Masters Research Essay: Total Factor Productivity Growth in Finland $1960 - 1999$

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Abstract

Our study encompasses two main objectives. One, using the 2001 OECD National Account database we compute Total Factor Productivity growth (TFPG) for Finland for the years from 1960 until 1999 using a chained Fisher index. We report that over the 4 decades Finland's TFPG has grown on a 2.4% annual basis. Additionally our results echo other studies in estimating a slow-down in productivity growth for the 1974-1991 period with TFPG registering a scant 0.7% average annual growth rate compared to a torrid 3.35% pace set in the previous period from 1960-1973. On a more positive note we document that TFPG growth accelerates quite dramatically for the decade of the 90s with an average growth rate of some 4.7% for the period from 1993 until 1999 suggesting that the Finnish economy might reprise its high growth performance last seen in the pre-1973 period

Second, we compute estimates of technical progress for Finland by specifying a Normalized Quadratic profit function analogous to the one used by Diewert & Fox (1999) in the producer context. We obtain as well estimates of producer (own and cross) supply price elasticities for the 1960-99 period. We estimate demand price elasticities in the consumer context by utilizing a Normalized Quadratic model once again for the same period and report finding rising trends in the annual producer and consumer price elasticities for Finland over the forty years period. It is alleged that these trends in elasticities in tandem with the high tax rates would lead to rising increasing deadweight losses over time which in combination with high inflation rampant in the decade of the 70s and 80s might possibly account for the productivity slump found in that period.[1]

[1]This essay was undertaken as part of the Economics 594 Summer Economics course at the University of British Columbia. The author would like to extend a special vote of thanks to Professor Erwin Diewert for his help and assistance in obtaining the necessary data and for providing the SHAZAM code for the computation of the various indices as well as the NQ producer/consumer models. For all errors, omissions we would like to blame the balmy Vancouver weather!

1 Introduction

The transformation of Finland from a nation whose economy revolved around agriculture and natural resources in the early decades of the 20th century to a modern diversified industrial economy that it is today has been nothing short of dazzling. This track record of high year on year economic growth is all the more impressive when seen against the backdrop of its historic past where Finland was often forced to lived under the influence more than one regional hegemon[2]. Meanwhile events of the 20th century such as the two world wars conspired to strew heavy obstacles in its path to economic prosperity[3] . On the economic front the two big oil price shocks threw a major spanner in the works causing severe disruption in the Finnish economy in the 1970s and 80s. Add to this the fall to grace of the Eastern-bloc countries which (especially the Soviet Union) extracted a severe toll over the Finnish economy in the late 80s and early 90s its no wonder the Finnish economic performance has been such a roll coaster in the post-war era. Despite these travails (or may be in spite of them) Finland's economy has roared back in the decades of the 90s spurred on in no small measure by the cylical rebound of the world economy since the recession years of the early 90s in addition to the well-documented success of its technology companies among which we must surely accord the pride of place to the Finnish mobile phone giant, Nokia.

This transformation of the Finnish economy has in turn made it subject of active research by scholars,policy-makers alike who strove to unravel the elixir of economic growth. One factor that is typically cited in accounting for the rapid growth of an economy is the productivity growth. The importance of productivity growth in engendering an improvement in the societal life-styles has been understood for nearly half a century dating at least as far back to the path-breaking papers by a number of scholars in the 1950s. Solow (1956) in particular has outlined an elegant model of economic growth wherein one may decompose the growth in an economy as arising from the confluence of growth in factor inputs (capital K and labour L) and productivity growth. This growth accounting framework has served as the dominant paradigm for scholars investigating macro-economic growth performance in the ensuing decades. Empirical studies on the growth performance of countries in the tradition of Solow, Denison, Jorgenson adduced a critical role played by productivity growth in explicating the economic growth of modern industrial states and in the process made productivity a hot button topic for policy makers and public alike.

Before we trot out policies aimed at spurring the rate of productivity growth it behooves us to have some knowledge of productivity performance which brings up the issue of how one might go about measuring productivity given the enormous complexity of modern economies. The daunting task of summarizing the underlying technical change through the use of a single number render topic of

[2]In centuries past Finland lay under the yoke of Sweden before falling prey to Russian imperialism in the 19th century.

[3]Finland suffered extensive damage to its infrastructure during WWII and was required to remit over $220 million to the former Soviet Union as war reparations

productivity (or what is given the more technical name of Total Factor Productivity) measurement a weighty topic in its own right given the number and subtlety of the concerns in measuring productivity growth. Fortunately or unfortunately for us the theoretical issues associated with productivity measurements have been thrashed out in other papers on this subject and the reader is refered to the fecund contributions by Diewert who has added much to the literature on productivity measurement. Our paper on the other hand utilizes the methodology of other researchers tailored to the specific case for Finland in the latter half of the 20th century. Thus the contribution of our paper to the burgeoning literature on TFP growth may be seen insofar as we shed some light on the following set of questions:

- How has the Finnish economy fared in terms of productivity growth over the 40 year post-war period from 1960 until 1999?
- Has this measured TFP growth/ technical progress been uniform over time and if not what have been some of the major trends in the TFP performance for Finland?
- What has been the responsiveness/elasticities of the various input & outputs and what if any trends have they exhibited over time?
- Finally what bearing would our findings on elasticities have in explicating the actual trends in the TFP growth in Finland over the four decades?

In forming our estimates of total factor productivity growth we shall piggy back the methodology contained in Diewert & Fox (1996,1998,1999)who motivate a multiple input-output model where they have two outputs (an aggregate of consumption, investment & government and a total exports output) and three inputs (labour, imports and an exogenously given capital input). Secondly we employ the services of a Normalized Quadratic (NQ) model developed by Diewert & Wales (1988a,1988b, 1993) in the producer and consumer context to get estimates and identify trends for the annual producer and consumer price elasticities as well the magnitudes of technical progress for the period going from 1960 until 1999. Knowledge of elasticities is a useful input in general equilibrium models of type used by Diewert & Lawrence (1999) in evaluating the size of deadweight losses due to taxes. Data for the construction of our model is derived from OECD sources principally the 1999 and 2001 editions of the Statistical Compendium published by the OECD.

The organization of the paper will be as follows. Section 2 provides a bird's-eye view of the recent economic & political history of Finland to situate the ensuing discussion on Finland's TFP growth performance. In Section 3 begins the formal side of things as we lay out the theoretical groundwork that undergirds the analytical framework for our paper i.e. we enter into a brief discussion on the chained Fisher index used in calculating the TFP growth for Finland and introduce the Normalized Quadratic (NQ) producer/consumer model which is the theoretical workhorse that we shall flog in arriving at our estimates of technical progress, elasticities. Section 4 concerns itself with elaborating the empirical methodology in operationalizing the theoretical model presented in section 3.

Section 5 is a prelude to our main results where we comment on a few trends seen in a couple of macroeconomic indicators for Finland. Section 6 showcases our main results for TFP growth and the producer/consumer models. This is followed by brief discussion on our results in section 7 and discourse a little bit on some of the limitations of the scope of our study in section 8. Finally section 9 concludes. A bibliography and an appendix containing an overview of index theory and profit/cost functions is included in the annex of our paper where may also be found the tables and figures utilized in our study.

2 Finland: A *primer*

Finland has over the course of forty years effected a remarkable transformation from a primarily natural resource/farm based economy to a modern industrialized one boasting a per capita income of around USD 23,000[4], a highly skilled work force that has been its linchpin for its booming technology sector. This rapid transformation termed a 'Cinderella' story by many commentators has been all the more impressive given the tumult suffered in the wake of major world events such as the two world wars , oil price shocks of the 1970s, the meltdown of the Soviet Union in the late 80s followed by the world wide recession in the early 90s . Of course things have not been all doom and gloom in this part of the world as there have been some spectacular success stories. For instance the decade of the 90s witnessed the coming of age of its technology sector with the most notable success story of this period being the glittering success of the Finnish cellphone giant Nokia. Companies such as Nokia and the subsidiaries that it spawned have helped to foment a spectacular turnaround in the Finnish economy that has allowed the country to assume the profile not unlike to that of the 'Tiger' economies of south-east Asia.

Finland is situated in the northern tip of Europe bordered by Baltic Sea in the South, the Gulf of Bothnia in the west and the Gulf of Finland in the east, and encompasses a total land area of some 337,030 sq km . It shares as well extensive land borders with Norway (729 km), Sweden (586 km) and Russia (1,313 km). Its geographical location has played no small part its economic and political history with Finland being a colony of first Sweden from the 12th until the 19th century and later coming under the domination of imperial Russia from 1809 onwards until its independence in 1917. This as we alluded to earlier has had ramifications for Finland's international relations in the 20th century spilling over into the realm of Finland's economic relations with both the European countries along with Russia accounting for a lion's share in its foreign trade[5].

[4]Purchasing Power Parity rates taken from *EIU country data* for 2000. This is still slightly lower than the figures of USD 30,000, 36,000, 26,000 for its Scandinavian counterparts of Denmark, Norway and Sweden

[5]EU countries excluding Russia combined for a total of 58% of total trade for Finland in 1999. Among these the largest trading partner was Germany that accounted for 15% of its exports and a similar amount of imports. Russia's share in the exports came to 8% based on the figures reported by the Finnish National Board of Customs and Statistics Finland.

Returning to the subject of geography we note over 76% of the country is under forest cover[6] which would probably go some way in explaining why the forestry industry has played and continues play an important role in its economy accounting for 23% of its total exports in 1999 according to the National Board of Customs of Finland. Finland also abounds in several natural resources such as copper, zinc, iron ore, silver which provide sustenance to its mineral extraction and metallurgical industries.

Ethnic groups represented in Finland comprise of Finns, Swedes, Sami or Lapp, Romany and Tatar with the two dominant groups in terms of population shares being the Finns with some 93% followed by Swedes with 6% . Official languages are Finnish and Swedish though English is becoming an increasingly popular language particularly among firms with international ties and among the younger generation. In addition Finland contains a small minority of Russian speakers whose association with Finland dates at least as far back as the Russian colonization of Finland beginning in the 19th century.

Politically Finland is a republic nominally headed by a president (Tarja HALONEN since March 2000) and led by a prime minister (Paavo Lipponen since April 1995) with the former elected for 6-year term of office and the latter a 4-year term. The main political parties are the Center Party or Kesk ; Leftist Alliance (Communist) composed of People's Democratic League and Democratic Alternative; National Coalition (conservative) Party; Social Democratic Party or SDP; Swedish People's Party or SFP. Currently the government in power is center-right-left alliance composed of a coalition of the SPD, National Coalition and the Leftist Alliance.

Finally we note that the Finland stand alone among the Nordic countries in having embraced the Euro which officially replaced its erstwhile currency ,the Finnish Markka, in 1999 and came into circulation in 2002.

3 Theoretical Background

In this section we afford the reader a brief look at the underlying theory related to the measurement of Total Factor Productivity Growth (TFPG) and introduce the normalized quadratic (NQ) profit and cost function which will be used extensively throughout the paper. For a thorough treatment on the subject of index measurements of productivity as well as the NQ functional forms we refer the reader to the numerous articles listed in the bibliography at the end of the paper.

3.1 Total Factor Productivity

Definition: Total Factor Productivity (TFP) is defined as the rate of transformation of total input (X) *into total output* (Y)*.*

We shall dwell on this relationship a little bit later when we comment on the productivity trends for Finland for the time period from 1960 until 1999.

[6] CIA World Factbook on Finland, http://www.cia.gov/cia/publications/factbook/geos/fi.html

This definition which is lifted from Diewert & Nakamura (2001) can be represented in the 1-output , 1-input case as the following:

$TFP = \frac{y_1^t}{x_1^t} \equiv a^t$, where a^t is termed as the output-input coefficient in period t.

Continuing in this 1-output/1-input vein ,Diewert et al specify 4 ways in which one may measure the growth in TFP over two periods t and s written as:

- ratio of the output-input coefficient, a, for the two periods t,s :
  $\{y_1^t x_1^t / y_1^s x_1^s\} \equiv a^t a^s \quad \longrightarrow TFPG(1)$
- ratio of output growth to input growth for the two periods t,s:
  $\{y_1^t y_1^s / x_1^t x_1^s\} \quad \longrightarrow TFPG(2)$
- ratio of growth in real revenue (R)/cost (C) ratio for the periods, t,s :
  $[R^t/R^s\ p_1^t/p_1^s]/[C^t/C^s w_1^t/w_1^s] \quad \longrightarrow TFPG(3)$
  where $R^t \equiv p_1^t y_1^t$ and $C^t \equiv w_1^t x_1^t$, $t = 1, ..T$
- ratio of growth in margins (m) after controlling for price change:
  $[(1+m^t)/(1+m^s)][(w_1^t w_1^s)/(p_1^t p_1^s)] \quad \longrightarrow TFPG(4)$
  where our period t margin m^t is defined as :$1 + m^t \equiv R^t/C^t$, $t = 1, ..T$

Of course it can be shown that under the constant returns to scale and perfect competition our 4 definitions of measured TFPG can be simplified to the form of $TFPG(1)$ which implies that our computations for the TFPG should be independent of the chosen specification . This is doubtless reassuring as we would not like our estimates to differ based on the manner in which we go about calculating change in the TFP for our chosen country, Finland. For our purpose $TFPG(2)$ will serve as our definition of choice when computing change in TFP over time.

Generalizing our definitions for TFPG for the M-output, N-input case mandates the use of indexes to aggregate multiple outputs and inputs i.e. when $M \succeq 1$& $N \succeq 1$ we need to utilize indexes that allow us to aggregate multiple outputs and inputs in order to use our formula $TFPG(2)$. Diewert (1998) gives an overview as well as the likely biases inherent in some of the commonly used index formulations for price and quantity indices. In the present case we proceed in blithe fashion to three of the commonly used quantity indices that allow us to aggregate over multiple outputs (Y), namely:

- Laspeyres quantity index , Q_L; $\quad Q_L \equiv \frac{\sum_{t=1}^M p^s q^t}{\sum_{t=1}^M p^s q^s} = \frac{Y(p^s,q^t)}{Y(p^s,q^s)}$
- Paasche quantity index, $\quad Q_P$; $\quad Q_P \equiv \frac{\sum_{t=1}^M p^t q^t}{\sum_{s=1}^M p^t q^s} = \frac{Y(p^t,q^t)}{Y(p^t,q^s)}$
- Fisher quantity index, $\quad Q_F$; $\quad Q_F \equiv \{Q_L \times Q_P\}^{1/2}$

Analogously we can define the Laspeyres (Q_L), Paasche (Q_P) and Fisher (Q_F) indices for the case of aggregating over multiple inputs (X). Putting the output and input indices together we can now define three formulations for measuring TFPG which are:

- Laspeyres Total Factor Productivity Growth Index;
  $TFPG_L \equiv \{Y(p^s, q^t)/Y(p^s, q^s)\}/\{X(p^s, q^t)/X(p^s, q^s)\}$
- Paasche Total Factor Productivity Growth Index;
  $TFPG_P \equiv \{Y(p^t, q^t)/Y(p^t, q^s)\}/\{X(p^t, q^t)/X(p^t, q^s)\}$
- Fisher Total Factor Productivity Growth Index;
  $TFPG_F \equiv \{TFPG_L \times TFPG_P)^{1/2}$

The usefulness of the $TFPG_F$ index is underscored by the fact that it can after a little bit of manipulation[7] be made equivalent to the $TFPG(1)$ defined previously which is helpful as both the Laspeyres and Paasche Indices contain terms that are not directly observable for instance ,$Y(p^s, q^t)$, in the case of output and $X(p^t, q^s)$ in the case of inputs. More important the Fisher index has a host of desirable properties associated with it that leads Diewert to dub it a superlative index[8] that enbolden us to utilize the $TFPG_F$ as our chosen specification when we go about measuring TFPG for Finland. To create our aggregate output (Y) and input (X) indices we follows the example of Diewert & Wales (1992) and Kohli (1998) in specifying output as an index of three outputs (consumption, exports and imports[9]) and two inputs (labour and capital). We shall expand on this a little later in section 4 when setting forth our strategy for operationalizing the chained Fisher index.

3.2 Normalized Quadratic Profit Function

Diewert and Wales (1987) motivate a variable profit function (V) that they refer to as the Normalized Quadratic (NQ) profit function that can be represented in matrix form as the following:

$$V(k, p, t) \equiv b^t pk + (1/2)[p^t Bp/\alpha^t p]k + ctp^t \qquad (E1)$$

where our $b^T = [b_1, b_2, b_3, b_4]$ and $c^T = [c_1, c_2, c_3, c_4]$ are vectors, t is a time trend, $t = 1, ..40$, k is some exogenously given capital and $B = [b_{ij}]$ is a matrix satisfying:

- $B = B^T$; i.e. B is symmetric
- $Bp^* = 0_N$ for some $p^* >> 0_N$

Our $\alpha^T = [\alpha_1, \alpha_2, \alpha_3, \alpha_4]$ is predetermined and positive i.e. $\alpha > 0_N$ & matrix B is constrained to be positive semi -definite in order that the profit function be convex in p. Some advantages of sticking with this functional form for the profit function (V) are that the NQ function is 'flexible' and one can impose curvature i.e. have our profit function be convex in prices (p) without loss to its flexibility property .When we set out to estimate our NQ function we shall additionally assume constant returns to scale (CRS).To extract our net supply function ($y(k, p, t)$) from our NQ profit function we make use of the well

[7] See Diewert & Nakamura 2002

[8] Refer to Diewert's 1998 survey article on Index numbers and the Appendix where we present a primer on index number

[9] Imports being entered as a negative output or an input

known Hotelling's Lemma result which states that one can obtain the net supply function by differentiating the profit function, i.e. $y(k,p,t) = \nabla V_p(k,p,t)$.This implies that our net supply function estimation equation takes the form:

$y^t(k,p,t)/k^t = b{+}Bv^t{-}(1/2)v^{tT}Bv^t\alpha{+}ct{+}\epsilon^t$ $(E2)$

where we have divided both sides by capital (k^t); $v^t \equiv (\alpha^T p^t)^{-1}p^t$, stands for our period t normalized prices and ϵ^t is a normally distributed error term.

3.2.1 Technical Progress and Producer Supply Elasticities

To examine what if any technical progress occurred in our model over time we calculate values of technical progress by following Diewert and Wales' (1992) lead in defining technical progress for period t (T^t) as :

$T^t \equiv pt^T c^* k^t/V^{t*}$; $t = 1,..T$ $(E3)$

An important feature that we might be interested in knowing something about is the responsiveness of the producers to changes in prices, i.e. we might be interested in investigating the producer supply elasticities based on the NQ profit function. This is easily done by first differentiating $(E2)$ with respect to p and then using our period t elasticity formula to get the following:

$[e^t_{ij}] = [\{p^t_j/y^{t*}_i\}\partial y_i(k^t,p^t,t)/\partial p_j]$; $t = 1,..T$ $(E4)$

3.2.2 Normalized Quadratic Profit Function with Linear Splines (NQLS)

Thus far we have not allowed for the possibility that there might be exogenous shocks buffeting the economy that would discontinuously change our previous estimates of technical progress (T) used in the NQ models. To give just one instance such an exogenous disturbance, it is certainly plausible that the Finnish economy (not unlike other OECD countries) experienced a major shock in 1973 that followed from the steep rise in the world price of oil , a considerable setback to the economic performance of the Finnish economy at the time. In order to be able to handle just such exogenous shocks we now modify our NQ model specified above by modeling technical progress (T^t) by adding linear splines that allows us to specify 'break points' in time (t^*) when we suspect an exogenous shock might have buffeted the economy. Accordingly we modify our NQ equation $(E1)$ as follows:

$V(k,p,t) \equiv b^t pk{+}(1/2)[p^t Bp/\alpha^t p]k{+}d(k,p,t)$ $(E5)$

where the new term $d(k,p,t)$ is defined as:

$d(k,p,t)$
$\equiv ctp^t k$ $\quad for \quad 1 \prec t \prec t^*$
$\equiv ctp^t k^* + (t - t^*)kf^T p$ $\quad for \quad t^* < t \prec t^{**}$
$\equiv ctp^t k^* + (t^{**} - t^*)kf^T p + (t - t^{**})kg^T p$ $\quad for \quad t^{**} \prec t \prec T$

t^*, t^{**} are break points signaling the advent of an exogenous disturbance, and c^t,f^t & g^t are vectors to be estimated by our regression. Our spline model in turn would lead us to rewrite our expression for getting our period t technical progress (T^t) above to the following:

$T^t \equiv \partial lnV(k^t,p^t,t)/\partial t = p^{tT} c^* k^t/V^{t*}$; $t = 1..t^*$
$T^t \equiv \partial lnV(k^t,p^t,t)/\partial t = p^{tT} f^* k^t/V^{t^*}$; $t = t^{*+1}..t^{**}$

$T^t \equiv \partial lnV(k^t, p^t, t)/\partial t = p^{tT} g^* k^t / V^{t^*}; \qquad t = t^{**+1}, ..T$

where t^*, t^{**} are our aforementioned break points in our spline model (see Diewert & Wales (1992)).

3.3 Normalized Quadratic Consumer Models

Just as we did in the producer context with profit functions we continue to shamelessly pillage the work by Diewert & Wales[10] developed in a series of articles by defining a normalized quadratic expenditure function specified as :

$$E(u,p) \equiv a \bullet p + b \bullet pu + u(1/2)(p \bullet Cp/p \bullet g) \qquad (E6)$$

where in our expenditure function $(e(u,p))$ $a \equiv [a_1, a_s, a_3, a_4]$ and $b \equiv [b_1, b_2, b_3, b_4]$ are parameter vectors that need to be estimated and $C \equiv [c_{ij}]$ is an $N \times N$ symmetric matrix that needs to be estimated as well while our vector $g \equiv [g_1, ..g_N]$ is determined in advance by having our vector $g_i = 1/N$ for $i = 1, 2, ...N$. In our theoretical model we thus have N consumer goods and $p \equiv [p_1..p_N]$ is the vector of prices incident on the consumer and finally u denotes the level of utility attained by the household. Given that utility is by very definition unobservable we can 'cardinalize' utility by following money-metric scaling by picking a reference price $p^* >> 0$ and requiring that the parameters our expenditure function $(e(u,p))$ satisfy:

- $a \bullet p^* = 0;$
- $b \bullet p^* = 1;$
- $Cp^* = 0_N;$

A useful result that we make use of is the Shephard's Lemma which allows us to extract the consumer demand functions (termed Hicksian) by simply differentiating the expenditure function such as the one specified in $(e(u,p))$ with respect to the price components. Thus to get our Hicksian demand function $x(u,p)$ we may differentiate $(E6)$ to get:

$$x(u,p) = a + bu + uCp/p \bullet g - u(1/2)p \bullet Cpg/(p \bullet g)^2 \qquad (E6_a)$$

If we now replace our indirect utility (u) by the specification ,$u = [y - a \bullet p]/[b \bullet p + (1/2)(p \bullet Cp/p \bullet g)]$,in the Hicksian demand function above we can write out our $x(u,p)$ in the following manner:

$$x(u,p) = a + [b + Cp/p \bullet g - (1/2)p \bullet Cpg/(p \bullet g)^2][y - a \bullet p][b \bullet p + (1/2)(p \bullet Cp/p \bullet g)]$$

3.3.1 Expenditure Shares and Consumer Elasticities

Using Diewert & Wales' notation we define the vector of normalized prices ,$P \equiv p/p \bullet g$, and normalized income ,$Y \equiv y/p \bullet g$, we may write our previous equation $(E7)$ as : $x(u,p) = a + [b + CP - (1/2)P \bullet CPg][Y - a \bullet p][b \bullet P + (1/2)(P \bullet CP]$. From this we can get our estimated equations for the expenditure share which we now state as:

[10] Refer to Diewert & Wales (1993,1988a,1988b,1987)

$s_i \equiv p_i x_i/y = [\hat{p}/y]\{a + [b + CP - (1/2)P \bullet CPg][Y - a \bullet p]\}[b \bullet P + (1/2)(P \bullet CP]$
for $i = 1, 2, ...N$ $\quad (E8)$

where are shares should sum to 1 in each period which implies that we will be dropping one of our shares,s_i, when we estimate our consumer regression model. As we are interested in knowing how responsive consumers are to changes in prices i.e. we'd be interested in calculating the consumers' elasticities and turn our attention to estimating elasticities. This is done in two steps. First we need to differentiate our Hicksian demand function, $x(u, p)$, as specified in eq.$(E6_a)$ with respect to the price component p $[\partial x_i(u, p)/\partial p_j]$ which in turn we shall plug into our elasticity formula to get our price elasticity $e_{ij} \equiv [\partial x_i(u, p)/\partial p_j]p_j/x_i(u, p)$ for $i, j = 1, ..N$.

If we wish to know something about the income elasticity in the consumer context we can follow a similar tack by first differentiating eq. $(E6_a)$ with respect to u (remember we've imposed money metric scaling) which we then apply to our income elasticity formula that takes the form:$e_{iu} \equiv [\partial x_i(u, p)/\partial u]u/x_i(u, p)$ for $i = 1, ..N$.

3.4 Normalized Quadratic Expenditure Functions with Linear Splines ($NQLS$)

We may also incorporate the use of linear splines in the consumer context just as we did in the producer context.This is would be beneficial if we posit that there might be discontinuous breaks in the consumers' utility in the period under question. Accordingly the NQ model eq. $(E6)$ would be modified as follows:

$E(u, p) \equiv a \bullet p + b \bullet pu + u(1/2)(p \bullet Cp/p \bullet g) + d(u, p) \qquad (E9)$

where the terms a, C and g remain as they were in eq.$(E6)$ while our new term $d(u, p)$ is defined :

$$d(u, p) = \begin{cases} ub \bullet p & \text{for } 0 \le u \le u^* \\ u^* b \bullet p + (u - u^*)f \bullet p & \text{for } u^* < u \end{cases}$$

where the new terms $b \equiv [b_1, b_2]$ and $f \equiv [f_1, f_2]$ are parameter vectors that will be estimated and the u^*, a break-point level of utility selected by looking at the zig-zag pattern of the utility level,u.

Now that we have assembled the theoretical tools we need to go about our investigation, the stage is set for us to wallow in our data which we proceed to do so in the next section.

4 Data & Empirical Strategy

We shall now elaborate a little bit on the data sources used for the generation of results and expand a wee bit on the methodology chosen in our paper in the next two subsections. First we look at the data assembled from the sundry sources and second we describe the manner in which we chose to operationalize our TFPG indexes along with the producer and consumer models.

4.1 Data

Our data for the TFPG estimates as well as the producer and consumer models come in the main part from the OECD economic database. Most of the data was stored in digital format and recorded in the Statistical Compendium (SC) CD-ROM published by the OECD for various years. We made use of the 1999 and 2001 editions of the SC CD-ROM which itself is derived from the data furnished by the Finnish national statistical agency. The period for the study extended from 1960 till 1999. For the producer models we collected data comprising of domestic output, exports, imports, labour and capital inputs. Domestic output was itself an aggregate on household consumption, government consumption and investment. On the consumer side our data was collected with aim of modeling two commodities namely consumption and leisure demand. We now present a cursory look at the data sources which we have placed into various sub-headings to aid in the exposition of the material.

4.1.1 Basic National Accounts Data

This data series under 'Main Aggregates' heading the SC cd-rom comprised of annualized information on:Government final consumption expenditure,Private final consumption expenditure,Increases in stocks, gross fixed capital formation,total exports and imports. The data series were collected in nominal term i.e. at current prices as well as in 1990 prices. From the same source we collected data on the cost side which consisted of data on: Indirect taxes, subsidies, consumption of fixed capital or depreciation, compensation of employees and operating surplus. All the figures were reported in the local currency (Finnish Markkas) for the years 1960-1999.

4.1.2 Labour & Population Statistics

Using the Labour Force Statistics series put out by the OECD we gathered annual information data on the size of a few labour force categories included as:Civilian employment, Armed forces, Wage earners and salaried employees, Self-employed workers and unpaid family workers. One of the problems in collecting information on the total labour force in this fashion is that we do not make any allowances for the heterogenous worker contributions to production for hours worked. To get around this problem we developed indices for the real quantity of labour input by using the available data on wage indices for the years 1960-1999 which were used to divide the total worker compensation to obtain an implicit labour input. The data for the wage index is developed using the index of manufacturing wage[11] published under the Main Economic Indicators: Historical Statistics 1960-1991 & Monthly publications of the OECD for the various years.

[11] 1990 as the base year

4.1.3 Capital & Interest Rate Series

Information on price/opportunity cost of financial capital i.e. on the nominal interest rates that producers are typically faced with is collected in our present context by looking at the interest rates on Long-term (= 5years) government bonds for the period in question. Our source was the one used previously in the labour case when we collected our wage index, namely the Main Economic Indicators published by the OECD using data reported by the Bank of Finland. We require this information on nominal interest rates as it will be used in tandem with the data on the gross fixed capital formation to give us an estimate of the capital inputs used in the producer case.

4.1.4 Taxes and Tax Rates

Data for taxes and tax rates on our labour and capital inputs were collected using the Statistical Compendium cd-rom 1999 & 2001 editions. Our purpose in collecting the tax data was to ferret our the relevant consumption, labour, business and trade tax (and subsidy) rates i.e. we wished to learn the appropriate tax rates incident on output, exports/imports, labour and capital for the Finnish economy over the period in question. With this in mind we collected our tax data under the various headings: 1000:*taxes on income*, *profits and capital gains*, 1100:*taxes on individuals*, 2000:*social security contributions*, 3000:*taxes on payroll and work force*, 4000: *taxes on property,* 5000: *taxes on goods and services* and 5123: *import tax/duties.* The tax data that we do collect are for the years going from 1965 until 1999 . We interpolate the tax rates for the missing years namely 1961-64 by using the prevalent tax rates from 1965. This is justified to the extent that tax codes are typically slow to change and we may use tax rates for the missing years using the ones from 1965.

4.2 Strategy Used for our Data

In order to able to generate the TFPG estimates and crank out our results from the producer and consumer models we need to make adjustments and modifications to our collected data. We will thus give an account of the actual strategy followed in this part of the paper. All data were modified and later producer & consumer regressions were carried out using SHAZAM.

4.2.1 Data Massage

Our first order of business in transforming our data was to create the price indices for the national account data described in the previous section. This is done by dividing data series given in current prices with those national accounts data specified in real terms i.e. using 1990-price levels , so that we now obtain price indexes for government, private consumption, exports, imports, inventories and capital formation. Following this we renormalize price and quantities so that our prices index equals 1 in period I i.e. in 1960 our price index will have a value of 1 and used these normalized prices to calculate our normalized quantity indices.. As the inventories price index evinces considerable variability we discard it and combine the inventories quantity index with the gross capital formation index and use the price index for the gross capital formation as the index for the two commodities.This is reported in $Table\ 1(a)$ at the end of the paper.

Next we calculate the tax and subsidy rates for our economy for the time period in question. Subsidy rates are simple enough we merely divide the total value of subsidies by the value of the sum of the government, private consumption, export & imports, gross fixed capital formation. This assumes that subsidy rates are equi-proportionate for the economy as a whole. Consumption tax rates are computed by first subtracting import duties and property taxes from the indirect taxes and then dividing this by the value of the total private consumption at current prices. Import tax rates are computed in an analogous fashion by dividing the total import duties by the total imports[12]. For the missing years $1960 - 64$ we interpolated the tax rates using 1965's tax rates as was mentioned earlier. Before we report tax rates on capital income and assets we need to turn to the our labour series data.

We formed a ratio of the total civilian employment to the total number of salaried employees for all the years in question and inflated our employment compensation data series by this ratio to allow for the inclusion of self-employed and unpaid workers in our picture. However we need to adjust for the wage differential existent among waged labour on the one hand and self-employed /unpaid workers on the other. We assume somewhat arbitrarily that the latter make only 70% of the wages of the former and thus get new series on worker compensation. This is a completely ad-hoc assumption on our part motivated by the paucity of actual data on the differential between self-employed versus waged labour. Moreover this is the ratio used as well by Diewert & Lawrence (1999) in the case of New Zealand and we decided to go with this number in the interest of making our results comparable with theirs.To get our labour tax rates we now divide the annual labor tax collected by this new adjusted employee compensation. Additionally we generated a wage index by using the manufacturing wage index[13] collected earlier and normalizing the same in SHAZAM so that we have the wage index equalling 1 in 1960.

For generating our capital inputs we first computed the gross profits using our producer prices that include the tax rates and subsidies i.e. we have :

$GrossProfits = P_C * (1 - TX_C) + SUB) * Q_C + P_I * (1 + SUB) * Q_I + P_G * (1 + SUB) * Q_G + P_{EX} * (1 + SUB) * Q_X - P_{IM} * (1 + TX_{IM}) * Q_M - P_L * Q_L$

where $C \equiv Consumption, Sub \equiv SubsidyRate, I \equiv Investment, G \equiv Governement, EX \equiv Exports, IM \equiv Imports, L \equiv Labour$

Our next step was to compute the rate of growth of capital investment where

[12] Note: our Import tax rates are *zero* for $1998 - 99$.

[13] A possible wrinkle with the wage index is that it was constructed only for the manufacuting industry and so we're assuming all sectors of the economy are being proxied by this index which may not innocuous given that services are by far the dominant sector in advanced OECD countries. Thus we are implicitly assuming that this index of manufacturing wages described the conditions prevalent in the Finnish labor market

we assumed a constant geometric rate of growth in the investment.. Using our values of gross fixed capital formation for the years 1960 and 1999 we computed that capital increased an average at the rate of 1.74% per annum. For our average real rate of return (ARR) on capital assets we assume a value of 8% based on the range of 5-12% estimated in other studies on OECD countries when excluding land & inventories. Similarly for our estimate for depreciation rates for our economy we took a mid-point of 8% whereby we assumed that there is a 50-50 breakdown in investment between structures versus machinery & equipment for Finland. These allowed us to calculate a new value of capital stock and in turn for our capital-output ratios as well as net profit rates. The details are contained in our SHAZAM code[14]. Tax rates on capital income are computed by ratio of the total capital taxes plus property taxes and dividing this by value of capital stock calculated previously. Internal rates of return (IRR) are calculated as well by taking the ratio of the net profits[15] to the value of capital stock for various years. To get a price index for capital we set P_K equal to the normalized investment price index, P_I multiplied by the IRR plus depreciation rate. Now we are ready to tackle our TFP growth rates for our economy.

4.2.2 Total Factor Productivity Growth Indexes

To obtain our aggregate output (Y) we use three normalized prices & quantity outputs namely domestic output (which itself is an aggregate index of the normalized prices & quantities for private consumption, government consumption and investment) plus exports and imports. Imports are entered as negative as is standard practice[16]. Analogously we create an index for inputs (X) where we used the normalized price and quantities of labour and capital described earlier.

We stated in section 3 that are our estimates for the $TFPG$ would be given by taking the ratio of the output growth (∇Y) to the ratio of input growth (∇X) where Y and X are our indexes for aggregate output and inputs.All chained Fisher[17] indexes are implemented[18] and tabled under $Tables$ $1(b)$ & $1(c)$.

Our formula for the $TFPG_F$ was specified as :

$TFPG_F \equiv \{TFPG_L \times TFPG_P)^{1/2}$ where $TFPG_L \equiv \{Y(p^s,q^t)/Y(p^s,q^s)\}/\{X(p^s,q^t)/X(p^s,q^s)$ & $TFPG_P \equiv \{Y(p^t,q^t)/Y(p^t,q^s)\}/\{X(p^t,q^t)/X(p^t,q^s)\}$

Our results are for $TFPG_F$ for the various years are tabulated in $Table$ 2.

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[14] available upon request

[15] Gross Profits - Depreciation

[16] Imports are viewed as inter-mediate inputs

[17] Refer to the Appendix on the desirable properties of Fisher Price and Quantity Indexes. Furthermore we use chained indexes as they revealed a lower spread for the Fisher- Divisia quantity indices as shown the Appendix figures 1-2,

[18] Using the INDEX command in SHAZAM

4.2.3 Producer Regression Models: $NQLS$

We implement a Normalized Quadratic Linear Spline ($NQLS$) model of the profit function to get at our producer supply elasticities. The ingredients of the model consist of indexed outputs and inputs comprising of outputs, exports, imports and labour derived in the previous section when we sought to get our TFPG estimates for Finland. The producer model of choice ,implemented using SHAZAM code, was conditioned by the following characteristics: constant returns to scale (CRS); we adjusted for heteroskedasticy by dividing all terms of the regression dependent variables by the capital input (K); curvature conditions were imposed apriori i.e. we force our model to be convex in prices; all prices were positive and set equal to 1 in period 1 i.e. 1960 : technical progress is modeled using linear splines.For all the equations we appended an error term and used our non-linear option in SHAZAM to execute our statements.

The use of splines merits a bit of a digression as it required us to posit break points that signal exogenous disturbances or shocks to the Finnish economy. These were chosen by 'eyeballing' the data by looking at the 'zig-zag' pattern of data between observed and predicted values from our non-spline (NQ) regression model. We chose as our break points for our model the years 1974, 1978, 1989 & 1993. The years 1973,1978 were the years of the infamous 'Oil Shock' when the OPEC cartel of oil exporting countries reduced their oil production quotas in a bid to ratchet up the world oil prices. Finland was especially hard hit given the prominence of energy in its imports from the rest of the world. For the other two break-points we turned to the years associated with intense political upheavals occurring the Communist-bloc nations. The implosion of the communist Eastern bloc countries especially the abject collapse of the Soviet Union in the years between 1989-1991 bolsters our case in picking 1989 as one of the break points. Finally the year 1993 was when the Finnish economy reached its nadir following a bout of severe recession brought on by the world recession of 1991-92 as well as the collapse of the Soviet era markets and the protracted weakness besetting the financial sector. The upshot of this was the coming to power of the new centrist parties led Paavo Lipponenin which embarked on a program of far reaching political and economic reforms in the following year .

4.2.4 Consumer Regression Model: NQ

We motivate a Normalized Quadratic (NQ) model in the consumer context just as we did for the producer context. To reiterate the advantages from employing NQ models are that these models are flexible (Refer to Diewert (1974)) and one can impose the appropriate curvature conditions without loss to the flexibility property.There were two goods in our consumer regression model, namely consumption and leisure per capita. Our data in the consumer context is derived from the OECD data used previously in computing the TFPG. To get the consumption per capita we simply formed a ratio of the normalized quantity of consumption divided by the total population for the various years. Leisure in the case of Finland was gotten by first taking an *average* of the per pop stock of economic time, H ≡product of labour supplied and the ratio of the total civilian employment divided by the population of 15-64 age group in Finland. Leisure per capita is equal to H after subtracting the normalized quantity of labour. The prices for these two goods are the normalized price of consumption P_C and $P_{Leisure}$ where $P_{Leisure}$ is equal to the wage adjusted for the labour tax rate i.e. $P_{Leisure} \equiv Wage(1 - TX_L)$.

Thus with our 2-good model the functional form for our consumer model will take the form:

$$e(u,p) \equiv a{\bullet}p{+}\{b{\bullet}p{+}(1/2)(p{\bullet}Cp/p{\bullet}g)\}u \qquad (E10)$$

where $a \bullet p = 0; b \bullet p = 1; Cp = 0_N$, for prices in our base period and a,b,p are vectors while C is 2×2 symmetric matrix and finally our scalar utility, u, in the above equation was solved using the substitution from eq.($E7$), all estimated using the non-linear option in SHAZAM. The results from running the producer and consumer models in SHAZAM are tabled under $Tables$ 3(a) & 3(b) respectively at the end of the paper.

4.2.5 *NQLS* Consumer Model

To operationalize the NQLS model in the consumer case we need to select an exogenous level of utility ,u^*, that represents a break-point i.e. where we believe that the utility experiences a discontinuous break. We picked the break-points by looking at the behavior of the predicted and observed values from our Normalized Quadratic consumer model. In the single linear spline case we set the break-point equal to 6.04 using our Fisher Index approximation of the period utility. This corresponds to period $t = 29$ or the year 1988 which leaves the years from 1989 until 1999 above this chosen break-point level of utility and years 1960 until 1988 below it. The use of the Fisher index as an approximation for the utility level is justified on the grounds that the Fisher index is a 'superlative'[19] index.

5 A Few Preliminaries

As a prelude to our main results on TFPG and producer/consumer elasticities we present a few statistical tidbits from our OECD data set for Finland. The reader should note that the graph for these sections have been included at the end of the paper.

5.1 Population & Labour Force Trends

From *figure I* it may be seen that population & labour force for Finland over the four decades have evinced a gradual increasing trend. The average rates of growth over the period for the population and labour force growth were

[19] See Diewert (1976) where he defines a 'superlative 'index to be one that is true for cost -of-living index for a utility based demand system and is flexible in that it can provide a second order approximation around any arbitrary point in the system

computed to be 0.4% and 0.47% respectively. Similarly we report that the growth trend for the population of people aged between 15 and 64 that form the potential labour force has been a somewhat higher 0.58%.

Average Population Growth Rate	0.4%
Average Population$_{15-64}$Growth Rate	0.58%
Avg. Labour Force Growth Rate	0.47%

5.2 Inflation Rate

Rates of Inflation for Finland are calculated by taking the percentage change in the annual consumer price index [20]. Looking at the behavior of consumer price inflation over the time period we are struck by the very high level of inflation recorded in the decade of the 70s where inflation rockets to a some 20% in 1974 and which continues to remain in double digit territory for the rest of the decade until 1982. As the Table below reveals the decade of the 70s was notable not only for the high average inflation of some 12% but also for the greater variability in the annual inflation rates given by the standard deviation of 4.09 for this period thereby painting a turbulent macroeconomic picture for Finland in these years. It is only after recession year of 1981-82 that we see a secular decline in inflation (see $figure\ II$) with rates declining for each year for the rest for the period till we reach 1990 when we see another upturn in inflation although not of the same magnitude as seen in the 1973-74 period. The decade of the 90s may be characterized as a low-inflation period where inflation has been quite tame with an average of around 2% for the this period. One possible explanation for these trends would undoubtedly rest with the price of oil. World prices of oil rose very sharply in the years 1973-74 and again in 1978-79 which wrecked havoc on the economies of countries heavily oil-dependent such as Finland. Imports of petrol and petroleum derivatives have a prominent place in total Finnish imports with oil constituting 11% of total imports in 1999[21] and thus changes in the world oil prices will be of enormous consequence for the economy of Finland. Thus the relatively low average levels and variability (standard deviation of 1.4) in inflation seen in the 90s in contrast with other periods may be explained in part by the glut in energy production and depressed world prices. Of course the monetary policy stance of Finnish central bank in the 90s play an important role in accounting for the inflation trends observed in this period[22].

[20] CPI was computed by taking the ratio of the consumption expenditures in current values divided by the expenditures recorded using the 1990 price levels

[21] National Board of Customs, Finland

[22] Refer to "Inflation Targeting: A strategy for U.S. monetary policy?" by Bernanke and Mishkin for an indepth-look at the central banks' approach aimed at amerliorating inflation expectations

Time Period	Avg. Inflation (%)	Std. Dev	Min(%)	Max(%)
1960s	4.7	2.49	1.68	9.26
1970s	11.58	4.09	6.7	19.6
1980s	6.39	2.66	3.12	11.77
1990s	2.08	2.0	1.4	5.6

5.3 IRR and Real (Exogenous) Interest Rates

We plotted as well the internal rate of return (IRR) and the Real (Exogenous) Interest rates over the same period and tabulate the results in the following table. The average IRR for the period as a whole is found to be 3.7% while that for the real interest rate was a figure of 2.35%. More relevant from our point of view was the break down in the averages for the various decades. Before we turn to actual numbers the reader should refer to $figure\ III$ for the trends in these two interest rate series. The real interest rate shows the more variable trends with rates plunging quite dramatically into negative territory (going as low as -11% in 1974) during the periods of high inflation in the decade of the 70s. This is followed by a period of rising real interest rates (1980-81) also saw negative interest rates) until another drastic turnaround in 1990 when real interest rates dip once more into negative territory (-6%). Thereafter real interest rates have stayed in positive territory meandering between 5 and 10% in the 90s. A similar story may be related for the case of the IRR which hovers around 3% for the bulk of the period from 1960 until 1992. 1977 and 1992 are two years of relatively low IRR, coming in at 0.6 and 0.7% respectively. However after 1992 we get a far better IRR performance with increases in the IRR to over 5% in 1995 and to nearly 11% in 1999. The average for this sub-period being a relatively high 6.02% which is double that of the decades of the 60s,70s and 80s.

Internal Rate of Return

Time Period	Avg. IRR(%)	Std. Dev	Min (%)	Max (%)
1960s	2.97	0.0951	2.07	4.93
1970s	2.88	0.097	0.064	4.09
1980s	3.37	0.056	2.27	4.26
1990s	6.02	3.64	0.7	10.75

Real Interest Rates

Time Period	Avg. RR (%)	Std. Dev	Min (%)	Max (%)
1960s	3.17	2.29	-1.21	6.13
1970s	-2.37	4.05	-11	1.53
1980s	3.23	4.02	-6	7.13
1990s	5.71	0.216	3.31	9.63

5.4 Terms of Trade

Given the importance of international trade for the Finnish economy[23] we calculated the terms of trade to examine trends if any over the period from 1960 until 1999. Improving terms of trade work to improve economic welfare as the economy has to export fewer commodities to balance trade which can then be directed towards increasing domestic consumption.We depict the behavior of the terms of trade over time for Finland in $figure\ IV$. From the chart we report a falling TOT trend for the years from 1965 until 1974, repeated in 1977 until 1980. It was stated earlier that this period coincided with the oil price hikes and thus would have increased the index of prices of imports relatively more than the export prices. Moreover with world commodity prices for natural resources such as forestry and minerals also showing greater fluctuations we should not be surprised to see a general downward trend for much of this period. From the 1981 onwards things take a more positive turn with an increasing trend thereby signalling an improvement in terms of trade (terms of trade are greater than unity for period from 1987 onwards) i.e. price of exports rising faster viz a viz imports. The only exceptions are the years 1991-92 when Finland was in the midst of its deep recession concomitant with the collapse of its major export markets such as the former Soviet Union

5.4.1 Real and After Tax Wage Rates

To see how the real wage growth and after tax wage growth fared in the Finnish economy over this period we calculated the real wage growth by taking a ratio of the wage index divided by the consumer price index and the real after tax wage rate growth by taking the ratio of the implicit price of labour adjusted by the labour tax rate divided by the consumer price index.Based on the magnitudes calculated we report that real wages nearly *tripled* over the period going from 1960 to 1999 while the real after tax wage rate increased by some 85% over the same period. A further point of note is seen by eyeballing the trends in the two wage series (refer to $figure\ V$)that shows the real wage growth diverged quite substantially from much of the period from the after tax wage series with the exception of the first couple years going from 1960 until 1968. Additionally we discern approximately four sub-periods when the trends in the two wage series appeared to change. First between the years 1960 until 1968 when the two wage series were almost identical, second from 1968-77 when the real wage series grows at faster clip compared with the after tax wages series, third from 1978 until 1987 when the two series move almost parallel to each other and finally the period from 1988 until 1999 when real wage rate resumes growing at a relatively faster clip compared with the after tax wage series.

[23]For instance we calculated that the proportion of export plus imports in the Finnish GDP amounted to some 54% on average for the period as a whole underlying Finland's the importance of foreign trade to its economy

6 Main Results

We now consider our results gotten from index measurements for the TFPG as well as those obtained from running our producer and consumer regression models..

6.1 Total Factor Productivity

Table 2 reports are estimates for the TFPG computed using the chained Fisher index. Before we consider the actual magnitudes computed it is instructional to consider the behavior of the aggregate output and input indexes for the period in question. We depict this in *Chart* 1 included at the end of the paper. Glancing at the chart reveals that the output index (Y) has outpaced the input index (X) for the period which augurs well for the overall productivity growth of the economy. Moreover we may discern some interesting trends in the case of the aggregate output and input indexes which will be relevant to our discussion on the TFPG. For one thing ,the trend for the output index exhibits some sharp fluctuations around 1973-74 and again in 1978-79 which should alert us to the possible impact of the oil price shock buffeting the economy in those years. The late 80s and early 90s displays a V-shaped trend with the output index declining quite rapidly in late 80s to recover in equally dramatic fashion in the early 90s (reflecting the macro-economic turmoil of those years in Finland) and then increases at a smart pace from 1994 onwards. The input index (X) does not exhibit such variability from year to year and rises uniformly throughout the period .

The above discussion on the behavior of the output index (Y) dovetails quite neatly with the index measures of the estimated TFP growth. For the period as whole the TFP growth for Finland was a sizzling 2.4% ranging from a maximum of 7.2% in 1998 to a low of -0.65% in the years 1974-75. For the period from 1960-73 we calculate that TFP grew at an average of 3.35% per annum while for the post-1973 period until 1999 the TFP the average stood at 1.97% per annum.Of special interest are the results reported for the decade of the 90s which were associated with spectacular success of Finnish Hi-tech companies most notably the cell-phone company ,Nokia. For these years we report that growth in TFP were worthy of the hype with the economy's TFP growing at an annual pace of 4.7% on average. The real slow-growth period is the 1974-91 period when the economy evinced a scant 0.74% growth in TFP[24]. We hazard the view that part of the pick-up reflects the overhang from the steep recession that plagued the economy in the early 90s following the double whammy from worldwide recession and the collapse of the Soviet Union and the rest could be ascribed to the productivity pick-up symbolized by the booming Finnish technology sectors or the 'Nokia' effect.

Table: Productivity Summary

[24] which is comparable for the magnitude of 1.15% reported by the Diewert & Fox (1998) for the 1974-92 period

Time Period	Average TFP Growth (%)
1960-1999	2.4
Pre- 1973	3.35
Post -1999	1.97
1992-1999	4.65
1974-1991	0.7

How does this compare with other studies on OECD countries' TFP performance? A slew of articles[25] on the subject TFP growth for the OECD countries for comparable time periods have reported finding a slowdown in TFP growth after the onset the first oil price shock in 1973. For instance Diewert & Fox (1997, 1998) using a methodology very similar to one followed here reported that TFP growth for the countries for the period from 1960-1992 grew at a faster clip in the pre-1973 period compared to post-1973. They estimated that TFP growth for the 18 OECD countries in their sample (that included Finland) grew on average 2.57% per annum in the years 1960-73 which was nearly a percentage & half higher than the 1.09% found for the 1974-92 time period. For Finland they reported an average TFP growth of 2.57% for the pre-1973 period and a 1.15% growth for TFP for the post-1973 period ending in 1992. A shortcoming of their paper was the fact that their period of study ended in the early 90s and as such were unable to answer the question whether TFP growth performance in the 90s fared better or worse compared to earlier periods. This has been remedied in our paper albeit only for the Finnish case.

6.2 Producer Regression Model ($NQLS$)

Our results from running NQLS models in the producer context are presented in $Table$ $3(a)$ where we ran our model using the years 1974,1978, 1989 and 1993 as our break-points. We imposed curvature properties beforehand ensuring that our model satisfied the convexity[26] property in prices. The R^2-statistics for the model are remarkably high with model typically accounting for more than 90% of all variation in the data using the dependent variables. R^2s for our model ranged from a low of 0.647 or 65% for output and to a high of 0.98 or 98% for labour indicating a good fit of our model as we can account for a large proportion of variation in our data based on the NQLS model. This allows us to be repose a high degree of confidence in the estimates of technical progress as well as producer supply elasticities that were computed on the basis of our NQ models.

6.2.1 Technical Progress

It should be mentioned at the outset that the use of linear splines does have the disadvantage in that our estimated technical progress measures are discontinu-

[25] Surveyed in Diewert & Fox (1998)

[26] The determinantal conditions for convexity, namely that the Hessian matrix be positive semi-definite are met in our NQLS model.

ous around the break-points. Thus by using our four break-points we effectively model our technical progress by using 5 segments[27] for the four decades from 1960 until 1999. Diewert & Wales (1993) advance a method to mitigate this problem by using quadratic splines which is not done in the present case. The second point of note is that our the magnitudes for technical progress are typically expected to be triple the ones estimated TFP growth[28] obtained previously . Finally we are implicitly assuming that technical progress in our economy has been neutral i.e. the improvement in the technical progress does not affect the marginal rate of technical substitution over time. This is fairly strong assumption as we are failing to take into account that technical progress may be biased toward output and/or inputs which sounds more plausible for the Finnish economy. Nevertheless extricating the biases inherent in our estimated values for the technical progress is not attempted in our present venture and the reader must interpret our estimates of technical progress computed by using the $NQLS$ model as standing for neutral technical change over time.

Bearing these points in mind we plot our estimated values for technical progress along with the TFPG estimates in $Table$ 2a and graph the same in *chart* 2. Use of linear splines allows us to better observe the technical progress across the various time periods of the Finnish economy. From our table we identify the decade of the 60s and the 90s as the periods that witnessed the most rapid rates of technical progress with magnitudes in the high-teens for much of the two time periods. One feature that immediately grabbed our attention from examining our chart is the tremendous rebound in technical progress for the years 1993 onwards where the estimated values go from low single digits to the high octane double-digit rates last seen in the decade of the 60s. In other words are estimated magnitudes of technical progress reveal a similar pattern found for the TFP growth, namely blockbuster rates of technical progress for the 60s and the 90s interspersed by a period of relatively lack-lustre growth in the period going from 1974 until 1992.

6.2.2 Producer Supply Elasticities

Own Price Elasticities Elasticities give us an indication of the responsiveness of the producers to prices. The NQLS models allow us to compute separately the elasticities of domestic output, exports, imports and labour which we report in $Table$ 4 and plot in *charts* 3 to 6 respectively. Elasticity of domestic output is positively related to price in line with economic theory and shows a moderately decreasing trend over the years from 1960-1999 suggesting that producers of domestic output have become less responsive to prices over time . Despite this we report that the average elasticity for the case of domestic output is 1.54 i.e. a 10% increase in prices would bring on a disproportionate increase domestic output in this case 15.4%. Thus we would classify domestic output to be elastic given the taxonomy provided by economic theory that stipulates that

[27] 1960-1973, 1974-1978, 1979-1989, 1990-1993, 1994-1999

[28] This owes the manner in which we computed the TFPG using the ratio of output and input growth rates where the share of capital (K) is normally only a third

elasticity greater than 1 is elastic or very responsive to price change.

Similar to the case for domestic output, export elasticities are also positive in magnitude which accords with economic theory. However unlike the case of domestic output, exports price elasticity has become relatively inelastic from the mid-80s onwards i.e. a 1% increase price has led to a less than 1% increase in exports. The average export elasticity for the four decades is calculated to be 0.77 (less than 0.5 for 90s). We hazard a conjecture that these trends may be explained by making use of the insights stemming from modern trade theory models. These models purport that industrializing countries will tend to specialize in intra-industry (as opposed to inter-industry trade) over time which mandates that countries specialize or develop a niche within a particular industry. This jives quite well with the recent economic history of Finland as has been alluded to in our background on Finland which has gone from being an exporter of natural resource products such as forestry & raw minerals to one exporting electrical & communication equipment such as cell-phones, engineering products over time. The highly differentiated nature of these products imparts a degree of monopoly power to their producers and thus would tend to diminish the elasticity of these products as is indeed seen in our present case of Finland bespeaking of the transformation of the Finnish economy over the course of forty years.

We tell a similar story when looking at the case of import price elasticity as it mirrors the trends remarked upon in the case of exports. The elasticities are of a negative sign which is what we would expect from economic theory (imports are regarded as inputs) and displays a declining trend for the time period under study with elasticities going from a value of -3.05 at the start of the period in 1960 to some -0.32 in 1999. Average price elasticity for the four decades is some -1.05 which reflects the high values in the decade of the 60s. For the 90s it may be easily verified from looking at Table 4 that the average import price elasticity is below -0.5 .Thus import elasticity goes from being high elastic to being inelastic i.e. price changes effect a less than proportionate response by importers. Our story for exports would be applicable in this context as well given the nature of Finnish trade whose composition is of an intra-industry variety[29].

Of particular importance are the estimates for own price elasticity of labour in model as it is germane to the issue of deadweight losses that will be taken up in Section 7. Based on our NQLS model we report an increasing trend in the labour own price elasticity over the years from 1960 until 1999. Thus our estimate of labour own price elasticity goes from -0.83 in 1960 to some -1.5 in 1999 with the period average standing at some -1.2 indicating that labour market in Finland has become more efficient i.e. with labor growing more responsive to price over time. Diewert & Fox (1998) suggest the increasing trend seen in the case of labour own price elasticity owes something to the nature of technical progress bias against labour reported in their study for the 18 OECD

[29]According to Statisitics Finland we have Finland exporting and importing primarily capital intensive goods which accounting for over 60% of total trade

countries. Technical progress was reported to be labour reducing in their model i.e. technical change lowers the labour requirement for producers over time. We impute a similar process is in play for Finland which would explain why labour's own price elasticity is growing over time suggesting greater flexibility on the part of employers and employees.

Cross Price Elasticities In addition to the own price elasticities of producers we would be interested in learning something about the cross price elasticities generated using our NQLS model with particular interest to ascertaining whether our outputs/ inputs remain complements or substitutes. In the case of producer output we say that two outputs are complements if their cross-price elasticities are positively signed i.e. if there is a price increase of one output the quantity produced of the other output increases as well. Analogously we may state that two inputs are complements if their cross price elasticities are negative i.e. if the price of an input increases then the quantity demanded of the other decreases. We report the average cross-price elasticities for the four decades in our model , first commenting on the complementarity/ substitutability of outputs followed by a look at the inputs.

Our two outputs were domestic output and exports. The estimated average output cross-price elasticities are positive are tabled under *Table 4a* . Our results would suggest complementarity but the actual magnitudes remain very small ranging in value between 0.018 to 0.0224. Such small values would indicate that a price increase for the domestic output would exercise a very small influence on the quantity of exports. Given the highly open nature of the Finnish economy where exports form a significant proportion of the economy we'd expect domestic output and exports to be highly complementary. One *possible* explanation could be the terms of trade effect whereby increasing terms of trade ,signifying an increase in the price of exports versus imports, would act to reduce the need to produce more exports in order to balance trade. In other words improving terms of trade would make exports not very responsive to changes in output.

Turning now to the case of the two inputs, namely imports and labour we report our finding a negative cross price elasticity between imports and labour. This suggest that our two inputs are complements i.e. a price increase (decrease) of one input would tend to reduce (increase) the demand for the other input as well. Moreover unlike the case of outputs the inputs have a relatively high cross-price elasticity which suggests that they are fairly good complements. The average cross price elasticity for the period as whole ranges between -1.609 to -0.46.

Finally we examine the cross price elasticity between output and inputs i.e. we look at the responsiveness of domestic outputs and exports with respect to the prices of labour & imports. The intuition from economic theory would stipulate that higher input price should reduce the quantity of output produced while higher output prices would tend to increase the quantity demanded for inputs. This is in fact what we found in the case of Finland. The cross price elasticity of domestic output with respect to imports and labour is -0.45 & -1.11 respectively. For exports we report the cross-price elasticity with respect to imports and labour to be -0.56 and -0.24 respectively. When we look at the cross price elasticities of inputs , imports and labour, we report the cross price elasticity of import to be 1.56 and 0.57 w.r.t. domestic output and exports respectively while the figures for labour are 1.6 and 0.12 w.r.t. to domestic output and exports respectively.

6.3 Consumer Regression Model (NQ)

In our consumer model the results of which are reported under $Table$ 3(b), we motivated a Normalized Quadratic (NQ) & NQ Linear Spline ($NQLS$) 2-good model. The two goods being consumption and leisure per capita. Additionally in NQ models of choice we imposed curvature properties a priori and allowed for the possibility of taste changes over time. A look at the R^2-statistic for our NQ consumer regression model reveals a figure of 0.712 suggesting that our model fits the data quite well as we are able explain nearly 71.2% of the variation in the expenditure share of consumption per capita via our model. Another point of interest would be to compare the estimated value of the scalar utility in our NQ econometric model (using eq. $E8$) with the indexed value of the utility obtained by taking the chained Fisher index of consumption per capita and leisure per capita presented in Table$Table$ 6.The utility obtained refers to the consumers' indirect utility which is function of income and prices. Given that the Fisher index is defined as a superlative index [30] i.e. it is exact for homothetic preferences, we may obtain an index for the indirect utility across two periods by using the Fisher Index. As $Table$ 6 shows the ,u, obtained econometrically from our NQ model and the one obtained by forming a chained Fisher index approximate each other quite closely. This is reassuring as it implies that we may calculate an index for utility if in place of estimation through econometric means.

NQLS model however does not fare as well as the NQ model as may be seen from the results tabulated in $Table$ 3(b) where we have listed the results from the NQ model by way of a contrast. The R^2-statistic for the NQLS is reported to 0.66 which we interpret to imply that we can explain some 66.7% of the variation in the dependent variables (expenditure share on consumption) on the basis of this model. Additionally when consider $Table$ 6 we find that in contrast with the NQ model our econometrically estimated values for the scalar utility do not approximate the Fisher Index level of utility very well. For instance in the years after 1988 which was our chosen break-point the Indexed utilities and the econometrically estimated u differ quite markedly from each other.

6.3.1 Own/Cross Price Elasticities

Own/cross price elasticity estimates ,which tell us how responsive consumers are to price change, for our NQ & $NQLS$ models are tabled under $Table$ 5(a)

[30](Refer to Diewert 1976)

& 5(b) and we plot the results in chart 7&8. The signs obtained are in line with what we would expect from economic theory. Examining the plot of the consumer own price elasticity reveals a somewhat declining trend for the period as whole with consumer own elasticity going from -0.83 in 1960 to some -0.67 in 1999. A similar trend is exhibited by the NQLS model where the consumer own price elasticity go from -1.39 in 1960 to -0.69 in 1999. An interesting point of note is the finding that for period until the break-point the estimates elasticities for the *spline* model have a greater magnitude in comparison to the NQ model. In any event the trends would suggest that the consumer own price elasticity is trending lower over time i.e. consumers are somewhat less responsive to the consumption prices in 90s than they were in the early 60s. The average for the period as a whole is -0.74 for the NQ and -1.08 for the NQLS. We classify an elasticity below 1 (ignoring the sign) as inelastic i.e. a price increase leads to a less than proportionate response to quantity consumed but we should bear in mind that we are employing a highly aggregated level of consumption. Thus consumption own price elasticities would be expected to exhibit higher and more variable elasticities compared to one estimated at the aggregate level.

Looking at the own price elasticities of leisure we report an increasing trend in the elasticities for the both the NQ and NQLS models i.e. the magnitudes of the estimated elasticities are growing progressively larger for the much of the period with the exception for the years 1993-1996 for the NQ and 1988-1999 for the NQLS model. An added difference in the case of the own price elasticity for leisure is the actual magnitude for any given year is more than double that found for the case of consumption. For the period as whole we report an average own price elasticity for the NQ and NQLS models as being -2.6 and -3.25 respectively. We find as well that the elasticities estimated for spline model are significantly greater than the ones obtained for the NQ model and the trend for the years from 1989 for the NQLS model evinces a dramatic decrease going from -5 in 1987 to -1.55 in 1988 and then decreasing gradually to a value of -1.11 in 1999.

The consumer's cross price elasticity are estimated to answer the question whether agents view consumption and leisure as substitutes or complements. A finding of a positively signed value for the consumer's cross price elasticity would indicate that agents in Finland regard consumption and leisure as substitutes. Table 6a reveals that cross-price elasticity is positive with two average cross price elasticities for the NQ ranging in magnitude between 0.73 & 2.6 while those for the NQLS model range from 1.07 to 3.25 for the period as a whole. This would suggest that leisure and general consumption are regarded as fairly good substitutes in Finland.

6.3.2 Income Elasticities

To examine if consumption is a normal or inferior good we estimated income elasticities of consumption and leisure through our NQ model which are tabled under $Table$ 5(c) and graphed in *charts* 9-10. The average income elasticities are summarized under Table 6a. Economic theory would describe a good as being normal if its consumption increases with the increase in income of the agent. A positive sign would imply that the good in question is a normal while a negative sign would indicate that the good is an inferior one. We report finding that general consumption is a normal good in Finland with the average income elasticity of consumption equal to 0.87. Similarly leisure is regarded as normal good as well with an average income elasticity equal to 1.59 over the forty years. *Charts* 9-10 reveal as well that income elasticities as estimated by our model have evinced a more or less stable trend with only a very slight incline being observed in the trends.

One wrinkle for the Linear Spline model are that the income elasticities will change discontinuously around the break-points as attested by the behavior on the *chart* 10 where the trends shift quite disjointedly for the break-point years. Moreover the trends for the two goods, consumption and leisure differ with income elasticity diminishing over time for consumption and rising over time for leisure. Nevertheless both consumption and leisure are found to normal goods in our $NQLS$ model with the average income elasticities for the two goods being estimated to be 0.33 & 3.6 for consumption and leisure respectively.

7 Discussion

Knowledge of elasticities naturally begets the question as to what we ought to do with it and why we went to all this bother in trying to get our Normalized Quadratic profit and expenditure function to crank out those elasticity estimates for us. In our introduction we had stated that one of the motivations for engaging in this research exercise was to get a handle on the productivity performance for Finland over the last four decades. The salient facts on the productivity performance of Finland paint a picture not altogether different from those obtained by other scholars who investigated the productivity performance of the OECD countries as whole. The period of the 60s (*swinging Sixties*?) and the decade of the nineties (*infectious greed*!) were associated with relatively high productivity growth while the period from the 1974 until the 1991 were characterized by fairly laggard growth. To say something about how countries might take steps to boost productivity requires that we construct a model based on economic theory that can account for the TFP growth performance. Such an endeavour would undoubtedly be a formidable one and one that we will not embark upon as the quest to explain the varying processes at work in the Finnish economy would be akin to Frodo's quest for the mythic ring in Tolkien's saga, requiring a trilogy of essays! In the interest of expediency we eschew that route and offer in its stead a qualitative explanation for the productivity swings by weaving a story that ties together the macroeconomic turbulence visited upon the Finnish economy along with the detrimental impact of deadweight losses from taxes incident on Finns.

7.1 Of Oil Shocks and Communist blocs

The rapid increase in annual TFP growth seen in the decade of the 60s were brought to an abrupt end by the spikes in the world oil prices in the years 1973-74 and 1978-79 when prices of crude oil quadrupled almost overnight. Finland's economy had been and continues to be a big oil guzzling one with energy being one of the chief imports of Finnish economy. Dramatic increases in world oil price would therefore have the knock-on effect of pushing up the prices of other goods and commodities given the primacy of oil in the economies of many developed countries especially in 70s. Inflation (see chart) measured by the consumer price inflation skyrocketed with our index of consumer price inflation displaying a steep rise to 20% in 1974-75 in the aftermath of the first oil price shock in 1973 and again to over 11% in the early 80s in the wake of the second shock in 1979. We may well ask what the costs of such high inflation were for the decade of the 70s and early 80s for Finland in terms of its impact in TFP growth actually measured? Bouts of high double digit inflation are typically abhorred by governments and citizens alike due to the deleterious impact of inflation on the future decision making by agents. For instance in the consumer context high inflation would adversely impact upon future savings and consumption decisions. On the producer side inflation would wreck havoc to resource allocation decisions given the perverse incentives abetted by soaring inflation for investments.This may be evidenced in our Finnish context by the calculated negative real interest years in the mid to late 70s and early 80s. As such firms in light of the uncertainty in payoffs would scale back or altogether postpone investment leading to sub-optimal levels of investment being undertaken in the economy. More important these affects of inflation may be expected to have long-term consequence and there are not expected to have merely evanescent impact on the Finnish economy.The low-levels of capital investment would thus appear to give us some insight into the causes for the stagnant TFP growth in those selfsame years.

The other major drag on the Finnish economy was the deep recession that Finland suffered as a direct consequence of the fall of the Soviet Union in late 80s /early 90s. The loss of the Soviet Union as a viable partner and the continued economic crisis that dogged the new Russia deprived the Finnish economy of a major trading partner and outlet for its exports. Not to mention the Soviet collapse nullified the barter agreement in place for many years that allowed Finland to exchange its manufacturing exports for Soviet oil and gas. The precipitous decline in the fortunes of the Soviet economy and the severe recession that followed in its wake created the right conditions for the Finnish economy's low, almost non-existent TFP growth in the late 80s & early 90s. Nevertheless the wrenching recession did not stymie economic performance for too long given that TFP growth and economic growth returned with a bang in the period after 1993 as we reported earlier. Thus we would infer from this that the recession hit years of the early 90s were more a consequence of a shortfall in aggregate demand and therefore of less significance in terms of explaining the relatively low rates of TFP growth viz-a-viz the periods of high inflation in the 70s. Of course high inflation and the consequent negative real interest rates constitute at best a partial explanation for the TFP growth slowdown in Finland during the period from 1974 until 1992. For the other half of this story we need to consider an additional candidate, namely high tax rates for the Finnish economy. We shall focus our attention on this subject in the next section on *Deadweight losses*.

7.2 Deadweight Losses / Excess Burden from Taxes

Taxes in Finland in common with other Scandinavian countries notably Sweden, Denmark have been relatively quite high in comparison to other OECD countries[31]. This feature owes part to the political and social priorities embraced by the Finnish populace for the creation of welfare state coterminous with the ascendancy of left leaning parties such as the Communist Party and the Social democrats from the mid-60s until the mid-80s. Increasing spending necessitates the imposition of increased taxation which economic theory informs has an onerous affect on the economy. How do we come by this idea? To start with we know that raising revenue to finance government expenditure on providing goods and services come with benefits and costs. Benefits would be the utility gains accruing to the individual(s) from consuming the services funded by taxes while costs would be in the form of opportunity costs imposed on members of society in remitting taxes to government. Opportunity costs of taxes typically *exceed* the revenue raised by taxes due to the perverse nature of incentives stemming from the incidence of a tax and thus are referred to as Deadweight Losses or Excess Burden of taxation. To crystallize this point we cite a quote from Diewert & Lawrence's 1996 CJE article-"*These incentive costs will be incurred if people turn to less preferred substitutes or employ less satisfactory methods of production*"

This point is made more emphatically with the help of a diagram and this is done in *charts* $11 - 12$ where we have modelled the distortionary impact of a proportional tax on producers and consumers ,the cost of the tax being shown as a net surplus loss.What factors would help determine the size of the deadweight loss for an economy? We can infer that the responsiveness of economic agents to taxes will be one of the important factors affecting deadweight losses. This was seen in the diagram presented in *chart* 12 where we envisioned a scenario affecting the supply of labour in the event of tax imposed on labour. The demand for labour is drawn as a horizontal line which signifies perfect elasticity in the demand. Consequently the burden of the tax falls squarely on the shoulders of our suppliers of labour. The shaded region delineates the deadweight loss suffered due to a tax. Now if the labour supply schedule were to be drawn with a flatter slope indicating a greater elasticity or responsiveness on the part of labour suppliers then we can easily see that the deadweight loss would be greater. A tax drives a wedge between buyers and sellers thereby introducing distortions in the market which is not fully offset by the revenue raised. Greater the responsiveness of buyers and sellers to the price wedge created by a tax, the

[31] see Joumard & Suyker (2002)

greater would be the deadweight loss suffered. In short one of the key factors affecting the deadweight loss (or what is also referred to as the excess burden) is the elasticity of agents to price increase effected by a tax. Rising trends in elasticity of our economic agents would signal that this excess burden from a tax would be growing correspondingly larger for our economy with repercussions on the other economic parameters including among other things the nature of the productivity growth.

To give the reader a snapshot of the tax rate structure in Finland over the four decades we tabulate the average tax rates from 1960 until 1999 in *Table* 7. In addition we present the actual tax rates for the five categories of consumption, import, labour, capital income and asset tax rates for the various years and graph the same in *Chart* 13. Of interest would be the *average* tax rates for consumption, labour and business/capital income which are computed to be 21.5%, 35.8% and 40.71% respectively. A glance at *Chart* 13 reveals as well that the tax rates for consumption and labour have been increasing over time whereas the business taxes show a roughly secular rise from 1970 onwards , peaking in 1991 at over 65% followed by a fall in early 90s and a slight rise in the years after 1993. We had shown earlier that Finnish economy's elasticities for general output as well as exports and import had been falling over time. However we computed previously that the magnitude of the average elasticity was hovering around 1 for the case of *general output* and actually increased in the case of *labour*. This would be a suggest that the tax burden has been rising over time for labour that would not bode well for Finnish TFP growth.

A natural extension would be to actually emulate Diewert & Lawrence (1996)[32] by quantifying these estimates of excess burden with the aid of a GE model used by the aforementioned authors in their research.We have not applied their methodology for the Finnish case. Despite the lack of quantitative estimates we would still be willing to bet that the excess burden on producers and labour has been increasing over time with the possible exception being period after 1994 when the centrist government of Paavo Lipponen embarked on its wide ranging policy reform that included among other things a revamping of the size of the government and deep tax cuts (see Joumard & Suyker (2002 for an overview).

7.3 Did we Overlook anything?

Yup! A number of scholars most notably Diewert[33] have instigated a major research agenda designed to explain what has come to be referred to as the 'productivity paradox'. TFP growth performance did not appear to pick up very much in the period from 1974 until 1991 which were also the years which saw the widespread adoption of the computer in the workplace. The well documented improvements in the technical performance of computers coupled by

[32]Diewert & Lawrence's CJE 1996 article forms a part of their larger research program on the New Zealand economy which is available as a New Zealand Treasury Discussion paper (1999)

[33]See Diewert & Fox (1998,1999)

falling prices would suggest that TFP growth should have taken off or at least shown greater rates of increase during this period when the computer went from being an exotic & unwieldy piece of office equipment to the versatile equipment that it is today. Yet productivity as measured by TFP growth in Finland for example showed no signs of catching fire as we saw in our case with average TFP growth for the 1974-1991 period being a paltry 0.7% per annum. Hence the moniker of a 'productivity paradox given by researchers to describe the anomaly of lack-luster TFP growth in the face of rapid technical progress in computing.

Among the factors singled our for attention are measurements errors that have historically been regarded as being somewhat trivial by productivity researchers. Diewert & company underline the fact that measurement errors may not have been as innocuous as they seem and in particular point to the flaws in the methodology of statistical agencies that leads them to traditionally neglect the *benefits* of new products and processes while recording the costs. The benefits would be the expansion of consumer choice sets, the 'reduction' in the price of quality with the inclusion of newer products that is not captured by statistical agencies in their consumer indexes. Additionally Diewert points to the other sources of measurement errors such as classification errors on the part of businesses and firms in the manner in which they fail to recognize business expenditures as consumption expenditures, the difficult problem of measuring the productivity of the service sectors, the related topic of properly accounting for the government service sector productivity and finally they revisit the whole topic of inflation and its role in understating productivity in OECD countries.

Actually the growing role of the government sector along with the services and how might go about accounting for it deserves a whole paper devoted to it. Why it is relevant to the issue of productivity growth is that the traditional way of measuring government (service sector productivity as well) output is to assume to that government output is equal to the magnitude of the costs of its inputs. This would without a doubt lower our estimates for productivity growth if we hark back to specific formula used for computing TFP growth set out in section 3 where one of our definitions of TFPG was :

$[(1+m^t)/(1+m^s)][(w_1^t w_1^s)/(p_1^t p_1^s)] \ \equiv TFPG(4)$

i.e. ratio of growth in margins after controlling for price change where our period t margin m^t is defined as :$1+m^t \equiv R^t/C^t$, $t=1,..T$. Now if we assume that factors are paid their marginal product so that period t margins m^t is equal to zero so that the revenue/cost ratio (R^t/C^t) is 1 then TFPG would be equal to the growth of price of inputs divided by price of outputs. Government TFP growth in this case would be zero as the prices of input and output in the equation above would cancel out leaving us with the implausible result that there has been no improvements productivity of the government sector. If the government sector looms large in the domestic economy as is believed to be the case in Finland then we would be guilty in underestimating productivity for the economy as whole[34]. Ditto for the service sector. Such a state of affairs should

[34]If we go back to $Table1$(a) on the behavior of aggregate price indices we can see that the

not be allowed to persist and future studies on TFP growth performance should explore ways for a more effectively measuring TFP growth for government and the service sector.

To investigate these additional channels by which productivity might have languished in the 1974-91 period would require us to interrogate data on the pace of new product entry in the post-1973 period versus earlier time periods, survey or published data on current Finnish practices of composition of business expenditures for firms over time, current size and methodology in the classification of the service sector inputs. We do not pursue these issues at this time and leave it as a topic for other researchers interested in an in-depth appraisal of the TFP growth performance of Finland. However we wish to touch on the subject of inflation and the misallocation of capital as applied to the Finnish case in slightly greater detail and do so in the following subsection.

7.4 Inflation *redux*

To motivate our discussion on the substantial influence that inflation has on the economy and in particular the channel by which inflation impacts on the productivity performance we refer the reader to *Chart* 14 that plots inflation and TFP growth for the Finnish economy for the period 1960 until 1999. Looking at the chart we are at once struck by coincidence between the period of the sharp decline in the TFP growth in the mid-70s and the high double digit inflation beginning in1973. Furthermore years with high inflation i.e. inflation in the double digits are associated with a declining or non-increasing trend in TFP trend in subsequent years. It thus seems no surprise that the period of the 90s were coterminous with relatively strong rates of growth in TFP for Finland given the quiescence of inflation in these years when TFP growth chalked up an impressive pace of 4.6% per annum on average while inflation remained south of 2% on average. Previously we alluded to the destructive impact of inflation on producers' allocation of capital or investment decisions as well as the detrimental role of taxes in creating deadweight losses in lowering TFP growth. We will now use a stylistic example sponged from Diewert & Fox (1998) that formalizes the relationship between inflation, taxes & productivity growth.The authors' story proceeds as follows.

Let there be a firm that purchases a durable input at a price ,P, and for which it employs a depreciation allowances in period t, D_t, as permitted by the tax authorities where we have:$P = D_1 + D_2 + ...D_T$, and T, denotes the useful life of this asset. Immediately one can see that the problem with this system of depreciation allowances is that the value of the depreciation allowance for a time period in the future will be less than the price made in the present i.e. if we take the present value of the depreciation allowances , (V) from period 1 until

price index for government expenditures has shown a relatively high rate of increase compared to the other aggregate indices. This increase in the price of government sector might be taken to imply that government sector productivity has been rising quite rapidly as well. We credit this insight to Benson Sim who put forward this hypothesis in his July 17th, 2002 presentation for the ECON 594 class.

the end of the period we will get that: $V \equiv D_1 1 + r_1 + D_2(1 + r_1)(1 + r_2) + D_3(1 + r_1)(1 + r_2)(1 + r_3) + ...D_T(1 + r_1)....(1 + r_T) \prec P$

where r_t is the firm's period opportunity cost of capital that is assumed to be positive for $t \equiv 1...T$. This implies that our firm's purchase price, P, would typically exceed the present value of depreciation allowances by the amount, $(P - V)$ which would leave it with a tax liability of $\tau(P - V)$, with τ, the tax rate on inputs. Diewert & Fox point out in a such a position the firm would have to balance its tax liability by in effecting charging itself an internal price P_I, which differs from the purchase price P, and is related to the original price as : $P_I = P + \tau(P_I - V)$ which would get us the specification:

$$P_I \equiv P + (1-\tau)^{-1}\tau(P-V) \succ P \qquad (E10)$$

where eq.$(E10)$ follows from the fact that $0 \prec \tau \prec 1$. Continuing the use of their notation if we make a few simplifying assumptions that set all the interest rates, r, equal to a constant for the period and use a straight line depreciation we may manipulate eq. $(E10)$ to take the following functional form of a ratio of $P_I P$ as follows:

$$P_I P = 1 + (1-\tau)^{-1}\tau[1 - \{1 - (1+\tau)^{-T}\}/rT] \qquad (E11)$$

From the specification of the price ratios of $P_I P$ will be increasing in the useful life of the input (T) and increasing with the nominal interest rate of capital (r). Accordingly the price wedge driven by the tax (τ) will be proportionately greater when we allow for inflation as inflation drives up the nominal interest rate resulting in the increase in the wedge or distortion between the actual and internal price for the firm. Why is this important? Well if the firm were to be a profit maximizing or cost minimizing one then the tax rate on inputs in tandem with the high inflation would cause it to diminish the use of this input or in effect reduce its investment that uses this input. This would be bad news when it comes to productivity as the impact of inflation acting in concert with tax rates given the nature of depreciation balances acts to reduce the level use of an intermediate input. To show how this effect materializes the authors make use of the assistance of a simple two-sector equity finance model where one sector produces a product for final demand and uses an input (an intermediate investment good) that is produced by a second sector. In their model the authors have labour, L, as one variable resource used and an intermediate investment good, I, that is produced by a second sector.The production function ,F, for the first sector in their model is given as:

$$Y = F(L_1, I)$$

which gives us the maximum value of output ,Y, produced by the first sector using L_1 unit of labour stock and I, units of the intermediate investment good produced by their second sector.For this second sector we have the case that the intermediate investment good is produced using L_2 units of the variable stock of labour ,L, not utilized by the first sector i.e. we have: $L \equiv L_1 + L_2$. The production function of this second sector producing the investment is given simply as : $I = L_2$. Thus putting all this together we have our production function for the first sector given as :

$$Y = F(L_1, I) \Longrightarrow Y = F(L_1, L - L_1)$$

and our objective is therefore to maximize this objective function in L_1 which is a simple planning problem that takes the form of :

$$\max_{L_1} \; F\left(L_1, L - L_1\right)$$

Solving this planning problem for L_1 and denoting the optimal value as ,L_1^*, that gives us as well the optimal value of the intermediate investment good as $I^* = L - L_1^*$ (and output of the for the first sector as ,$Y^* = F(L_1^*, I^*)$), the authors then go on to show in their paper that a tax levied @ t,on the production of the intermediate input I would have the effect of reducing the output produced (Y) by first sector as it must cut back on the use of the taxed input. The interesting result from this model is not that output falls in the presence of a tax but that its fall may be of magnitude that would help explain the steep fall in the productivity that was observed in the post-1973 period. This is done by approximating the loss in output through the use of duality theory where they motivate a unit cost function,c, that is a dual of the production function F where $c(w,p) \equiv \min_{L_1,I}\{wL_1 + pI : F(L_1, I) = 1\}$, w & p stand for the prices of the two inputs labour ,L, and the intermediate investment good I after the imposition of a tax t. i.e. we have $p = (1+t)w$. Solving this cost minimization program leads the authors to derive a second order approximation for the loss generated by the tax rate t in output as function of the of the optimal output ($Y(0) = Y^*$) that they specify as follows:

$$[Y(0) - Y(t)]Y(0) = (1/2)t^2 \varepsilon s_I s_L \qquad (E12)$$

where they defined $s_I \equiv \frac{I(0)}{Y(0)}$ as the (optimal) intermediate goods share of output and $s_L \equiv \frac{L}{Y(0)}$ is the variable resource's (optimal) share of output while ε stands for the price elasticity of demand for intermediate inputs by sector 1.In order to estimate the loss in output resulting from a tax on input in the event of tax we may use the substitute for the t the ratio of the internal price charged by the firm to the actual purchase price of an intermediate input, P_I/P, as our approximation of the price wedge introduced due to a business tax. Thus to get our numerical estimate we will use the assumptions from Diewert & Fox (1998) in ascribing a value for $\varepsilon = 1$, $s_I = 0.1$ and $s_L = 1$ and use $T = 20$ i.e. we examine the case where the elasticity of sector 1 for the intermediate input is 1,share of intermediate good is 10%, share of labour in general output is 1 and where the useful life of intermediate asset is 20 years. We do however differ from Diewert& Fox in choosing as our business tax rate (t) a value equal to 0.41or 41% which is the average tax rate for the Finland over the forty years.Using this we estimate the loss of output using different values for the nominal interest rate ranging from $r = 0.03$ to $r = 0.2$. The periods of high double digit inflation in the mid-70s following the two oil price shocks would thus be good candidates for the $r = 0.2$ given that inflation rocketed to over 15% for this period.

Our results from this simple simulation are reported under *Table* 7a. Of special interest are the value calculated for $r = 0.2$ signifying very high inflation similar to ones experienced in Finland in mid-70s. We compute using our model that output loss would have been in the neighborhood of 1.4% which is not a trivial amount by any means. Thus the impact of inflation coupled with the nature of depreciation rates and taxes in our stylized example has been shown to impose *substantial* costs in our economy under a simple equity finance model. Clearly it would be tantamount to hubris if were to believe that inflation and the burden from taxes were the sole causes of the abrupt slowdown in TFP growth . However as the authors point the importance of performing such a simulation is to lend credence to the view that misallocation of capital due to business taxes in cahoots with high inflation may well have played a sinister role in undermining the TFP growth for decades of 70s and 80s. The resurgence in the TFP growth in the 90s in an era of lower business taxes coupled with nascent inflation should not therefore be a total surprise and suggests that reducing the tax burden on firms and labour continue to be a top priority for policy makers.

8 A Few Limitations

8.1 No Sectoral Break-down

The results obtained for the TFP growth so far have been remiss on a few points. Clearly if one of the primary motivations of this paper is to shed some light on the productivity performance of Finnish economy over this period then it is incumbent on us to provide a break down of the productivity by looking at the various sub-sectors making up the Finnish economy. This would be especially helpful from the point of view of prescribing effective policy guidelines to government mandarins, corporate leaders etc. This would be emulating Diewert & Lawrence (1999) who assemble a database consisting of over 44 outputs and some 20 inputs which allows them to compute the productivity for the economy as a whole and provide a sectoral breakdown for New Zealand that gives the comparative performance of various sectors over time.

A second major drawback is that we have not provided a formal i.e. a quantitative breakdown of the contributions of productivity, terms of trade etc. on the economic growth of Finland. The seminal papers that have attempted to do this would be Kohli (1978,1989) and Diewert & Morrison (1986) that set out the methodology for just such a task by specifying a GDP function which enables them to extricate separately the impact of labor, capital, terms of trade, non-traded sectors on economic performance. This is especially important in light of the fact that we have period when terms of trade were increasing but productivity showed little or no growth (80s). Given the beneficial influence of rising terms of trade for the economy we would wish to know if and by how much of the economic growth was the consequence of the terms of trade versus TFP growth. Clearly such a task is worth doing in light of the availability of the theoretical framework pioneered by the aforementioned researchers and should motivate future research on this issue.

8.1.1 An Crude Approximation

We can give a foretaste of this by making a crude approximation of the contributions of the TFP growth and the combined factor inputs (labour and capital) on the real economic growth in Finland for the period from 1960-1999. Just as a reminder we computed our combined factor inputs by forming a chained Fisher quantity index (X) which is an aggregate of labour and capital. Thus we can make a crude estimation of the contribution the combined factor inputs and TFP growth in the real Gross Domestic Product (GDP)[35] for Finland for the period under investigation. This we have tabulated in the table shown below:

Table: Contribution of TFP Growth and Combined Factor Inputs to RGDP growth

Time Period	Real GDp Growth (%)	Contrib. of TFP Growth(%)	Contrib.of Inputs (L,K) %
1960-1999	3.47	52.3	47.7
1961-1970	5.10	56.3	43.7
1971-1980	3.51	58.9	41.1
1981-1990	2.59	31.2	68.8
1991-1999	2.6	78.1	21.9

As may be seen from our table the contribution of TFP growth has been comparatively lower in the decades of the 80s with most of the growth in real economic growth being the result of a factor accumulation rather than due to any underlying improvement in factor productivity. The turnaround in productivity reported in the 90s does show up in the tables when we consider the contribution of factor productivity which accounts for a whopping three-quarters of all growth in the real economic output and underscores not only the productivity rebound but all the importance of the productivity as the real engine of growth in the 90s[36].

8.2 Whither Tax Reform?

It has been argued in our paper that TFP growth performance has been compromised by the incidence of high tax rates compounded by the impact of inflation and accounting practices. With monetary policy currently under the aegis of the European Central Bank based in Frankfurt one area that has to be an important focus for the Finnish policy makers and public alike is tax reform. We had

[35] We're used a growth-accounting framework to extract the separate contributions of factor growth and productivity growth on Real GDP growth

[36] Of course this itself is testament to the depths of the recession suffered in Finland in the early 90s. As a result there was substantial excess capacity in those years and accordingly when conditions amerliorated in years after 1993, productivity was the first to rebound as firms held back on investment and hiring and instead tried to meet rising demand through greater capacity utilization. It is starting from 1996 when firms grew more sanguine about business conditions that we observed a move towards factor accumulation by companies looking to expand capacity to meet demand and thereby a greater contribution of the factor accumulation to the growth in Real GDP

underlined the propinquity of tax policy reform (instigated by the new right-left coalition government) in the early 90s and the rekindling of the TFP growth as evidence of the culpability of excess burden in diminishing TFP growth operating through high tax rates particularly on labour and business. To chart the future direction of reforms of the tax system in place in Finland would be a little outside the scope of this paper but we can draw attention to the work by Joumard & Suyker (2002) that sheds light on the pertinent issues. In their paper the authors provide an in-depth look at the tax structure in Finland with reference to the system of tax policy reforms instituted by the Finnish government in the mid-90s. J & S present two cogent arguments that would conceivably stay the hands of reform minded Finnish policy-makers. The first point is the future financial pressures on the pension system in Finland due to an aging population and two, the extent of tax policy reform already undertaken by the Finnish government in the corporate tax area which were driven by the competitive pressures on Finland to make its ICT sectors more competitive[37] with respect to other countries in the EU as well as the US. As such the authors point to the fact that further reform in light of these pressures would involve a delicate balancing act on the part of Finnish policy makers who must balance the need to keep Finland competitive in the area of attracting high tech investment all the while cognizant of the need to take remedial action to shore up the pension system in light of future demographic trends. With these issues in the foreground, Joumard & Suyker still recommend a policy of lowering taxes on labour[38] through shifting the burden on the hitherto comparatively less taxed property as well as the phasing in of additional consumption taxes[39].

9 Some Final Thoughts

In summary we have used the services of chained Fisher index of output and inputs to compute the Total Factor Productivity Growth (TFPG) for Finland for the period from 1960 until 1999. For the period as whole we reported that Finland's TFP growth was 2.4% on average. More important we pointed out the TFP growth was higher in the pre-1973 period as compared to the post-1973 with average TFP growth in the former period standing at some 3.35% as compared to a figure of 1.9% in the latter sub-period. This would appear to be in keeping with the results reported in a number of other studies on TFP growth performance that found productivity growth dipped in the period after

[37] Accession to the conditions of the Maastricht Treaty as well as the general upturn in the economic conditions after the withering recession in 1991-93 were other important agents of change for the Finnish fiscal policy

[38] especially for highly skilled labour in light of their relatively greater mobility which would make them more elastic to higher taxes. We noted earlier that labour elasticities have evinced a tendency to rise over time and thus the deadweight losses operating through high labour taxes would be expected to be quite large for Finland

[39] All this of makes the assumption that the fiscal policy iwill not be hamstrung by a weak economy due to slackness of demand in Finland's export markets of Europe, North America and Asia which under the current climate of uncertainty is far from given

the first oil price shock in 1973-74 as compared to the period prior to 1973. Moreover we have identified the period from 1974 until 1991 as being especially bereft of strong TFP growth with the average TFP growth being a lowly 0.7% growth. However this is not the entire story as far as Finland is concerned. The decade of the 90s has been found to be decidedly propitious to Finland's TFP growth performance, which has staged a remarkable comeback. TFP grew on average at a sizzling pace of 4.7% in the Nineties once the economy shrugged off its torpor due to the deep recession suffered in the early 90s.

The second aspect of our study was to employ Normalized Quadratic (NQ) producer and consumer models to calculate the magnitudes in the annual elasticities of the output/inputs from the producer and consumer vantage points. Trends in the producer and consumer elasticities were commented upon with the view to possibly suggesting the detrimental impact of deadweight losses operating on the economy due to presence of high taxes. Additionally our NQ producer models allowed us to ferret out the rates of technical progress for Finland over the four decades from 1960 until 1999. The trends observed there appeared to mirror those found in the case of the TFP growth over the same period i.e. rates of technical progress[40] appeared to lag considerably in the 70s and 80s echoing the pattern seen for the TFP growth before storming back in the 1990s after the recession years from 1990-92.

Our findings from the elasticity estimates provided tacit support for the view that the deadweight losses or excess burden may have been culpable in the abrupt drop in the TFP growth seen in the period after 1973 and especially in the period from 1973 until 1991 when we factored in inflation and tax rates in a simple back of the envelope calculation. Hence a natural extension of our findings would be to quantify the extent of deadweight losses suffered by calibrating a general equilibrium (GE) of the type used by Diewert & Lawrence (1996). This would allow us to test our hypothesis that Finland has been subject to large and growing deadweight losses over time that is relevant to explicating the TFP growth slowdown in the 1970s and 80s. Should these results pan out one of the policy prescriptions stemming from this study would be that Finland implement a more business –friendly tax structure by taxing outputs and property[41] rather than capital and labour inputs so as to sustain its high TFP growth trajectory. The easing the tax burden would in turn require a continued retrenchment in government spending through reduced spending as well as continued policy reform to reduce the regulatory burden on companies operating in Finland. The sterling performance of the technology sector which came about through the confluence of deliberate policies such as the liberalization of the telecom sector and lowering of business taxes as well as the stabilization of inflation should be an object lesson on the management of the Finnish economy for policymakers in their bid to ensure that Finland continues to reprise its reputation ,burnished in the 90s, as an economic & technological powerhouse within the OECD and

[40] We assumed in our model that all technical progress computed is of a neutral kind i.e. where technical progress leaves the marginal rates of technical substitution unchanged.

[41] recommendation from a recent working paper by Joumard & Suyker (2002) on Tax Reform for Finland

retain the bragging rights to being labelled a '*Nordic Tiger*'.

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Appendix

10 An Overview of Index Number Theory

The following abridged discussion on construction indexes for price and quantities borrows quite heavily from Professor Diewert's numerous contributions in this area. In particular the reader is referred to Diewert's own review of the relevant issues pertaining to the theory of index numbers contained in Diewert (1987,1992b, 1993a, 1995a,1995b,1998, 2000, & 2002). The analytical framework for the study and construction of index numbers in the literature has been categorized under the following five main headings:

- The fixed basket approach
- The test or axiomatic approach;
- The stochastic or statistical approach;
- The economic approach and
- The approach of Divisia.

We omit the discussion the Divisia approach and the reader is referred to Diewert's treatise on the subject to obtain an overview of this last approach.(Refer to Diewert 2002). Instead we embark on a very cursory examination of the index theory and why the Fisher price/quantity index might be regarded as being the 'best' functional form for an index.

10.1 Fixed basket approach

Traditionally scholars in interested in the behavior of prices over time have approached the problem by constructing a common basket of products that a representative consumer might use and looked at the prices of the components of these typical basket of goods over time. Two most common ones used by major statistical agencies are the Laspeyres and Paasche price indices which are constructed in the following fashion for the bilateral case i.e. when we have two periods, the base period 0 and period 1 for $n = 1, ..N$ commodities as:

Laspeyres Price Index, $P_L(p^0, p^1, q^0) \equiv p^1 \cdot q^0 / p^0 \cdot q^0 \equiv \sum_{n=1}^{N} p_n^1 q_n^0 / \sum_{n=1}^{N} p_n^0 q_n^0$

Paasche Price Index, $P_P(p^0, p^1, q^1) \equiv p^1 \cdot q^1 / p^0 \cdot q^1 \equiv \sum_{n=1}^{N} p_n^1 q_n^1 / \sum_{n=1}^{N} p_n^0 q_n^1$

where q^0, q^1 refer to the basket in periods 0 and 1 respectively. If we use our knowledge of expenditure shares of commodity n in periods 0 and 1 as $s_n^0 \equiv p_n^0 q_n^0 / \sum p_n^0 . q_n^0$ & $s_n^1 \equiv p_n^1 q_n^1 / \sum p_n^1 . q_n^1$,respectively then our two price index formulas may be restated in share form as follows:

Laspeyres Price Index, $P_L(p^0, p^1, q^0) \equiv \sum_{n=1}^{N} (p_n^1 / p_n^0) s_n^0$

Paasche Price Index, $P_P(p^0, p^1, q^1) \equiv \sum_{n=1}^{N} [(p_n^1 / p_n^0)^{-1} s_n^1]^{-1}$

From this new share based specification of the two indices we have the fact that the Laspeyres price index is a (base period)share weighted arithmetic average while the Paasche price index is a harmonic average of (period 1) share weights. A well known result in from the field of inequalities is that the harmonic average is less than or equal to the arithmetic average. This would imply that $P_P \preceq P_L$ where the inequality will be a strict one when prices in period 1 are not all a constant multiple of the prices in period 0 & the expenditure shares are constant over the two periods. In other words one would estimate different magnitudes for the price index depending on whether one chose the Laspeyres or the Paasche as the formula for the computation of an index with the Paasche being typically smaller than the Laspeyres. One way to circumvent this problem would be to take an average of two and Diewert (1993a) that the one 'symmetric' average of the two indices is given by a geometric average of the two indices which is our Fisher price index ,$P_F \equiv [P_L \times P_P]^{1/2}$. Furthermore such an average of the two indices is a superior one to the Laspeyres and Paasche as these basket do not allow for the possibility of a substitution bias and thus would tend to over or underestimate the price changes over time. We will take this up again when we discuss the economic approach.

10.2 Test or Axiomatic Approach

This approach to the study of index numbers requires that any index formulation being essentially a weighted average of prices or quantities satisfy a battery of mathematical properties or axioms that would allow us to repose greater confidence that the chosen index is well-behaved. These tests which come to 20 in are listed below and the greater the number of tests our specific index satisfies the more well-behaved it is regarded to be. These tests are:

T1. Positivity test: $P(p^0, p^1, q^0, q^1) > 0$;

T2. Continuity:$P(p^0, p^1, q^0, q^1)$ is a continuous function of its arguments;

T3. Identity test: $P(p^0, p^1, q^0, q^1) = 1$;

T4. Fixed basket test: $P(p^0, p^1, q^0, q^1) = p^1 \bullet q / p^0 \bullet q$

Homogeneity tests:

T5. Proportionality in current prices: $P(p^0, \lambda p^1, q^0, q^1) = \lambda P(p^0, p^1, q^0, q^1)$ for $\lambda > 0$;

T6. Inverse proportionality in base period prices: $P(\lambda p^0, p^1, q^0, q^1) = \lambda^{-1}$ $P(p^0, p^1, q^0, q^1)$ for$\lambda > 0$;

T7. Invariance to proportional changes in current quantities: $P(p^0, p^1, q^0, \lambda q^1)$ $= P(p^0, p^1, q^0, q^1)$ for $\lambda > 0$;

T8. Invariance to proportional changes in base quantities: $P(p^0, p^1, \lambda q^0, q^1)$ $= P(p^0, p^1, q^0, q^1)$ for$\lambda > 0$;

Invariance and symmetry test:

T9. Commodity reversal test: $P(p^{0*}, p^{1*}, q^{0*}, q^{1*}) = P(p^0, p^1, q^0, q^1)$, where $*$ denotes a different ordering of the commodities;

T10. Invariance to changes in the units of measurement: $P(^*p_1^0, \ldots, ^*p_N^0; ^*p_1^1, \ldots, ^*p_N^1; q10, \ldots, *n{-}1qn0; *1{-}1q11, \ldots, *n{-}1qn1) = P(p_1^0, \ldots, p_N^0; p_1^1, \ldots, p_N^1; q_1^0, \ldots, q_N^0; q_1^1, \ldots, q_N^1)$ for $*1 > 0, \ldots, *n > 0$;

T11. Time reversal test: $P(p^0, p^1, q^0, q^1) = 1/P(p^1, p^0, q^1, q^{01})$;

T12. Quantity reversal test: $P(p^0, p^1, q^0, q^1) = P(p^0, p^1, q^1, q^0)$;

T13. Price reversal test: $Q(p^0, p^1, q^0, q^1) = Q(p^1, p^0, q^0, q^1)$;

Mean value tests:

T14. Mean value test for prices: $Min\{p_i^1/p_i^0 : i = 1, \ldots, N\} \preceq P(p^0, p^1, q^0, q^1) \preceq Max\left\{p_i^1/p_i^0 : i = 1, \ldots, N\right\}$;

T15. Mean value tests for quantities: $Min\{q_i^1/q_i^0 : i = 1, \ldots, N\} \preceq Q(p^0, p^1, q^0, q^1) \preceq Max\left\{q_i^1/q_i^0 : i = 1, \ldots, N\right\}$;

T16. Paasche and Laspeyres bounding tests: $P(p^0, p^1, q^0, q^1)$ lies between $P_L(p^0, p^1, q^0, q^1) = p^1 \cdot q^0/p^0 \cdot q^0$ and $P_P(p^0, p^1, q^0, q^1) = p^1 \cdot q^1/p^0 \cdot q^1$;

Monotonicity tests:

T17. Monotonicity in current prices: $P(p^0, p^1, q^0, q^1) < P(p^0, p^2, q^0, q^1)$ if $p^1 < p^2$;

T18. Monotonicity in base prices: $P(p^0, p^1, q^0, q^1) > P(p^2, p^1, q^0, q^1)$ if $p^0 < p^2$;

T19. Monotonicity in current quantities: $Q(p^0, p^1, q^0, q^1) < Q(p^0, p^1, q^0, q^2)$ if $q^1 < q^2$;

T20. Monotonicity in base quantities:$Q(p^0, p^1, q^0, q^1) > Q(p^0, p^1, q^2, q^1)$ if $q^0 < q^2$.

The Fisher Index satisfies all the 20 twenty test unlike the Laspeyres and Paasche indices which fail the crucial time reversal tests. Furthermore Diewert has shown that the only index that does satisfy the twenty tests is in fact the Fisher Index which would be a second reason for picking the Fisher as 'best' index for price and quantities.

10.3 Stochastic or Statistical Approach

The approach towards the construction of index numbers follows from the line of work instigated by Jevons and Carli in the 19th century and may be further divided into two separate approaches: an unweighted and a weighted stochastic approach. The price index developed by Jevons and Carli are given as:

Jevon Price Index, $P_J(p^0, p^1\} \equiv_{i=1}^{N} \left(p_i^1/p_i^0\right)^{(1/N)}$

Carli Price Index, $P_C(p^0, p^1\} \equiv \sum_{i=1}^{N} (1/N) p_i^1/p_i^0$

where both approaches consider the relative price ratios for the various commodities as being of the form: $p_i^1/p_i^0 = \alpha + \varepsilon_i$, α here standing for the common inflation rate and ϵ_i being a normally distributed error term. Such an approach would make this approach amenable to estimation using the OLS method and hence its name. In terms of tests or axioms satisfied the Carli index which is an arithmetic average of the relative price ratios does not satisfy the time reversal test and is in fact upwardly biased while the Jevons ,which is a geometric mean of the relative price ratios , fares better as it does pass the time reversal test.However the fact that both these price indexes are unweighted to be more exact accord equal weight to the various commodity price ratios had led to calls for the development of a weighted stochastic price index. One such index is the Törnqvist-Theil price index that is stated in the log from as :

$lnP_T(p^0, p^1, q^0, q^1) \equiv \sum_{n=1}^{N} (1/2)(s_n^0 + s_n^1) ln(p_n^1/p_n^0)$

where the weights used here are the average of the expenditure shares for the commodity n over the two periods 0 & 1. Our P_T the satisfies time reversal property and the specification chosen for the price index can be interpreted as the expected value of the logarithmic price ratios. Additionally in the next section on economic approach when we introduce the notion of Diewert (1976) of 'superlative' indexes we shall refer to the result that Fisher , Theil and Walsh are all superlative indexes. This is turn gives another boost to choice of Fisher as the index method of choice in aggregating over price and quantity.

10.4 Economic Approach

Konüs defined a the true cost of single household as the ratio of the cost minimization across to periods to attain some reference utility level where it was assumed that our household had well defined preferences over the N commodities and the price vectors $p^0 \equiv \left[p_1^0, ..p_N^0\right]$ & $p^1 \equiv \left[p_1^1, ..p_N^1\right]$ are all assumed to be positive and our utility u is a function of the common quantity vector, $q \equiv [q_i, .q_N]$ for the two period 0 and 1 i.e. we have our true cost of living for the two periods given as :$P_K(p^0, p^1, q) \equiv C[u = f(q), p^1]/C[u = f(q), p^0]$ where $C(u, p)$ is our cost function for the household for a particular period. Now we may use the results owing to Diewert (1983) that the Laspeyres price index which used the basket of goods from the base period as overstating the true cost of living index given by the Konus index while the Paasche index which uses the period 1 basket of goods and services as understating the true cost. These shortcomings of the two indices in turn created the need to once again obtain a specific price index that would take the average of the Paasche and Laspeyres and thus enable us to come closer to the true cost of living index. The Fisher index was earlier defined to be a geometric mean of the Laspeyres and Paasche and would thus by definition lie midway between them which would appear to make it a candidate for our true cost of living index. Once again the important contribution by Diewert (1976) proved that this indeed was the case that the Fisher was exact for the true cost of living index under a system of homothetic preferences where the cost functions are 'flexible' i.e. the unit cost functions can approximate any linearly homogenous system of cost function to the second order. This property of flexibility and exactness was termed superlative and was found to hold for the Fisher, Törnqvist and Walsh indices earning them the sobriquet of superlative indices.

Tying all this together we have the Fisher as being our best index owing to the fact that it is a symmetric mean of the Laspeyres and Paasche (Fixed-basket), it satisfied all the 20 test set out for an index (Test), it is a superlative index (Economic) and shares this property with the Tornqvist-Theil index and thus approximates the Theil index that was the best from the stochastic approach.

11 Producer Models: Profit Functions

We define as a variable profit t function as follows:

$V(p, k) \equiv max_y \left\{p^T y : k = F(y)\right\}$

where p^T denotes the transpose of the price vector, $F(y)$ is termed the capital requirements function, that gives the minimum amount of capital k required to produce the vector of net outputs, y.

Moreover we shall assume that :

- $p_i > 0$ for $i = 1, ..N$, where N is the number of variable goods
- $k \succ 0$
- If commodity i is an output then $y_i \succ 0$; else if $y_i \prec 0$ then the commodity is an input.

Properties of Profit functions

- $V(p, k)$ defined above is linearly homogenous of degree *one* for a fixed k
- Convex and continuous in the price p
- Convexity implies that $\nabla_p y(p, k) = \nabla^2_{pp} V(p, k)$ is a positive semi-definite matrix
- Positive homogeneous of degree *one* in k & concave and continuous in k for every given p

Furthermore from the well-known Hotelling's Lemma result the derivative of the profit function $V(p, k)$ with respect to the price of a commodity i yields us the supply or demand function for the ith commodity, i.e. we have: $y_i(p, k) = \nabla_{p_i} V(p, k)$

(Refer to Diewert, 1986, for proofs).

12 Cost Functions

Similar to the model of profit function outlined above we have the cost function that are defined analogously to the various profit functions defined earlier i.e. our cost function solves the minimization problem as:

$C(y, p) \equiv min_x\{p^T x : f(x) \succeq y\}$ where our cost function characterized by:

- $C(y, p)$ is linearly homogenous in input prices p
- $C(y, p)$ is concave in its price , $p \Longrightarrow \nabla_p y(y, p) = \nabla^2_{pp} C(y, p)$, which is a negative semi-definite matrix
- $C(y, p)$ is non –decreasing in p and increasing in the output level y for fixed prices

Turning to the consumer context we define a cost function for a typical consumer as :

Cost function , $C(u, p) \equiv min_x\{p^T x : f(x) \succeq u\}$ where u refers to a scalar level of utility.

Analogous to the Hotelling's Lemma we have the Shephard's Lemma result in the consumer context, i.e. we can obtain the consumer's system of commodity demand by differentiating the $C(u, p)$ w.r.t commodity price. Thus we state:

$$x(u, p) = \nabla_p C(u, p)$$

To 'observe' utility we set cost function equal to the period t expenditure Y^t i.e. set $C(u,p) = Y$ to get $u = g(y,p)$, where $u = g(y,p)$ is our indirect utility function so we can write our Shephard's Lemma result as:

$$x(u,p) = \nabla_p C(g(y,p),p)$$

Moreover with homothetic preferences we have $C(u,p) = c(p)u$; where $c(p)$ is the unit cost function. Thus we have $x(u,p) = u\nabla_p c(p)$.