# The Nonlinear Skeletons in the Closet

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Forthcoming in Michael Belongia and Jane Binner (eds), *Money, Measurement, and Computation*, Palgrave

May 2004

## 1. Introduction

Although nonlinearity is the rule in economic theory, nonlinearity tends to make life difficult for econometricians. While there have been many advances in nonlinear econometrics in recent years, some problems produced by nonlinearity remain "skeletons in the closet" in empirical economic research. In this paper, we open the door to that closet and take a look at two of the biggest skeletons. For a deeper look into that closet, see Barnett and Binner (2004).

### 1.1 Cointegration Does Not Necessary Produce Linear Models

In the literature on cointegration, a linear combination of the cointegrated variables is integrated of order zero, i.e. stationary. That linear combination often is specified to follow a linear ARMA process or is used within linear structural models. However, there is no reason to expect that such a linear combination is a linear process. For example, if processes are integrated of order 1, but also are nonlinear processes, there may indeed be a linear combination of those processes that is stationary---but is also a nonlinear stochastic process. Linear combinations of nonlinear processes are nonlinear. We explore some of the more popular cases in which cointegration is used as a basis for the construction of linear models, and run tests for nonlinearity of the linear combination of the cointegrated variables. The results are discussed in section 2 of this paper.

### 1.2 Curvature is not Sufficient for Regularity

In producing structural models of demand and supply systems, violations of first and second order regularity conditions are known to invalidate inferences produced by integrable estimated structures, especially during forecasting and simulation. Those regularity conditions typically consist both of monotonicity conditions and curvature restrictions. For example, indirect utility functions must be monotonically decreasing and quasiconvex, in order for applications of Roy's identity to be valid in deriving estimable consumer demand systems from the indirect utility function. The ability to impose or test such conditions within nonlinear systems of equations have been available since the asymptotic properties of the maximum likelihood estimator for nonlinear systems of equations were derived by Barnett (1976). Yet in much of the recent applied literature, regularity has been equated with satisfaction of curvature conditions alone---with no mention of monotonicity conditions, which are neither imposed nor verified.

We believe this practice of "overlooking" the monotonicity conditions reflects the fact that the relevant models often violate the monotonicity conditions, once the curvature conditions have been imposed. The source of those likely violations can be found in the fact that flexibility of many parametric specifications collapses, when monotonicity as well as curvature conditions are imposed. For example, the translog collapses to Cobb-Douglas, although translog subject to curvature alone does not collapse in flexibility.

We investigate the empirical implications of this practice by exploring the regularity properties of a model often viewed as especially attractive by those who are emphasizing curvature conditions alone. However, we investigate monotonicity as well as curvature. The model is the symmetric generalized McFadden model (often now called generalized quadratic). Imposition of curvature on that model does not assure that isoquants (or indifference curves in the consumer demand version) will have negative slope throughout the region of the data. Without valid monotonicity, the second order conditions and thereby the duality conditions that produce the model---fail. It is even possible (and happened in one of our cases) that some first derivatives of the model can become complex valued, so that the elasticities are not defined. In addition to investigating the overlooked monotonicity conditions, we also introduce risk and estimate the Euler equations by generalized method of moments to connect our application with the current state of the art. The resulting model is deeply nonlinear. Our results are discussed in section 3 of this paper.

# 2. Time Series Cointegration Tests and Nonlinear Stationary Residuals

## 2.1 Introduction

Modern macroeconomic theory emphasizes the interactions among representative agents (households and firms) who are, in general, assumed to behave according to nonlinear decision rules, which are obtained as solutions to dynamic optimization problems. Consequently, it is reasonable to posit the existence of nonlinear relationships among macroeconomics variables.

During the last decade, as theoretical macroeconomics has been concerned with microeconomic foundations, cointegration has become one of the most important characterizations of macroeconomic time series. For example, real business cycle research now commonly assumes balanced growth between output, consumption, and investment, and stable long run money demand with unit income elasticity. See, for example, King and Watson (1996). These assumptions imply the existence of cointegration relations among the key business cycle variables. Cointegration studies have, however, rarely explored the possibility of nonlinear relationships among macroeconomic variables. I(1) cointegration analysis focuses on non-stationary economic variables that are integrated of order one, meaning that their first differences are stationary. The existence of cointegration implies that some linear combination of these integrated variables is stationary. The integrated variables may be nonlinear stochastic processes, a hypothesis that is seldom entertained as either a feature of the data generating process or as a convenient statistical description of the data in the cointegration literature. This section addresses this gap between the implications of modern dynamic macroeconomic theory and the cointegration literature by applying tests for nonlinearity to the stationary linear combinations produced from cointegration.

A number of studies have tested for the existence of nonlinearity in macroeconomic data (Hinich and Patterson, 1987, Barnett and Chen, 1988, Barnett and Hinich, 1992, Brock and Sayers, 1988). However, most of the existing nonlinearity tests are univariate, and some of the available nonlinearity tests are not invariant to prior linear filtering of the data.[2] In this section, we investigate the application of univariate nonlinearity tests to stationary linear combinations of non-stationary (and possibly nonlinear) macroeconomic time series, which have been identified through cointegration analysis. Thus, rather than testing the first differences of individual economic time series for nonlinearity, we test the long run relationships between those series.

The remainder of this section is organized into subsections as follows: in Section 2.2, we review the relevant aggregation theory and index number theory; in Section 2.3, we present the results of the cointegration analysis; in Section 2.4, we present the results of the nonlinearity tests; and Section 2.5 concludes.

### 2.2 Aggregation and Index Number Theory

In this section, we briefly review the monetary aggregation theory motivating the choice of variables in our empirical analysis. For more extensive reviews, see Barnett (1987), Barnett (1990), Barnett (1997), Barnett, Fisher, and Serletis (1992) and Anderson, Jones, and Nesmith (1997 a), in which the most general conditions under which monetary aggregates exist are discussed. Also see Belongia and Chalfant (1989) and Belongia (1996). For a unified and systematic development and presentation of the theory, see Barnett and Serletis (2000).

Arrow and Hahn (1971) showed that if monetary assets are valued in general equilibrium, there exists a derived utility function containing monetary assets. If we assume that a representative agent exists and that current period monetary assets are blockwise weakly separable in that agent's utility function, a conditional second stage monetary services allocation decision exists. In that second stage, the representative agent can be viewed as solving a current period decision problem of the following form:

$$\text{Max } V(\mathbf{m}_t) \text{ subject to } \mathbf{m}_t^T \boldsymbol{\pi}_t = M_t$$

where V is the category subutility function of the (weakly separable) block of real monetary assets, $\mathbf{m}_t$ is the vector of monetary assets, $\boldsymbol{\pi}_t$ is the vector of user costs for monetary assets, and $M_t$ is total expenditure on monetary assets allocated in the prior first stage decision. If V is linearly homogeneous, then it is the economic monetary aggregate, and the economic agent views the economic aggregate as an elementary good, that we call monetary services.[3]

In aggregation theory, monetary assets are viewed as durable goods, and thus the opportunity cost for each monetary asset is its user cost. Monetary assets provide a flow of services to the representative agent over the decision period and the equivalent rental price of this flow is the user cost. Barnett (1978, 1980) shows that under perfect certainty the user cost of monetary assets in a household intertemporal optimization model is:

$$\pi_{it} = p^* \frac{R_t - r_{it}}{1 + R_t}$$

where $R_t$ is the risk-free rate of return on a completely illiquid asset, called the benchmark rate, $r_{it}$ is the own rate of return on monetary asset i, and p* is a true cost-of-living index .

For the remainder of this section, we assume that monetary assets are weakly separable from the other decision variables in the representative agent's utility function, that the category sub-utility function is linearly homogeneous, and that the representative agent is an optimizing price taker. Under these assumptions, the economic monetary aggregate can be tracked by the Törnqvist-Theil discrete time approximation to the continuous time Divisia index. Diewert (1976) proved that this index is superlative, in the sense that it can provide a second order approximation to any arbitrary economic aggregate in discrete time.

The Törnqvist-Theil monetary services index is defined as follows:

$$Q_t = Q_{t-1} \prod_i \left(\frac{m_{it}}{m_{i,t-1}}\right)^{\frac{1}{2}\{s_{it} + s_{i,t-1})}$$

where $s_{it}=\pi_{it}m_{it}/M_t$ is the $i^{th}$ expenditure share and

$$M_t = \sum_{j=1}^{n} \pi_{jt} m_{jt}$$

is the total expenditure on monetary services (assets).

Under these aggregation assumptions, there exists a demand function for monetary services that is a function of the opportunity cost of monetary services, called the dual user cost. This dual user cost can be tracked by a user cost index, $\Pi_t$, that satisfies Fisher's weak factor reversal formula:

$$\Pi_t = \Pi_{t-1}\left(\frac{M_t \;/\; M_t}{Q_t \;/\; Q_{t-1}}\right).$$

Under these aggregation assumptions, the economic aggregates contain all information about the component monetary assets and user costs that is relevant to the behavior of other variables. Consequently, the dispersion of the component growth rates contains no additional macroeconomic information. See Barnett and Serletis (1990), and Anderson, Jones, and Nesmith (1997 a) for discussions of these issues.

Although the aggregation conditions are typically maintained in macroeconomics, the assumptions for exact aggregation are strong. Barnett and Serletis (1990) have argued that a test for the existence of additional information in measures of component dispersion would be a diagnostic test for the existence of aggregation error. They also suggest adding such measures to an economic model as a correction for aggregation error.

The log change of the monetary services index is

$$DQ_t = \sum_{i=1}^{n} \bar{s}_{it} Dm_{it}$$

where $\bar{s}_{it} = \frac{1}{2}(s_{it} + s_{i,t-1})$, and D is the log change operator defined by $Dx_t = \log(x_t) - \log(x_{t-1})$. The log change (growth rate) of the monetary services index is a weighted average of the log changes (growth rates) of the component asset stocks. Because the weights are average expenditure shares, Theil (1967) observed that the growth rate of the Törnqvist-Theil index has the interpretation of a share weighted mean of component growth rates. This is the intuition behind stochastic index number theory; the average shares induce a valid probability measure, and the growth rate of the quantity indexes are means of component quantity growth rates.[4] Theil (1967) defined the Törnqvist-Theil second moments, which have a direct interpretation as measures of component dispersion. In particular, the monetary services quantity variance, K, is defined as:

$$K_t = \sum_{i=1}^{n} \bar{s}_{it} (Dm_{it} - DQ_t)^2 .$$

Törnqvist-Theil second moments were first used empirically with monetary data in Barnett, Offenbacher and Spindt (1984). Törnqvist-Theil second moments have been found to contain information relevant to macroeconomic variables in two empirical studies, Barnett and Serletis (1990) and Barnett, Jones, and Nesmith (1995). These papers suggest that aggregation error is present in monetary models and that inclusion of the monetary services second moments in monetary models can correct for that aggregation error. We include the Törnqvist-Theil quantity variance in our cointegration analysis as a correction for aggregation error.

### 2.3 Cointegration Analysis

Cointegration has been widely used to study monetary variables, for example Johansen (1992 b, 1995 b). Only a few such studies have used theoretically consistent monetary aggregates. For example using data for the United States, Barnett and Xu (1995) and Chrystal and MacDonald (1994) found that monetary services indexes are cointegrated with income, prices, and interest rates. Jones (1998) conducted an extensive I(2) analysis of theoretically consistent monetary systems. Additional studies that used monetary services indexes for other countries include Serletis and King (1993) for Canada, and la Cour (1995) for Denmark. The cointegration analysis in this paper extends these previous studies in two ways. Our system includes the dual user cost index and the monetary services quantity variance which can provide a correction for certain types of aggregation error.

### 2.3.1 Cointegration Theory

Granger (1981, 1983) first defined the concept of cointegration. Let X(t) be a vector stochastic process, $X(t) = (x_1(t), \ldots, x_n(t))^T$, and let $d = (d_1, \ldots, d_n)^T$ be a vector of integer values, where each of the system variables, $x_i(t)$, is integrated of order $d_i$, denoted $x_i(t) \sim I(d_i)$. By this we mean that the $d_i^{th}$ difference of $x_i(t)$ is stationary, denoted $\Delta^{d_i} x_i(t) \sim I(0)$, for all i. In this paper, we assume that d is a vector of zeros and ones, and therefore the system is said to be I(1). There may exist some linear combinations of elements of X(t) that are I(0), which is the definition of linear I(1) cointegration. Specifically, an I(1) system, X(t), is said to be cointegrated if there exists a non-zero vector $\beta_i$ such that $\beta_i^T X(t)$ is I(0), and $\beta_i$ is called a linear cointegration vector. Granger (1991) has suggested a number of generalizations of cointegration, including nonlinear cointegration, in which there is a nonlinear function, f, such that $f(x_1(t), \ldots, x_n(t))$ is stationary. If any of the variables in the system are I(0), there will be a trivial cointegrating vector consisting of zeros in all entries, except in the entry corresponding to the stationary variable. Non-trivial linear cointegration is an interesting property, because linear combinations of non-stationary processes are generally non-stationary. Although cointegration produces stationary stochastic processes, $\beta_i^T X(t)$, these processes may be nonlinear, because, like non-stationarity, nonlinearity is a dominant property.

Johansen (1988, 1991) provides a maximum likelihood estimation procedure for determining the number of significant (linear) cointegration vectors, under the assumption that X(t) is a vector of processes that are multivariate Gaussian as well as I(1). We use the Johansen procedure in the following analysis.

The Johansen procedure starts from the vector error correction model (VECM):

$$\Delta \boldsymbol{X}(t) = \mu + \sum_{i=1}^{q-1} \Gamma_i \Delta \boldsymbol{X}(t-i) - \alpha\beta^T \boldsymbol{X}(t-1) + \varepsilon_t ,$$

where X(t) is the p dimensional I(1) vector stochastic process, q is the lag length of the underlying vector autoregression (VAR), $\mu$ is a p by 1 vector of constants, the $\Gamma_i$'s are p by p coefficient matrices of short-run effects, $\alpha$ and $\beta$ are p by r matrices and $\varepsilon_t$ is the serially independent Gaussian residual of the underlying VAR, for t = 1,...,T and X(1-q),...,X(0) fixed.[5]

The hypothesis, $Hr_0$, that there are at most $r_0$ linear cointegrating vectors, can be stated as the reduced rank condition on the $\alpha$ and $\beta$ matrices, that $r = r_0$ is strictly less than p. The r estimated cointegration vectors comprise the rows of $\beta^T$. This reduced rank hypothesis can be tested and the $\alpha$ and $\beta$ matrices can be estimated using the maximum likelihood procedure described in Johansen and Juselius (1990). The number of significant cointegrating vectors can be determined using either the trace test or the maximum eigenvalue test statistics described in Johansen and Juselius (1990).

### 2.3.2 Empirical Results

Details of our empirical results are available in Barnett, Jones, and Nesmith (1999). In this section, we summarize and discuss some of those results.

The system of variables estimated are: monetary service indexes (at both the M2 and L levels of aggregation, denoted MSIM2 and MSIL), the dual user cost indexes (at M2 and L levels of aggregation, denoted DUALM2 and DUALL), the monetary services quantity variance (KM2 and KL), industrial production (IP), and the consumer price index (CPI). The monetary services indexes, their dual user cost indexes, and their second moments are calculated using the monthly seasonally adjusted quantity, own rate, and benchmark rate data from Thornton and Yue (1992), for the periods 1960:1 - 1992:12.[6] IP proxies for monthly output as the income variable, and the CPI proxies for the appropriate monthly output deflator.[7] We estimate the model in log form (except for the quantity variances which are not logged), and the system is estimated both with and without K. All variables have been seasonally adjusted, based on the X-11 procedure, and deterministic seasonal dummies are not included in the system. The lag length, q, in the VECM is set by a sequence of Sims' corrected likelihood ratio tests. The likelihood ratio tests chose a lag length of three for all four estimations.

In order to implement the VECM analysis we test the hypothesis that the system is I(1). One method of insuring that the system is not stationary is to test the hypothesis that there are trivial cointegration vectors, $\beta_i$, satisfying $\beta_i^T X(t) = x_i(t)$, i.e. that $x_i(t)$ is itself stationary. This test rejects stationarity for all of the variables tested, insuring that the system is at least I(1). Univariate augmented Dickey-Fuller (ADF) tests confirm that the variables are non-stationary, with the possible exception of the quantity variances. The results of univariate testing are sensitive to the lag lengths in the regression from which the ADF tests are conducted. Most variables appear to be I(1), but at lag lengths significantly larger than three there is some evidence that the CPI is integrated of higher order.

In this section, we maintain that the system is I(1). This maintained hypothesis is consistent with most previous research on cointegration, see Miyao (1996) for a survey. In contrast, King. Plosser, Stock, and Watson (1991) and Friedman and Kuttner (1992) have assumed that US monetary aggregates and the price level are I(2), but that real monetary aggregates are I(1). Recently, Johansen (1995), Jorgenson, Kongsted, and Rahbek (1996), and Parulo (1996) have developed maximum likelihood estimation procedures that can be used to test for the existence of I(2) roots within the VECM framework. These I(2) procedures generalize the I(1) procedures used in this paper. Jones (1998) conducted a systematic I(2) VECM analysis of US monetary systems using both conventional and aggregation theoretic monetary aggregates, for the period 1954-1997. Jones found that previous empirical characterizations from I(1) analyses are not robust in an I(2) VECM analysis, and that nominal monetary aggregates and prices may be I(2).

Most published cointegration studies report the results of various whiteness tests. We implemented the following univariate residual tests: a corrected Jarque-Bera (normality) test; LM tests for ARCH, and LM tests for serial correlation, as discussed in L.G. Godfrey (1988). We also implemented the multivariate LM serial correlation tests, as described in Johansen (1995b). We eliminated models at the M1 and M3 levels of aggregation based on evidence of serially correlated residuals. The M2 and L models performed very well on all serial correlation tests, but the univariate residuals showed evidence of both non-normality and ARCH. As noted in Johansen (1995 b), the rejection of normality is not serious, because the asymptotic properties of the maximum likelihood estimator only depend on the i.i.d. assumptions. But for the same reason, the finding of ARCH may be more damaging.[8]

Based on the above findings, we assume that X(t) = (MSIM2 or MSIL,DUALM2 or DUALL, CPI, IP,KM2 or KL) is an I(1) vector stochastic process, and it is therefore appropriate to implement the Johansen estimation procedure. In Barnett, Jones, and Nesmith (1999), we report the trace test statistic and the maximum eigenvalue test statistic for the number of significant cointegration vectors in the various models. Both tests indicate the existence of a unique cointegration vector at both levels of aggregation for the models without the monetary services quantity variances. When the quantity variances are added to the models, two cointegration vectors are significant at each level of aggregation.

The cointegration analysis produces stationary linear combinations of variables, from the I(1) system. However, the stationary linear combinations are not necessarily linear stochastic processes. In testing for nonlinearity, data is typically differenced until it appears stationary and the transformed data is then tested for nonlinearity, whereas cointegration analysis provides a model based method of producing stationary series. In the remainder of this section, we will apply Hinich's bispectrum test to the stationary linear combinations produced by this cointegration analysis.

The interpretation of cointegration relationships can be tenuous, although it is possible to state and hypothesis test identifying restrictions, in the model. In the system we are examining, the logical approach would be to attempt an identification of the cointegration relations as linearized money demand functions in implicit form. The variables in our system are consistent with the aggregation theory discussed in section 2.2, and would therefore be more amenable to such an identification than an analysis using theoretically inconsistent monetary aggregates. Such an analysis, although interesting, is beyond the scope of this paper.

## 2.4 Testing for Nonlinearity

In this section, we review bispectrum theory and estimation. Basic discussions on higher order polyspectra can be found in Nikias and Raghuveer (1987), Mendel (1991), Brillinger and Rosenblatt (1967 a, b), and Brillinger (1965). The first use of the bispectrum in economics was by M. D. Godfrey (1965). The following sections have drawn upon Hinich and Patterson (1989), Stokes (1991), and Barnett and Hinich (1992).

### 2.4.1 Nonlinear Testing Statistical Theory

We assume that X={x(t)} is a real, mean zero, third order stationary stochastic process.[9] Third order stationarity implies that the mean function, $c_x(t) = E[x(t)]$, is zero for all t, the covariance function $c_{xx}(t_1,t_2) = E[x(t_1)x(t_2)]$, is a function only of $\tau_1=(t_1-t_2)$, and the general third order moments, $c_{xxx}(t_1,t_2,t_3) = E[x(t_1)x(t_2)x(t_3)]$, are a function of only two variables, $\tau_1=(t_1-t_2)$ and $\tau_2=(t_2-t_3)$.

If $c_{xx}(\tau_1) = 0$ for all $\tau_1 \neq 0$, then the process is said to be white noise. If the distribution of $\{x(n_1), \ldots ,x(n_T)\}$ is multivariate normal for all $n_1,\ldots,n_T$, then X is said to be Gaussian. X is said to be a pure white noise process if $\{x(n_1), \ldots ,x(n_T)\}$ are independent random variables for all values of $\{n_1,\ldots,n_T\}$. Priestly (1981) and Hinich and Patterson (1985) have emphasized that stochastic independence and whiteness are not the same. All pure white noise processes are white, but white noise processes are not, in general, pure white noise processes, although for

Gaussian processes the conditions are equivalent. If the process is not Gaussian, then, in general, the third order moments are not zero. Testing for pure white noise using only the covariance function implicitly assumes that the process is Gaussian. Such a procedure ignores the potential existence of nonlinear serial dependence, which can only be detected using higher order moments. It is thus necessary to test residuals for both Gaussianity and nonlinearity in addition to whiteness, since structural disturbances often are assumed to be pure white noise.[10]

The following definitions of the power spectrum and bispectrum are for discrete stochastic processes, although these definitions can be generalized. See, for example, Hinich and Messer (1995). The power spectrum (second order cumulant polyspectrum), $P(\omega)$, is defined as the Fourier transform of the covariance function, i.e.

$$P(\omega) = \sum_{\tau=-\infty}^{\infty} c_{xx}(\tau)\exp[-i(\omega\tau)], \text{ for frequencies (in radians) } |\omega| < \pi .$$

The bispectrum (third order cumulant polyspectrum) is defined as the second order Fourier transform of the third order moment function, i.e.

$$B_{xxx}(\omega_1,\omega_2) = \sum_{r=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} c_{xxx}(r,s)\exp[-i(\omega_1 r + \omega_2 s)],$$

for frequencies in the principal domain, $\Omega$, which is defined as

$$\Omega = \{(\omega_1,\omega_2): 0 \le \omega_1 \le \pi, \omega_2 \le \omega_1, 2\omega_1 + \omega_2 \le 2\pi\}.$$ [11]

For extensive discussion of the principal domain and the symmetries of the bispectrum see Hinich and Messer (1995).

The bispectrum can be interpreted by considering the Cramér spectral representation of X,

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t} dZ(\omega),$$

where, $E[dZ(\omega)] = 0$, $E[dZ(\omega_1)dZ^*(\omega_2)] = \begin{cases} 0, & \omega_1 \ne \omega_2 \\ 2\pi P(\omega)d\omega, & \omega_1 = \omega_2 = \omega \end{cases}$, and $E[dZ(\omega_1)dZ(\omega_2)dZ^*(\omega_3)] = \begin{cases} 0, & \omega_1 + \omega_2 \neq \omega_3 \\ B(\omega_1, \omega_2)d\omega_1 d\omega_2, & \omega_1 + \omega_2 = \omega_3 \end{cases}$.[12]

As noted by Nikias and Raghuveer (1987), the power spectrum describes the contribution to the expectation of the product of two Fourier components whose frequencies are the same, whereas the bispectrum describes the contribution to the expectation of the product of three Fourier components where one frequency is equal to the sum of the other two.

It is now easy to motivate the use of the bispectrum in our analysis. If W={w(t)} is a mean zero, third order stationary, pure white noise process, it will have both a flat power spectrum and a flat bispectrum. If X is a linear process (i.e. it is the output of a linear time invariant filter applied to a pure white noise process) such that $x(t) = \sum_{n=0}^{\infty} a(n)w(t-n)$, then the normalized squared-skewness function, $\Gamma(\omega_1, \omega_2)$, will be equal to a constant for all frequency pairs in the principal domain, where

$$\Gamma^2(\omega_1, \omega_2) = \frac{|B_{xxx}(\omega_1, \omega_2)|^2}{P_{xx}(\omega_1)P_{xx}(\omega_2)P_{xx}(\omega_1 + \omega_2)}.$$[13]

If, in addition, X (W) is Gaussian, then $\Gamma(\omega_1, \omega_2)=0$ for all frequency pairs in the principal domain.

### 2.4.2 Estimation

The mathematical theory relating the normalized squared skewness function to linearity and Gaussianity has been used to derive testing procedures by Hinich (1982) and Rao and Gabr (1980). The procedure used in this paper is the one derived in Hinich (1982). Details of the Hinich test are also discussed in Hinich and Patterson (1985, 1989) and Ashley, Hinich, and Patterson (1986).

The conventional methods of bispectrum estimation are reviewed in Nikias and Raghuveer (1987). The bispectrum can be estimated consistently from a finite sample {x(1), ..., x(N)} by the following procedure. Segment the record of N observations into K (non-overlapped) blocks of L observations each, L is called the block-length.[14] The parameter K/N = 1/L, is the resolution bandwidth.[15] Define, for k=1,...,K, the bi-periodogram as

$$G_k(f_i, f_j) = \frac{1}{L} X_k(f_i) X_k(f_j) X_k^*(f_i + f_j), \text{ where } X_k(f) = \sum_{n=(k-1)L+1}^{kL} x(n)\exp[-i2\pi fn / N]. \text{ A}$$

consistent and asymptotically normal estimator for the bispectrum is

$$\hat{B}_{xxx}(f_i, f_j) = \frac{1}{K}\sum_{k=1}^{K} G_k(f_i, f_j), \text{ where } 2f_i + f_j < N \text{ and } 0 < f_j < f_i < N, \text{ and } f_i = i/L\ (i=1,2,...,L).$$

See Hinich and Messer (1995) for details on the estimator.[16] This type of estimator is analogous to the direct estimator of the power spectrum described in Welch (1967), and Groves and Hannan (1968), in which the data record is segmented into frames, and periodograms are computed frame by frame, and then averaged at each frequency. Hence, the power spectrum estimator is

$$\hat{P}_{xx}(f_i) = \frac{1}{K}\sum_{k=1}^{K} I_k(f_i), \text{ where the periodogram is defined as } I_k(f_i) = \frac{1}{2\pi L} X_k(f_i) X_k^*(f_i),$$

k=1,2,...,K.[17] In the bispectrum case, bi-periodograms are computed frame by frame and then averaged at each frequency pair. It is the final averaging step which leads to consistency of the estimator in both cases. The variance is reduced by averaging over more frames, but at a cost of reduced resolution.[18]

As will be detailed below, we estimate the bispectrum over a range of values for the block-length, L, in accordance with a suggestion of Stokes (1991). The suggested range of block-lengths is $(N/3)^{1/2}$ to $(N)^{1/2}$, which for our sample size (N=396) corresponds to a range of block-lengths, between 12 and 19. See Stokes (1991) for an example using a well known gas data model. The setting L=12, corresponds to $N^{.42}$, and is the closest to Hinich's suggestion of $N^{.4}$.

The Hinich test for nonlinearity produces a test statistic Z, which is distributed as the standard normal under the null hypothesis of constant skewness (linearity). The Hinich Gaussianity test also produces a test statistic G, which is a standard normal under the null of zero skewness (Gaussianity). Both tests are one sided, and the null is rejected if the test statistics are large.

The Hinich test is extremely conservative. If the stochastic processes X is linear, then all of its polyspectra of order greater than two are constant. The Hinich test is based only on the bispectrum. A rejection of its null would be a strong result, because the null includes all linear processes and some nonlinear processes. Consequently, the Hinich test cannot confirm linearity. It can only fail to reject it. In principle, we could test for nonlinearities using polyspectra of higher order than the bispectrum, but estimating even the trispectrum would not be feasible for the sample sizes of most economic data sets.

The conservatism of the Hinich test has been reflected in empirical studies. For example, Barnett, Gallant, Hinich, Jungeilges, Kaplan, and Jensen (1994) find that the Hinich test was much less likely to reject its null than other competing tests, such as the BDS test (Brock, Dechert, Scheinkman, and LeBaron, 1996) and the Kaplan (1993) test. In particular, Hong (1996) notes that the third order cumulants of an ARCH (autoregressive conditional heteroskedastic) process can be identically zero, in which case the bispectrum test would fail to reject linearity. Barnett, Gallant, Hinich, Jungeilgies, Kaplan, and Jensen (1994) demonstrate that empirically the Hinich test has low power against ARCH. Nevertheless, Ashley, Hinich, and Patterson (1986) show that the Hinich nonlinearity test does have substantial power (at reasonable sample sizes) against many commonly considered forms of nonlinear serial dependence.

The Hinich test has been applied previously in economic analysis. Barnett and Hinich (1992) find that Divisia monetary aggregate data exhibit deep nonlinearity at the M1 level of aggregation. Hinich and Patterson (1987) examine trade by trade stock market data for evidence of nonlinearity.

### 2.4.3 Cointegration and Nonlinearity

Cointegration in an I(1) economic system implies the existence of stationary linear combinations of the I(1) system variables. If **X**(t) is I(1) and $\boldsymbol{\beta}$ is a cointegration vector, then the derived stochastic process $\zeta_t = \boldsymbol{\beta}^T\mathbf{X}(t)$ is stationary, where T designates transpose. According to the Wold decomposition, a stationary stochastic process with no deterministic components can be represented as a moving average process, MA($\infty$). See Brockwell and Davis (1991, pp 187) and Engle and Granger (1987). Consequently, the derived stochastic process can be represented as the output of a linear time invariant filter applied to a white noise input process:

$$\zeta_t = \boldsymbol{\beta}^T\mathbf{X}(t) = \sum_{n=0}^{\infty} a(n)u(t-n) ,$$

where U = {u(t)} is a white noise process. The derived stationary stochastic process is not a linear stochastic process, unless U is pure white noise.

The Hinich bispectrum test for nonlinearity was designed to test stationary stochastic processes for nonlinearity, and hence can be applied to the stationary linear combinations we found in Section 3. We implement the Hinich bispectrum test to investigate the possibility that the components of $\boldsymbol{\beta}^T\mathbf{X}(t)$ are stationary nonlinear stochastic process. The results are reported in Barnett, Jones, and Nesmith (1999).

If the null hypothesis of linearity is rejected, then there is neglected structural nonlinearity in the economic system. Linear combinations of nonlinear stochastic processes are, in general, nonlinear stochastic processes. Thus, if any of the components of **X**(t) are nonlinear, we would expect the components of the derived process, $\zeta_t$, to also be nonlinear. Rejection of the null could also occur if the maintained hypothesis of the test is invalid. The maintained hypotheses of the Hinich test is third-order stationarity of the process, and absolute summability of the second and third-order moments. If the conclusions of the maximum likelihood analysis were in error, then the maintained hypothesis of the Hinich test might be invalid. The maximum likelihood analysis is based on the Gaussian likelihood function, although the asymptotic properties of the analysis only depend on the assumption that the errors are independent and identically distributed. The existence of nonlinear serial dependence in the error structure of the VECM could invalidate the estimation and rank identification of the model. Although most published cointegration studies report the results of tests for serial correlation in the residuals of the VECM, as well as the results of ARCH tests, most studies do not report the results of tests for general nonlinear serial dependence in the residuals of the VECM.

### 2.5 Empirical Results

The results of the Hinich bispectrum test for our models are reported in Barnett, Jones, and Nesmith (1999). We find that in the linear combinations which do not include the quantity variance, there is no evidence of nonlinearity. In the linear combinations which do include the quantity variance, we find evidence of nonlinearity. Specifically, for the more significant linear combination, which heavily weights the quantity variance, we find some evidence of nonlinearity at every level of aggregation. This is not surprising, because the univariate tests detect nonlinearity in the quantity variances (See Barnett, Jones, and Nesmith (1999)). We find the strongest evidence of nonlinearity in the second linear combination at the L level of aggregation. This result does not hold at the M2 of aggregation; the second linear combination at the M2 level of aggregation shows no evidence of nonlinearity.

In this section, we report on our tests for the existence of nonlinearity in the cointegration relations of a system containing money demand variables, by applying the Hinich bispectrum test. We find some evidence of nonlinearity in spite of that test's inherent conservatism, and therefore demonstrate that the issue is empirically relevant. Although the testing of identification restrictions is beyond the scope of the current paper, the most probable source of the cointegration we investigate would be the existence of long run money demand relations. This interpretation would be more reasonable in our study than in many other studies, due to our use of aggregation theory in the selection of the variables in our system. There are many avenues for further research on this topic. The testing of nonlinearity in cointegration relations could be extended to other contexts, notably systems including consumption and investment. In addition, greater attention could be paid to formal identification of these relations. In future applications of cointegration analysis, we suggest that nonlinear cointegration models be investigated, as suggested by Granger (1991). Approaches that have the ability to remove nonlinear structure by modeling the source of the nonlinearity within the equations should also be investigated. We conclude that stationarity of the linear combination of cointegrated variables should not be viewed as sufficient for linearity of the derived process without testing that null.

## 3. Technology Modeling: Curvature is not Sufficient for Regularity

### 3.1. Introduction

Recently there has been a growing tendency to impose curvature, but not monotonicity, on specifications of technology. This practice is especially common with the currently popular generalized McFadden (also called generalized quadratic) model. We believe that this practice of overlooking monotonicity should not be taken lightly. Regularity requires satisfaction of both the curvature and the monotonicity conditions. Without both satisfied, the second order conditions for optimizing behavior fail, duality theory fails, and the specification should be viewed as compromised in a serious manner.

An earlier practice with "flexible functional forms" was to impose neither monotonicity nor curvature, but check those conditions at each data point *ex post*. Experience in that tradition has suggested that when violations of regularity occur, they are much more likely to occur through violations of curvature conditions than through violations of monotonicity conditions. Based upon those results, the more recent approach of imposing curvature alone seems constructive and reasonable. But once curvature is imposed without the imposition of monotonicity, the earlier observation may no longer apply. Permitting a highly parameterized function to depart from the neoclassical function space is usually fit-improving, regardless of whether or not the neoclassical null would pass a hypothesis test. With curvature imposed, the only way that an estimator's fit can be improved spuriously in that manner is through violations of monotonicity. In short, violations of monotonicity that had not occurred prior to imposition of curvature can be induced by imposition of curvature.

In addition, a further complication can be produced by imposing curvature. In those studies that follow this recent approach, the technology usually is specified as a composite function, including an outer function and inner aggregator subfunctions. Curvature is imposed both on the outer and the inner subfunctions. But if the outer function violates monotonicity in the level of the inner aggregate, then curvature of the composite function can fail. This unfortunate possibility follows from the fact that changing the sign of a concave function can produce a convex function. Hence the currently growing approach to imposing curvature may not only result in violations of monotonicity, but in violations of curvature itself.

We believe that the recent literature that takes curvature seriously is to be welcomed as a constructive step in a positive direction. See, e.g., Diewert and Wales (1987,1988, 1995). But the potential damage done by violations of monotonicity should not be underestimated, when curvature is imposed. Barnett and Pasupathy (2003) and Barnett (2002) explore that issue in their papers, and we discuss their results below.

One productive manner in which this issue could be investigated would be to set the model's parameters at various levels that are consistent with plausible elasticities and then produce the regular region within which the model satisfies monotonicity, conditionally upon imposition of curvature. Experiments of that sort have been done with the earlier flexible functional forms by Caves and Christensen (1980), Barnett and Lee (1985), and Barnett, Lee, and Wolfe (1987). Such experiments have not been conducted with the newer generalized McFadden model, although we currently are working on that experiment. While we believe that such experiments should be conducted, we believe that a logically prior investigation would display the isoquants and investigate regularity of technologies estimated with actual data. If problems are revealed by that econometric investigation, then the more complicated experiment becomes warranted. Barnett and Pasupathy (2003) and Barnett (2002) provide the results of an exploration of properties of the model conditional upon estimation of the parameters, and we discuss their results below.

Since Barnett and Pasupathy (2003) and Barnett (2002) condition their investigation upon estimation of the parameters, rather than upon a large range of possible settings of the parameters, we believe that estimation in a manner consistent with the state of the art is appropriate, and the state of the art currently could be viewed as generalized method of moments (GMM) estimation of Euler equations for production under risk. Barnett and Pasupathy (2003) and Barnett (2002) apply an approach to the estimation of technology parameters when financial assets are included among either the inputs or outputs of the technology and explores both the monotonicity and curvature properties of the resulting technology. The relevant technologies are those of the financial intermediaries that produce inside money as output services and the nonfinancial firms that demand financial services as inputs to production technology. Virtually every firm in the economy falls into one of those two groups. The need for GMM estimation results from the inclusion of monetary assets in the model, with interest paid at the end of the period and thereby unknown at the start of the period.

The specification of technology that Barnett and Pasupathy (2003) and Barnett (2002) use is the currently popular generalized McFadden (generalized quadratic) model. That model can be estimated subject to imposition of global curvature, but monotonicity can be imposed only at one point without compromising the model's flexibility. Barnett and Pasupathy (2003) and Barnett (2002) impose curvature globally on all inner and outer subfunctions of the technology, but they impose monotonicity only at a point. They do check for monotonicity at each data point.

### 3.2. Financial Intermediaries

One of the recent approaches to modeling financial intermediaries is to model them as profit maximizing neoclassical multiproduct firms, which produce financial services, such as demand deposits and time deposits, as outputs by employing financial and non financial factors as inputs. Early work that used this approach was based on the assumption of perfect certainty. See Hancock (1985, 1987, 1991), Barnett (1987), and Barnett and Hahm (1994). Barnett and Zhou (1994a,b) extended this approach to the case of uncertainty. Barnett, Kirova, and Pasupathy (1995) introduced capital accumulation and relaxed the assumption of "no retained earnings." They also rigorously nested exact monetary output aggregates within the transformation function of the financial intermediary and report the behavior of the resulting exact monetary aggregate.

The resulting model can be viewed as a step in the direction of exploring technological change and economies of scale and scope in financial intermediation in a manner that is invariant to central bank policy intervention, and in a manner that can produce inside money aggregates that are consistent with the theory that produced the policy invariant Euler equations. In our current investigation of the regularity properties of the model, we use Barnett, Kirova, and Pasupathy's (1995) model and estimates as an illustration of the implications of the generalized Mc Fadden model under state-of-the-art application and estimation.

### 3.3. Financial Firm's Production Model

The theoretical model builds on the ground work set forth by Barnett and Zhou (1994a) in their adaptation and application of Hancock's (1991) specification of the variable profit function. The financial firm uses real resources such as labor, capital, and other material inputs, plus a monetary input in the form of cash, in the production of the services of the produced liabilities. The output of the firm in Barnett, Kirova, and Pasupathy's (1995) application consists of demand deposits and time deposits, which are liabilities to the firm.

Let $Y_t$ be the real balances of the asset (loan) portfolio, $y_{i,t}$ the real balances of the *i*th produced account (liability) type, $C_t$ the real balances of cash holdings, $z_{j,t}$ the quantity of *j*th real input (including labor), and $K_t$ the quantity of capital stock of the financial firm at time *t*.

In the model, $y_{i,t}$ constitute the outputs of the financial firm, while $C_t$, $z_{j,t}$, and $K_t$ are the inputs. Let $R_t$ be the portfolio rate of return, which is unknown at the beginning of period *t*, and let $h_{i,t}$ be the holding cost per dollar of the *i*th liability. All financial transactions are contracted at the beginning of the period. Interests on the deposits are paid at the end of the period. The cost per unit of the *j*th real input, $w_{j,t}$, is incurred at the beginning of the period. Let $P_{K,t}$ be the cost of capital and $P_t$ be the general price index, which is used to deflate nominal to real terms.

Variable profits (net of investment expenditure), $\pi_t$ , at the beginning of period *t*, can be represented by

$$\begin{aligned}\pi_t &= (1+R_{t-1})Y_{t-1}P_{t-1} - Y_t P_t + C_{t-1}P_{t-1} - C_t P_t \\ &\quad + \sum_{i=1}^{I}[y_{i,t}P_t - (1+h_{i,t-1})y_{i,t-1}P_{t-1}] - \sum_{j=1}^{J} w_{j,t}z_{j,t} - P_{K,t}I_t\end{aligned}$$

The first two terms in the above equation represent the change in variable profits from rolling over the loan portfolio during period *t*. The third and fourth terms represent the change in the nominal value of excess reserves. The fifth term represents the change in the firm's variable profits from the change in the issuance of produced financial liabilities. The sixth term constitutes payments for real inputs, and the last term is the expenditure on investments.

Portfolio investment , $Y_t$, is constrained by total available funds. The constraint is given by

$$Y_t P_t = \sum_{i=1}^{I}[(1-k_{i,t})y_{i,t}P_t] - C_t P_t - \sum_{j=1}^{J} w_{j,t}z_{j,t} - P_{K,t}I_t$$

where $k_{i,t}$ is the required reserves ratio on the *i*th produced liability. This equation implies that the total deposits $\sum_{i=1}^{I} y_{i,t}P_t$ are allocated to required reserves, excess reserves, payment for all real inputs used in production, investment in capital, and investment in loans.

The time to build approach is adopted to model capital dynamics. Capital accumulation based on this approach is given by:

$$K_t = I_{t-1} + (1-\delta)K_{t-1}$$

where the depreciation rate δ is a constant and is assumed to be given. Gross investment at time *t-1*, $I_{t-1}$, becomes productive only in period *t*. By substitution, Barnett, Kirova, and Pasupathy (1995) derive the variable profits at time *t*.

The financial firm maximizes the expected value of the discounted intertemporal utility of its variable profits stream, subject to the firm's technological constraint. The firm's optimization problem is then given by:

$$Max \quad E_t\left[\sum_{s=t}^{\infty}\left(\frac{1}{1+\mu}\right)^{s-t} U(\pi_s)\right]$$

$$s.t. \quad \Omega(y_{1,s},\ldots,y_{I,s},C_s,z_{1,s},\ldots,z_{J,s},K_s)=0 \quad \forall s \geq t$$

where $E_t$ is the expectation at time *t*, $\mu$ is the subjective rate of time preference, $U$ is the utility function, $\pi_s$ is the variable profit at time *s*, and $\Omega$ is the transformation function.

The Hyperbolic Absolute Risk Aversion (HARA ) class of utility functions can be represented by:

$$U(\pi_t) = \frac{1-\rho}{\rho}\left(\frac{h}{1-\rho}\pi_t + d\right)^{\rho},$$

where $\rho$, *h* and *d* are parameters to be estimated. Barnett, Kirova, and (1998) used the power function special case of the HARA class utility function:

$$U(\pi_t) = \frac{1}{\rho}\pi^{\rho}.$$

Using Bellman's method and the Benveniste and Scheinkman equation, Barnett, Kirova, and Pasupathy (1995) obtained the Euler equations. Since closed form algebraic solutions rarely exist for Euler equations, Barnett, Kirova, and Pasupathy (1995) can solve only numerically for $(y_{1,t},...,y_{I,t},C_t,z_{1,t},...,z_{J,t},K_t)$ from the system of Euler equations and the transformation function, Ω. The parameter estimation can be done through the estimation of Euler equations under rational expectations by using Hansen and Singleton's (1982) Generalized Methods of Moments (GMM) estimation.

### 3.4 Output Aggregation for Financial Intermediatries

The financial firm's outputs consist of demand deposits and time deposits. The financial firm's output of demand and time deposits are important in determining the level of inside money in the economy. In this section, we find the aggregation-theoretic exact quantity output aggregate that measures the firm's produced service flow. Relative to the money markets, our aggregation in this case is on the supply side. In a later section below, when we investigate monetary service factor demand by nonfinancial manufacturing firms, we will be producing demand side monetary aggregates, as also is relevant to consumer demand for monetary services.

Generating the exact quantity aggregate consists of first identifying the components over which aggregation is admissible and then determining the aggregator function defined over the identified components. The first step determines the existence of an exact aggregate, and the second step produces that aggregate in the manner that is consistent with microeconomic theory. The second step cannot be applied unless the first step succeeds in identifying the existence of an admissible cluster of components. The condition for the existence of an admissible component group is *blockwise weak separability*. In accordance with the definition of weak separability, a component grouping is admissible if and only if the group can be factored out of the rest of the economy's structure through a subfunction. Then the economic structure can be represented in the form of a composite function, with the goods in the separable block being the only goods in the inner function of the structure. If this condition is satisfied, an exact quantity aggregate exists over the goods in the block, and the aggregator function that produces the exact quantity aggregate over those goods is the inner ("category") function itself. Without weak separability, no such inner function exists and hence no aggregate exists.

Let $\mathbf{y} = (y_{1t}, \ldots, y_{It})'$ be the firm's output vector, and let $\mathbf{x} = (x_{1,t}, \ldots, x_{J,t})'$ be the input vector, so that transformation function can be written as $\Omega(\mathbf{y}, \mathbf{x}) = 0$ An exact supply side aggregate exists over all of the firm's outputs if and only if **y** is weakly separable from **x** within the function $\Omega$. In accordance with the definition of weak separability, there then exist two functions $H$ and $y_0$ such that

$$\Omega(\mathbf{y}, \mathbf{x}) = H(y_0(\mathbf{y}), \mathbf{x}),$$

where the output aggregator function, $y_0(\mathbf{y})$, is a convex function of **y**. Although weak separability alone is sufficient for the existence of an aggregate, a considerable ( although unnecessary) simplification is available if we also assume that $y_0(\mathbf{y})$ is linearly homogeneous in **y**.

If we can test for weak separability of the transformation function and then estimate the resulting aggregator function $y_0(\mathbf{y})$, we obtain the econometrically estimated exact output aggregate. The related literature on statistical index numbers, such as Divisia and Laspeyres, seeks to produce nonparametric approximations that can track the level of $y_0(\mathbf{y})$ over time without the need to estimate the parameters of the aggregator function $y_0$ itself.

In Barnett, Kirova, and Pasupathy (1995), the econometric estimate of the aggregator function is obtained by estimating the Euler equations using the generalized method of moments (GMM) technique.

### 3.5 Testing for Weak Separability of Output for Financial Intermediaries

The conventional parametric approach to testing weak separability is adopted in Barnett, Kirova, and Pasupathy (1995), since weak separability is a strictly nested null hypothesis within our parametric specification of technology. To minimize the biases that can be produced from specification error, Barnett, Kirova, and Pasupathy (1995) use a flexible functional form for technology. Unfortunately flexible functional forms need not satisfy the regularity conditions imposed by economic theory, including the monotonicity and curvature conditions. Hence we must consider methods for testing and imposing those conditions, at least locally, as well as methods for testing and imposing global blockwise weak separability of the technology in its outputs.  For existence of aggregator functions, the weak separability must be global.  Hence we must test and impose weak separability globally. Barnett, Kirova, and Pasupathy (1995) use the Generalized McFadden functional form to specify the technology of the firm.  That specification, which also was used in the case of stochastic choice by Barnett and Zhou (1994a), was originated by Diewert and Wales (1991), who also originated the Generalized Barnett functional form.  That latter specification was applied by Barnett and Hahm (1994) in the perfect certainty case, but has not yet been adapted to the case of stochastic choice.

Barnett, Kirova, and Pasupathy (1995) assume that the transformation function, $\Omega$, is linearly homogeneous. Instead of specifying the form of the full transformation function $\Omega$, and then imposing weak separability in **y**, Barnett, Kirova, and Pasupathy (1995) directly impose weak separability by specifying $H(y_0,\mathbf{x})$ and $y_0(\mathbf{y})$ separately.  The specification for $\Omega$ is then obtained by substituting $y_0(\mathbf{y})$ into $H(y_0,\mathbf{x})$.  Since $y_0(\mathbf{y})$ and $H(y_0,\mathbf{x})$ are both specified to be flexible, the full technology $\Omega$ is flexible, subject to the separability restriction.

The function $H$ is specified to be the symmetric generalized McFadden functional form. Further details of the specification and estimation are in Barnett, Kirova, and Pasupathy (1995).

**3.6 Empirical Application for Banks**

Barnett, Kirova, and Pasupathy (1995) apply their approach to estimating the technology of commercial banks.  The outputs of that aggregated financial firm in their application consist of demand deposits and time deposits. Demand deposits and time deposits account for the major portion of the fund-providing functions of the bank's balance sheet.  The inputs used in the production process include both financial and nonfinancial inputs.  The financial input in the form of cash is excess reserves.  The nonfinancial inputs includes labor, materials, and physical capital. The output vector is given by $\mathbf{y}' = (D_t, T_t)$ and the input vector is $\mathbf{x}' = (C_t, L_t, M_t, K_t)$, where $D_t$ is demand deposits, $T_t$ is time deposits, $C_t$ is excess reserves, $L_t$ is labor input, $M_t$ is material inputs, and $K_t$ is capital.

The data used for estimating the model was mainly obtained from the Federal Reserve Bank Functional Cost Analysis (FCA) Program. Data on the National Average Banks for the years 1966-1992 were used in the estimation. Labor inputs consist of two groups: managerial and non-managerial. Data on expenditure and quantity for the two categories of labor were obtained from FCA. Material inputs are divided into three categories: printing and stationery, telephone and telegraph, and postage, freight and delivery. Physical capital is made up of structures (bank buildings), furniture and equipment, and computers. Data on expenditure on the various types of material inputs and physical capital were obtained from the FCA, while the corresponding price indices were obtained from the *Survey of Current Business*. A quantity aggregate and the corresponding price aggregate must be constructed for each of the three nonfinancial inputs.

Data on the nominal quantity of demand deposits and time deposits, net interest rate on demand deposits and time deposits, and the bank's portfolio rate of return were obtained from the FCA data set. The required reserves ratio was obtained from the Federal Reserve *Bulletin*. Nominal dollar balances of all financial goods were converted to real balances by deflating the nominal balances using Fisher's ideal price index.

### 3.7 Results for Banks

The parameter estimates were obtained by estimating the system of Euler equations using the GMM estimation procedure on mainframe TSP (version 7.02). This estimation process allows for heteroskedasticity and autocorrelation in the disturbance terms. Barnett, Kirova, and Pasupathy (1995) specified a second order moving average serial correlation. Bartlett kernels were specified for the kernel density. Discount window rate, federal funds rate, composite bond rate, lagged value of excess reserves, lagged value of interest paid on time deposits, and a constant were chosen as instruments.

The parameter estimates are reported in Barnett, Kirova, and Pasupathy (1995). Barnett and Pasupathy (2003) subsequently showed that there were violations of monotonicity, despite the fact that curvature had been imposed.

### 3.8 Manufacturing Firms

As discussed above, the supply side model used by Barnett, Kirova, and Pasupathy (1995) is an extension of the model previously produced and estimated by Barnett and Zhou (1994a), and an analogous model on the consumer demand side has been produced and estimated by Poterba and Rotemberg (1987) and by Barnett, Hinich, and Yue (1991). But this modern approach to modeling, with nested demand-side monetary aggregator functions and GMM estimation of Euler equations, had not been attempted for manufacturing firms prior to Barnett, Kirova, and Pasupathy (1995). However, in the perfect certainty case, relevant theory is available in Barnett (1987) and a positive contribution to dynamic modeling of firm demand for money, although without nested exact quantity aggregation, has been made by Robles (1993). The potential importance of this under-researched area has been emphasized by Drake and Chrystal (1994). We now provide Barnett, Kirova, and Pasupathy's (1995) model of a manufacturing firm that employs a monetary asset portfolio as inputs. They assume rational expectations under risk, and they investigate the existence of an exact aggregation-theoretic monetary asset input aggregate.

There is no unanimous agreement among economists about the specific role that money plays in the production process. But regardless of the explicit role of money in the operation of a manufacturing firm, a derived production function always exists that absorbs that motive into the firm's technology, even if no direct role exists for money inside the factory's physical production activities. When Barnett, Kirova, and Pasupathy (1995) enter the monetary asset portfolio into the firm's technology as factors of production, the technology should be understood to be the derived technology of the firm, and not necessarily the physical technology of the factory.

### 3.9 The Manufacturing Firm Model

Barnett, Kirova, and Pasupathy's (1995) model is based on Barnett's (1987) monetary aggregation-theoretic approach, extended to include uncertainty and capital accumulation. Perfect competition in all markets and risk neutrality of the firm are assumed. The objective of the firm is to maximize the expected discounted value of its future variable profit flow, subject to its production technology. The firm uses $L_t$ real units of labor, $K_t$ real units of capital goods, a vector $\boldsymbol{\varepsilon}_t$ of monetary assets, and a vector $\mathbf{x}_t$ of other variable inputs as factors of production in producing $\mathbf{y}_t$ real units of output quantities during period $t$. The firm's technology is given by the production function:

$$y_t = F(L_t, K_t, x_{1t}, ..., x_{Nt}, \varepsilon_{1t}, ..., \varepsilon_{Jt})$$

The production technology is assumed to be concave in its arguments in accordance with the properties of a neoclassical production function. In addition, $F$ is assumed to be monotonically increasing in all its arguments.

Let $y_t$ be real output with a price $p_t^y$, $x_{nt}$ be the $n$th variable input with a price $p_t^{x_n}$, where n=1,...,N, and $w_t$ be the wage rate of labor $L_t$. The $j$th component of the real balances of monetary assets, held by the firm in period $t$ is $\varepsilon_{jt}$, where $j = 1, \ldots, J$. Real balances $\varepsilon_t$ are defined to equal nominal balances divided by $p_t^*$, the true cost of living index. The return on holding nominal money balances of type j is $r_{jt}$ and is paid to the firm at the end of the period.

As the firm operates over time, it retains part of the earnings and uses them to finance its expansion and development. It is assumed that there exist markets for new and used capital. Capital accumulation is given by:

$$K_t = (1 - \delta)K_{t-1} + I_t$$

where $I_t$ is gross investment and $\delta$ is the physical depreciation rate of capital. Investment becomes productive instantaneously, but capital installation is costly to the firm. Thus the total costs of purchasing and installing $I_t$ are given by $p_t^I\left[I_t + C(I_t)\right]$, where $p_t^I$ is the price of capital goods and $C(I_t)$ is a convex function, representing the costs of adjustment associated with installing capital.

Extending Barnett's (1987, eq. 4.3) formula to include q theory capital dynamics, the firm's variable profits during period $t$ are:

$$\pi_t = y_t p_t^y - \sum_n x_{nt} p_t^{x_n} - w_t L_t + \sum_j \left[ (1 + r_{jt-1}) p_{t-1}^* \varepsilon_{jt-1} - p_t^* \varepsilon_{jt} \right] - p_t^I \left[ I_t + C(I_t) \right]$$

where

$$I_t = K_t - (1-\delta)K_{t-1}$$

The first term represents revenues from production during period $t$ . The second term is the cost of other variable inputs $x_t$, and the third term represents costs of labor. The fourth term in the equation represents the flow of funds from rolling over the firm's portfolio of monetary assets, where the first part of the term is the nominal value of the monetary asset portfolio, available at the beginning of the period as a result of last period holdings. The last term is the total cost of purchasing and installing capital during the period.

Summing over each period's discounted profit flow and substituting into to eliminate $I_t$, Barnett, Kirova, and Pasupathy (1995) obtained the intertemporal profit flow function. It is assumed that the manufacturing firm chooses the levels of output and real and monetary factors of production to maximize the expected discounted intertemporal profit flow, subject to its production technology. The first order conditions of this stochastic optimal control problem were derived by Barnett, Kirova, and Pasupathy (1995), applying Bellman's dynamic programming method.

### 3.10 Demand-Side Monetary Aggregation and Weak Separability

By estimating the parameters of the Euler equations by GMM, we can investigate properties of technology, such as returns to scale. If the firm's monetary inputs are weakly separable from output, we also can investigate the resulting exact demand-side monetary aggregate.

The approach to identifying and generating an exact theoretical demand-side monetary aggregate for a manufacturing firm is described by Barnett (1987) in the case of perfect certainty. Barnett, Kirova, and Pasupathy (1995) extend his approach to the case of risk. The procedure involves two steps. First one tests the existence condition for an exact aggregate. The existence condition is blockwise weak separability of the monetary assets in the production function from the firm's other factor inputs. Under this existence condition, it becomes possible to factor those monetary assets as a subfunction out the rest of the firm's structure.

### 3. 11 Flexible Functional Form Specification and Regularity Conditions

As in the case of the financial intermediary, Barnett, Kirova, and Pasupathy (1995) specify the technology of the firm to be Diewert and Wales's (1991) symmetric generalized McFadden flexible functional form, but now with exact nested input aggregation for financial assets rather than exact nested output aggregation. Hence the null hypothesis of exact monetary input aggregation is imposed directly on the production function F by specifying separately flexible functional forms for $H(\mathbf{y}_0,z_0)$ and $z_0(\mathbf{z})$, where $z_0 = z_0(\mathbf{z})$ is nested into $H(\mathbf{y}_0,z_0)$ to assure the desired weakly separable structure for F. It is further assumed that the production function is linearly homogeneous in its components. The extension to nonconstant returns to scale is a subject for future research.

### 3.12 Data and Empirical Application

The model is applied empirically to the aggregate US manufacturing sector with data for the period 1949-1988. Real input resources include capital, labor, and materials. Monetary inputs include two types of assets: cash on hand and in US banks and US government securities.

The data comes primarily from two sources. Data on output and factor inputs in US Manufacturing for the period 1949-1988 is acquired from the Division of Multifactor Productivity of the Bureau of Labor Statistics. The data consists of quantity and price Törnqvist indices. Capital input is defined as the flow of services from physical assets, which include equipment, structures, inventories, and land. Labor input is defined as the paid hours of all persons engaged in the sector. Materials input consists of all commodity inputs exclusive of fuel inputs.

The source of data on money balances held by manufacturing firms is the *Quarterly Financial Report for Manufacturing, Mining, and Trade Corporations* for the period 1949-1988. To convert to real units, Barnett, Kirova, and Pasupathy (1995) deflate the nominal balances by the Fisher ideal index approximation to the true cost-of-living index, computed as in section 2 above. The rates of return on cash on hand and in banks and government securities are from the City Bank database. Barnett, Kirova, and Pasupathy (1995) use the 6-month commercial paper yield as the rate of return on cash on hand and in banks and the 3-month Treasury-bill rate as the rate of return on government securities. The reason for the non-zero rate of return for cash on hand and in banks is that it does not consist solely of currency. Cash on hand and in banks in our data source is defined to be the sum of manufacturing sector holdings of currency, demand deposits, and time deposits. Separate data on such holdings of currency are available only for the most recent observations.

Barnett, Kirova, and Pasupathy (1995) use an external nominal bond rate for the rate of return on capital, $R_t$, since data was unavailable to compute an internal rate of return. Data on Moody's Baa bond rate is obtained from the City Bank database.

**3.13 Results for Manufacturing Firms**

Barnett, Kirova, and Pasupathy (1995) estimated the model using the GMM estimator in the TSP mainframe version 4.2B. The following variables are used as instruments in the estimation procedure: a constant, total US population, and lagged values of the prices of capital and materials, of the rate of return on cash and securities, and of the Moody's Baa bond rate. The results are robust to heteroscedasticity and autocorrelation.

The GMM estimates of the manufacturing firm's production function are reported in Barnett, Kirova, and Pasupathy (1995). Barnett (2002) used the parameter estimates to explore the regularity properties of the manufacturing firm's production technology. Monotonicity of the monetary aggregator function was imposed only at the point of local flexibility, and checked at all data observations. No violations were observed at the data points. Concavity of the monetary aggregator function was imposed globally.

Monotonocity of the composite production function F was imposed at only one point, and checked at all data points after estimation. Violations of monotonicity were found. But even more strangely, Barnett (2002) found that the violations of monotonicity induced violations of curvature of the composite function, even when the inner and outer functions in the model's weakly separable technology were forced to satisfy curvature globally.

**4. Conclusions**

In the results described in this paper, we find that nonlinearity produces difficult unsolved problems. In the time series literature, we find that cointegration need not imply the existence of a linear model, since the resulting cointegrated linear combination of stochastic processes may be a nonlinear stochastic process, and indeed we find precisely that result in our empirical tests. Similarly in the structural modeling literature, we find that the now common practice of imposing curvature without monotonicity may induce violations of monotonicity that would not otherwise have occurred. We do not offer solutions to these difficult problems, either in the time series case or in the structural modeling case. We only point out that nonlinearity can and does produce difficult problems that disappear when linearity is assumed. We believe that the preferred approach to research in this area is to face and to try to solve the difficult problems, rather than to hide them under unwarranted assumptions of linearity.

The research on which we report in this paper is ongoing. This paper is therefore in the form of a progress report on joint work into this inherently difficult subject. Many of our prior results in this area are collected together in Barnett and Binner (2004). We also are working on the related issues produced by bifurcation of nonlinear macroeconometrics models, and have so far found Hopf, transcritical, and singularity bifurcation boundaries located disturbingly close to the parameter point estimates in two highly regarded macroeconometric models: the Bergstrom continuous time model of the UK economy and the Leeper and Sims Euler equations model of the US economy. See Barnett and He (2002) and He and Barnett (2004).

In related research, we have run a competition among tests for nonlinearity and found low robustness of inferences across tests, largely resulting from the existence of many different definitions of “nonlinear process” in use in that literature. See Barnett, Gallant, Hinich, Jungeilges, Kaplan, and Jensen (1995,1997).

[1] Barry E. Jones and Travis Nesmith wish to thank the Federal Reserve Bank of St. Louis for partially supporting this research. Any views expressed in this article are those of the authors and not necessarily those of the Federal Reserve Bank of St. Louis or the Federal Reserve System. William Barnett acknowledges partial support from National Science Foundation grant SES9223557. The authors wish to thank Melvin Hinich for providing us with the code for his bispectrum test and for his helpful technical advice. The authors wish to thank Richard Anderson, John Keating, Lisbeth la Cour, Yi Liu, Houston Stokes, and Haiyang Xu for their helpful comments and suggestions. Any remaining errors are ours.

[2] Barnett, Gallant, Hinich, Jungeilgies, Kaplan, and Jensen (1994, 1995) study the power of several competing univariate nonlinearity tests with artificial and monetary data respectively. Hinich and Wilson (1992) analyzes the cross bispectrum, which can detect multivariate nonlinear relationships.

[3] The extension to nonhomogeneous V requires the use of the distance function. See Barnett (1987) and Anderson, Jones, Nesmith (1997 a).

[4] The Törnqvist-Theil user cost and expenditure share indexes have a similar interpretation. See Theil (1967). Diewert (1995) presents measures of "functional form error", of which the second moments are a specific example.

[5] Similar tests and representation theorems for I(2) systems have recently been worked out in Johansen(1992 a, 1992 b, 1995 b), Parulo(1996), and Jorgensen et al. (1996).

[6] The Federal Reserve Bank of St. Louis has extensively revised the monetary services quantity and user cost indexes, for the period 1960-present. These revisions are detailed in Anderson, Jones, and Nesmith (1997a,b). All underlying non-confidential source data, as well as the quantity and user cost indexes, and the Divisia second moments are publicly available from the Federal Reserve Economic Database (FRED) at http://www.stls.frb.org/research/msi/index.html. The monetary services index, at the M2 and L level of aggregation, is reported in the Federal Reserve Bank of St. Louis' monthly publication Monetary Trends.

[7] If we strictly followed Barnett and Xu (1995), we would use GDP and the GDP deflator, but the larger number of observations at the monthly frequency is desirable for the Hinich bispectrum test. The use of industrial production as a proxy for GDP follows Christiano (1986), Fisher (1989), and Serletis (1987).

[8] Lee and Tse (1996) find that generalized autoregressive conditional heteroskedasticity (GARCH) causes over rejection of the null of no cointegration, but they state that the problem is generally not very serious.

[9] The assumptions of real and mean zero can be relaxed, see Hinich and Messer (1995).

[10] The following comment by Johansen (1995 b) is particularly relevant, "The methods derived [for cointegration] are based on the Gaussian likelihood but the asymptotic properties of the model depend only on the i.i.d. assumption of the errors." (pp. 29)

[11] It is assumed that $c_{xx}$ and $c_{xxx}$ are absolutely summable.

[12] Here, and throughout this section, the * notation denotes the complex conjugate operation.

[13] The operation | | denotes complex modulus, because the bispectrum is in general complex. This result is due to Brillinger (1965) and is based on the fact that under the stated assumptions

$B_{xx}(\omega_1, \omega_2) = c_{www}(0,0)A(\omega_1)A(\omega_2)A^*(\omega_1 + \omega_2)$, where A( ) denotes the filter transfer function.

[14] Melvin Hinich, in personal correspondence, has suggested that the block-length be set to insure that $\ln(L)/\ln(N) \approx .4$. Consistency of the estimators requires that the parameter $e = \ln(L)/\ln(N) < .5$.

[15] If the last frame is incomplete, it is dropped from the calculation of the estimator.

[16] For highly kurtotic stochastic processes, Hinich and Messer (1995) state that the use of the asymptotic distribution may not be warranted.

[17] We employ a trapezoidal taper in-order to reduce side lobe distortion. Some modification of these formulas is therefore required.

[18] Koopmans (1974) called this tradeoff the Grenander uncertainty principle. For a discussion of power spectral estimation, see Kay and Marple (1981).

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