ABOUT A "NEW" INEQUALITY INDEX

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1. INTRODUCTION

Recently some authors (e.g. Slottje et al.(1989)) dealt with an inequality index based on the length of the Lorenz curve, attributing the same to Kakwani (1980). As a matter of fact such an index was proposed for the first time by Amato (1968).

2. SOME RESULTS

Let us suppose that income $X \in [0,\infty)$ is a random variable with differentiable cumulative distribution function $F(x)$ and first moment distribution function ${}_1F(x)$

$$(1) \qquad F(x) = \int_0^x dF(t); \qquad {}_1F(x) = (1/\mu)\int_0^x t\, dF(t)$$

where μ exists, is finite and $\neq 0$. Equations (1) define the Lorenz curve in the orthogonal plane $(F(x); {}_1F(x))$. The length ℓ of this curve is

$$(2) \qquad \ell = (1/\mu)\int_0^\infty (x^2 + \mu^2)^{\frac{1}{2}} f(x)\, dx$$

and given that $\ell = \sqrt{2}$ in the case of perfect equality and $\ell = 2$ in the case of perfect inequality, a normalised measure may be defined as

$$(3) \qquad L = (\ell - \sqrt{2})/(2 - \sqrt{2}) \qquad 0 \leq L \leq 1$$

Kakwani (1980,p.83) labelled measure (3) as "new", but in effect it is by no means new in so far as it was already introduced in literature by Amato (1968,p.261). More or less at the same time as the latter, other two Italian statisticians carried out studies on L. More explicity Scala (1969), within a Paretoan Distribution, tested its sensitivity in comparison with other indices including the Gini concentration ratio. Lombardo(1969)dwelled on certain characteristics of the length ℓ of the Lorenz curve, subsequently demonstrating (Lombardo,1979) that its sampling distribution is nor-

mal, and pointing out that by variate transformations, such a result is valid also for L.Another interesting contribution was made by De Simoni (1979) who extended the study of index (3) to the multivariate case. The index L was rediscovered, albeit in a slightly different context, by Piesch (1975,p.99) and subsequently by Dagum (1980,p.338). Whereas Kakwani (1980,p.84-85,101-103) must take the credit for having studied in depth other most interesting aspects since he derived the bounds on L and demonstrated its aptitude for measuring the degree of poverty.

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RÉSUMÉ

L'Auteur montre qu'un indice de la disparité du revenu proposé par Amato (1968) a été successivement redécouvert par nombreux spécialistes qui en ont approfondi et étendu certains aspects.