A MODEL TO DISTRIBUTE MARK-UP AMONGST QUOTATION COMPONENT ITEM PRICES: AN OUTLINE

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The outline of a proposed new unbalanced bidding model is discussed. Background is provided as regards the role of item price loading, otherwise known as unbalanced bidding. Three types of loading are described, namely those of 'front-end loading', 'back-end loading' and 'quantity error exploitation' (otherwise known as 'individual rate loading'). It is proposed that one single mathematical model could embrace all three of the above types and that the aspect of risk may be addressed partially by means of using the quadratic programming techniques employed within the field of Modern Portfolio Theory. MPT is a field pioneered by Markowitz in 1959 and was developed to identify optimum portfolios of investments, typically equities. It is hypothesized that MPT presents a basis by which to distinguish Efficient Item Pricing combinations from inefficient ones and thereby provide a scientific tool by which rational contractors may optimally price a project's items. A brief history of unbalanced bidding describes the field that was pioneered in the 1960's by Marvin Gates and Robert Stark, as well as the subsequent contributions by the leading researchers in the field.

Keywords: unbalanced bidding, bidding models, item price loading, modern portfolio theory, portfolio analysis, construction industry, mathematical models, bidding strategies.

INTRODUCTION

Traditionally it is standard practice in the construction industry for contractors to compete for work using lump-sum tender prices. In order for a contractor to arrive at his tender price it is common practice that he estimates the cost of each item contained in the Bills of Quantities (BoQ) supplied to him by the project's professional quantity surveyor (PQS). He then does a summation of these costs and then adds an overall mark-up to arrive at the aforementioned tender price.

The building developer conditionally awards the contract to a tenderer, on condition that his pricing of the Bills' items is found fair and reasonable (as assessed by the quantity surveyor). The contractor is then asked to submit his priced Bills for inspection, for the purposes of this assessment by the PQS.

This process requires that the contractor has to decide the individual prices of all the BoQ items that in summation need to be equal to his tender price. This presents the contractor with the opportunity (of which many avail themselves) to price some items using a high mark-up, and others using a low mark-up (to compensate). The incentive for this uneven distribution of mark-up relies on the role by which the individual item

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prices form part of the contract and by which they are used for the valuation of interim payments, escalation compensation and design variations. The manipulation of prices in this way is able to generate benefits for the contractor which, correspondingly, represent additional expense and risk to the developer.

Loading the prices of items that will arise early in the construction schedule obviously improves a contractor's cashflow. This process is known as 'front-end loading'.

'Back-end loading', on the other hand, is the process of loading the prices of items that will arise late in the schedule and also loading those items that fall within workgroups that are expected to have a high rate of escalation.

A third style of loading is called 'quantity error exploitation' or otherwise as 'individual rate loading'. This process entails loading the price of items whose final quantities are expected to exceed the initial quantities contained in the BoQ.

It is obvious how any one of these loading strategies might be pursued in isolation but the challenge becomes more complex when one envisages the pursuit of all three in combination. Not only does it become obvious that some sophisticated mathematical modelling would be beneficial but this is further complicated by the need to recognise that unbalanced bidding generates its own risks. A rational contractor may not be prepared to pursue an extreme example of loading (that may be expected to generate a high additional profit) if he were to know that it comes along with extreme risk. Thus a contractor is faced with a classic risk versus return decision that offers no single 'optimal' solution but rather might present a series of 'efficient' options each of which represents a different risk and return scenario.

HISTORY OF RESEARCH

Item price loading – as described above – is common practice in the construction industry (see, for example, McCaffer, 1979; Green, 1986; Kaka and Price, 1991; Kenley, 2003). However, the extent of benefit believed possible by loading far exceeds that presently enjoyed by contractors (Cattell, 1987).

The practice of item price loading is otherwise known as 'unbalancing the bid'. This practice is believed to have first been commented on by Gates (1959 and 1967) who suggested that unbalanced bidding had more to offer a contractor in the short-term as a strategy than any other bidding strategy. He proposed a simple method that addressed a contractor's need for an accelerated cashflow as well as a means to benefit from loading the prices of items whose quantities are anticipated to increase as a result of a likely variation order. His approach largely steered clear of any sophisticated mathematics and he went so far as to comment that he felt that unbalanced bidding, at least in so far as the manner in which he advocated it, was the least mathematically involved of all the bidding strategies that he was then proposing. The significance of his work was more so that he is thought to have been first to formally comment on unbalanced bidding as a strategy and furthermore that he had identified it as being of great potential significance.

Stark (1968, 1972 and 1974) then approached the problem largely as it had been defined by Gates. His approach involved greater mathematical sophistication and he advocating a simple linear programming solution. As with Gates, he recognised there to be some risk for a contractor to pursue an unbalanced bid and his suggestion was to adopt a discipline of using sensitivity analyses. His approach could largely be described as being of a deterministic nature.

Whilst Stark's efforts took account of the combined benefits of cashflow and quantity estimation errors, Ashley and Teicholz's subsequent research (1977) didn't build upon this but rather, seemingly independently, proposed a simple linear unbalancing model for the sole purpose of improving a contractor's cashflow. This practice is otherwise known as 'front-end loading' (Harris and McCaffer, 2001). In this initial research, they recognised that some benefit could be derived from "quantity error exploitation" but they concluded that it appeared too difficult to systematically model. Teicholz and Ashley (1978) went on to enhance their earlier efforts in combination with using the contribution of Stark's (1974). They proposed a more sophisticated 'optimal model' (resembling that of Gates') which expanded on their initial effort to now include quantity error exploitation.

Diekmann, Mayer and Stark (1982) took Stark's original deterministic model and, without reference to the works of Ashley and Teicholz, added a probabilistic formulation to take account of the risk. They ignored other risks however and also failed to take account of the item price loading benefits other than that of cashflow.

Tong and Lu (1992) developed a method that was focused solely on optimizing the advantage of what they called 'error exploitation unbalancing' (referred to by Green (1986) as 'individual rate loading' and by Cattell (1987) as 'loading for anticipated quantity variation orders'). In other words, this method ignored the other benefits in the areas of cashflow and escalation.

Cattell (1984) found that there were three benefits that could be derived from item price loading: namely, what he called cashflow, variations and escalation. He incorporated all three into one mathematical model but as commented on by Taylor and Bowen (1987) this early work failed to properly take account of estimation risk.

Green (1986) was next to identify all three categories or opportunities for unbalancing: namely, what he described as front-end loading, individual rate loading, and back-end loading. Although he identified them, his research (1989) concluded that it was difficult to envisage how 'individual rate loading' could be applied in a systematic and optimizing method. Likewise, he steered clear of formulating a method to address what he had described as 'back-end loading'.

Cattell's (1987) later research was without reference to the earlier work of Gates, Stark, Ashley and Teicholz, and Green. He formulated a model that not only took into account the benefits of cashflow and anticipated quantity variation orders, but also brought into the equation the benefit that can be derived from escalation –the practice that Harris and McCaffer (2001) refers to briefly, in principle, as 'back-end loading'. Cattell then proposed that this model take account of risk by applying mathematical techniques (such as Quadratic Programming) drawn from the field of Modern Portfolio Theory (pioneered by Markowitz, 1959), originally designed for use for optimizing investment portfolios – typically comprising equities. The research came to conclude that no single unit pricing combination can be found to be optimal but rather that not one, but a series of alternative item pricing combinations could be identified as all being *efficient. Efficient* pricing, within this context, implies making the maximum profit for any accepted degree of risk. Thus, Cattell's model could be used to identify the most efficient item pricing combination to suit any specified acceptable level of risk. The research illustrated the technique by which to discern *efficient* item price combinations from *inefficient* ones.

Bidding models

Unbalanced bidding models are not to be confused with what are more commonly known simply as ‘bidding models’. The latter field of research was founded by Friedman (1956) and by Gates (1959). Bidding models (as opposed to ‘unbalanced bidding models’) provide mathematical techniques for the use of a contractor to predict his probability of winning a tender at any particular bid price, and hence provide him with the mechanism to determine his optimum bid price. Gates’ model contradicted Friedman’s model as regards some fundamental mathematics. The Gates versus Friedman debate has raged on ever since with many dozens of researchers falling largely into one or other of these two camps. Most bidding models are derived from either Friedman’s or Gates’ original models. Abdel-Razek (1987) is one of those who has provided a synopsis of the early stages of this ‘battle’, Crowley (2000) provides a more recent assessment, and Skitmore (2002) provides a quantitative comparison.

OUTLINE OF THE MODEL

The nett present value to a contractor of the worth of a project PV_{proj} may be valued as being

$$PV = \sum_{j=1}^{J}\sum_{n=1}^{N}\left(\frac{1}{1+r_j}\right)^n \left[\left(\square_{nj}(Q_j+Q'_j)\right)\left[(1-R_n)P_j - C_j + \square_{nj}(fP_j - C_j)\right] + (1-i)R_n\,\square_{nj}(Q_j+Q'_j)P_j\right]$$

(Cattell, 1987)

where j = item number

n = month number

J = number of items

N = duration of project in months

r_j = discount rate appropriate to the risk of item j

i = interest rate earned on retained funds

$\square_{nj}$ = proportion of Q_j to be built in month n

Q_j = bill quantity of item j

Q'_j = additional quantity of item j due to variation

R_n = proportion retained in month n

P_j = bill price per unit of item j

C_j = unit cost of item j

$\square_{nj}$ = adjustment for inflation = $\dfrac{\text{index}_n - \text{index}_0}{\text{index}_0}$

f = "Haylett" factor e.g. 0.85

This equation therefore provides a basis to determine the contribution of each item to a project's worth to a contractor and therefore this equation's derivatives provide a measure of the added benefit that is to be derived from any increase in any item's price.

Thus if one considers that the overall strategic objective as regards unbalanced bidding is one in which the contractor is having to decide where (i.e. to which items) to allocate the distribution of the overall tender price, the above equation (and its logical derivatives) can be used to identify which items offer a greater added contribution for any price allocation than others.

Clearly it would be unrealistic to allocate all of the tender price to the item that this model identifies as being the one that is the most worthy beneficiary. Gates (1959) and Stark (1968) both proposed that item price loading needs to be restrained by 'unit rate constraints'. Both of them suggested that some (if not all) items have a limited acceptable range of prices, dependent on the prices given to other items. For example, the excavation of hard rock should be priced at more than the excavation of soft rock. Whilst these proposals appear reasonable, these constraints need to be applied to this model.

A further obvious constraint may be stated as follows:

$$TP = \sum_{j=1}^{J} Q_j \, P_j$$

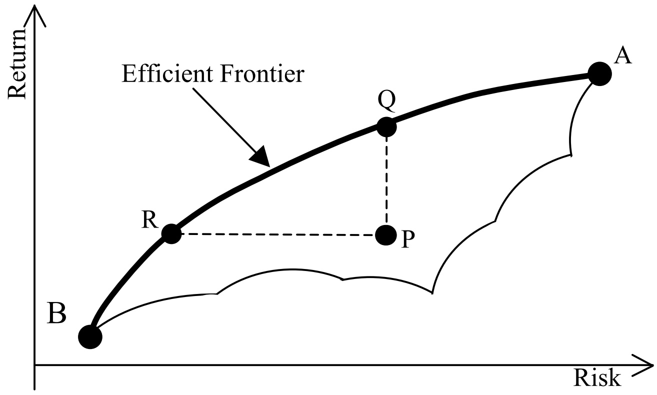
where TP = the contractor's tender price.

Working within these constraints, a contractor is able to quantify the PV benefits of numerous item price combinations.

THE APPLICATION OF MODERN PORTFOLIO THEORY

Not only do different item price combinations generate different expected returns for a contractor, but they also generate different levels of risk. 'Risk' in this context refers to "the degree of confidence one has that the expected return will be close or equal to the actual return" (Branch, 1976). Risk is, in other words, a measure of the variance by which the actual return may be different from the expected return. Some pricing combinations can be expected to offer a high return but suffer from a high risk, whilst others may offer a lower return with a lower risk. It is reasonable to expect that some rational contractors (who may be more risk averse) might prefer lower-return : lower-risk combinations rather than other higher-return : higher-risk combinations. Thus one can realise that item price loading cannot reasonably have the objective of identifying a single *optimal* item price combination that should be considered preferable to all others.

This trade-off between risk and return is a classic decision that is presented to almost all financial investors and it is handled especially well by techniques developed within the field of Modern Portfolio Theory (otherwise known as portfolio analysis) which was founded by Markowitz (1952 and 1959). MPT is based on the logic that whilst someone might reasonably prefer a low-risk : low-return option, it is not logical that, given choices, a rational investor would prefer a low-return : high-risk option. Thus, MPT will typically make use of quadratic programming to identify what are termed *efficient* choices (i.e. ones that may rationally be considered attractive) as distinquished from others that are hence termed *inefficient*.



The above chart illustrates how the full range of options may be plotted showing their respective risks and returns. If all of the options were to fall within the envelope AB, it is logical that any rational investor would prefer both options Q and R to the alternative P. Option Q offers a higher return for the same amount of risk, whilst R offers a lesser risk for the same return. Thus all of the options that are described by the Efficient Frontier (shown as the heavy line) are preferable to all of those that otherwise fall within the envelope AB.

In the instance of item price loading, it is suggested that quadratic programming may be used to identify this Efficient Frontier and therefore the full range of item price combinations that all would represent rational choices. Equipped with these choices, a contractor could decide which risk : return combination best suits his objectives. Thus he will have identified his best suited (loaded) item prices.

The benefits of diversification

The traditional use of MPT (for the purposes of investing in equities or other securities) produces its benefits for reason that there is not a perfect correlation between the variances of alternative investments. As Vergara (1977) points out, MPT relies upon the benefit that diversification reduces risk. Markowitz (1959) however says that whilst the benefits of diversification are obvious, it is not enough to simply pursue any form of diversification but rather that the diversification has to be of the 'right kind'. He quotes the example of a portfolio comprising of only sixty different US railway securities and argues that this portfolio would fail to offer the benefits comparable to another portfolio (even if it were of similar size) which comprised a spread of unrelated securities participating across many different sectors of the global economy. If the US railway industry were to perform badly, the 'portfolio' comprising only railway securities would perform badly, regardless that it comprises as many as sixty different securities. Markowitz thus argues that a well diversified portfolio comprises the right mix of components – the right mix largely being dependent upon the characteristic that they would share low covariances amongst themselves.

If, on the other hand, all investment options were to be perfectly correlated (in other words, they were to share a covariance of 1.0) investors would not be able to derive any greater benefit from making up a portfolio that they would enjoy from investing in any single one of the portfolio's component investments.

MPT does more than advocate the merits of simple diversification but does well to illustrate the further benefits of a mathematically balanced, optimised mix.

The application of MPT to item price loading

A BoQ may be considered to have some similarities to a portfolio of investments. Ordinarily though, an investor in equities may have a vast array of choice comprising thousands of different securities between which to choose and thus he has the ability to select a small subset of these available investments so as to form his chosen portfolio. Should this investor be rational and well-informed and hence pursuant of an 'efficient' portfolio, it is necessary that he should not only be choosing the 'correct' securities, but also just as importantly, he needs to be choosing the right weighting between these securities in order to accomplish what Markowitz would describe as the 'right mix'.

In the case of a construction project, a contractor has no choice but to participate in all of the items contained in the project's BoQ. He does however have some choice as regards the weighting between these component items. He has no choice as regards the quantity of each item but he does have some choice as regards the items' prices. By loading some item's prices more than others, a contractor is thus able to adjust his 'mix' between the BoQ component items. Thus it is hypothesised that he might employ similar mathematical apparatus as that advocated by MPT by which he might similarly pursue the identity of a project's full series of alternative *efficient* item pricing combinations. Having identified his full range of *efficient* alternatives, he would then be very well informed and well equipped to make his choice of the one price combination which represents his best risk versus return trade-off.

One reason for suggesting that MPT has application for this purpose is the observation that whilst different item price combinations generate different degrees of risk, this is for reason that each item price combination comprises different weightings being given to the variance or risk associated with each item. This is similar to a portfolio of securities. Furthermore, and just as importantly, whilst all items thus do not share the same variance, they also do not share the same covariances between them. It is logical that some pairs of items may share a covariance of 1.0 whilst others may have variances quite independent of one another (that is, a covariance of 0.0). It is this independence between items that suggests the potential for the application of MPT.

THE PRACTICE

Kenley (2003) has questioned the morality of item price loading but nevertheless has acknowledged it as having considerable impact as well as having widespread use. He has however found that its use is with limited levels of mathematical sophistication. Green (1989) has likewise commented on the significant value of unbalancing, and in particular from the practice of front-end loading. In a survey of his (1986), he identified extensive use of what he called 'individual rate loading' and 'front-end loading' although he noted that, in practice, it was conducted in an unscientific manner. Kaka and Price (1991) have also commented on the 'significant' effect of front-end loading in their efforts to account for this practice when they forecast developers' cashflows.

Warning

Kenley (2003) has warned of the dangers to contractors of pursuing unbalanced bidding. This is largely founded on his experience in which some contractors' contract managers have lost track of the extent or nature of their own estimators' initial price loading. Kenley has found that there is often poor communication internally between a contractors' estimators and their contract managers. He has found that this has at times given rise to these contract managers enjoying a false sense of optimism early on in projects when they find it surprisingly easy to contain a project's costs far below the BoQ rates. This is said to sometimes give rise to the projection of this perception of good fortune through to the end of a project. Kenley has quoted examples of where these initial periods of surplus cashflow have led to the vast expansion of some contractors' overheads to the extent that some have indulged themselves with the acquisition of luxury yachts, racehorses, sports cars and the like.

Problems obviously then arise later when these managers cannot understand why items built later in these projects are beginning to suffer from increasing lesser profit margins to the extent that work done near the end of these projects is typically having to be built at a loss. Ultimately some contractors are described as having failed altogether and long established businesses have reportedly gone bankrupt due to these short-comings in their cashflow management.

Discussions with Kenley have clarified that these problems should not be blamed on the practice of unbalanced bidding (nor on other judicious 'cash farming' techniques that he has identified). Instead, it is more reasonable to blame this on poor communication within these contractor's management teams as well as on their poor and unsophisticated systems which lack the ability to keep contract management suitably informed. Kenley has argued that the survival of contractors is highly dependent on these sophisticated systems and management techniques with regards to their cashflow management, especially in an environment where this is made more challenging through the widespread use of item price loading.

The Role of the PQS

Kenley (2003) has described the practice of item price loading as 'ethically questionable', 'dubious' and 'illegitimate'. Whilst many researchers have largely steered clear of such judgement, the opinion of Stark (1968 and 1974) is that unbalanced bidding provides contractors with an efficiency that is healthy not only for them but for the industry as a whole as well as for clients. This argument appears to be founded on the logic that the process of tendering is there to serve the purpose of ensuring that the most efficient contractor wins, something which, upon a superficial assessment at least, would appear to clearly benefit clients. The further pursuit of this argument would appear to suggest that contractors who fail to utilise item price loading will be failing to derive a considerable profitable opportunity that's inherent within the prospective benefits of any project. The consequence of this is then that any contractor not practicing item price loading will be suffering an 'opportunity cost' - thereby effectively rendering them uncompetitive within any environment in which item price loading has become the norm.

Another perspective on this would suggest that although item price loading might provide increased efficiency for contractors, it generates considerable additional 'hidden costs' for clients. Thus whilst competition within an environment in which unbalanced bidding may be commonplace, may result in lower tender prices, these lower bids may not necessarily represent lower effective costs for the clients. Nevertheless, whilst item price loading has considerable benefits to offer, it appears illogical to expect rational contractors not to pursue this opportunity. Once doing so, it would appear sensible that any rational contractor would furthermore wish to refine this process to the extent that he is able to maximise this opportunity. It's likewise logical that any contractors not doing the same would be rendered uncompetitive. Thus, the optimised practice of item price loading might become the norm. In this scenario it would be naive for a client to expect anything other than unbalanced bids and thus he will have need to factor the 'hidden costs' that are associated with such bids into his budgets. The role of the PQS in protecting his client will become more challenging as the practice of unbalanced bidding becomes more widely practiced, more sophisticated and more extreme. Furthermore, sophisticated item pricing loading models are normally designed to optimally exploit any BoQ errors caused by the PQS and thus it may be argued that, within this scenario, it will become more important for clients to invest in a good PQS as well as in a well prepared BoQ.

A possible means by which a client might protect himself from the effects of item price loading would be for him to insist that all bidding contractors are to submit, for comparison, a fully priced BoQ. This would facilitate that the PQS could be able to compare bids not solely on the basis of their bottom-line tender price but rather by comparison of the present value of the anticipated cashflows that each contractor's priced BoQ is likely to generate. The PQS would furthermore be able to run sensitivity analyses on these priced BoQ testing for variations in item quantities and thereby should be able to highlight cases of any quantity error exploitation.

Another method by which a PQS might be able to circumvent and / or moderate some of the benefits of unbalanced bidding has been suggested by Kenley (2003) to be that clients might insist that contractors bids also incorporate a commitment as regards their cashflow drawdown. Interim payments would therefore be derived from this agreed schedule rather than from the use of item prices. As Kenley has proposed, this might necessitate that the client would want to pre-select the contractors who will participate in such a tender, so as to be assured of their reputation and sound standing. If a PQS were to advocate this practice, he would have helped remove much of the incentive for unbalanced bidding. By doing so, the PQS might hope for more balanced bids, and therefore might expect to have alleviated much of his client's risk.

Clearly, the PQS has a significant role to play to help circumvent both much of the cost as well as the risk of unbalanced bidding.

CONCLUSION

The practice of unbalanced bidding has been described and an outline of an item price loading model has been proposed that offers two significant advantages over prior models. One advantage is that the model manages to provide a formulation that comprehensively incorporates all three forms of loading: front-end loading, back-end loading and quantity error exploitation. The other advantage is that it offers a solution as regards the problem of risk. In particular, it suggests that there is no single item price loading combination that may be considered *optimal* but rather many different combinations can be considered *efficient.* The model identifies the path by which *efficient* price combinations can be discerned from *inefficient* ones thereby providing rational contractors with an informed risk versus return decision as to which efficient item price combination would best suit their objectives.

FURTHER RESEARCH

Whilst only an outline of this proposed model has been described, the further development of this model is the subject of on-going research. In particular, one area of emphasis of this research is now needing to consider the mathematics of identifying the risk associated with different item price combinations. Another relates to the problem of finding a practical means by which to assemble the vast quantities of data needed for such analysis. Finally, the research is intended to embrace an initial practical test.

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