Tests of seasonal integration and cointegration in multivariate unobserved component models

Fabio Busetti*
Bank of Italy, Research Department

October 22, 2004

Abstract

The paper considers tests of seasonal integration and cointegration for multivariate unobserved component models. First, the locally best invariant (LBI) test of the null hypothesis of a deterministic seasonal pattern against the alternative of seasonal integration is derived for a model with Gaussian i.i.d. disturbances and deterministic trend. Then the null hypothesis of seasonal cointegration is considered and a test for common nonstationary components at the seasonal frequencies is proposed. The tests are subsequently generalized to account for stochastic trends, weakly dependent errors and unattended unit roots. Asymptotic representations and critical values of the tests are provided, while the finite sample performance is evaluated by Monte Carlo simulation experiments. Finally, the tests are applied to the series of industrial production of the four largest countries of the European Monetary Union. It is found that Germany does not appear to cointegrate with the other countries at most seasonal frequencies, while there seems to exist a common nonstationary seasonal component between France, Italy and Spain.

KEYWORDS: Common Components, Cramér-von Mises distribution, Locally best invariant test, Seasonal unit roots.
JEL classification: C12, C32.

*Address for correspondence: Fabio Busetti, Banca d'Italia - Research Department, Via Nazionale 91, 00184 Rome, Italy. Tel: +39 0647923245. Fax: +39 064747820. Email: fabio_busetti@yahoo.com

1. Introduction

Economic time series are often characterized by a slowly changing, as opposed to fixed, seasonal pattern. Models with seasonal unit roots, or unit roots at the seasonal frequencies, can account for this kind of behavior. Statistical tests for the presence of seasonal unit roots in quarterly time series have been proposed by Hylleberg et al. (1990). The tests have been extended to monthly data and seasonal trends in Beaulieu and Miron (1993) and Smith and Taylor (1998), respectively. In a multivariate set-up, Lee (1992), Ahn and Reinsel (1994), Johansen and Schaumburg (1999), Cubadda (2001), Ahn et al. (2004), Cubadda and Omtzigt (2004) have proposed likelihood-based tests for the rank of the seasonal cointegration space, which extend the VAR framework of Johansen (1988, 1991, 1995) to seasonal time series. Empirical applications are given in, *inter alia*, Engle et al. (1993), Kunst (1993), Reimers (1997), Huang and Shen (2002). Franses and McAleer (1998) is a comprehensive survey of this literature.

In all those articles the tests are constructed from the autoregressive representation of linear time series. This paper, on the other hand, considers testing for seasonal integration and cointegration within the unobserved component (UC) model

$$\mathbf{y}_t = \boldsymbol{\mu}_t + \mathbf{s}_t + \boldsymbol{\varepsilon}_t, \tag{1.1}$$

where $\mathbf{y}_t = (y_{1t}, ..., y_{Nt})'$ is a $N \times 1$ vector time series, which is made up of a trend $\boldsymbol{\mu}_t$, a seasonal component $\mathbf{s}_t$ and an irregular term $\boldsymbol{\varepsilon}_t$. Specifically, we test for the presence of common non-stationary components in the seasonal patterns $\mathbf{s}_t$; deterministic seasonality will emerge as a special case. The tests are derived in the multivariate LBI framework of Nyblom and Harvey (2000) and may be viewed as a generalization to multivariate models of the CH test of seasonal stability of Canova and Hansen (1995) and subsequent developments by Caner (1998), Busetti and Harvey (2003), Taylor (2003a,b).

An important difference between our tests and those constructed within the VAR framework is that they reverse the role of the null and the alternative hypotheses, i.e. in our case the model is "more stationary" under the null hypothesis than under the alternative one. This parallels the difference between Nyblom and Harvey (2000) and the rank tests of Johansen (1988, 1991, 1995), and also between the KPSS stationarity test of Kwiatkowski et al. (1992) and the Dickey-Fuller-type unit root tests. A formal comparison of the properties of our tests of seasonal cointegration with those based on vector autoregressions involves a number of problems (reversal of the null and alternative hypotheses, evaluation under different data generating processes, different ways to allow for serial correlation) and goes beyond the scope of this paper; however it is interesting that for the data considered in section 7 similar evidence on the number of common seasonal components is obtained in both frameworks.

The relationships between unobserved component and ARIMA models are examined in Harvey (1993, ch. 5). In general, the choice of setting up a UC model or a VAR largely depends on the type of information one would like to extract from the data. In the context of modelling seasonal time series there are at least three major reasons for which UC models can be attractive. First, UC models have a structural interpretation in the sense that they are specifically constructed to break up the series into components of interests, trend, cycle, seasonal. Smoothed estimation and confidence intervals for the various components can be easily computed by the Kalman filter. In section 7 we will see that imposing seasonal cointegration restrictions enhances efficiency and yields narrower confidence bands for the estimated trend and seasonal components. Second, UC models are in general more parsimonious in terms of estimated parameters than vector autoregressions, in particular for monthly series where the inclusion of lagged variables of at least order 12 could lead to a large loss of degrees of freedom. Third, seasonal fluctuations, when they are stochastic, typically change very slowly[1] and thus the UC framework that takes the null of deterministic seasonality (or that of "more seasonal cointegration" than under the alternative) is probably more appealing than the autoregressive framework where the null hypothesis is that of seasonal unit roots (or that of "less seasonal cointegration"). Furthermore, in the case of small signal-to-noise ratio (that is when the variance of the disturbance driving the seasonal component is small relative to that of the irregular component) the ARIMA representation of a UC model has moving average roots that lie near the unit circle and thus many lags would be needed for an autoregressive approximation.

One possible drawback of the unobserved component models considered in this paper is that they allow for contemporaneous but not for polynomial seasonal cointegration, where the latter implies that linear combinations of contemporaneous and lagged seasonally integrated series are stationary. If it is found that a UC model yields a good representation of the data, then the case for polynomial cointegration becomes less stringent. However it is worth mentioning that Cubadda

[1] Quoting Harvey (1993, p.146), "... *In relatively short time series, consisting of only a few years, it is difficult to detect a change in the seasonal pattern, which suggest that the variance of the seasonal component will be zero or close to zero ...*"

(2001) and Ahn et al. (2004) provide empirical examples where polynomial seasonal cointegration is detected. Finally, testing hypotheses on the cointegration coefficients can be performed in unobserved component models (see Harvey and Koopman, 1997, and section 7), although such tests may appear more easily formulated within the vector autoregression framework *a la* Johansen.

The seasonal cointegration tests developed in this paper are applied to the series of the index of industrial production of the four largest countries of the European Monetary Union. We find evidence that Germany does not cointegrate with the other countries at most seasonal frequencies, while there seems to be a common non-stationary seasonal component between France, Italy and Spain.

In summary, the paper proceeds as follows. Section 2 reviews the definition of seasonal integration and cointegration. Section 3 introduces the LBI test of seasonal stability against seasonal integration. Section 4 derives the test of seasonal cointegration when the trend is a deterministic function of time and the disturbances are Gaussian white noise. Section 5 shows how to modify the tests to allow for the presence of stochastic trends and for serial correlation in the error term. The finite sample properties of the tests are evaluated by Monte Carlo simulation experiments in section 6. Section 7 applies the tests to the series of industrial production of European countries and Section 8 concludes.

2. Seasonal integration and cointegration

Seasonal integration and cointegration are defined following, *inter alia*, Hylleberg et al. (1990), Gregoir (1999) and Cubadda (1999). Let $\Delta(\lambda)$ be the difference operator at frequency $\lambda \in [0, \pi]$, that is

$$\Delta(\lambda) = \begin{cases} 1 - \cos\lambda\, L, & \lambda \in \{0, \pi\}, \\ 1 - 2\cos\lambda\, L + L^2, & \lambda \in (0, \pi), \end{cases}$$

where L is the usual lag operator, $L^k x_t = x_{t-k}$, $k = 0, 1, 2, ...$ The operator $\Delta(\lambda)$ is a simple linear filter with zero gain only at the spectral frequency $\lambda \in [0, \pi]$; in other words it removes unit roots at that frequency.

A real-valued vector time series process $\mathbf{y}_t$ is said to be integrated of order d at frequency $\lambda \in [0, \pi]$, denoted $I(d; \lambda)$, if its d-th λ-difference, $\Delta(\lambda)^d \mathbf{y}_t$, is a linear process with a continuous and positive definite spectrum at λ. The process $\mathbf{y}_t$ is said to be (contemporaneously) cointegrated of order d, b at frequency λ, $CI(d, b; \lambda)$, if (i) each component of $\mathbf{y}_t$ is $I(d; \lambda)$ and (ii) there exists a nonzero vector $\boldsymbol{\alpha}$ such that $\boldsymbol{\alpha}'\mathbf{y}_t$ is $I(d-b)$, where $d \geq b > 0$. [2] In the context of seasonal time series, the interest lies in the seasonal frequencies $\lambda(h) = 2\pi h/s$, $h = 1, ..., [s/2]$, where s is the number of seasons and the notation $[x]$ denotes the biggest integer that is smaller than or equal to x. The period of $\lambda(1)$ is one year. This is denoted as the fundamental frequency, while the other frequencies are called harmonics. Therefore a process is said to be seasonally integrated (cointegrated) if it is $I(d; \lambda(h))$ $(CI(d, b; \lambda(h)))$ at one of the seasonal frequencies $\lambda(h)$, $h = 1, ..., [s/2]$. In this paper we concentrate on the cases $d = b = 1$.

A seasonally integrated linear process can be represented in terms of a nonstationary stochastic seasonal component. Likewise, a seasonally cointegrated process implies the existence of common stochastic seasonal components. In the following sections we consider an unobserved component model where the coefficients of an otherwise deterministic seasonal component are stochastic and evolve as random walks. The objective is to make inference on the rank of the disturbances driving those random walks. The case of rank zero corresponds to deterministic seasonality, full rank to seasonal integration, while seasonal cointegration occurs otherwise.

3. The multivariate LBI test against seasonal integration

Let s be the number of seasons and $\lambda(h) = 2\pi h/s$, $h = 1, ..., [s/2]$, be the seasonal frequencies. Denote by $\mathbf{z}_t(h)$, $h = 1, ..., [s/2]$, the spectral indicator variable associated with each of the $\lambda(h)$, that is $\mathbf{z}_t(h) = (\cos \lambda(h)t, \sin \lambda(h)t)'$ for $h < s/2$ and, when s is even, $\mathbf{z}_t(s/2) = \cos \lambda(s/2)t = \cos \pi t$.

We consider a model where each individual series y_{it} is characterized by a deterministic trend $\mathbf{x}_t'\boldsymbol{\beta}_i$, where $\mathbf{x}_t$ is a $k \times 1$ vector of non-stochastic regressors and $\boldsymbol{\beta}_i$ are fixed coefficients, a seasonal component of the form $\sum_{h=1}^{[s/2]} \mathbf{z}_t(h)'\boldsymbol{\gamma}_{it}(h)$, where $\boldsymbol{\gamma}_{it}(h)$ are random walk coefficients[3], and a white noise disturbance term. Note that we work with the trigonometric representation of the seasonal component but an equivalent formulation in terms of the (perhaps more usual) seasonal dummy variables can be obtained; cf. e.g. Harvey (1989, p. 40-43) and Canova

[2] This definition does not cover the case of polynomial cointegration at frequency $\lambda \in (0, \pi)$, that occurs if there is a polynomial vector $\boldsymbol{\alpha}(L) = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 L$ such that $\boldsymbol{\alpha}(L)\mathbf{y}_t$ is $I(d-b)$; see Hylleberg et al. (1990, p.230). Polynomial cointegration however is not allowed in the unobserved component representation considered in this paper.

[3] For $h < s/2$ the stochastic coefficients $\boldsymbol{\gamma}_{it}(h)$ are 2-dimensional column vectors, while they are scalar when $h = s/2$ for s is even.

and Hansen (1995).

The vector representation of our model is the following:

$$\mathbf{y}_t = \boldsymbol{\mu}_t + \mathbf{s}_t + \boldsymbol{\varepsilon}_t, \tag{3.1}$$

$$\boldsymbol{\mu}_t = \mathbf{X}_t \boldsymbol{\beta}, \tag{3.2}$$

$$\mathbf{s}_t = \sum_{h=1}^{[s/2]} \mathbf{Z}_t(h) \boldsymbol{\gamma}_t(h), \tag{3.3}$$

$$\boldsymbol{\gamma}_t(h) = \boldsymbol{\gamma}_{t-1}(h) + \boldsymbol{\eta}_t(h), \quad h = 1, ..., [s/2], \tag{3.4}$$

$$\boldsymbol{\eta}_t(h) \sim IID\left(\mathbf{0}, \boldsymbol{\Sigma}_\eta(h)\right), \quad h = 1, ..., [s/2], \tag{3.5}$$

$$\boldsymbol{\varepsilon}_t \sim IID\left(\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon\right), \tag{3.6}$$

where $\mathbf{X}_t = (\mathrm{I}_N \otimes \mathbf{x}_t'), \boldsymbol{\beta} = (\boldsymbol{\beta}_1', ..., \boldsymbol{\beta}_N')', \mathbf{Z}_t(h) = (\mathrm{I}_N \otimes \mathbf{z}_t(h)'), \boldsymbol{\gamma}_t(h) = (\boldsymbol{\gamma}_{1t}(h)', ..., \boldsymbol{\gamma}_{Nt}(h)')'$, $h = 1, ..., [s/2]$. It is also assumed that $\boldsymbol{\eta}_t(h)$ is independent of $\boldsymbol{\eta}_s(l)$ for $h \neq l$, i.e. the seasonal components at different frequencies are orthogonal, and also independent of the irregular disturbance $\boldsymbol{\varepsilon}_s$, for all t, s; $\otimes$ denotes the Kronecker product. The model is the multivariate analogue of that considered in Canova and Hansen (1995) and Busetti and Harvey (2003).[4]

The objective of this paper is to test for the presence of unit roots at the seasonal frequencies $\lambda(h)$, $h = 1, ..., [s/2]$. Specifically, if $\boldsymbol{\Sigma}_\eta(h)$ is of full rank the process displays seasonal integration at frequency $\lambda(h)$; if rank $\boldsymbol{\Sigma}_\eta(h) = 0$, the seasonal component at that frequency is deterministic; seasonal cointegration occurs otherwise.

Consider first the case of a model where $\boldsymbol{\Sigma}_\eta(l) = 0$ for $l \neq h \in \{1, 2, ..., [s/2]\}$, i.e. all seasonal components are deterministic except at the frequency $\lambda(h)$. The following proposition provides the locally best invariant (LBI) test, under Gaussianity, of $\mathrm{H}_0 : \boldsymbol{\Sigma}_\eta(h) = 0$ against $\mathrm{H}_A : \boldsymbol{\Sigma}_\eta(h) = q^2 \left(\boldsymbol{\Sigma}_\varepsilon \otimes \mathrm{I}_{a(h)}\right)$, where $q^2 > 0$ and $a(h) = 1$ if $h = s/2$ and $a(h) = 2$ otherwise. The proof is contained in the working paper version Busetti (2003).

Proposition 3.1. *Let* $\mathbf{y}_t$ *be generated from the model (3.1)-(3.6) with* $\boldsymbol{\Sigma}_\eta(l) = 0$ *for* $l \neq h \in \{1, 2, ..., [s/2]\}$, *and let* $\mathbf{e}_t$ *be the OLS residuals from regressing* $\mathbf{y}_t$ *on*

[4] Our representation of stochastic seasonality (3.4)-(3.5) corresponds to the one given in Harvey (1989, p. 42), that, under the restriction that the variance driving the seasonal component is the same for all frequencies, is implemented in the package STAMP 6.0 of Koopman et al. (2000).

$(\mathbf{x}_t', \mathbf{z}_t')'$, $t = 1, ..., T$. *Under Gaussianity, the LBI test of* $H_0 : \mathbf{\Sigma}_\eta(h) = 0$ *against* $H_A : \mathbf{\Sigma}_\eta(h) = q^2 (\mathbf{\Sigma}_\varepsilon \otimes I_{a_h})$ *rejects when*

$$\xi_{0,N}(h) = a(h) trace \left(\widehat{\mathbf{\Sigma}}_\varepsilon^{-1} \mathbf{C}(h) \right) > c \tag{3.7}$$

where $\widehat{\mathbf{\Sigma}}_\varepsilon = T^{-1} \sum_{t=1}^T \mathbf{e}_t \mathbf{e}_t'$, $\mathbf{C}(h) = T^{-2} \sum_{t=1}^T \left(\mathbf{S}_t^A(h) \mathbf{S}_t^A(h)' + \mathbf{S}_t^B(h) \mathbf{S}_t^B(h)' \right)$, $\mathbf{S}_t^A(h) = \sum_{s=1}^t \mathbf{e}_s \cos \lambda(h) s$, $\mathbf{S}_t^B(h) = \sum_{s=1}^t \mathbf{e}_s \sin \lambda(h) s$, *and* c *is an appropriate critical value.*

Under $H_0 : \mathbf{\Sigma}_\eta(h) = 0$, *with* $\mathbf{x}_t$ *satisfying assumption A1 of Busetti and Harvey (2003) and with* $\mathbf{\Sigma}_\eta(l) = 0$ *also for* $l \neq h$, *the limiting distribution of* $\xi_{0,N}(h)$ *is Cramér-von Mises with* $a(h)N$ *degrees of freedom,*

$$\xi_{0,N}(h) \xrightarrow{d} \int_0^1 \mathbf{B}_{a(h)N}(r)' \mathbf{B}_{a(h)N}(r) dr \equiv CvM\left(a(h)N\right), \tag{3.8}$$

where $\mathbf{B}_k(r) = \mathbf{W}_k(r) - r\mathbf{W}_k(1)$, $r \in [0,1]$, *denotes a* k*-dimensional Brownian bridge process and* $\mathbf{W}_k(r)$ *a* k*-dimensional Brownian motion.*

Remark 1. *When* s *is even the LBI statistic at the Nyqvist frequency* $\lambda(s/2) = \pi$ *can be written without the terms* $\mathbf{e}_s \sin \lambda(h) s$ *as they are identically zero, that is* $\mathbf{S}_t^B(s/2) = 0$.

The test can be viewed as the extension to multivariate series of the CH test of seasonal stability of Canova and Hansen (1995). Its limiting distribution is independent of the form of the deterministic regressors $\mathbf{x}_t$, as long as they satisfy the mild regularity conditions of assumption A1 of Busetti and Harvey (2003); polynomial trends and level and/or slope shifts do not change the distribution.

Although it is locally most powerful for the alternative hypothesis of same signal-to-noise ratio for all series, the test is consistent against any alternative in which $\mathbf{\Sigma}_\eta(h)$ is different from zero; see remark 2 in the next section. Monte Carlo results in Busetti (2003) show that the power losses from departures of the LBI set-up are typically rather small.

A joint test against seasonal integration at all frequencies is obtained by taking the sum of (3.7) over h, that is by the statistic

$$\overline{\xi}_{0,N} = \sum_{h=1}^{[s/2]} \xi_{0,N}(h). \tag{3.9}$$

From the additivity property of independent Cramér-von Mises random variables (cf. Busetti and Harvey, 2001, p.136), the null limiting distribution of (3.9) is Cramér-von Mises with $(s-1)N$ degrees of freedom,

$$\overline{\xi}_{0,N} \xrightarrow{d} CvM\left((s-1)N\right).$$

As $T \to \infty$ (3.9) diverges (and thus the joint test rejects the null hypothesis of deterministic seasonality) if there is a unit root for at least one of the seasonal frequencies.[5]

Upper tail percentage points for a $CvM(k)$, for $k \leq 12$, are tabulated in Canova and Hansen (1995). Additional critical values are contained in Table 1 below, in the columns headed $K = 0$. Specifically, the first 6 rows of the table (labelled one frequency) contain the quantiles of $CvM(2N)$, $N = 1, 2, ..., 6$, the following 6 rows refer to $CvM(3N)$ and the final rows to $CvM(11N)$. The quantiles have been obtained by direct simulation of the functional (3.8) for sample sizes of 1000 over 50000 Monte Carlo replications. The random number generator of the matrix programming language Ox 2.20 of Doornik (1998) was used. For other values of k large enough, the quantiles of a $CvM(k)$ can be obtained by a Gaussian approximation via a standard Central Limit Theorem; cf. Hadri (2000) and Harvey (2001).

Extending model (3.1)-(3.3) to allow for fixed seasonal slopes will make the distribution of the LBI test change to second level Cramér-von Mises; see Busetti (2003).

4. Tests of seasonal cointegration

A simple test of seasonal cointegration of order 1,1 for a known $N \times R$ full rank cointegration matrix, $\boldsymbol{\alpha}$, is just the (multivariate) LBI test (3.7) applied to $\boldsymbol{\alpha}'\mathbf{y}_t$. The test, however, would not be valid if $\boldsymbol{\alpha}$ is estimated; cf. Nyblom and Harvey (2000) where the same problem, but at frequency zero, is considered.

In general, we consider the data generating process (3.1)-(3.6) under the restriction that the seasonal component at frequency $\lambda(h)$ is driven by reduced rank random walk coefficients, i.e. that

$$\boldsymbol{\Sigma}_\eta(h) = \left(\overline{\boldsymbol{\Sigma}}_\eta(h) \otimes I_{a(h)}\right)$$

[5] A test of stability at any subset of the seasonal frequencies can also be constructed in an obvious way, that is by summing over the relevant frequencies, and critical values are obtained from a Cramér-von Mises distribution with the appropriate number of degrees of freedom.

where $\overline{\mathbf{\Sigma}}_\eta(h)$ is a $N\times N$ symmetric positive semidefinite matrix with $rank(\overline{\mathbf{\Sigma}}_\eta(h)) = K$, $0 \leq K < N$. We take this restriction as the null hypothesis:

$$H_{0,K} : rank(\overline{\mathbf{\Sigma}}_\eta(h)) = K.$$

It can be easily seen that under $\mathrm{H}_{0,K}$ the vector time series $\mathbf{y}_t$ is seasonally cointegrated at frequency $\lambda(h)$, $CI(1,1;\lambda(h))$, with $R = N - K$ linearly independent cointegrating vectors[6]. The alternative hypothesis is

$$H_{A,K} : rank(\overline{\mathbf{\Sigma}}_\eta(h)) > K,$$

i.e. that the cointegration space has a lower dimension than under the null hypothesis. As in the previous section, we first maintain that $\mathbf{\Sigma}_\eta(l) = 0$ for $l \neq h \in \{1,2,...,[s/2]\}$, i.e. that all seasonal components are deterministic except that at the frequency $\lambda(h)$.

The test statistic is the sum of the R smallest eigenvalues of $a(h)\widehat{\mathbf{\Sigma}}_\varepsilon{}^{-1}\mathbf{C}(h)$,

$$\xi_{K,N}(h) = \sum_{j=K+1}^{N} \ell_j(h), \tag{4.1}$$

where $\ell_1(h) \geq \ell_2(h) \geq ... \geq \ell_N(h) \geq 0$ are the N ordered eigenvalues. Notice that $\xi_{0,N}(h)$ is the statistic (3.7) of the previous section. The following proposition provides the limiting distribution of $\xi_{K,N}(h)$ under $H_{0,K} : rank(\overline{\mathbf{\Sigma}}_\eta(h)) = K$, $1 \leq K < N$; the case $K = 0$ has been dealt with in the previous section. The proof, that extends Nyblom and Harvey (2000), is in Busetti (2003).

Proposition 4.1. *Under $H_{0,K} : rank(\overline{\mathbf{\Sigma}}_\eta(h)) = K$, $1 \leq K < N$, with $\mathbf{x}_t$ satisfying assumption A1 of Busetti and Harvey (2003) and with $\mathbf{\Sigma}_\eta(l) = 0$ for $l \neq h$, and $h \neq s/2$,*

$$\xi_{K,N}(h) \xrightarrow{d} Tr\left(\mathbf{C}^*_{22}(h) - \mathbf{C}^*_{12}(h)'\mathbf{C}^*_{11}(h)^{-1}\mathbf{C}^*_{12}(h)\right), \tag{4.2}$$

[6] Under $\mathrm{H}_{0,K}$ there exists a full rank $N \times K$ matrix $\mathbf{\Theta}$ such that $\mathbf{\Theta\Theta}' = \overline{\mathbf{\Sigma}}_\eta(h)$. Thus we can write $\boldsymbol{\eta}_t(h) = \left(\mathbf{\Theta} \otimes I_{a(h)}\right)\boldsymbol{\eta}^*_t$ where $\boldsymbol{\eta}^*_t$ is $IID\left(0, I_{a(h)K}\right)$. Let $\boldsymbol{\alpha}$ be a $N \times 1$ vector belonging to the (R-dimensional) left null space of $\mathbf{\Theta}$, i.e. such that $\boldsymbol{\alpha}'\mathbf{\Theta} = \mathbf{0}$. This is a seasonal cointegration vector since it annihilates the stochastic seasonal component at frequency$\lambda(h)$,
$\boldsymbol{\alpha}'\mathbf{s}_t = (\boldsymbol{\alpha}' \otimes \mathbf{z}'_t(h))\left(\mathbf{\Theta} \otimes I_{a(h)}\right)\sum_{j=1}^t \boldsymbol{\eta}^*_j = (\boldsymbol{\alpha}'\mathbf{\Theta} \otimes \mathbf{z}'_t(h))\sum_{j=1}^t \boldsymbol{\eta}^*_j = 0.$

with

$$\begin{aligned}
\mathbf{C}_{11}^{*}(h) &= \int_0^1 \left(\int_0^r \overline{\mathbf{W}}_K^A(s)ds\right)\left(\int_0^r \overline{\mathbf{W}}_K^A(s)ds\right)' dr + \int_0^1 \left(\int_0^r \overline{\mathbf{W}}_K^B(s)ds\right)\left(\int_0^r \overline{\mathbf{W}}_K^B(s)ds\right)' dr \\
\mathbf{C}_{12}^{*}(h) &= \int_0^1 \left(\int_0^r \overline{\mathbf{W}}_K^A(s)ds\right)\mathbf{B}_R^A(r)'dr + \int_0^1 \left(\int_0^r \overline{\mathbf{W}}_K^B(s)ds\right)\mathbf{B}_R^B(r)'dr \\
\mathbf{C}_{22}^{*}(h) &= \int_0^1 \mathbf{B}_R^A(r)\mathbf{B}_R^A(r)'dr + \int_0^1 \mathbf{B}_R^B(r)\mathbf{B}_R^B(r)'dr
\end{aligned}$$

where $\overline{\mathbf{W}}_K^A(r)$, $\overline{\mathbf{W}}_K^B(r)$ *are independent* K*-dimensional demeaned Wiener processes and* $\boldsymbol{B}_R^A(r)$, $\boldsymbol{B}_R^B(r)$ *independent* R*-dimensional Brownian bridges.*

Remark 2. *The test is consistent for* $H_{A,K} : rank(\overline{\boldsymbol{\Sigma}}_\eta(h)) > K$ *as at least one of the eigenvalues in (4.1) is* $O_p(T)$*.*

Remark 3. *When* s *is even, the limiting null distribution of* $\xi_{K,N}(s/2)$ *is that of Nyblom and Harvey (2000).*

As in the previous section, a joint test for seasonal cointegration at all frequencies is obtained by taking the sum of (4.1) over h, that is by the statistic

$$\overline{\xi}_{K,N} = \sum_{h=1}^{[s/2]} \xi_{K,N}(h). \tag{4.3}$$

Since the statistics for each individual frequency are asymptotically independent, the limiting distribution of (4.3) under the joint null hypothesis $\overline{\mathrm{H}}_{0,K}$: $rank(\overline{\boldsymbol{\Sigma}}_\eta(h)) = K$, $h = 1, ..., [s/2]$, can be obtained by simulating percentage points from the sum, over h, of independent random variables, each with asymptotic representation given by proposition 4.1 (taking into account remark 3 which applies for s even). A non-rejection of $\overline{\mathrm{H}}_{0,K}$ in the joint test implies seasonal cointegration with $R = N - K$ linearly independent cointegrating vectors at each of the seasonal frequencies. The cointegrating vectors however are allowed to differ across frequencies.

Upper tail percentage points for the limiting null distributions of $\xi_{K,N}(h)$, $\overline{\xi}_{K,N}$ are provided in Table 1; for the joint test statistic $\overline{\xi}_{K,N}$ we provide values appropriate to quarterly and monthly data. The columns headed $K = 0$ correspond to the tests of the previous section where the distribution is a CvM with an appropriate number of degrees of freedom, while those for $1 \leq K < N$ are appropriate for the tests of seasonal cointegration. The quantiles have been obtained by direct simulation of the functional (4.2) for sample sizes of 1000 over 50000 Monte Carlo replications. The critical values for testing at frequency π are given in Nyblom and Harvey (2000).

5. Stochastic trends, serial correlation and unattended unit roots

In the previous sections we have considered testing for seasonal integration and cointegration in the multivariate unobserved component model (1.1) under the assumptions that (i) the trend $\boldsymbol{\mu}_t$ is a deterministic function of time and the irregular component $\boldsymbol{\varepsilon}_t$ is a white noise, (ii) the seasonal components at all frequencies except the frequencies under test are deterministic. These restrictions are relaxed in the following two subsections.

5.1. Stochastic trends and serial correlation

In an unobserved component model, in general, the trend is allowed to be stochastic. A flexible form of the trend function which is typically adequate for many economic time series is the local linear trend of Harvey (1989), where both the level and the slope are stochastic and evolve as random walks.

Testing seasonal integration and cointegration in a model with a stochastic trend can be carried out by two strategies: either by removing the stochastic trend by appropriate differencing or by estimating a fully parametrized model and constructing the test from the model's residuals (the latter approach will be denoted as parametric).

An $I(1;0)$ trend is annihilated by applying the standard first difference operator. However, the resulting irregular component is no longer a white noise but a moving average process. The statistics of the previous section are thus no longer appropriate, but the test can be run after a nonparametric modification that allows the irregular component to follow a weakly dependent process.[7]

Specifically, suppose that the irregular component $\boldsymbol{\varepsilon}_t$ is a weakly dependent process and let $\boldsymbol{\Omega}(\lambda)$, $\lambda \in [0, \pi]$, denote its multivariate spectrum (multiplied by 2π). Then it suffices to replace $\widehat{\boldsymbol{\Sigma}}_\varepsilon$ in the statistics defined in section 3,4 by a consistent estimator, say $\widehat{\boldsymbol{\Omega}}(h)$, of the spectrum at frequency $\lambda(h)$, e.g.

$$\widehat{\boldsymbol{\Omega}}(h) = \sum_{j=-m}^{m} k(j,m)\widehat{\Gamma}(j)\left(\cos\lambda(h)j - i\sin\lambda(h)j\right)$$

where $k(.,.)$ is a kernel function, e.g. the Newey-West kernel $k(j,m) = 1 - |j|/(m+1)$, $\widehat{\Gamma}(|j|) = T^{-1}\sum_{t=j+1}^{T} e_t e_{t-j}'$ is the sample autocovariance of the OLS

[7] If the data are not differenced the test will still be consistent but suffer from a loss of power in finite samples for the problem of the unattended unit roots; see the next subsection.

residuals at lag $j \geq 0$, and $\widehat{\Gamma}(-|j|) = \widehat{\Gamma}(|j|)'$. Alternative options for the kernel may be found in, *inter alia*, Priestley (1989) and Andrews (1991). Setting the bandwidth parameter m such that $m \to \infty$ and $m/T^{1/2} \to 0$ as $T \to \infty$ ensures that $\hat{\mathbf{\Omega}}(h) \xrightarrow{p} \mathbf{\Omega}(\lambda(h))$ under the null and remains stochastically bounded under the alternative hypothesis of stochastic seasonality, thereby ensuring consistency of the test; see Stock (1994, p.2797-2799).

Thus we have the following *spectral nonparametric statistic* for seasonal integration and cointegration,

$$\xi^*_{K,N}(h) = \sum_{j=K+1}^{N} \ell^*_j(h), \qquad 0 \leq K < N, \tag{5.1}$$

where $\ell^*_1(h), ..., \ell^*_N(h)$ are the N ordered eigenvalues of $a(h)\widehat{\mathbf{\Omega}}(h)^{-1}\mathbf{C}(h)$, with $a(h)$, $\mathbf{C}(h)$ defined in proposition 3.1. Note that, as $\widehat{\mathbf{\Omega}}(h)$ and $\mathbf{C}(h)$ are positive definite hermitian matrices, the eigenvalues of $\widehat{\mathbf{\Omega}}(h)^{-1}\mathbf{C}(h)$ are real and positive. By extending the arguments of Nyblom and Harvey (2000) it is straightforward to show that the limiting null distributions of (5.1) are as given by Propositions 3.1 and 4.1.

A *parametric approach* to deal with a stochastic trend (and possibly other stochastic components) can also be employed. The idea, that extends the univariate case considered in Busetti and Harvey (2003), is to fit a fully parametric model where the nuisance parameters are estimated under the alternative hypothesis of seasonal integration (or cointegration). The model is put in state space form and the Kalman filter is run *under the null* hypothesis: the resulting Kalman filter innovations, the model's residuals, are used to compute the statistic $\xi_{K,N}(h)$ of the previous section. The limiting null distribution will be unchanged; see Busetti and Harvey (2003) and the empical analysis of section 7.

5.2. Unattended unit roots

Busetti and Taylor (2003) and Taylor (2003a) have considered the effect of unit root behavior at some frequency on the stability tests at other frequencies; this situation is termed "unattended unit roots". They show that the power of the tests is vastly reduced in the presence of unattended unit roots; indeed, under the null hypothesis, the test statistics converge in probability to zero. However, a simple way to avoid this reduction in power is to prefilter the data so as to annihilate any unattended unit roots.

In the context of testing for seasonal integration and cointegration at frequency $\lambda(h)$, $h \in \{1, ..., [s/2]\}$, one may wish to guard against the effects of unit roots at the other seasonal frequencies $\lambda(l)$, $l \neq h$. This is accomplished by computing the tests after the filter $\nabla_s(\lambda(h)) \equiv (1 + L + ... + L^{s-1})/\Delta(\lambda(h))$ has been applied to the data; the seasonal sum operator in the numerator is just the product, over frequencies, of the first difference filters $\Delta(\lambda(h))$ of section 2: $\prod_{h=1}^{[s/2]} \Delta(\lambda(h)) = 1 + L + ... + L^{s-1}$. For example, the test at frequency π for quarterly data will be computed on the transformed data $(1 + L^2)\,\mathbf{y}_t$.

Since the application of the prefilter $\nabla_s(\lambda(h))$ transforms a white noise into a moving average process, the tests need to be computed with some correction for serial correlation, as in the previous subsection, even if the irregular component is a white noise. The resulting process will be strictly non-invertible at all seasonal frequencies except $\lambda(h)$, that is the spectrum at $\lambda(h)$ is a positive definite matrix.

Consequently, we suggest using the statistic (5.1) where the OLS residuals are computed from the regression of $\nabla_s(\lambda(h))\mathbf{y}_t$ on $\mathbf{w}_t = (\nabla_s(\lambda(h))\mathbf{x}_t', \mathbf{z}_t')'$; note that the prefiltered regressors $\nabla_s(\lambda(h))\mathbf{z}_t$ span the same space as $\mathbf{z}_t$. If the data generating process also contains a unit root at frequency zero, as when the trend $\boldsymbol{\mu}_t$ is a random walk process, the data should be prefiltered by $(1 - L)\,\nabla_s(\lambda(h))$. Further insights on the effect of pre-filtering on unit root inference are given in Franses (1991).

6. Monte Carlo results

In this section we use Monte Carlo simulation methods to investigate the finite sample size and power properties of the tests for seasonal integration and cointegation considered in the previous sections. We generate quarterly data from the DGP (3.1)-(3.6), setting

$$\boldsymbol{\Sigma}_\varepsilon = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}.$$

We focus on the properties of the tests at the fundamental frequency $\lambda(1) = \pi/2$ for the cases of

(A) seasonal integration: $\boldsymbol{\Sigma}_\eta(1) = q_1^2\,(\boldsymbol{\Sigma}_\varepsilon \otimes \mathrm{I}_2)$,

(B) seasonal cointegration: $\boldsymbol{\Sigma}_\eta(1) = q_1^2 \left(\begin{pmatrix} 1 & 1 \end{pmatrix}' \begin{pmatrix} 1 & 1 \end{pmatrix} \otimes \mathrm{I}_2 \right)$,

where the square root of the signal-to-noise ratio, q_1, takes on the values 0, 0.025, 0.050, 0.075, 0.1, 0.5. The results are reported in Table 2. (A) is the LBI set-up, while (B) corresponds to a common seasonal component with cointegration vector equal to (1, −1). [8] As concerns frequency π we set $\mathbf{\Sigma}_{\eta}(2) = q_2^2 \mathbf{\Sigma}_{\varepsilon}$, with the square root of the signal-to-noise ratio q_2 equal to $0, 0.5$. Setting $q_2 > 0$ allows us to see the effect of unattended unit roots.

The results are for quarterly series of length $T = 100$. For each configuration of the parameters of the data generating process we compute the following four statistics for testing at frequency $\pi/2$:

(1) $\ell_1(1) + \ell_2(1)$, to test the null hypothesis $K = 0$ with data in levels,
(2) $\ell_1^*(1) + \ell_2^*(1)$, to test the null hypothesis $K = 0$ with prefiltered data,
(3) $\ell_2(1)$, to test the null hypothesis $K = 1$ with data in levels,
(4) $\ell_2^*(1)$, to test the null hypothesis $K = 1$ with prefiltered data.

When the prefilter $\nabla_4(\lambda(1)) = 1 + L$ is applied to the data, the eigenvalues $\ell_j^*(h), j, h = 1, 2$, are computed using a spectral estimate $\widehat{\mathbf{\Omega}}(h)$ with a Newey-West kernel with bandwidth $m = 4$. The empirical rejection frequencies, reported in percentages, are based on 100,000 replications and refer to tests run at the 5% significance level.

Consider first the results in panel (A) of Table 2 for the case of no unattended unit root, $q_2 = 0$. The LBI test (1) of $K = 0$ at frequency $\pi/2$ appears slightly oversized; in fact, the prefiltered test (2) computed with bandwidth $m = 4$ has a size very close to the nominal 5% and, although prefiltering is not advisable as $q_2 = 0$, it does not suffer from a big power loss with respect to (1).

As expected, the tests for seasonal cointegration (3)-(4) display lower power than in the LBI setup (1)-(2) where the null hypothesis is that of deterministic seasonality, but they are clearly consistent since the eigenvalues $\ell_2(1)$, $\ell_2^*(1)$ diverge as the sample size grows to infinity. For a signal-to-noise ratio as small as 0.01 ($q_1 = 0.1$) the power of the seasonal cointegration test (3) is around 25% in a sample of 100 observations; however if the sample size is enlarged to $T = 200$ power goes up to 67% (detailed Monte Carlo results for the case $T = 200$ are available upon request).

It is interesting to examine the effect of unattended unit roots at frequency π. When $q_2 = 0.5$ the power of test (1) in the levels is very low; on the other hand, the rejection frequencies of the prefiltered test (2) are largely comparable to those of the LBI test where $q_2 = 0$. Analogous effects apply to the tests (3)-(4)

[8] Additional simulations, covering departures from the homogeneity assumption $\mathbf{\Sigma}_{\eta}(1) = q_1^2 (\mathbf{\Sigma}_{\varepsilon} \otimes \mathrm{I}_2)$ in the data generating process, are contained in the working paper version Busetti (2003). It turns out that power is not much influenced by homogeneity, which may be a reflection of the result that the distribution of the statistic under the alternative hypothesis depends only on the rank of $\mathbf{\Sigma}_{\eta}(1)$ (cf. Nyblom and Harvey, 2000).

of seasonal cointegration.

The case of a data generating process with perfect correlation in $\mathbf{\Sigma}_\eta(1)$, that is with seasonal cointegration, is examined in the panel (B) of Table 2. Consider first the case $q_2 = 0$. Even for large values of q_1 the rejection frequencies of the seasonal cointegration tests (3)-(4) never exceed 5.3%; that is, the empirical size of the test, defined as maximum probability of rejecting the null hypothesis when it is true, turns out to be close to the nominal size even in a sample of $T = 100$. The finite sample power of the seasonal cointegration tests (3)-(4) is in the figures of panel (A) of the table 2.A-B and has already been discussed. As concerns the power of the tests (1)-(2) of the null hypothesis $K = 0$, it is somehow lower than the corresponding figures of panel (A) but higher than the power of the seasonal cointegration tests (3)-(4) for the same case. The unattended unit root, $q_2 > 0$, has the effect of reducing power in a qualitatively similar way as in panel (A) of the table.

7. Application: industrial production in the euro area

Figure 1 shows the logarithm of the monthly index of industrial production in the four largest countries of the European Monetary Union: Germany, France, Italy and Spain. The data refer to the period 1985M1-2001M12 with the base year being 1995; the source is Eurostat. The series are characterized by large seasonal swings and it also appears, from visual inspection, that the seasonal patterns are not constant over time. The main questions we want to address are whether the seasonality of industrial production is deterministic and whether there are co-movements at the seasonal frequencies.

We first apply the tests of seasonal integration and cointegration to each combination of the four countries by using the spectral nonparametric statistic (5.1) computed on the prefiltered observations $\mathbf{y}_t^* \equiv (1 - L)\,\nabla_{12}(\lambda(h))\mathbf{y}_t$. The results are displayed in Table 3 for values of the bandwidth parameter $m = 10, 15$; the number of observations is 204. The choice of m reflects the usual trade-off between size and power of the tests; given that the filter would turn a white noise irregular component into a moving average of at least order 10, it seems appropriate to include at least 10 lags for the spectral estimation. The figures that appears shaded and in italics indicate rejection at 5% significance level of the null hypothesis that there are K non-stationary seasonal components, that is seasonal cointegration with $R = N - K$ cointegrating vectors. The rows of the table indicate to which subset of the four countries, Germany, France, Italy and Spain, the tests are applied; the columns indicate the seasonal frequency to which the figures refer. The last 3 columns contain the joint test at all seasonal frequencies.

Consider first the last 4 rows of the table, where the tests are applied to the multivariate series of industrial production of Germany, France, Italy and Spain. For each of the seasonal frequencies $\pi/6$, $\pi/3$, $\pi/2$, $2\pi/3$ the results point to the existence of two common seasonal components across the four countries (the null hypothesis $K = 2$ is not rejected), while a single non-stationary component is detected at the higher frequencies $5\pi/6$ and π. This is also indirectly confirmed by looking at the bivariate and trivariate analyses contained in the upper part of the table. However, if we exclude Germany (row labelled FR-IT-SP) the evidence is for a single non-stationary component at each of the seasonal frequencies (as the 1% critical value for testing at a single frequency and for the joint test are 1.277 and 4.082 respectively, the results for $\pi/3$ and for the joint test are also consistent with $K = 1$). To summarize, Germany appears to cointegrate with the other countries at some but not all of the seasonal frequencies, while France, Italy and Spain seem to be characterized by a single non-stationary seasonal component.[9]

A further step in the analysis is to fit a model to the data. We start with the univariate *Basic Structural Model* (BSM) of Harvey (1989, p. 47), where μ_t is a stochastic linear trend, s_t is a non-stationary seasonal component and the irregular term ε_t is a white noise. We do not provide details except for saying that the model's diagnostics are satisfactory for Germany, Spain and Italy (that however fails the normality test of the residuals due to the presence of a few outliers) but not for France, in which case the model passes all standard diagnostics only if estimated over the subsample 1991M1-2001M12. The extracted trend of the four series are depicted in figure 1, while figure 2 plots, for each month, the extracted seasonal component for France, Italy and Spain. The latter graph may perhaps visually confirm the finding of table 3 of common components in the seasonal fluctuations among the three countries.

Then we fit a multivariate unobserved component model to the three series of industrial production of France, Italy and Spain, that -in the light of the spectral nonparametric tests reported in table 3- seem to be characterized by seasonal cointegration at all frequencies. We specify a model made up of a *smooth trend* (or integrated random walk), a nonstationary seasonal component with-

[9] This result for France, Italy and Spain contrasts with most of other empirical analyses where the seasonal integration and cointegration properties of the data are generally found to be different across frequencies (cf. Ghysels and Osborne, 2001).

out cointegration restrictions and an irregular term[10]. We restrict the sample to 1991M1-2001M12, as the model would not fit French data for the earlier part of the sample. The estimated cross correlations of the seasonal disturbance range from 0.870 to 0.997, corroborating the evidence for seasonal cointegration of table 3. The parametric test is obtained by the Kalman filter residuals from this model but computed under the restriction of common seasonality (that is setting all cross-correlations equal to 1 and leaving the other parameters at their estimated values; see section 5). In this case the null hypothesis of $K = 1$ (a single common component) is not rejected at the 5% significance level at each frequency except $\pi/6$ for which there is borderline rejection; the joint test statistic at all seasonal frequencies is equal to 3.518 against a 5% asymptotic critical value of 3.571. The results of the parametric and non-parametric tests are therefore consistent with each other. The attraction of the parametric test is that it can be used for model building, according to a general-to-specific strategy.[11]

The model is re-estimated by imposing the seasonal cointegration restriction of a single common component. The maximum likelihood estimates and main

[10] Furthermore we assume that the variance of the disturbances driving the nonstationary seasonal components is the same for all frequencies: in terms of (3.3)-(3.5), we have $\mathbf{\Sigma}_\eta(h) = \mathbf{\Sigma}_\eta(l)$ for all $h, l \in \{1, 2, ..., 6\}$. Harvey (1989, p.43) argues that, as a rule, very little is lost by imposing this restriction. In this case, however, the seasonal cointegration vectors are not allowed to differ across frequencies. The smooth trend is specified as $\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \boldsymbol{\beta}_{t-1}$, $\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \zeta_t$, $\zeta_t \sim IID\left(\mathbf{0}, \mathbf{\Sigma}_\zeta\right)$.

[11] Following a referee's suggestion we have also fitted the multivariate Basic Structural Model to the series of Germany, Italy and Spain over the entire sample 1985M1-2001M12. The estimated correlations among the seasonal components have been found equal to 0.84 for Italy and Spain, 0.58 for Germany and Spain and 0.24 for Germany and Italy. The results of the parametric tests are again in line with the non-parametric analogues of table 3: the joint test implies a 5% rejection of the null hypothesis $K = 1$ (the statistic being equal to 3.909) and a non-rejection of $K = 2$.

diagnostics are reported in the following table.

Tri-variate Model	FR	IT	SP
std. dev. irregular (∗100)	0.71	1.83	1.13
std. dev. slope (∗100)	0.14	0.18	0.25
std. dev. seasonal (∗100)	0.08	0.20	0.17
std. error of model (∗100)	1.20	2.72	2.23
seasonal R^2	0.32	0.49	0.49
normality test	0.24	19.17	0.33
Durbin-Watson	1.81	2.00	1.71
Box-Ljung test (p-value)	0.41	0.02	0.07

The model fits the data reasonably well, although Italy still fails the normality test of the residuals due to the presence of outliers. The standard error is smallest for France, where the seasonal fluctuations also change most slowly. The seasonal R^2 statistic is interpreted as the goodness of fit improvement with respect to a random walk plus deterministic seasonality model. The factor loadings are estimated as $\Theta = (1,\ 2.44,\ 2.13)'$, that is $\mathbf{s}_t = \Theta s_t$, where s_t is a scalar nonstationary seasonal component that is shared by the three series.

The cointegration space is two-dimensional with its elements orthogonal to the Θ matrix. Testing hypotheses on the cointegrating vectors can be done either non-parametrically or within the model. For example, since the loads for Spain and Italy are similar, one could test whether the two countries are seasonally cointegrated with vector $(1, -1)$. The non-parametric test is simply the univariate seasonal stability test of Canova and Hansen (1995) applied to the series of the difference between Italian and Spanish industrial production. A parametric test is instead constructed by comparing the likelihood of the estimated model with the resulting likelihood after restricting the loads for Italy and Spain to be the same; this likelihood ratio test has a $\chi^2(1)$ limiting distribution under the null hypothesis. In this case, both the non-parametric and the parametric test strongly reject the null hypothesis of cointegration vector equal to $(1, -1)$.

One of the advantage of unobserved component models is that they allow easy extraction of the salient characteristics of the data. In our case, the estimated trend and seasonal components (March, August and December) for Italy are graphed in figure 3. The thick lines are the components (with 90% confidence intervals) from the multivariate model with seasonal cointegration, while the thin solid line and the dashed line correspond to the multivariate unrestricted model and the univariate model respectively; note for example that the seasonal drop of Italian industrial production in August has reduced from over 70% to about 55% over the years, while the December effect never exceeded 5%. In most periods the trend and seasonal components of the univariate and multivariate unrestricted models lie within the confidence interval of our preferred seasonal cointegrated model. Imposing seasonal cointegration, however, allows efficiency gains with respect to leaving the model unrestricted; in particular, the variance of the trend component is reduced by around 30% and that of the seasonal by about 10% with respect to the unrestricted model; the confidence bands are also much narower than in the univariate case. Similar plots apply to the series of France and Spain but, to save space, are not presented.

Finally in table 4 we compare the results of our tests of seasonal cointegration with the tests based on vector autoregressions, using the complex reduced rank regression approach of Cubadda (2001). The code for computing the VAR-type tests has been kindly provided by Gianluca Cubadda. We restrict attention to testing at frequencies $\pi/2$ and π for the tri-variate quarterly series of industrial production of France, Italy and Spain. We fit two VARs, with 5 and 8 lags respectively, and compare the results with the UC-based tests constructed with bandwidth parameter m equal to 5 and 8 (clearly there is no relation between the lag order of the VAR and the bandwidth parameter in the UC framework, except that they both need to grow at a much slower rate than the sample size). Although the VAR set-up allows for polynomial cointegration at frequency $\pi/2$, it is interesting that the evidence from the VAR(5) model is very much in line with the tests based on the unobserved component framework; in both cases there is indication of seasonal cointegration with a single common component. We think this is a useful further robustness check on the co-movement properties of the data.

8. Conclusions

The paper has proposed tests of seasonal integration and cointegration in the framework of multivariate unobserved component models. The tests have been derived under the assumption of Gaussian white noise disturbances and then extended to models with stochastic trends, weakly dependent errors and unattended unit roots. The finite sample properties of the tests have been investigated by Monte Carlo simulation experiments. The application of the tests to the series of industrial production across the main countries of the European Monetary Union has provided evidence of seasonal cointegration with a single common component for France, Italy and Spain. Germany appears to cointegrate with the other countries at some but not all of the seasonal frequencies.

ACKNOWLEDGMENTS

I wish to thank Tim Bollerslev, Gianluca Cubadda, Andrew Harvey, Robert Taylor, and three anonymous referees for helpful comments. Gianluca Cubadda kindly provided the Gauss code for computing the complex reduced rank regression cointegration test in table 4. Earlier versions of this paper were presented at the European Congress of the Econometric Society, Stockholm 2003, at the ESF-EMM Conference on Econometric Methods for the Modelling of Nonstationary Data, Policy Analysis and Forecasting, Rome 2003, and at the Bank of Italy Economic Seminar. The views expressed here are mine and do not necessarily represent those of the Bank of Italy.

REFERENCES

References

[1] Ahn, S.K., Cho, S. and B.C. Seong (2004), Inference on seasonal cointegration: Gaussian reduced rank estimation and tests for various types of cointegration, *Oxford Bulletin of economics and Statistics* 66, 261-284.

[2] Ahn, S.K. and G.C. Reinsel (1994), Estimation of partially non-stationary vector autoregressive models with seasonal behavior, Journal of Econometrics 62, 317-50.

[3] Andrews, D.W.K. (1991), Heteroscedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica* 59**,** 817-58.

[4] Beaulieu, J.J. and J.A. Miron (1993), Seasonal unit roots in aggregate US data, *Journal of Econometrics*, 55, 305-328.

[5] Busetti, F. (2003), Tests of seasonal integration and cointegration in multivariate unobserved component models, Banca d'Italia Discussion Papers, n. 476, downloadable from http://www.bancaditalia.it/ricerca/consultazioni/temidi.

[6] Busetti, F. and A.C. Harvey (2001), Testing for the presence of a random walk in series with structural breaks. *Journal of Time Series Analysis 22, 127-50.*

[7] Busetti, F. and A.C. Harvey (2003), Seasonality tests, *Journal of Business and Economic Statistics 21*, 420-436.

[8] Busetti, F. and A.M.R. Taylor (2003), Testing against stochastic trend and seasonality in the presence of unattended breaks and unit roots, *Journal of Econometrics* 117, 21-53.

[9] Caner, M. (1998), A locally optimal seasonal unit root test, *Journal of Business and Economic Statistics*, 16, 349-56.

[10] Canova, F. and B.E. Hansen (1995), Are seasonal patterns constant over time? A test for seasonal stability, *Journal of Business and Economic Statistics*, 2, 292-349.

[11] Cubadda, G. (1999), Common cycles in seasonal non-stationary time series, *Journal of Applied Econometrics*, 14, 273-291.

[12] Cubadda, G. (2001), Complex reduced rank models for seasonally cointegrated time series, *Oxford Bulletin of economics and Statistics* 63, 497-511.

[13] Cubadda, G. and P. Omtzigt (2004), Small-sample improvements in the statistical analysis of seasonally cointegrated systems, *Computational Statistics and Data Analysis*, forthcoming.

[14] Doornik, J.A. (1998), *Object-oriented matrix programming using Ox 2.0*, Timberlake Consultants Press, New York.

[15] Engle, R.F., Granger, C.W.J., Hylleberg, S. and H.S. Lee (1993), Seasonal cointegration: the Japanese consumption function, *Journal of Econometrics*, 55, 275-298.

[16] Franses, P.H. (1991), Moving average filters and unit roots, *Economic Letters* 37, 399-403.

[17] Franses, P.H. and M. McAleer (1998), Cointegration analysis of seasonal time series, *Journal of Economic Surveys*, 12, 651-678.

[18] Ghysels, E. and D.R. Osborn (2001), *The econometric analysis of seasonal time series*, Cambridge University Press, New York.

[19] Gregoir, S. (1999), Multivariate time series with various hidden unit roots, part I: integral operator algebra and representation theory, *Econometric Theory* 15, 435-468.

[20] Hadri, K. (2000), Testing for stationarity in heterogeneous panel data, *Econometrics Journal*, 3, 148-161.

[21] Harvey, A.C. (1989), *Forecasting, structural time series models and the Kalman filter*, Cambridge, Cambridge University Press.

[22] Harvey, A.C. (1993), *Time Series Models*, Harvester Wheatsheaf.

[23] Harvey, A.C. (2001), Testing in unobserved components models. *Journal of Forecasting*, 20, 1-19.

[24] Harvey, A.C. and S.J. Koopman (1997), *Multivariate structural time series models*, in System Dynamics in Economic and Financial Models, C. Heij, J.M. Schumacher, B. Hanzon and C. Praagman editors, New York, John Wiley and Sons.

[25] Huang, T. and C. Shen (2002), Seasonal cointegration and cross-equation restrictions on a forward-looking buffer stock model of money demand, *Journal of Econometrics* 111, 11-46.

[26] Hylleberg, S., R.F. Engle, C.W.J. Granger and B.S. Yoo, 1990, Seasonal integration and cointegration, *Journal of Econometrics* 44, 215-238.

[27] Johansen, S. (1988), Statistical analysis of cointegration vectors, J*ournal of Economic Dynamics and Control* 12, 131-154.

[28] Johansen, S. (1991), Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive model, *Econometrica* 59, 1551-1580.

[29] Johansen, S. (1995), *Likelihood-based inference in cointegrated vector autoregressive models*, Oxford University Press, Oxford.

[30] Johansen, S. and E. Schaumburg (1999), Likelihood analysis of seasonal cointegration, *Journal of Econometrics*, 88, 301-339.

[31] Kunst, R.M. (1993), Seasonal cointegration in macroeconomic systems: case studies for small and large European countries, *Review of Economics and Statistics*, 75, 325-330.

[32] Koopman, S.J., Harvey, A.C., Doornik, J.A. and N. Shephard (2000), *Stamp 6.0: structural time series analyser, modeller and predictor*, London, Timberlake Consultants.

[33] Kwiatkowski, D., Phillips, P.C.B., Schmidt, P. and Y. Shin (1992), Testing the null hypothesis of stationarity against the alternative of a unit root: how sure are we that economic time series have a unit root?, *Journal of Econometrics* 44, 159-178.

[34] Lee, H.S. (1992), Maximum likelihood inference on cointegration and seasonal cointegration, *Journal of Econometrics*, 54, 1-47.

[35] Nyblom, J. and A.C. Harvey, 2000, Tests of common stochastic trends, *Econometric Theory* 16, 176-199.

[36] Reimers, H.E. (1997), Seasonal cointegration analysis of German consumption function, *Empirical Economics*, 22, 205-231.

[37] Smith, R.J. and A.M.R. Taylor (1998), Additional critical values and asymptotic representations for seasonal unit root tests, *Journal of Econometrics* 85, 269-288.

[38] Stock, J.H. (1994) Unit roots, structural breaks and trends, in R.F. Engle and D.L. McFadden (eds.), *Handbook of Econometrics* 4, Elsevier Science, 2739-2840.

[39] Taylor, A.M.R. (2003a), Robust stationarity tests in seasonal time series processes, *Journal of Business and Economic Statistics* 21, 156-163.

[40] Taylor, A.M.R. (2003b), Locally optimal tests against seasonal unit roots, *Journal of Time Series Analysis* 24, 591-612.

	N	K=0			K=1			K=2			K=3			K=4			K=5		
		0.90	**0.95**	**0.99**	**0.90**	**0.95**	**0.99**	**0.90**	**0.95**	**0.99**	**0.90**	**0.95**	**0.99**	**0.90**	**0.95**	**0.99**	**0.90**	**0.95**	**0.99**
one frequency (≠π for s even)	**1**	0.602	0.738	1.073															
	2	1.065	1.236	1.624	0.445	0.559	0.842												
	3	1.482	1.688	2.095	0.789	0.933	1.277	0.299	0.378	0.595									
	4	1.892	2.116	2.587	1.105	1.270	1.650	0.543	0.651	0.911	0.214	0.262	0.404						
	5	2.287	2.526	3.036	1.411	1.595	1.988	0.772	0.901	1.218	0.388	0.456	0.638	0.164	0.197	0.285			
	6	2.684	2.941	3.460	1.710	1.911	2.330	0.998	1.148	1.492	0.555	0.642	0.868	0.299	0.344	0.457	0.132	0.157	0.220
all frequencies (s=4)	**1**	0.839	0.992	1.361															
	2	1.482	1.688	2.095	0.554	0.682	0.974												
	3	2.092	2.323	2.771	1.000	1.155	1.498	0.364	0.444	0.660									
	4	2.684	2.941	3.460	1.411	1.592	1.993	0.665	0.774	1.043	0.257	0.307	0.446						
	5	3.263	3.533	4.118	1.813	2.014	2.443	0.948	1.081	1.402	0.472	0.540	0.724	0.196	0.230	0.320			
	6	3.832	4.132	4.750	2.205	2.432	2.908	1.231	1.387	1.744	0.678	0.761	0.982	0.361	0.407	0.522	0.158	0.184	0.249
all frequencies (s=12)	**1**	2.482	2.733	3.246															
	2	4.596	4.914	5.552	1.798	1.994	2.415												
	3	6.621	7.004	7.789	3.321	3.571	4.082	1.248	1.389	1.710									
	4	8.612	9.058	9.940	4.785	5.079	5.707	2.313	2.504	2.890	0.905	1.001	1.224						
	5	10.592	11.073	12.007	6.223	6.566	7.254	3.359	3.585	4.026	1.695	1.824	2.107	0.696	0.760	0.909			
	6	12.566	13.078	14.146	7.662	8.030	8.817	4.393	4.638	5.169	2.474	2.626	2.957	1.314	1.400	1.596	0.567	0.612	0.712

Table 1. Asymptotic critical values for the tests of seasonal integration and cointegration.

(A) Data generating process with *seasonal integration*			q_1					
			0	0.025	0.050	0.075	0.100	0.500
q_2=0	K=0	(1) level	5.52	13.21	39.15	66.79	83.43	100.00
		(2) prefiltered	5.01	11.54	34.64	61.05	78.35	99.76
	K=1	(3) level	0.33	0.94	4.69	13.28	24.53	94.38
		(4) prefiltered	0.25	0.75	3.53	10.21	19.30	77.34
q_2=0.5	K=0	(1) level	0.04	0.16	1.66	8.32	21.73	99.11
		(2) prefiltered	5.05	10.73	31.37	56.98	75.23	99.73
	K=1	(3) level	0.00	0.00	0.03	0.24	1.12	64.34
		(4) prefiltered	0.23	0.63	3.01	8.90	17.10	76.36

(B) Data generating process with *seasonal cointegration*			q_1					
			0	0.025	0.050	0.075	0.100	0.500
q_2=0	K=0	(1) level	5.52	10.68	25.80	41.32	53.22	92.14
		(2) prefiltered	5.01	9.16	21.93	35.64	46.20	78.55
	K=1	(3) level	0.33	0.69	1.48	2.14	2.73	5.33
		(4) prefiltered	0.25	0.52	1.05	1.71	2.16	4.04
q_2=0.5	K=0	(1) level	0.04	0.16	1.47	5.90	12.85	74.43
		(2) prefiltered	5.05	8.61	20.20	33.52	44.14	78.21
	K=1	(3) level	0.00	0.00	0.00	0.01	0.02	0.09
		(4) prefiltered	0.23	0.48	1.00	1.57	2.03	3.94

Table 2. Percentage rejection frequencies for the Monte Carlo simulations of section 6.

		$\lambda=\pi/6$		$\lambda=\pi/3$		$\lambda=\pi/2$		$\lambda=2\pi/3$		$\lambda=5\pi/6$		$\lambda=\pi$		all λ's	
		m=10	m=15	m=10	m=15	m=10	m=15	m=10	m=15	m=10	m=15	m=10	m=15	m=10	m=15
GE-FR	K=0	*3.543*	*3.046*	*4.128*	*3.341*	*4.459*	*3.585*	*3.131*	*2.451*	*2.232*	*1.802*	*1.615*	*1.167*	*19.108*	*15.391*
	K=1	*0.781*	*0.653*	*0.936*	*0.709*	*0.823*	*0.647*	*0.641*	0.522	0.417	0.360	0.134	0.114	*3.732*	*3.006*
GE-IT	K=0	*3.566*	*3.242*	*7.177*	*6.123*	*2.905*	*2.222*	*3.422*	*2.656*	*3.482*	*2.777*	*1.705*	*1.224*	*22.257*	*18.244*
	K=1	*0.937*	*0.783*	*1.334*	*0.982*	0.367	0.301	*0.677*	0.554	*0.632*	0.501	0.110	0.086	*4.057*	*3.206*
GE-SP	K=0	*3.681*	*3.647*	*4.852*	*4.066*	*5.737*	*4.861*	*2.662*	*2.072*	*3.746*	*2.798*	*1.697*	*1.239*	*22.375*	*18.684*
	K=1	*0.820*	*0.722*	*0.895*	*0.710*	*1.082*	*0.838*	*0.597*	0.482	0.517	0.424	0.204	0.165	*4.116*	*3.339*
FR-IT	K=0	*3.268*	*2.756*	*4.051*	*3.282*	*2.958*	*2.246*	*3.249*	*2.399*	*3.061*	*2.366*	*0.993*	*0.763*	*17.580*	*13.813*
	K=1	*0.609*	0.534	*0.731*	*0.598*	0.390	0.314	0.308	0.246	0.458	0.368	0.079	0.064	*2.574*	*2.124*
FR-SP	K=0	*3.013*	*2.552*	*3.233*	*2.446*	*3.271*	*2.541*	*3.109*	*2.340*	*3.460*	*2.536*	*0.879*	0.709	*16.965*	*13.125*
	K=1	0.291	0.285	0.272	0.236	0.356	0.305	0.546	0.420	0.379	0.301	0.212	0.171	*2.057*	1.719
IT-SP	K=0	*3.197*	*2.653*	*6.862*	*6.104*	*3.308*	*2.516*	*2.989*	*2.261*	*3.531*	*2.592*	*1.004*	*0.772*	*20.890*	*16.897*
	K=1	0.363	0.327	*1.161*	*0.906*	0.500	0.397	0.347	0.289	0.240	0.207	0.212	0.172	*2.824*	*2.299*
GE-FR-IT	K=0	*4.354*	*3.858*	*8.457*	*7.549*	*4.929*	*3.946*	*4.433*	*3.435*	*3.874*	*3.106*	*1.816*	*1.318*	*27.863*	*23.211*
	K=1	*1.357*	*1.146*	*1.666*	*1.252*	*1.084*	0.860	*0.994*	0.815	0.902	0.730	0.218	0.179	*6.221*	*4.983*
	K=2	0.399	0.350	0.282	0.232	0.231	0.189	0.305	0.243	0.252	0.224	0.073	0.058	*1.542*	1.295
GE-FR-SP	K=0	*4.267*	*4.094*	*5.373*	*4.435*	*7.126*	*6.538*	*4.296*	*3.363*	*4.101*	*3.112*	*1.853*	*1.360*	*27.015*	*22.901*
	K=1	*1.169*	*1.027*	*1.146*	0.896	*1.330*	*1.049*	*1.335*	*1.110*	0.844	0.703	0.295	0.243	*6.119*	*5.028*
	K=2	0.291	0.284	0.157	0.144	0.245	0.207	0.445	0.339	0.306	0.253	0.073	0.062	*1.517*	1.289
GE-IT-SP	K=0	*4.439*	*4.287*	*10.605*	*9.669*	*6.548*	*5.566*	*4.305*	*3.437*	*4.422*	*3.336*	*1.950*	*1.426*	*32.267*	*27.720*
	K=1	*1.182*	*0.997*	*1.545*	*1.156*	*1.340*	*1.042*	*1.168*	*1.017*	0.792	0.640	0.325	0.262	*6.350*	*5.114*
	K=2	0.139	0.132	0.136	0.127	0.190	0.154	0.318	0.258	0.150	0.133	0.094	0.072	1.027	0.877
FR-IT-SP	K=0	*3.784*	*3.269*	*7.177*	*6.374*	*4.105*	*3.232*	*3.773*	*2.802*	*4.048*	*3.009*	*1.216*	0.944	*24.103*	*19.631*
	K=1	0.849	0.765	*1.406*	*1.117*	0.832	0.688	0.691	0.545	0.703	0.575	0.269	0.219	*4.750*	*3.908*
	K=2	0.201	0.193	0.225	0.194	0.286	0.239	0.143	0.123	0.206	0.175	0.056	0.046	1.117	0.970
GE-FR-IT-SP	K=0	*4.972*	*4.792*	*10.983*	*10.094*	*8.170*	*7.677*	*5.454*	*4.387*	*4.776*	*3.644*	*2.026*	*1.488*	*36.381*	*32.084*
	K=1	*1.617*	*1.386*	*1.796*	*1.363*	*1.631*	*1.293*	*1.570*	*1.328*	1.140	0.938	0.386	0.315	*8.140*	*6.623*
	K=2	0.568	0.512	0.372	0.320	0.465	0.388	0.598	0.466	0.489	0.430	0.148	0.119	*2.641*	2.235
	K=3	0.121	0.119	0.089	0.087	0.168	0.136	0.143	0.122	0.129	0.115	0.053	0.045	0.703	0.623

Table 3. Results of the tests for seasonal cointegration. The figures shaded and in italics correspond to rejection of the null hypothesis at 5% significance level.

VAR TEST OF SEASONAL COINTEGRATION							UC TEST OF SEASONAL COINTEGRATION						
	frequency $\pi/2$		5% c.v.	frequency π		5% c.v.		frequency $\pi/2$		5% c.v.	frequency π		5% c.v.
H_0	VAR(5)	VAR(8)		VAR(5)	VAR(8)		H_0	$m=5$	$m=8$		$m=5$	$m=8$	
$R=0$	63.9	58.5	56.4	38.4	44.8	24.3	$K=0$	3.45	2.91	1.69	1.30	0.98	1.00
$R\leq 1$	31.0	21.6	30.9	15.2	12.5	12.5	$K=1$	0.86	0.75	0.93	0.12	0.13	0.38
$R\leq 2$	5.5	0.6	13.2	1.9	1.7	3.9	$K=2$	0.20	0.20	0.38	0.05	0.06	0.12

Table 4. VAR vs UC tests of seasonal cointegration on the quarterly series of industrial production of France, Italy and Spain.

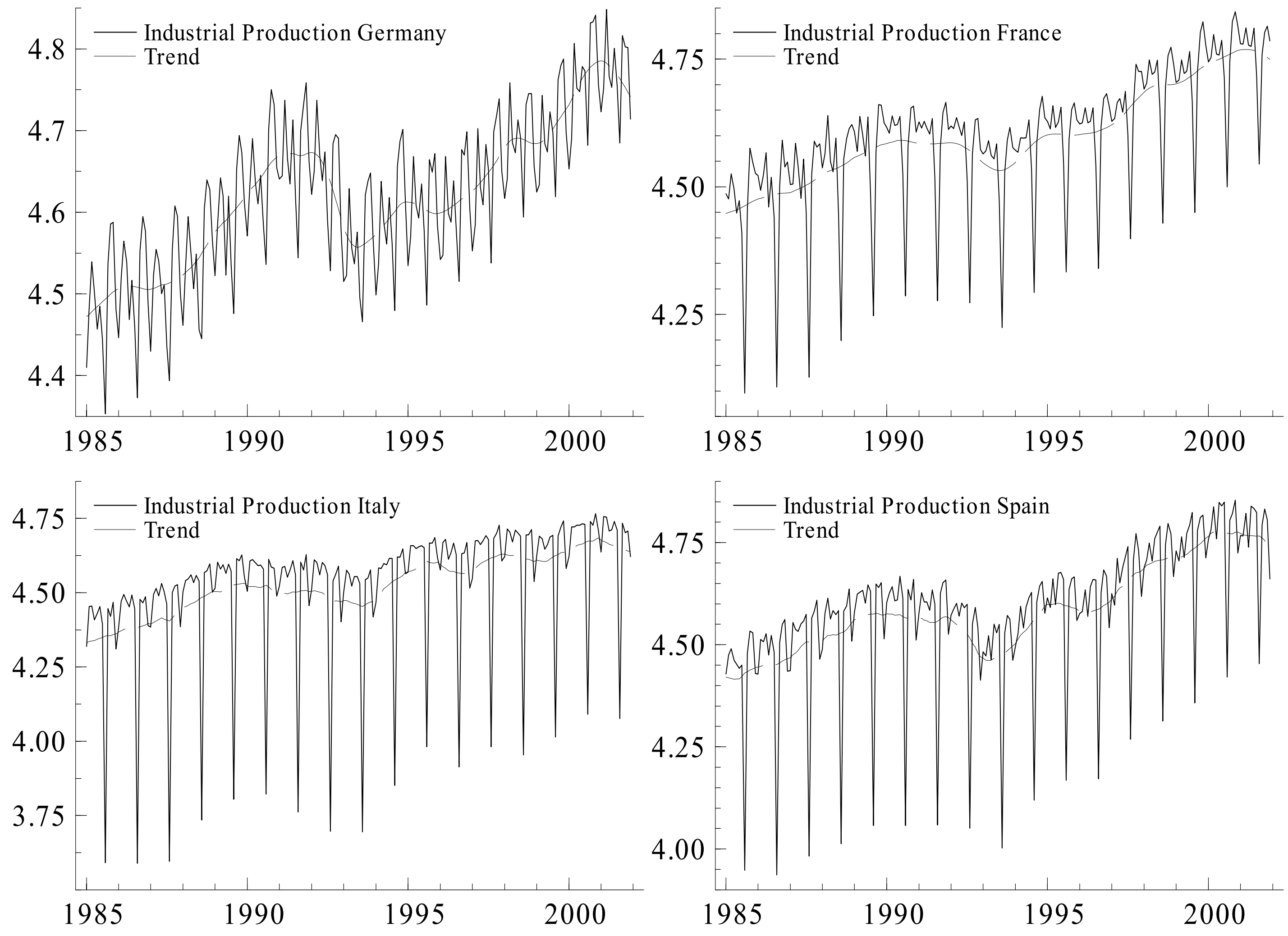

FIGURE 1. Index of Industrial Production for Germany, France, Italy and Spain, 1985-2001; source: Eurostat.

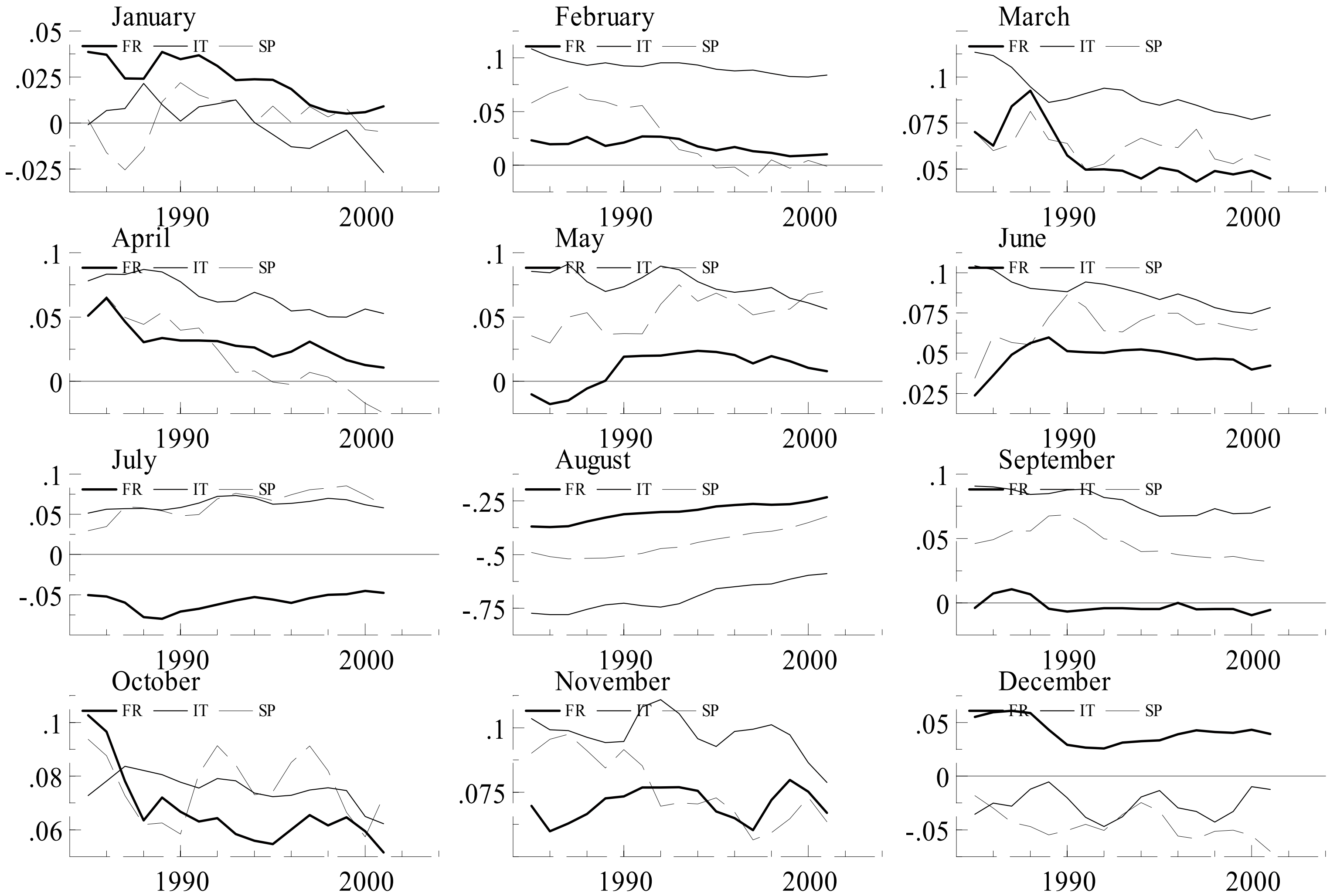

FIGURE 2. Extracted seasonal components from the Basic Structural Model.

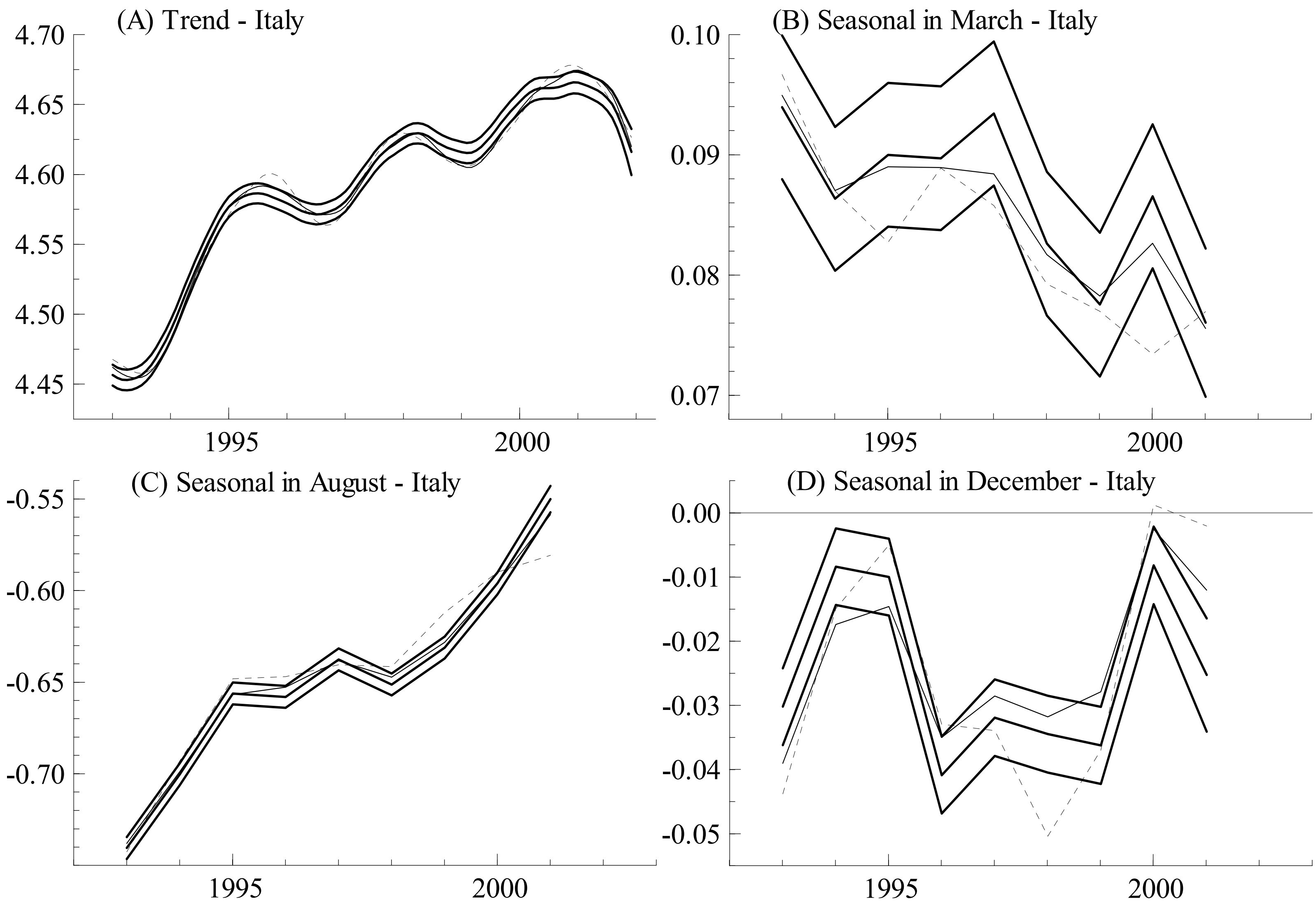

FIGURE 3. Extracted trend and seasonal components (March, August and December) for Italy. The thick lines are the estimated components (with 90% confidence intervals) in the model with seasonal cointegration, while the thin solid and the dashed lines correspond to the multivariate unrestricted model and the univariate model respectively.