Dhaka Univ. J. Sci., 51(2): 269-278 2003(July)

Application of Local Influence Diagnostics to the Linear Logistic Regression Models

Monzur Hossain* **and M. Ataharul Islam**
Department of Statistics, University of Dhaka

Received on 25.01.2002. Accepted for Publication on 11.06.2002

Abstract

This paper focuses the development of the diagnostics for the perturbations of case-weights and explanatory variables (one or more) in a linear logistic regression model. The effect of specific perturbation scheme on the estimation of parameters is also assessed. In addition, the interpretation of the value of curvature diagnostics is highlighted in this paper. This paper also demonstrates and extends the utility of the diagnostics for dichotomous outcome variables. For illustration, a sub-set of the Framingham Heart Study data set is used.

Key words and phrases: *Local influence, diagnostics, logistic regression model, and curvature diagnostics.*

1. Introduction

The linear logistic regression model is considered as one of the most important and widely applicable techniques in analyzing categorical outcome variables. To assess the fit of a model, it is necessary to identify the influential elements. Cook (1986) developed some local influence diagnostics procedures for linear regression models. Extension of the Cook's approach are made for different models (see Weissfeld, 1990; Weissfeld and Scneider, 1990 (1990a); Escobar and Meeker, 1992; others). However, no attempt has been made so far in order to provide a detailed and clear understanding about the extension of Cook's approach for the linear logistic regression models.

The application of the diagnostics is widely discussed by researchers for the linear regression, the Weibull and other parametric regression models. In this paper, an attempt is made to extend the procedures of diagnostics for the linear logistic regression models. Although a brief discussion on the use of the diagnostics is provided for the generalized linear models by Thomas And Cook (1989), a detailed extension of the diagnostics for the logistic regression models is necessary to understand the extent and pattern of influence on the estimates of interest. This paper demonstrates and extends the utility of the diagnostics for dichotomous outcome variables. Pregibon (1981) demonstrated an approach to identify the outliers and their effect on the maximum likelihood fit of a logistic regression model.

* Assistant Director, Bangladesh Bank, Motijheel, Dhaka-1000, Bangladesh. E-mail: monzur_h@yahoo.com

In this paper it is shown that the local influence techniques have a clear and concise advantage and flexibility over the global influence. The case-deletion approach is an example of global influence since it causes major perturbations of a model. It is noteworthy that the influence of a group of cases or variables cannot be

detected by a global influence method. The global influence method also fails to identify the nature of the influential elements. This paper focuses the development of the diagnostics for the perturbations of case-weights, explanatory variables (one or more) in a linear logistic regression model. The effect of perturbations on the estimates of parameters is also assessed. In addition, the interpretation of the value of curvature diagnostics is highlighted in this paper.

2. The Logistic Regression Model

We consider the logistic regression model of the dichotomous outcome variable Y taking values 1 and 0 with probabilities π_i and $1-\pi_i$ respectively (see Hosmer and Lemehow;1989) as

$$\pi_i = \Pr(Y_i = 1 | X = X_i) = \frac{e^{X_i\beta}}{1+e^{X_i\beta}}$$

and

$$1-\pi_i = \Pr(Y_i=0|X=X_i) = \frac{1}{1+e^{X_i\beta}} \qquad (2.1)$$

where X_i is the ith row of X and β be the vector of parameters. The unperturbed log-likelihood function of the logistic regression model is defined by

$$L(\beta) = \sum_{i=1}^{n}\left[Y_i X_i\beta - \log(1+e^{X_i\beta}) \right]. \qquad (2.2)$$

Introducing different types of perturbation schemes to (2.2) we can assess the local influence on the parameter estimates as well as influential elements can be detected which is discussed in section 3. The maximum likelihood estimates can be obtained by maximizing log-likelihood for which the score vector is

$$U(\hat{\beta}) = \frac{\partial L(\beta)}{\partial \beta_j} = 0$$

$$\Rightarrow \frac{\partial L(\beta)}{\partial \beta_j} = \sum_{i=1}^{n} (y_i - \hat{\pi}_i)x_{ij} = 0;\ j = 0,1,\ldots\ldots..,p \qquad (2.3)$$

and the information matrix is defined by $I(\beta) = -\frac{\partial^2 L(\beta)}{\partial\beta\partial\beta'} = X'QX$; where Q= diag $\pi_i(1-\pi_i)$. The solution to the likelihood equations is obtained using a numerical iterative method such as Newton Raphson method. The following set of iterative equations is solved for estimating β:

$$\beta^{t+1}=\beta^{t}+I(\hat{\beta})^{-1}U(\hat{\beta}),\ t=0,1,2,..... \tag{2.4}$$

3. Local Influence Technique

Let $L(\beta|\omega)$ be the log-likelihood for the perturbed data where ω be a $n\times 1$ vector of small perturbations and let $\hat{\beta}_{\omega}$ be the maximum likelihood estimate from the perturbed data. Let ω_0 represents null perturbation so that $L(\beta|\omega_0) = L(\beta)$. For a unit direction vector u, Cook(1986) defined curvature diagnostic as

$$C(u)=2\ |u^{\prime}\ Hu| \tag{3.1}$$

where u is the eigen vector component of H and (i,j)th element of H is defined by $H=\left[\frac{\partial^2 L(\hat{\beta}\mid\omega)}{\partial\omega_i\omega_j}\right]_{n\times n}$. The matrix H can be more easily computed by using the relation

$H=\Delta^{\prime}\ I^{-1}\ \Delta$ evaluated at ω_0 and $\hat{\beta}$ where $I^{-1}=I(\hat{\beta})^{-1}$ is the variance-covariance matrix and Δ matrix of order $(p+1)\times n$ is defined by

$$\Delta=\frac{\partial^2 L(\beta\mid\omega)}{\partial\beta\,\partial\omega} \tag{3.2}$$

evaluated at ω_0 and $\hat{\beta}$.

The maximum curvature diagnostic C_{max} assesses local influence on the parameter estimates and it is obtained from (3.1) by considering eigen vector u_{max} of the influence matrix H corresponding to the largest eigen value. The actual effect of the locally influential elements can be determined by perturbing the data in the direction indicated by u_{max}. It also gives the measure of local change in the estimates of regression coefficients as measured by the likelihood displacement.

The likelihood displacement for assessing the influence of ω is defined by

$$D(\omega)=2\{L(\hat{\beta})-L(\hat{\beta}|\omega)\}$$

which compares $\hat{\beta}_{\omega}$ and $\hat{\beta}$ with respect to the unperturbed log-likelihood. Escobar and Meeker (1992) showed that the likelihood displacement could be approximated by taking half of the diagonal elements of H for perturbing a single case i.e. $D(\omega) \approx \frac{1}{2} H_{ii}$. (3.3)

By plotting $\frac{1}{2} H_{ii}$ against case number, we can also detect the most influential case(s) along with other influential cases.

The local influence diagnostic procedures particularly depend on the range of scale of the perturbation of interest. There is no standard rule for the range of ω. Usually it may range between 0 to 2 (Cook, 1986) with a null perturbation $\omega_i = 0$ for additive perturbation and $\omega_i = 1$ for multiplicative perturbation. The perturbation range depends on the nature of the selected variables and the underlying models of interest. It is noteworthy that successful diagnostics depend on the choice of ω_i.

4. Local Influence Diagnostic Procedures

The diagnostic procedures for the perturbation of case-weights, all explanatory variables with special case of individual and more than one explanatory variables, and individual coefficient of the logistic regression model are proposed in this section. The diagnostics for the minor perturbations of the above mentioned elements are used to observe the changes on the estimates and to detect influential cases.

4.1 Case-weights

Let $\omega'=(\omega_1,........,\omega_n)$ be a vector of weights while $\omega_0'=(1,........,1)$ represents $n \times 1$ vector of null perturbation. To assess the influence for the case-weight perturbations, the perturbed log-likelihood is defined by

$$L(\beta|\omega) = \sum_{i=1}^{n} \omega_i \left[Y_i X_i \beta - \log(1 + e^{X_i\beta}) \right]. \qquad (4.1)$$

Let we consider that the intercept is included in the model, so the score-vector $U(\hat{\beta}_\omega)$ is solved as

$$\frac{\partial L(\beta \mid \omega)}{\partial \beta_j} = \sum_{i=1}^{n} \omega_i (Y_i - \hat{\pi}_i) X_i = 0 \text{ ; where } \hat{\pi}_i = \frac{e^{x_{ij}\hat{\beta}_j}}{1 + e^{x_{ij}\hat{\beta}_j}}; j = 0,1,.....,p \qquad (4.2)$$

The (i,j)th elements of Δ matrix of order $(p+1) \times n$ is given by

$$\Delta_{ij} = \frac{\partial^2 L(\beta \mid \omega)}{\partial \beta_j \partial \omega_i} = \left[y_i x_{ij} - \frac{e^{x_{ij}\beta_j}}{1 + e^{x_{ij}\beta_j}} x_{ij} \right] = (y_i - \pi_i) x_{ij} \text{ ; } j = 0,1,...,p \text{ ; } i = 1,2,....,n \qquad (4.3)$$

evaluated at ω_0 and $\hat{\beta}$. The influence matrix H of order $n \times n$ is obtained by following the relation

$H = \Delta' I^{-1}\Delta$ under the perturbation scheme. Thus the maximum curvature C_{max} can be easily computed as defined in (3.1).

4.2 Explanatory Variables

In this section, we consider a general method for perturbing the whole design matrix X i.e. for the modification of all explanatory variables. Let β be a vector of parameters and the perturbed log-likelihood $L(\beta|\omega)$ is obtained by replacing explanatory variables X with Z which is defined as

$$Z = X + WV \tag{4.4}$$

where $W = (\omega_{ij})$ is a $n\times(p+1)$ matrix of perturbations and the scaling factor $V = \text{diag}(v_1, v_2, \ldots\ldots, v_{p+1})$ is used to convert the perturbations ω_{ij} to the appropriate size and units so that $\omega_{ij} v_j$ is consistent with the ijth element of X. Under this perturbation scheme, the perturbed log-likelihood will take the form

$$L(\beta \mid \omega) = \sum_i \left[y_i \left(x_{ij} + \omega_{ij} v_j \right) \beta_j - \ln\left(1 + e^{\left(x_{ij} + \omega_{ij} v_j \right)\beta_j} \right) \right] \tag{4.5}$$

where i=1,2,....,n and j=0,1,.......,p and to obtain the estimates of the parameters, we solve the following first differential iterative equations by Newton-Raphson Method:

$$\begin{aligned} &\frac{\partial L(\beta \mid \omega)}{\partial \beta_j} = 0 \Rightarrow \sum_i (y_i - \hat{\pi}_{\omega i})(x_{ij} + \omega_{ij} v_j) = 0 \\ &\Rightarrow \sum_i (y_i - \hat{\pi}_{\omega i}) z_{ij} = 0; \quad \text{where } \hat{\pi}_{\omega i} = \frac{e^{(x_{ij} + \omega_{ij} v_j)\hat{\beta}_j}}{1 + e^{(x_{ij} + \omega_{ij} v_j)\hat{\beta}_j}}. \end{aligned} \tag{4.6}$$

To compute the curvature diagnostics, we partition the Δ matrix of order $(p+1)\times n(p+1)$ into (p+1) sub-matrices as $\Delta = (\Delta_1, \Delta_2, \ldots\ldots\ldots, \Delta_{p+1})$ (4.7)

where kth (k=1,2,....,p+1) sub-matrix Δ_k of order $(p+1)\times n$ is defined by

$$\begin{aligned} \Delta_k &= \frac{\partial^2 L(\beta \mid \omega)}{\partial \beta_j \partial \omega_{ik}} \\ &= \frac{\partial}{\partial \omega_{ik}} \left[\sum_i (y_i - \pi_{\omega i})(x_{ij} + \omega_{ij} v_j) \right] \\ &= \sum_i \left((y_i - \pi_{\omega i}) - \pi_{\omega i}(1 - \pi_{\omega i}) z_i \beta_j \right) v_j; \; i = 1,2,\ldots,n \text{ and } j = 0,1,\ldots.,p \end{aligned} \tag{4.8}$$

evaluated at ω_0 and $\hat{\beta}$. Thus the influence matrix H is calculated by considering Δ of (4.7) through the relation $H = \Delta' I^{-1} \Delta$. In this application, H is a very large $n(p+1)\times n(p+1)$ matrix.

a) Individual Explanatory Variable

The above results can be restricted to the situations where only one explanatory variable is of interest by setting $v_j = 0$ for the unperturbed variables. In particular, let only the first column of X is perturbed.

Thus $v_j = 0$ for $j \neq 1$ and the influence matrix is defined by $H = \Delta'_1 I^{-1} \Delta_1$ (4.9)

where Δ_1 is a $(p+1) \times n$ matrix and C_{max} is calculated by obtaining u_{max} from H.

b) More than One Explanatory Variables

The procedure of calculating the maximum curvature diagnostic C_{max} is discussed in this section for partial perturbation of the whole design matrix. This is the extension of situation (a) discussed above for perturbing two or more but less than p+1 explanatory variables. For example, the procedure of calculating the curvature diagnostics is discussed for the following situations: (1) Suppose, we want to perturb the first two columns of Z. Thus $v_j = 0$ for $j\neq1,2$ and Δ is partitioned as $\Delta = (\Delta_1,\Delta_2)$; (2) Similarly, if we want to perturb columns 1, 2 and 3, so Δ will be of order $3\times3n$ that is $\Delta = (\Delta_1, \Delta_2, \Delta_3)_{3\times3n}$.

4.3 Individual Coefficient

For examining the sensitivity of the ith coefficient to each of the perturbation scheme discussed above, a curvature diagnostic is extended for the logistic regression model on the basis of Cook's (1986) method suggested for linear regression. First we rearrange the columns of X as $X = (X^{(1)}, X^{(2)})$ so that the first column $X^{(1)}$ corresponds to the coefficient β_1 of interest. The curvature is defined by

$$C_u(\beta_1) = 2\left|u'\Delta'\left(I^{-1} - \ell\right)\Delta u\right| \tag{4.10}$$

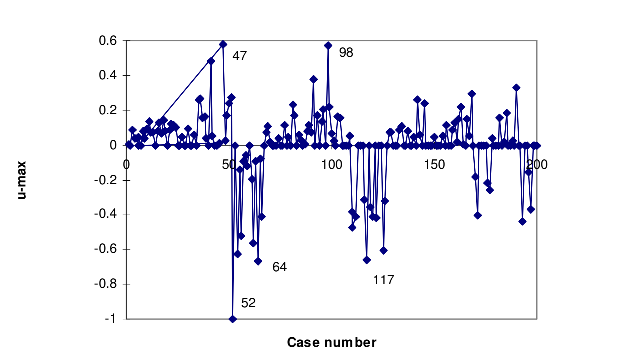
where u is the eigenvector of $\Delta'\left(I^{-1} - \ell\right)\Delta$ corresponding to the largest eigenvalue and the symbols denote usual meaning except ℓ which is given by $\ell = \begin{pmatrix} 0 & 0 \\ 0 & P^{-1} \end{pmatrix}$ (4.11)

where P^{-1} is obtained from the partition of the information matrix $I = \begin{pmatrix} M & N \\ O & P \end{pmatrix}$.

Under specific perturbation scheme, the sensitivity of the other coefficients can be investigated by following the above procedure.

5. Illustration

The application of the proposed diagnostics is shown by fitting a logistic regression model to the Framinghalm Heart Study data sub-set which consists of a random sample of 200 individuals out of 669 individuals (taken from Kahn et al. 1989). The response is binary, presence or absence of coronary heart disease (CHD), and explanatory variables are age, systolic blood pressure, diastolic blood pressure, cholesterol, Framingham relative weight (FRW), and cigarette smoked per day(CIG). To identify the influential cases and to assess influence on the estimate of β, we first use the local influence diagnostics for the null perturbations of the case-weights. Index plot of U_{max} (Fig.-1) and value of 1/2 of H_{ii} (Table 4.1) demonstrates that the case 52 is the most influential along with the cases 41, 47, 68, 98,117, 193 etc.



Since the case 52 has larger variance component, the greatest local change in β essentially depends on the weight given to case 52. The next attempt has been made only to perturbing the most influential case 52 and we consider ω_{52}=0.8. The influential nature of the cases remain unchanged for this minor perturbations. The effect for the deletion of the case 52 and other influential cases on the estimates is also assessed in Table 4.2 by maximum curvature diagnostic and components of u_{max} are shown in Table 4.3. Deletion of the cases 41, 47 has changed the influential nature of the case 52 and C_{max} value is found to be comparatively higher for deletion of the cases 41, 47 and 52.

In Table 4.1, the application of the diagnostics for the perturbation of an individual explanatory variable is also demonstrated to see the effect of minor perturbations on the estimates of parameters and on the nature of the influential cases. Under the perturbation scheme, we decrease the value of CIG into 5 for the following influential cases: 41, 47, 52, 54, 64, 98, 117, 125, 171, 190, 193 and 197 because the number of cigarette smoked per day is at least 20 by each of the cases. For such perturbations, the C_{max} value is found to be 4.77, which is much lower than for other perturbations and indicates the extent of influence on the estimates. It is also evident from the components of u_{max} that all the influential cases have changed their direction of deviation. Thus the perturbation scheme considered here is found to be influential to the model fitting and diagnostics suggest that the joint influence or 'masking effect' of the cases is due to their higher number of cigarette smoked per day.

Major perturbation such as case deletion is assessed by Curvature diagnostics and in this situation u_{max} is also used to indicate the direction of variation of the cases (Table 4.2 and 4.3). Although the traditional diagnostics for case deletion such as Cook's D, DFBETA and DFFITS etc. are not used here, we can derive conclusion from the results that local influence diagnostics can be used to assess the global influence on the estimates.

Under the null perturbations and the perturbations of explanatory variable CIG, the sensitivity of coefficients is also examined by the Curvature diagnostics. In both situations, coefficient β_6 shows sensitivity as the C_{max} value is higher (Table 4.4).

Here usually the question arises about the value of C_{max} in order to identify the influence contained in data. The decision can be taken from empirical experience of using diagnostics and understanding the nature of the observations. However, there is arbitrariness in making decision on the basis of curvature diagnostics. This issue is addressed in this paper and an alternative approach is proposed by using Chi-square calibration of C_{max}. The alternative approach of the Chi-square calibration for C_{max} is discussed below.

Escobar and Meeker (1992) showed that $D(\omega)$ can be expressed approximately as ½ $u'Hu$ and suggested that if $D(\omega)>\chi^2_{(1-\alpha,p)}$, the perturbation ω results in a $\hat{\beta}_\omega$ that lies outside of the null perturbation approximate likelihood-ratio-based $100(1-\alpha)\%$ confidence region for β . Since $C_{max}=2u'Hu$, from the above results we may conclude that $C_{max}>4\chi^2_{(1-\alpha,p)}$ indicates influence on the estimates which provides approximate likelihood-ratio-based $100(1-\alpha)\%$ confidence region for β.

On the basis of the chi-square calibration, we can summarize the results of C_{max} in Tables 4.1, 4.2 and 4.4. For the C_{max} values in Table 4.1 and Table 4.2, consider $\alpha=0.05$ and $p=7$ for which the χ^2 value is 2.167. Since the C_{max} values for the case-weight perturbations are greater than 8.67 in Table 4.1, so the perturbation ω results in a $\hat{\beta}_\omega$ that lies outside of the null perturbation which provides 95% confidence region for β. Similarly, the C_{max} values for the perturbation of the explanatory variable CIG indicates no influence on the estimates (Table 4.1), while C_{max} values in Table 4.2 indicates influence on the estimates for the deletion of the selected cases.

The sensitivity of the coefficients for the specific perturbations is also examined by the proposed chi-square calibration for C_{max}. For the C_{max} values in Table 4.4, we consider $\alpha=0.05$ and $p=1$ for which the value of χ^2 is 0.0039. Since C_{max} values of β_1, β_5 and β_6 are greater than 0.0136, these coefficients are found sensitive to the model fitting for the null perturbation. Similarly all the coefficients individually shows sensitivity to the model fitting for the perturbation of CIG. But for this perturbation scheme, the overall effect was found less influential on the estimates (Table 4.1).

5. Summary and Conclusion

The application of the proposed diagnostics for the logistic regression model under different perturbation schemes has been discussed in this paper. The diagnostics can be employed in order to detect influential cases that produce the greatest local changes on the estimates. The advantages of local influence analysis over global influence analysis are also explored. From our empirical results, we see that locally influential cases are also globally influential. This paper also provides insight into the interpretation of the value of curvature diagnostic on the basis of chi-square calibration. It is noteworthy that relatively larger values of curvature diagnostic indicate the influence of the perturbations. This is also investigated by the chi-square calibration. On the basis of the results obtained from the curvature diagnostics for the perturbation of explanatory variable, CIG reveals that increased number of cigarettes can influence the estimates. The overall effect of these perturbations has less influence on the estimates while individually the coefficients show sensitivity to the model fitting. Deletion of the influential cases resulted in small changes in the parameter estimates as well as in the value of curvature diagnostics. These changes are not large enough to affect the inferences drawn from the analysis.

Table 4.1 Influence eigen vector components, half of the H_{ii} values and estimated parameters for null perturbation, case-weight perturbation (w_{52}=0.8) , and perturbation of explanatory variable CIG.

Case #	Eigen vector component			Half of H_{ii} values			Parameter estimates		
	U_1 (null perturb-ation)	U_2 (w_{52}=0.8)	U_3 (perturb-ation of CIG)	V_1 (null perturb-ation)	V_2 (w_{52}=0.8)	V_3 (perturb-ation of CIG)	For null perturbation	For w_{52}=0.8	For perturbation of CIG
41	0.48	0.46	0.11	0.02	0.02	-0.001	$\hat{\beta}_0$=-7.91	$\hat{\beta}_0$=-7.88	$\hat{\beta}_0$=-7.13
47	0.58	0.57	0.15	0.02	0.02	-0.003	$\hat{\beta}_1$=0.011	$\hat{\beta}_1$=0.011	$\hat{\beta}_1$=0.002
52	-0.99	-0.99	-0.14	0.09	0.09	-0.002	$\hat{\beta}_2$=0.001	$\hat{\beta}_2$=0.001	$\hat{\beta}_2$=0.002
54	-0.62	-0.63	-0.10	0.02	0.02	-0.001	$\hat{\beta}_3$=0.039	$\hat{\beta}_3$=0.039	$\hat{\beta}_3$=0.036
64	-0.66	-0.67	-0.13	0.03	0.02	-0.003	$\hat{\beta}_4$=0.011	$\hat{\beta}_4$=0.010	$\hat{\beta}_4$=0.011
98	-0.57	-0.67	0.11	0.03	0.03	-0.001	$\hat{\beta}_5$=0.001	$\hat{\beta}_5$=0.001	$\hat{\beta}_5$=0.002
117	-0.66	-0.66	-0.17	0.03	0.03	-0.002	$\hat{\beta}_6$=-0.010	$\hat{\beta}_6$=-0.011	$\hat{\beta}_6$=-0.032
125	-0.60	-0.60	-0.15	0.03	0.03	-0.001			
171	-0.40	-0.40	-0.16	0.01	0.03	-0.002			
190	-0.33	-0.32	0.46	0.01	0.01	0.006			
193	-0.44	-0.43	-0.17	0.00	0.00	-0.000			
197	-0.37	-0.37	-0.14	0.01	0.01	-0.003			
C_{max}							15.28	14.82	4.77

Table 4.2 : Change on M.L.Es with the ith case deleted.

Case i	M.L.Es with case i deleted							
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	C_{max}
None	-7.9115	0.0113	0.00108	0.0388	0.01095	0.00137	-0.0101	15.276
41	-6.7607	0.0265	-0.00798	0.0530	0.00791	-.01008	-0.0129	19.654
47	-6.8977	0.0252	-0.00795	0.0555	0.00853	-.01161	-0.0128	19.544
52	-6.7030	0.0275	-0.00815	0.0531	0.00765	-.01037	-0.0133	19.744
54	-6.9345	0.0308	-0.00900	0.0533	0.00687	-.00723	-0.0108	12.424
64	-7.1346	0.0326	-0.00984	0.0557	0.00710	-.00781	-0.0107	12.511
98	-7.6769	0.0276	-0.00680	0.0548	0.00737	-.00451	-0.0067	13.873
117	-7.7492	0.0243	-0.00634	0.0505	0.00918	-.00258	-0.0118	12.452
125	-7.4093	0.0218	-0.00557	0.0477	0.00908	-.00298	-0.0116	12.558

Table 4.3 : The components of u_{max} for the case i deleted.

Selected Case (i)	u_{1i} (41deleted)	u_{2i} (47 deleted)	u_{3i} (52 deleted)	u_{4i} (98 deleted)	u_{5i} (117 deleted)	u_{6i} (125 deleted)
41	-	0.515	0.47	0.456	0.408	0.40
47	0.72	-	0.70	0.56	0.53	0.52
52	0.31	0.312	-	-0.999	-0.999	-0.999
54	-0.97	-0.9519	-0.976	-0.675	-0.66	-0.66
64	-0.999	-0.999	-0.999	-0.71	-0.69	-0.68
98	0.276	0.778	0.7067	-	0.52	0.52
117	-0.926	-0.933	-0.921	-0.68	-	-0.65
125	-0.838	-0.8417	-0.834	-0.599	-0.59	-
171	-0.610	-0.618	-0.6088	-0.444	-0.415	-0.41
190	0.444	0.459	0.439	0.35	0.326	0.31
193	0.085	0.082	0.085	0.063	0.628	0.065
197	-0.534	-0.541	-0.532	-0.375	-0.36	-0.365

Table 4.4 : Examining the sensitivity of an individual coefficient with null perturbation and for the perturbation of variable CIG.

Coefficients	Value of C_{max} (for null perturbation)	Value of C_{max} (for perturbation of CIG)
β_1	0.27016	0.052948
β_2	0.00565	0.032853
β_3	0.01335	0.07266
β_4	0.01306	0.265725
β_5	0.12098	0.32563
β_6	14.7515	15.89744

References

Cook, R.D. (1986). Assessment of local influence, Journal of Royal Statistical Society, Series B; 48, 133-169

Escobar, LA and Meeker, W.Q. (1992), Assessing influence in regression analysis with censored data. Biometrics 48, 507-528

Hosmer, D. W. and Lemeshow, S. (1989). Applied logistic regression. John Wiley and Sons, New York.

Kahn, H. A. and Sempos, C. D. (1989). Statistical methods in epidemiology. Oxford University Press, New York.

Pregibon, D (1981). Logistic regression diagnostics. Annals of statistics, 9, 705-724.

Thomas, W. and Cook, R.D. (1989). Assessing influence on regression coefficients in generalized linear model. Biometrika 76, 741-749.

Weissfeld, L. A. and Schneider, H. (1990a). Influence diagnostics for normal linear model with censored data. Australian Journal of Statistics, 32, 11-20.

Weissfeld, L. A. and Schneider, H. (1990b). Influence diagnostics for the Weibull model fit to censored data, Statistics and Probability Letters, 9, 67-73.

Weissfeld, L. A. (1990). Influence diagnostics for the proportional hazards models. Statistics and Probability Letters, 10, 411-417.