

Econometric Estimation of Parameters of Preservation of Perishable Goods in Cold Logistic Chains

Miroslav Verbič

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Abstract

Paper discusses the parameters of preservation of perishable goods in cold logistic chains. The key parameters are the intensity of deterioration of goods, the conservation effect of perishable goods and the delay of activation of the conservation effect. The values of these parameters tell us the quantity of the product being deteriorated in the logistic chain and the extent to which the deterioration can be alleviated. Econometric estimation thus presents us with the quantity effects of preservation procedures, whereas the financial effects can be derived using the proper price categories in the calculation of the net present value or the annuity stream. In this way one can determine whether the implementation of preservation procedures is more rational than the purchase of attainable insurance policy.

Key Words: cold chains, Cold Chains Management, econometric estimation, intensity of deterioration, conservation effect, Input – Output analysis, Laplace transforms, MRP

JEL Classification: C13, C15, D21, M11

1 Introduction

Production and logistics facilities in the supply chain of perishable goods are situated between the origin and the supply market or in a part of the latter. Any changes in time-distance or temperature in the chain could increase the costs or cause the net present value of the activities and their added value in the supply chain to be perturbed. In reality, these perturbations can be robust (Bogataj, 1998; Bogataj and Bogataj, 2004). To control such perturbation in a system, one needs to be able to evaluate its effects on stability of perishable goods in supply chains and to implement appropriate measures to preserve the required quantity and quality of the goods. The preservation of stability in cold chains, i.e. the determination of conditions that need to be fulfilled in order to keep the behaviour of logistic chain within prescribed limits even after a robust perturbation and time delays, is the foremost task of logistics management of cold chains (Bogataj *et al.*, 2004, p. 1).

The purpose of the paper is to examine the parameters of preservation of perishable goods in cold logistic chains. The parameters to be estimated are the intensity of deterioration of goods at given levels of production or distribution in the supply chain and the conservation effect of perishable goods, dependent on the state of the system and activated with time delay. Since we are interested in the values of these parameters and their stability, the primary objective of the paper is to develop a framework for econometric estimation of parameters at different levels of production and distribution in supply chains. Therefore the global logistics management of cold chains and the contributions of leading researchers from this field are being presented in the following chapter. The third chapter discusses the preservation of perishable goods in cold logistic chains, while in the fourth chapter the foundations for econometric estimation of the aforementioned parameters of preservation are being developed. This is illustrated in detail with a numerical example of a food-processing company in the penultimate chapter. The final chapter concludes our work with some key findings.

2 Global Logistics Management of Cold Chains

Global Cold Chains Management (CCM) is a process of planning, implementing and controlling efficient and effective flow and storage of perishable goods, related services and information from one or more points of origin to the points of production, distribution and consumption in order to meet customers' requirements on a worldwide scale (Bogataj *et al.*, 2004, pp. 1-2). It is therefore a process of integrating the existing business activities with special activities for perishable goods conservation along value chains, where more suppliers of certain raw materials or more productions cells of certain semi-products appear in order to

create value for the end user. Cold Chains Management is one of the main viewpoints of Supply Networks Management, which help companies to develop the visibility of flows in a network and to understand management tools needed to supply network logistics. It is essential in the global marketplace to get the right goods at the right time to the right place (*cf.* Baldiwala, 2001); to do the things right, which is the essence of efficiency, and to do the right things, which is the core of successfulness. Visibility is especially important in cold chains and other chains of perishable goods, where the quality and quantity of the product at the end of the supply chain on the required level is maintained by controlling the temperature already in production cells and particularly in transportation and by adding preservatives.

Such preservation procedures are gaining its importance, since the global market of refrigerated products and prepared meals is expanding rapidly in the context of globalization and integration processes due to decreasing tariffs, permanent improvements in transport efficiency and developments in communication and information technology. Nonetheless, the expanding trade in perishable goods, which is under the influence of perturbations in the form of time delays and temperature deviations, requires regulation of quality to protect final customers from decreases in quality of such goods (Bogataj *et al.*, 2004, pp. 2-3). The mutual activity of both types of perturbations can easily decrease the quality of perishable goods to the level, which can be health-hazardous for the consumer. The corresponding EU regulations and directives have led to the introduction of cold traceability, which requires tools and equipment to trace miscellaneous groups of perishable goods that are being produced and distributed under different cooling requirements.

To manage the chain flows and to achieve optimal control in the procedures of cooling and improvement, R. W. Grubbström and his associates are developing since 1960s a specific approach, combining Laplace transforms, Input – Output analysis and Material Requirements Planning (MRP), but basically limited to production and inventories (*cf.* Grubbström, 1967; Grubbström and Molinder, 1994; Grubbström, 1996; 1998; Grubbström and Tang, S.a.). M. Bogataj and L. Bogataj with their associates have expanded Grubbström's approach in the time domain and frequency space to the area of distribution and recently also to the area of recycling (*cf.* Bogataj, 1994; 1996; 1998; Bogataj and Bogataj, 2004). As they found out (Bogataj in Bogataj, 2001; 2004), only the analysis in the space of square integrable Lebesgue functions (the space of L^2 -functions or the L^2 -space) is mathematically correct. Nevertheless, the purpose of research work presented in this paper is much more constrained, since it only covers partial discussion of cold logistic chains with additional evaluation of parameters of preservation in distance functions (*cf.* Bogataj and Bogataj, 2001; Bogataj *et al.*, 2004), where we focus our attention to econometric estimation of these parameters.

3 Preservation of Perishable Goods in Cold Logistic Chains

The influences of transportation and production parameters and their perturbations on the chain performance, especially on the net present value or the annuity stream of supply chain operations, are relatively easily estimated by material requirements planning models, complemented with Input – Output approach in production, distribution and/or recycling formalization of a supply chain. The latter process, which in fact represents reverse logistics, is being an option hitherto. For a certain time unit the following expression has to hold (Bogataj *et al.*, 2004, pp. 3-4):

$$\mathbf{S} = (\mathbf{I} - \mathbf{H})\mathbf{P} - \mathbf{F}, \quad (1)$$

where \mathbf{S} is a vector of inventory per time unit; \mathbf{I} unit matrix; \mathbf{H} Input – Output matrix; \mathbf{P} vector of gross production per time unit; \mathbf{F} vector of deliveries per time unit; $\mathbf{H} \cdot \mathbf{P}$ internal demand per time unit and $(\mathbf{I} - \mathbf{H}) \cdot \mathbf{P}$ net production per time unit. Expression (1) is therefore a representation of flows where the dimension of the space, which describes the production and distribution is equal to the number of activity cells in a chain. The demand¹ \mathbf{D} has to meet the delivery \mathbf{F} with a certain delivery service level β , i.e. with a certain probability $p = 1 - \beta$ of running out of goods when faced with effective demand.

In the global integrated supply network the location of the activity cells influences storage and distribution costs, especially the response time in control actions (*cf.* Bogataj and Bogataj, 2001). Demand is stochastic by nature and disturbances appear at unexpected locations, at unexpected time and with unexpected magnitude. Evaluation of spatial interactions and influences of location on the uncertainty of supply in case of two sequential activity cells was undertaken in Bogataj and Horvat (1996) and Bogataj (1996; 1998; 1999) using classical location theory approaches. The analysis of perturbations of lead time in production and inventories, given by Grubbström (1967; 1996; 1998), was expanded by the use of the concepts such as uncertain supply and perturbations of lead time to study the entire global supply network (Bogataj, 1996; 1998; 1999; Bogataj and Bogataj, 2001). Such perturbations are gaining on its importance due to increasing distances between activity cells in the process of globalization. In such circumstances, *homo oeconomicus* is seeking a trade-off between weaknesses and advantages amid the optimization of the ongoing process of activities and the purchase of available insurance policy in case of perturbations.

¹ The final demand is exogenous in the presented logistic chain model of perishable goods.

If the process of optimization is chosen, then in the case of perishable goods in our supply chain at any of n stages the development of the state of the system² $x_k(t)$ is described by a set of first-order linear differential-delay equations (DDE) of the form (Bogataj *et al.*, 2004, p. 4):

$$\frac{d\bar{x}_k(t)}{dt} = \mathbf{A}_k \bar{x}_k(t) + \mathbf{B}_k(t) \bar{x}_k(t - \tau_k), \quad k = 1, \dots, n, \quad (2)$$

with the initial condition: $\bar{x}_k(\theta) = \bar{\varphi}(\theta)$, and with the condition of equality between the initial $-\tau_k \leq \theta \leq 0$

value at stage k , i.e. $\bar{x}_k(0) = \bar{x}_{k,0}$, and the final value at stage $k - 1$, i.e. $\bar{x}_{k-1}(f) = \bar{x}_{k-1,f}$: $\bar{x}_{k,0}(\theta) = \bar{x}_{k-1,f}(\theta)$. The vector of state of the system \bar{x}_k contains the following two

components: $\bar{x}_k(t) = \begin{bmatrix} x_{1,k}(t) \\ x_{2,k}(t) \end{bmatrix}$, where $x_{1,k}(t)$ is the quantity of good products at time t at stage

k and $x_{2,k}(t)$ is the quantity of deteriorated products at time t at stage k . Expression (2) contains three parameters of preservation of perishable goods, which are crucial in Cold Chains Management. Time delay τ_k represents the time that elapses at stage k before the conservation effect activates at that stage. The matrix of coefficients \mathbf{A}_k describes the intensity of deterioration of goods at stage k of production or distribution part of the supply chain. The matrix of coefficients $\mathbf{B}_k(t)$ presents the conservation effect of perishable goods at stage k that is activated with the delay τ_k and depends on the state of the goods in the system.

By combining expressions (1) and (2) for n stages of logistic chain, we obtain the following differential-delay equation (*cf.* Bogataj *et al.*, 2004, pp. 5-6):

$$\begin{bmatrix} \dot{\bar{x}}_p(t) \\ \dot{\bar{x}}_1(t) \\ \dot{\bar{x}}_2(t) \\ \vdots \\ \dot{\bar{x}}_n(t) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_n \end{bmatrix} \begin{bmatrix} \bar{x}_p(t) \\ \bar{x}_1(t) \\ \bar{x}_2(t) \\ \vdots \\ \bar{x}_n(t) \end{bmatrix} + \begin{bmatrix} \mathbf{I} - \mathbf{H} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{P}(t) \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{F}(t) \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{B}_n \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \bar{x}_1(t - \tau_1) \\ \bar{x}_2(t - \tau_2) \\ \vdots \\ \bar{x}_n(t - \tau_n) \end{bmatrix}, \quad (3)$$

² State of the system, presented in this paper, belongs to the state space of continuous functions (*cf.* Hale, 1977).

where each dimension of the balance vector $\bar{x}_p(t)$ equals the number of stages and the total quantity of goods at stage k equals the sum of good and deteriorated products:

$$\left[\begin{array}{ccccc|ccccc} -\mathbf{1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{1} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{1} \end{array} \right] = \left[\begin{array}{c} \bar{x}_p(t) \\ \bar{x}_{1,1}(t) \\ \bar{x}_{1,2}(t) \\ \vdots \\ \bar{x}_{n,2}(t) \end{array} \right].$$

In addition, expression (3) can be presented in the following compact form:

$$\dot{\hat{\mathbf{X}}}(t) = \hat{\mathbf{A}}\hat{\mathbf{X}}(t) + (\hat{\mathbf{I}} - \hat{\mathbf{H}})\hat{\mathbf{P}}(t) + \hat{\mathbf{B}}(t)\hat{\mathbf{X}}(t, \tau_1, \tau_2, \dots, \tau_n) - \hat{\mathbf{F}}(t), \quad (4)$$

where the initial condition: $\hat{X}_0(\theta) = \hat{\Phi}(\theta)$, and sequential conditions: $\bar{x}_{k,0}(\theta) = \bar{x}_{k-1,f}(\theta)$
 $-\max\{\tau_{kj}\} \leq \theta \leq 0$, $-\tau_k \leq \theta \leq 0$

have to be satisfied. Moreover, the mass-preservation condition has to be fulfilled as well: $\hat{\mathbf{G}}\hat{\mathbf{X}}(t) = 0; t \in [0, t_f]$, where $\hat{\mathbf{G}}$ is the matrix of preservation of the quantity of goods in compact presentation.

The expression (4) is correct in the absence of disturbances in temperature or in time delays of the system. However, these can appear at any stage k and can influence the deterioration matrix \mathbf{A}_k with effect $\Delta\mathbf{A}_k$. In such cases the supply chain needs to be additionally cooled and/or the products have to be additionally treated to maintain its quality and quantity, which can be described by the differential-delay equation (5). Further results in the conservation matrix $\mathbf{B}_k(t)$ as a consequence of additional treatment $\Delta\mathbf{B}_k(t)$ usually appear with time delays of these actions $\Delta\tau_k$ for any stage k (cf. Bogataj *et al.*, 2004, 6-8):

$$\begin{aligned}
\begin{bmatrix} \dot{x}_p(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix} &= \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_1 + \Delta\mathbf{A}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_2 + \Delta\mathbf{A}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_n + \Delta\mathbf{A}_n \end{bmatrix} \begin{bmatrix} \bar{x}_p(t) \\ \bar{x}_1(t) \\ \bar{x}_2(t) \\ \vdots \\ \bar{x}_n(t) \end{bmatrix} + \\
&+ \begin{bmatrix} \mathbf{I} - \mathbf{H} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{P}(t) \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{F}(t) \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} + \\
&+ \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_1 + \Delta\mathbf{B}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_2 + \Delta\mathbf{B}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{B}_n + \Delta\mathbf{B}_n \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \bar{x}_1(t - \tau_1 - \Delta\tau_1) \\ \bar{x}_2(t - \tau_2 - \Delta\tau_2) \\ \vdots \\ \bar{x}_n(t - \tau_n - \Delta\tau_n) \end{bmatrix}, \tag{5}
\end{aligned}$$

The linear differential-delay equation (5) can also be written in the following condensed form:

$$\begin{aligned}
\dot{\hat{\mathbf{X}}}(t) &= \widehat{(\mathbf{A} + \Delta\mathbf{A})} \hat{\mathbf{X}}(t) + \widehat{(\mathbf{I} - \mathbf{H})} \hat{\mathbf{P}}(t) + \\
&+ \widehat{(\mathbf{B}(t) + \Delta\mathbf{B}(t))} \hat{\mathbf{X}}(t, \tau_1 + \Delta\tau_1, \tau_2 + \Delta\tau_2, \dots, \tau_n + \Delta\tau_n) - \hat{\mathbf{F}}(t), \tag{6}
\end{aligned}$$

where the corresponding mass-preservation condition, initial condition and sequential conditions, similar to the already stated conditions in the system without perturbations, have to be fulfilled. The analysis of perturbations in supply chains can be undertaken separately for individual parameters, while the mutual activity of perturbations in the parameters discussed or any combination of them can be performed thereupon.

The problem of optimization at this stage of development of the system can be developed in different ways, which provide us with different objective functions. Our problem of optimization can be presented in the cost minimization form or in the net present value or annuity stream maximization form. Since both approaches are solved at the abstract theoretical level and presented in Bogataj *et al.* (2004, pp. 8-11), they are not specifically discussed in the present paper. It is worth stressing though, that the net present value or annuity stream maximization approach is more suitable, given that the cost minimization procedure is not able to capture the complex dynamics of flows in cold logistic chains.

4 Basics of Econometric Estimation of Parameters of Preservation of Perishable Goods in Cold Logistic Chains

In the previous chapter we have presented the concept of cold logistic chain in its basic and in its perturbed form. We have discovered that there are the following three key parameters of preservation of perishable goods in supply chains: (1) the time delay τ_k of activation of the conservation effect; (2) the coefficients of intensity of deterioration of perishable goods \mathbf{A}_k at stage k in the supply chain, which represent the transition between good and deteriorated products and (3) the coefficients of conservation of perishable goods $\mathbf{B}_k(t)$ at stage k that is activated with the aforementioned delay τ_k and depends on the state of the goods in the system. Since the conservation effects depend on time $(t - \tau_k)$, the specific treatment of the time delay τ_k would complicate the analysis considerably, but also unnecessarily. That is why we decided to adopt a constant value of the time delay of activation of conservation effect in the present paper and not to estimate the values at each stage of the supply chain specifically. Let us now discuss the remaining two parameters of preservation of perishable goods in cold logistic chains more thoroughly, where we originate in the basic linear differential-delay equation (2) and consider the real Banach space of all continuous linear maps $\Lambda : \mathfrak{R}^m \rightarrow \mathfrak{R}^m$ (cf. Itô, 1993, pp. 163-167), denoted by $L(\mathfrak{R}^m, \mathfrak{R}^m)$.

The matrix of coefficients of intensity of deterioration of goods at stage k in the supply chain, $\mathbf{A}_k \in L(\mathfrak{R}^m, \mathfrak{R}^m)$, in principle has for elements, representing the levels of transition between good and deteriorated products without cooling and/or adding preservatives. Let us write the expression $\mathbf{A}_k \bar{x}_k(t)$ from the differential-delay equation (2) in the basic matrix form, which is valid for every stage of activity k :

$$\mathbf{A}_k \bar{x}_k(t) = \begin{bmatrix} a_{11,k} & a_{12,k} \\ a_{21,k} & a_{22,k} \end{bmatrix} \begin{bmatrix} x_{1,k}(t) \\ x_{2,k}(t) \end{bmatrix} = \begin{bmatrix} a_{11,k}x_{1,k}(t) + a_{12,k}x_{2,k}(t) \\ a_{21,k}x_{1,k}(t) + a_{22,k}x_{2,k}(t) \end{bmatrix}, \quad (7)$$

where the coefficients $a_{ij,k}$ represent the level of transition from one form of products to another and i corresponds to the index of good products, while j stands for the index of deteriorated goods. Since deteriorated products do not convert to good products, $a_{12,k} = 0$ needs to be valid and since deteriorated goods stay in deteriorated form, there is no transition from one form of a product to another and $a_{22,k} = 0$ has to hold. At the end of the k -th stage of development of the logistic chain there remains $(1 + a_{11,k})x_{1,k}(t)$ of good products and $(1 + a_{21,k})x_{1,k}(t)$ of deteriorated products, where $a_{11,k} \leq 0$ and $a_{21,k} \geq 0$. If we denote the degree of intensity of deterioration of goods at stage k by q_k and bear in mind the mass-preservation condition at each stage of activity, then we can write:

$$\mathbf{A}_k \bar{x}_k(t) = \begin{bmatrix} -q_k & 0 \\ q_k & 0 \end{bmatrix} \begin{bmatrix} x_{1,k}(t) \\ x_{2,k}(t) \end{bmatrix} = \begin{bmatrix} -q_k x_{1,k}(t) \\ q_k x_{1,k}(t) \end{bmatrix}. \quad (8)$$

The coefficients of intensity of deterioration of goods at stage k in the supply chain, $a_{ij,k}$, are therefore being reduced to the degree of intensity of deterioration of goods at stage k , q_k , which can be subjected to direct econometric estimation.

The matrix of coefficients of conservation of goods at stage k that is activated with the aforementioned delay and dependent on the state of the goods in the system, $\mathbf{B}_k(t) \in L(\mathfrak{R}^m, \mathfrak{R}^m)$, in principle has four elements as well, representing alleviation of transition levels between the two forms of products by cooling and/or adding preservatives. Let us write the expression $\mathbf{B}_k(t) \bar{x}_k(t)$ from the differential-delay equation (2) in the basic matrix form, which is also valid for every stage of activity k :

$$\mathbf{B}_k(t) \bar{x}_k(t) = \begin{bmatrix} b_{11,k}(t) & b_{12,k}(t) \\ b_{21,k}(t) & b_{22,k}(t) \end{bmatrix} \begin{bmatrix} x_{1,k}(t) \\ x_{2,k}(t) \end{bmatrix} = \begin{bmatrix} b_{11,k}(t)x_{1,k}(t) + b_{12,k}(t)x_{2,k}(t) \\ b_{21,k}(t)x_{1,k}(t) + b_{22,k}(t)x_{2,k}(t) \end{bmatrix}, \quad (9)$$

where the coefficient $b_{ij,k}$ represent changes in the level of transition from one form of products to another and i corresponds to the index of good products, while j stands for the index of deteriorated goods. Since deteriorated products do not convert to good products, there are no changes in this transition and $b_{12,k} = 0$ needs to be valid. Since deteriorated goods stay in deteriorated form, correspondingly the relationship $a_{22,k} = 0$ has to hold. At the end of the k -th stage of development of the logistic chain there remains $b_{11,k}(t)x_{1,k}(t - \tau_k)$ more good products and $b_{21,k}(t)x_{1,k}(t - \tau_k)$ less deteriorated products, then there would remain without cooling and/or adding preservatives, where $b_{11,k} \geq 0$ and $b_{21,k} \leq 0$. If we denote the degree of conservation of goods at stage k by r_k and bear in mind the mass-preservation condition at each stage of activity and the constant time delay³ τ_k , then we can write:

$$\mathbf{B}_k(t) \bar{x}_k(t - \tau_k) = \begin{bmatrix} r_k & 0 \\ -r_k & 0 \end{bmatrix} \begin{bmatrix} x_{1,k}(t - \tau_k) \\ x_{2,k}(t - \tau_k) \end{bmatrix} = \begin{bmatrix} r_k x_{1,k}(t - \tau_k) \\ -r_k x_{1,k}(t - \tau_k) \end{bmatrix}. \quad (10)$$

The coefficients of conservation of goods at stage k that is activated with the time delay τ_k and depends on the state of the goods in the system, $b_{ij,k}$, are therefore being reduced to the degree

³ The delay of activation of the conservation effect τ_k can be understood as an instrumental variable to vary the values of the remaining two parameters, i.e. the degree of intensity of deterioration of perishable goods q_k and the degree of conservation of perishable goods r_k at stage k .

of conservation of goods at stage k , r_k , which contains the effects of preservation procedures and can be subjected to indirect econometric estimation.

The sum of (non-positive) effect of degree of intensity of deterioration of goods $\mathbf{A}_k \bar{x}_k(t)$ at stage k and (non-negative) conservation effect $\mathbf{B}_k(t) \bar{x}_k(t)$ at stage k represents the (non-positive) overall effects of preservation of perishable goods at stage k in the supply chain that is expressed in the differential-delay equation (2) as a flow of perishable goods $\dot{\bar{x}} = \frac{d\bar{x}_k(t)}{dt}$.

Bearing the mass-preservation condition in mind, the (non-positive) flow of deteriorated products is just a mirror image of the (non-negative) flow of good products, therefore it suffices to consider merely the version of the corresponding differential-delay equation for good products for the purpose of econometric estimation of parameters of preservation of perishable goods in cold logistic chains. Considering also the apparent validity of $A_k = q_k \leq 0$ and $B_k = r_k \geq 0$, we thus obtain the following expression:

$$\frac{dx_k(t)}{dt} = q_k x_k(t) + r_k x_k(t - \tau_k), \quad k = 1, \dots, n. \quad (11)$$

Since the supply system is already discretized over the stages of activity k , only the formal discretization of the expression (11) over time t is required. Thus the following difference equation is obtained (*cf.* Sage and White, 1977, pp. 96-98; Winston, 1994, pp. 606-612):

$$t \rightarrow T \in \{T_1, T_2, \dots, T_E\} \quad \frac{\Delta x_{k,T}}{\Delta T} = x_{k,T+1} - x_{k,T}; \quad \Delta T = 1 \quad (12)$$

$$\Delta x_{k,T} = q_{k,T} x_{k,T} + r_{k,T} x_{k,T}(T - \Psi_k), \quad k = 1, \dots, n,$$

where it is presumed that the discretized time delay Ψ_k is identical at every stage of activity k through all observations T , i.e. over the entire temporal horizon T_1, \dots, T_E .

Considering constant time delay of activation of the conservation effect, we can perform a further simplification of the differential-delay equation (11) and express the overall effects of preservation of perishable goods in cold logistic chains with the degree of retention of perishable goods at stage k , which is equal to $s_k = q_k + r_k$; $s_k \leq 0$. Thus the following differential-delay equation is obtained:

$$\frac{dx_k(t)}{dt} = s_k x_k(t, \tau_k), \quad k = 1, \dots, n, \quad (13)$$

which can be analogously discretized in the form of the following difference equation:

$$t \rightarrow T \in \{T_1, T_2, \dots, T_E\} \quad \frac{\Delta x_{k,T}}{\Delta T} = x_{k,T+1} - x_{k,T}; \quad \Delta T = 1 \quad (14)$$

$$\Delta x_{k,T} = s_k x_k(t, \Psi_k), \quad k = 1, \dots, n.$$

It is understandable that the degree of retention of goods at stage k , s_k , in case of absence of preservation procedures reduces to the degree of intensity of deterioration of goods at stage k , q_k . In this manner should the aforementioned (in)directness of econometric estimation of parameters of preservation of perishable goods be understood. The degree of intensity of deterioration of goods at stage k can be observed and thus econometrically estimated in the period before the implementation of preservation procedures, while the degree of retention of goods at stage k can be observed and thus econometrically estimated in the period after the implementation of preservation procedures. Under the *ceteris paribus* assumption, i.e. under the assumption of unchanged time delay of activation of the conservation effect, the degree of conservation of perishable goods can be calculated as the difference between the degree of retention of goods and the degree of intensity of deterioration of goods at stage k .

5 Implementation of Procedures of Preservations of Perishable Goods

Econometric estimation of parameters of preservation of perishable goods in cold logistic chains will be illustrated with numerical example of a food-processing company ‘X’, producing and distributing perishable product ‘PP’ in a supply chain of three stages⁴. The company is interested to find out to what extent the procedures of preservation of perishable goods can decrease on average the degree of intensity of deterioration of goods. Hence we will first describe the logistic chain and the necessary data in more detail and then proceed to econometric estimation of required parameters and to interpretation of the acquired results.

5.1 Description of the Cold Logistic Chain

The goods have already been distributed at all stages at the initial moment T_0 , as it is presented by the vector $\widehat{\mathbf{X}}(0)$. This means that we have $x_{1,T}$ of boxes of the product ‘PP’ at the first activity cell, namely the production, packing and storage of the product in the warehouse

⁴ In the subsequent numerical example several stages of activity in the supply chain were combined due to simplification to obtain three stages only. Be believe that this is sufficient to demonstrate econometric estimation of parameters of preservation of perishable goods in cold logistic chains.

of the company; $x_{2,T}$ of boxes at the second activity cell, that is the transportation of the product in containers on the road, to be precise and $x_{3,T}$ of boxes at the third activity cell, that is the reload of the product to palettes and storage in the warehouse of the main buyer, specifically the wholesaler who takes care of retail. In the temporal horizon $(-\Psi, 0)$ there is no deterioration of the initial goods. Such logistic chain without the implementation of procedures of preservation is presented in Figure 1.

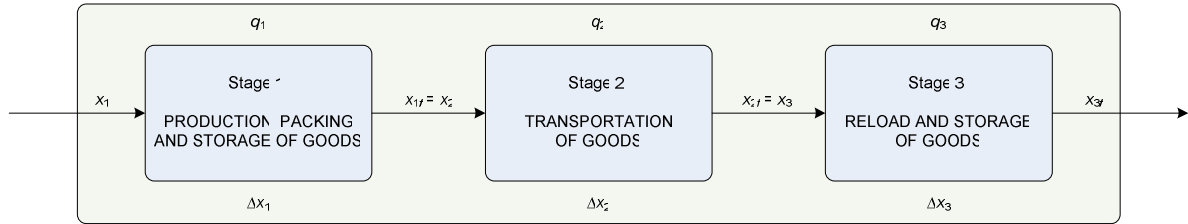


Figure 1: Plain logistic chain from the source to the supply market

Since the product is an essential foodstuff, the logistic chain described above is being carried out continually throughout the year, while for the purpose of econometric estimation of parameters of perishable goods we are observing the flow of goods in discrete hourly time intervals ΔT_k . In each observation, i.e. at every hour there is a different number of boxes $x_{1,T}$ of the product ‘PP’ entering the supply chain, dependent on the orders that are arriving continuously and are managed electronically. At every stage k ; $k = 1, 2, 3$ of the logistic chain the number of boxes of good products⁵ decreases due to deterioration of perishable goods with the degree of intensity of deterioration q_k or remains the same at the most, when $q_k = 0$. At the end of the third stage of the logistic chain there remains $x_{3f,T} = x_{3,T} + \Delta x_{3,T}$; $\Delta x_{3,T} \leq 0$ of boxes of good products in the wholesaler’s warehouse, which are paid to the company ‘X’ as invoiced. Owing to the aforementioned mass-preservation condition, the number of boxes with deteriorated goods remaining at stage k is a residue to the initial state of the system at stage k , which represents a non-positive flow of perishable goods. The quality control of the goods is located at the end of the third stage of the supply chain, while the company ‘X’ checks the quantity of the product ‘PP’ internally at the end of each stage as well.

⁵ A single deteriorated product ‘PP’ automatically indicates that the box with such a product is unserviceable regardless of the number of the good products remaining in the box and is as such separated and discarded at the expense of the company ‘X’.

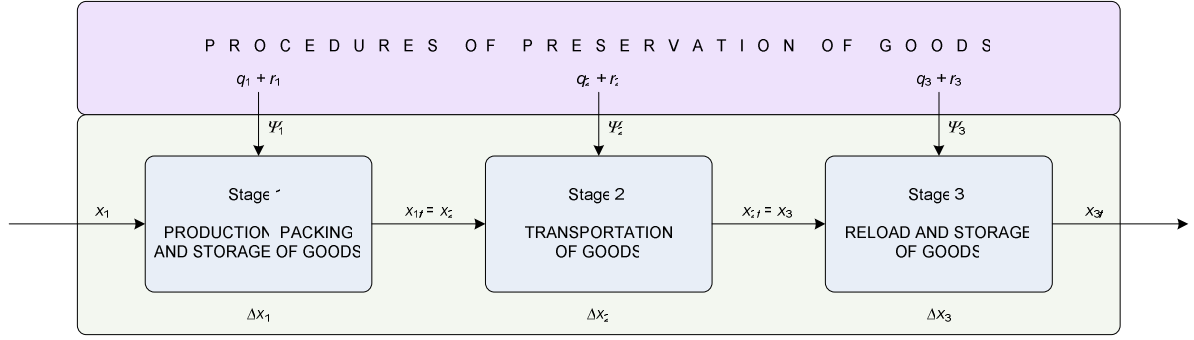


Figure 2: Cold logistic chain from the source to the supply market

After the company ‘X’ eventually ascertained from delivery notes and transport documents that a relatively large share of the perishable goods is being deteriorated compared to its competitors and that it could harm its goodwill, decision was made at time T_{i^*} to implement procedures of preservation of goods, i.e. cooling during transportation and in the wholesaler’s warehouse and adding preservatives at the stage of production, packing and storage of goods. Thus we obtain a cold logistic chain, presented in Figure 2. Additional preservation procedures occur sequentially with time delay Ψ_k at stage k ; where Ψ_k is in our case constant in the temporal horizon $[T_1, \dots, T_{i^*}, \dots, T_E]$. The supply chain is being observed till the final state T_E , which is similarly to the starting state T_1 just one of the cross-sections of the system, whither the product ‘PP’ is being distributed to all activity cells.

5.2 Description of the Database

The above described supply chain is being observed for two consecutive months in hourly time intervals. As was already brought up, the initial state of the system $\widehat{\mathbf{X}}(0)$ is given. Each time interval then represents an observation, described by the following categories: (1) the number of boxes $x_{1,k,T}$ of good product; (2) the change $\Delta x_{1,k,T}$ in the quantity of good product; (3) the number of boxes $x_{2,k,T}$ of deteriorated product; (4) the change $\Delta x_{2,k,T}$ in the quantity of deteriorated product; (5) the state of preservation procedures of perishable goods $D_{k,T}$, taking the value 1, if the procedures are already being carried out and 0 otherwise and (6) the time delay Ψ_k , representing the time passed before the activation of preservation procedures. Due to several aforementioned simplifications only the data on the number of boxes of good product and the state of preservation procedures is needed, while the change in the quantity of the good product can be computed as the appurtenant difference, $\Delta x_{k,T} = x_{k,T} - x_{k,T-1}$. The procedures of preservation are implemented instantly at the beginning of the second month.

The data required for the representative purpose of the method applied in this paper was generated by the use of a version of the Monte Carlo method, based on a general algorithm, described in Rubinstein (1981). A requirement was being set for the generated data to follow multivariate normal distribution with a pre-defined correlation matrix. To describe the method briefly, let us state that a set of variables, which are orthogonal multivariate normals with means of zero and variances of one, should be created by the factor procedure. Thereupon we define the target correlation matrix and calculate the appropriate Cholesky factor matrix. The latter is an upper triangular matrix, which behaves as a ‘square root’ of the target matrix. The independent standard normals are then post-multiplied by the Cholesky factor matrix to give a new data matrix. A standard deviation other than one is achieved by multiplying this new matrix by the desired standard deviation, while a nonzero mean is achieved by adding the desired mean to the matrix, after the standard deviation is already set. Thus the necessary variables for further analysis are produced.

5.3 Estimation of Parameters of Preservation of Perishable Goods

As already mentioned, we are interested in finding out to what extent the procedures of preservation of goods can decrease on average the degree of intensity of deterioration of perishable goods q_k . In other words, we want to determine the average degree of conservation r_k of perishable product ‘PP’ at stage of activity k . Thus we are going to estimate the parameters of preservation of perishable goods at each stage of activity using expressions, developed in the fourth chapter. In the period $[T_1, \dots, T_{i^*-1}]$ only the degree of intensity of deterioration of goods q_k is observed, which is equal to the degree of retention of perishable goods s_k . Whilst in the period $[T_{i^*}, \dots, T_E]$ only the degree of conservation of goods s_k is observed, which is not equal to the degree of retention of perishable goods s_k any more, since additional procedures of preservation are in effect through (directly) unobservable degree of conservation of goods r_k .

The problem is solved by implementation of a dummy variable D_T , which represents the state of preservation procedures of perishable goods and takes the value 1 for observations $T \geq T_{i^*}$ and 0 otherwise. If this variable is being multiplied by the variable x_k and the entire temporal horizon $[T_1, \dots, T_{i^*}, \dots, T_E]$ is observed, the corresponding coefficient at the newly-created variable $D_T \cdot x_{k,T}$ represents the degree of conservation of perishable goods r_k . Formally, we are dealing with the following system of population regression functions⁶:

⁶ The regression equations of the system of equations (15) do not contain constant terms, since we assume that all relevant regressors of the dependent variable are included. Such specification is also in accordance with the nature of difference equations (12) and (14).

$$\begin{aligned}
\Delta x_{1,T} &= \beta_{1,1}x_{1,T} + \beta_{2,1}D_T \cdot x_{1,T} + u_{1,T}, \\
\Delta x_{2,T} &= \beta_{1,2}x_{2,T} + \beta_{2,2}D_T \cdot x_{2,T} + u_{2,T}, \\
\Delta x_{3,T} &= \beta_{1,3}x_{3,T} + \beta_{2,3}D_T \cdot x_{3,T} + u_{3,T},
\end{aligned} \tag{15}$$

where $\Delta x_{k,T} \leq 0$ is the flow of good products of perishable goods ‘PP’ at stage k and time T in the number of boxes; $x_{k,T} \geq 0$ is the quantity of the product ‘PP’ at the end of stage k and time T in the number of boxes; D_T is the aforementioned dummy variable and $u_{k,T}$ is normally-distributed random variable at stage of activity k and time T . The regression coefficient $\beta_{1,k}$ therefore represents the degree of intensity of deterioration of goods at stage k in the supply chain, while the regression coefficient $\beta_{2,k}$ represents the degree of conservation of goods at stage k in the supply chain. At constant time delay Ψ_k of activation of the conservation effect the sum of both regression coefficients corresponds to the degree of retention of perishable goods at stage k of the cold logistic chain. Moreover, the condition of equality between the initial value at stage k and the final value at stage $k - 1$ is preserved, taking this time the following form: $x_{k,T} = x_{k-1,T} + \Delta x_{k-1,T}$; $k = 2, 3$.

The supply chain is being observed a month before the implementation of preservation procedures of perishable goods and a month after the cooling and addition of preservatives is already implemented. Since the observed months are July and August and the observation interval ΔT is hourly, we obtain 1488 observations. Originating from the system of population regression equations (15), the results of econometric estimation are as follows:

$$\begin{aligned}
\Delta x_{1,T} &= -0.0082x_{1,T} + 0.0031D_T \cdot x_{1,T} + e_{1,T}, \\
&(-22.13) \quad (7.628) \quad s_e = 0.0515 \\
&\quad \quad \quad R_*^2 = 0.9988 \\
\Delta x_{2,T} &= -0.0351x_{2,T} + 0.0189D_T \cdot x_{2,T} + e_{2,T}, \\
&(-18.61) \quad (5.418) \quad s_e = 0.0982 \\
&\quad \quad \quad R_*^2 = 0.9952 \\
\Delta x_{3,T} &= -0.0162x_{3,T} + 0.0067D_T \cdot x_{3,T} + e_{3,T}, \\
&(-25.20) \quad (8.019) \quad s_e = 0.0890 \\
&\quad \quad \quad R_*^2 = 0.9991
\end{aligned} \tag{16}$$

where in brackets below the regression coefficients β_j ; $j = 1, 2, 3$ are the values of the corresponding t -statistics; $e_{k,T}$ represents normally-distributed random error at stage k ;

$k = 1, 2, 3$ and time T ; s_e symbolizes the value of standard error of regression and R_*^2 stands for modified determination coefficient of multiple regression⁷.

5.4 Interpretation of the Values of Parameters of Preservation of Perishable Goods

Let us briefly comment the results of econometric estimation, presented by expression (16). The degree of intensity of deterioration of goods at the stage of production, packing and storage in the company's warehouse equals -0.0082 , which means that without procedures of preservation on average, *ceteris paribus*, 0.82 per cent of boxes of the perishable product becomes deteriorated at this stage. At the stage of transportation of goods on the road the degree of intensity of deterioration of goods equals -0.0351 , which means that without procedures of preservation on average, *ceteris paribus*, 3.51 per cent of boxes of the perishable product becomes deteriorated at this stage. At the stage of reload and storage in the wholesaler's warehouse the degree of intensity of deterioration of goods equals -0.0162 , which means that without procedures of preservation on average, *ceteris paribus*, 1.62 per cent of boxes of the perishable product becomes deteriorated at this stage.

After the implementation of procedures of preservation the loss decreases as expected, which is reflected in the degree of conservation of goods. The latter amounts to 0.0031 at the stage of production, packing and storage in the company's warehouse, which means that preservation procedures decreased on average, *ceteris paribus*, the ascertained loss by 0.31 percentage points at this stage. At the stage of transportation of goods the degree of conservation of goods amounts to 0.0189, which means that preservation procedures decreased on average, *ceteris paribus*, the ascertained loss by 1.89 percentage points at this stage. At the stage of reload and storage in the wholesaler's warehouse the degree of intensity of deterioration of goods amounts to 0.0067, which means that preservation procedures decreased on average, *ceteris paribus*, the ascertained loss by 0.67 percentage points at this stage.

It can be seen that both the loss of the perishable product 'PP' and the decrease of that loss are the highest at the stage of transportation of goods, which can be explained by difficult conditions in transportation and corresponding procedures of preservation, respectively. Namely, the degree of retention of perishable goods, indicating 'net effects' of deterioration and preservation of the perishable product at constant time delay Ψ_k of activation of the conservation effect, is equal to -0.0051 at the stage of production, packing and storage in the

⁷ The determination coefficients R_*^2 of the quasi-behavioural regression equations, i.e. regression equations without constant terms, are calculated using the uncentred formula (*cf.* Verbeek, 2000, p. 21).

company's warehouse, -0.0162 at the stage of transportation of goods and -0.0095 at the stage of reload and storage in the wholesaler's warehouse. By comparing this figures to degrees of intensity of deterioration of goods, estimated earlier, it can be established that the decrease in the quantity loss of the perishable product 'PP' is substantial. The reduction of the loss of the product due to deterioration on average amounts to more than 40 per cent, while in the aforementioned second stage of the supply chain this reduction is greater than 50 per cent, representing a decrease from 3.51 to 1.62 per cent of boxes lost due to deterioration.

However, quantity reduction of the loss of perishable goods does not automatically imply the optimal financial position for the company, even though it usually represents an improvement in business activities. To ascertain whether this is the case, a cost-benefit analysis of the procedures of preservation of perishable goods should be carried out. Bearing in mind the complexity of dynamic flows in cold logistic chains, an analysis of net present value or annuity stream would be more suitable, though. No earlier can it be adequately established whether the mathematical optimization of the supply chain is more favourable from the financial point of view than the purchase of attainable insurance policy on the actuary market.

6 Conclusion

Paper discusses the parameters of preservation of perishable goods in cold logistic chains from the viewpoint of their econometric estimation, which is indispensable when selecting appropriate procedures of preservation and also when examining perturbations in supply chains. Herein a specific approach is applied, combining Laplace transforms, Input – Output analysis and Material Requirements Planning (MRP) to transform a Cold Chains Management (CCM) problem to a parametric problem of linear programming.

The key parameters are the delay of activation of the conservation effect, the intensity of deterioration of goods and the conservation effect of perishable goods. The latter two can be combined to obtain the retention effect of perishable goods. The values of these parameters tell us the quantity of the product being deteriorated in the logistic chain and the extent to which the deterioration can be alleviated. Econometric estimation thus presents us with the quantity effects of preservation procedures, whereas the financial effects can be derived using the proper price categories in the calculation of the net present value or the annuity stream. In this way one can determine whether the implementation of preservation procedures is more rational than the purchase of attainable insurance policy.

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