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A Methodological Survey of Recent Studies for the Measurement of Inequality of Economic Welfare Carried out by Some Italian Statisticians

1. *Introduction*

Starting from the beginning of the 1970's, following the contributions of Atkinson [1] and Sen [2], there has been a renewed interest in the determination of measures [3] of the welfarian effects of inequality in the distribution of income and in the problems associated with it, among scholars of different cultural backgrounds and, in particular, among the welfare economists. These economists, noting that the traditional synthetic measures of inequality of income are based directly on the modalities observed and termed *objective measures* do not take into consideration the effective economic content of the phenomenon, believe it opportune to draw a sharp distinction [4] between these measures and the ones that they call *normative*, these being based instead on functions chosen to describe the economic welfare and on their properties.

In reality, the situation is not as clear and well-defined as the welfare economists show, both because of the rather confused evolution of the concept of the function of economic welfare of incomes and because the boundaries between the two types of measures above mentioned do not result to be clearly marked.

Recently, a further motivation for studying the problem has come from Italian statisticians. We shall devote our attention, therefore, to the new contributions of Professor Ferreri and Professor Benedetti. In particular we shall try to show the groundlessness of the above-mentioned distinction in the light of the results obtained by these two Italian authors, who, we believe, have notably contributed to the theoretical statistical elaboration, that is coherent with a rational formulation of problems of economic and social importance.

[1] A. B. Atkinson [1].
[2] A. Sen [13], [14].
[3] As long ago as 1920 H. Dalton [5] had indicated the principle criteria for the construction of a measure of the welfarian effects of inequality in the distribution of income.
[4] A. Sen [14], p. 2.

2. *The Methodological Approach of Ferreri*

Ferreri proposes [5] a new measure, Daltonian in conception, which is termed individual income-based degrees of economic welfare. The measure is aimed at showing the welfare effects produced by single income levels on the collective economic welfare, which Ferreri prefers to call « collective equivalent income », and which he suggests is equal to the sum of individual equivalent incomes. These incomes are considered by Ferreri « utilitarian effects of incomes, which are measurable at the level considered by Dalton » [6].

If we now consider the income levels of n people, indicated by

$$x_1, x_2, x_3, \ldots, x_n \tag{1}$$

where $x_i \geqslant 0$ $(i = 1, 2, \ldots, n)$ and $x_j \leqslant x_{j+1}$ $(j = 1, 2, \ldots, n-1)$, a simple and immediate expression of the degree of concentration of total income in the single units of the collective can be obtained by the relation

$$g_i = x_i / \sum_{j=1}^{n} x_j \quad \text{where} \quad g_j \leqslant g_{j+1} \quad ; \quad \sum_{i=1}^{n} g_i = 1 \tag{2}$$

on which are based many of the traditional indexes of concentration. It would represent, therefore, an *objective* measure which fails to reflect the content and the economic significance of the phenomenon under study.

Nevertheless, starting from these g_i, Ferreri establishes an index which, in accordance with the welfare theory of Dalton [7], does not limit itself only to the consideration of the differences between single individual levels of income, but also indicates the effects associated with it. In fact, Ferreri observes [8] that according to the so-called « classical approach » the normative measures often are derived from individual utility functions which are assumed as being equal for all the income receivers considered. That is:

$$U(x) = U(x_i) \quad ; \quad \forall_i \tag{3}$$

and moreover it is supposed that

$$U'(x) > 0 \quad ; \quad U''(x) < 0 \tag{4}$$

[5] C. Ferreri [6].

[6] C. Ferreri [6], p. 368.

[7] In fact, Dalton affirmed that « for the economist is primarily interested, not in the distribution of income as such, but in the effects of the distribution of income upon the distribution and total amount of economic welfare, which may be derived from income »; in H. Dalton [5], p. 348.

[8] C. Ferreri [8] p. 16-17.

$$W = \sum_{i=1}^{n} U(x_i) \tag{5}$$

where (4) indicates that $U(x)$ must be an increasing and strictly concave function, while (5) signifies that the collective economic welfare W is additive, that is, equal to the sum of the individual utilities.

At this point we think it is necessary to turn our attention to the rather confused evolution that the concept of the function of economic welfare of income has experienced and which is also showed by the fact that the so-called « classical approach » described in (3), (4), and (5), even if inspired by Dalton's approach, actually differs from it as far as measurement is concerned. Ferreri observes [9] that « Dalton by basing his procedure on the acceptance of (3) and (4), has clearly expressed it in terms of levels of individual economic welfare $w(x_i)$ (and not $U(x_i)$ [10]) which is measurable on a ratio scale with zero determined by a level of income; and such welfare is not considered as equivalent to the cardinal utilities of income measurable on an interval scale ».

According to Ferreri this distinction has not been taken into sufficient consideration by Atkinson [11] who, in proposing his measure of inequality, assumes that the measure proposed by Dalton [12] is not invariant with respect to positive linear transformations [13]. However, as we have already mentioned, Dalton actually considered the individual economic welfare of income as measurable by means of a ratio scale (in which case the drawback referred to by Atkinson does not hold) and not by means of an interval scale (used on the contrary by the researchers who habitually refer to the classical formulation, and this is the case of Atkinson [14]).

After this brief but necessary digression we come back to the function $U(x_i)$, concerning which, in addition to (3), (4), and (5), it is supposed that

$$U(x_i) > 0 \quad , \quad \forall_i \tag{6}$$

Hypothesis (6) excludes the possibility, considered by Dalton [15], of negative economic welfare. Nevertheless, assumption (6) may be accepted, as Ferreri affirms [16], « paying attention only to economic welfare, that is, excluding the

[9] C. Ferreri [7] p. 154.

[10] When Ferreri refers to Dalton's approach he indicates individual function of economic welfare with w, while when he considers the so-called « classical approach » he uses U to indicate the individual utility of income so as to stress the difference between individual utility and individual economic welfare from the point of view of measurement. Nevertheless, in order to avoid possible confusion for the reader in this paragraph we shall continue to use the symbol U, while in that which follows we shall use the symbol w.

[11] A. B. Atkinson [1], pp. 249-250.

[12] H. Dalton [5].

[13] A. Sen [14], p. 37, about Atkinson's criticisms of Dalton, has also shown some perplexities.

[14] For a more detailed critical analysis of the approach of Atkinson the reader is referred to C. Ferreri [8], pp. 18-23.

[15] H. Dalton [5], p. 350.

[16] C. Ferreri [6], p. 375.

so-called extra-economic welfare, since we think that the individual welfare of income should not be considered so much as a level that separates welfare from poverty but rather as a magnitude that increases positively from a minimum level (corresponding to a certain minimum level of income) with respect to which an economic welfare more or less elevated is indicated ». According to the author, therefore, the acceptance (6) is equivalent to considering « levels $U(x_i)$ as individual levels of equivalent income linked to the function $U(x)$ and therefore to be considered in the same way as levels (1) »; for this reason they could be taken as degrees g_i, expressed in this case in terms of individual utilities, as

$$(7) \quad g_i = U(x_i) / \sum_{i=1}^{n} U(x_i) \; ; \; \sum_{i=1}^{n} g_i = 1 \; ; \; g_j \leqslant g_{j+1} \; , \; j = 1, 2, \ldots, n-1$$

Since (7) is defined in terms of individual utilities of economic welfare of income, it is evident that it can be measured only by incomes x_i; therefore, the first step is that of proceeding to the explication of $U(x)$.

With regard to (7) Ferreri [17] points out that « such relations cannot but produce the results deduced or expected with the criteria indicated by the welfare economists, thus rendering useless the research of alternative measures to the traditional statistical criteria for measuring concentrability at least on the basis of (3) and (4) ».

In order to explicate $U(x)$, the well-known increasing function with isoelastic marginal utility [18] is taken into consideration

$$(8) \qquad U(x) = k\,x^r \qquad k > 0 \quad ; \quad 0 < r < 1$$

The motives that induced Ferreri to use (8) and to assume $0 < r < 1$ are rigorously analysed in his work [19], to which the interested reader is referred for a much deeper understanding of the problem. Here we shall limit ourselves to noting that for $r \to 0$, (8) converges at k, producing a result independent of the level of income, while for $r \to 1$, it reduces to a straight line and therefore conditions (4) are not satisfied. Thus in both cases $U(x)$ would no longer be a function of utility with the characteristics that have previously been described. On the basis of (8), (7) becomes:

$$(9) \qquad g_{r,i} = x_i^r / \sum_{i=1}^{n} x_i^r \qquad 0 < r < 1 \quad ; \quad i = 1, 2, \ldots, n$$

where $g_{r,i}$ can be considered as degrees of concentration of the collective economic welfare of income.

[17] C. Ferreri [5], p. 375.

[18] This function was also used by R. Bentzel [3], but with prevalent reference to consumption rather than income.

[19] C. Ferreri [6], p. 378, note 1 and pp. 387-389.

The use of degrees $g_{r,i}$ to examine inequality of income puts in evidence some particularly significant aspects from the economic point of view that are not brought out with other measures. In this regard, Ferreri stresses [20] that « relations (9) as a part of the whole is, above all, a system of indicators that expresses, together with the relative order, the economic welfare level of the respective income receivers considered in themselves and within the distributive structure of economic welfare ». With the aim of demonstrating these economic peculiarities, let us consider the transfer of a certain quantity h of income from a level of income x_j to a lower level x_i, at the same time considering that the income levels are ordered in a non-decreasing way. This type of transfer, in relation to collective economic welfare

$$W = k \sum_{i=1}^{n} x_i^r \tag{10}$$

obtained from (5) on the basis of (8), leads to a variation of the type

$$\Delta W = k\,[(x_i + h)^r + (x_j - h)^r - x_i^r - x_j^r] \tag{11}$$

where it is assumed that $0 < h < x_j - x_i$ because for $h = 0$ no transfer of income would result, while taking for valid (10), (11) would cancel out for $h = x_j - x_i$, since this would correspond to an exchange in the order of incomes between the individuals that occupy positions *i-th* and *j-th*; moreover, for $h > x_j - x_i$, a transfer of income from x_i to x_j would result, but this is not taken into consideration by the hypothesis outlined above.

Thus, with the restrictions $0 < r < 1$ and $0 < h < x_j - x_i$, a transfer h of income from x_j to x_i $(i < j)$ produces, in accordance with the Pigou-Dalton condition[21], an increase in the collective economic welfare and simultaneously a decrease in the degree of concentration, not only for the person who experiences loss of quantity h, but also in the degree of concentration for all the other individuals with the exception of the *i-th* that has received quantity h.

In order to make it clear, Ferreri [22] notes that from (9), on the basis of (10) and (11), one obtains the relation

$$\Delta g_{r,k}/g_{r,k} = -\,(\Delta W/W + \Delta W) \qquad (k \neq i,\ j) \tag{12}$$

from which it is deduced that: i) the relative variation of each degree of concentration not included directly in the transfer of income is equal and opposite to the relative variation that the collective economic welfare W undergoes when a transfer h from x_j to x_i $(i < j)$ is carried out; ii) the absolute variation $\Delta g_{r,k}$ becomes higher in proportion to the increase in the corresponding degree of concentration $(\forall r,\ 0 < r < 1)$.

[20] C. Ferreri [8], p. 27.
[21] A. C. Pigou [12], H. Dalton [5].
[22] C. Ferreri [6], p. 381.

These results are, we think, of notable interest since they demonstrate the fact that, through the degrees of concentration $g_{r,k}$ it is also possible to take account of the economic-welfare effects caused by a particular transfer of welfare rather than by another, and in particular, they allow us to observe the variations in the fraction of collective economic welfare of the poorest and therefore to achieve information about the changes in the social conditions.

If it is true that, in line with the Pigou-Dalton condition, a transfer of income from a level x_j to a lower level x_i causes an increase in the collective economic welfare, it is likewise true that this does not necessarily entail an improvement in the social conditions, since, as Ferreri demonstrates [23] with clear and suitable numerical examples, transfers from the richer classes [24] to the intermediate classes, in terms of degrees of concentration of equivalent income, produce a worsening in the standard of living of the poorest, and consequently in their social conditions.

Moreover from the system of values $g_{r,i}$ expressed in (9) one may derive additional information to which Kondor refers [25] and that he believes necessary for ordering [26] the different distributions of income when, for example, a single index of inequality cannot do this in the best way, since it fails to reflect the social preferences of the individuals belonging to the collective examined. In regard to this Ferreri [27] believes that an ordering of these distributions effected, for example, only through the average individual economic welfare without considering the variations of the degrees $g_{r,i}$, is inadequate. This is because an increase in this average value, even if linked to an increase in the fraction of income possessed by the poorest individuals, is likely to cause (with respect to welfare) a decrease in their degree of individual welfare, with consequent deterioration of the social conditions.

3. *The Methodological Approach of Benedetti*

Another Italian scholar influential in the debate mentioned at the beginning of this paper is Benedetti who, with his recent article [28] has given a precise meaning in economic-welfare terms to an index that already existed in the field of classical descriptive statistics but with little substantial significance.

After having considered n individuals classified on the basis of a non-decreasing order of their incomes

$$0 \leqslant x_1 \leqslant x_2 \leqslant \ldots \leqslant x_n ,$$

[23] C. Ferreri [6], pp. 382-386.

[24] Ferreri considers the classes « not so much (or at least not only) with respect to professionality or economic category of the incomes, but rather in relation to the order of quantity of the incomes »; for an explanation of the motives for this choice, the reader is referred to C. Ferreri, [6], pp. 386-387.

[25] Y. Kondor [10], p. 309.

[26] For some new and interesting proposals concerning ordering see C. Ferreri, [7], pp. 161-164; [8], pp. 36-39.

[27] C. Ferreri [7], p. 152.

[28] C. Benedetti [2].

Benedetti assumes with $w(x_i)$ the function of individual economic welfare and supposes that this function is continuous, non-linear and, moreover, that $w(x) = w(x_i)$ that is, is equal for all the individuals, with $w'(x) > 0$, and with $w''(x) < 0$ for $0 \leqslant x \leqslant nM$, and with $w(0) = 0$. Therefore, in accordance with the Daltonian approach, he assumes that, the collective economic welfare of the n individuals considered, who possess the total welfare $\sum_{i=1}^{n} x_i = nM$ (where M is the arithmetic mean of x_i), is additive, that is, $W = \sum_{i=1}^{n} w(x_i)$. Also in this case, within the hypotheses formulated and in respect of the Pigou - Dalton condition, the collective economic welfare increases when a transfer of income $h > 0$ is effected from x_j to x_i $(x_i < x_j)$; on the other hand there is a decrease when the transfer is from x_i to x_j $(x_i < x_j)$. In the first case Benedetti supposes that $h \leqslant min\,(x_j - x_{j-1},\ x_{i+1} - x_i)$, while in the second case $h \leqslant min\,(x_i - x_{i-1},\ x_{j+1} - x_j)$ since «any graduated succession of n real non-negative numbers with constant sum nM can be obtained with transfers subject to these limitations» [29].

On the basis of the constraints $\sum_{i=1}^{n} x_i = nM = constant$, $n = constant$, and therefore also $M = constant$, $x_i \geqslant 0$, the maximum of W is when $x_1 = x_2 = \ldots = x_n = M$, that is, $W_{max} = nw(M)$, while the minimum of W is when $x_1 = x_2 = \ldots = x_{n-1} = 0$, $x_n = nM$, that is $W_{min} = w(nM)$. After these premises let us introduce the index of inequality of n individual situations of welfare proposed by Benedetti:

$$(13) \qquad I = [W_{max} - W(x_i)] / (W_{max} - W_{min}) = \\ = [nw(M) - \sum_{i=1}^{n} w(x_i)] / [nw(M) - w(nM)]$$

It is evident that index I assumes the value 0 when W is maximum and the value 1 when W is minimum, thus $0 \leqslant I \leqslant 1$.

After establishing the structure of the index of inequality Benedetti assumes that $w(x) = ax + bx^2$ where $b < 0$, $a > 0$, $a > -2bnM$ and, considering that, for example $w(M) = aM + bM^2$, $w'(M) = a + 2bM$, $w'(0) = a$ and operating similarly for $w(x_i)$ and $w(nM)$, after simple algebraic elaborations (13) transforms into:

$$(14) \qquad I = (M_2^2 - M^2) / (n-1)\,M^2 = \\ = \sigma^2 / (n-1)\,M^2$$

where $M_2 = (\sum_{i=1}^{n} x_i^2 / n)^{1/2}$.

Thus, in order to obtain a higher sensitivity of the index to the values closest to unit, Benedetti proposes in the place of I the use of its positive root of

[29] C. Benedetti [2], p. 8.

any whole order not inferior to 2; if we take, for example, the square root of I, the well known index of relative variability is obtained

$$I' = I^{1/2} = \sigma / M (n-1)^{1/2} \tag{15}$$

that retains all the properties proper to I and which was seen mostly from a formal point of view without the economic-welfare aspects that now can be attributed to it on the basis of the hypotheses put forward. This result shows once again how unjustified is the distinction between objective and normative measures.

The results obtained can be expressed in more general form by assuming that the function of economic welfare is represented by a polynomial of degree k ($k \geqslant 2$). Thus, considering the assumption previously formulated, (13) becomes:

$$I = \frac{\left\{\left[\sum_{j=2}^{k} (M_j^j - M^j)\right]\left[w^{(j)}(0)/j!\right]\right\}}{\left\{\left[\sum_{j=2}^{k} M^j (n^{j-1} - 1)\right]\left[w^{(j)}(0)/j!\right]\right\}} \tag{16}$$

where $M_j = \left[(1/n)\sum_{i=1}^{n} x_i^j\right]^{1/j}$. It is evident that, for the motives already discussed, also for (16) one can take any positive root of order not inferior to 2.

4. *Some Considerations on the Contributions of the two Italian Statisticians*

Drawing the conclusion of our analysis of the measures proposed by the two Italian statisticians, we shall concentrate on the hypotheses they have adopted and on the results obtained.

With regard to the objection that one might raise [30] as to the excessive simplifications introduced by the equality of the welfare function for all the individuals considered and by the assumption of additivity of the collective welfare function, it seems to us that both the authors are in agreement in maintaining that such simplified hypotheses are usually formulated when methods and indexes are proposed, that are suitable for effective application to empirical analyses.

In addition, it seems opportune to note that Ferreri's adoption of function (8) can be seen in relation to the wide use that, within the economic theory of welfare, is made of functions of increasing utilities with isoelastic marginality of the type $U(x) = a + (b/r)x^r$ ($r < 1$, $r \neq 0$, $b > 0$) of which $U(x) = kx^r$ constitutes a particular case for $a = 0$, $(b/r) = k > 0$, $0 < r < 1$.

[30] Y. Kondor [10], p. 310, note 1.

On the other hand concerning the adoption of a polynomial of degree $k \geqslant 2$ as a function of individual welfare Benedetti maintains [31] that « this can be considered the least arbitrary hypothesis since it is supported by the famous theorem of Weierstrass according to which, for each continuous function $f(x)$ in an interval $[a, b]$ there exists a polynomial $P(x)$ such that $|f(x) - P(x)| < \delta$, where $a \leqslant x \leqslant b$ and δ is a positive number that is arbitrarily small. Thus one may suggest to approximate each function $w(x)$ (with $w(0) = 0$, $w'(x) > 0$, $w''(x) < 0$) in the interval $[0, nM]$ with a polynomial of a suitable degree k''.

It is evident that the assumption $k = 2$, that is $w(x) = ax + bx^2$, is a further simplification that however may well be compensated by the simplicity, and therefore facility of use of I'.

Bentzel [32], with regard to the choice of the welfare function, affirms that such a choice involves a degree of arbitrariness but « the degree of arbitrariness involved in such a procedure seems to be much smaller than that involved in traditional measures of inequality, since these measures are defined without any regard to the theory behind the analysis. In addition these measures do not show the properties we are interested in ».

Still with the goal of demonstrating the groundlessness of the sharp distinction between normative and objective measures drawn by the welfare economists, we should observe also that the measure of inequality of economic welfare proposed by Benedetti can also be considered from the point of view of concentration. This is because included in the structure of the index I ($0 \leqslant I \leqslant 1$) are both equal distribution (which corresponds to the maximum economic welfare) and maximum concentration, which corresponds to the minimum economic welfare.

Ferreri [33] has also shown how, operating in terms of equivalent income, the measure of inequality of welfare proposed by Dalton can be seen from the point of view of concentration.

In fact Dalton considered a relation of this type

$$I_D = n\, w(M) \Big/ \sum_{i=1}^{n} w(x_i) \tag{17}$$

in which the numerator is « the total economic welfare attainable under an equal distribution » and the denominator « the total economic welfare attainable under the given distribution » [34].

From (17) one deduces that $I_D = 1$ in the case of equal distribution, while $I_D > 1$ for all the unequal distributions of income; therefore the first step that Ferreri takes in order to achieve an index which assumes values 0 and 1 (corr-

[31] C. Benedetti [2], p. 11.
[32] R. Bentzel [3], p. 263.
[33] C. Ferreri [9], pp. 333-334, note 35.
[34] H. Dalton [5], p. 349.

esponding to equal distribution and maximum concentration respectively), consists in the following:

$$I'_D = 1 - (1/I_D) = 1 - \left[\sum_{i=1}^{n} w(x_i) / n\, w(M) \right] \tag{18}$$

that can also be written as

$$I'_D = 1 - (M_r^r/M^r) \text{ where } M_r = \left[(1/n) \sum_{i=1}^{n} x_i^r \right]^{1/r} \tag{19}$$

indeed, if one supposes that the function of economic welfare is of the type

$$w(x_i) = k\, x_i^r \qquad k > 0, \quad 0 < r < 1 \tag{20}$$

and considering that $w(x_i) > 0$, $w'(x_i) > 0$, $w''(x_i) < 0$, $W = \sum_{i=1}^{n} w(x_i)$, the result is:

$$\begin{cases} W_{max} = n\, w(M) = n\, k\, M^r \\ \sum_{i=1}^{n} w(x_i) = k\, n M_r^r \\ W_{min} = w(nM) = k\, n^r\, M^r \end{cases} \tag{21}$$

from which, by substituting the first two relations of (21) in (18) one obtains (19). As, in terms of equivalent incomes $0 \leqslant I_D' < 1$, and wishing to consider, according to the point of view of concentration also $I'_D = 1$ in the case of maximum concentration (that one expects in correspondence with the minimum economic welfare), Ferreri proposes to adopt [35] an index of the type (13) instead of (18). In this case let us substitute (21) in (13) to obtain

$$C_D = (n\, k\, M^r - n\, k\, M_r^r) / (n\, k\, M^r - n^r\, k\, M^r) \tag{22}$$

developing this, one has

$$C_D = [1/(1 - n^{r-1})]\ [1 - (M_r^r/M^r)] \tag{23}$$

but $[1 - (M_r^r/M^r)] = I'_D$ so that we may write

$$C_D = [1/(1 - n^{r-1})]\, I'_D\,. \tag{24}$$

[35] C. Ferreri [9], p. 333.

Therefore in the light of the results reached by Ferreri and Benedetti, we can conclude this survey by reaffirming that the sharp distinction drawn by the welfare economists between the traditional synthetic measures of inequality of incomes and those based on the functions chosen to describe economic welfare respectively termed objective and normative measures is not justified. As we have seen, within the hypotheses formulated, the former is not always distinguishable from the latter.

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