The Partial Distribution:

Definition, Properties and Applications in Economy

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Abstract. In this discussed draft, we want to present the Partial Distribution (F.Dai, 2001) for discussing. We compare the partial distribution with lognormal and levy distribution. Though the levy distribution is better to describe the prices distribution of stock and stock indexes in a moderately large volatility range, the lognormal is better in a region of low values of volatility. We shall try to elucidate that the Partial Distribution is better than lognormal distribution in many respects. From partial distribution, we can acquire lots of interesting results, such as, describing the probability that stock price become zero if corresponding company collapses or the commodity price become zero if it lapses, expressing the average selling price of a commodity or stocks as the cost and average profits, and offering the accurate analytic model of American puts options pricing, etc. there are some related studies in appendix.

Index Terms. partial distribution, economic analysis, commodity pricing, American puts option, accurate pricing formula

Ⅰ. INTRODUCTION

Financial time series typically exhibit strong fluctuations that cannot be described by a Gaussian distribution. Recent empirical studies of stock market indices examined whether the distribution of returns can be described by a (truncated) Levy-stable distribution with some index $0<\alpha\leq 2$. While the Levy distribution cannot be expressed in a closed form. In a study **[** P. Gopikrishnan et al., Phys. Rev. E **60**, 5305 (1999)**]** it was found that the tails of $P(r)$ exhibit a power-law decay, with an exponent $\alpha\cong 3$, thus deviating from the Levy distribution. Ofer Biham et al. (2001) studied the distribution of returns in a generic model that describes the dynamics of stock market indices. For the distributions $P(r)$ generated by this model, we observe that the scaling of the central peak is consistent with a Levy distribution while the tails exhibit a power-law distribution with an exponent $\alpha>2$, namely, beyond the range of Levy-stable distributions. The results are in agreement with both empirical studies and reconcile the apparent disagreement between their results.

Salvatore Micciche et al. (2002) investigated the historical volatility of the 100 most capitalized stocks traded in US equity markets. An empirical probability density function (pdf) of volatility is obtained and compared with the theoretical predictions of a lognormal model and of the Hull and White model. The lognormal model well describes the pdf in the region of low values of volatility whereas the Hull and White model better approximates the empirical pdf for large values of volatility. Both models fail in describing the empirical pdf over a moderately large volatility range.

Since 90's of 20th century, the U.S. stock market soar first and slump later, and the fluctuation is violent, and the volatility is large. At the same time, we can use the Levy model. But, this does not mean that the U.S. stock market or other market will be in the violent fluctuation forever. Even if the stock market is generally in the violent fluctuation, the stock price is also in the low values of volatility at some periods of time. In fact, we need to use the most proper model to analyze the probability distribution of stocks price in case the stock price behavior is in the region of low values of volatility. In this case, people accustom to the use the lognormal model. However, when a company collapses, the price of its stock will be the zero. The lognormal model can't describe the possibility of zero price of a stock. The partial distribution $P(\mu, \sigma^2)$ (F. Dai, 2001) can do this. So the partial distribution should be applied to describe the price distribution of commodities and stocks at the low values of volatility. When value of μ is lower, partial distribution have a sharper peak than lognormal distribution.

In addition, Levy distribution and the truncated Levy distribution is usually applied to describe the price behavior in symmetry. Because of the price is non-negative, the distribution of price is generally non-symmetry. The non-symmetry can be reflected in lognormal or partial distribution.

In another hand, there is no accurate analytic formula for American Puts option pricing now. The American put value must be at least its exercise value since it can be exercised immediately. Geske and Johnson (1984), however, obtained a solution as an n-fold compound option, using Geske's (1979) compound option formula and the equivalent martingale/risk neutrality assumption. Bunch, D. S. and H. Johnson (1992) given a simple and numerically efficient valuation method for American puts option. Y.Tian (1993, 1999) given the binomial option pricing models. Up to now lots of numerical approximation procedures were proposed for pricing American put options. Because of various difficulties in calculating the price of American put options, however, intensive efforts are still needed for developing new accurate formula to this problem. Here, we shall present an accurate analytic formula for American Puts option pricing based on partial distribution.

Ⅱ. DEFINITION AND BASIC CONCLUSIONS

Definition (Partial Distribution) Let X is a non-negative stochastic variable, and it follows the distribution of density

$$f(x)=\begin{cases} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \Big/ \int_0^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx & x \ge 0 \\ 0 & x<0 \end{cases} \tag{1}$$

then X is said to have a Partial Distribution, and denotes $X \in P(\mu, \sigma^2)$. The partial distribution is a kind of truncated normal distribution.

By means of [1], we have

Theorem 1 For any $x \in [0,\infty]$, the following equations are correct approximately:

1) $\int_0^x e^{-\frac{t^2}{2}} dt = \sqrt{\frac{\pi}{2}(1-e^{-\frac{2}{\pi}x^2})}$

2) $\int_0^x e^{-\frac{(u-\mu)^2}{2\sigma^2}} du = \sqrt{\frac{\pi}{2}}\sigma \times (\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}} + \operatorname{sgn}(x-\mu)\sqrt{1-e^{-\frac{2}{\pi}(\frac{x-\mu}{\sigma})^2}})$

where, $\operatorname{sgn}(x)=\begin{cases} 1 & x>0 \\ 0 & x=0 \\ -1 & x<0 \end{cases}$;

3) $\int_0^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \sqrt{\frac{\pi}{2}}\sigma(\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+1)$

Theorem 2 Let X follow the partial distribution $P(\mu, \sigma^2)$, thus
1) The expected value $E(x)$ is as follows

$$E(X) = \langle x \rangle = \mu + \sqrt{\frac{2}{\pi}} \frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+1} \tag{2}$$

where, $R(x) = \sqrt{\frac{2}{\pi}} \frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+1}$ (3)

2) The variance $D(x)$ is as follows

$$D(X) = \langle x^2 \rangle = \sigma^2 + E(x)[\mu - E(x)] \tag{4}$$

Ⅲ. PROPERTIES ANALYSIS

We shall show the basic properties of Partial Distribution by comparing with lognormal and levy.

A. The properties of lognormal distribution $Ln(\mu, \sigma^2)$.

1) Stochastic variable is non-negative, and the probability is zero at x=0, namely, $p(0)$=0.
2) The shape of distribution curve is relevant to parameter σ and variable x.

3) Expectation is $\langle x \rangle = e^{\mu+\frac{\sigma^2}{2}}$, and variance is $\langle x^2 \rangle = e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$

B. The properties of Levy distribution $L_\alpha(a,x)$.

1) Levy distribution is the probability distribution function $\zeta_\alpha(a,x)$ with a characteristic function:

$$\hat{l}_\alpha(a,k) = e^{-a|k|^\alpha} \quad (0<\alpha\leq 2)$$

2) Gaussian (α=2, $a=\sigma^2/2$) and Cauchy (α=1) distribution are the only two levy distributions that can be inverted and expressed as elementary functions.
3) When $0<\alpha<2$, both expectation and variance of levy does not exist.
4) The center of the Levy distribution gets sharper and higher and the tails get fatter as $\alpha\to 0$ and $x\to 0$.
5) the most applications of truncated levy distribution is on both ends at the same time, the discussion on truncating a single side is not much more.

C. The properties of Partial Distribution $P(\mu, \sigma^2)$.

1) Stochastic variable is non-negative, and the probability is non-zero at x=0, namely,

$$p(0)= \sqrt{\frac{2}{\pi}} \frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+1}$$

2) The shape of distribution curve is relevant to parameter σ and μ.

3) The expectation $E(x)=\mu+R(x)$, i.e. $E(x)>\mu$.
4) The variance $D(x)=\sigma^2+E(x)(\mu-E(x))$, 即 $D(x)<\sigma^2$.
5) When $\boldsymbol{x}\rightarrow\boldsymbol{\mu}$, Partial distribution is sharper than Gaussian and lognormal distribution as $\boldsymbol{\mu}$ is less.
6) If $\boldsymbol{\mu}$ is big enough, the Partial distribution is approximately near to Gaussian distribution.

The curve shapes of Partial distribution are shown in figure 1.

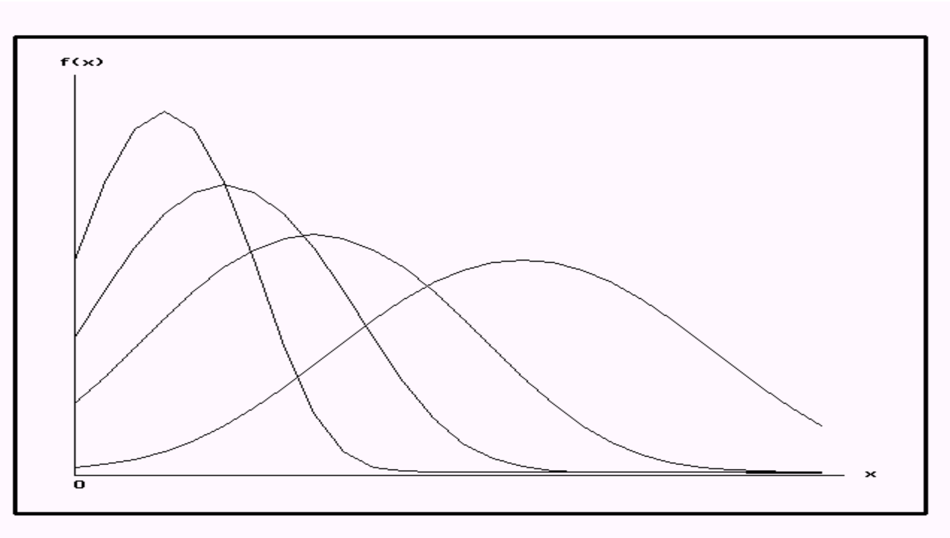


Figure 1. The curve shapes of Partial distribution

Ⅳ. APPLICATIONS OF PARTIAL DISTRIBUTION

A. Pricing analysis for commodity or stock

Denote:

μ——the cost price of commodity, or the average of holding price of all traders in the market to a stock.

σ——the standard variance of cost price of commodity.

x——the market price variable of a commodity or a stock.

thus, $R(x)$ in expression (3) is an average selling profits.

Comparing with Levy distribution,

1) Partial distribution can describe the non-negative characteristic of price of commodities or stocks.
2) Partial distribution can explain that the price of commodity may be zero.
3) The expectation and the variance of partial distribution can be expressed as elementary functions.
4) The average selling profits $R(x)$ in (3) can be estimated.
5) If measure the risk by the variance, we can deduce to that the risk of market price is smaller than the risk of cost price.

Go a step further, if we let $\boldsymbol{Y}$ be a future on underlying $\boldsymbol{S}$, and $\boldsymbol{S}\in P(\mu, \sigma^2)$, thus $\boldsymbol{Y}\in P(E(\boldsymbol{S}), D(\boldsymbol{S}))$. We have

$E(\boldsymbol{Y})= E(\boldsymbol{S})+R(\boldsymbol{Y})$,

$D(\boldsymbol{Y})= D(\boldsymbol{S})+ E(\boldsymbol{Y})(E(\boldsymbol{S})-E(\boldsymbol{Y}))$, where $R(\boldsymbol{Y})=\sqrt{\dfrac{2D(S)}{\pi}}\dfrac{e^{-\frac{[E(S)]^2}{2D(S)}}}{\sqrt{1-e^{-\frac{2[E(S)]^2}{\pi D(S)}}}+1}$.

Because $E(\boldsymbol{Y})>E(\boldsymbol{S})$, then $D(\boldsymbol{Y})<D(\boldsymbol{S})$, this means that the risk of future is less than the risk of its underlying.

B. The formulas of American call and put option pricing

We will use the following notation:

t—current time.

$S(t)$—market price of the underlying at t.

Z—strike price of option on $S(t)$.

T—time of expiration of option.

r—risk-free rate of interest to maturity T.

$S(t)e^{r(T-t)}$—forward value of $S(t)$($\hat{E}(S(T))$,the expected value in a risk-neutral world).

C_S—value of call option to buy one share.

P_S—value of put option to sell one share.

1) The price of call option at time t is

$$C_S(t)=(S(t)-Ze^{-r(T-t)})\times$$

$$\left[\frac{\sqrt{1-e^{-\frac{2}{\pi}\frac{(Ze^{-r(T-t)})^2}{D[S(t)](T-t)}}}+\mathrm{sgn}(S(t)e^{r(T-t)}-Z)\sqrt{1-e^{-\frac{2}{\pi}\frac{(S(t)-Ze^{-r(T-t)})^2}{D[S(t)](T-t)}}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Ze^{-r(T-t)})^2}{D[S(t)](T-t)}}}}\right]+\sqrt{\frac{2D[S(t)](T-t)}{\pi}}\left[\frac{e^{-\frac{(S(t)-Ze^{-r(T-t)})^2}{2D[S(t)](T-t)}}-e^{-\frac{(Ze^{-r(T-t)})^2}{2D[S(t)](T-t)}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Ze^{-r(T-t)})^2}{D[S(t)](T-t)}}}}\right] \quad (5)$$

When the call option is brought forward to execute at any time $\tau\in[t,T]$, the price of underlying stock, $S(\tau)$, becomes a constant to the option contract, thus $D[S(\tau)]=0$. According to (5) and theorem 1, the current value of the option is

$C_S(\tau)=S(\tau)-Ze^{-r(T-\tau)}$, if $S(\tau)>Ze^{-r(T-\tau)}$;

$C_S(\tau)=0$, if $S(\tau)\leq Ze^{-r(T-\tau)}$;

namely, $C_S(\tau)=max\{S(\tau)-Ze^{-r(T-\tau)}, 0\}$. At this time, the intrinsic value of the call option is $max\ \{S(\tau)-Z, 0\}$, thus

$$C_S(\tau)\geq max\{S(\tau)-Z, 0\} \quad (6)$$

2) The price of put option at time t is

$$P_S(t)=(Ze^{-r(T-t)}-S(t))\times$$

$$\times\left[\frac{1-\mathrm{sgn}[S(t)e^{r(T-t)}-Z]\sqrt{1-e^{-\frac{2}{\pi}\frac{(S(t)-Ze^{-r(T-t)})^2}{D[S(t)](T-t)}}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Ze^{-r(T-t)})^2}{D[S(t)](T-t)}}}}\right]+\sqrt{\frac{2D[S(t)](T-t)}{\pi}}\left[\frac{e^{-\frac{(S(t)-Ze^{-r(T-t)})^2}{2D[S(t)](T-t)}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Ze^{-r(T-t)})^2}{D[S(t)](T-t)}}}}\right] \quad (7)$$

When the put option is brought forward to execute at any time $\tau\in[t, T]$, the price of underlying, $S(\tau)$, becomes a constant to the option contract, thus $D[S(\tau)]=0$. According to (7) and theorem 1, the current value of the option is

$P_S(\tau)=Ze^{-r(T-\tau)}-S(\tau)$, if $S(\tau)<Ze^{-r(T-\tau)}$

$P_S(\tau)=0$, if $S(\tau)\geq Ze^{-r(T-\tau)}$

i.e. $P_S(\tau)= max\{Ze^{-r(T-\tau)}-S(\tau), 0\}$. At this time, the intrinsic value of the put option is $max\{Z-S(\tau), 0\}$, thus,

$$P_S(\tau)\leq max\{Z-S(\tau), 0\} \tag{8}$$

3) put-call parity between call and put option prices. According to (5) and (7), we have the following put-call parity:

$C_S(t)-P_S(t)=S(t)-Ze^{-r(T-t)}-R_S(t)$, i.e. $C_S(t)+Ze^{-r(T-t)}+R_S(t)=S(t)+P_S(t)$

where, $R_S(t)=\sqrt{\dfrac{2D[S(t)](T-t)}{\pi}}\dfrac{e^{-\frac{d^2}{2}}}{\sqrt{1-e^{-\frac{2d^2}{\pi}}}+1}$, $d=\dfrac{Ze^{-r(T-t)}}{\sqrt{D[S(t)](T-t)}}$.

Because we can calculate the partial distribution of $S(t)$ and the $D[S(t)]$ at any time as t goes on, the American put option could be priced.

***C.* Fitting analysis for partial distribution.**

The methods of estimating the parameters is from 2) in appendix.

1) The fitness *of DJX*. We take the close points of ***1/100DJ INDU*** as sample data.

Time: Jun. 19, 2002 -Dec. 24, 2002. Trading days: n=132. Length of each field: Δ=0.862121 2120.

Number of fields: m=25. The estimated results of parameters are as follows:

- The parameters estimated in partial distribution $P(\mu,\sigma^2)$:

$\hat{\mu}$=84.84577713; $\hat{\sigma}^2$=28.65615031

- The parameters estimated in lognormal distribution $Ln(\mu_l,\sigma_l^2)$:

$\hat{\mu}_l$=4.440767790, $\hat{\sigma}_l^2$=0.002746603980;

- The fiducial test:

Partial distribution: χ^2=24.90088870< $\chi^2_{0.025}$(22)=36.781;

Lognormal distribution: χ^2_l=30.43529668< $\chi^2_{0.025}$(22)=36.781.

2) The fitness *of MSFT*. We take the close prices of ***MICROSOFT CP*** as sample data.

Time: Jan. 29, 2002-Dec. 24, 2002.Trading days: n=230. Length of each field: Δ=0.467609(US \$).

Number of fields: m=46. The estimated results of parameters are as follows:

- The parameters estimated in partial distribution $P(\mu,\sigma^2)$:

$\hat{\mu}$=53.58500013; $\hat{\sigma}^2$= 24.62632700;

- The parameters estimated in lognormal distribution $Ln(\mu_l,\sigma_l^2)$:

$\hat{\mu}_l$=3.976827406; $\hat{\sigma}_l^2$=0.008770254161;

- The fiducial test:

Partial distribution: χ^2=58.08095341< $\chi^2_{0.025}$(43)=62.990;

Lognormal distribution: χ_l^2 =58.02536391< $\chi_{0.025}^2$ (43)=62.990.

D. Comparison research between *DF* pricing and *BS* pricing

Suppose *r* (risk-free rate of interest) =0.07. Here we compare results from *DF* formulas with those of *BS* (*Black-Scholes*) formulas in options pricing.

1) The comparative analysis for *DJX*.

Time: Dec. 24, 2002

Product: The option contract on *DJX*

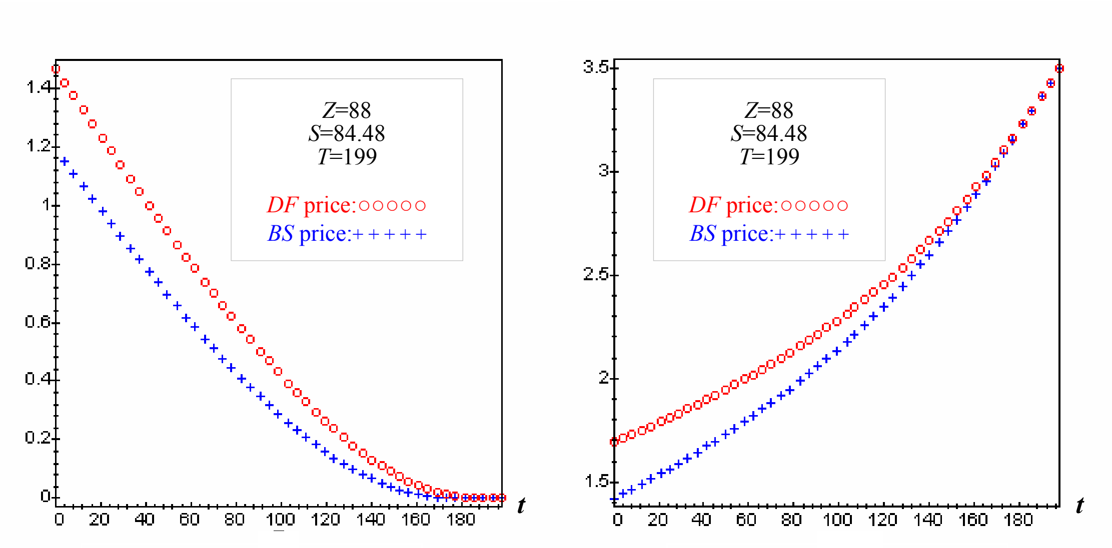
Maturity: Expires After: Fri 19-Dec-2003.

Underlying: Close point of *DJX* at current date, 84.48.

In table 1, the prices are of Dec. 24, 2002, which were the closing prices traded actually in the United States option market (*TP*), the call and put options prices calculated by *DF* pricing formulas, (*DF*), and the call and put options prices calculated by *BS* pricing formulas, (*BS*). From table 1, we see the *DF* prices are closer to the actual trading prices than the *BS* prices. If taking the strike price, *Z*=88, and *T*=199 for example, the variety of call and put option prices calculated by *DF* formulas and *BS* formulas are respectively shown in figure 2(a) and 2(b).

Table 1. Comparison of options prices of ***DJX***

Strike prices	Call options prices			Put options prices		
	TP	*DF*	*B-S*	*TP*	*DF*	*B-S*
76.0	13.60	11.34	11.33	5.50	.0108	.0001
80.0	11.00	7.542	7.484	7.20	.0668	.0086
84.0	8.70	4.015	3.831	9.00	.3900	.2053
88.0	7.10	1.457	1.197	11.00	1.682	1.421
92.0	5.70	.3180	.1812	13.00	4.393	4.256



(***a***): price curve of call option (***b***): price curve of put option

Figure 2. Comparison between curves of options prices of ***DJX***

2) The comparative analysis for *MSFT*.

Time: Dec. 25, 2002.

Product: The option contract on *MSFT*.

Maturity: Expires After: Fri,16-Jan-2004.

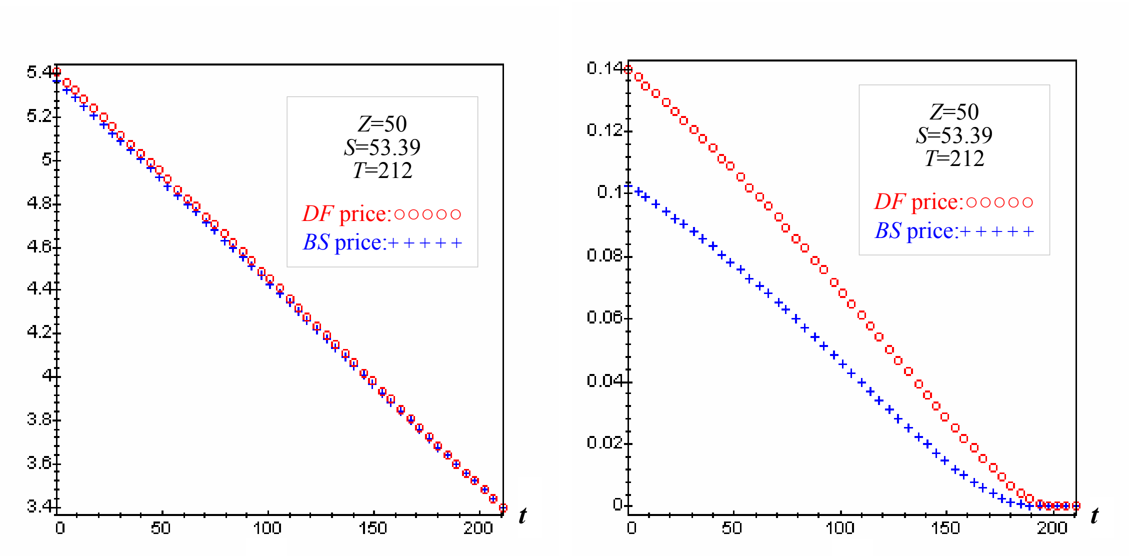
Underlying: Close price of *MSFT* at current date, 53.39$.

In table 2, there are the prices on Dec.24, 2002, which were the closing prices traded actually in the United States option market (TP), the call and put options prices calculated by *DF* structure formulas, (*DF*) , and the call and put options prices calculated by *BS* formulas, (*BS*).

According to the data from table 2, it is difficult to know whether the *DF* formula is better than *BS* formula or not, we should do further empirical research. Taking the strike price, Z=50, and *T*=212 for example, the variety of call and put option prices calculated by *DF* formulas and B*S* formulas are respectively shown in figure 3(a) and 3(b).

Table 2. Comparison of options prices of ***MSFT***

Strike prices	Call options prices			Put options prices		
	TP	*DF*	*BS*	*TP*	*DF*	*BS*
50.0	11.40	5.406	5.369	7.70	.1399	.1028
55.0	9.20	1.702	1.706	9.90	1.248	1.252
60.0	6.80	0.2213	0.2466	12.50	4.580	4.605
65.0	5.10	0.0212	0.0151	15.90	9.192	9.186
70.0	3.80	0.0007	0.0004	19.50	13.98	13.98



(***a***): price curve of call option (***b***): price curve of put option

Figure 3. Comparison between curves of options prices of ***MSFT***

In the table 1 and table2, we can see that most of the prices calculated from *DF* formulas are more near to real options prices (*TP*) than those from *BS* calculated formulas.

E. Possible researches on following topics

1) Price analysis for other kind of derivatives.
2) Structure analysis for disk distribution.
3) Estimating analysis for worth of manpower.
4) The analysis and decision for the comparison construction of the foreign exchange rate.
5) Structure and characteristic analysis for credit grade.
6) The diaphaneity analysis for information of commodity.
7) Analysis for the degree of happiness and its distribution.
8) Structure and characteristic analysis for society health status.
9) Composing and distribution analysis of society potential power.
10) Structure and characteristic analysis of society moral behavior.

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APPENDIX
(Related studies about partial distribution)

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1) *Chinese Journal of Management Science*, Vol. 9, No.1, 2001, pp. 62-69

A New Kind of Pricing Model for Commodity and Estimating Indexes System for Price Security

DAI Feng JI Guangpo

***ABSTRACT*:** In this paper, the concept and the expression of the partial distribution and the partial process are put forward. By them, the variations of commodity price could be described, analyzed, and researched. The relations of the cost price, the market price, the risk of cost price and the risk of market price of commodity are advanced in analytic expressions, which can be applied to pricing and to measuring the risks of the spots, the securities, the futures and the options in market. The Security Indexes are designed in analytic expressions. So we can use them to measure and evaluate the development security and investment security in the society, the economy, the finance, and so on.
***KEY WORDS*:** the partial distribution, pricing model, logical price, the Security Index

2) *Proceedings of SCI 2003*, Orlando, USA.

The PD-Fitness Analysis of Price Structures on Chinese Stocks Market

Feng DAI Xiaohong CHEN Kewei SUN

***ABSTRACT*:** In this paper, we present a special method of estimating the parameter in the Partial Distribution (PD), and accomplish the fitness analysis and test about the stock indices and stock price by using the dealing data of Shanghai and Shenzhen Stock Markets. We have got the conclusion, from the fiducial test, that the Partial Distribution is better than the lognormal distribution used commonly describing the structure of stock prices. And we get the indices of market risk and price safety at the same time.
***KEY WORDS*:** partial distribution, Chinese stock market, stock index and price, fitness analysis, fiducial test

3) *Chinese Journal of Management Science*, Vol. 11, No.1, 2003, pp. 33-37

A New Kind of Method of Optimal Pricing for Commodity

Dai Feng Xu Wei-xuan Liu hui Xu Hua

***ABSTRACT*:** Based on **PD** and deciding the field of income, the method of optimal pricing for commodity in market is given. So we can calculate the optimal selling price and the expect profits of a commodity. And the way to estimate parameters in **PD** presented at the same time, so the method in this paper is of more practicability.
***KEY WORDS*:** PD-the partial distribution, commodity price, optimal pricing

4) *IEEE 2003 IEMC Conference Proceedings*, pp. 311-315

The Model of Optimal Pricing for Assets Based On the Partial Distribution and Its Empirical Research

Feng DAI Hui LIU Zi-fu QIN

***ABSTRACT*:** We give the concepts and the expressions of the Partial Distribution and Partial Process in this paper, and the way of estimating the parameters in Partial Distribution. Based on the Partial Distribution, we put forward a new kind of pricing method for assets (capitals, stocks or commodities), and present the model of optimal pricing (MOP) on the method. Also, we give some examples to show that the Partial Distribution is, at sometimes, better than the lognormal distribution used commonly in describing the behavior of stock prices, and

that the pricing result, calculated by MOP, is better than any of other pricing result.
***KEY WORDS*:** partial distribution, assets pricing, optimal model, parameter estimating, empirical research

5) *Already accepted by Chinese Journal of Management Science*

Analyzing Method of the Behavior Properties of Stocks Price Based on Partial Distribution

DAI Feng LIU Hui QIN Zi-fu

***ABSTRACT*:** Based on Partial Distribution [2]-[3], this paper gives out the concepts and analytic expressions of the behavior indexes of security price, extreme limit price, balanced price, focus price for the first time, particularly gives out the calculating methods of extreme limit price and focus price, the former is beneficial to judge the reversal position of price movement trend under the different significance levels, the latter is beneficial to judge that the focus of current price of security market is reasonableness or not.
***KEY WORDS*:** Partial distribution; security; price behavior index; extreme limit price; focus price

6) *Waiting for publication*

The Optimal Decision Making for Expanding the Scale of Production

Feng DAI Song-tao WU Zi-fu QIN

***ABSTRACT*:** Based on the partial distribution [6]-[8], we give the basic conditions and put forward the optimal method of decision making for expanding the production scales under the cases that the variable cost, fixed cost or total cost would be separately divided after expanding. The method is widespread suitable for all kinds of expanding the production scales. Two examples show, by the method, we can not only resolve the problems of expanding the general merchandise production scale, and can still resolve the problems of dividing stock capitals scale.
***KEY WORDS*:** Partial Distribution; merchandise production, stock capital dividing, scale expanding; optimal decision making

7) *Waiting for publication*

***DF* Structure Models for Options Pricing**

Feng DAI Zi-fu QIN

***ABSTRACT*:** Based on the Partial Distribution, we presents the concepts and expressions of *DF* process and *DF* structure and put forward the *DF* structure models of pricing options on a non-dividend-paying underlying for the first time. The *DF* structure models are able to price the call and put options exercised at any time, so it is applicable to pricing the American and European options. Finally, examples are given to compare the options priced by *DF* formulas and by *Black-Scholes* formulas, they show, as a whole, that the *DF'* prices of options are closer to the trading prices than *Black-Scholes'* prices in many cases.
KEY WORDS*:** Partial Distribution; ***DF structure; options pricing; analytic formula; non-dividend-paying

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Choosing Models of Optimal Opportunity for Executing American Options

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***ABSTRACT*:** Based on the structure models of options pricing on non-dividend-paying stock [20], this paper presents the choosing models and methods of optimal opportunity of executing American options for the first time. By using the models and methods, we can find the optimal opportunity and criterion to execute American options, i.e. the product of price of options and probability of the price realization maximizes. So we can decide an American option should be executed or not in any time.
***KEY WORDS*:** American option; structure pricing; strike price; optimal criterion; analytic formula