Extraction of Common Signals from Series with Different Frequency

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Abstract

The extraction of a common signal from a group of time series is generally obtained using variables recorded with the same frequency or transformed to have the same frequency (monthly, quarterly, etc.). The statistical literature has not paid a great deal of attention to this topic. In this paper we extend an approach based on the use of dummy variables to the well known trend plus cycle model, in a multivariate context, using both quarterly and monthly data. This procedure is applied to the Italian economy, using the variables suggested by an Italian Institution (ISAE) to provide a national dating.

Keywords: Business cycle; State-space; Time Series; Trend; Turning Points

1 Introduction

In the statistical analysis the extraction of common signals, as a common trend or a common cycle, from a set of time series, is generally obtained using series with the same frequency (monthly, annual, quarterly and so on). If the available data possess different frequency, for example a first group of monthly series and a second one of quarterly series, one of the two sets is transformed to obtain series with the same frequency, with simple aggregation (transforming the monthly

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series in quarterly series) or disaggregation (transforming the quarterly series in monthly ones). For example, in Italy, the ISAE (Istituto di Studi ed Analisi Economica) extracts the common cycle from 2 quarterly and 4 monthly time series, transforming the quarterly series in monthly with a redistribution algorithm, and then applying the methodology developed by Altissimo et al. (2000) to create a coincident indicator.

The Kalman filter routines contain alternative methods. For example, Azavedo et al. (2003) insert the GNP quarterly series with other monthly indicators in a state-space model, using the STAMP routines (Koopman et al., 2000); in each step of the Kalman filter, the quarterly series are forecasted 2-periods ahead. Anyway, also in this case, they create artificial data.

The possibility to work with both the kinds of data has not received adequate attention in the statistical literature, maybe because the results derived from forecasting techniques are considered a good approximation of the reality. Recently, Mariano and Murasawa (2003) deal with this problem applying a model à la Stock and Watson (1991) to estimate a coincident indicator, in which the "holes" of the quarterly series are not estimated, but are inserted in the Kalman filter without interpolations, using dummy variables.

The primary purpose of this work is applying the idea of Mariano and Murasawa (2003) to extract a common component from a set of time series with different frequency; in particular, we deal with the six series used by ISAE, extending this approach to the trend plus cycle model (see, for example, Harvey, 1985, 1990). This is one of the most used models in literature, because of its flexibility and the possibility to have, as a particular case in the univariate framework, the well known Hodrick-Prescott filter (Hodrick and Prescott, 1997). Extending this model to the multivariate case, we obtain a sort of multivariate Hodrick-Prescott filter, alternative to that proposed by Laxton and Tetlow (1992). The last one considers a local common trend model (without the common cyclical component) relative to a main variable, whereas the other variables are used as regressors; the cycle is the residual series obtained as difference between the main series and the trend. Anyway, considering that the multivariate Hodrick-Prescott filter, in Laxton and Tetlow (1992) version, has a state-space representation (see Boone, 2000), as the model we will use, we can consider them as models belonging to the same family.

The second purpose is to check if the results of this model are similar to those obtained with the corresponding multivariate and univariate models with monthly data; of course, using separate univariate models, we can not obtain a common signal, but it is possible to identify separately the turning points of the single series and synchronize them, for example with the Harding and Pagan (2002) method. A comparison among several parametric and non parametric approaches, in terms of turning points, was made by Bruno and Otranto (2004), who obtain alternative dating of the Italian economy, using the same six variables suggested by ISAE for the period January 1972-September 2002. In this paper we use the same variables and the same time span, but maintaining the distinction between quarterly and monthly series.

In the next section we describe the model proposed, emphasizing the technique based on dummy variables to avoid artificial data; in section 3 we develop the application on the Italian economy, applying three alternative models (a multivariate model with quarterly and monthly data, a multivariate model only with monthly data and six separate univariate models with monthly data, with turning points successively synchronized). Final remarks follow.

With regard to the notation, we will indicate with $\mathbf{I}_h$ the identity matrix of dimension hxn and with $\mathbf{0}_{h,s}$ a matrix of dimension hxs with all the elements equal to zero.

2 The Model Proposed

Let us consider n_1 time series recorded with frequency s_1 and n_2 with frequency s_2 (s_1, s_2 equal to 1 if the series are annual, 4 if the series are quarterly, 12 if they are monthly and so on); we suppose that $s_1 > s_2$. The purpose is to extract a common cycle from this $n_1 + n_2 = n$ series. Let us denote with y_{it} the i-th time series ($i = 1, ...n$) observed at time t ($t = 1, ..., T$). Supposing that the index t is referred to the s_1 frequency, we adopt a simple additive trend plus cycle model for each component (Harvey, 1985):

$$y_{it} = \mu_{it} + \psi_t + \varepsilon_{it}, \tag{1}$$

in which μ_{it} represents the proper stochastic trend of the variable i, ψ_t is the cycle common to all the variables, ε_{it} are Independent Identically Normally (IIN) distributed disturbances with 0 mean and variance σ_i^2; in addition we suppose that the cycle is the only common element among the variables, so that the n trends are considered mutually independent, as well as the n series of disturbances.

The trends and the common cycle are unobserved variables, which follow dynamics expressed by separate equations; each trend follows a linear model as:

$$\begin{aligned} \mu_{it} &= \mu_{it-1} + \beta_{it-1} + \eta_{it} \\ \beta_{it} &= \beta_{it-1} + \varsigma_{it} \end{aligned} \tag{2}$$

where β_{it} is the slope of the trend and η_{it} and ς_{it} are uncorrelated IIN disturbances with zero mean and variances respectively equal to δ_i^2 and ν_i^2. It is equivalent to an IMA(2,1) process. If δ_i^2=$\nu_i^2 = 0$ the trend is deterministic, whereas, if $\nu_i^2 = 0$ and $\delta_i^2 > 0$, the model is equivalent to a random walk with drift. The case with $\nu_i^2 > 0$ and $\delta_i^2 = 0$ represents a trend stationary in the second differences and has the characteristic to be relatively smooth, which is a generally accepted idea of a trend component; the well known Hodrick-Prescott filter corresponds to a model as (1) without ψ_t, with $\delta_i^2 = 0$ and the ratio ν_i^2/σ_i^2 fixed (Harvey and Jaeger, 1993). For example, Hodrick and Prescott (1997) suggest the value 1/1600 for quarterly series; some authors suggest other values (for example, Pedersen, 2001) or to estimate it (for example, Otranto and Iannaccone, 2005). In this case, the cycle is represented by the disturbance ε_{it}. In our application we will estimate all the parameters.

Modeling the cyclical component explicitly, we put (Harvey, 1985):

$$\begin{aligned} \psi_t &= \rho\left[\cos(\lambda)\,\psi_{t-1} + \sin(\lambda)\psi^*_{t-1}\right] + \omega_t \\ \psi^*_t &= \rho\left[-\sin(\lambda)\,\psi_{t-1} + \cos(\lambda)\psi^*_{t-1}\right] + \omega^*_t \end{aligned} \quad (3)$$

where ψ_t^* is an unobservable variable which appears by construction, $0 \leq \lambda \leq \pi$ is the frequency of the cycle, 0$\leq \rho \leq 1$ is a damping factor on the amplitude of the cycle; ω_t and ω_t^* are uncorrelated IIN disturbances with 0 mean and the same variance κ^2 (this assumption is not forced, because, assuming it, generally there is not a real loss in goodness of fit; see Harvey, 1985).

The n equations expressed by (1) can be grouped in the vector $\mathbf{y}_t$, whereas the trends and the slopes respectively in the vectors μ_t and β_t, the disturbances in the vector ε_t. A compact form to express these relationships is the following state-space model:

$$\begin{aligned} \mathbf{y}_t &= \mathbf{A}\xi_t + \varepsilon_t \\ \xi_t &= \mathbf{B}\xi_{t-1} + \mathbf{w}_t \end{aligned} \quad (4)$$

where the unobservable vector state is given by:

$$\xi_t = \left[\begin{array}{cccc} \mu_t' & \beta_t' & \psi_t' & \psi_t^{*\prime} \end{array}\right]'.$$

The fixed matrices $\mathbf{A}$ and $\mathbf{B}$ are expressed by:

$$\mathbf{A} = \begin{bmatrix} \mathbf{I}_n & \mathbf{0}_{n,n} & \mathbf{c} & \mathbf{0}_{n,1} \end{bmatrix}; \qquad \mathbf{B} = \begin{bmatrix} \mathbf{I}_n & \mathbf{I}_n & \mathbf{0}_{n,1} & \mathbf{0}_{n,1} \\ \mathbf{0}_{n,n} & \mathbf{I}_n & \mathbf{0}_{n,1} & \mathbf{0}_{n,1} \\ \mathbf{0}_{1,n} & \mathbf{0}_{1,n} & \rho\cos(\lambda) & \rho\sin(\lambda) \\ \mathbf{0}_{1,n} & \mathbf{0}_{1,n} & -\rho\sin(\lambda) & \rho\cos(\lambda) \end{bmatrix},$$

whereas ε_t and $\mathbf{w}_t$ are nx1 and $2n$x1 vectors, containing respectively the disturbances in (1) and those in (2)-(3); they are mutually uncorrelated and IIN with zero means and covariance matrices $\mathbf{\Sigma}$ and $\mathbf{Q}$, expressed by diagonal matrices with elements respectively given by:

$$\sigma^2 = \begin{bmatrix} \sigma_1^2, & \sigma_2^2, & \cdots & \sigma_n^2 \end{bmatrix}; \qquad \mathbf{q}^2 = \begin{bmatrix} \delta_1^2, & \cdots & \delta_n^2, & \nu_1^2, & \cdots & \nu_n^2, & \kappa^2, & \kappa^2 \end{bmatrix}.$$

We obtain the case of smooth trend when the first n elements of $\mathbf{q}^2$ are equal to zero. The nx1 vector $\mathbf{c}$, contained in $\mathbf{A}$, is composed by n constants, representing multiplicative factors relative to the common cycle ψ_t to model the single equation (1). An alternative approach is that of Harvey and Koopman (1997), named similar cycle model, in which each equation in (1) has a proper cyclical component ψ_{it}, but the n cycles have the same damping factor ρ and frequency λ. Being one of the purpose of this paper the extraction of a common component, we prefer to adopt our specification, allowing $\mathbf{c}$ to differentiate the presence of the common cycle on the single series.

Now, let us suppose that the first n_1 variables contained in $\mathbf{y}_t$ are recorded with frequency s_1 and the remaining n_2 with frequency s_2. Let us suppose also, for the sake of simplicity, that the last n_2 variables are stock variables, so that their values represent the total amount of the variable at that time (which is the case of the successive application of section 3). For the case of flow variables, it is possible to use the hypotheses adopted by Mariano and Murasawa (2003).

Following Mariano and Murasawa (2003), we can treat the n_2 variables with lowest frequency as variables recorded with frequency s_1 with missing values. For example, let $s_1 = 12$ (monthly frequency) and $s_2 = 4$ (quarterly frequency). In addition, let x_t^* one of the n_2 quarterly series; then, it is observed at time t, $t+3$, $t+6$, $t+12$, ..., whereas in the other dates it is missing. To avoid the estimation of missing values, we can suppose that, for all t:

$$x_t = \begin{cases} x_t^* & \text{when } x_t \text{ is observable} \\ z_t & \text{otherwise} \end{cases}$$

where z_t are random variables IIN with distribution not depending by unknown coefficients. Using this hypothesis, the missing values will not affect the maximum likelihood estimators because z_t and $\mathbf{y}_1$, $\mathbf{y}_2$, ...$\mathbf{y}_T$ are independent by construction. In this case the likelihood function L can be rewritten as:

$$L(\rho, \lambda, \kappa^2, \sigma^2, \mathbf{q}^2 | \mathbf{y}_1^*, \mathbf{y}_2^*, ...\mathbf{y}_T^*) = L(\rho, \lambda, \kappa^2, \sigma^2, \mathbf{q}^2 | \mathbf{y}_1, \mathbf{y}_2, ...\mathbf{y}_T) \prod_{i \in M} f(z_i),$$

where M denotes the set of time instants in which the quarterly data are not observed and $\mathbf{y}_t^*$ is the vector containing the n variables, with the last n_2 elements missing if $t \in M$, equal to $\mathbf{y}_t$ otherwise. In other terms, the likelihood function of the unknown parameters given the full data set $\mathbf{y}_1^*$, $\mathbf{y}_2^*$, ...$\mathbf{y}_T^*$, is equivalent to the likelihood of the same parameters given the only data observed $\mathbf{y}_1$, $\mathbf{y}_2$, ...$\mathbf{y}_T$ up to scale. As z_t does not affect the estimation procedure, it can be anything, so that we suppose, as in Mariano and Murasawa (2003), that f is the Normal distribution with mean $\mathbf{0}_{n_2,1}$ and covariance matrix $\mathbf{I}_{n_2}$ and that its realizations in our data set are always equal to zero.

The state-space representation is the same of (4), but the matrix $\mathbf{A}$**,** when x_t is not observable, will change in:

$$\mathbf{A}_1 = \begin{bmatrix} \mathbf{I}_{n_1} & \mathbf{0}_{n_1,n+n_2} & \mathbf{c}_1 & \mathbf{0}_{n_1,1} \\ & \mathbf{0}_{n_2,2(n+1)} & & \end{bmatrix},$$

where $\mathbf{c}_1$ is an n_1x1 vector with the elements equal to the first n_1 elements of $\mathbf{c}$, whereas the covariance matrix of ε_t will change in a diagonal matrix $\mathbf{\Sigma}_1$ with elements given by:

$$\begin{bmatrix} \sigma_1^2, & \cdots & \sigma_{n_1}^2, & 1, & \cdots & 1 \end{bmatrix}.$$

Let:

$$\gamma = \begin{cases} 1 & \text{when all the } n \text{ variables are observed} \\ 0 & \text{otherwise} \end{cases}$$

The final model can be written as:

$$\begin{aligned} \mathbf{y}_t &= [\gamma \mathbf{A} + (1-\gamma)\mathbf{A}_1]\xi_t + \varepsilon_t \\ \xi_t &= \mathbf{B}\xi_{t-1} + \mathbf{w}_t \\ \\ \varepsilon_t &\sim IIN(\mathbf{0}_{n,1}, \gamma\mathbf{\Sigma} + (1-\gamma)\mathbf{\Sigma}_1) \\ \mathbf{w}_t &\sim IIN(\mathbf{0}_{2(n+1),1}, \mathbf{Q}) \end{aligned} \tag{5}$$

Note that the dummy variable γ is not present in the state equation, so that the trends and the common cyclical components are estimated for each time t; at the same time, these estimations, that in a classical state space model constitute the estimation of missing values too, do not enter in the likelihood function.

3 Extracting the Italian Business Cycle

In Italy ISAE has been establishing a business cycle dating, based on the NBER methodology.

In this section, we use the method proposed in this work (hereafter the Quarterly-Monthly Multivariate Model-QMMM) to extract the common cycle and to have a dating, comparing them with the ISAE results. Then, we estimate the same multivariate model, but using monthly variables (reconstructing the quarterly series) and six univariate models. In order to establish the dating of turning points, we adopt the automatic Bry and Boschan (1971) procedure for all the models. In the following sub-sections we describe briefly the data used, the other methods and finally we compare the results.

3.1 The Data Used

The six (seasonally adjusted) variables used are:

1. monthly index of industrial production (total industry excluding construction);
2. monthly quantity of goods (tons) transported on railways;
3. monthly percentage of overtime hours in large industrial firms;
4. monthly imports of investments goods (quantity);
5. quarterly investments in machinery and equipment at constant prices;
6. quarterly value added of service sectors, excluding mainly non-market sectors (education, health services, public administration) at constant prices.

The source of the seasonally adjusted data is the Italian National Statistical Institute (Istat); the data were seasonally adjusted with the TRAMO-SEATS routine (Gómez and Maravall, 1997) with additive components. For this reason we will

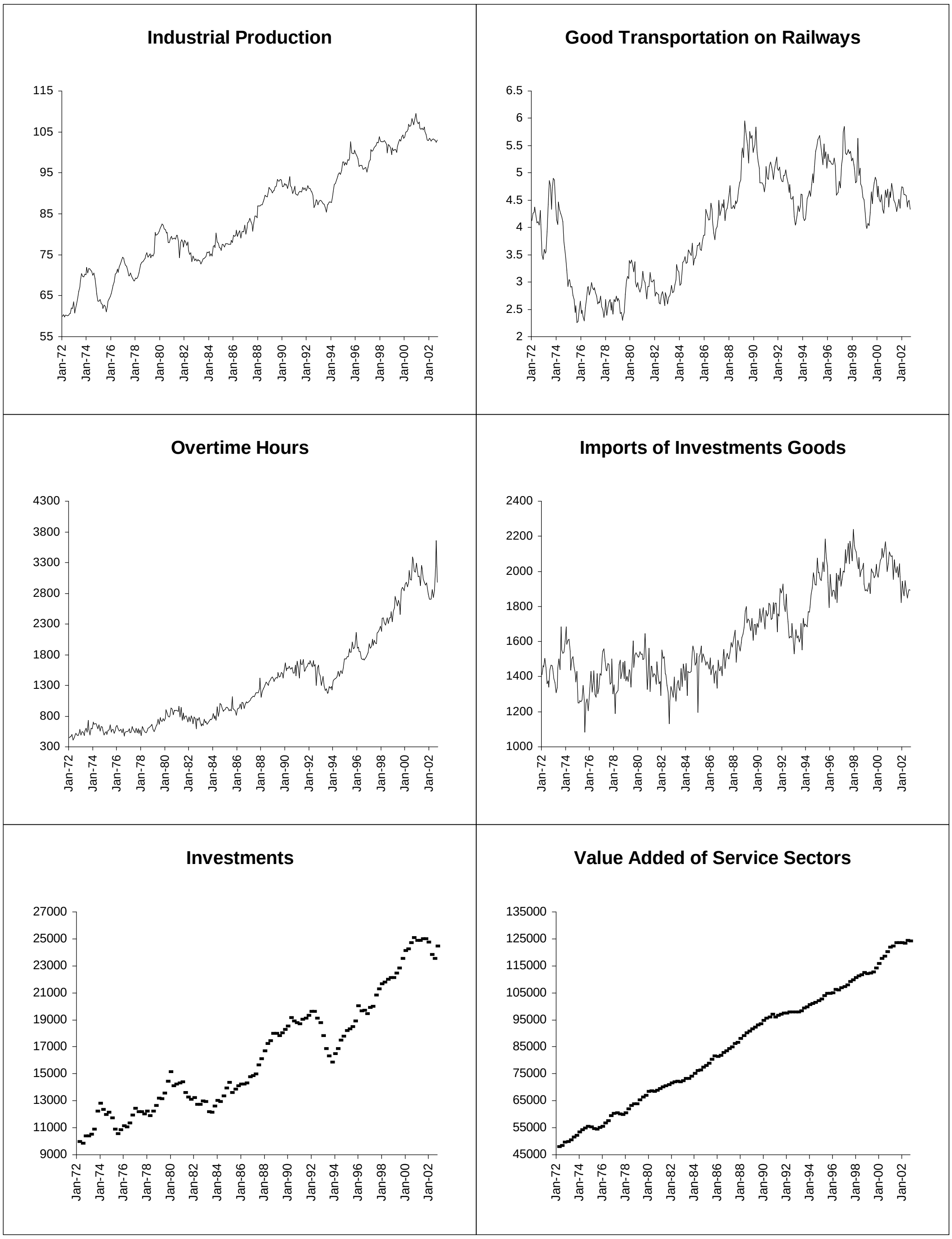

Figure 1: Seasonally adjusted series.

not adopt the logarithmic transformation. They are shown in Figure 1; we can note the similar dynamics, but the different degree of irregularity.

Altissimo et al. (2000) have selected these variables from a set of 183 time series referring to the Italian economy, in successive steps in which they made several restrictions, based on their coincidence behavior and capability to insure the representability of various aspects of economic activity (in fact the series selected represent the supply side, the demand side, the service sector, the labour market).

The last two variables are disaggregated in monthly frequencies with the procedure contained in the software Winrats 32 (Doan, 2000). This procedure assumes that the monthly data are generated by the process:

$$y_t^m = y_{t-1}^m + u_t$$

where $u_t \sim \mathsf{NID}(0, \sigma^2)$. The quarterly data y_t^q are assumed to be observed without error. Moreover, the higher frequency data sums to the lower frequency values across every quarter. The procedure `DISTRIB.SRC` then estimates (with maximum likelihood) the y_t^m's which produce the correct y_t^q's.

Note that they do not insert a typical coincident variable, such as the Gross Domestic Product (GDP), among the variables selected; in this work we accept the choices made by Altissimo et al. (2000) and refer them for other details; anyway they use the GDP to express the judgemental aggregation of the turning points of the single series.

3.2 The ISAE Procedure

The ISAE procedure is based on the NBER methodology. In practice, the turning points of the six series selected are detected with the Bry and Boschan procedure. The dating for the whole economy is obtained aggregating the turns of the single time series, based on a judgemental assessment. The results of this automatic procedure and the judgemental assessments supplied by business cycle experts, provide the Italian business cycle turning points. As noted in Bruno and Otranto (2004), this is not an official dating in a strict sense, but it is considered by the users as a likely picture of the Italian business cycle dynamics. This aspect is of paramount importance because the apparent turning points can be excluded by the dating; at the same time the limit of this procedure is that it needs the subjective judgement of an experienced business cycle analyst. In this way it is very difficult to reply the ISAE results with a purely statistical procedure, but this dating can be assumed as a benchmark to evaluate the other methods proposed in this work. Bruno and Otranto (2004) have used the previous six variables to experiment several parametric and nonparametric procedures to extract the business cycle turning points in an automatic way for the period January 1972- September 2002, using the ISAE dating as benchmark. Their results show that the methods provide similar results with respect to the ISAE chronology in the period 1972-1983, characterized by the two oil shocks, and 1993-2002, whereas they detect various extra-cycles in the middle period, not indicated by ISAE. As said in the Introduction, in this work we will use the same period. A sub-product of this procedure is a coincident indicator of the Italian cycle. Hereafter we will indicate this model with ISAE.

3.3 The Monthly Multivariate Model

To evaluate the performance of the model (1)-(2)-(3), estimated with both monthly and quarterly data, with respect to a classical case in which all the series have the same frequency, we have estimated the analogous model (4), or the model (5) with $\gamma = 1$ for each t, using the monthly disaggregation explained in sub-section 3.1. The main interest in this case is to verify if the artificial data can produce extra-cycles or loose cycles detected by the contemporaneous use of monthly and quarterly series. Hereafter we will indicate this model with MMM.

3.4 Univariate Indirect Approach

Another possibility is to estimate six separate univariate models for the monthly series:

$$\begin{aligned} y_t &= \mu_t + \psi_t + \varepsilon_t \\ \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t \\ \beta_t &= \beta_{t-1} + \varsigma_t \\ \\ \psi_t &= \rho \left[\cos(\lambda)\, \psi_{t-1} + \sin(\lambda) \psi^*_{t-1} \right] + \omega_t \\ \psi^*_t &= \rho \left[-\sin(\lambda)\, \psi_{t-1} + \cos(\lambda) \psi^*_{t-1} \right] + \omega^*_t \end{aligned} \tag{6}$$

In practice, in an univariate framework, this one is equivalent to the model (1)-(2)-(3), providing separate cycles for each series. For each cycle, we extract the turning points following the Bry-Boschan procedure; then we aggregate, in an indirect way, the turning points with the procedure of Harding and Pagan (2002).

In practice, this procedure consists in finding, for every t, a 6x1 vector of distances for the nearest peak (trough) for each time series considered. The median of this vector is interpreted as the mean distance from the nearest peak (trough) for the whole economy and the local minima of this series are candidate to be a peak (trough) for the whole economy. Then, the turning points are selected so that they alternate and the cycles and single phases last not less than 15 and 5 months respectively. This approach is useful because more similar to the ISAE one, being conducted in terms of single univariate analysis, but, at the same time, it uses the same trend model of the multivariate approaches considered in this paper. Hereafter we will indicate this model with UIA.

3.5 Empirical Results

From preliminary analysis, the variances δ_i^2 have resulted near to zero, so we have imposed the first equation in (2) as deterministic (as in the Hodrick-Prescott procedure), but not fixing the ratios ν_i^2/σ_i^2, that we will estimate for each variable. The same holds for MMM and UIA. The final estimates are shown in Table 1.

The first macroscopic difference of the QMMM model with respect to the others, is relative to the estimation of the variances of the quarterly series; anyway, this is not unexpected because the monthly transformation is a deterministic one, which disaggregates the levels of the original series; in practice we introduce an artificial reduction of the variance in the monthly series. In the rest of the estimates, the multivariate models show similar variances in the trend components (excluding the 5th variable). The univariate models provide different variances for the trend component. In Figure 2 the trends of each variable obtained with the three different approaches are shown. Note that the dynamics of the trends deriving from the multivariate approaches are very similar; the only difference can be found in the investments series, in which the MMM approach provides a more irregular trend. The univariate models show the main differences with respect to the multivariate models for the original monthly variables; they have a very smooth behavior. This is due to the fact that, not being the constraint of a common cycle, the univariate models provide smooth trends, assigning large part of the variance to the irregular or cyclical components (see Table 1). The last two variables (the quarterly transformed series) show a different behavior with components similar to that obtained from the multivariate approaches. Vice versa, the multivariate approaches assign to the trend components some movements that are assigned to the cyclical component in the univariate approaches; of course, this is due to the presence of a common component, representing the business cycle.

Table 1: Estimated parameters

	QMMM	MMM	UIA					
			y_1	y_2	y_3	y_4	y_5	y_6
σ_1	0.432	0.336	0.422					
σ_2	0.134	0.133		0.055				
σ_3	74.361	74.368			70.751			
σ_4	59.636	58.682				54.099		
σ_5	271.87	0.301					0.082	
σ_6	292.59	0.142						0.068
c_1	0.149	0.799						
c_2	0.001	0.003						
c_3	1.565	11.054						
c_4	4.237	23.349						
c_5	1.226	1.220						
c_6	1.089	1.354						
ν_1	0.001	0.000	0.000					
ν_2	0.063	0.064		0.001				
ν_3	8.153	8.872			1.734			
ν_4	4.632	2.538				0.341		
ν_5	25.742	36.019					34.715	
ν_6	51.095	49.539						47.992
ρ	0.953	0.963	0.970	0.952	0.971	0.944	0.963	0.963
λ	0.087	0.080	0.082	0.000	0.130	0.117	0.082	0.082
κ	5.579	1.128	0.830	0.178	29.576	30.28	0.330	0.334

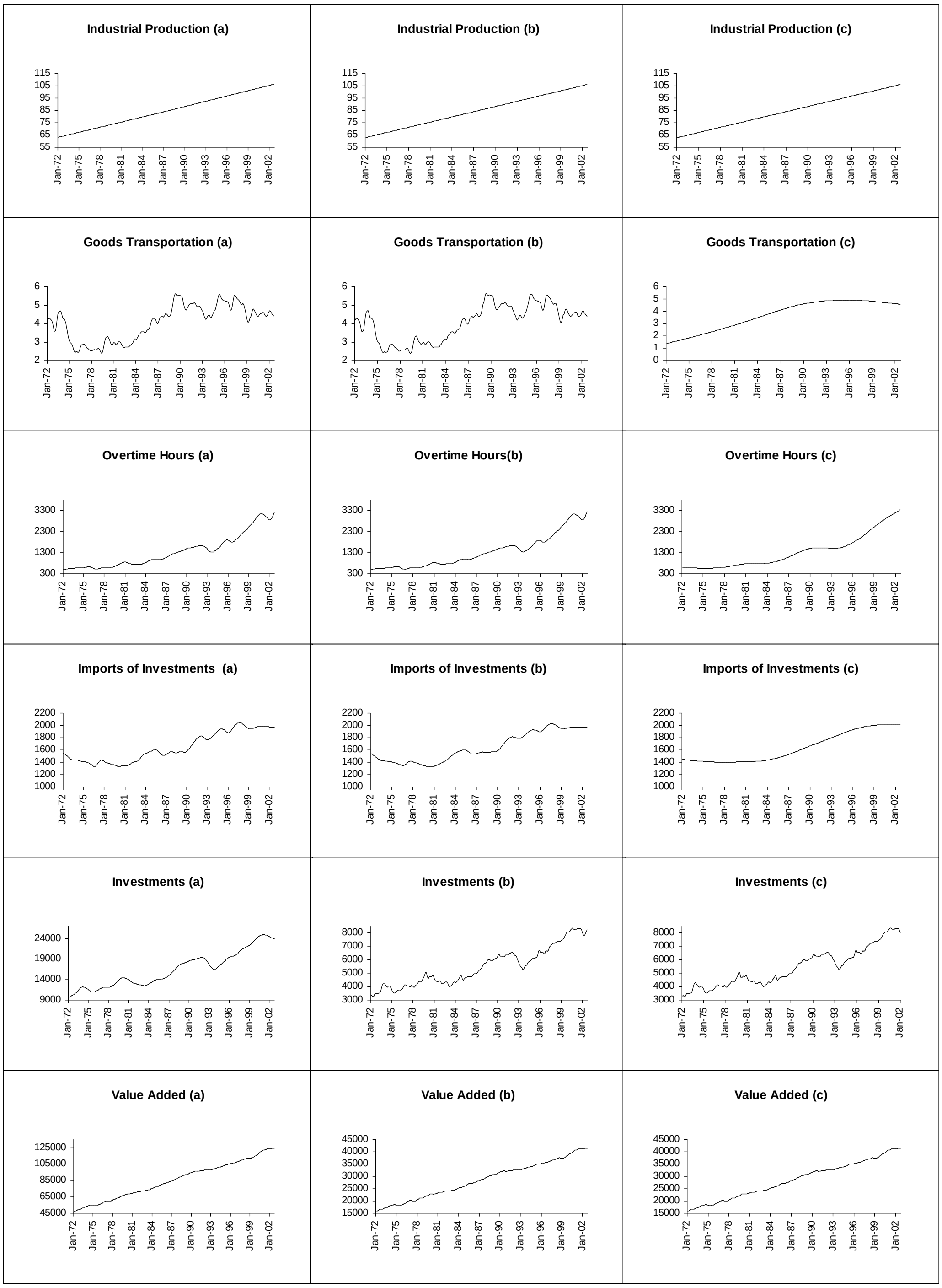

Figure 2: Trends extracted with the QMMM (a), MMM (b), UIA (c) models.

The difference between the trend component of a multivariate approach and the trend of the univariate approach constitutes an autonomous transitory part, proper of the dynamics of the series analyzed.

It is interesting to observe that the trend of industrial production is a straight line, which implies that its fluctuations are totally due to the cyclical component; in addition it is a deterministic trend, as shown by the estimates of the standard deviation ν_1.

With respect to the cycle, the frequency λ is 0.087 in the QMMM approach and 0.080 in MMM, which implies a period $2\pi/\lambda$ for the common cycle corresponding to 6 years for the former, 6.5 years for the last. Note that the only variables with a similar period of the cycle, in the univariate approach, are the industrial production, the investments and the value added of service sectors; the others have different behavior, with the extreme case of goods transportation, with $\lambda = 0$. Furthermore, the anomalous behavior in cyclical terms of this series is confirmed by the null coefficient c_2 in the multivariate models, which in practice eliminates the ψ_t component. This is confirmed when we exclude this variable in the multivariate models; the results are the same of Table 1 with the same inference on trends and cycles. In other terms, using trend plus cycle models, the transportation goods do not provide any relevant information about the common cycle and its use seems unappropriate. In effect, the trend plus cycle model can be inefficient and difficult to estimate. As noted by Harvey and Jaeger (1993), in general it is difficult to pick out the cycle in an unrestricted model, as in this case, especially when the variance of the cycle κ^2 is small; in fact, in this case the likelihood function is very flat. In addition, a λ very small (or null, as in the goods transportation) could indicate that a simple local trend model (which corresponds to model (6) without the cyclical component) is more appropriate. Anyway, "the fact that the cycle model would be rejected on grounds of parsimony does not mean that it does not provide a valid description of the data. Furthermore, if we feel a priori that the underlying trend should be smooth then the cycle model is to be preferred over the more parsimonious local linear trend" (Harvey and Jaeger, 1993, p. 238).

The variances of the cyclical component of the two multivariate approaches are quite different, but this does not imply different dynamics; in Figure 3 we can note that the cyclical components obtained with the two multivariate models have a very similar dynamics. In addition there is a strong feeling of a certain degree of similarity in the phases of growth and recession with respect those deriving from the ISAE composite indicator, as shown in the graph at the top of the Figure. Anyway, a more clear comparison is obtained using the turning points deriving

Table 2: Turning Points

Turning Points	ISAE	QMMM	MMM	UIA
Trough			jun-72	
Peak	mar-74	jan-74	jan-74	jan-74
Trough	may-75	aug-75	aug-75	jun-75
Peak	feb-77	dec-76	dec-76	nov-76
Trough	dec-77	dec-77	dec-77	dec-77
Peak	mar-80	mar-80	mar-80	jan-80
Trough	mar-83	may-83	may-83	mar-83
Peak			aug-84	nov-84
Trough			oct-85	nov-86
Peak		aug-89	aug-89	nov-88
Trough				jul-90
Peak	mar-92			jan-92
Trough	jul-93	aug-93	aug-93	aug-93
Peak	nov-95	aug-95	aug-95	sep-95
Trough	nov-96	dec-96	dec-96	nov-96
Peak		dec-97	dec-97	nov-97
Trough		may-99	may-99	may-99
Peak	dec-00	dec-00	dec-00	sep-00
Similarity with respect to the ISAE dating				
		0.168	0.222	0.211

from each approach, obtained by the Bry and Boschan routine. (We acknowledge the use of the RATS routine developed by G. Bruno for the Bruno and Otranto, 2004 work). This is made in Table 2.

The procedures proposed all capture the two recessions in 1973-74 (first oil shock) and 1977; but in the analyzed period the MMM procedure identifies the beginning of a period of growth from June 1972 to January 1974, whereas the others consider the full first period of the time series (until the beginning of 1974) as a growth one. This is a first difference between the two multivariate methods, probably due to the use of quarterly data; in fact, in the univariate analysis with monthly variables, the estimated cycles of investments and value added of service sector show a deep trough with a successive peak (top of Figure 4); we have also extracted the cycle directly from the original quarterly series and this behavior is not present (bottom of Figure 4). The feeling is that the disaggregated series have

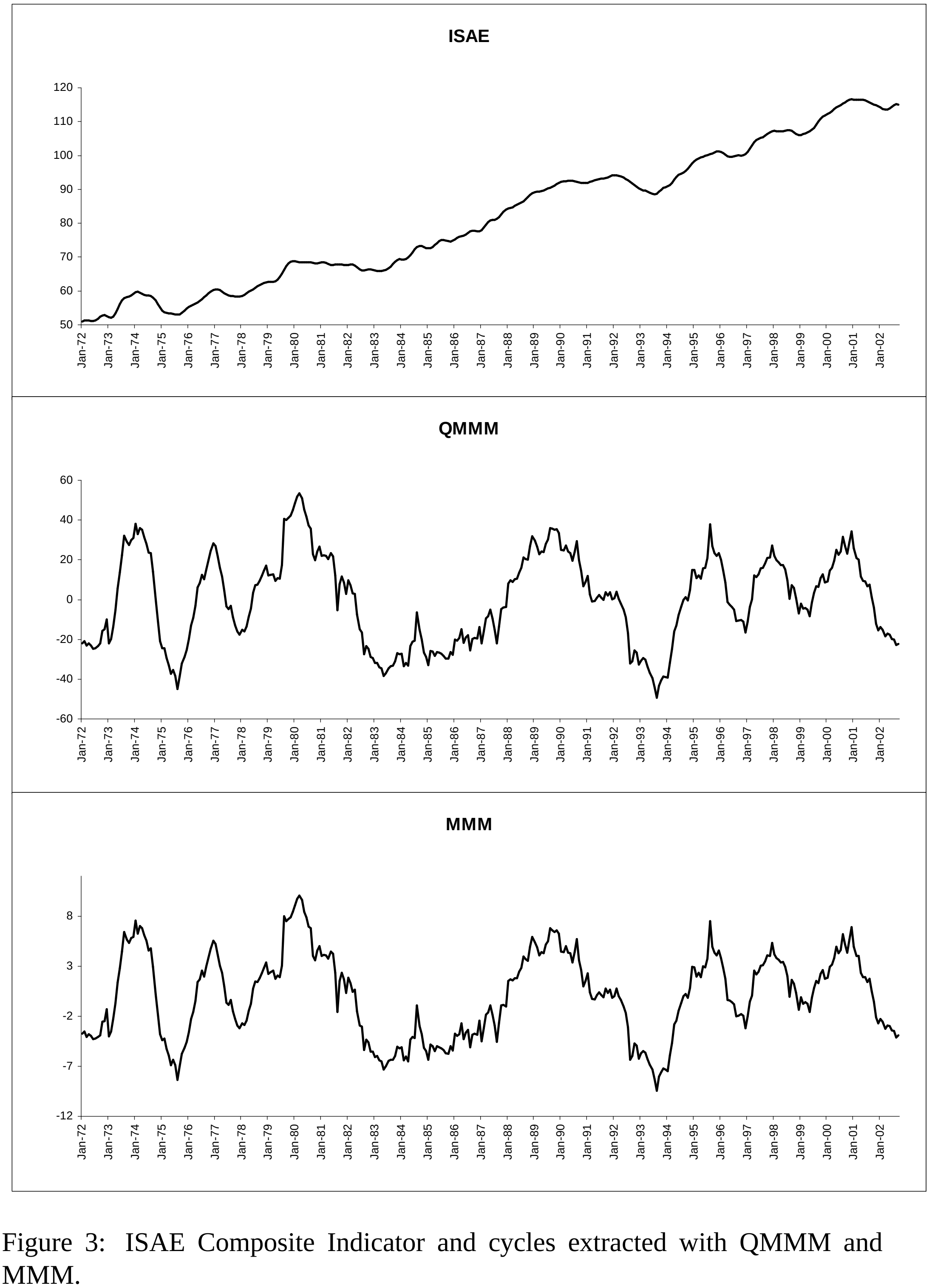

Figure 3: ISAE Composite Indicator and cycles extracted with QMMM and MMM.

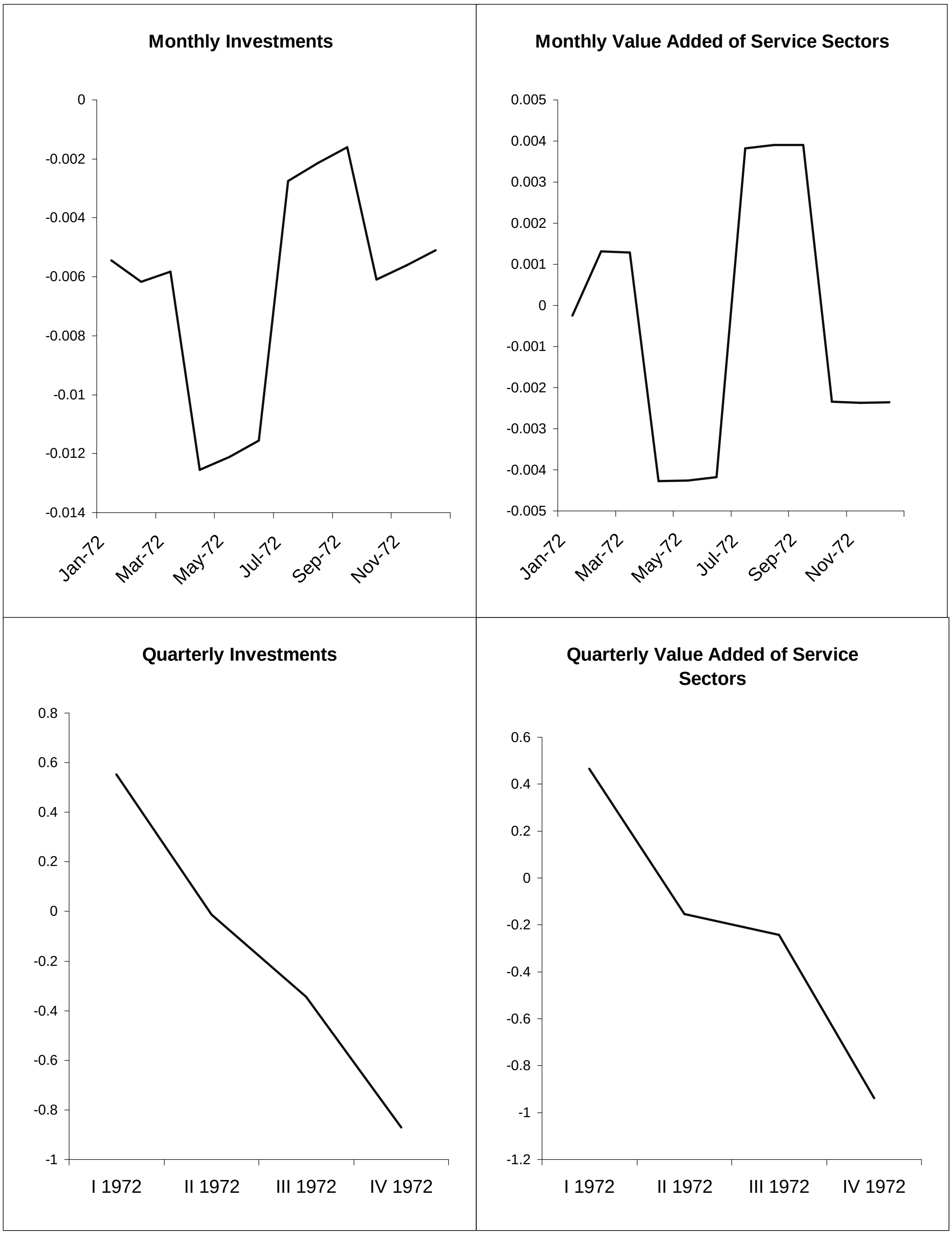

Figure 4: Details of the cyclical components in 1972.

produced the trough in the multivariate analysis with monthly variables. On the other side, the synchronization of turning points derived from the six univariate analysis does not provide this trough, not being present in the other series (except in the good transportation on railways). We do not show all the turning points obtained with the univariate analysis (available on request).

All the methods agree in detecting a peak in the first half of 1980, starting a 3-years long recession. In this period another difference between QMMM and MMM arises; in fact MMM shows an extra-cycle in the period 1983-1985, whereas QMMM establishes a long growth period without interruptions starting from 1983 until August 1989 (whereas ISAE until March 1992). In this case the difference is explained by the censoring rules of the Bry-Boschan routine; in fact, after the first screening, it identifies a peak in August 1984 for QMMM too, whereas the trough is placed in January 1985. This last one is dropped to ensure the constraint of the minimum phase duration of six periods and, as a consequence, the peak of August 1984 is deleted to ensure the alternation of turning points.

This period is a puzzling one due to the difficult to establish a precise dating; in fact UIA follows a proper behavior with more extra-cycles. Bruno and Otranto (2004) registered the same difficulties using various parametric and non parametric methods. During the nineties', the turning points derived from the three approaches are consistent with the ISAE dating, establishing a recession in 1995-96, as well as a peak at the end of 2000; but they find an extra-cycle between the end of 1996 and the middle of 1999.

From the simple list of turning points is not easy to evaluate the best performance in terms of detection of turning points among the three parametric methods. For this reason, we have calculated a loss function measuring the degree of similarity between the dating of a particular parametric method and the ISAE dating. This loss function is obtained as:

$$\frac{1}{T}\sum_{t=1}^{T}\left|P_t^M - P_t^{ISAE}\right| \tag{7}$$

where P_t^M is a dummy variable assuming value 0 if at time t the parametric method M has identified a recession period (t is located between a previous peak and a subsequent trough), 1 if it has identified a growth period (t is located between a previous trough and a subsequent peak).

In the bottom of Table 2 the values assumed by (7) are showed; QMMM has a better performance with respect to MMM and UIA, clearly considering the ISAE dating as the correct one.

4 Final Remarks

In this paper we have extended the idea of Mariano and Murasawa (2003) to extract a common cyclical component from a group of series composed by monthly and quarterly data, without transforming them to obtain homogeneous frequencies. Differently from Mariano and Murasawa (2003), who use the Stock and Watson (1991) procedure, we have extended the trend plus cycle model of Harvey (1985) to the multivariate case; this is one of the most used and flexible models created for this kind of analysis and provides directly a common cyclical component.

The extension is quite natural and does not imply methodological difficulties, whereas some problem arises in terms of estimation; in fact, nevertheless the good fitting of the model to the data, there are some problems in terms of convergence of the estimation algorithms, due to the flatness of the likelihood function. This problem is common to the multivariate and univariate approaches, and was pointed out by Harvey and Jaeger (1993). In this case, from our point of view, a useful alternative could be some Bayesian estimation procedure in the state space framework (see, for example, Carter and Kohn, 1994) to improve the estimation step. Anyway, this problem is relatively important with respect to the possibility to extract a common signal representing the cycle. In fact, in our application to the Italian economy, the final results seem consistent with the cyclical dynamics found by ISAE (Figure 3).

Another purpose was to verify the differences between our approach and the analogous one, obtained using monthly data (with a disaggregation of the quarterly series). This analysis was conducted in terms of cyclical component and detection of turning points. A part the differences in terms of estimation, the cyclical components obtained with the two approaches are very similar and the only difference consists in two extra-cycles, detected by the MMM approach. In this case the QMMM approach is more consistent with the ISAE judgemental evaluation, and this is confirmed by the loss function (7).

The univariate analysis suggests some doubts about the coincident behavior of the six variables selected by Altissimo et al. (2000); in this case, it seems that only the industrial production, the investments and the value added of service sectors have a similar cyclical frequency, consistent with the dynamics deduced by the multivariate models, whereas the goods transportation variable does not seem useful to determine the common cycle in all the parametric approaches here proposed.

Finally, we want stress the utility of the exercise developed in this work, with the purpose to diffuse the idea that the contemporaneous use of data with different frequency in multivariate models can be easily implemented and provide good results, without the creation of artificial data. In addition it could be extended to all the multivariate models which can be represented in a state-space form, being the specification adopted in (4) very general.

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