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A LOOK AT THE BONFERRONI INEQUALITY MEASURE IN A RELIABILITY FRAMEWORK

G.M. Giorgi, M. Crescenzi

1. INTRODUCTION

Bonferroni (1930) proposed a measure (*B*) which has suitable properties (Giorgi, 1998) for studying wealth and income inequality and, compared with some analogous indices, it is also more sensitive to low levels of income distribution (Giorgi and Mondani, 1995). This peculiarity makes *B* very useful in constructing a poverty measure (Giorgi and Crescenzi, 2001). For further information about *B*, see also Tarsitano (1990), Giorgi and Mondani (1994) and for some biographical aspects on Carlo Emilio Bonferroni see Benedetti (1982).

In this paper we look at the Bonferroni curve (*BC*) and index (*B*) from a different point of view, and we consider the possibility of applying *BC* and *B* not to evaluate inequality, but to analyse lifetesting and reliability. Other authors, e.g. Chandra and Singpurwalla (1981), Klefsjö (1984) and more recently Pham and Turkkan (1994), studied the Lorenz (1905) curve from a similar point of view.

It is well known that the probabilistic theory of reliability gives solutions in optimizing particular objective functions such as mean life, survival probability and has been applied for solving many scientific problems. From a practical point of view, for instance, let us consider the case of a firm which wants to evaluate the reliability of its own equipment in terms of component lifetime, to make replacements in adequate time. The analysis of lifetesting provides techniques which can be also used in longitudinal demographic surveys or in medical follow-up studies where one could be interested in the lifetime of a group of patients, in the same clinical conditions, undergoing the same surgical operation or the same pharmacological treatment.

The Bonferroni curve and index, some basic notions in reliability, and the Scaled Total Time on Test Transform curve (T_4) are briefly reviewed in sections 2.1, 2.2 and 2.3, respectively.

After having investigated the existing relationship between T_4 and *BC*, we show, in section 3, how the Bonferroni curve may be applied in reliability theory. In this framework we propose a criterion, based on the *BC*'s graph, to assign a survival distribution $F(x)$ to the class of *NBUE* (see definition 2.4). Moreover we show that the Bonferroni ordering allows to construct an *OHR*-ordering in the same way reported by Pham and Turkkan (1994) for the Lorenz curve (*LC*). To clarify this argument, an application to the Pareto type I model has been developed in section 5.

Then we focus (section 4) on the strong uniform consistency of the empirical Bonferroni curve not only when the sample is complete, but also in the case, usual in reliability (see Kaplan and Meier, 1958), in which observations are randomly censored. This will allow, if the sample is large, to extend the empirical results to the corresponding theoretical population.

2. MAIN DEFINITIONS AND PROPERTIES

2.1. *Bonferroni curve, ordering and the index B*

Let us suppose that the nonnegative and absolutely continuous random variable $X \in [0, +\infty)$ is the lifetime of an item. Its cumulative distribution function $F(x) = \int_0^x f(t)\,dt$ is continuous and differentiable at least twice and $\mu = \mathrm{E}(X) = \int_0^{+\infty} x f(x)\,dx \neq 0$ is finite.

The first incomplete moment and the partial mean of the probability distribution are:

$$ {}_1F(x) = \frac{1}{\mu} \int_0^x t\, f(t)\,dt, \tag{1} $$

$$ \mu_x = \frac{\int_0^x t\, f(t)\,dt}{\int_0^x f(t)\,dt} = \mu\left(\frac{{}_1F(x)}{F(x)}\right), \tag{2} $$

and furthermore allow:

$$ B[F(x)] = \frac{\mu_x}{\mu} = \frac{{}_1F(x)}{F(x)}. \tag{3} $$

The *Bonferroni Curve* is defined in the orthogonal plane $[F(x), B[F(x)]]$ within a unit square (see Giorgi, 1998) and denoting $p = F(x)$, the parametric expression of *BC* is

$$ B(p) = \frac{{}_1F\left[F^{-1}(p)\right]}{p} = \frac{1}{p\,\mu} \int_0^p F^{-1}(t)\,dt, \qquad p \in (0,1], \tag{4} $$

where $F^{-1}(t) = Q(t) = \inf\{x : F(x) \geq t\}$.

When $p \to 0$, $B(p)$ takes the form 0/0. Consequently, we cannot say in general that *BC* starts from the origin of the orthogonal plane, as it depends entirely on the definition of X.

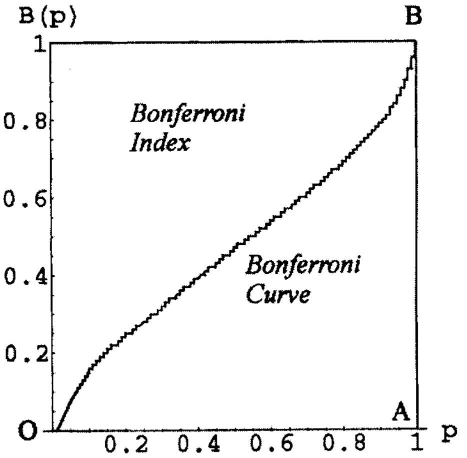

Figure 1 - An example of the Bonferroni diagram when $X \in [0, +\infty)$.
Source: Authors' tabulations of Bank of Italy data for 1995.

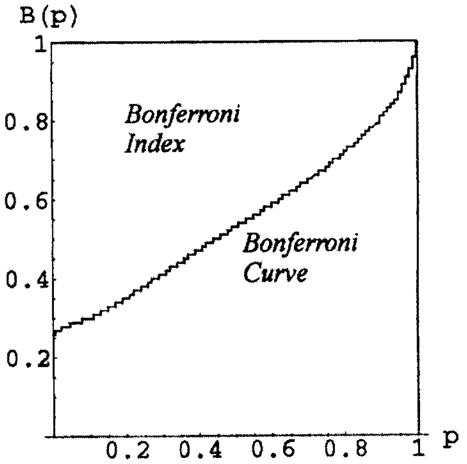

Figure 2 - An example of the Bonferroni diagram when $X \in (0, +\infty)$.
Source: Authors' tabulations of Bank of Italy data for 1995. (Personal incomes greater than 7 millions of Italian Lire)

Since $\frac{\partial B(p)}{\partial p} > 0$, the graph of $B(p)$ is strictly increasing but nothing can be said about its second derivative sign. The Bonferroni curve could be convex in some parts and concave in others. When $P(X = \mu) = 1$ for a fixed value of μ, the BC tends to the so-called equidistribution line which connects the point (0,1) to (1,1). In the case corresponding, in economics, to the maximum concentration, the Bonferroni curve will approach the sides $\overline{OA}$ and $\overline{AB}$ and of the unit square in figure 1.

From a geometrical point of view, the Bonferroni index B is the area delimited by the ordinate axis, the equidistribution line and BC, therefore

$$B = 1 - \int_0^1 B(p)\,dp\,, \qquad B \in [0,1]. \tag{5}$$

Definition 2.1. Bonferroni ordering

$$X \succcurlyeq_B Y \quad \text{if} \quad B_X(p) \geq B_Y(p) \qquad \forall p \in [0,1]\,.$$

This ordering is equivalent to the Lorenz one (see Dagum, 1985, p. 158), and it is partial since not all the pairs of r.v.'s can satisfy the condition $X \succcurlyeq_B Y$. In fact there will be r.v. whose Bonferroni curves are intersecting in one or more points.

2.2. *Hazard rate and classes of distributions*

The *reliability function* (or *survival function*) of the r.v. X defined above is given by $S(x) = 1 - F(x)$ and its *hazard rate* (Patel, 1983) is:

$$hr(x) = \frac{f(x)}{1 - F(x)} = -\frac{S'(x)}{S(x)} = -\frac{\partial[\ln S(x)]}{\partial x}\,. \tag{6}$$

It may be noted that the hazard rate function corresponds to the quantity called, in demography, "force of mortality". A probabilistic interpretation of $hr(x)$ is that $hr(x)dx$ represents the conditional probability that a unit of age x will fail in the interval $(x, x + dx)$.

An *Increasing Hazard Rate (IHR)* indicates that, for a unit of age x, it is more probable to fail in a given increment of time than at an earlier age. This model explains the circumstance of units subjected to time deterioration. Instead a *Decreasing Hazard Rate (DHR)* means that the unit is improving with age. The really observable situations are usually mixtures of these two cases.

A typical trend of the hazard rate is U-shaped and involves a lot of early failures, due to errors in the production phase, then an interval of time in which failures tend to disappear (with a purely accidental nature) until the final deterioration raises it again. Now the reference to the force of mortality tendency that concerns human populations, where infant mortality explains the initial stage, is clear. From (6) we can easily deduce that exponential is the only distribution for which $hr(x)$ is constant in x.

The c.d.f. $F(x)$ is in one to one correspondence to the hazard rate (Patel, 1983, p. 590), since

$$F(x) = 1 - \exp\left\{-\int_0^x hr(u)\,du\right\}.$$

If the p.d.f. $f(x)$ is truncated, so that $l < x < r$, the hazard rate (6) is

$$hr(x \mid l < x < r) = \frac{f(x)}{S(x) - S(r)} = hr(x)\frac{S(x)}{S(x) - S(r)}\,.$$

When $r \to \infty$, $hr(x \mid l < x < r) = hr(x)$ and this means the left-hand truncation does not change the hazard rate, whereas the right-hand truncation increases it.

Now let us introduce three important definitions (see Patel, 1983, pp. 591-592) and some relationships which will subsequently be useful.

Definition 2.2. A c.d.f. $F(x)$ is *IHR (DHR)* if $hr(x)$ its is a non decreasing (non increasing) function in x.

With less restrictive conditions, we can introduce more general classes of distributions as the *New Better (Worse) than Used* [*NBU (NWU)*] and the *New Better (Worse) than Used in Expectation* [*NBUE (NWUE)*].

Definition 2.3. A distribution $F(x)$ is *NBU (NWU)* if $S(x+y) \le (\ge)\ S(x)\ S(y)$ for $x \ge 0$, $y \ge 0$. An intuitive interpretation is that the survival probability at time $x+y$, for a unit of age x, would be $\le (\ge)$ than the survival probability for a new unit of age y. As a direct consequence, if $F(x)$ is *NBU (NWU)* it is better to replace (not replace) the unit of age x with a new one.

Definition 2.4. A $F(x)$ is *NBUE (NWUE)* if the mean μ is finite and $\int_0^\infty S(y)\,dy \le (\ge)\, \mu S(x)$ for $x \ge 0$. Hence, $F(x)$ *NBUE (NWUE)* implies that a used unit of age x has a mean remaining life $\varepsilon(x) = \frac{1}{S(x)} \int_0^\infty S(y)\,dy \le (\ge)$ than a new unit.

The classes of life distributions discussed above satisfy the following relations:

$$IHR \subset NBU \subset NBUE \qquad \text{from which} \qquad IHR \Rightarrow NBU \Rightarrow NBUE\,;$$

$$DHR \subset NWU \subset NWUE \qquad \text{from which} \qquad DHR \Rightarrow NWU \Rightarrow NWUE\,.$$

2.3. *The scaled total time on test transform curve*

If the assumptions in section 2.1 for X hold, the *Scaled Total Time on Test Transform Curve* (T_4) of $F(x)$ is (see Bergman and Klefsjö, 1986):

$$T_4(p) = \frac{1}{\mu} \int_0^{F^{-1}(p)} S(t)\,dt, \qquad p \in [0,1]. \tag{7}$$

The mathematical relation that links the $T_4(p)$ to the Bonferroni curve (4) is:

$$T_4(p) = pB(p) + [(1-p)/\mu]\, F^{-1}(p) \qquad \text{for} \qquad p \in (0,1]\,.$$

In figure 3 the T_4 curve for 4 different populations is represented. We can immediately see the similarity of its graph with *BC*'s one.

The T_4 curve, like the Bonferroni's one, is defined within the unit square and always has a positive first derivative and a second derivative with a variable sign. These common characteristics legitimate the comparison between the T_4 curve and *BC* rather than with the Lorenz curve (*LC*).

The *Lorenz Curve* in the orthogonal plane $[F(x), L[F(x)]]$ is

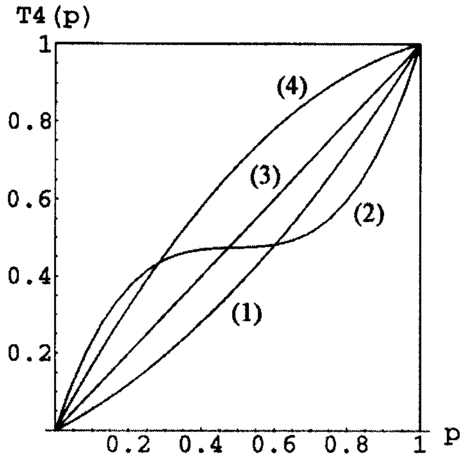

Figure 3 - Examples of T_4 curves: (1) T_4 convex; (2) T_4 with a variable second derivative sign; (3) $T_4(p) = p$; (4) T_4 concave.

$$L[F(x)] = {}_1F(x) \tag{8}$$

a strictly increasing and convex function defined within the triangle $\overset{\Delta}{OAB}$ in figure 1. A main result of this work (see section 4) is that the *BC* graph could give important information for assigning the c.d.f. $F(x)$ to one of the classes defined in section 2.2.

In figure 3, the curve (3) coincides with the diagonal of the unit square; this happens if and only if, as already said, the model is exponential.

Now consider the following results (see Bergman and Klefsjö, 1986, p. 298) which will subsequently be of use.

Proposition 1. The c.d.f. $F(x)$ is *IHR (DHR)* if and only if the T_4 curve is concave (convex).

Proposition 2. The c.d.f. $F(x)$ is *NBUE* if and only if the T_4 curve is completely above the principal diagonal of the unit square.

3. THE BONFERRONI CURVE AND INDEX IN RELIABILITY

To understand the mathematical relationship between the curves discussed in the section above, we introduce the following proposition.

Proposition 3. The Bonferroni curve always lies above the Lorenz curve and below the T_4.

Proof. Obviously,

$$L(p) = pB(p) \leq B(p)\,, \qquad p \in (0,1]\,.$$

Furthermore, under the assumption $F(0) = 0$ and $F^{-1}(0) = 0$,we have

$$T_4(p) = \frac{1}{\mu}\int_0^{F^{-1}(p)} [1 - F(t)]\,dt = \frac{1}{\mu}\int_0^p (1 - t)\,dF^{-1}(t).$$

So, it is easy to show that

$$T_4(p) = \frac{1}{\mu}\left\{(1 - p)\,F^{-1}(p) + \int_0^p F^{-1}(t)\,dt\right\}$$

$$\geq \frac{1}{\mu}\left\{(1 - p)\frac{1}{p}\int_0^p F^{-1}(t)\,dt + \int_0^p F^{-1}(t)\,dt\right\} = \frac{1}{\mu p}\int_0^p F^{-1}(t)\,dt = B(p), \qquad p \in (0,1].$$

Therefore we conclude that the Bonferroni curve always lies below the T_4.

In the light of this result, the Bonferroni index B, traditionally used to study income inequality, can be related to the overall degree of *IHR* or *DHR* for the probability distribution of the r.v. $X \in [0, +\infty)$.

Looking at figure 3, the area above the T_4 curve, the ordinate axis and the line connecting points (0,1) and (1,1), is given by the Gini concentration index R (Klefsjö, 1984, section 3). As a consequence, because of proposition 3, the area lying between the T_4 and Bonferroni curves is $B - R$ (≥ 0).

The limit cases of maximum *IHR* or *DHR* correspond to the cases where the Bonferroni index B takes the extreme values (0 and 1) of its range respectively, and the same is true for R. So small values of these inequality measures (B and R) will be usually associated to distributions with an increasing hazard rate (6).

But the main result is that we find a sufficient condition in the Bonferroni curve to say the c.d.f. F is *NBUE*. In fact, when the Bonferroni curve lies completly above the diagonal of the unit square, the T_4 is in the same position (proposition 3) and it implies (proposition 2) that $F(x)$ is *NBUE*. The graphical determination of a *NBUE* distribution is also possible evaluating *LC* and R (see Pham and Turkkan, 1994, par. 3.3), but our criterion (based on *BC*) achieves a more easily classification.

This is the only case where the Bonferroni curve assures that $F(x)$ belongs to one of the distribution classes defined in section 2.2. When *BC* crosses the principal diagonal or is at the lower side of the unit square, nothing may be said about the position of T_4, and therefore about $F(x)$. But if we know the Bonferroni ordering (definition 2.1) of distributions, we can establish an ordering on the *IHR* degree of c.d.f. following the via proposed by Pham and Turkkan (1994) for *LC*.

The condition that *BC* for c.d.f. F_2 lies above the *BC* for c.d.f. F_1, identifies a geometric relationship which is equivalent to the Lorenz partial ordering. Hence

$$B_2(p) \geq B_1(p)\,, \qquad \forall p \in [0,1]$$

implies (Pham and Turkkan, 1994, p. 81)

$T_{4,2}(p) \geq T_{4,1}(p), \quad \forall p \in [0,1]$ from which $F_2 \underset{OHR}{\geq} F_1$.

The $\underset{OHR}{\geq}$ ordering means that F_2 is more *IHR* or less *DHR* than F_1.

In section 5, we report an example of the *OHR*-ordering for a Pareto/I(m, θ) family with a shape parameter θ.

4. ESTIMATION OF TOTAL TIME ON TEST TRANSFORM AND BONFERRONI CURVES UNDER RANDOM CENSORSHIP

By following a nonparametric large sample approach, Csörgo *et al.* (1998, section 2) and Zitikis (1998) proved that the T_4 and Bonferroni sample curves are strong uniformly consistent estimators when data are complete. This result is useful, when $n \to +\infty$, for extending all the empirical conclusions about polygonals to the corresponding population curves. The problem we now face is that, in lifetesting, we cannot often observe the lifetime for all the sample items. In medical follow-up studies, for example, we need to know the lifetimes of patients who underwent the same operation, but it may happen that contacts with some individuals are lost before their death or that someone dies for accidental causes and therefore these observations cannot be considered. Some other cases are excluded because the study needs to be terminated before a given time. In all these circumstances we are faced with incomplete data and, in the problem formalization, we consider the units not observed as having been "censored" by chance.

Some important results can be proved also in the case of a random censorship. Csörgo *et al.* (1987) have investigated the estimation of T_4 and Lorenz curves; instead here we examine the Bonferroni curve.

The nonnegative and absolutely continuous r.v. $X \in [0, T_F)$ is the lifetime of an item and its cumulative distribution function is $F(x)$. $T_F = \inf\{t : F(t) = 1\}$ denotes the maximum available time for observation and the mean life, finite and nonzero, is

$$\mu = \mathrm{E}(X) = \int_0^{+\infty} tf(t)\,dt.$$

To obtain a mathematical representation of the random censorship, we introduce a sequence of n i.i.d. r.v. $V_1, \ldots, V_n$ with $G(t) = P\{V \leq t\}$, $t \geq 0$. These random variables censor the n sample observations $X_1, \ldots, X_n$ on the right. What we really can observe, in practice, is:

$$(W_1, \delta_1), \ldots, (W_n, \delta_n) \tag{9}$$

where, for any $k = 1, \ldots, n$, $W_k = \min\{X_k, V_k\}$ and

$$\delta_k = \begin{cases} 1 & \text{if } X_k \leq V_k \\ 0 & \text{if } X_k > V_k \end{cases}.$$

Let us assume that the sequences of r.v. $\{X_k\}$ and $\{V_k\}$ are mutually independent.

If $H(t) = P\{W_i \le t\}$, $t \ge 0$, is the c.d.f. of the observed minima $W_1, \dots, W_n$, then the indipendence of $\{X_k\}$ and $\{V_k\}$ is equivalent to saying that:

$$1 - H(t) = [1 - F(t)]\,[1 - G(t)]\,, \qquad t \ge 0\,. \tag{10}$$

Now, if $T_G = \inf\{t : G(t) = 1\}$ and $T_H = \inf\{t : H(t) = 1\}$, we have:

$$T_H = \min\{T_F, T_G\}\,.$$

In this model, the estimator $\hat{F}_n(\cdot)$ of $F(\cdot)$ satisfies (see Csörgo *et al.*, 1987, p. 80):

$$\hat{S}_n(t) = 1 - \hat{F}_n(t) = \begin{cases} \displaystyle\prod_{(\ell : W_{(\ell)} \le t)} \left(\frac{n-\ell}{n-\ell+1}\right)^{\delta_{(\ell)}} & \text{if } t < W_{(n)} \\ 0 & \text{if } t \ge W_{(n)} \end{cases} \tag{11}$$

where $W_{(\ell)}$ is the statistic of rank ℓ in the non decreasing ordering of $W_1, \dots, W_n$.

If we denote with $Z_{(1)} \le \dots \le Z_{(\nu_n)}$ the non censored $W_{(\ell)}$'s, which correspond to $\delta_{(\ell)} = 1$ ($\ell = 1, \dots, n$), the natural estimator of the Bonferroni curve (3) in the model under random censorship is $\hat{B}_n(\cdot)$ which fulfils the equation:

$$\hat{\mu} \cdot \hat{B}_n(u) = \begin{cases} Z_{(1)} & \text{if } 0 < u \le \hat{F}_n(Z_{(1)}) \\ \dfrac{\sum_{k=1}^{i} Z_{(k)}[\hat{F}_n(Z_{(k)}) - \hat{F}_n(Z_{(k-1)})]}{\hat{F}_n(Z_{(i)})} & \\ & \text{if } \hat{F}_n(Z_{(i-1)}) < u \le \hat{F}_n(Z_{(i)}),\ i = 2, \dots, \nu_n \\ \dfrac{\sum_{k=1}^{\nu_n} Z_{(k)}[\hat{F}_n(Z_{(k)}) - \hat{F}_n(Z_{(k-1)})] + Z_{(n)}[1 - \hat{F}_n(Z_{(\nu_n)})]}{\hat{F}_n(Z_{(i)})} & \\ & \text{if } \hat{F}_n(Z_{(\nu_n)}) < u \le 1 \end{cases} \tag{12}$$

Note that the expression $\hat{B}_n(\cdot)$ is the Bonferroni empirical curve for the distribution of the r.v. $W_1, \dots, W_n$. Under random censorship, computing $\hat{B}_n(\cdot)$ presents a serious problem: we are not able to estimate the mean life μ. The intuitive explanation of these difficulties is that we do not know what criterion the censorship will adopt to remove the units from our observation.

In spite of this, we can propose (12) as the estimator of the unscaled Bonferroni curve. Remembering that I) the asymptotic behaviour of the Bonferroni process $\sqrt{n}\{\hat{B}_n(u) - B(u)\}$, $0 < u \le 1$, is equivalent to the Lorenz one (Zitikis, 1998, pp. 673-674); II) in the model discussed above, the sample unscaled Lorenz curve is a strong uniformly consistent estimator for the population curve (Csörgo *et al.*, 1987, pp. 92-94). If the quantile function $Q(\cdot)$ is continuous on $[0, p_0]$ we may conclude that:

$$\sup_{0\le u\le p_0} |\hat{\mu}\hat{B}_n(u) - \mu B(u)| \xrightarrow{a.s.} 0, \qquad p_0 \le F(T_G), \tag{13}$$

which is the strong uniform consistency of the estimator presented in (12).

The Total Time on Test Transform curve, computed for the r.v. $W_1, \ldots, W_n$ is also a strong uniformly consistent estimator (Csörgo *et al.*, 1987, p. 93) for the unscaled version of (7). Therefore the following idea is to use the estimator (14) to classify, under random censorship, the c.d.f. $F(\cdot)$ as seen in section 3.

Of course, the unscaled Bonferroni curve will not be defined within the unit square but in a rectangle with a base equal to one and unknown height. It means that we can only preserve the result about the *OHR*-ordering, opportunely comparing the unscaled *BC*s.

5. PARETO/I DISTRIBUTION

We present now an application of some theoretical results developed in section 3 (see Chandra and Singpurwalla, 1981, p. 120, for *LC*). In particular we show in which way the Bonferroni curve can be used to establish an *OHR*-ordering.

Let $X \in [m, +\infty)$ an absolutely continuous r.v. with a Pareto/I(m, θ) distribution, and suppose we know its minimum lifetime $m > 0$. The p.d.f. is

$$f(x) = \frac{\theta m^\theta}{x^{\theta+1}}, \qquad \theta > 0.$$

In this model, the mean μ exists as finite only if $\theta > 1$ and, in this case, $\mu = \dfrac{m\theta}{\theta - 1}$.

The Bonferroni curve (4) is

$$B(p,\theta) = \begin{cases} \dfrac{1-(1-p)^{\frac{\theta-1}{\theta}}}{p} & \text{if } p \in (0,1] \\[2ex] \dfrac{\theta-1}{\theta} & \text{if } p = 0 \end{cases}$$

where, because of the 1st De L'Hôpital theorem, we have

$$\lim_{p\to 0^+} \frac{1}{p}\left(1-(1-p)^{\frac{\theta-1}{\theta}}\right) = \lim_{p\to 0^+} \frac{(\theta-1)}{\theta}(1-p)^{-1/\theta} = \frac{\theta-1}{\theta}.$$

The Pareto/I(m, θ) distributions make up a Bonferroni ordering when the shape parameter θ value is altered, in the sense that $\theta_2 \ge \theta_1 \Rightarrow B_2(p) \ge B_1(p)\ \forall p \in [0,1]$.

So, in the Pareto/I(m, θ) family, we individuate an *OHR*-ordering only by observing the position of the Bonferroni curves. In fact

$$\theta_2 \ge \theta_1 \Rightarrow B_2(p) \ge B_1(p) \quad \forall p \in [0,1] \Rightarrow \text{Pareto/I}(m,\theta_2) \underset{OHR}{\ge} \text{Pareto/I}(m,\theta_1)$$

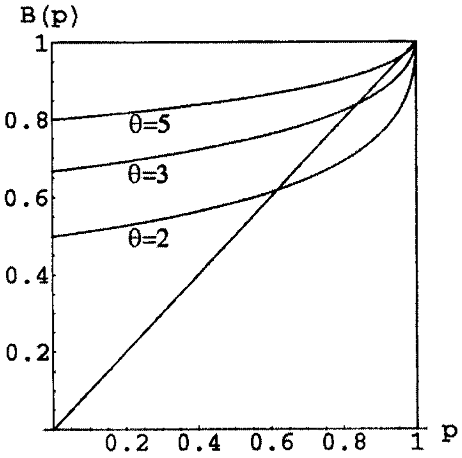

Figure 4 - The *BC* in a Pareto/I(m, θ) model for three different values of θ.

6. SOME FINAL REMARKS

The mathematical similarity between the Bonferroni curve (*BC*) and the Scaled Total Time on Test Transform (T_4) curve gives the idea of connecting economy and reliability; two important fields in statistical applications. In this paper we have proposed a way to use the Bonferroni curve and index for solving some problems related to lifetesting. In this sense we have shown the aspects that make the Bonferroni curve of interest in the reliability applications. We also have presented some asymptotic results which are valid in large sample theory.

Dipartimento di Statistica, Probabilità
e Statistiche applicate
Università di Roma "La Sapienza"

GIOVANNI MARIA GIORGI
MICHELE CRESCENZI

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RIASSUNTO

Uno sguardo alla misura di disuguaglianza di Bonferroni in affidabilità

In questo articolo viene proposta un'interpretazione della curva (BC) e dell'indice (B) di Bonferroni nell'ambito della teoria dell'affidabilità. La stretta relazione tra BC e la curva Scaled Total Time on Test Transform (T_4) fornisce lo spunto per mostrare alcune possibili applicazioni dell'indice di disuguaglianza di Bonferroni in affidabilità. Nel lavoro vengono considerati criteri per classificare le distribuzioni di sopravvivenza ed un ordinamento basato sulla funzione di rischio. Lo studio include anche il caso di censura casuale, frequente nei problemi di analisi di sopravvivenza.

SUMMARY

A look at the Bonferroni inequality measure in a reliability framework

In this paper we give an interpretation of the Bonferroni curve (BC) and index (B) from the reliability theory point of view. The close relationship betweeen BC and the Scaled Total Time on Test Transform curve (T_4) allows us to show some applications of the Bonferroni inequality measure in a reliability set-up. Criteria for classifying a survival distribution and an ordering based on the hazard rate function are considered. The study also concerns the case of a random censorship which is usual in lifetesting.