3.3.3. The Pension System

The old age insurance provided by the social security system is designed as a pay-as-you go pension system that provides retired workers with a basic retirement income. The benefit calculation is based on “average indexed monthly earnings” (AIME). To compute this figure, monthly wages up to the social security tax ceiling ($ 65,400 in 1997) are discounted to the retirement year. Only the best 35 years of a working life are taken into account. If social security contributions have been paid during less than 35 years, a zero wage is assumed for the remaining periods.[17]

The actual benefit paid to retirees is highly regressive in AIME. In 1997, it was 90% of the first $ 455, plus 32% of AIME between $ 456 and $ 2,741, plus 15% of AIME above $ 2,741. Thus, applying the 1997 figures, the highest possible social security benefit is $ 1,547.37, or about 28% of the corresponding gross wages up to the social security tax ceiling. Couples can receive the social security income based on the individual contributions or 150% of the higher pension, whatever is more.

Social security pensions are only partially subject to income tax. If gross income, including half of the pensions, lies below $ 25,000 per year ($ 32,000 for couples), social security

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[17] Again, a simplified method for calculating the current value of the “pension account” is employed. Using a similar approach as French (2000), I update AIME for an individual with an employment history of t years as:

if $t \le 35$: $$AIME_{t+1} = (AIME_t \cdot t + \min\{\text{gross monthly earnings}, \$\ 65,400\})/(t+1)$$

if $t > 35$: $$AIME_{t+1} = (AIME_t \cdot 34 + \max\{\min\{\text{gross monthly earnings}, \$\ 65,400\}, AIME_t\})/35,$$

income is not taxed at all. Otherwise, up to 85% of social security pensions can be included in the tax base.

4. Estimation

4.1. The Method of Simulated Moments

I use a Method of Simulated Moments (MSM) strategy to estimate most of the structural parameters of the model. It is based on the Generalized Method of Moments (GMM) approach developed by Hansen and Singleton (1982). Like with GMM, the MSM estimator $\hat{\theta}$ is the combination of parameters that makes a (large) set of theoretical moments derived from the model $\mu = \mu(\theta)$ as close as possible to their empirical counterparts $\tilde{\mu} = \tilde{\mu}(x,\theta)$, which depend on the data x. A quadratic form $Q(\theta) = (\mu(\theta) - \tilde{\mu}(x,\theta))'W(\mu(\theta) - \tilde{\mu}(x,\theta))$ is used to define the distance between μ and $\tilde{\mu}$. Under some regularity conditions, $\hat{\theta} = \arg\min_{\theta} Q(\theta)$ is consistent and asymptotically normal for any positive definite weighting matrix W. If W is chosen optimally, the parameters of the model are estimated efficiently.

The Method of Simulated Moments introduced by Pakes and Pollard (1989) and Duffie and Singleton (1993) extends GMM to by allowing for moments obtained by numerical integration. If it is difficult to derive a an appropriate set of moment conditions for estimation analytically, one can alternatively simulate the model for a given parameter constellation and compute the required moments numerically. Both the optimal weighting matrix and the asymptotic covariance matrix of the estimator are computed in a similar way as for GMM. It is also possible to compute an overidentification criterion to test the specification of the model. The MSM estimator was shown to exhibit the same asymptotic optimality properties as its GMM counterpart.[18]

The MSM approach has successfully been applied to the estimation of life cycle models using micro data. Gourinchas and Parker (2002) were the first to propose a MSM strategy tailored to this class of models. Their approach was followed by Cagetti (2003) and French (2000). These authors first estimate all parameters of the model that can be obtained without simulation using traditional estimation techniques. Using these first stage estimates as calibrations for the actual MSM estimation reduces the dimension and thus the computational complexity of the minimisation operation associated with the estimation process. The moments are chosen such that the empirical moments can be calculated immediately from

[18] A rigorous yet accessible treatment of simulation based estimation can be found in Gouriéroux and Montford (1996).

cross sectional or panel data sets and are independent of the parameter vector θ, i.e. $\tilde{\mu} = \tilde{\mu}(x)$. The simulated moments for a certain parameter combination can then be obtained by simulating a large number of life cycle histories, generating the same variables as in the empirical data set and computing the same moments on these simulated data.

Gourinchas and Parker (2002) match mean log consumption by age to estimate the parameter of relative risk aversion and the discount rate. Doing a similar exercise, Cagetti (2003) chooses the median of wealth for each age group as his set of moments. In his model of retirement behaviour, French (2000) uses six moment conditions per age group, matching both the mean and the median of wealth as well as two labour market participation indicators conditional on health (which he models as a two-state variable).

4.2. Simulation and Numerical Optimisation

Both for the simulation process and for the further experiments with the model undertaken in section 6, it is necessary to solve the agent's optimisation problem and to obtain aggregate variables that result from this optimal behaviour. This section describes the methods used to this end and gives some details about the implementation of the MSM procedure.

As is already clear from the exposition section 2, the model is solved in discrete time. It is designed to describe individuals aged between 20 and 80 years. The period length is chosen to be 1/24 of a year, i.e. approximately a fortnight, which I hope is short enough to allow for a realistic job search behaviour. The total number of periods is therefore 1440.

For the calculation of the optimal programs, the state space is discretised. The relevant state variables are age j and employment situation s, which are discrete by definition, as well as the continuous variables wealth a, skill level q, job match m, and the vector of institutional states $\Omega = (\omega_{qual}, \omega_{dur}, \omega_{ben}, \omega_{pens})$, consisting of an employee's qualification status for unemployment benefits ω_{qual}, the remaining benefit duration ω_{dur} and its amount ω_{ben} for an unemployed, and the claims to a state pension ω_{pens}. For the continuous states, appropriate grids are chosen (see appendix for details). Values off the grids are computed using linear interpolation. Because of the nonlinearity of the tax/benefit system, it is not possible to reduce the effective number of state variables during the computation of the optimal program normalising wealth and income variables by a measure of permanent income (which would be q in this case), as is often done in the literature.

The value function is computed on the grids using backward induction. Starting with the last period, where there is "no tomorrow", it is obtained by solving the optimisation problem given by equation (4), making use of next period's value function, which is already known.

The evaluation of the expectation in equation (4) is easy for employed agents (i.e. $s_j = e$), as the probabilities and values of keeping the job and becoming unemployed are known, and it is trivial for retired agents, who do not face any uncertainty whatsoever. If the current state is unemployment, the expected value is obtained by integrating over all possible match qualities of offered jobs. This integral is approximated by a weighted sum over the nodes of the match quality grid.

Up to two controls must be chosen as a the solution of (4). One of them is a rule whether or not to accept a job. It can be obtained in the most straightforward way by comparing the value of the job to the value of unemployment for the relevant combination of states. The other control is the optimal level of consumption (alternatively: saving), which is the solution of a one-dimensional optimisation problem. The strategy for computing this value is described in the appendix. As it would be too memory consuming to store the optimal controls for each period and each combination of states on the grid, they are kept for only one out of 24 periods (i.e. one set of controls per year) up to period 1056 (age 64), and the missing values are obtained by interpolation. The loss of precision due to this measure is rather small, as the behaviour of an individual in any given circumstances (as defined by the states) changes little over time spans shorter than a year.

Having computed the optimal program, it is straightforward to simulate histories. If these simulations are used to compute moments for the MSM procedure, it is important that the individual histories are non-random in the sense that the same individuals facing the same idiosyncratic shocks can be observed for different parameter constellations.

The Method of Simulated Moments estimation of a parameter vector requires that a quadratic form of the deviations of the simulated moments from their empirical counterparts be minimised. Unfortunately, experiments reveal that standard (local) optimisation strategies fail to find the minimum of the objective function.[19]

An algorithm that is very successful in solving the problem turned out to be an evolutionary strategy. The main feature of algorithms of this kind is that they do not only consider one point of the parameter space at a time. Instead, they evaluate a large number (a generation) of "individuals", which are characterised by a combination of parameters (i.e. a potential solution to the original problem, the "object variables") and a set of "strategy parameters".

[19] I have experimented with the „Amoeba" algorithm of Nelder and Meade (1965) as well as gradient-based strategies.

From each generation, the fittest individuals (i.e. those whose object variables yield the best results) are chosen as "parents" for the next generation. Their "children" are generated through recombination and mutation. The object variables are mutated simply by adding a normally distributed random noise vector, the covariance matrix of which is given by the strategy parameters. An individual thus implements both a point in the parameter space and a preferred direction of movement.[20,21]

Although the evolutionary strategy is rather good at finding the right region in the parameter space, final convergence is rather small. Therefore, a mixed strategy is employed. The evolutionary algorithm is applied for 40 generations, and the best combination of parameters found thus far is used as the starting point for a local optimisation run of the Nelder and Mead (1965) algorithm.[22]

4.3. Data

To estimate the structural parameters of the model, adequate data for are needed for the three countries under consideration. Ideally, one would like to use large household-level panel datasets, which are comparable between the countries, are publicly available, cover a relatively long time span, and provide reliable information on all variables of the model (in particular on demographics, income, consumption, wealth, and employment). Unfortunately, actually available data are seldom ideal.

In what follows, the datasets used for the estimation are briefly described and the construction of the variables that enter the estimation process is discussed.

4.3.1. Germany

The most widely used German micro dataset is the German Socio-Economic Panel Study (GSOEP) administered by the German Institute for Economic Research (DIW) in Berlin.[23] It was started in 1984 as a longitudinal survey of households and individuals. The first sample (A) covered almost 6,000 households. In the following years, additional sub-samples of the population were drawn. Sample B and D cover foreigners and immigrants, respectively.

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[20] Details can be found e.g. in Rudolph (1997). I choose a $(\mu+\lambda)$ strategy with $\mu=24$ and $\lambda=160$.

[21] A big advantage of evolutionary strategies over for example simulated annealing algorithms is the fact that they support parallel computing in a very natural way.

[22] A good description of this algorithm is given in Lagarias et al. (1998).

[23] A more detailed description of the (G)SOEP data can be found in Haisken-DeNew and Frick (2003).

Sample C, created in 1990, is representative of the East German population. In 1998 and 2000, two new samples of the whole German population were drawn.

Individuals are interviewed annually, and most the questions refer to the past calendar year. Whenever possible, face-to-face interviews are made. The topics covered in every wave include demographics, education, employment, income, and health. In addition, every year a certain topic is treated in greater detail.

The GSOEP provides rather detailed information on household composition, demographics, and educational attainment. Income variables and information on the employment status are available on a monthly basis. Unfortunately, comprehensive data on household wealth and (consumption) expenditure are not collected on a regular basis.

4.3.2. The United Kingdom

The British Household Panes Study (BHPS) is a representative sample of the population of England, Wales and southern Scotland. The study was started in 1991, when about 10,000 individuals in 5,000 households were interviewed. In 1997 an low income sample was drawn and in 1999 and 2001 additional sub-samples form Scotland, Wales, and Northern Ireland were created.

The annual interviews are conducted from September on, with the questions referring to the twelve month from September last year to past August. Besides covering demographic, employment, income, and health variables, it also features some information on savings, durables, and food consumption.

4.3.3. The United Stated

The Panel Study of Income Dynamics (PSID) at the University of Michigan, which was started in 1968, is a representative sample of US households. In the 1990ies, it covered more than 8,000 families. From 1990 to 1995, a particular immigrant sub-sample was included. After 1996, the ample size was reduced and a new immigrant sample was added.

Most interviews are conducted over the phone. The core topics include demography, employment, and income. The only measure of consumption available is expenditure for food. Comprehensive data on health, savings, and wealth are only available for a few waves.

4.3.4. The Cross National Equivalent File

The Cross National Equivalent File (CNEF) is a dataset based on GSOEP, BHPS, PSID, and the Canadian Survey of Labour and Income Dynamics, created at Cornell University. It consists of variables derived from the respective national surveys. The main goal of the CNEF is to provide internationally comparable micro data. The CNEF only covers a subset of the topics addressed by the individual studies, mainly focusing on demographic and income variables.[24] Where they are not provided in the original surveys, net incomes, taxes, and social security payments are constructed using micro-simulation.

4.3.5. Case Selection Construction of the Variables

I focus on household heads aged twenty or above. For all three countries, only the waves from 1991 to 1997 are selected. The income and employment variables thus cover the time from January 1990 to December 1996 for Germany and the US and from September 1990 to August 1997 for Britain. The selected time span can be seen as a compromise between the wish to have a sufficiently large number of observations and the necessity to restrict attention to a period of a rather stable institutional environment. At the end of the nineties, both in Britain and in Germany, major reforms were put in place by the newly elected Labour and Social Democrat governments. A longer time span may also help to reduce the influence of business cycle effects.

In the case of Germany, individuals living in the eastern part of the country or in Berlin are excluded. The model developed here is unlikely to do justice to the specific situation that households in this region are facing.

Among the variables needed for estimation, family size, the number of children in the household, the household head's age and labour income are directly available from the CNEF. Three further variables, referring to the employment situation of the household head, can be constructed from the original survey: The number of weeks working (full time) and unemployed searching for work can easily be constructed from the employment calendars available in all three studies. A variable indicating whether an individual is currently self-employed or has been self-employed in the last job before the current unemployment spell is only available in the PSID and the BHPS. For Germany, only information on the current job is available, so this variable needs to be constructed exploiting the longitudinal dimension of the dataset. Household food consumption is included as the only available measure of consumption expenditure.[25] Unfortunately, this variable is not available in the GSOEP dataset.

[24] More information on the data and the CNEF project can be found in Burkhauser et al. (2001)

[25] see Attanasio and Weber (1995) for a discussion on why food consumption is not an ideal measure of overall consumption.

Apart from these individual level variables, some macro variables are required. A wage index and a price index are added. The real riskless interest rate is computed from the short-term return on treasury bills.

For the estimation procedure, the variables just described as well as their first lags are used. Thus only cases for which all these variables are available in two consecutive years are kept.

For the estimation of labour market parameters, attention is restricted to certain working-age households: First, all observations where the household head is currently self-employed or as been self-employed before the current unemployment spell are dropped.[26] Finally, to exclude households that do not fully participate in the labour market, I drop all observations for which the sum of the weeks working and the weeks unemployed is below 48.[27]

The resulting number of household-year observations is 13,017 for Germany, 9,391 for the UK, and 15,922 for the US.

4.4. Estimation Procedure and Calibrations

Following Gourinchas and Parker (2002), I employ a two-stage approach to estimate the model. In the first stage, structural parameters of the model that can easily be obtained from the data without recurring to the structural model are estimated. These estimators are then used to calibrate the structural model, which is used to estimate the remaining parameters by MSM.

The eleven parameters to be estimated are listed in Table 1. The parameter obtained most easily is the real interest rate r, which is computed as the average real short-term return on treasury bills during the relevant time period.

δ_e and λ can be computed immediately from the micro data as the sample counterparts of the actual moments. δ_e is constructed as the growth rate of the average discounted wage income of individuals fully employed in two consecutive years. λ is be computed as the probability of being unemployed at least one week this year conditional on having been employed the whole last year.

Log-linearisation of the Euler equation

$$u'(c_t, f_j) = (1+r)\beta u'(c_{t+1}, f_{j+1})\varepsilon_{t+1} \tag{10}$$

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[26] This is justified by the observation that self-employed individuals face a higher unemployment risk. Moreover, the job-search model used here might be inappropriate for this group.

[27] This threshold is somewhat arbitrary, however experiments reveal that for higher thresholds the number of resulting cases strongly decreases, whereas lowering the threshold only has very few observations.

with a utility function u as defined in equation (6) and a mean-one error term ε_{t+1} yields

$$\Delta \ln c_{t+1} = \alpha + \frac{\phi}{\gamma} \Delta \ln f_{j+1} + \eta_{t+1} \tag{11}$$

for some constant α and a mean-zero error term η. Equation (11) is estimated by OLS to obtain the ratio ϕ/γ. Although ϕ cannot be identified from this regression, having this ratio reduces the dimension of the second stage estimation problem by one. For the UK and the US, the same data as for the other first and second stage estimations can be used, but without any restrictions on the employment status of the household head. In the German data, a (food) consumption variable is only available for the years 2000 and 2001. Thus, for the estimation of equation (11), only these two waves are used.

Table 1: Parameters to be estimated

parameter	meaning	estimated in stage
δ_e	growth rate of skills during employment	1 (sample moment)
δ_u	skill loss per period during unemployment	2
δ_f	skill decline at job loss	2
λ	job destruction rate	1 (sample moment)
ω	job offer rate	2
σ_m	standard deviation of log match quality	2
β	discount rate	2
γ	relative risk aversion	2 [a]
ϕ	elasticity of utility w.r.t. family size	1 (OLS)
r	real interest rate	1 (macro data)
χ	scale parameter	2

[a] only estimated for the UK; constrained to equal 3.0 for Germany and the US; see next section and appendix.

As mentioned above, the first stage estimators are used to parameterise the numerical model used to perform the second step estimation. To compute the actual tax burden of a household, some information on the household composition is required. As a practical solution, the average family size and number of children found in the data for each year of age of the household head are used in the model.

The moments chosen to perform the second stage MSM estimation are mean gross labour income, mean weeks unemployed per year, and annual growth of mean consumption, all conditional on the unemployment duration during the previous year, and by age group.[28] Last year's unemployment duration can take on five values, representing (1) no unemployment, (2) less than three months, (3) between three and six months, (4) more than six months, and (5) the whole year. There are 14 age groups, each covering three years, the first including household heads between 23 and 25 years, the last one those between 62 and 64. The maximum possible number of moments is thus 3 x 5 x 14 = 210.

Still, the actual number of moments used can be smaller for two reasons. First, in the case of Germany, there is no data available on household consumption, which already reduces the potential number of moments to 140. Second, due to the small number of observations for some age/unemployment duration combinations, it is not always possible to construct three independent moments for these groups. The actual number of moments used for the estimation turns out to be 210 for the UK, 161 for the US, and 136 for Germany.

As initial conditions for the simulation, every agent begins his existence at the age of 20 years without a job, with assets corresponding to one month of the average wage, with a skill level of one, and without any claims to unemployment benefits.

5. Results

Performing the second stage estimation for the thee countries, it turned out that no reasonably precise result for the coefficient of relative risk aversion could be obtained in the case of Germany and the US. As a practical solution, this parameter was thus constrained to be equal to 3.0. The results of the unconstrained estimation and a detailed description of the problems can be found in the appendix.[29]

The results of the first and second step estimation are presented in Table 2.

The estimators for Germany all exhibit realistic values and imply a plausible lifecycle-behaviour. Although individual productivity only grows at a rate of δ_e=1.6% per annum during employment, because of the low probability of being laid off and the moderate depreciation of human capital in this case, the average labour income of a cohort grows

[28] Further information on the choice of moments is given in the appendix.

[29] For the constrained estimation the number of generations computed by the evolutionary optimisation algorithm was reduced to 30.

substantially over the lifecycle, as can bee seen in Figure 2 (a). Average consumption starts declining around age 55, which is likely to be a consequence of the high degree of impatience implied by the relatively high estimated discount rate of 1-β=12%.

As Figure 1 (a) shows, the simulated unemployment rates by age quite closely resemble those found in the data. The higher unemployment rate of older individuals is fully due to an increase of long term unemployment. The share of those who are unemployed for less than 6 months is almost constant at just below 1% of the cohort, whereas the rate of those unemployed for more than a year increases steadily with age.

Table 2: Estimation results (standard deviations in parenthesis)

parameter	Germany	United Kingdom	United States
δ_e [per annum]	0.016 (0.002)	0.024 (0.003)	0.027 (0.003)
δ_u [per annum]	0.091 (0.031)	0.023 (0.009)	-0.023 (0.019)
δ_f	0.208 (0.013)	0.499 (0.005)	0.240 (0.040)
λ [per annum]	0.040 (0.003)	0.044 (0.002)	0.065 (0.003)
ω [per month]	23.37 (1.719)	6.199 (0.419)	23.88 (20.44)
σ_m	0.112 (0.019)	0.281 (0.004)	0.019 (0.008)
β [per annum]	0.880 (0.005)	0.934 (0.002)	0.985 (0.049)
γ	3.0 [a]	5.340 (0.069)	3.0 [a]
ϕ/γ	0.468 (0.038)	0.521 (0.021)	0.349 (0.019)
r [per annum]	0.033 (0.012)	0.036 (0.009)	0.014 (0.009)
χ	1981 (70.52)	374.1 (5.264)	1231 (101.9)

[a] by constraint

The generous German pension system allows most households to maintain a negative net asset position during most of their working live, retiring with only little wealth.

Reservation match qualities and hazard rates of "typical" agents depicted in Figure 3 (a1) and (a2). The three lines stand for household heads of different ages, namely 30, 45, and 60 years, and describe their behaviour during an unemployment spell of up to four years. Typical means that these agents are characterised by average assets holdings for their age group, average productivity, an average wage, and claims to an average pension and maximum-duration unemployment benefits before they are laid off. These figures show that the probability of accepting a job decreases rapidly, immediately after the beginning of the unemployment spell. Starting from a hazard rate of about 5 to 10% per week, which is comparable to the results obtained for Britain and the United States, the probability of finding a suitable job is reduced to almost zero within the first 36 months of the spell for all age groups. The figures also show that the switch from unemployment compensation to unemployment assistance benefits, which takes place after 12, 22, and 32 months for agents aged 30, 45, and 60 years, respectively, hardly affects the willingness to accept a job.

Looking at the results for Great Britain, two parameters stand out. First, the share of human capital lost at a job break-up δ_f is at almost 50% and thus maybe implausibly high. It implies that a worker who loses a job experiences a skill decline of some 52% during the first year. This is at odds with the data, which suggest that such a person would typically find a job paying about 79% of the last wage. Together with the job destruction rate λ and the growth rate of individual productivities δ_e it also implies a very flat labour income profile over the lifecycle, with a maximum expected wage growth rate of 0.05% per year. The average labour income profile depicted in figure 2 confirms this. Second, the estimated coefficient of relative risk aversion γ of 5.3 is relatively high, though not necessarily implausible.

Figure 1 (b) shows that both the actual and the simulated unemployment rates increase with age. The simulation suggests that this increase is fully explained by the increase in long-term unemployment. The number of agents unemployed for less than six months (one year) remains rather stable at around 1.5% (2.2%) of the cohort, while the share of those being unemployed for a longer time increases from less than 1% at age 23 to over 9% at age 60.

Figure 3 (b1) and (b2) again shows the change of the reservation wage and the resulting hazard rate from unemployment for typical agents of different ages over a time of four years. Normally, a 30 year old agent has relatively little wealth, so that when he loses contribution-based unemployment insurance after six months, he immediately qualifies for income-based benefits. Thus for such a household, the incentives to accept a job do not change immediately after this six-months period. The hazard rate slowly increases from about 2% to 5% per week during the first four years of the spell.

The situation looks very similar for a typical 45 year old agent. After losing the contribution-based benefit, he quickly runs down his assets to a level that allows him to apply for income-based benefits. Thus, he is subject to almost the same incentives as his 30 year old counterpart, and shows a very similar behaviour.

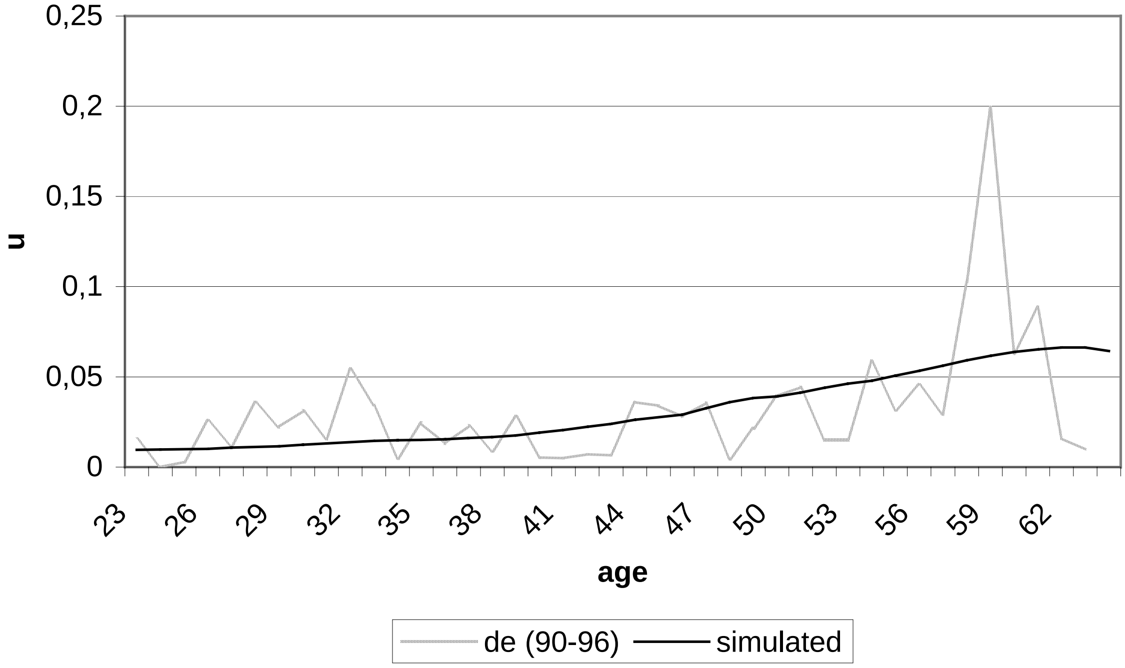
Agents nearing the age of 64 need to save for retirement. As the basic state pension considered in this model is rather low, older households with little wealth have a strong incentive to work, i.e. to be able to save rather than running down the assets holdings while being unemployed. Also, as the maximum duration of a new job decreases with age, the option value of unemployment declines. This explains the high hazard rates from unemployment of an agent who lost his job at the age of sixty. After four years of unemployment, when the worker is only a few months away from retirement, the probability of accepting a job becomes as high as 30% per week.

As older households also have accumulated significant assets, agents do not qualify for any further benefits after the first six months of unemployment. The additional incentive to accept a job created by the expiration of contribution-based benefits can clearly be seen in Figure 3 (b1) and (b2). During the first six months of unemployment, the hazard rate almost doubles from 3% to 5% per week.

The US estimates are slightly less precise than those for the other two countries. Like in the estimation of the German model, the risk aversion parameter γ was constrained to be 3. The other stage-two estimators exhibit plausible values, but some exhibit rather large variances. In particular, the distribution of the job offers, which is governed by the parameters ω and σ_m, is could not be estimated tightly.

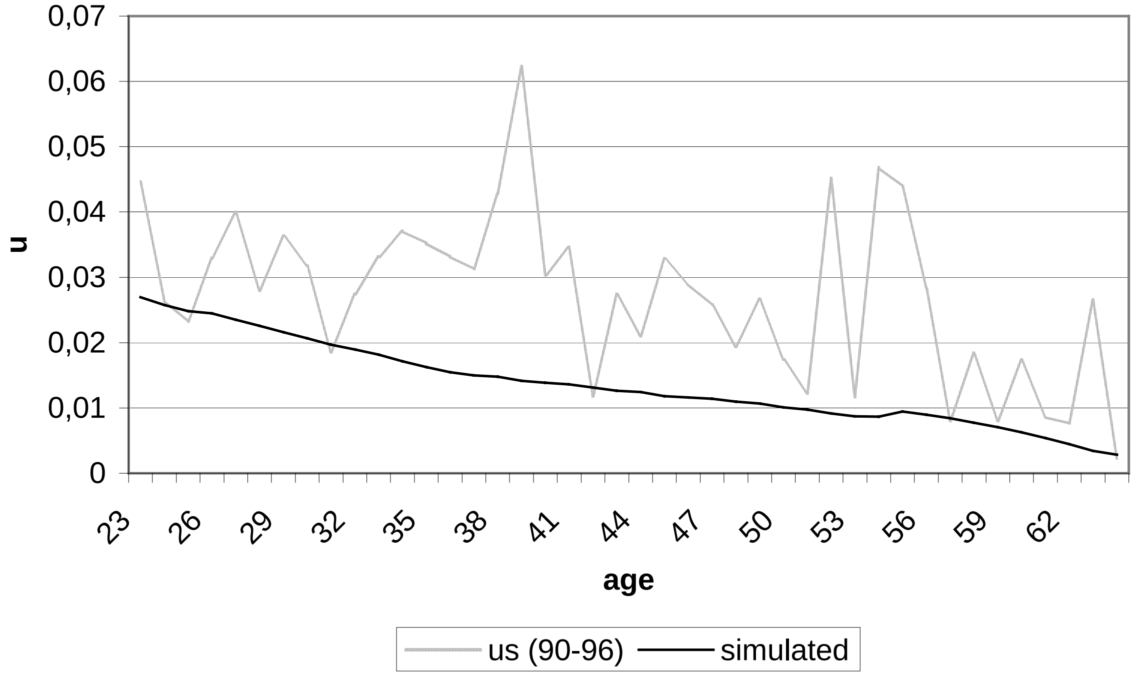
The depreciation rate of productivity during unemployment δ_u is estimated with a negative value of -2.3%. This is interesting because it means that the growth rate of productivity (and thus potential wages) is also positive during unemployment and only slightly lower than on the job. A worker who is laid off immediately loses about one quarter of his former productivity, but then does not further fall behind much further compared to employed workers in terms of potential wages during his unemployment spell. A natural explanation would be that unemployed individuals do not lose much human capital during the spell, but they benefit from general productivity improvements.

Figure 1: Empirical and simulated unemployment rates by age.



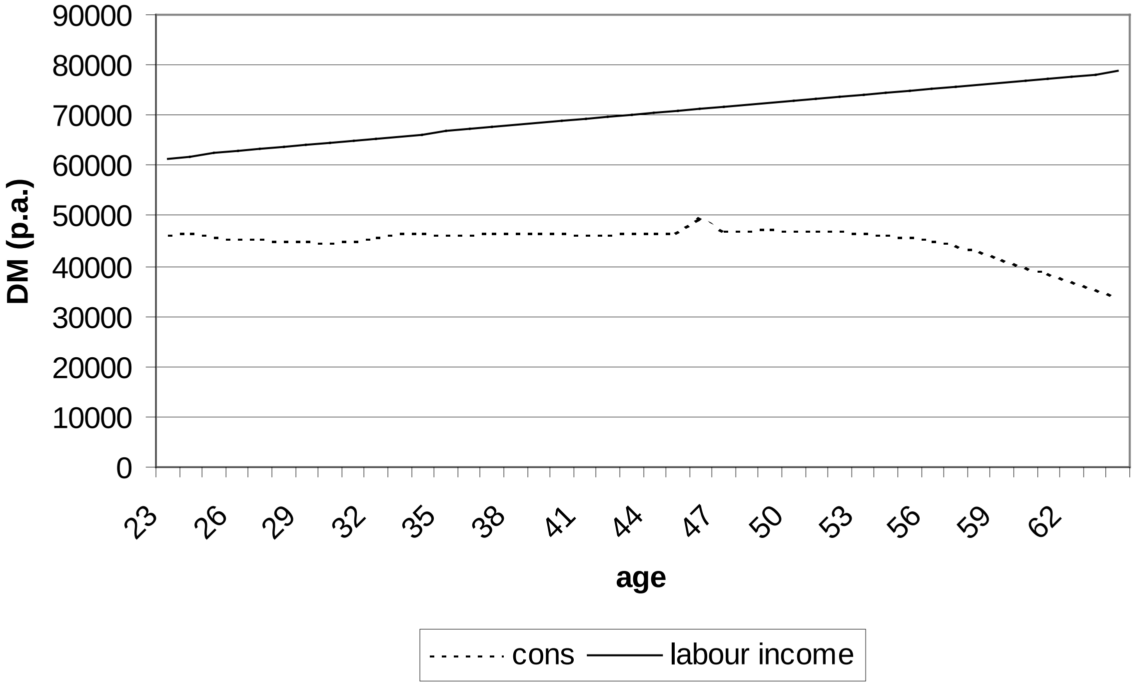
(a) Germany

(b) United Kingdom

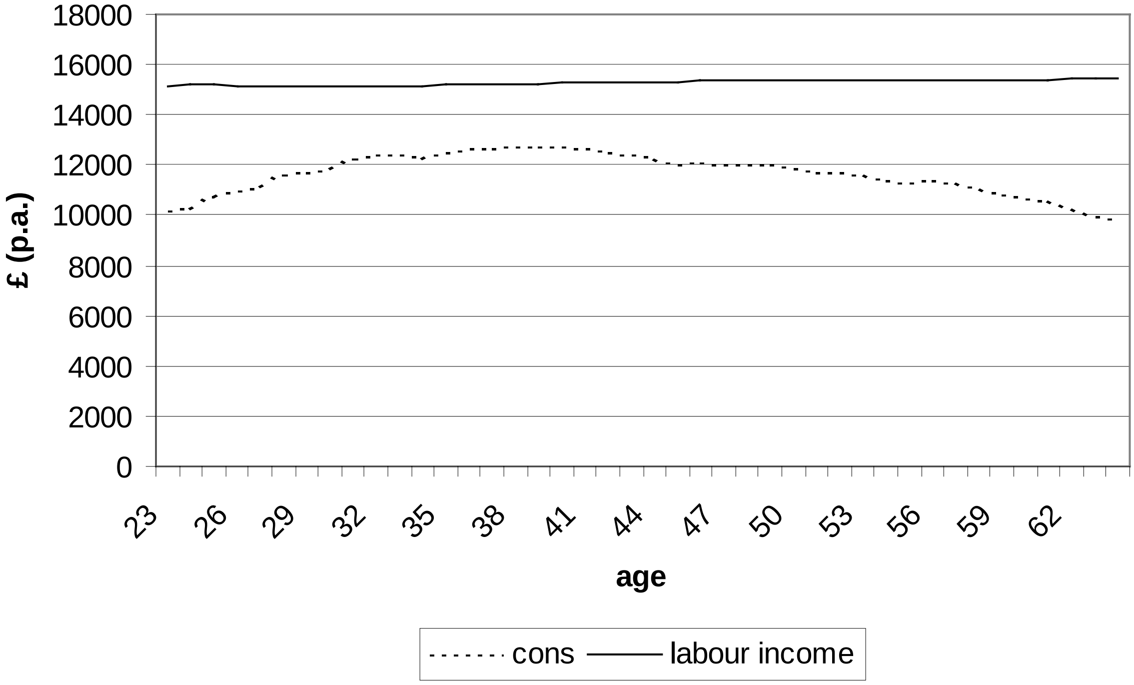


(c) United States

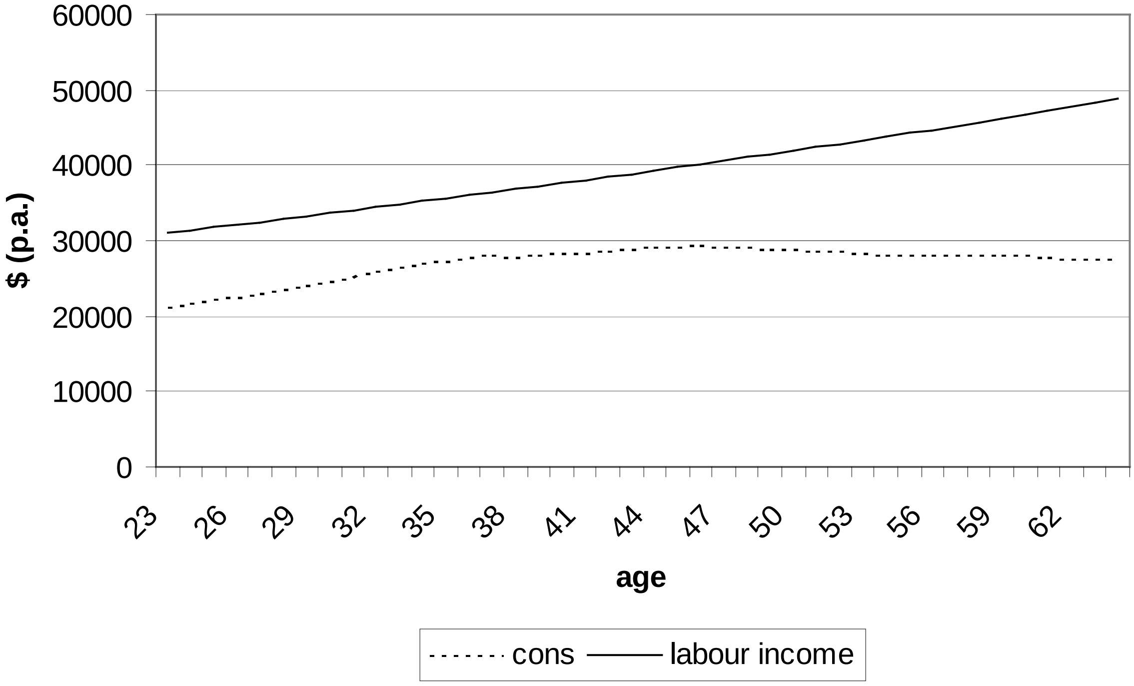
Figure 2: Average gross labour income and consumption.



(a) Germany



(b) United Kingdom



(c) United States

Figure 1 (c) again compares the simulated to the actual unemployment rated by age. The simulated values follow the same trend as those obtained from the dataset, but are lower. The observation that the unemployment rate decreases with age in the US, while is has the opposite trend in Germany and the UK, can at least partly be explained by the fact that the simulated long-run unemployment is virtually zero for the US.

Figure 3: Reservation match quality and hazard rates

(a1) reservation match, Germany

(a2) hazard rate, Germany

(b1) reservation match, UK

(b2) hazard rate, UK

(c1) reservation match, US

(c2) hazard rate, US

Figure 3 (c1) and (c2) show the reservation match qualities and hazard rates from unemployment for the same "typical" agents as before. The first six months of an unemployment spell, during which unemployment benefits are paid, are characterised by a fast decrease of the reservation wage and a corresponding increase of the hazard rate from unemployment. After the benefit duration expires, unemployed individuals aged 30 or 45 years are not subject to any significant changes of their incentives to accept a job for about two years. As a consequence, their hazard rate remains almost constant at a rather high value (20% and 23% per week, respectively). The strong increase of the hazard rates for those groups of agents, which occur about 2½ to 2 years after the beginning of the unemployment spell, marks the time by which households have run down their assets to a point where further dissaving is hardly possible and consumption can almost only be paid out of current wage income. Agents who are close to retirement are even more likely to accept a job than their younger counterparts, because the option value of unemployment is rather low and the time to accumulate wealth for retirement is running out.

6. Simulation

6.1. Simple Policy Experiments

In this subsection, some simple policy simulations are presented. The intention of this effort is not to present any thought-through policy alternatives. Without even a simple government budget constraint, the model presented in this paper is not well suited to rank different policies in terms of any efficiency criteria, let alone perform sophisticated welfare comparisons. The main focus is to highlight the consequences of the incentives created at the micro-level by different unemployment insurance policies for aggregate unemployment. Thus, the policies discussed are chosen not because of their feasibility or political relevance, but rather because they hopefully provide some insight into how simple policy modifications can affect the labour market outcome.

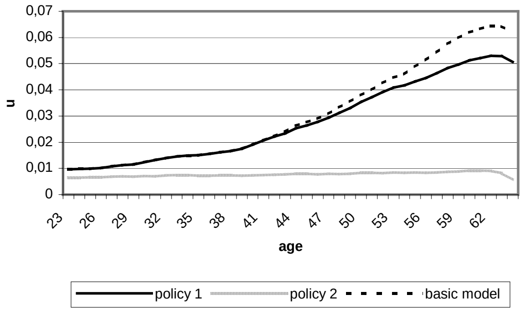
For Germany, the following two policy experiments are conducted. The first one restricts the maximum duration for unemployment compensation to twelve months. This change only affects unemployed individuals older than 42 years, who enjoy maximum benefit durations of 18 to 32 months. The second policy replaces unemployment assistance, which guarantees a defined replacement ratio for an infinite time, by social assistance.

Figure 4 shows the results of the policy experiments. Panel (a) depicts the resulting unemployment rates by age, and panels (b) to (d) show the hazard rates from unemployment again for the typical agents aged 30, 40, and 60.

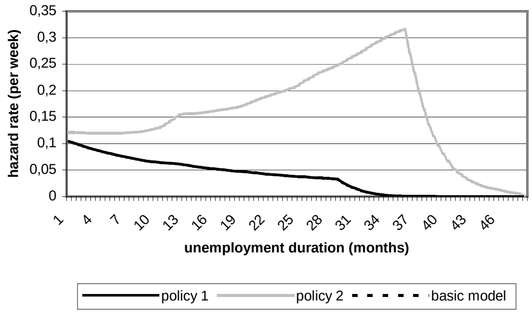
As expected, policy 1 does not affect the behaviour and unemployment rates of younger agents. The total reduction of the unemployment rate of 0.3% is rather small. However, (long-term) unemployment among the group workers older than 50 years is reduced by about one percentage point. Figure 4 (c) and (d) show that the incentive to accept a job is only slightly increased compared to the base scenario. Even with the reduced maximum duration of unemployment compensation, unemployment assistance is still attractive enough to induce older individuals to practically retreat from the labour market after about one year of unemployment.

Policy 2, on the other hand, reduces the overall unemployment rate to less than 1% and almost eliminates long-term unemployment. The incentive effect of the abolition of the relatively generous unemployment assistance is obvious in Figure 4 (b) to (d). The hazard rates start at a level similar to or above those in the baseline case, but do not immediately start to decrease. Rather, they remain almost constant until they pick up shortly before unemployment compensation is exhausted, which is the case after 12 months in Figure 4 (b), 22 months in Figure 4 (c) and 32 month in Figure 4 (d). It is interesting to note that these rather long benefit durations are not very harmful as long as the benefit is finally replaced by a sufficiently unattractive programme.

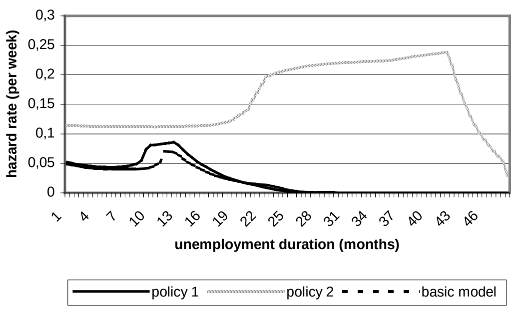
Figure 4: Policy experiments (Germany).



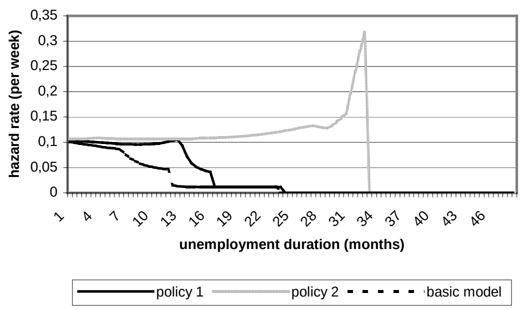
(a) unemployment rates



(b) hazard rates (age 30)



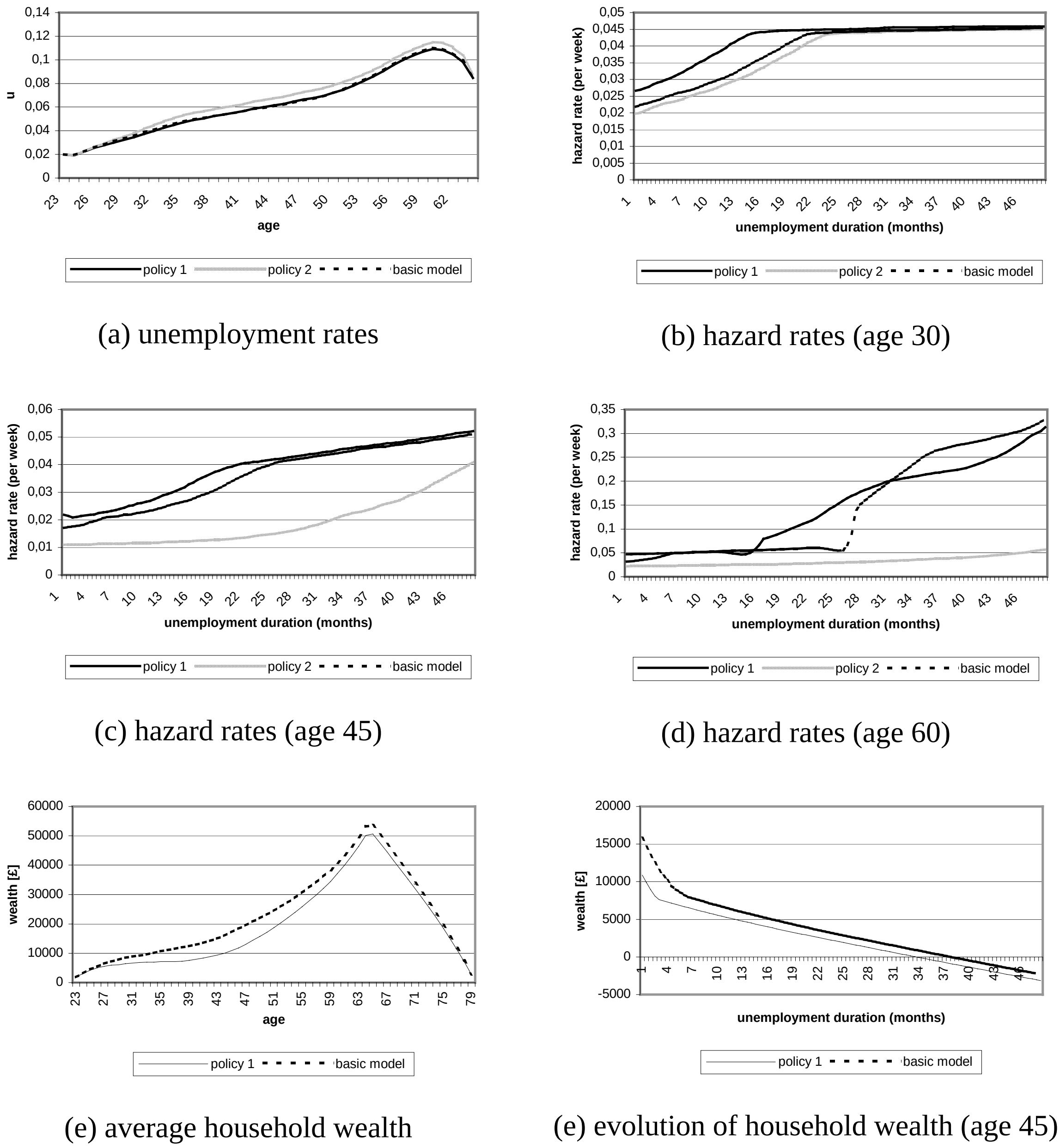
(c) hazard rates (age 45)



(d) hazard rates (age 60)

Consider now the following simple policy modifications for the UK. First, the replacement of the contribution-based jobseeker’s allowance by the means-tested income-based jobseeker’s allowance without any change of taxes or social insurance contributions. Such a policy would basically eliminate any special treatment of unemployed individuals, offering only a social assistance level of benefits to needy households. Second, the extension of the non means-tested unemployment insurance to unlimited benefit duration for all unemployed agents (without any contribution requirements).

Figure 5: UK policy experiments.



(a) unemployment rates

(b) hazard rates (age 30)

(c) hazard rates (age 45)

(d) hazard rates (age 60)

(e) average household wealth

(e) evolution of household wealth (age 45)

Figure 5 (a) shows the effect of these two experiments on the unemployment rates. While policy 1 hardly affects the unemployment rates, policy 2 tends to increase unemployment among workers older than 40 years. To highlight the mechanisms behind these outcomes, panes (b) to (d) of Figure 5 show the hazard rates from unemployment for a typical worker of age 30, 45, and 60, very much like in Figure 3 (b2).

It may be surprising that the policy 1 hazard rates for 30 and 45 year old agents (Figure 5 (b) (c)) differ from those in the baseline case even after several years of unemployment, although the only difference between the two scenarios is that in the basic model, unemployment compensation is not subject to a means test during the first six months of the unemployment spell.. The explanation is simple, however. The lack of a benefit that is not means-tested under policy 1 discourages agents from accumulating wealth beyond the threshold level of £ 8,000 for a long time, which is shown in Figure 5 (e).[30] The substantial wealth of 45 year old agents in the basic scenario would make them ineligible for benefits after half a year of unemployment, which creates a strong incentive to find a job. However, the typical 45 year old agent chooses to run down his assets from about £ 16,000 to £ 8,000 during the first six month of the spell. Still, at any duration he holds a larger wealth than the individual under policy 1, and therefore has a slightly higher reservation wage (see Figure 5 (f)). It is often argued that unemployment insurance has a negative effect the capital supply in an economy as it reduces the need to self-insure against job loss by saving. The comparison of policy 1 to the base case suggests that in the presence of a means-tested social assistance benefit, unemployment insurance with a benefit close to that of social assistance may actually stimulate saving by reducing the negative incentive effect created by the means-test.

Figure 5 (d) shows the effects of the policy experiments on older households, which have already accumulated assets for retirement. Whereas in the basic model and under policy 1 agents cannot hope for much support, as benefits are means-tested in the long run, policy 2 makes unemployment benefits available for an unlimited time, thereby clearly reducing the incentive to leave unemployment.

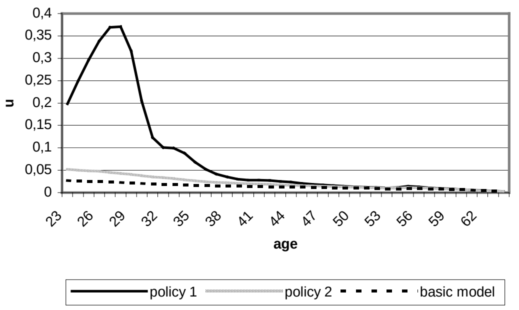
The two alternative policies considered for the US are first an infinite duration of the unemployment benefit and second an extension of the benefit duration from six months to one year. The unemployment rates and hazard rates that result under these policies are shown in Figure 6.

The effect of an unlimited benefit duration is quite extreme: the overall unemployment rate increases to 8%, and unemployment is strongly concentrated among younger workers. Agents who newly enter the labour market must still find an employment to become eligible for the

[30] The effect that individuals who are likely to become eligible for a benefit conditional on a means test adjust their wealth accumulation has been analysed by Hubbard, Skinner, and Zeldes (1995).

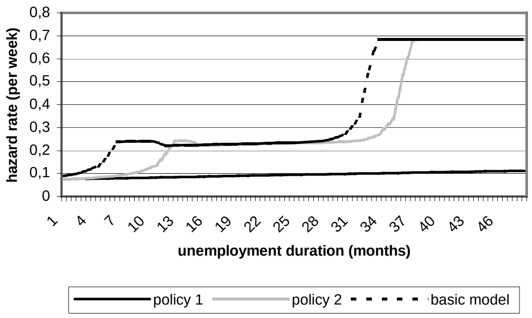
unemployment benefits. However, workers who lose their job before the age of 30 are not willing to accept any offer they can realistically expect. Agents older than thirty years behave differently. Immediately after being laid off, they accept the same range of jobs as under the base scenario. But whereas the hazard rates in the base case pick up quickly as the end of the unemployment benefit payments are nearing, the hazard rates under policy 1 only increase slowly during unemployment. The fact that hazard rates from unemployment rise at all with the duration of the spell is related to the estimated parameters that control the evolution of individual skills over time. Given the estimate for δ_u of –2.4% per annum, workers are not hurt by a long unemployment spell, as their individual productivity improves almost at the same pace as on a job. This implies that the ratio between potential wage income and the fixed unemployment benefit increases. This also explains why there is so little long-term unemployment in the simulated American economy.

Figure 6: US policy experiments.

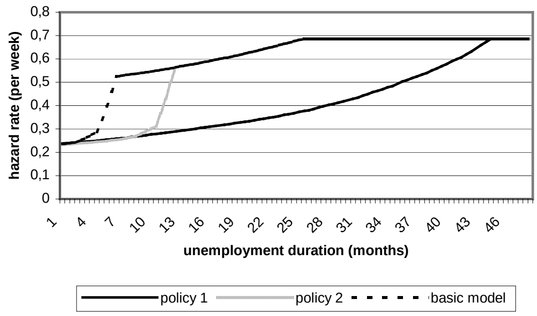


(a) unemployment rates

(b) hazard rates (age 30)



(c) hazard rates (age 45)



(d) hazard rates (age 60)

Policy 2, which simply doubles the benefit duration, also approximately doubles the unemployment rates for each age group. Figure 6 (b) to (c) show that this results simply from the agents' taking more time to wait for a good job immediately after they become unemployed. After the end of the benefit duration, they accept exactly the some offers as under the basic model. The increase of the average wages due to longer search is almost exactly offset by the increase of unemployment, and the wage sum hardly changes.

6.2. Hartz IV

In summer 2004, the German parliament passed the "Fourth Law for Modern Services on the Labour Market"[31]. This law, which is often referred to as "Hartz IV", implements central suggestions made by a labour market reform commission named after its chairman Peter Hartz.

The core of the reform is the substitution of a social assistance benefit paid at a fixed rate for unemployment assistance, which guaranteed a replacement ratio of over 50% for an infinite duration.

As a first step, on January 1st 2005, social assistance and unemployment assistance were replaced by a common benefit called unemployment compensation II (UC 2)[32]. This new means-tested benefit is paid at the rates of former social assistance (typically 331 Euro to 345 Euro for a household head). All recipients who are able to work are required to register with the unemployment agency. To avoid stark reductions of the benefit after claimants run out of unemployment compensation (UC), they receive "transitional payments" for two years. During the first year, this payment amounts to 2/3 of the difference between the UC and the UC 2 benefit, during the second year it is 1/3 of this difference.[33]

In a second step, starting in February 2006, the benefit durations of UC will be reduced for newly unemployed workers. From then on, the maximum duration will be one year for persons younger than 55 years, and 18 months for older claimants.

To analyse this reform, the new labour market policies are embedded into the model economy resembling Germany in 1997, which has been used in the estimation process.

Figure 7 (a) shows the simulated effects of the first and second stage of the reform (labelled "Hartz 05" and "Hartz 06", respectively) on the unemployment rates by age. For both policies, the unemployment rate is reduced to about 0.7% to 1.1% for all age groups, and long-term unemployment is virtually eliminated. Figure 7 (b) to (d) again illustrate the hazard rates under the two policies for agents of age 30, 45, and 60. It can be seen that the reduction

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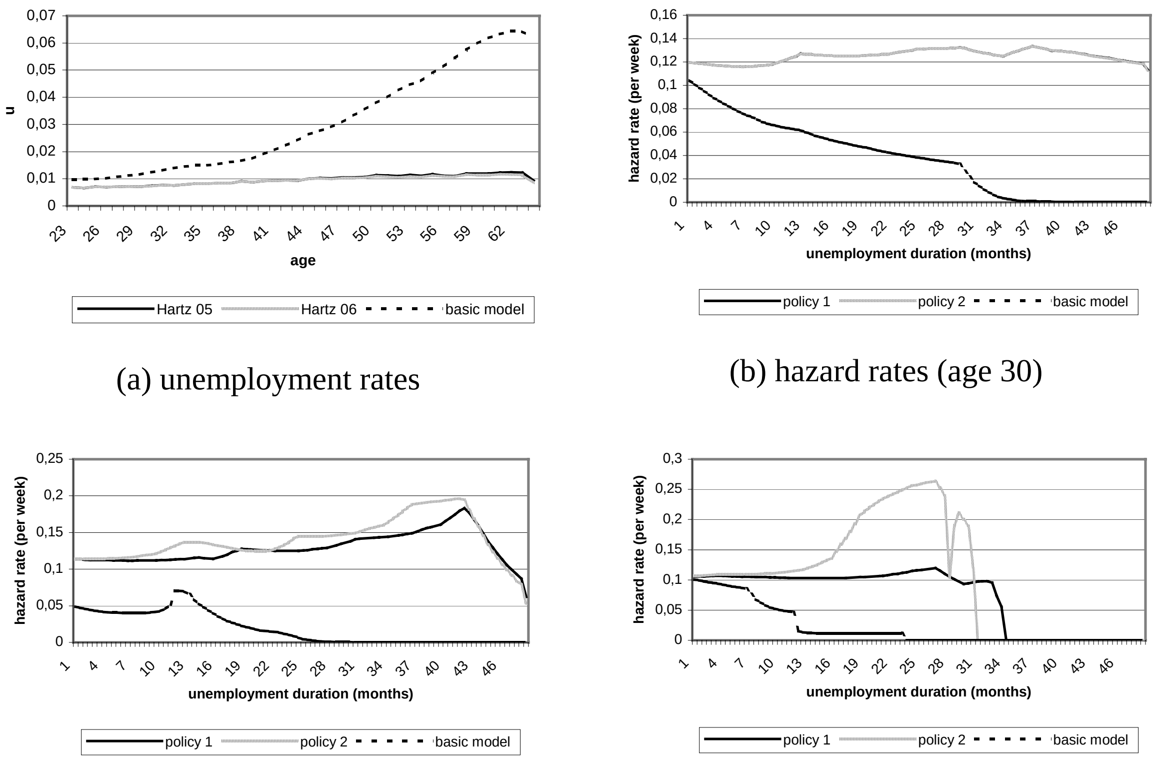
[31] "Viertes Gesetz für moderne Dienstleistungen am Arbeitsmarkt", Bundesgesetzblatt 2003, Teil I Nr. 66

[32] Arbeitslosengeld II

[33] The transitional payments are subject to a monthly ceiling of 160 Euro for the claimant + 160 Euro for her spouse + 60 Euro per dependent child.

of the maximum benefit duration does affect the willingness to accept jobs, but this effect does not have a strong influence on the unemployment rates, as 98% of the unemployment spells are shorter than one year even under the “Hartz 05” regime.

Figure 7: Hartz IV reform



(a) unemployment rates

(b) hazard rates (age 30)

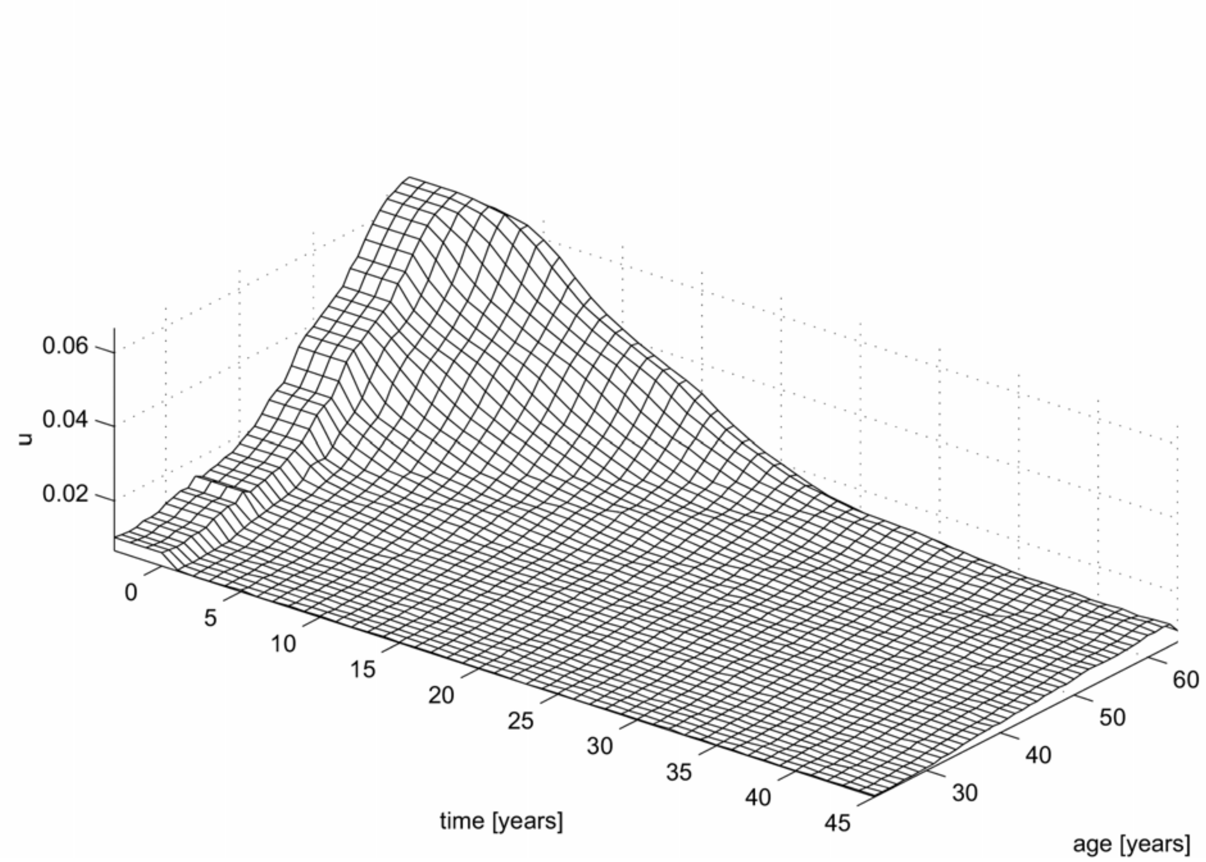
(c) hazard rates (age 45)

(d) hazard rates (age 60)

These figures suggest that – if the model is appropriate – the Hartz IV reform could be very successful in reducing the unemployment rates of the group of workers considered here (i.e. West-German household heads). However, the values presented thus far are (long-run) equilibrium results. In reality, it may take a long time for the existing unemployment to go down.

Figure 8 shows the simulated transition of the age-specific unemployment rates to the new equilibrium after an unanticipated introduction of the “Hartz 05” policy in period t=0. Within the first year after the change takes effect, the overall unemployment rate drops by roughly 0.4 percentage points from 3.1% to 2.7%. From then on, the further reduction of the unemployment rate takes place very slowly. Apparently, the policy change has little effect on the behaviour of the long-term unemployed. For many of them the reform comes too late in the sense that their skill level has already depreciated so much that even with a strongly reduced unemployment benefit, their reservation wage is still too high to find a job. It takes 5 years for the overall unemployment rate to go down by one percentage point, and even after this time, unemployment rate among individuals older than 60 years has hardly changed.

Figure 8: Transition to the new equilibrium after the introduction of the Hartz IV reform

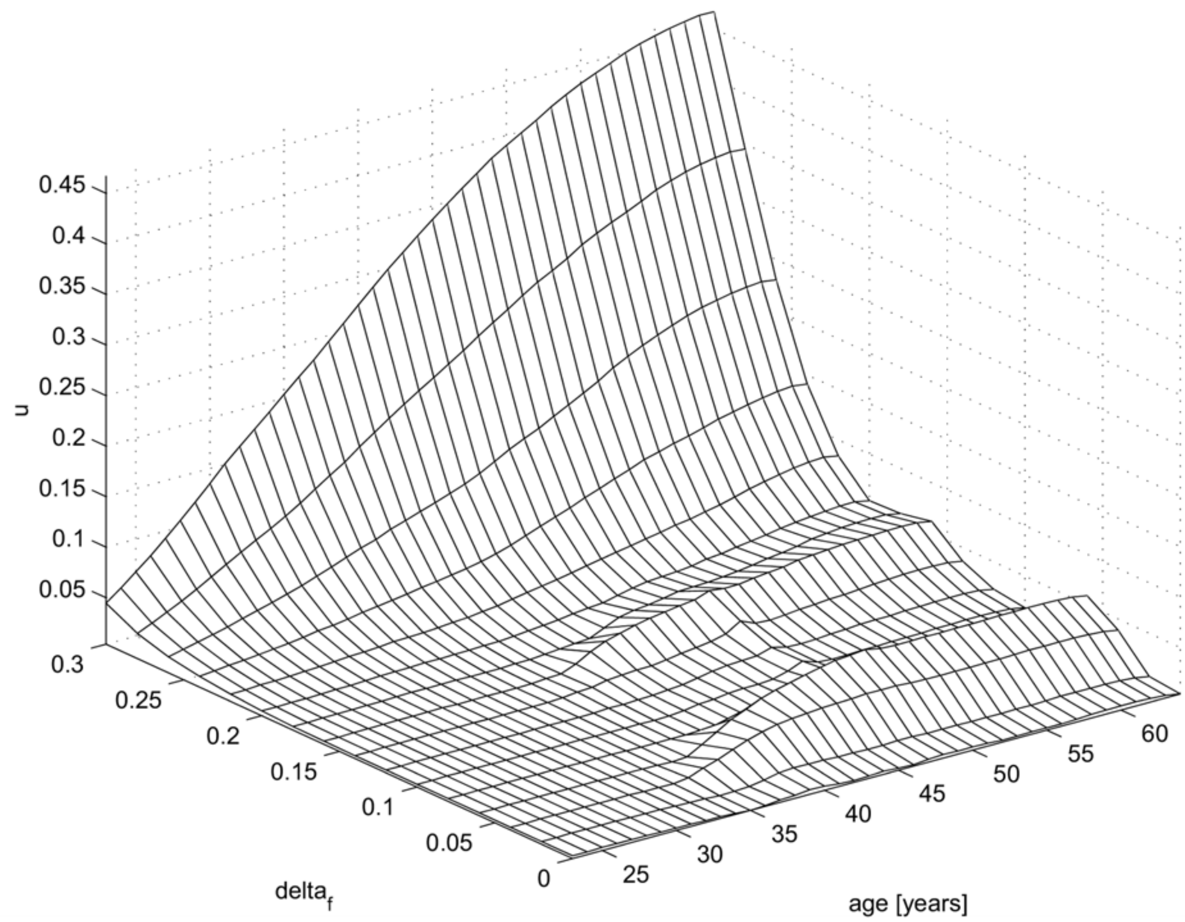


6.3. *The Effects of Turbulence*

Ljungqvist and Sargent (1998, 2003) argue that the rising unemployment in European countries could be explained as the consequence of an increase in economic turbulence. The authors present a model in which workers are characterised by different skill levels that govern their productivities on a job, and thus their wages. Productivities change stochastically between discrete levels. During employment, the skills improve by one level with a certain probability per period, while during unemployment they decline at a rate twice as high. After an exogenous separation between employer and employee, the worker is assigned a new skill level that is drawn from a probability distribution over the interval between his former skill and the lowest possible skill level. If the variance of the distribution is zero, an agent who loses his job does not suffer an immediate decline of his skills, whereas with an increasing variance, the expected value of the skill loss increases. Ljungqvist and Sargent associate this variance with economic turbulence.

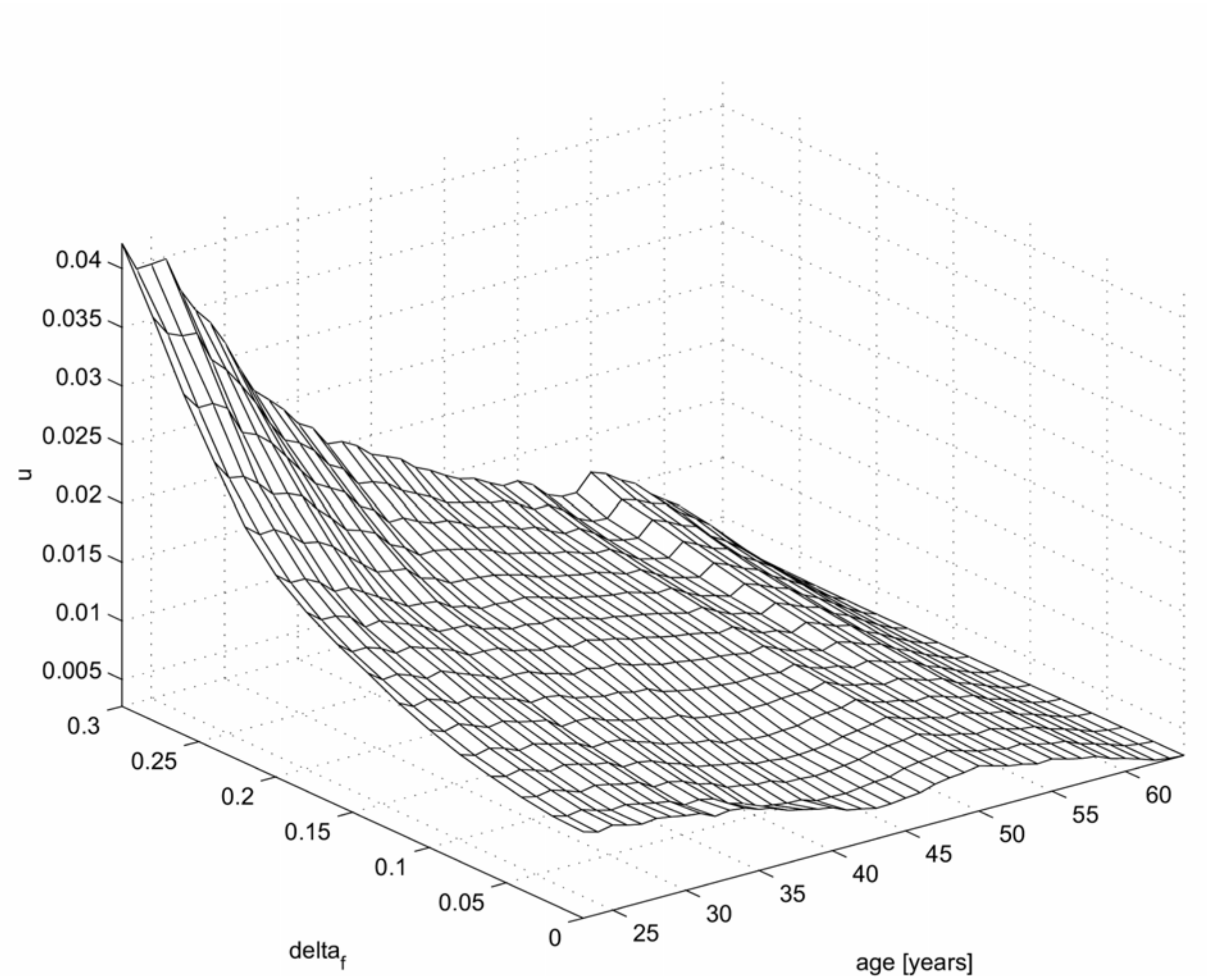
Economies that provide generous unemployment benefits that are tied to past wage income are more vulnerable to an increase of turbulence. If agents are guaranteed high replacement ratios, their reservation wages do not fall enough with the drop of their skill levels, and the unemployment rate increases. Ljungqvist and Sargent claim that an increase of turbulence that occurred over the last decades hit the European economies with their generous social security systems much harder than the United States.[34]

Figure 9: Relationship between the turbulence parameter δ_f and the unemployment rate



(a) Germany

[34] den Haan et al. (2001) criticise the turbulence hypothesis on the grounds that an increase of turbulence also worsens the outside option of employed workers, which reduces endogenous separations if they also cause a skill decline. Depending on the parameterisation of the model, this positive effect on the unemployment rate might dominate. As our model abstracts from endogenous separations, this question cannot be addressed here.



(b) United States

In the model presented here, the parameter of (deterministic) skill decline after job loss δ_f is very close to the definition of turbulence in the Ljungqvist and Sargent (1998) model. Figure 9 (a) and (b) illustrate how an increase of this parameter affects the unemployment rate-age profiles for Germany and the United States, respectively. The simulated US economy exhibits only a weak reaction to the variation of δ_f between zero end 30%.

For Germany, the increase of the unemployment rate in the rage between δ_f=0 and the estimated value of δ_f=0.2 is slightly stronger. However, the figure suggests that any further increase of turbulence would lead to an unbearable rise of unemployment. For example, a value of δ_f=0.24, which is close to the parameter estimated for the US, would imply an overall unemployment rate of more than 7%, which is more than twice as high as for the estimated value of δ_f=0.2. For a turbulence parameter of 30%, the unemployment rate is 30%. This strong effect is most likely caused mainly by the social security system with its high replacement ratios and the infinite benefit duration.

It is not clear, however, whether the relatively worse performance of the German labour market for lower degrees of turbulence should solely be attributed to the unemployment insurance system. The estimated parameters include a much lower depreciation of skills during unemployment δ_u for the US than for Germany, which ceteris paribus reduces long-run unemployment.

7. Summary and Conclusions

This paper presented structural estimates of a life-cycle model with unemployment. The central new feature of the model is that agents are characterised by an individual productivity or skill level that improves on the job and declines during unemployment. In combination with unemployment insurance system, this feature of the model can help to explain the structure of unemployment observed in actual economies.

Using panel data from Germany, the United Kingdom, and the United States, the model was estimated following a method of moments strategy. Most of the parameters obtained exhibit plausible values and imply a realistic life-cycle behaviour, however it was not possible to obtain realistic estimates of the risk aversion coefficient for Germany and the United States.

The model allows us to analyse the effects of policy changes on the life-cycle behaviour of agents as well as on macro-variables like the unemployment rate. Without introducing a budget constraint for the government, however, it is not possible to assess the welfare effects of policy changes. Introducing this kind of macro-constraints would therefore be an interesting extension of the model.

Another interesting extension could be the introduction of endogenous separations between employer and worker.[35] As den Haan et al. (2001) argue, strong declines of an agent's skills during unemployment, which in the present model lead to an increase of the unemployment rate if unemployment compensation is sufficiently generous, are likely to reduce en employed agent's outside option and thus job destruction rates. The introduction of endogenous job destruction might therefore lead to different conclusions concerning the effects of parameter changes.

Finally, it would be interesting to allow for (imperfect) monitoring of the job acceptance behaviour of unemployed agents. Whereas this model assumes that individuals can freely decide whether or not to accept a job, in reality unemployment agencies often require

[35] see e.g. Pissarides (2000), Chapter 2

unemployed workers to accept certain job offers. Individuals who do not comply are sanctioned, if their misconduct is detected.

Appendix

Summary of the Tax and Benefit Systems

This section summarises the tax and benefit systems as they were implemented in the simulations.

For the use with the simulated model, the taxes and benefits must be adapted in two ways. First, they have to be adjusted to the period length of ½ month. While this is rather innocuous in the case of benefits and proportional taxes, it tends to increase the progressivity of income taxes. Second, as the simulation makes use of *average* demographics, the rules must be generalised to allow for non-integer variables like 1.3 children and 0.6 of a spouse.

It should be noted that any effect of inflation on taxes and benefits is ignored, i.e. income taxes are paid on real interest income and there is no real decline of unemployment benefits tied to former nominal wages.

Germany

Social Security Contributions

Paid on labour income, at equal rates by employer and employee. For each side, the rates are 20.85% on the first DM 6,150 and 13.4% on the income between DM 6,150 and DM 8,200

Income Tax

deductions and exemptions (p.a.):

on labour income:	DM 2,000
on capital income:	DM 6,100 (x 2 if married)
on pensions:	DM 200
for aged persons:	40% of non-labour income (max. DM 3,720) if older than 63
for parents:	DM 3,456 per child (x 2 if married)
social security contributions	

income tax (p.a.):

income x	tax	
0-12,095	0	
12,096-55,727	(86.63y+2590)y	where y=(x-12,042)/10,000
55,728-120,041	(151.91z+3346)z+12,949	where z=(x-55674)/10,000
120,042-	0.53x-22,842	

splitting: couples pay 2 x tax((common income)/2)

none of the benefits described below are taxable except pensions, which are only partially taxed (between 27% for 65 year old tax-payers and 12% for 79 year olds)

the income tax is increased by a surtax of 5.5%

Benefits

child benefit:

paid monthly at the rate of DM 220 for the first and second child, DM 300 for the third, and DM 350 for any further child

unemployment compensation:

replacement ratio: normally 60%; 67% if at least one dependent child

contribution requirements: at least 12 months since last spell

For a contribution period of 12 months the benefit duration is 6 months. For each further 4 months of contribution payments the benefit duration is increased by 2 months up to the maximum duration.

age	<42	42-44	44-49	49-54	>=54
max. ben. duration (months)	12	18	22	26	32

unemployment assistance:

paid to persons who run out of unemployment insurance

replacement ratio: normally 53%; 57% if at least one dependent child

no contribution requirements, no maximum duration

means test: wealth limit: DM 1,000 x age (x 2 if married)

social assistance:

basic rate: DM 534 per month

household member	head	further adult	avg. child
benefit (% of basic rate)	100%	80%	65%

means test: wealth limit: DM 2,500 +
DM 2,000 (if older than 60) +
DM 1,200 (if married) +
DM 500 for all family members other than claimant

pensions:

during their working life, individuals accumulate “remuneration points” (RP): one year of contributions paid from an “average” income is worth one point; the exact formula used to compute the points for a year is:

(min{labour income,12 x 8,100})/52,143

During unemployment, contributions corresponding to 80% of the last wage are assumed.

The monthly pension payment amounts to: DM 40.51 x RP’, where RP’ are adjusted remuneration points: if the average points earned per contribution year are below 0.75, RP is increased by 50% (up to a maximum of 0.75 average points) to obtain RP’.

United Kingdom

Social Security Contributions

Jobs are subject to social security taxes if the wage is above the threshold level of £ 62 per week. Employers pay a proportional tax, the rate of which depends on the wage.

wage (per week)	<62	62-110	110-155	155-210	>210
tax rate	0%	3%	5%	7%	10%

Employees who earn a wage above the threshold level pay 2% on the first £ 62 per week plus 10% on the rest of their wage income up to a wage limit of £ 465.

Income Tax

deductions and exemptions (per annum):

£ 4,045 (£ 5,220 if older than 65, £ 5,400 if older than 75)

income tax rates:

income	tax rate
0-4,100	20%
4,100-26,100	23% (20% on capital income)
26,100-	40%

tax credits:

15% of £ 1,830 (£ 3,185 if older than 65, £ 3,225 if older than 75) for married couples

family credit:

paid at a weekly rate of £ 58.20 (plus £ 16.66 for each child on average) to an adult working full-time

The benefit is reduced by 70 pence for each pound that disposable income exceeds £ 77.15 per week. An income of 0.4% per week is assumed on assets above £ 3,000 (£ 6,000 if older than 60).

means-test: wealth must be below £ 8,000 (£ 12,000 if older than 60)

Couples always file separately. All benefits except child benefits are taxed in principle, but they are low enough to usually remain untaxed.

Benefits

child benefit:

£ 11.05 per week for the first child, £ 9 for every further child

general benefit rates (per week):

single	£ 49.15 (£ 38.90 if younger than 25)
couple	£ 77.15
child	£ 20.97 (on average)
family premium	£ 10.80

unemployment compensation (contribution-based jobseeker's allowance)

paid at the general benefit rates for a maximum duration of six months

contribution requirement: contributions corresponding to fifty times the social security contribution threshold of £ 62 per week

unemployment assistance (income-based jobseeker's allowance)

paid at the general benefit rates to unemployed not eligible for the contribution-based allowance

means-test: wealth must be below £ 8,000 (£ 12,000 if older than 60)

an income of 0.4% per week is assumed from assets above £ 4,250 (£ 5,500 if married) for individuals younger than 60, and from income above £7,250 (£ 8,500 if married) for older individuals

pensions:

The full basic state pension amounts to £ 64.70 per week. A spouse receives a derived pension of £ 38.70.

Council Tax and Council tax benefit:

following OECD (1999b), a council tax of £ 12.95 per week is assumed

The benefit amounts to 80% of the difference between disposable income and need up to the amount of the tax. Need is given by the general benefit rates.

The means-test and assumed asset income is the same as for family credit.

United States

Social Security Contributions

The social security tax rates, including unemployment insurance, amount to:

annual wage income bracket	employee	employer
0-7,000	7.65%	12.85%
7,000-9,500	7.65%	12.05%

9,500-65,400	7.65%	7.65%
65,400-	1.45%	1.45%

Federal Income Tax

deductions and exemptions (per annum):

exemptions	$ 2,650 per household member if income is above $ 121,200 ($ 181,800 for couples), the exemption is phased out in a linear fashion over an income range of $ 125,000
standard deductions	if younger than 65: $ 4,150 ($ 6,900 for couples) if older than 65: $ 5,150 ($ 8,500 for couples)

tax rates:

income (single)	income (couple)	tax rate
0-24,650	0-41,200	15%
24,650-59,750	41,200-99,600	28%
59,750-124,650	99,600-151,750	31%
124,650-271,050	151,750-271,050	36%
271,050-	271,050-	39.6%

unemployment benefits are taxed, social security income is partially taxed (up to 85% are included in the tax base if income is above $25,000 for singles and $32,000 for couples)

Michigan State Tax

deductions and allowances:

exemptions	$ 2,500 per household member
older than 65	$ 900 (also available to spouse)
unemployment	$ 900 if unemployment insurance benefit amounts to more than 50% of gross income

tax rate:

the tax rate in 4.4%

Social security income is not taxable

Earned Income Credit

Earned Income Credit (EIC) is available to working individuals who are between 25 and 65 years old or have children and hold assets worth less than $ 2,250. The general design of the credit is as follows: the EIC increases linearly from 0 to C for gross incomes between 0 and I_1, remains constant for incomes between I_1 and I_2, and decreases linearly to zero for incomes between I_2 and I_3.

number of children	I_1	I_2	I_3	C
0	4,340	5,450	9,770	332
1	6,500	11,950	25,760	2,210
>1	9,140	11,950	29,290	3,656

Benefits

unemployment compensation:

The maximum benefit duration is six months. The benefit is paid at a replacement ratio of 50%, subject to a lower limit of $ 377 per month and an upper bound of $ 1,300.

contribution requirement: at least half a year at an income above $ 3,084.

food stamps:

available to low income households

means-test: wealth must be below $2,000 ($3,000 if older than 60)

income test: both “basic income” (disposable income) and “counted income” (basic income minus allowances: $134, 20% of labour income, $177 per child) must be below certain limits

the monthly rates are:

household size	basic income limit	counted income limit	benefit
1	893	671	125
2	1,176	905	230
3	1,479	1,138	329

To compute the actual allotment, the benefit rates are reduced by 30% of the labour income.

supplemental security income (SSI):

available to persons older than 65 years

means-test: wealth must be below $2,000 ($ 3,000 for couples)

benefit amount: $484 per month ($ 726 for couples)

the benefit is reduced one-for-one for every dollar earned above the monthly limit of $ 10

pensions (“social security”):

The pension is computed from an individual's "Average Indexed Monthly Earnings" (AIME) during the working life. AIME is the average monthly income up to a limit of $ 5,450 of the best 35 contribution years.

The monthly pension is highly regressive in AIME:

90% of the first $ 455 of AIME, 32% of the next $ 2,286, 15% of the rest.

The derived pension of a spouse is 50%.

Discrete Approximation and Grids

The model includes up to five continuous state variables. For the numerical computation, the value functions and choice variables were computed for a (finite) number of values that lie on 5-dimensional grids. The construction of these grids along the five dimensions is sketched in this appendix.

Assets

For all tree countries, the asset grid is composed of 120 points covering the range of –4 x APW to 36 x APW, where APW is the average annual production wage as reported by the OECD. These limits are far beyond the values typically realised in the simulations. For improved accuracy, the points are concentrated around zero. The two main reasons for doing this are first that a large share of the simulated population holds relatively little wealth and second the fact that the tax and benefit system (in particular means-tested benefits) has most distorting effect on the saving decision of individuals with little wealth. Also the value functions are more concave for low asset values.

Country	Germany	United Kingdom	United States
1997 APW	DM 58,338	£ 16,760	$ 28,584

Skills

The grids for the individual skill level are designed to cover all values that can be reached during a working live given the first stage estimate of δ_e. It is composed of seven equally spaced values between zero and one plus five to eight values above one. To save memory and computation time, the grid size is reduced for younger agents taking advantage of their lower maximum skill level.

Match Quality/Unemployment Benefit

The match quality is a variable only relevant for employed agents. For unemployed, the variable is used to store the level of the unemployment benefit. Ten grid points of the match quality represent the medians of quantiles of the distribution of job offers, which are chosen such that (1) the probability weights of the respective quantiles decrease in a linear fashion and (2) the highest of these values represents a match quality for which one has to wait two years on average. This set of points is amended by two higher values corresponding to 50% increases of the match quality.

The unemployment benefit is computed on an equally spaced grid (Germany: 10 values, US: 6 values, UK: 1 value, as the benefit only depends on demographics) that covers the admissible range.

Unemployment Insurance

A further state variable is used to describe the current eligibility status for unemployment compensation if employed (i.e. to what degree the contribution criterion has been met) or the remaining benefit duration if unemployed. For the US and the UK, grid sizes of four to six points suffice, but for Germany, with its maximum benefit duration of 32 months, up to nine points are used.

Pension

The last state variable to be discussed here represents the claims to the pension system (for the UK, where all agents are assumed to receive the same pension payments, this variable takes on only one value). As this variable is likely to become more important as agents are approaching their retirement age, finer grids are used for older agents. For the first 360 periods (age 20 to 34), only 3 values are used, for the next 360 periods (age 35 to 50) five values, and for the rest of the lifecycle a nine-pint grid is employed.

Computation of the Optimal Asset Level

Given the complexity of the state space, it is essential to compute the value function efficiently in order to keep the computation time at an acceptable level. Since it has to be done for all periods and states, the fast calculation of next period’s wealth level a' (equivalently: consumption or saving) as the optimal control is central to the runtime performance of the program.

The optimal wealth a' is the solution of the maximisation problem given by the right hand side of equation (4), or, ignoring the other state variables to simplify notation,

$$a' = \arg\max_{\alpha} \{\mathrm{u}(x - \alpha / R) + \beta\, \mathrm{EV}(\alpha)\}, \tag{12}$$

where u is again the period utility function, x is disposable wealth and EV is next period's value function, which is has been computed already. Since the value function is computed on a grid and linear interpolation is used, EV is effectively a piecewise linear function. Using the definition of u (6), it is possible to find a closed form solution for a' once it is known in which linear segment of EV it is to be found. If EV is assumed to be concave, it is also easy to decide whether a' must lie to the left, to the right, or in a kink of the function EV.

Let n be the size of the grid describing the asset state variable and take the number of states associated with the other variables as constant. a' can then be calculated for all states in $\mathrm{O}(n \log n)$ time using a simple nested-intervals algorithm. However, one can do better than that. For any combination of states, next period's optimal decision in the resulting situation is likely to be similar to this period's optimal choice. It is therefore a promising strategy to take next period's optimal decision, which is already known, as a starting point and then simply search for the segment of EV containing a' by moving along the grid in the right direction. In spite of having a worst-case time complexity of $\mathrm{O}(n^2)$, this algorithm exhibits $\mathrm{O}(n)$ runtime if optimal controls are "similar enough" across periods and clearly outperforms the nested-intervals approach.

Unfortunately, due to the complicated structure of the institutional framework (including means tests, effective marginal tax rates above 100%, and the like), the value function cannot be expected to be globally concave in wealth. To deal with this problem, the value function EV is first turned into a concave function EV^c by calculating the convex upper contour line of its graph. This can be done in $\mathrm{O}(n)$ time using an algorithm that simply drops points from the polygon representing EV lying within its convex hull. As EV and EV^c are the same for all levels of disposable income x, all other states being equal, the total time complexity remains at $\mathrm{O}(n)$. The optimal controls are then first calculated from EV^c. Only if a' lies in a segment of EV^c that has been created by "stretching out" a non-concave area of EV, the respective segments of the original value function are searched for the optimal level of a'.

The Choice of Moments and the Weighting Matrix

To identify the structural parameters of the model, a set of simulated population moments is matched to its empirical counterpart obtained from the data. The moments chosen are basically the expected values of gross labour income x^y, periods unemployed x^u, and consumption growth x^c, all conditional on periods unemployed last year $x^{u,-1}$, by age group. There are 24 periods per year. Each observation of a household in a certain year is taken as a single independent data point for the calculation of the moments, i.e. I do not exploit the panel dimension of the samples in this respect. Let $j = 1...J$ index the age groups and $i_j = 1...I_j$ the observations belonging to age group j. The moments used are thus

$$m_{jkm} = \mathrm{E}[x^k \mid j, x^{u,-1} \in G_m]\,\mathrm{prob}(x^{u,-1} \in G_m)\,, \tag{13}$$

where j indicates the age group, $k = y,u,c$, and $m = 1...5$ index groups of unemployment durations, defined in terms of periods unemployed by $G_1 = \{0\}$, $G_2 = (0,6]$, $G_3 = (6,12]$, $G_4 = (12,24)$, and $G_5 = \{24\}$. Making use of the household weights ω_{ij} for the samples used, the empirical moments are calculated as

$$\tilde{m}_{jkm} = \sum_{\substack{i \in \{1...I_j\} \\ x_{ij}^{u,-1} \in G_m}} x_{ij}^k \omega_{ij} \Big/ \sum_{i=1}^{I_j} \omega_{ij}\,.$$

An optimal weighting matrix for the estimation is the inverse of the covariance matrix of the moments. In principle, the variances of both the empirical and the simulated moments would have to be taken into account. Choosing a number of simulated cases much greater than the number of observations in the data set, the variance of the simulated moments becomes very small and can be ignored without much loss in precision.[36] As all observations – and therefore as a special case observations for different age groups – are treated as independent, the estimated weighting matrix $\tilde{W} = (w_{jkm,j'k'm'})^{-1}$ has a block diagonal structure. The typical (non-zero) element of $\tilde{W}^{-1}$ is computed as

$$w_{jkm,jk'm'} = \sum_{i=1}^{I_j} (1(x_{ij}^{u,-1} \in G_m) x_{ij}^k - \tilde{m}_{jkm})(1(x_{ij}^{u,-1} \in G_{m'}) x_{ij}^{k'} - \tilde{m}_{jk'm'})\omega_{ij}\,,$$

where 1() is the indicator function, which is 1 if its argument is true, 0 otherwise.

It can happen that there are not enough observations per year and unemployment duration group for the estimated covariance matrix to be regular. In such cases, moments are dropped until the matrix has full rank.

As both the moment vector and the weighting matrix can be computed from the data without simulation, once obtained, they can be used for the entire estimation process. This means that there is no need to apply a two step estimation procedure like in Hansen and Singleton (1982).

[36] I simulate 50,000 complete histories and therefore obtain 50,000 x 42 = 2.1 Mio individual-year observations for agents aged 23 to 64. This has to be compared to a total of about 9,500 to 15,000 cases in the data sets. To obtain a numerical approximation of the derivative of the moments with respect to the parameters, which is used for the computation of the variances of the estimators, 200,000 histories are simulated.

Unconstrained Estimation for Germany and the US

Table 3 reports the results of the unconstrained MSM estimation for Germany and the United States. The values reported in column 3 for the US show that the rate of relative risk aversion γ is estimated far less precisely than the other parameters. Moreover, as the 90% confidence interval stretches from about –4 to +5, the point estimate does not reveal much reliable information about the actual magnitude of the risk aversion parameter.

Constraining γ to be equal to 3 interestingly hardly affects the estimates obtained for the other parameters. Comparing the results to those of the constrained estimation reported in Table 2, none of them are significantly different, and most are almost the same. The parameter ω exhibits the biggest deviation between the two estimations, but it is estimated rather imprecisely in both cases.

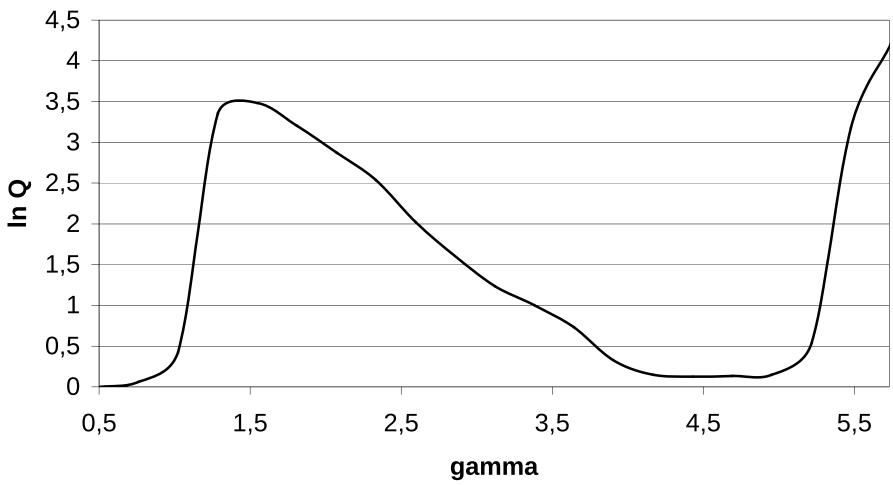
Table 3: Unconstrained Second stage estimation results (standard deviations in parenthesis)

parameter	Germany I	Germany II	United States
δ_u [per annum]	0.213	0.110	-0.024
	(0.085)	(0.031)	(0.012)
δ_f	0.154	0.199	0.235
	(0.042)	(0.016)	(0.044)
ω [per month]	26.95	18.52	32.78
	(11.32)	(2.322)	(15.26)
σ_m	0.218	0.127	0.019
	(0.047)	(0.002)	(0.010)
β [per annum]	0.846	0.868	0.985
	(0.125)	(0.006)	(0.025)
γ	0.507	3.390	0.517
	(0.587)	(0.166)	(2.710)
χ	1548	1945	1220
	(185.3)	(8.610)	(110.4)

Column one of Table 3 shows the results of the unconstrained estimation for Germany (labelled “Germany I”). Similarly to the unconstrained US estimation, the point estimate of the parameter γ is has a very low value and exhibits a large variance. In column two of Table 3, an alternative estimation (“Germany II”) is presented, which was obtained using an only slightly different modelling of the unemployment insurance system. The results in this case are not, as one would expect, close to the Germany I estimators. The most evident difference occurs for the parameter γ, which is estimated at 3.39 in the Germany II case.

The reason for this obvious lack of robustness can be seen in Figure 10. The graph shows the profile of the log criterion function used for the estimation of the Germany I scenario along its γ dimension, evaluated at the minimising parameters (i.e. $\ln \hat{Q}(\gamma)$ where $\hat{Q}(\gamma) = Q(\hat{\theta}_1, \hat{\theta}_2, ..., \gamma, ...\hat{\theta}_7)$, in the notation of section 4.1). One can clearly see that the criterion function has two local minima in this direction, one at rather low values of around 0.5 and another one for a higher level of risk aversion. The minimisation algorithm sketched in section 4.2 is unable to correctly choose between these two local minima, as the function value of Q is almost the same in both cases. Comparing the Germany I and Germany II estimation, one can see that the different choice of the γ parameter has a considerable effect on the other estimators as well. The best solution to this problem would be to evaluate the objective function Q more accurately, i.e. for a greater number of parameter combinations. Yet, because of the computational cost associated with evaluating Q, this is impracticable. Therefore, γ is simply constrained at the plausible value 3.0.[37] The resulting estimators are very close to those obtained in the unconstrained Germany II case.

Figure 10: Projection of the criterion function for unconstrained estimation (Germany)



[37] This value lies within the 99% confidence interval for γ in the Germany II estimation.

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